Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.9-P-x-d+e-x-^m-a+b-x+c-x^2-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [400]. This is test number [38].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (400)	% 0.00 (0)
Mathematica	% 98.50 (394)	% 1.50 (6)
Maple	% 97.00 (388)	% 3.00 (12)
Maxima	% 72.50 (290)	% 27.50 (110)
Fricas	% 83.00 (332)	% 17.00 (68)
Sympy	% 35.25 (141)	% 64.75 (259)
Giac	% 83.75 (335)	% 16.25 (65)
Mupad	% 48.75 (195)	% 51.25 (205)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

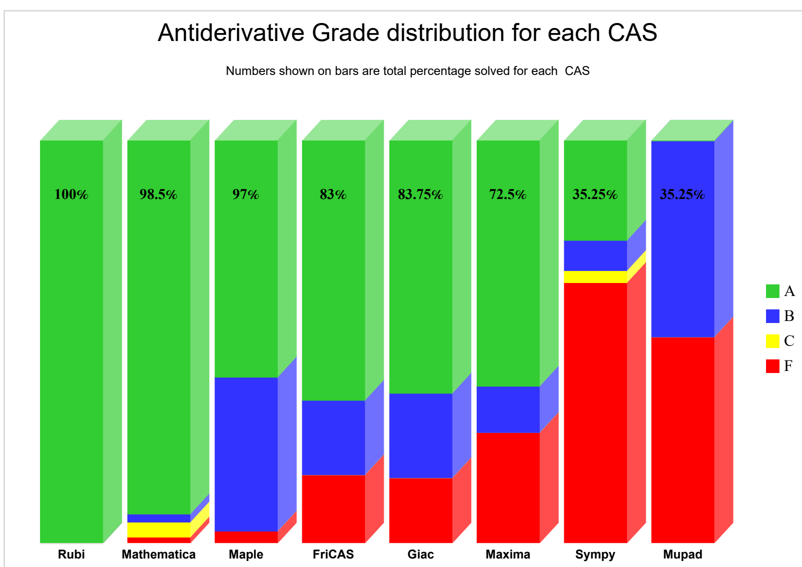
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

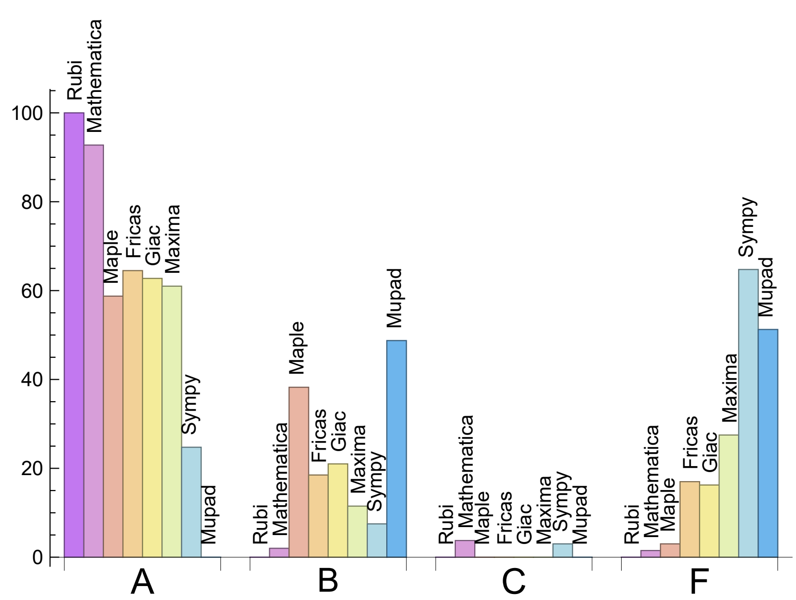
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	92.75	2.00	3.75	1.50
Maple	58.75	38.25	0.00	3.00
Maxima	61.00	11.50	0.00	27.50
Fricas	64.50	18.50	0.00	17.00
Sympy	24.75	7.50	3.00	64.75
Giac	62.75	21.00	0.00	16.25
Mupad	0.00	48.75	0.00	51.25

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	6	100.00 %	0.00 %	0.00 %
Maple	12	100.00 %	0.00 %	0.00 %
Maxima	110	31.82 %	1.82 %	66.36 %
Fricas	68	36.76 %	63.24 %	0.00 %
Sympy	259	74.90 %	24.71 %	0.39 %
Giac	65	40.00 %	21.54 %	38.46 %
Mupad	205	99.51 %	0.49 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

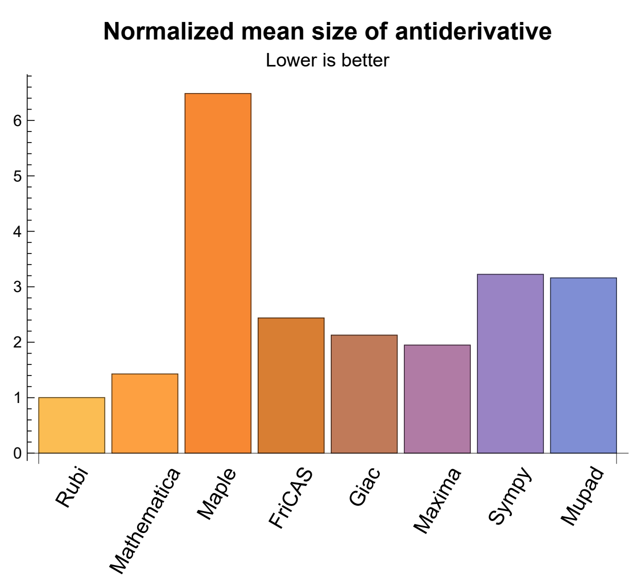
1.3 Performance

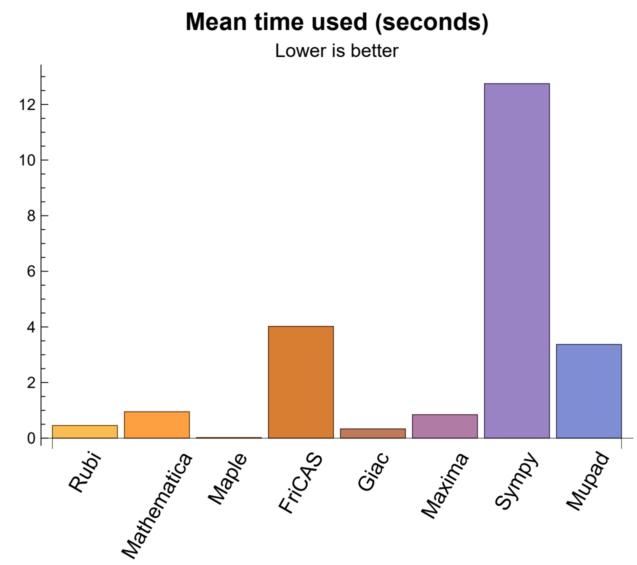
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.46	238.80	1.00	166.50	1.00
Mathematica	0.95	684.25	1.43	132.00	0.92
Maple	0.02	3404.39	6.48	209.00	1.46
Maxima	0.84	363.92	1.95	149.50	1.05
Fricas	4.02	455.78	2.44	172.00	1.40
Sympy	12.75	398.38	3.22	221.00	1.56
Giac	0.33	461.91	2.13	180.00	1.16
Mupad	3.37	773.44	3.16	185.00	1.29

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the antiderivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {259, 263, 264, 265, 271, 383, 398, 399, 400}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

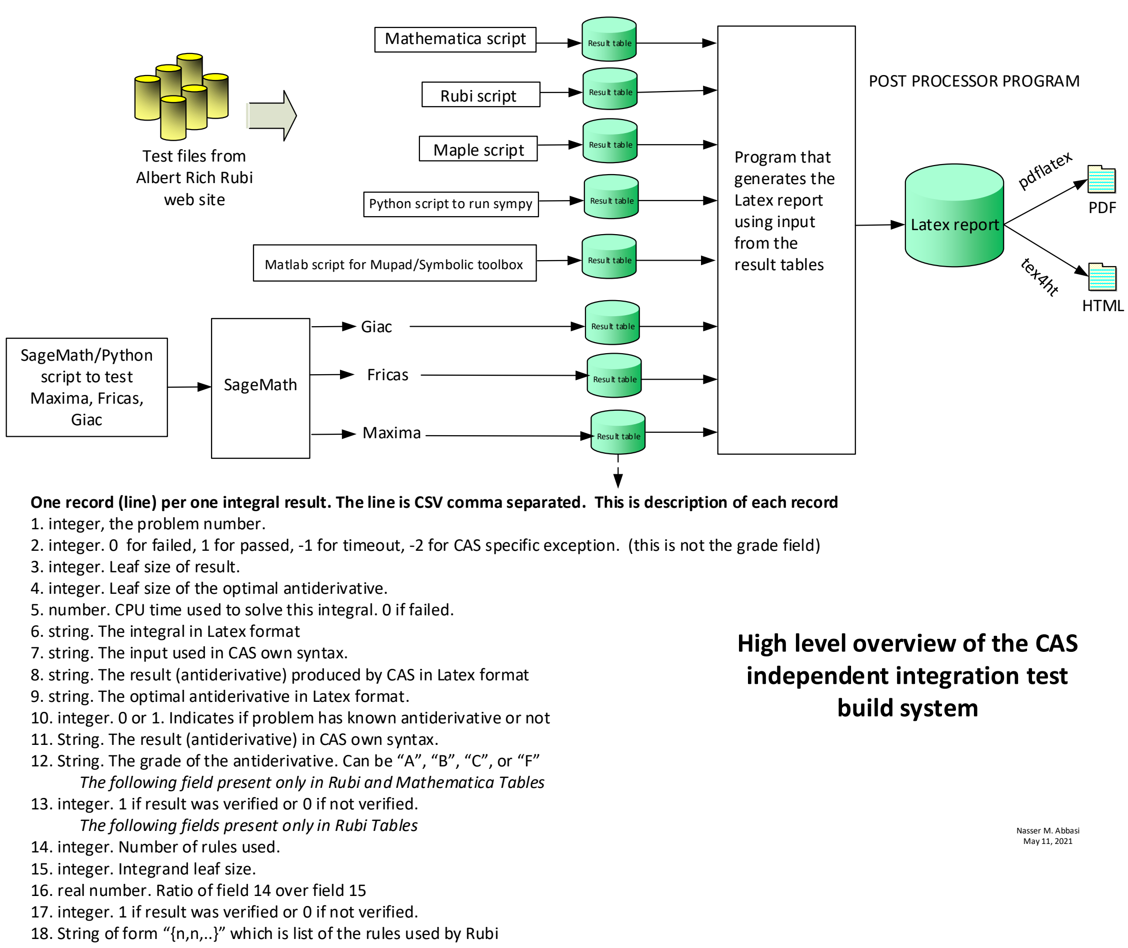
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

B grade: { 39, 40, 41, 42, 202, 203, 204, 278 }

C grade: { 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 275, 366 }

F grade: { 136, 137, 138, 272, 273, 274 }

2.1.3 Maple

A grade: { 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 52, 53, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 134, 135, 140, 141, 142, 143, 144, 145, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 181, 182, 183, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 229, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 254, 256, 257, 258, 275, 276, 277, 279, 283, 284, 285, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 311, 312, 313, 314, 318, 319, 320, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 374, 375, 376, 380, 381, 382, 386, 387, 388, 389, 392, 393, 394 }

B grade: { 4, 5, 6, 7, 15, 38, 39, 40, 41, 42, 48, 49, 50, 51, 54, 55, 56, 57, 61, 62, 63, 64, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 107, 108, 109, 112, 113, 114, 127, 133, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 178, 179, 180, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 249, 252, 253, 255, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 278, 280, 281, 282, 286, 287, 288, 309, 310, 315, 316, 317, 322, 323, 358, 359, 360, 361, 362, 365, 366, 367, 368, 369, 372, 373, 377, 378, 379, 383, 384, 385, 390, 391, 395, 396, 397 }

C grade: { }

F grade: { 136, 137, 138, 139, 272, 273, 274, 370, 371, 398, 399, 400 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 88, 89, 90, 91, 93, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 140, 141, 142, 143, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 275, 276, 277, 279, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 361, 362, 363, 364, 374, 375, 376, 380, 381, 382, 386, 387, 388, 392, 393, 394 }

B grade: { 8, 9, 15, 16, 17, 39, 40, 41, 42, 56, 62, 63, 64, 84, 85, 86, 87, 92, 94, 95, 96, 97, 98, 106, 107, 112, 113, 114, 131, 177, 252, 253, 278, 317, 323, 343, 358, 359, 360, 367, 368, 369, 377, 383, 389, 395 }

C grade: { }

F grade: { 6, 7, 99, 136, 137, 138, 139, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 280, 281, 282, 283, 284, 285, 286, 287, 288, 365, 366, 370, 371, 372, 373, 378, 379, 384, 385, 390, 391, 396, 397, 398, 399, 400 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 52, 53, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 148, 149, 150, 151, 152, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 179, 180, 181, 186, 187, 188, 189, 199, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 255, 256, 257, 275, 276, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 312, 313, 314, 320, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 373, 374, 375, 376, 380, 381, 382, 386, 387, 388, 392, 393, 394 }

B grade: { 31, 38, 39, 40, 41, 42, 48, 50, 51, 54, 57, 58, 64, 65, 66, 67, 107, 112, 113, 114, 145, 146, 147, 155, 156, 157, 177, 178, 182, 183, 184, 185, 196, 197, 198, 232, 233, 234, 235, 236, 237, 238, 253, 258, 277, 278, 310, 311, 315, 316, 317, 318, 319, 322, 323, 360, 365, 366, 367, 368, 369, 372, 377, 378, 379, 383, 384, 385, 389, 390, 391, 395, 396, 397 }

C grade: { }

F grade: { 49, 55, 56, 61, 62, 63, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 136, 137, 138, 139, 153, 154, 158, 159, 190, 191, 192, 193, 194, 195, 200, 201, 202, 203, 204, 205, 206, 207, 230, 231, 239, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 370, 371, 398, 399, 400 }

2.1.6 Sympy

A grade: { 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 53, 59, 60, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 110, 111, 115, 118, 119, 120, 140, 141, 142, 143, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 276, 279, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 307, 314, 321 }

B grade: { 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 58, 116, 117, 126, 132, 144, 145, 146, 147, 148, 149, 150, 151, 156, 157, 177, 275, 277, 278 }

C grade: { 1, 2, 3, 304, 305, 306, 311, 312, 313, 318, 319, 320 }

F grade: { 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 47, 48, 49, 54, 55, 56, 57, 61, 62, 63, 64, 65, 66, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 107, 108, 109, 112, 113, 114, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 152, 153, 154, 155, 158, 159, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 280, 281, 282, 283, 284, 285, 286, 287, 288, 308, 309, 310, 315, 316, 317, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

2.1.7 Giac

A grade: { 1, 2, 3, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 88, 89, 90, 91, 92, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 122, 124, 125, 126, 127, 130, 131, 132, 133, 135, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 186, 187, 188, 189, 199, 208, 209, 210, 211, 214, 215, 216, 217, 220, 221, 222, 223, 226, 227, 228, 229, 235, 236, 240, 241, 242, 243, 244, 246, 247, 248, 249, 252, 253, 254, 255, 279, 280, 281, 282, 284, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 334, 335, 336, 338, 344, 345, 346, 350, 351, 352, 353, 354, 358, 359, 360, 361, 365, 366, 372, 373, 374, 375, 376, 377, 380, 381, 382, 383, 386, 387, 388, 389, 392, 393, 394, 395, 396 }

B grade: { 39, 40, 41, 42, 61, 63, 64, 65, 84, 85, 87, 94, 95, 97, 98, 99, 107, 112, 114, 121, 123, 128, 129, 134, 147, 158, 159, 178, 184, 185, 196, 197, 198, 203, 212, 218, 224, 232, 233, 234, 237, 239, 245, 250, 251, 256, 257, 275, 276, 277, 278, 285, 286, 287, 288, 327, 328, 329, 330, 331, 332, 333, 337, 339, 340, 341, 342, 343, 347, 348, 349, 355, 356, 357, 362, 363, 364, 367, 368, 369, 379, 390, 391, 397 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 83, 86, 93, 96, 106, 113, 136, 137, 138, 139, 190, 191, 192, 193, 194, 195, 200, 201, 202, 204, 205, 206, 207, 213, 219, 225, 230, 231, 238, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 283, 370, 371, 378, 384, 385, 398, 399, 400 }

2.1.8 Mupad

A grade: { }

B grade: { 8, 9, 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 103, 104, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 180, 182, 183, 184, 185, 186, 187, 188, 189, 208, 209, 210, 236, 254, 258, 275, 276, 277, 278, 279, 284, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 367, 368, 369, 372, 373, 374, 375, 376 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 10, 11, 14, 15, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 112, 113, 114, 136, 137, 138, 139, 178, 179, 181, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 280, 281, 282, 283, 285, 286, 287, 288, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 370, 371, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	226	371	338	211	1231	197	-1
normalized size	1	1.00	0.96	1.57	1.43	0.89	5.22	0.83	-0.00
time (sec)	N/A	0.470	0.506	0.059	0.996	0.681	22.934	0.224	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	190	304	202	173	670	160	-1
normalized size	1	1.00	1.02	1.63	1.09	0.93	3.60	0.86	-0.01
time (sec)	N/A	0.227	0.299	0.017	0.976	0.866	12.771	0.213	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	121	154	116	108	343	85	-1
normalized size	1	1.00	0.97	1.23	0.93	0.86	2.74	0.68	-0.01
time (sec)	N/A	0.069	0.133	0.007	0.964	0.973	7.108	0.202	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	103	384	171	112	0	0	-1
normalized size	1	1.00	0.70	2.59	1.16	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.221	0.023	1.086	0.893	0.000	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	109	439	197	190	0	0	-1
normalized size	1	1.00	0.64	2.58	1.16	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.227	0.028	1.014	0.675	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	114	318	0	258	0	0	-1
normalized size	1	1.00	0.77	2.13	0.00	1.73	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.221	0.018	0.000	0.852	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	112	453	0	304	0	0	-1
normalized size	1	1.00	0.57	2.31	0.00	1.55	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.298	0.017	0.000	0.832	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	109	116	945	320	0	0	601
normalized size	1	1.00	0.61	0.64	5.25	1.78	0.00	0.00	3.34
time (sec)	N/A	0.210	0.203	0.009	0.542	0.915	0.000	0.000	4.670

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	144	152	1378	399	0	0	960
normalized size	1	1.00	0.62	0.65	5.89	1.71	0.00	0.00	4.10
time (sec)	N/A	0.264	0.225	0.011	0.575	1.072	0.000	0.000	5.243

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	174	374	390	178	1268	166	-1
normalized size	1	1.00	0.74	1.58	1.65	0.75	5.37	0.70	-0.00
time (sec)	N/A	0.657	0.453	0.031	0.979	0.614	24.443	0.368	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	139	301	253	145	891	131	-1
normalized size	1	1.00	0.73	1.58	1.32	0.76	4.66	0.69	-0.01
time (sec)	N/A	0.378	0.177	0.012	0.983	0.957	18.032	0.316	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	103	234	150	109	484	97	270
normalized size	1	1.00	0.72	1.64	1.05	0.76	3.38	0.68	1.89
time (sec)	N/A	0.199	0.107	0.011	0.979	0.971	10.172	0.292	5.012

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	67	108	70	71	262	52	148
normalized size	1	1.00	0.77	1.24	0.80	0.82	3.01	0.60	1.70
time (sec)	N/A	0.051	0.040	0.006	0.988	0.640	4.560	0.344	4.399

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	83	149	138	155	0	0	-1
normalized size	1	1.00	0.81	1.45	1.34	1.50	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.158	0.014	0.994	0.845	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	95	355	317	221	0	0	-1
normalized size	1	1.00	0.58	2.18	1.94	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.218	0.016	0.995	0.915	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	103	116	608	244	0	0	109
normalized size	1	1.00	0.57	0.64	3.38	1.36	0.00	0.00	0.61
time (sec)	N/A	0.205	0.197	0.012	1.015	0.786	0.000	0.000	3.795

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	139	152	975	320	0	0	204
normalized size	1	1.00	0.59	0.65	4.17	1.37	0.00	0.00	0.87
time (sec)	N/A	0.249	0.221	0.010	1.061	0.923	0.000	0.000	3.776

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	173	208	217	202	248	257	242	206
normalized size	1	0.99	1.19	1.24	1.15	1.42	1.47	1.38	1.18
time (sec)	N/A	0.313	0.087	0.002	0.446	0.746	0.117	0.170	0.088

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	173	150	148	141	171	173	171	143
normalized size	1	0.99	0.86	0.85	0.81	0.98	0.99	0.98	0.82
time (sec)	N/A	0.216	0.060	0.001	0.448	0.741	0.095	0.152	3.614

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	79	80	94	97	100	80
normalized size	1	1.00	1.00	0.92	0.93	1.09	1.13	1.16	0.93
time (sec)	N/A	0.106	0.029	0.001	0.452	0.747	0.082	0.154	3.561

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	40	42	40	39
normalized size	1	1.00	1.00	0.85	0.83	0.87	0.91	0.87	0.85
time (sec)	N/A	0.029	0.013	0.002	0.440	0.835	0.072	0.157	0.025

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	143	136	210	159	161	148	170	175
normalized size	1	0.99	0.94	1.45	1.10	1.11	1.02	1.17	1.21
time (sec)	N/A	0.245	0.074	0.008	0.448	1.608	0.638	0.151	3.623

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	151	142	234	169	250	185	240	192
normalized size	1	0.99	0.93	1.53	1.10	1.63	1.21	1.57	1.25
time (sec)	N/A	0.205	0.154	0.010	0.451	0.872	1.257	0.163	0.090

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	154	176	257	177	273	206	167	185
normalized size	1	0.99	1.13	1.65	1.13	1.75	1.32	1.07	1.19
time (sec)	N/A	0.199	0.100	0.010	0.471	0.790	5.288	0.152	0.095

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	301	335	385	360	432	445	423	332
normalized size	1	0.99	1.10	1.27	1.18	1.42	1.46	1.39	1.09
time (sec)	N/A	0.535	0.131	0.002	0.450	0.756	0.134	0.159	0.140

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	216	241	268	257	302	311	302	244
normalized size	1	1.00	1.11	1.24	1.18	1.39	1.43	1.39	1.12
time (sec)	N/A	0.313	0.090	0.001	0.441	0.436	0.124	0.168	3.724

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	144	151	154	172	180	181	140
normalized size	1	1.00	1.12	1.18	1.20	1.34	1.41	1.41	1.09
time (sec)	N/A	0.159	0.051	0.001	0.447	0.806	0.097	0.152	3.695

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	69	75	74	76	83	76	74
normalized size	1	1.00	1.03	1.12	1.10	1.13	1.24	1.13	1.10
time (sec)	N/A	0.040	0.031	0.001	0.438	0.896	0.081	0.151	0.038

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	295	285	490	377	379	359	416	422
normalized size	1	0.99	0.96	1.65	1.27	1.28	1.21	1.40	1.42
time (sec)	N/A	0.640	0.169	0.008	0.485	0.616	0.937	0.158	3.685

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	289	272	527	392	553	416	497	575
normalized size	1	0.99	0.93	1.80	1.34	1.89	1.42	1.70	1.97
time (sec)	N/A	0.525	0.281	0.012	0.478	0.967	2.785	0.185	0.121

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	292	274	563	402	608	474	397	495
normalized size	1	0.99	0.93	1.91	1.36	2.06	1.61	1.35	1.68
time (sec)	N/A	0.495	0.122	0.014	0.494	0.909	14.200	0.160	3.825

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	400	459	553	512	618	646	606	490
normalized size	1	0.99	1.14	1.37	1.27	1.53	1.60	1.50	1.21
time (sec)	N/A	0.691	0.208	0.001	0.470	0.742	0.161	0.202	4.050

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	288	329	388	367	432	447	432	343
normalized size	1	1.00	1.14	1.34	1.27	1.49	1.55	1.49	1.19
time (sec)	N/A	0.424	0.130	0.001	0.454	0.750	0.147	0.155	3.938

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	196	223	222	249	265	261	187
normalized size	1	1.00	1.16	1.32	1.31	1.47	1.57	1.54	1.11
time (sec)	N/A	0.187	0.070	0.000	0.440	0.769	0.114	0.185	0.099

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	100	111	108	111	122	111	103
normalized size	1	1.00	1.15	1.28	1.24	1.28	1.40	1.28	1.18
time (sec)	N/A	0.058	0.031	0.001	0.432	0.769	0.086	0.186	0.057

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	487	498	880	672	674	685	764	741
normalized size	1	0.99	1.02	1.80	1.37	1.38	1.40	1.56	1.51
time (sec)	N/A	1.098	0.474	0.009	0.487	0.865	1.467	0.167	3.877

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	483	641	928	691	932	748	838	1511
normalized size	1	0.99	1.32	1.91	1.42	1.92	1.54	1.72	3.11
time (sec)	N/A	0.980	0.397	0.017	0.487	0.931	4.949	0.196	3.986

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	463	438	978	701	1025	816	727	1290
normalized size	1	0.99	0.94	2.10	1.50	2.20	1.75	1.56	2.77
time (sec)	N/A	0.967	0.225	0.017	0.530	0.926	25.279	0.174	3.936

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	62	76	82	78	73	111	85
normalized size	1	1.00	3.65	4.47	4.82	4.59	4.29	6.53	5.00
time (sec)	N/A	0.032	0.037	0.006	0.458	0.951	0.368	0.175	0.077

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	62	76	82	78	73	111	85
normalized size	1	1.00	3.65	4.47	4.82	4.59	4.29	6.53	5.00
time (sec)	N/A	0.019	0.016	0.005	0.466	0.844	0.367	0.155	3.836

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	90	157	160	120	153	216	252
normalized size	1	1.00	5.29	9.24	9.41	7.06	9.00	12.71	14.82
time (sec)	N/A	0.046	0.042	0.007	0.452	0.872	0.587	0.183	3.777

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	90	157	160	120	153	216	252
normalized size	1	1.00	5.29	9.24	9.41	7.06	9.00	12.71	14.82
time (sec)	N/A	0.025	0.025	0.006	0.444	0.890	0.610	0.199	0.049

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	237	223	399	244	592	1008	279	277
normalized size	1	0.99	0.93	1.66	1.02	2.47	4.20	1.16	1.15
time (sec)	N/A	0.471	0.243	0.011	0.982	0.904	5.462	0.170	3.990

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	166	155	256	161	404	638	176	181
normalized size	1	0.99	0.92	1.52	0.96	2.40	3.80	1.05	1.08
time (sec)	N/A	0.262	0.167	0.007	0.990	0.791	3.150	0.157	3.899

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	86	133	86	206	337	91	97
normalized size	1	1.00	0.92	1.43	0.92	2.22	3.62	0.98	1.04
time (sec)	N/A	0.117	0.087	0.006	0.972	0.584	1.660	0.158	3.780

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	56	59	48	125	156	48	56
normalized size	1	1.00	1.02	1.07	0.87	2.27	2.84	0.87	1.02
time (sec)	N/A	0.053	0.041	0.003	0.966	0.802	0.486	0.159	3.731

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	120	247	123	262	0	125	840
normalized size	1	1.00	0.90	1.86	0.92	1.97	0.00	0.94	6.32
time (sec)	N/A	0.162	0.102	0.008	0.969	14.211	0.000	0.159	6.490

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	188	462	255	904	0	270	1199
normalized size	1	1.00	0.88	2.16	1.19	4.22	0.00	1.26	5.60
time (sec)	N/A	0.355	0.312	0.010	1.035	69.906	0.000	0.170	6.773

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	277	754	495	0	0	489	2980
normalized size	1	1.00	0.91	2.47	1.62	0.00	0.00	1.60	9.77
time (sec)	N/A	0.651	0.309	0.015	1.052	0.000	0.000	0.197	9.186

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	233	484	287	931	952	289	303
normalized size	1	1.00	1.08	2.24	1.33	4.31	4.41	1.34	1.40
time (sec)	N/A	0.505	0.204	0.015	0.984	0.924	34.457	0.174	4.014

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	175	323	188	631	593	184	195
normalized size	1	1.00	1.20	2.21	1.29	4.32	4.06	1.26	1.34
time (sec)	N/A	0.245	0.138	0.013	0.965	0.967	18.398	0.170	0.229

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	102	134	113	337	318	112	191
normalized size	1	1.00	1.05	1.38	1.16	3.47	3.28	1.15	1.97
time (sec)	N/A	0.082	0.095	0.009	0.972	0.970	6.411	0.218	0.139

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	76	62	195	116	60	60
normalized size	1	1.00	0.99	1.10	0.90	2.83	1.68	0.87	0.87
time (sec)	N/A	0.043	0.052	0.008	0.965	0.812	0.654	0.183	0.100

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	195	742	293	1024	0	350	1493
normalized size	1	1.00	0.86	3.28	1.30	4.53	0.00	1.55	6.61
time (sec)	N/A	0.435	0.224	0.017	1.002	64.284	0.000	0.193	7.675

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	371	320	1036	604	0	0	608	2094
normalized size	1	0.99	0.86	2.77	1.61	0.00	0.00	1.63	5.60
time (sec)	N/A	0.950	0.413	0.023	1.039	0.000	0.000	0.190	9.909

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	524	524	466	1588	1030	0	0	957	2828
normalized size	1	1.00	0.89	3.03	1.97	0.00	0.00	1.83	5.40
time (sec)	N/A	1.552	0.630	0.027	1.215	0.000	0.000	0.185	14.480

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	281	402	379	1138	0	348	920
normalized size	1	1.00	1.34	1.92	1.81	5.44	0.00	1.67	4.40
time (sec)	N/A	0.304	0.247	0.013	1.001	1.434	0.000	0.269	1.767

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	175	211	283	253	806	391	254	230
normalized size	1	1.12	1.35	1.81	1.62	5.17	2.51	1.63	1.47
time (sec)	N/A	0.231	0.139	0.010	0.995	1.216	141.179	0.160	3.959

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	137	157	160	470	240	152	128
normalized size	1	1.00	1.05	1.21	1.23	3.62	1.85	1.17	0.98
time (sec)	N/A	0.108	0.101	0.009	0.983	1.197	32.420	0.185	0.150

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	90	96	98	314	156	84	88
normalized size	1	1.00	0.92	0.98	1.00	3.20	1.59	0.86	0.90
time (sec)	N/A	0.063	0.070	0.007	0.971	1.255	1.234	0.162	3.843

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	321	1598	655	0	0	715	2392
normalized size	1	1.00	0.91	4.53	1.86	0.00	0.00	2.03	6.78
time (sec)	N/A	0.734	0.424	0.023	1.097	0.000	0.000	0.177	9.900

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	571	566	498	2159	1196	0	0	1107	6848
normalized size	1	0.99	0.87	3.78	2.09	0.00	0.00	1.94	11.99
time (sec)	N/A	1.925	0.764	0.031	1.239	0.000	0.000	0.235	6.657

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	753	753	672	2737	1835	0	0	1532	8774
normalized size	1	1.00	0.89	3.63	2.44	0.00	0.00	2.03	11.65
time (sec)	N/A	3.143	1.078	0.037	1.274	0.000	0.000	0.201	7.244

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	437	647	599	1864	0	636	669
normalized size	1	1.00	1.87	2.76	2.56	7.97	0.00	2.72	2.86
time (sec)	N/A	0.291	0.306	0.013	1.042	1.457	0.000	0.178	4.382

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	288	350	464	457	1378	0	475	402
normalized size	1	1.13	1.38	1.83	1.80	5.43	0.00	1.87	1.58
time (sec)	N/A	0.542	0.297	0.011	1.018	0.833	0.000	0.166	4.069

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	266	333	323	1062	0	328	287
normalized size	1	1.00	1.18	1.48	1.44	4.72	0.00	1.46	1.28
time (sec)	N/A	0.398	0.161	0.012	1.005	0.913	0.000	0.166	0.227

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	171	182	208	636	298	194	164
normalized size	1	1.00	1.04	1.10	1.26	3.85	1.81	1.18	0.99
time (sec)	N/A	0.136	0.134	0.010	0.981	1.202	139.971	0.157	3.936

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	112	113	133	430	196	109	116
normalized size	1	1.00	0.89	0.90	1.06	3.41	1.56	0.87	0.92
time (sec)	N/A	0.078	0.088	0.010	0.976	0.675	2.078	0.180	3.895

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	29	30	29	46	29	29	30
normalized size	1	1.00	0.67	0.70	0.67	1.07	0.67	0.67	0.70
time (sec)	N/A	0.050	0.019	0.008	0.953	0.929	0.129	0.148	0.035

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	27	24	23	40	20	23	23
normalized size	1	1.00	0.90	0.80	0.77	1.33	0.67	0.77	0.77
time (sec)	N/A	0.040	0.021	0.008	0.959	0.970	0.124	0.211	0.035

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	24	23	33	20	23	25
normalized size	1	1.00	0.79	0.83	0.79	1.14	0.69	0.79	0.86
time (sec)	N/A	0.025	0.009	0.005	0.963	0.962	0.127	0.153	0.031

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	20	10	12	14
normalized size	1	1.00	1.00	0.93	0.86	1.43	0.71	0.86	1.00
time (sec)	N/A	0.011	0.007	0.005	0.955	0.866	0.114	0.148	3.797

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	28	26	25	41	24	26	32
normalized size	1	1.00	0.90	0.84	0.81	1.32	0.77	0.84	1.03
time (sec)	N/A	0.039	0.010	0.008	0.956	1.023	0.159	0.154	0.042

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	30	34	49	31	35	38
normalized size	1	1.00	1.00	0.91	1.03	1.48	0.94	1.06	1.15
time (sec)	N/A	0.045	0.016	0.009	0.952	0.711	0.161	0.184	3.808

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	39	38	41	61	42	43	47
normalized size	1	1.00	0.87	0.84	0.91	1.36	0.93	0.96	1.04
time (sec)	N/A	0.063	0.016	0.010	0.971	1.068	0.176	0.162	0.038

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	22	12	13	12	18	8	12	12
normalized size	1	1.83	1.00	1.08	1.00	1.50	0.67	1.00	1.00
time (sec)	N/A	0.007	0.012	0.004	0.951	1.037	0.115	0.150	0.031

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	21	21	25	20	21	21
normalized size	1	1.00	1.00	0.78	0.78	0.93	0.74	0.78	0.78
time (sec)	N/A	0.014	0.011	0.006	0.965	0.833	0.127	0.183	3.830

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	387	362	661	436	855	1088	475	-1
normalized size	1	0.99	0.93	1.69	1.12	2.19	2.79	1.22	-0.00
time (sec)	N/A	0.830	0.457	0.018	0.466	0.968	28.367	0.227	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	279	256	446	305	595	738	321	-1
normalized size	1	1.00	0.91	1.59	1.09	2.12	2.64	1.15	-0.00
time (sec)	N/A	0.498	0.646	0.009	0.463	1.046	21.001	0.206	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	153	230	169	329	384	180	-1
normalized size	1	1.00	0.87	1.31	0.97	1.88	2.19	1.03	-0.01
time (sec)	N/A	0.268	0.417	0.007	0.449	1.003	11.876	0.208	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	98	111	96	190	170	87	-1
normalized size	1	1.00	0.92	1.05	0.91	1.79	1.60	0.82	-0.01
time (sec)	N/A	0.064	0.204	0.005	0.451	0.946	6.903	0.184	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	224	1265	362	0	0	278	-1
normalized size	1	1.00	1.09	6.14	1.76	0.00	0.00	1.35	-0.00
time (sec)	N/A	0.392	0.436	0.018	0.606	0.000	0.000	0.224	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	303	264	2818	478	0	0	0	-1
normalized size	1	0.98	0.86	9.15	1.55	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.509	0.258	0.016	0.647	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	295	318	4432	927	0	0	923	-1
normalized size	1	1.00	1.07	14.97	3.13	0.00	0.00	3.12	-0.00
time (sec)	N/A	0.546	0.561	0.016	0.703	0.000	0.000	0.353	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	382	5565	1772	0	0	1719	-1
normalized size	1	1.00	1.22	17.72	5.64	0.00	0.00	5.47	-0.00
time (sec)	N/A	0.505	0.825	0.015	0.841	0.000	0.000	0.503	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	312	439	7237	3404	0	0	0	-1
normalized size	1	1.00	1.40	23.12	10.88	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.429	1.305	0.020	1.140	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	432	583	8546	5793	0	0	4212	-1
normalized size	1	1.00	1.35	19.74	13.38	0.00	0.00	9.73	-0.00
time (sec)	N/A	0.743	1.619	0.025	1.525	0.000	0.000	0.656	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	462	481	794	525	1177	1916	652	-1
normalized size	1	1.00	1.04	1.72	1.14	2.55	4.15	1.41	-0.00
time (sec)	N/A	1.134	0.541	0.018	0.456	1.482	72.414	0.274	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	345	346	552	380	831	1304	452	-1
normalized size	1	1.00	1.00	1.60	1.10	2.40	3.77	1.31	-0.00
time (sec)	N/A	0.524	1.095	0.010	0.452	1.344	54.496	0.264	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	212	209	287	211	477	768	264	-1
normalized size	1	1.00	0.98	1.35	0.99	2.24	3.61	1.24	-0.00
time (sec)	N/A	0.271	0.633	0.005	0.447	1.022	27.893	0.254	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	125	146	131	262	348	129	-1
normalized size	1	1.00	0.91	1.07	0.96	1.91	2.54	0.94	-0.01
time (sec)	N/A	0.083	0.257	0.006	0.448	1.195	17.013	0.234	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	348	2420	632	0	0	551	-1
normalized size	1	1.00	1.07	7.42	1.94	0.00	0.00	1.69	-0.00
time (sec)	N/A	0.766	1.211	0.013	0.789	0.000	0.000	0.293	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	428	392	5121	708	0	0	0	-1
normalized size	1	0.99	0.91	11.85	1.64	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.901	0.525	0.017	0.783	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	488	480	435	7817	1299	0	0	1036	-1
normalized size	1	0.98	0.89	16.02	2.66	0.00	0.00	2.12	-0.00
time (sec)	N/A	0.921	0.655	0.017	0.878	0.000	0.000	0.433	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	475	469	517	9835	2415	0	0	1900	-1
normalized size	1	0.99	1.09	20.71	5.08	0.00	0.00	4.00	-0.00
time (sec)	N/A	0.845	1.233	0.021	1.095	0.000	0.000	0.612	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	511	511	575	12481	4326	0	0	0	-1
normalized size	1	1.00	1.13	24.42	8.47	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.092	2.154	0.025	1.503	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	507	639	14169	6650	0	0	4408	-1
normalized size	1	1.00	1.26	27.95	13.12	0.00	0.00	8.69	-0.00
time (sec)	N/A	0.860	2.268	0.029	1.893	0.000	0.000	1.327	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	403	696	17026	10724	0	0	6122	-1
normalized size	1	1.00	1.72	42.14	26.54	0.00	0.00	15.15	-0.00
time (sec)	N/A	0.552	2.472	0.038	2.729	0.000	0.000	1.123	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	531	863	19093	0	0	0	7936	-1
normalized size	1	1.00	1.62	35.89	0.00	0.00	0.00	14.92	-0.00
time (sec)	N/A	0.889	2.495	0.051	0.000	0.000	0.000	1.118	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	150	181	166	333	510	168	-1
normalized size	1	1.00	0.89	1.08	0.99	1.98	3.04	1.00	-0.01
time (sec)	N/A	0.102	0.311	0.009	0.453	1.317	32.916	0.213	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	323	252	528	349	559	796	314	-1
normalized size	1	0.99	0.78	1.62	1.07	1.72	2.45	0.97	-0.00
time (sec)	N/A	0.664	0.355	0.015	0.452	1.221	22.204	0.255	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	222	164	339	230	381	518	206	-1
normalized size	1	1.00	0.74	1.52	1.03	1.71	2.32	0.92	-0.00
time (sec)	N/A	0.372	0.233	0.010	0.453	0.684	15.782	0.230	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	135	96	172	126	199	282	110	227
normalized size	1	0.99	0.71	1.26	0.93	1.46	2.07	0.81	1.67
time (sec)	N/A	0.179	0.105	0.006	0.436	0.873	9.024	0.211	5.171

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	76	61	124	150	58	107
normalized size	1	1.00	0.85	1.03	0.82	1.68	2.03	0.78	1.45
time (sec)	N/A	0.048	0.038	0.006	0.435	0.602	3.496	0.197	4.559

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	125	453	218	0	0	138	-1
normalized size	1	1.00	0.96	3.48	1.68	0.00	0.00	1.06	-0.01
time (sec)	N/A	0.174	0.219	0.012	0.564	0.000	0.000	0.223	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	218	923	419	0	0	0	-1
normalized size	1	1.00	1.30	5.49	2.49	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.235	0.363	0.015	0.578	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	224	254	1574	896	1088	0	848	-1
normalized size	1	1.00	1.13	7.00	3.98	4.84	0.00	3.77	-0.00
time (sec)	N/A	0.292	0.455	0.016	0.672	22.087	0.000	0.263	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	228	246	516	346	758	0	339	-1
normalized size	1	1.00	1.07	2.25	1.51	3.31	0.00	1.48	-0.00
time (sec)	N/A	0.325	0.450	0.016	0.457	1.223	0.000	0.253	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	177	327	227	530	0	219	-1
normalized size	1	1.00	1.19	2.19	1.52	3.56	0.00	1.47	-0.01
time (sec)	N/A	0.184	0.284	0.010	0.448	0.891	0.000	0.247	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	102	163	126	278	209	116	151
normalized size	1	1.00	1.02	1.63	1.26	2.78	2.09	1.16	1.51
time (sec)	N/A	0.087	0.137	0.005	0.437	1.143	18.840	0.250	5.280

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	74	69	61	181	87	63	68
normalized size	1	1.00	1.21	1.13	1.00	2.97	1.43	1.03	1.11
time (sec)	N/A	0.036	0.062	0.006	0.430	0.634	8.859	0.201	4.332

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	137	862	453	721	0	294	-1
normalized size	1	1.00	0.99	6.25	3.28	5.22	0.00	2.13	-0.01
time (sec)	N/A	0.141	0.177	0.015	0.622	4.004	0.000	0.294	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	285	1663	1085	1573	0	0	-1
normalized size	1	1.00	1.19	6.96	4.54	6.58	0.00	0.00	-0.00
time (sec)	N/A	0.418	0.608	0.017	0.767	7.179	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	372	404	2584	2254	2853	0	1440	-1
normalized size	1	0.99	1.08	6.91	6.03	7.63	0.00	3.85	-0.00
time (sec)	N/A	1.028	1.184	0.018	1.017	33.904	0.000	0.394	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	50	47	83	68	194	48	59
normalized size	1	1.00	0.75	0.70	1.24	1.01	2.90	0.72	0.88
time (sec)	N/A	0.042	0.032	0.004	0.435	0.788	17.131	0.211	4.222

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	71	72	118	103	638	80	93
normalized size	1	1.00	0.73	0.74	1.22	1.06	6.58	0.82	0.96
time (sec)	N/A	0.057	0.050	0.004	0.441	0.895	37.219	0.264	4.281

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	92	96	153	137	1880	112	115
normalized size	1	1.00	0.72	0.76	1.20	1.08	14.80	0.88	0.91
time (sec)	N/A	0.087	0.065	0.006	0.451	0.855	78.297	0.266	4.375

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	54	79	78	60	94	54	45
normalized size	1	1.00	0.51	0.75	0.74	0.57	0.89	0.51	0.42
time (sec)	N/A	0.114	0.065	0.013	0.948	1.009	2.203	0.226	0.050

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	48	65	64	54	75	48	40
normalized size	1	1.00	0.59	0.79	0.78	0.66	0.91	0.59	0.49
time (sec)	N/A	0.088	0.043	0.005	0.959	0.899	1.185	0.202	4.099

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	44	51	50	49	63	44	35
normalized size	1	1.00	0.71	0.82	0.81	0.79	1.02	0.71	0.56
time (sec)	N/A	0.052	0.027	0.008	0.962	0.802	0.552	0.194	0.034

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	60	55	58	88	0	99	61
normalized size	1	1.00	0.90	0.82	0.87	1.31	0.00	1.48	0.91
time (sec)	N/A	0.081	0.028	0.009	0.962	0.914	0.000	0.235	0.189

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	64	65	65	106	0	48	68
normalized size	1	1.00	0.90	0.92	0.92	1.49	0.00	0.68	0.96
time (sec)	N/A	0.070	0.101	0.011	0.969	1.001	0.000	0.320	0.115

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	55	74	76	89	0	180	77
normalized size	1	1.00	0.71	0.96	0.99	1.16	0.00	2.34	1.00
time (sec)	N/A	0.067	0.064	0.012	0.971	0.775	0.000	0.257	0.111

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	58	79	78	76	0	54	110
normalized size	1	1.00	0.67	0.91	0.90	0.87	0.00	0.62	1.26
time (sec)	N/A	0.104	0.055	0.014	0.959	0.852	0.000	0.214	0.059

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	53	65	64	72	0	49	105
normalized size	1	1.00	0.75	0.92	0.90	1.01	0.00	0.69	1.48
time (sec)	N/A	0.084	0.050	0.006	0.963	0.865	0.000	0.196	4.067

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	51	50	67	114	44	100
normalized size	1	1.00	0.87	0.93	0.91	1.22	2.07	0.80	1.82
time (sec)	N/A	0.044	0.029	0.005	0.959	0.764	15.959	0.235	0.037

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	88	58	83	0	82	106
normalized size	1	1.00	0.96	1.66	1.09	1.57	0.00	1.55	2.00
time (sec)	N/A	0.060	0.023	0.009	0.963	0.732	0.000	0.208	0.136

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	98	84	103	0	168	157
normalized size	1	1.00	0.95	1.31	1.12	1.37	0.00	2.24	2.09
time (sec)	N/A	0.077	0.042	0.012	0.965	1.120	0.000	0.274	4.145

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	78	107	124	119	0	196	180
normalized size	1	1.00	0.80	1.10	1.28	1.23	0.00	2.02	1.86
time (sec)	N/A	0.123	0.083	0.011	0.981	0.820	0.000	0.245	4.169

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	91	105	87	0	53	212
normalized size	1	1.00	0.86	1.25	1.44	1.19	0.00	0.73	2.90
time (sec)	N/A	0.085	0.068	0.014	0.951	0.851	0.000	0.184	0.057

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	58	77	91	83	0	48	200
normalized size	1	1.00	0.97	1.28	1.52	1.38	0.00	0.80	3.33
time (sec)	N/A	0.075	0.051	0.008	0.962	0.780	0.000	0.194	0.050

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	30	27	50	40	180	25	185
normalized size	1	1.00	0.73	0.66	1.22	0.98	4.39	0.61	4.51
time (sec)	N/A	0.053	0.017	0.005	0.425	0.771	77.502	0.336	4.107

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	58	133	81	103	0	91	218
normalized size	1	1.00	0.79	1.82	1.11	1.41	0.00	1.25	2.99
time (sec)	N/A	0.085	0.048	0.009	0.972	0.578	0.000	0.214	0.135

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	91	143	107	134	0	233	270
normalized size	1	1.00	0.96	1.51	1.13	1.41	0.00	2.45	2.84
time (sec)	N/A	0.169	0.055	0.013	0.977	0.592	0.000	0.517	4.312

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	75	140	147	149	0	183	301
normalized size	1	1.00	0.64	1.20	1.26	1.27	0.00	1.56	2.57
time (sec)	N/A	0.206	0.100	0.014	0.987	0.903	0.000	0.295	4.190

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	420	417	0	0	0	0	0	0	-1
normalized size	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.604	1.148	0.114	0.000	1.009	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	403	401	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.453	0.692	0.028	0.000	0.839	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.519	2.850	0.046	0.000	1.519	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	165	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.407	0.319	0.177	0.000	0.952	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	256	343	263	308	320	308	244
normalized size	1	1.00	1.01	1.35	1.04	1.21	1.26	1.21	0.96
time (sec)	N/A	0.326	0.094	0.001	0.436	0.710	0.136	0.155	0.126

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	163	223	165	187	197	187	149
normalized size	1	1.00	1.01	1.39	1.02	1.16	1.22	1.16	0.93
time (sec)	N/A	0.193	0.046	0.001	0.429	0.651	0.108	0.169	0.071

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	90	87	99	102	99	88
normalized size	1	1.00	1.00	0.94	0.91	1.03	1.06	1.03	0.92
time (sec)	N/A	0.099	0.023	0.001	0.430	0.834	0.089	0.155	4.086

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	40	42	40	39
normalized size	1	1.00	1.00	0.85	0.83	0.87	0.91	0.87	0.85
time (sec)	N/A	0.030	0.009	0.000	0.430	0.735	0.069	0.180	0.026

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	84	140	0	265	413	78	224
normalized size	1	1.00	1.04	1.73	0.00	3.27	5.10	0.96	2.77
time (sec)	N/A	0.100	0.092	0.006	0.000	0.772	1.213	0.154	0.194

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	98	146	0	511	376	108	172
normalized size	1	1.00	0.98	1.46	0.00	5.11	3.76	1.08	1.72
time (sec)	N/A	0.068	0.081	0.008	0.000	1.263	1.212	0.159	4.533

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	160	373	0	1199	774	217	401
normalized size	1	1.00	0.99	2.32	0.00	7.45	4.81	1.35	2.49
time (sec)	N/A	0.114	0.205	0.011	0.000	0.894	2.362	0.181	4.173

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	204	643	0	2103	1224	407	698
normalized size	1	1.00	0.99	3.12	0.00	10.21	5.94	1.98	3.39
time (sec)	N/A	0.190	0.408	0.015	0.000	0.967	4.220	0.172	4.358

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	591	591	585	1738	0	2150	4972	771	967
normalized size	1	1.00	0.99	2.94	0.00	3.64	8.41	1.30	1.64
time (sec)	N/A	1.428	0.651	0.012	0.000	2.912	118.420	0.171	5.462

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	345	1028	0	1273	2839	426	557
normalized size	1	1.00	0.99	2.95	0.00	3.66	8.16	1.22	1.60
time (sec)	N/A	0.677	0.380	0.009	0.000	1.110	47.193	0.161	4.684

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	173	510	0	654	1265	201	273
normalized size	1	1.00	0.98	2.88	0.00	3.69	7.15	1.14	1.54
time (sec)	N/A	0.350	0.205	0.006	0.000	0.933	14.462	0.194	0.530

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	95	196	0	302	488	89	132
normalized size	1	1.00	1.03	2.13	0.00	3.28	5.30	0.97	1.43
time (sec)	N/A	0.156	0.073	0.004	0.000	0.771	2.139	0.156	0.253

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	193	622	0	625	0	204	2467
normalized size	1	1.00	0.98	3.17	0.00	3.19	0.00	1.04	12.59
time (sec)	N/A	0.349	0.206	0.009	0.000	83.612	0.000	0.163	10.450

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	281	1125	0	0	0	449	3991
normalized size	1	1.00	0.89	3.56	0.00	0.00	0.00	1.42	12.63
time (sec)	N/A	0.761	0.536	0.013	0.000	0.000	0.000	0.196	14.713

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	504	1945	0	0	0	1002	12784
normalized size	1	1.00	0.99	3.82	0.00	0.00	0.00	1.97	25.12
time (sec)	N/A	1.251	0.766	0.019	0.000	0.000	0.000	0.206	6.819

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	398	1712	0	2771	0	540	742
normalized size	1	1.00	1.38	5.94	0.00	9.62	0.00	1.88	2.58
time (sec)	N/A	0.700	0.811	0.016	0.000	2.482	0.000	0.184	5.779

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	225	500	0	1413	1535	285	376
normalized size	1	1.00	1.26	2.81	0.00	7.94	8.62	1.60	2.11
time (sec)	N/A	0.266	0.462	0.012	0.000	1.270	60.262	0.167	5.039

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	114	194	0	632	459	125	203
normalized size	1	1.00	0.97	1.64	0.00	5.36	3.89	1.06	1.72
time (sec)	N/A	0.098	0.106	0.008	0.000	0.936	2.236	0.160	3.896

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	405	3202	0	0	0	860	13698
normalized size	1	1.00	1.00	7.87	0.00	0.00	0.00	2.11	33.66
time (sec)	N/A	1.087	0.905	0.023	0.000	0.000	0.000	0.193	6.700

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	650	4716	0	0	0	1437	26278
normalized size	1	1.00	0.97	7.01	0.00	0.00	0.00	2.14	39.05
time (sec)	N/A	2.559	2.206	0.038	0.000	0.000	0.000	0.327	8.926

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	60	53	51	75	60	51	55
normalized size	1	1.00	0.97	0.85	0.82	1.21	0.97	0.82	0.89
time (sec)	N/A	0.074	0.036	0.008	0.955	1.360	0.158	0.158	0.041

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	46	46	70	54	46	48
normalized size	1	1.00	1.00	0.84	0.84	1.27	0.98	0.84	0.87
time (sec)	N/A	0.066	0.026	0.007	0.950	0.805	0.151	0.159	0.043

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	45	43	60	53	43	59
normalized size	1	1.00	1.00	0.87	0.83	1.15	1.02	0.83	1.13
time (sec)	N/A	0.052	0.021	0.005	0.956	1.345	0.148	0.151	3.838

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	39	34	32	41	41	32	35
normalized size	1	1.00	0.95	0.83	0.78	1.00	1.00	0.78	0.85
time (sec)	N/A	0.033	0.022	0.005	0.945	1.268	0.137	0.157	3.833

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	48	47	72	54	48	58
normalized size	1	1.00	1.00	0.86	0.84	1.29	0.96	0.86	1.04
time (sec)	N/A	0.089	0.026	0.009	0.953	1.091	0.184	0.151	0.100

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	55	54	85	65	55	68
normalized size	1	1.00	1.00	0.90	0.89	1.39	1.07	0.90	1.11
time (sec)	N/A	0.128	0.023	0.011	0.958	1.199	0.198	0.150	4.132

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	60	63	98	71	63	75
normalized size	1	1.00	0.97	0.88	0.93	1.44	1.04	0.93	1.10
time (sec)	N/A	0.110	0.030	0.011	0.954	0.923	0.217	0.158	0.098

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	8	10
normalized size	1	1.00	1.00	1.10	1.00	1.00	0.70	0.80	1.00
time (sec)	N/A	0.011	0.006	0.004	0.430	0.822	0.095	0.149	0.045

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	27	27	36	27	29
normalized size	1	1.00	1.00	0.90	0.87	0.87	1.16	0.87	0.94
time (sec)	N/A	0.032	0.008	0.003	0.953	1.031	0.117	0.173	0.032

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	21	22	21	21
normalized size	1	1.00	1.00	0.96	0.91	0.91	0.96	0.91	0.91
time (sec)	N/A	0.035	0.005	0.005	0.953	0.655	0.110	0.151	0.042

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	18	17	25	14	18	17
normalized size	1	1.00	0.90	0.86	0.81	1.19	0.67	0.86	0.81
time (sec)	N/A	0.013	0.009	0.007	0.420	0.785	0.087	0.166	0.042

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	14	16	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	0.78	0.89	0.78
time (sec)	N/A	0.018	0.005	0.006	0.427	0.691	0.111	0.149	3.922

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	12	14	12
normalized size	1	1.00	1.00	0.93	0.86	0.86	0.86	1.00	0.86
time (sec)	N/A	0.017	0.006	0.006	0.426	0.861	0.114	0.156	3.851

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	31	22	21	21	22	21	17
normalized size	1	1.00	1.15	0.81	0.78	0.78	0.81	0.78	0.63
time (sec)	N/A	0.028	0.005	0.004	0.943	0.607	0.123	0.151	3.800

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	30	36	55	46	45	35
normalized size	1	1.00	1.00	0.62	0.75	1.15	0.96	0.94	0.73
time (sec)	N/A	0.045	0.075	0.003	0.956	0.898	0.123	0.190	0.111

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	19	19	15	19	17
normalized size	1	1.00	1.00	0.81	0.90	0.90	0.71	0.90	0.81
time (sec)	N/A	0.012	0.008	0.005	0.425	0.777	0.119	0.157	3.841

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	30	39	37	30	36
normalized size	1	1.00	1.00	0.87	0.77	1.00	0.95	0.77	0.92
time (sec)	N/A	0.023	0.027	0.006	0.958	0.644	0.136	0.165	3.835

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	33	33	31	11	11
normalized size	1	1.00	1.00	1.09	3.00	3.00	2.82	1.00	1.00
time (sec)	N/A	0.008	0.006	0.005	0.436	0.833	0.132	0.151	3.800

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	344	997	0	953	0	482	-1
normalized size	1	1.00	1.29	3.73	0.00	3.57	0.00	1.81	-0.00
time (sec)	N/A	0.238	0.861	0.011	0.000	1.201	0.000	0.275	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	267	613	0	605	0	297	-1
normalized size	1	1.00	1.26	2.89	0.00	2.85	0.00	1.40	-0.00
time (sec)	N/A	0.183	0.581	0.010	0.000	1.094	0.000	0.282	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	144	327	0	355	0	160	240
normalized size	1	1.00	0.92	2.08	0.00	2.26	0.00	1.02	1.53
time (sec)	N/A	0.124	0.208	0.008	0.000	0.880	0.000	0.217	4.262

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	86	136	0	203	0	84	-1
normalized size	1	1.00	0.83	1.31	0.00	1.95	0.00	0.81	-0.01
time (sec)	N/A	0.080	0.146	0.010	0.000	1.022	0.000	0.247	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	104	169	0	403	0	110	108
normalized size	1	1.00	1.06	1.72	0.00	4.11	0.00	1.12	1.10
time (sec)	N/A	0.077	0.728	0.008	0.000	1.218	0.000	0.270	4.210

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	107	137	0	242	0	193	127
normalized size	1	1.00	0.94	1.20	0.00	2.12	0.00	1.69	1.11
time (sec)	N/A	0.093	0.894	0.008	0.000	2.335	0.000	0.258	4.142

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	148	316	0	563	0	452	578
normalized size	1	1.00	0.89	1.89	0.00	3.37	0.00	2.71	3.46
time (sec)	N/A	0.107	1.933	0.009	0.000	17.769	0.000	0.284	4.525

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	199	555	0	978	0	805	1018
normalized size	1	1.00	0.90	2.52	0.00	4.45	0.00	3.66	4.63
time (sec)	N/A	0.142	1.738	0.012	0.000	52.069	0.000	0.313	5.063

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	930	927	1093	3543	0	2817	0	1702	3262
normalized size	1	1.00	1.18	3.81	0.00	3.03	0.00	1.83	3.51
time (sec)	N/A	3.013	2.418	0.024	0.000	2.454	0.000	0.306	14.702

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	584	581	436	2179	0	1791	0	1012	1881
normalized size	1	0.99	0.75	3.73	0.00	3.07	0.00	1.73	3.22
time (sec)	N/A	1.440	0.966	0.016	0.000	1.755	0.000	0.312	7.911

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	258	1117	0	1009	0	495	877
normalized size	1	1.00	0.80	3.47	0.00	3.13	0.00	1.54	2.72
time (sec)	N/A	0.504	0.476	0.010	0.000	1.162	0.000	0.237	5.624

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	173	453	0	465	0	212	320
normalized size	1	1.00	0.99	2.59	0.00	2.66	0.00	1.21	1.83
time (sec)	N/A	0.167	0.298	0.008	0.000	0.801	0.000	0.236	4.240

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	331	2549	0	0	0	0	-1
normalized size	1	1.00	1.03	7.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.778	0.786	0.018	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	453	486	6218	0	0	0	0	-1
normalized size	1	0.99	1.06	13.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.100	1.561	0.017	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	446	645	12139	0	0	0	0	-1
normalized size	1	1.00	1.44	27.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.875	3.584	0.018	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	603	601	439	19321	0	0	0	0	-1
normalized size	1	1.00	0.73	32.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.449	1.925	0.020	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	497	499	447	29161	0	0	0	0	-1
normalized size	1	1.00	0.90	58.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.855	3.901	0.026	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	824	826	1128	40336	0	0	0	0	-1
normalized size	1	1.00	1.37	48.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.335	6.333	0.036	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1169	1166	721	5881	0	4751	0	2977	-1
normalized size	1	1.00	0.62	5.03	0.00	4.06	0.00	2.55	-0.00
time (sec)	N/A	3.698	2.707	0.029	0.000	10.427	0.000	0.438	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	753	749	468	3769	0	3145	0	1852	-1
normalized size	1	0.99	0.62	5.01	0.00	4.18	0.00	2.46	-0.00
time (sec)	N/A	2.104	1.598	0.017	0.000	4.348	0.000	0.367	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	285	2026	0	1833	0	955	-1
normalized size	1	1.00	0.68	4.85	0.00	4.39	0.00	2.28	-0.00
time (sec)	N/A	0.645	0.785	0.011	0.000	1.034	0.000	0.294	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	392	862	0	839	0	417	-1
normalized size	1	1.00	1.66	3.65	0.00	3.56	0.00	1.77	-0.00
time (sec)	N/A	0.242	0.691	0.007	0.000	0.514	0.000	0.264	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	660	660	635	6715	0	0	0	0	-1
normalized size	1	1.00	0.96	10.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.825	2.210	0.017	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	754	750	756	14734	0	0	0	0	-1
normalized size	1	0.99	1.00	19.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.503	4.158	0.020	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	824	819	4162	26596	0	0	0	0	-1
normalized size	1	0.99	5.05	32.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.141	6.274	0.025	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	833	829	7806	40092	0	0	0	7319	-1
normalized size	1	1.00	9.37	48.13	0.00	0.00	0.00	8.79	-0.00
time (sec)	N/A	2.264	6.492	0.029	0.000	0.000	0.000	26.172	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1097	1096	46895	57957	0	0	0	0	-1
normalized size	1	1.00	42.75	52.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.123	6.622	0.057	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1226	1223	1111	76693	0	0	0	0	-1
normalized size	1	1.00	0.91	62.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.997	6.296	0.069	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	657	660	766	100754	0	0	0	0	-1
normalized size	1	1.00	1.17	153.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.216	6.243	0.133	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1062	1062	1221	126612	0	0	0	0	-1
normalized size	1	1.00	1.15	119.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.004	6.419	0.184	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	70	115	126	83	0	78	170
normalized size	1	1.00	0.49	0.80	0.88	0.58	0.00	0.55	1.19
time (sec)	N/A	0.138	0.050	0.016	0.951	0.745	0.000	0.263	5.541

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	65	98	109	78	0	73	153
normalized size	1	1.00	0.55	0.83	0.92	0.66	0.00	0.62	1.30
time (sec)	N/A	0.112	0.038	0.008	0.957	0.948	0.000	0.303	5.151

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	60	81	92	73	0	68	136
normalized size	1	1.00	0.65	0.87	0.99	0.78	0.00	0.73	1.46
time (sec)	N/A	0.066	0.028	0.007	0.957	0.894	0.000	0.263	4.879

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	86	95	96	115	0	126	-1
normalized size	1	1.00	0.85	0.94	0.95	1.14	0.00	1.25	-0.01
time (sec)	N/A	0.117	0.054	0.010	0.963	0.907	0.000	0.394	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	92	123	103	133	0	380	-1
normalized size	1	1.00	0.85	1.14	0.95	1.23	0.00	3.52	-0.01
time (sec)	N/A	0.116	0.086	0.014	0.974	0.909	0.000	0.719	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	93	125	114	149	0	0	-1
normalized size	1	1.00	0.81	1.09	0.99	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.085	0.013	0.986	0.847	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	80	134	155	93	0	88	-1
normalized size	1	1.00	0.51	0.85	0.98	0.59	0.00	0.56	-0.01
time (sec)	N/A	0.201	0.050	0.016	0.978	0.912	0.000	0.259	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	75	117	138	88	0	83	-1
normalized size	1	1.00	0.53	0.83	0.98	0.62	0.00	0.59	-0.01
time (sec)	N/A	0.121	0.046	0.006	0.927	0.906	0.000	0.270	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	70	100	121	83	0	78	-1
normalized size	1	1.00	0.60	0.86	1.04	0.72	0.00	0.67	-0.01
time (sec)	N/A	0.082	0.037	0.005	0.966	0.927	0.000	0.211	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	96	151	125	125	0	136	-1
normalized size	1	1.00	0.77	1.22	1.01	1.01	0.00	1.10	-0.01
time (sec)	N/A	0.144	0.062	0.008	0.988	0.913	0.000	0.283	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	103	179	132	143	0	570	-1
normalized size	1	1.00	0.79	1.37	1.01	1.09	0.00	4.35	-0.01
time (sec)	N/A	0.140	0.092	0.011	0.978	0.881	0.000	0.841	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	103	162	143	159	0	0	-1
normalized size	1	1.00	0.75	1.17	1.04	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.107	0.013	0.984	0.889	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	90	153	184	103	0	98	-1
normalized size	1	1.00	0.48	0.81	0.97	0.54	0.00	0.52	-0.01
time (sec)	N/A	0.156	0.062	0.017	0.978	0.886	0.000	0.201	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	85	136	167	98	0	93	-1
normalized size	1	1.00	0.52	0.83	1.02	0.60	0.00	0.57	-0.01
time (sec)	N/A	0.133	0.056	0.007	0.971	0.954	0.000	0.212	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	80	119	150	93	0	88	-1
normalized size	1	1.00	0.58	0.86	1.08	0.67	0.00	0.63	-0.01
time (sec)	N/A	0.092	0.048	0.006	0.967	0.895	0.000	0.187	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	106	207	154	135	0	146	-1
normalized size	1	1.00	0.72	1.41	1.05	0.92	0.00	0.99	-0.01
time (sec)	N/A	0.160	0.075	0.010	0.972	0.877	0.000	0.307	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	113	235	161	153	0	760	-1
normalized size	1	1.00	0.73	1.53	1.05	0.99	0.00	4.94	-0.01
time (sec)	N/A	0.163	0.116	0.012	0.999	0.962	0.000	1.097	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	113	199	172	169	0	0	-1
normalized size	1	1.00	0.70	1.24	1.07	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.114	0.015	0.994	0.940	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	693	692	588	1869	0	1435	0	822	-1
normalized size	1	1.00	0.85	2.70	0.00	2.07	0.00	1.19	-0.00
time (sec)	N/A	2.102	1.251	0.019	0.000	1.733	0.000	0.323	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	418	343	1069	0	861	0	457	-1
normalized size	1	1.00	0.82	2.55	0.00	2.05	0.00	1.09	-0.00
time (sec)	N/A	1.011	0.649	0.013	0.000	1.329	0.000	0.296	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	215	505	0	461	0	210	-1
normalized size	1	1.00	0.96	2.26	0.00	2.07	0.00	0.94	-0.00
time (sec)	N/A	0.303	0.234	0.008	0.000	1.107	0.000	0.269	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	96	185	0	227	0	98	-1
normalized size	1	1.00	0.83	1.59	0.00	1.96	0.00	0.84	-0.01
time (sec)	N/A	0.106	0.152	0.007	0.000	0.863	0.000	0.249	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	172	599	0	0	0	0	-1
normalized size	1	1.00	0.96	3.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.293	0.281	0.015	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	239	227	1671	0	0	0	0	-1
normalized size	1	0.99	0.94	6.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.371	0.348	0.015	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	367	3615	0	2034	0	2307	-1
normalized size	1	1.00	1.09	10.76	0.00	6.05	0.00	6.87	-0.00
time (sec)	N/A	0.656	1.100	0.017	0.000	174.731	0.000	0.545	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	502	715	2780	0	2937	0	1054	-1
normalized size	1	1.00	1.42	5.52	0.00	5.83	0.00	2.09	-0.00
time (sec)	N/A	1.176	1.595	0.020	0.000	28.674	0.000	0.350	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	288	412	1557	0	1769	0	580	-1
normalized size	1	1.00	1.43	5.39	0.00	6.12	0.00	2.01	-0.00
time (sec)	N/A	0.392	0.827	0.013	0.000	20.322	0.000	0.322	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	205	735	0	905	0	271	-1
normalized size	1	1.00	1.10	3.95	0.00	4.87	0.00	1.46	-0.01
time (sec)	N/A	0.227	0.734	0.009	0.000	14.447	0.000	0.284	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	113	249	0	429	0	122	143
normalized size	1	1.00	1.02	2.24	0.00	3.86	0.00	1.10	1.29
time (sec)	N/A	0.066	0.295	0.006	0.000	1.367	0.000	0.267	4.535

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	271	2079	0	1905	0	719	-1
normalized size	1	1.00	1.20	9.24	0.00	8.47	0.00	3.20	-0.00
time (sec)	N/A	0.266	0.493	0.014	0.000	34.395	0.000	0.286	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	418	487	4930	0	5098	0	0	-1
normalized size	1	0.99	1.16	11.71	0.00	12.11	0.00	0.00	-0.00
time (sec)	N/A	0.797	2.460	0.019	0.000	107.700	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	713	707	762	9126	0	0	0	5637	-1
normalized size	1	0.99	1.07	12.80	0.00	0.00	0.00	7.91	-0.00
time (sec)	N/A	2.670	5.291	0.024	0.000	0.000	0.000	0.882	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	60	96	97	73	0	68	-1
normalized size	1	1.00	0.50	0.80	0.81	0.61	0.00	0.57	-0.01
time (sec)	N/A	0.135	0.045	0.015	0.954	0.841	0.000	0.244	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	55	79	80	68	0	63	-1
normalized size	1	1.00	0.58	0.83	0.84	0.72	0.00	0.66	-0.01
time (sec)	N/A	0.099	0.031	0.008	0.972	0.757	0.000	0.221	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	50	62	63	63	0	58	-1
normalized size	1	1.00	0.71	0.89	0.90	0.90	0.00	0.83	-0.01
time (sec)	N/A	0.058	0.021	0.007	0.960	0.755	0.000	0.264	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	60	67	105	0	116	-1
normalized size	1	1.00	1.00	0.77	0.86	1.35	0.00	1.49	-0.01
time (sec)	N/A	0.097	0.046	0.009	0.973	0.766	0.000	0.572	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	82	67	74	123	0	48	-1
normalized size	1	1.00	0.99	0.81	0.89	1.48	0.00	0.58	-0.01
time (sec)	N/A	0.095	0.053	0.012	0.979	0.874	0.000	0.277	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	69	74	82	96	0	204	-1
normalized size	1	1.00	0.78	0.83	0.92	1.08	0.00	2.29	-0.01
time (sec)	N/A	0.088	0.039	0.035	0.977	0.685	0.000	0.328	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	69	115	97	97	0	67	-1
normalized size	1	1.00	0.67	1.12	0.94	0.94	0.00	0.65	-0.01
time (sec)	N/A	0.124	0.041	0.014	0.965	0.707	0.000	0.210	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	61	98	80	92	0	62	-1
normalized size	1	1.00	0.74	1.20	0.98	1.12	0.00	0.76	-0.01
time (sec)	N/A	0.108	0.033	0.009	0.969	0.595	0.000	0.271	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	50	81	63	87	0	57	-1
normalized size	1	1.00	0.79	1.29	1.00	1.38	0.00	0.90	-0.02
time (sec)	N/A	0.060	0.119	0.006	0.943	0.652	0.000	0.272	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	73	102	64	96	0	91	-1
normalized size	1	1.00	1.18	1.65	1.03	1.55	0.00	1.47	-0.02
time (sec)	N/A	0.074	0.023	0.009	0.965	0.606	0.000	0.581	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	74	109	96	106	0	168	-1
normalized size	1	1.00	0.85	1.25	1.10	1.22	0.00	1.93	-0.01
time (sec)	N/A	0.092	0.044	0.012	0.955	0.692	0.000	0.297	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	79	111	145	126	0	223	-1
normalized size	1	1.00	0.71	0.99	1.29	1.12	0.00	1.99	-0.01
time (sec)	N/A	0.155	0.056	0.013	0.967	0.637	0.000	0.313	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	71	163	202	117	0	67	-1
normalized size	1	1.00	0.83	1.90	2.35	1.36	0.00	0.78	-0.01
time (sec)	N/A	0.113	0.061	0.014	0.963	0.594	0.000	0.208	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	146	185	112	0	62	-1
normalized size	1	1.00	0.97	2.15	2.72	1.65	0.00	0.91	-0.01
time (sec)	N/A	0.095	0.126	0.009	0.967	0.865	0.000	0.237	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	76	51	0	28	49
normalized size	1	1.00	0.70	0.64	1.62	1.09	0.00	0.60	1.04
time (sec)	N/A	0.048	0.059	0.005	0.437	0.804	0.000	0.196	4.196

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	72	158	93	126	0	101	-1
normalized size	1	1.00	0.85	1.86	1.09	1.48	0.00	1.19	-0.01
time (sec)	N/A	0.094	0.051	0.010	0.962	0.681	0.000	0.434	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	111	165	125	141	0	233	-1
normalized size	1	1.00	1.01	1.50	1.14	1.28	0.00	2.12	-0.01
time (sec)	N/A	0.151	0.064	0.011	0.974	0.744	0.000	0.363	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	89	148	174	156	0	233	-1
normalized size	1	1.00	0.66	1.10	1.29	1.16	0.00	1.73	-0.01
time (sec)	N/A	0.213	0.075	0.013	0.991	0.829	0.000	0.349	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	219	324	0	465	0	0	1089
normalized size	1	1.00	1.05	1.56	0.00	2.24	0.00	0.00	5.24
time (sec)	N/A	0.424	0.505	0.013	0.000	33.082	0.000	0.000	5.748

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	906	905	15669	19955	0	0	0	0	-1
normalized size	1	1.00	17.29	22.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.700	14.990	0.192	0.000	0.802	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	668	668	9965	12761	0	0	0	0	-1
normalized size	1	1.00	14.92	19.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.188	14.410	0.071	0.000	0.962	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	749	746	13240	8221	0	0	0	0	-1
normalized size	1	1.00	17.68	10.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.518	14.039	0.085	0.000	0.730	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	712	711	8456	21038	0	0	0	0	-1
normalized size	1	1.00	11.88	29.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.271	14.268	0.129	0.000	0.710	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	992	989	12997	48427	0	0	0	0	-1
normalized size	1	1.00	13.10	48.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.916	14.666	0.252	0.000	0.613	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1363	1363	19853	88790	0	0	0	0	-1
normalized size	1	1.00	14.57	65.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.200	16.015	0.428	0.000	0.753	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1904	1904	29140	153623	0	0	0	0	-1
normalized size	1	1.00	15.30	80.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	6.243	18.387	0.804	0.000	0.822	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	724	724	9972	14084	0	0	0	0	-1
normalized size	1	1.00	13.77	19.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.779	14.550	0.079	0.000	0.699	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	992	8161	0	0	0	0	-1
normalized size	1	1.00	1.78	14.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.889	11.588	0.061	0.000	0.565	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	470	1080	4251	0	0	0	0	-1
normalized size	1	1.00	2.29	9.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.482	12.500	0.054	0.000	0.745	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	506	772	6053	0	0	0	0	-1
normalized size	1	1.00	1.52	11.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.649	7.280	0.072	0.000	0.649	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	684	680	1194	20481	0	0	0	0	-1
normalized size	1	0.99	1.75	29.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.192	12.156	0.132	0.000	0.691	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	944	942	12295	46697	0	0	0	0	-1
normalized size	1	1.00	13.02	49.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.256	15.038	0.261	0.000	0.681	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	510	508	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.833	2.277	0.155	0.000	0.928	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	496	494	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.672	1.462	0.046	0.000	0.820	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	590	588	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.755	3.464	0.069	0.000	0.947	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	119	36	59	75	221	141	58
normalized size	1	1.00	2.90	0.88	1.44	1.83	5.39	3.44	1.41
time (sec)	N/A	0.052	0.106	0.005	0.576	0.581	13.248	0.181	4.246

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	39	66	83	280	191	78
normalized size	1	1.00	0.74	0.85	1.43	1.80	6.09	4.15	1.70
time (sec)	N/A	0.072	0.130	0.005	0.576	0.631	173.947	0.244	4.395

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	43	51	98	123	483	314	120
normalized size	1	1.00	0.75	0.89	1.72	2.16	8.47	5.51	2.11
time (sec)	N/A	0.121	0.306	0.006	0.598	0.518	171.168	0.306	4.457

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	167	8419	1779	2467	2281	2383	2026
normalized size	1	1.00	8.35	420.95	88.95	123.35	114.05	119.15	101.30
time (sec)	N/A	0.420	0.449	0.004	0.500	0.437	1.527	0.230	4.865

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	19	18	18	20	20	18
normalized size	1	1.00	1.00	0.73	0.69	0.69	0.77	0.77	0.69
time (sec)	N/A	0.025	0.005	0.005	0.426	0.540	0.244	0.165	0.051

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	282	783	0	701	0	330	-1
normalized size	1	1.00	0.82	2.26	0.00	2.03	0.00	0.95	-0.00
time (sec)	N/A	0.812	0.739	0.014	0.000	1.045	0.000	0.314	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	199	532	0	499	0	228	-1
normalized size	1	1.00	0.81	2.17	0.00	2.04	0.00	0.93	-0.00
time (sec)	N/A	0.438	0.443	0.010	0.000	0.710	0.000	0.386	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	141	333	0	341	0	149	-1
normalized size	1	1.00	0.80	1.88	0.00	1.93	0.00	0.84	-0.01
time (sec)	N/A	0.235	0.264	0.009	0.000	0.757	0.000	0.268	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	134	220	0	733	0	0	-1
normalized size	1	1.00	0.86	1.42	0.00	4.73	0.00	0.00	-0.01
time (sec)	N/A	0.256	0.369	0.010	0.000	5.780	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	127	173	0	703	0	171	166
normalized size	1	1.00	0.91	1.24	0.00	5.06	0.00	1.23	1.19
time (sec)	N/A	0.235	0.399	0.012	0.000	3.717	0.000	0.372	4.460

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	137	241	0	783	0	352	-1
normalized size	1	1.00	0.86	1.52	0.00	4.92	0.00	2.21	-0.01
time (sec)	N/A	0.244	0.359	0.012	0.000	4.813	0.000	0.361	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	150	375	0	365	0	689	-1
normalized size	1	1.00	0.81	2.02	0.00	1.96	0.00	3.70	-0.01
time (sec)	N/A	0.320	0.310	0.014	0.000	5.552	0.000	0.292	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	212	591	0	525	0	1448	-1
normalized size	1	1.00	0.79	2.19	0.00	1.94	0.00	5.36	-0.00
time (sec)	N/A	0.488	0.522	0.013	0.000	10.830	0.000	0.300	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	299	859	0	727	0	2177	-1
normalized size	1	1.00	0.81	2.32	0.00	1.96	0.00	5.87	-0.00
time (sec)	N/A	0.817	0.731	0.018	0.000	42.469	0.000	0.508	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	212	208	206	237	230	230	196
normalized size	1	1.00	0.82	0.81	0.80	0.92	0.89	0.89	0.76
time (sec)	N/A	0.257	0.040	0.002	0.434	0.739	0.518	0.168	4.204

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	136	146	145	160	158	160	137
normalized size	1	1.00	0.87	0.93	0.92	1.02	1.01	1.02	0.87
time (sec)	N/A	0.166	0.032	0.002	0.432	0.670	0.153	0.157	4.108

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	84	79	83	87	90	77
normalized size	1	1.00	1.00	0.90	0.85	0.89	0.94	0.97	0.83
time (sec)	N/A	0.108	0.013	0.001	0.428	0.608	1.911	0.151	0.047

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	34	34	37	34	34
normalized size	1	1.00	1.00	0.83	0.81	0.81	0.88	0.81	0.81
time (sec)	N/A	0.025	0.002	0.002	0.423	0.642	0.288	0.148	0.026

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	179	286	228	230	235	228	260
normalized size	1	1.00	0.79	1.25	1.00	1.01	1.03	1.00	1.14
time (sec)	N/A	0.193	0.059	0.006	0.433	0.840	1.174	0.155	4.139

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	223	313	234	319	238	308	363
normalized size	1	1.00	0.98	1.37	1.03	1.40	1.04	1.35	1.59
time (sec)	N/A	0.191	0.087	0.012	0.435	0.781	1.134	0.173	4.181

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	204	336	240	360	248	216	297
normalized size	1	1.00	0.88	1.45	1.04	1.56	1.07	0.94	1.29
time (sec)	N/A	0.204	0.066	0.011	0.444	0.570	2.627	0.180	0.092

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	277	264	263	305	298	296	251
normalized size	1	1.00	0.71	0.68	0.67	0.78	0.76	0.76	0.64
time (sec)	N/A	0.386	0.043	0.002	0.435	0.701	0.203	0.157	4.247

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	201	186	185	206	206	206	175
normalized size	1	1.00	1.00	0.93	0.92	1.02	1.02	1.02	0.87
time (sec)	N/A	0.240	0.026	0.001	0.429	0.700	0.146	0.161	0.108

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	121	108	105	107	112	116	101
normalized size	1	1.00	1.00	0.89	0.87	0.88	0.93	0.96	0.83
time (sec)	N/A	0.160	0.016	0.001	0.431	0.673	0.136	0.156	4.173

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	45	44	44	56	44	44
normalized size	1	1.00	1.00	0.75	0.73	0.73	0.93	0.73	0.73
time (sec)	N/A	0.036	0.001	0.000	0.423	0.723	0.155	0.149	0.035

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	262	465	366	368	372	378	434
normalized size	1	1.00	0.74	1.32	1.04	1.05	1.06	1.07	1.23
time (sec)	N/A	0.316	0.122	0.006	0.436	0.870	0.997	0.170	0.079

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	342	500	372	490	393	459	939
normalized size	1	1.00	0.97	1.42	1.05	1.39	1.11	1.30	2.66
time (sec)	N/A	0.328	0.139	0.012	0.438	1.004	2.311	0.180	4.217

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	311	531	378	545	394	354	771
normalized size	1	1.00	0.88	1.50	1.07	1.54	1.11	1.00	2.18
time (sec)	N/A	0.343	0.102	0.014	0.449	0.807	4.872	0.161	0.127

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	344	558	390	587	401	345	560
normalized size	1	1.00	0.96	1.55	1.08	1.63	1.11	0.96	1.56
time (sec)	N/A	0.358	0.123	0.014	0.462	0.651	8.089	0.159	4.284

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	178	291	206	206	450	212	397
normalized size	1	1.00	0.81	1.32	0.93	0.93	2.04	0.96	1.80
time (sec)	N/A	0.190	0.121	0.007	0.964	0.844	2.579	0.167	4.182

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	130	191	141	141	303	145	223
normalized size	1	1.00	0.83	1.22	0.90	0.90	1.94	0.93	1.43
time (sec)	N/A	0.162	0.085	0.006	0.961	0.788	1.719	0.159	0.099

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	86	102	84	84	163	88	107
normalized size	1	1.00	0.87	1.03	0.85	0.85	1.65	0.89	1.08
time (sec)	N/A	0.108	0.054	0.005	0.956	0.778	0.849	0.155	0.070

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	50	44	43	43	61	43	45
normalized size	1	1.00	0.89	0.79	0.77	0.77	1.09	0.77	0.80
time (sec)	N/A	0.049	0.018	0.003	0.967	0.649	0.232	0.162	0.042

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	146	298	160	171	0	158	713
normalized size	1	1.00	0.87	1.77	0.95	1.02	0.00	0.94	4.24
time (sec)	N/A	0.194	0.110	0.012	0.964	1.001	0.000	0.216	6.389

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	233	538	294	416	0	355	312
normalized size	1	1.00	1.00	2.31	1.26	1.79	0.00	1.52	1.34
time (sec)	N/A	0.251	0.160	0.016	0.983	1.051	0.000	0.177	4.669

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	278	819	498	698	0	406	493
normalized size	1	1.00	0.88	2.58	1.57	2.20	0.00	1.28	1.56
time (sec)	N/A	0.290	0.434	0.015	0.998	1.356	0.000	0.179	4.763

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	209	283	212	350	444	206	333
normalized size	1	1.00	1.11	1.50	1.12	1.85	2.35	1.09	1.76
time (sec)	N/A	0.255	0.158	0.016	0.958	0.872	2.772	0.162	0.148

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	150	189	147	245	298	145	211
normalized size	1	1.00	1.07	1.35	1.05	1.75	2.13	1.04	1.51
time (sec)	N/A	0.208	0.113	0.012	0.960	0.803	1.961	0.156	0.114

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	96	106	90	147	165	94	115
normalized size	1	1.00	0.99	1.09	0.93	1.52	1.70	0.97	1.19
time (sec)	N/A	0.193	0.066	0.010	0.961	0.825	1.023	0.156	4.152

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	51	52	78	65	52	52
normalized size	1	1.00	0.94	0.81	0.83	1.24	1.03	0.83	0.83
time (sec)	N/A	0.077	0.037	0.007	0.952	0.769	0.192	0.153	4.148

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	186	691	289	479	0	284	330
normalized size	1	1.00	0.83	3.08	1.29	2.14	0.00	1.27	1.47
time (sec)	N/A	0.340	0.164	0.020	0.982	0.937	0.000	0.170	4.612

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	270	986	548	910	0	571	601
normalized size	1	1.00	0.86	3.15	1.75	2.91	0.00	1.82	1.92
time (sec)	N/A	0.498	0.262	0.023	1.015	1.124	0.000	0.204	4.835

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	363	1314	851	1499	0	595	887
normalized size	1	1.00	0.88	3.19	2.07	3.64	0.00	1.44	2.15
time (sec)	N/A	0.715	0.396	0.027	1.090	1.464	0.000	0.204	4.940

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	209	267	222	441	469	201	299
normalized size	1	1.00	1.22	1.56	1.30	2.58	2.74	1.18	1.75
time (sec)	N/A	0.336	0.201	0.015	0.968	0.837	8.078	0.208	0.152

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	146	179	155	302	304	144	203
normalized size	1	1.00	1.09	1.34	1.16	2.25	2.27	1.07	1.51
time (sec)	N/A	0.239	0.182	0.013	0.961	0.832	3.963	0.165	4.214

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	107	102	101	172	163	97	125
normalized size	1	1.00	1.04	0.99	0.98	1.67	1.58	0.94	1.21
time (sec)	N/A	0.145	0.085	0.011	0.956	0.760	2.311	0.185	0.120

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	53	47	56	75	61	46	55
normalized size	1	1.00	0.83	0.73	0.88	1.17	0.95	0.72	0.86
time (sec)	N/A	0.050	0.039	0.007	0.958	0.735	0.201	0.155	0.049

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	282	1437	571	1052	0	460	641
normalized size	1	1.00	0.86	4.37	1.74	3.20	0.00	1.40	1.95
time (sec)	N/A	0.496	0.305	0.025	1.030	1.128	0.000	0.247	4.793

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	389	1850	916	1734	0	762	965
normalized size	1	1.00	0.88	4.18	2.07	3.91	0.00	1.72	2.18
time (sec)	N/A	0.893	0.526	0.031	1.134	1.562	0.000	0.256	4.989

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	70	115	126	83	0	78	170
normalized size	1	1.00	0.49	0.80	0.88	0.58	0.00	0.55	1.19
time (sec)	N/A	0.155	0.154	0.017	0.965	0.808	0.000	0.204	1.716

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	65	98	109	78	0	73	153
normalized size	1	1.00	0.52	0.79	0.88	0.63	0.00	0.59	1.23
time (sec)	N/A	0.092	0.095	0.006	0.954	0.847	0.000	0.199	0.767

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	91	127	128	125	0	129	-1
normalized size	1	1.00	0.61	0.85	0.86	0.84	0.00	0.87	-0.01
time (sec)	N/A	0.240	0.147	0.010	1.006	0.887	0.000	0.224	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	98	152	132	143	0	531	-1
normalized size	1	1.00	0.66	1.02	0.89	0.96	0.00	3.56	-0.01
time (sec)	N/A	0.237	0.162	0.013	1.003	0.919	0.000	0.531	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	98	158	143	159	0	258	-1
normalized size	1	1.00	0.65	1.05	0.95	1.05	0.00	1.71	-0.01
time (sec)	N/A	0.228	0.148	0.015	1.002	0.839	0.000	0.244	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	98	165	160	173	0	304	-1
normalized size	1	1.00	0.62	1.04	1.01	1.09	0.00	1.92	-0.01
time (sec)	N/A	0.226	0.159	0.013	0.980	0.656	0.000	0.258	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	98	167	181	189	0	327	-1
normalized size	1	1.00	0.59	1.01	1.10	1.15	0.00	1.98	-0.01
time (sec)	N/A	0.234	0.176	0.013	1.022	0.839	0.000	0.373	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	98	188	222	203	0	387	-1
normalized size	1	1.00	0.59	1.14	1.35	1.23	0.00	2.35	-0.01
time (sec)	N/A	0.229	0.200	0.014	1.044	0.888	0.000	0.278	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	91	195	250	156	0	405	-1
normalized size	1	1.00	0.54	1.15	1.48	0.92	0.00	2.40	-0.01
time (sec)	N/A	0.218	0.169	0.016	1.050	0.825	0.000	0.264	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	96	216	301	171	0	456	-1
normalized size	1	1.00	0.49	1.11	1.55	0.88	0.00	2.35	-0.01
time (sec)	N/A	0.268	0.198	0.019	1.039	0.709	0.000	0.293	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	80	134	155	93	0	88	-1
normalized size	1	1.00	0.48	0.81	0.93	0.56	0.00	0.53	-0.01
time (sec)	N/A	0.194	0.184	0.018	0.977	0.806	0.000	0.191	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	75	117	138	88	0	83	-1
normalized size	1	1.00	0.51	0.80	0.94	0.60	0.00	0.56	-0.01
time (sec)	N/A	0.121	0.124	0.005	1.003	0.816	0.000	0.334	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	101	183	157	135	0	139	-1
normalized size	1	1.00	0.59	1.06	0.91	0.78	0.00	0.81	-0.01
time (sec)	N/A	0.268	0.183	0.010	1.002	0.615	0.000	0.231	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	108	208	161	153	0	707	-1
normalized size	1	1.00	0.63	1.21	0.94	0.89	0.00	4.11	-0.01
time (sec)	N/A	0.282	0.210	0.013	1.013	0.877	0.000	0.451	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	108	214	172	169	0	268	-1
normalized size	1	1.00	0.62	1.23	0.99	0.97	0.00	1.54	-0.01
time (sec)	N/A	0.273	0.216	0.015	1.001	0.924	0.000	0.303	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	108	221	189	183	0	314	-1
normalized size	1	1.00	0.60	1.22	1.04	1.01	0.00	1.73	-0.01
time (sec)	N/A	0.267	0.219	0.017	1.036	0.974	0.000	0.269	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	108	204	210	199	0	503	-1
normalized size	1	1.00	0.57	1.09	1.12	1.06	0.00	2.68	-0.01
time (sec)	N/A	0.265	0.228	0.017	1.042	0.956	0.000	0.421	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	108	225	251	213	0	406	-1
normalized size	1	1.00	0.55	1.15	1.29	1.09	0.00	2.08	-0.01
time (sec)	N/A	0.263	0.237	0.017	1.060	1.034	0.000	0.324	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	108	246	297	229	0	452	-1
normalized size	1	1.00	0.55	1.26	1.52	1.17	0.00	2.32	-0.01
time (sec)	N/A	0.268	0.253	0.017	1.072	1.056	0.000	0.345	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	108	267	348	243	0	489	-1
normalized size	1	1.00	0.55	1.37	1.78	1.25	0.00	2.51	-0.01
time (sec)	N/A	0.262	0.259	0.019	1.074	0.909	0.000	0.332	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	60	95	96	73	0	68	-1
normalized size	1	1.00	0.50	0.79	0.80	0.61	0.00	0.57	-0.01
time (sec)	N/A	0.136	0.117	0.012	0.974	0.875	0.000	0.194	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	55	79	80	68	0	63	-1
normalized size	1	1.00	0.54	0.78	0.79	0.67	0.00	0.62	-0.01
time (sec)	N/A	0.080	0.073	0.006	0.963	0.783	0.000	0.196	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	81	92	99	115	0	119	-1
normalized size	1	1.00	0.64	0.73	0.79	0.91	0.00	0.94	-0.01
time (sec)	N/A	0.210	0.102	0.008	0.981	0.739	0.000	0.232	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	88	96	103	133	0	339	-1
normalized size	1	1.00	0.70	0.76	0.82	1.06	0.00	2.69	-0.01
time (sec)	N/A	0.202	0.115	0.012	0.980	0.969	0.000	0.402	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	88	102	114	149	0	248	-1
normalized size	1	1.00	0.69	0.80	0.89	1.16	0.00	1.94	-0.01
time (sec)	N/A	0.208	0.129	0.012	0.984	0.964	0.000	0.255	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	88	109	131	163	0	285	-1
normalized size	1	1.00	0.65	0.81	0.97	1.21	0.00	2.11	-0.01
time (sec)	N/A	0.205	0.149	0.014	1.003	0.854	0.000	0.257	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	81	116	149	125	0	164	-1
normalized size	1	1.00	0.58	0.83	1.07	0.90	0.00	1.18	-0.01
time (sec)	N/A	0.191	0.135	0.013	1.024	0.946	0.000	0.283	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	74	132	114	102	0	72	-1
normalized size	1	1.00	0.60	1.06	0.92	0.82	0.00	0.58	-0.01
time (sec)	N/A	0.152	0.470	0.017	0.972	0.987	0.000	0.225	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	60	115	97	97	0	67	-1
normalized size	1	1.00	0.58	1.12	0.94	0.94	0.00	0.65	-0.01
time (sec)	N/A	0.102	0.185	0.007	0.970	0.862	0.000	0.216	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	55	98	80	92	0	62	-1
normalized size	1	1.00	0.67	1.20	0.98	1.12	0.00	0.76	-0.01
time (sec)	N/A	0.055	0.131	0.008	0.952	0.858	0.000	0.219	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	86	148	99	149	0	118	-1
normalized size	1	1.00	0.85	1.47	0.98	1.48	0.00	1.17	-0.01
time (sec)	N/A	0.151	0.354	0.011	0.977	1.007	0.000	0.396	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	104	152	116	157	0	225	-1
normalized size	1	1.00	0.96	1.41	1.07	1.45	0.00	2.08	-0.01
time (sec)	N/A	0.153	0.323	0.011	1.007	0.965	0.000	0.423	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	84	144	149	126	0	220	-1
normalized size	1	1.00	0.75	1.29	1.33	1.12	0.00	1.96	-0.01
time (sec)	N/A	0.146	0.298	0.013	0.985	0.919	0.000	0.247	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	95	151	217	141	0	271	-1
normalized size	1	1.00	0.69	1.10	1.58	1.03	0.00	1.98	-0.01
time (sec)	N/A	0.204	0.158	0.014	1.007	0.878	0.000	0.292	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	65	180	219	122	0	71	-1
normalized size	1	1.00	0.62	1.71	2.09	1.16	0.00	0.68	-0.01
time (sec)	N/A	0.131	0.687	0.023	0.983	0.863	0.000	0.216	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	60	163	202	117	0	66	-1
normalized size	1	1.00	0.70	1.90	2.35	1.36	0.00	0.77	-0.01
time (sec)	N/A	0.082	0.253	0.008	0.967	0.719	0.000	0.206	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	55	146	185	112	0	62	-1
normalized size	1	1.00	0.81	2.15	2.72	1.65	0.00	0.91	-0.01
time (sec)	N/A	0.052	0.229	0.007	0.983	0.891	0.000	0.304	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	190	110	126	0	92	-1
normalized size	1	1.00	0.94	2.24	1.29	1.48	0.00	1.08	-0.01
time (sec)	N/A	0.128	0.470	0.009	0.968	0.842	0.000	0.229	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	92	194	127	141	0	206	-1
normalized size	1	1.00	0.84	1.76	1.15	1.28	0.00	1.87	-0.01
time (sec)	N/A	0.153	0.408	0.011	1.006	0.855	0.000	0.399	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	97	200	178	155	0	228	-1
normalized size	1	1.00	0.72	1.48	1.32	1.15	0.00	1.69	-0.01
time (sec)	N/A	0.221	0.310	0.013	0.998	0.890	0.000	0.265	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	89	207	246	170	0	279	-1
normalized size	1	1.00	0.56	1.29	1.54	1.06	0.00	1.74	-0.01
time (sec)	N/A	0.283	0.236	0.014	1.013	0.916	0.000	0.280	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	316	1406	0	1373	0	465	-1
normalized size	1	1.00	0.89	3.97	0.00	3.88	0.00	1.31	-0.00
time (sec)	N/A	0.377	1.145	0.015	0.000	88.993	0.000	0.306	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	319	1453	0	1385	0	488	-1
normalized size	1	1.00	0.90	4.12	0.00	3.92	0.00	1.38	-0.00
time (sec)	N/A	0.386	1.111	0.018	0.000	95.777	0.000	0.448	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	588	588	537	5924	2292	4795	0	10960	4341
normalized size	1	1.00	0.91	10.07	3.90	8.15	0.00	18.64	7.38
time (sec)	N/A	0.364	0.382	0.076	0.745	1.202	0.000	0.621	8.392

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	432	391	3222	1414	2796	0	6223	2625
normalized size	1	1.00	0.91	7.46	3.27	6.47	0.00	14.41	6.08
time (sec)	N/A	0.243	0.242	0.035	0.641	0.971	0.000	0.392	6.050

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	261	1504	788	1448	0	3098	1425
normalized size	1	1.00	0.89	5.15	2.70	4.96	0.00	10.61	4.88
time (sec)	N/A	0.189	0.169	0.016	0.548	0.608	0.000	0.266	5.087

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	221	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.481	0.731	0.258	0.000	0.883	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	441	0	0	0	0	0	-1
normalized size	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.900	1.803	0.125	0.000	0.711	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	528	528	488	1244	0	3480	0	657	1027
normalized size	1	1.00	0.92	2.36	0.00	6.59	0.00	1.24	1.95
time (sec)	N/A	1.312	1.079	0.019	0.000	0.986	0.000	0.220	6.175

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	765	765	754	1960	0	2643	0	982	2779
normalized size	1	1.00	0.99	2.56	0.00	3.45	0.00	1.28	3.63
time (sec)	N/A	5.825	0.731	0.013	0.000	1.838	0.000	0.177	7.261

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	85	166	177	97	0	92	221
normalized size	1	1.00	0.41	0.80	0.85	0.47	0.00	0.44	1.06
time (sec)	N/A	0.352	0.305	0.033	0.990	0.864	0.000	0.210	6.306

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	75	132	143	87	0	82	187
normalized size	1	1.00	0.45	0.80	0.86	0.52	0.00	0.49	1.13
time (sec)	N/A	0.203	0.189	0.010	0.973	0.895	0.000	0.203	6.009

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	65	98	109	77	0	72	153
normalized size	1	1.00	0.52	0.79	0.88	0.62	0.00	0.58	1.23
time (sec)	N/A	0.116	0.100	0.007	0.959	0.872	0.000	0.189	5.378

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	189	403	500	304	0	144	-1
normalized size	1	1.00	1.01	2.16	2.67	1.63	0.00	0.77	-0.01
time (sec)	N/A	0.355	0.895	0.090	1.173	0.905	0.000	0.265	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	354	1084	0	378	0	0	-1
normalized size	1	1.00	1.78	5.45	0.00	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.250	1.276	0.032	0.000	0.990	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	334	2342	0	390	0	378	-1
normalized size	1	1.00	1.57	11.00	0.00	1.83	0.00	1.77	-0.00
time (sec)	N/A	0.237	1.397	0.031	0.000	0.961	0.000	0.262	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	95	185	206	107	0	102	-1
normalized size	1	1.00	0.41	0.80	0.89	0.46	0.00	0.44	-0.00
time (sec)	N/A	0.364	0.418	0.036	1.005	0.878	0.000	0.235	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	85	151	172	97	0	92	-1
normalized size	1	1.00	0.45	0.80	0.91	0.51	0.00	0.49	-0.01
time (sec)	N/A	0.227	0.251	0.010	0.984	0.698	0.000	0.216	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	75	117	138	87	0	82	-1
normalized size	1	1.00	0.51	0.80	0.94	0.59	0.00	0.56	-0.01
time (sec)	N/A	0.130	0.148	0.008	0.957	0.944	0.000	0.204	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	202	730	535	326	0	154	-1
normalized size	1	1.00	0.96	3.48	2.55	1.55	0.00	0.73	-0.00
time (sec)	N/A	0.304	0.938	0.020	1.310	0.985	0.000	0.282	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	354	1828	0	378	0	0	-1
normalized size	1	1.00	1.59	8.23	0.00	1.70	0.00	0.00	-0.00
time (sec)	N/A	0.322	1.833	0.023	0.000	0.897	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	376	3828	0	447	0	0	-1
normalized size	1	1.00	1.61	16.36	0.00	1.91	0.00	0.00	-0.00
time (sec)	N/A	0.319	2.239	0.022	0.000	0.993	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	75	147	148	87	0	82	-1
normalized size	1	1.00	0.41	0.79	0.80	0.47	0.00	0.44	-0.01
time (sec)	N/A	0.312	0.252	0.026	0.994	0.808	0.000	0.245	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	65	113	114	77	0	72	-1
normalized size	1	1.00	0.45	0.79	0.80	0.54	0.00	0.50	-0.01
time (sec)	N/A	0.203	0.143	0.010	0.974	0.870	0.000	0.392	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	55	79	80	67	0	62	-1
normalized size	1	1.00	0.54	0.78	0.79	0.66	0.00	0.61	-0.01
time (sec)	N/A	0.145	0.080	0.009	0.964	0.729	0.000	0.220	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	157	204	465	297	0	125	-1
normalized size	1	1.00	0.96	1.24	2.84	1.81	0.00	0.76	-0.01
time (sec)	N/A	0.230	0.467	0.019	1.138	0.857	0.000	0.275	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	313	510	0	330	0	276	-1
normalized size	1	1.00	1.76	2.87	0.00	1.85	0.00	1.55	-0.01
time (sec)	N/A	0.196	0.999	0.021	0.000	0.865	0.000	0.279	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	371	1194	0	390	0	378	-1
normalized size	1	1.00	1.63	5.26	0.00	1.72	0.00	1.67	-0.00
time (sec)	N/A	0.271	1.175	0.021	0.000	1.099	0.000	0.322	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	75	166	148	112	0	81	-1
normalized size	1	1.00	0.45	1.00	0.89	0.67	0.00	0.49	-0.01
time (sec)	N/A	0.241	0.433	0.030	0.989	0.827	0.000	0.261	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	65	132	114	102	0	71	-1
normalized size	1	1.00	0.52	1.06	0.92	0.82	0.00	0.57	-0.01
time (sec)	N/A	0.161	0.266	0.008	0.968	0.692	0.000	0.248	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	55	98	80	92	0	62	-1
normalized size	1	1.00	0.67	1.20	0.98	1.12	0.00	0.76	-0.01
time (sec)	N/A	0.087	0.150	0.007	0.958	0.991	0.000	0.215	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	174	489	777	333	0	112	-1
normalized size	1	1.00	1.05	2.95	4.68	2.01	0.00	0.67	-0.01
time (sec)	N/A	0.216	1.135	0.018	1.155	0.780	0.000	0.248	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	351	1214	0	392	0	295	-1
normalized size	1	1.00	1.63	5.65	0.00	1.82	0.00	1.37	-0.00
time (sec)	N/A	0.316	1.138	0.020	0.000	0.881	0.000	0.275	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	381	2600	0	452	0	397	-1
normalized size	1	1.00	1.52	10.40	0.00	1.81	0.00	1.59	-0.00
time (sec)	N/A	0.324	1.586	0.024	0.000	1.329	0.000	0.323	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	242	0	0	0	0	0	-1
normalized size	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.401	0.075	0.000	0.893	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	236	0	0	0	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.318	0.064	0.000	0.865	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	302	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.481	0.517	0.070	0.000	0.893	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [168] had the largest ratio of [.3571]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	34	0.147
2	A	6	5	1.00	32	0.156
3	A	5	5	1.00	27	0.185
4	A	5	5	1.00	34	0.147
5	A	5	5	1.00	34	0.147
6	A	5	5	1.00	34	0.147
7	A	8	6	1.00	34	0.176
8	A	4	4	1.00	34	0.118
9	A	5	4	1.00	34	0.118
10	A	7	4	1.00	34	0.118
11	A	6	4	1.00	34	0.118
12	A	5	4	1.00	32	0.125
13	A	4	4	1.00	27	0.148
14	A	4	4	1.00	34	0.118
15	A	7	5	1.00	34	0.147
16	A	4	4	1.00	34	0.118
17	A	5	4	1.00	34	0.118
18	A	2	1	0.99	25	0.040
19	A	2	1	0.99	25	0.040
20	A	2	1	1.00	23	0.043
21	A	2	1	1.00	18	0.056
22	A	2	1	0.99	25	0.040
23	A	2	1	0.99	25	0.040
24	A	2	1	0.99	25	0.040
25	A	3	2	0.99	27	0.074
26	A	3	2	1.00	27	0.074
27	A	3	2	1.00	25	0.080
28	A	3	2	1.00	20	0.100
29	A	2	1	0.99	27	0.037
30	A	2	1	0.99	27	0.037
31	A	2	1	0.99	27	0.037
32	A	3	2	0.99	27	0.074
33	A	3	2	1.00	27	0.074
34	A	3	2	1.00	25	0.080
35	A	3	2	1.00	20	0.100

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
36	A	2	1	0.99	27	0.037
37	A	2	1	0.99	27	0.037
38	A	2	1	0.99	27	0.037
39	A	1	1	1.00	32	0.031
40	A	1	1	1.00	31	0.032
41	A	1	1	1.00	34	0.029
42	A	1	1	1.00	33	0.030
43	A	5	4	0.99	27	0.148
44	A	5	4	0.99	27	0.148
45	A	5	4	1.00	25	0.160
46	A	5	4	1.00	20	0.200
47	A	5	4	1.00	27	0.148
48	A	5	4	1.00	27	0.148
49	A	5	4	1.00	27	0.148
50	A	6	5	1.00	27	0.185
51	A	5	5	1.00	27	0.185
52	A	4	4	1.00	25	0.160
53	A	3	3	1.00	20	0.150
54	A	6	5	1.00	27	0.185
55	A	6	5	0.99	27	0.185
56	A	6	5	1.00	27	0.185
57	A	5	5	1.00	27	0.185
58	A	3	3	1.12	27	0.111
59	A	3	3	1.00	25	0.120
60	A	4	4	1.00	20	0.200
61	A	7	6	1.00	27	0.222
62	A	7	5	0.99	27	0.185
63	A	7	5	1.00	27	0.185
64	A	4	4	1.00	27	0.148
65	A	4	4	1.13	27	0.148
66	A	4	4	1.00	27	0.148
67	A	4	4	1.00	25	0.160
68	A	5	4	1.00	20	0.200
69	A	6	5	1.00	17	0.294
70	A	5	5	1.00	17	0.294
71	A	4	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	3	3	1.00	14	0.214
73	A	6	5	1.00	17	0.294
74	A	6	5	1.00	17	0.294
75	A	6	5	1.00	17	0.294
76	A	3	3	1.83	16	0.188
77	A	3	3	1.00	18	0.167
78	A	7	6	0.99	29	0.207
79	A	6	6	1.00	29	0.207
80	A	5	5	1.00	27	0.185
81	A	5	5	1.00	22	0.227
82	A	7	6	1.00	29	0.207
83	A	7	6	0.98	29	0.207
84	A	7	6	1.00	29	0.207
85	A	7	6	1.00	29	0.207
86	A	5	5	1.00	29	0.172
87	A	6	6	1.00	29	0.207
88	A	8	6	1.00	29	0.207
89	A	7	6	1.00	29	0.207
90	A	6	5	1.00	27	0.185
91	A	6	5	1.00	22	0.227
92	A	8	6	1.00	29	0.207
93	A	8	6	0.99	29	0.207
94	A	8	7	0.98	29	0.241
95	A	8	6	0.99	29	0.207
96	A	8	7	1.00	29	0.241
97	A	8	6	1.00	29	0.207
98	A	6	5	1.00	29	0.172
99	A	7	6	1.00	29	0.207
100	A	7	5	1.00	22	0.227
101	A	6	5	0.99	29	0.172
102	A	5	5	1.00	29	0.172
103	A	4	4	0.99	27	0.148
104	A	4	4	1.00	22	0.182
105	A	6	5	1.00	29	0.172
106	A	6	5	1.00	29	0.172
107	A	4	4	1.00	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	5	5	1.00	29	0.172
109	A	4	4	1.00	29	0.138
110	A	4	4	1.00	27	0.148
111	A	4	4	1.00	22	0.182
112	A	4	4	1.00	29	0.138
113	A	4	4	1.00	29	0.138
114	A	5	5	0.99	29	0.172
115	A	3	3	1.00	22	0.136
116	A	4	4	1.00	22	0.182
117	A	5	4	1.00	22	0.182
118	A	5	4	1.00	29	0.138
119	A	4	4	1.00	29	0.138
120	A	3	3	1.00	27	0.111
121	A	5	5	1.00	29	0.172
122	A	5	5	1.00	29	0.172
123	A	4	4	1.00	29	0.138
124	A	5	4	1.00	29	0.138
125	A	4	4	1.00	29	0.138
126	A	3	3	1.00	27	0.111
127	A	4	4	1.00	29	0.138
128	A	4	4	1.00	29	0.138
129	A	5	5	1.00	29	0.172
130	A	4	3	1.00	29	0.103
131	A	4	3	1.00	29	0.103
132	A	2	2	1.00	27	0.074
133	A	5	5	1.00	29	0.172
134	A	5	4	1.00	29	0.138
135	A	6	5	1.00	29	0.172
136	A	6	4	0.99	27	0.148
137	A	6	4	1.00	29	0.138
138	A	5	5	1.00	31	0.161
139	A	6	6	1.00	70	0.086
140	A	2	1	1.00	20	0.050
141	A	2	1	1.00	20	0.050
142	A	2	1	1.00	20	0.050
143	A	2	1	1.00	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	6	5	1.00	20	0.250
145	A	4	4	1.00	20	0.200
146	A	5	5	1.00	20	0.250
147	A	6	5	1.00	20	0.250
148	A	6	5	1.00	30	0.167
149	A	6	5	1.00	30	0.167
150	A	6	5	1.00	28	0.179
151	A	6	5	1.00	23	0.217
152	A	6	5	1.00	30	0.167
153	A	6	5	1.00	30	0.167
154	A	6	5	1.00	30	0.167
155	A	6	6	1.00	30	0.200
156	A	5	5	1.00	28	0.179
157	A	4	4	1.00	23	0.174
158	A	7	6	1.00	30	0.200
159	A	7	6	1.00	30	0.200
160	A	7	6	1.00	20	0.300
161	A	7	6	1.00	20	0.300
162	A	5	5	1.00	18	0.278
163	A	4	4	1.00	17	0.235
164	A	7	6	1.00	20	0.300
165	A	7	6	1.00	20	0.300
166	A	7	6	1.00	20	0.300
167	A	1	1	1.00	16	0.062
168	A	6	5	1.00	14	0.357
169	A	6	5	1.00	16	0.312
170	A	3	2	1.00	18	0.111
171	A	5	3	1.00	16	0.188
172	A	5	3	1.00	19	0.158
173	A	6	5	1.00	23	0.217
174	A	5	3	1.00	19	0.158
175	A	2	2	1.00	23	0.087
176	A	4	4	1.00	17	0.235
177	A	1	1	1.00	19	0.053
178	A	7	5	1.00	22	0.227
179	A	6	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	5	5	1.00	22	0.227
181	A	4	4	1.00	22	0.182
182	A	4	4	1.00	22	0.182
183	A	3	3	1.00	22	0.136
184	A	4	4	1.00	22	0.182
185	A	5	4	1.00	22	0.182
186	A	7	6	1.00	32	0.188
187	A	6	6	0.99	32	0.188
188	A	5	5	1.00	30	0.167
189	A	5	5	1.00	25	0.200
190	A	7	6	1.00	32	0.188
191	A	7	6	0.99	32	0.188
192	A	7	6	1.00	32	0.188
193	A	7	6	1.00	32	0.188
194	A	5	5	1.00	32	0.156
195	A	6	6	1.00	32	0.188
196	A	8	6	1.00	32	0.188
197	A	7	6	0.99	32	0.188
198	A	6	5	1.00	30	0.167
199	A	6	5	1.00	25	0.200
200	A	8	6	1.00	32	0.188
201	A	8	6	0.99	32	0.188
202	A	8	7	0.99	32	0.219
203	A	8	6	1.00	32	0.188
204	A	8	7	1.00	32	0.219
205	A	8	6	1.00	32	0.188
206	A	6	5	1.00	32	0.156
207	A	7	6	1.00	32	0.188
208	A	7	6	1.00	32	0.188
209	A	6	6	1.00	32	0.188
210	A	5	5	1.00	30	0.167
211	A	7	7	1.00	32	0.219
212	A	7	7	1.00	32	0.219
213	A	7	7	1.00	32	0.219
214	A	9	6	1.00	32	0.188
215	A	7	6	1.00	32	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	6	5	1.00	30	0.167
217	A	8	7	1.00	32	0.219
218	A	8	7	1.00	32	0.219
219	A	8	8	1.00	32	0.250
220	A	9	6	1.00	32	0.188
221	A	8	6	1.00	32	0.188
222	A	7	5	1.00	30	0.167
223	A	9	7	1.00	32	0.219
224	A	9	7	1.00	32	0.219
225	A	9	8	1.00	32	0.250
226	A	6	5	1.00	32	0.156
227	A	5	5	1.00	32	0.156
228	A	4	4	1.00	30	0.133
229	A	4	4	1.00	25	0.160
230	A	6	5	1.00	32	0.156
231	A	6	5	0.99	32	0.156
232	A	4	4	1.00	32	0.125
233	A	5	5	1.00	32	0.156
234	A	4	4	1.00	32	0.125
235	A	4	4	1.00	30	0.133
236	A	4	4	1.00	25	0.160
237	A	4	4	1.00	32	0.125
238	A	4	4	0.99	32	0.125
239	A	5	5	0.99	32	0.156
240	A	6	5	1.00	32	0.156
241	A	5	5	1.00	32	0.156
242	A	4	4	1.00	30	0.133
243	A	6	6	1.00	32	0.188
244	A	6	6	1.00	32	0.188
245	A	4	4	1.00	32	0.125
246	A	6	5	1.00	32	0.156
247	A	5	5	1.00	32	0.156
248	A	4	4	1.00	30	0.133
249	A	4	4	1.00	32	0.125
250	A	4	4	1.00	32	0.125
251	A	5	5	1.00	32	0.156

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
252	A	5	4	1.00	32	0.125
253	A	5	4	1.00	32	0.125
254	A	2	2	1.00	30	0.067
255	A	5	5	1.00	32	0.156
256	A	5	4	1.00	32	0.125
257	A	6	5	1.00	32	0.156
258	A	3	3	1.00	47	0.064
259	A	8	7	1.00	34	0.206
260	A	7	6	1.00	34	0.176
261	A	7	6	1.00	34	0.176
262	A	7	6	1.00	34	0.176
263	A	7	6	1.00	34	0.176
264	A	8	7	1.00	34	0.206
265	A	9	7	1.00	34	0.206
266	A	8	6	1.00	34	0.176
267	A	7	6	1.00	34	0.176
268	A	6	5	1.00	34	0.147
269	A	6	5	1.00	34	0.147
270	A	7	6	0.99	34	0.176
271	A	8	6	1.00	34	0.176
272	A	6	4	1.00	30	0.133
273	A	6	4	1.00	32	0.125
274	A	5	5	1.00	34	0.147
275	A	3	3	1.00	42	0.071
276	A	2	2	1.00	46	0.043
277	A	2	2	1.00	69	0.029
278	A	2	2	1.00	75	0.027
279	A	6	4	1.00	16	0.250
280	A	6	5	1.00	33	0.152
281	A	5	4	1.00	31	0.129
282	A	5	4	1.00	30	0.133
283	A	7	5	1.00	33	0.152
284	A	7	6	1.00	33	0.182
285	A	7	5	1.00	33	0.152
286	A	5	4	1.00	33	0.121
287	A	6	5	1.00	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
288	A	7	5	1.00	33	0.152
289	A	2	1	1.00	36	0.028
290	A	2	1	1.00	36	0.028
291	A	2	1	1.00	34	0.029
292	A	2	1	1.00	29	0.034
293	A	2	1	1.00	36	0.028
294	A	2	1	1.00	36	0.028
295	A	2	1	1.00	36	0.028
296	A	2	1	1.00	38	0.026
297	A	2	1	1.00	38	0.026
298	A	2	1	1.00	36	0.028
299	A	2	1	1.00	31	0.032
300	A	2	1	1.00	38	0.026
301	A	2	1	1.00	38	0.026
302	A	2	1	1.00	38	0.026
303	A	2	1	1.00	38	0.026
304	A	6	5	1.00	38	0.132
305	A	6	5	1.00	38	0.132
306	A	6	5	1.00	36	0.139
307	A	6	5	1.00	31	0.161
308	A	6	5	1.00	38	0.132
309	A	6	5	1.00	38	0.132
310	A	6	5	1.00	38	0.132
311	A	7	6	1.00	38	0.158
312	A	7	6	1.00	38	0.158
313	A	7	6	1.00	36	0.167
314	A	7	6	1.00	31	0.194
315	A	7	6	1.00	38	0.158
316	A	7	6	1.00	38	0.158
317	A	7	6	1.00	38	0.158
318	A	8	6	1.00	38	0.158
319	A	8	6	1.00	38	0.158
320	A	6	6	1.00	36	0.167
321	A	5	4	1.00	31	0.129
322	A	8	6	1.00	38	0.158
323	A	8	6	1.00	38	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
324	A	7	5	1.00	38	0.132
325	A	7	5	1.00	33	0.152
326	A	9	7	1.00	40	0.175
327	A	9	8	1.00	40	0.200
328	A	9	8	1.00	40	0.200
329	A	9	7	1.00	40	0.175
330	A	9	7	1.00	40	0.175
331	A	9	7	1.00	40	0.175
332	A	7	5	1.00	40	0.125
333	A	8	6	1.00	40	0.150
334	A	8	5	1.00	38	0.132
335	A	8	5	1.00	33	0.152
336	A	10	7	1.00	40	0.175
337	A	10	8	1.00	40	0.200
338	A	10	8	1.00	40	0.200
339	A	10	7	1.00	40	0.175
340	A	10	8	1.00	40	0.200
341	A	10	7	1.00	40	0.175
342	A	10	8	1.00	40	0.200
343	A	10	7	1.00	40	0.175
344	A	6	4	1.00	38	0.105
345	A	6	4	1.00	33	0.121
346	A	8	6	1.00	40	0.150
347	A	8	7	1.00	40	0.175
348	A	8	7	1.00	40	0.175
349	A	8	6	1.00	40	0.150
350	A	6	4	1.00	40	0.100
351	A	7	5	1.00	40	0.125
352	A	6	5	1.00	38	0.132
353	A	5	5	1.00	33	0.152
354	A	7	7	1.00	40	0.175
355	A	7	7	1.00	40	0.175
356	A	5	5	1.00	40	0.125
357	A	6	5	1.00	40	0.125
358	A	6	5	1.00	40	0.125
359	A	5	4	1.00	38	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
360	A	5	4	1.00	33	0.121
361	A	5	4	1.00	40	0.100
362	A	5	4	1.00	40	0.100
363	A	6	5	1.00	40	0.125
364	A	7	5	1.00	40	0.125
365	A	5	4	1.00	35	0.114
366	A	5	4	1.00	36	0.111
367	A	2	1	1.00	38	0.026
368	A	2	1	1.00	38	0.026
369	A	2	1	1.00	36	0.028
370	A	4	2	1.00	38	0.053
371	A	5	3	1.00	38	0.079
372	A	6	5	1.00	38	0.132
373	A	6	5	1.00	53	0.094
374	A	11	5	1.00	35	0.143
375	A	9	5	1.00	35	0.143
376	A	7	5	1.00	33	0.152
377	A	9	7	1.00	35	0.200
378	A	9	7	1.00	35	0.200
379	A	7	5	1.00	35	0.143
380	A	12	5	1.00	35	0.143
381	A	10	5	1.00	35	0.143
382	A	8	5	1.00	33	0.152
383	A	10	7	1.00	35	0.200
384	A	10	8	1.00	35	0.229
385	A	10	7	1.00	35	0.200
386	A	10	4	1.00	35	0.114
387	A	8	4	1.00	35	0.114
388	A	6	4	1.00	33	0.121
389	A	8	6	1.00	35	0.171
390	A	6	4	1.00	35	0.114
391	A	7	4	1.00	35	0.114
392	A	9	5	1.00	35	0.143
393	A	7	5	1.00	35	0.143
394	A	5	5	1.00	33	0.152
395	A	6	4	1.00	35	0.114

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
396	A	7	4	1.00	35	0.114
397	A	8	4	1.00	35	0.114
398	A	7	5	1.00	26	0.192
399	A	7	6	1.00	24	0.250
400	A	11	8	1.00	29	0.276

Chapter 3

Listing of integrals

3.1 $\int (d+ex)^2 \left(A+Bx+Cx^2\right) \sqrt{d^2-e^2x^2}\,dx$

Optimal. Leaf size=236

$$\frac{d^2x\sqrt{d^2-e^2x^2}\left(10Ae^2+4Bde+3Cd^2\right)}{16e^2}-\frac{x\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+2Bd)+3Cd^2\right)}{8e^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}\left(e(10Ae+7\right.}{15e^3}$$

```
[Out] -1/15*d*(4*C*d^2+e*(10*A*e+7*B*d))*(-e^2*x^2+d^2)^(3/2)/e^3-1/8*(3*C*d^2+2*
e*(A*e+2*B*d))*x*(-e^2*x^2+d^2)^(3/2)/e^2-1/5*(B*e+2*C*d)*x^2*(-e^2*x^2+d^2
)^(3/2)/e-1/6*C*x^3*(-e^2*x^2+d^2)^(3/2)+1/16*d^4*(10*A*e^2+4*B*d*e+3*C*d^2
)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/16*d^2*(10*A*e^2+4*B*d*e+3*C*d^2)*
x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] time = 0.47, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1815, 641, 195, 217, 203}

$$\frac{d^2x\sqrt{d^2-e^2x^2}\left(10Ae^2+4Bde+3Cd^2\right)}{16e^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}\left(e(10Ae+7Bd)+4Cd^2\right)}{15e^3}-\frac{x\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+2\right.}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (d^2*(3*C*d^2 + 4*B*d*e + 10*A*e^2)*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) - (d*(4
*C*d^2 + e*(7*B*d + 10*A*e))*(d^2 - e^2*x^2)^(3/2))/(15*e^3) - ((3*C*d^2 +
2*e*(2*B*d + A*e))*x*(d^2 - e^2*x^2)^(3/2))/(8*e^2) - ((2*C*d + B*e)*x^2*(d
^2 - e^2*x^2)^(3/2))/(5*e) - (C*x^3*(d^2 - e^2*x^2)^(3/2))/6 + (d^4*(3*C*d^
2 + 4*B*d*e + 10*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int (d+ex)^2\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx = -\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \sqrt{d^2-e^2x^2}\left(-6Ad^2e^2-6de^2(Bd+2Ae)x \right.}{}$$

$$= -\frac{(2Cd+Be)x^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \sqrt{d^2-e^2x}}{}$$

$$= -\frac{\left(3Cd^2+2e(2Bd+Ae)\right)x\left(d^2-e^2x^2\right)^{3/2}}{8e^2} - \frac{(2Cd+Be)x^2\left(d^2-e^2x\right.}{5e}$$

$$= -\frac{d\left(4Cd^2+e(7Bd+10Ae)\right)\left(d^2-e^2x^2\right)^{3/2}}{15e^3} - \frac{\left(3Cd^2+2e(2Bd+Ae\right.}{8e^2}$$

$$= \frac{d^2\left(3Cd^2+4Bde+10Ae^2\right)x\sqrt{d^2-e^2x^2}}{16e^2} - \frac{d\left(4Cd^2+e(7Bd+10A\right.}{15e^3}$$

$$= \frac{d^2\left(3Cd^2+4Bde+10Ae^2\right)x\sqrt{d^2-e^2x^2}}{16e^2} - \frac{d\left(4Cd^2+e(7Bd+10A\right.}{15e^3}$$

$$= \frac{d^2\left(3Cd^2+4Bde+10Ae^2\right)x\sqrt{d^2-e^2x^2}}{16e^2} - \frac{d\left(4Cd^2+e(7Bd+10A\right.}{15e^3}$$

Mathematica [A] time = 0.51, size = 226, normalized size = 0.96

$$\frac{\sqrt{d^2-e^2x^2}\left(15\sin^{-1}\left(\frac{ex}{d}\right)\left(2d^3e(5Ae+2Bd)+3Cd^5\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(2e\left(5Ae\left(-16d^3+9d^2ex+16de^2x^2+6e^3x^3\right)+\right.\right.\right.}{240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(C*(-64*d^5 - 45*d^4*e*x - 32
*d^3*e^2*x^2 + 50*d^2*e^3*x^3 + 96*d*e^4*x^4 + 40*e^5*x^5) + 2*e*(5*A*e*(-1
6*d^3 + 9*d^2*e*x + 16*d*e^2*x^2 + 6*e^3*x^3) + B*(-56*d^4 - 30*d^3*e*x + 3
2*d^2*e^2*x^2 + 60*d*e^3*x^3 + 24*e^4*x^4))) + 15*(3*C*d^5 + 2*d^3*e*(2*B*d
 + 5*A*e))*ArcSin[(e*x)/d]))/(240*e^3*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.68, size = 211, normalized size = 0.89

$$-\frac{30\left(3\,Cd^6+4\,Bd^5e+10\,Ad^4e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(40\,Ce^5x^5-64\,Cd^5-112\,Bd^4e-160\,Ad^3e^2+48\left(2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/240*(30*(3*C*d^6 + 4*B*d^5*e + 10*A*d^4*e^2)*arctan(-(d - sqrt(-e^2*x^2
+ d^2))/(e*x)) - (40*C*e^5*x^5 - 64*C*d^5 - 112*B*d^4*e - 160*A*d^3*e^2 + 4
8*(2*C*d*e^4 + B*e^5)*x^4 + 10*(5*C*d^2*e^3 + 12*B*d*e^4 + 6*A*e^5)*x^3 - 3
2*(C*d^3*e^2 - 2*B*d^2*e^3 - 5*A*d*e^4)*x^2 - 15*(3*C*d^4*e + 4*B*d^3*e^2 -
 6*A*d^2*e^3)*x)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [A] time = 0.22, size = 197, normalized size = 0.83

$$\frac{1}{16}\left(3\,Cd^6+4\,Bd^5e+10\,Ad^4e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)+\frac{1}{240}\sqrt{-x^2e^2+d^2}\left(\left(2\left(\left(4\left(5\,Cxe^2+6\left(2\,Cde^9+Be^{10}\right)\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(3*C*d^6 + 4*B*d^5*e + 10*A*d^4*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) + 1/2
40*sqrt(-x^2*e^2 + d^2)*((2*((4*(5*C*x*e^2 + 6*(2*C*d*e^9 + B*e^10)*e^(-8))
*x + 5*(5*C*d^2*e^8 + 12*B*d*e^9 + 6*A*e^10)*e^(-8))*x - 16*(C*d^3*e^7 - 2*
B*d^2*e^8 - 5*A*d*e^9)*e^(-8))*x - 15*(3*C*d^4*e^6 + 4*B*d^3*e^7 - 6*A*d^2*
e^8)*e^(-8))*x - 16*(4*C*d^5*e^5 + 7*B*d^4*e^6 + 10*A*d^3*e^7)*e^(-8))
```

maple [A] time = 0.06, size = 371, normalized size = 1.57

$$\frac{5A\,d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}+\frac{B\,d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{4\sqrt{e^2}\,e}+\frac{3C\,d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}\,e^2}+\frac{5\sqrt{-e^2x^2+d^2}\,A\,d^2x}{8}+\frac{\sqrt{-e^2x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/6*C*x^3*(-e^2*x^2+d^2)^(3/2)-3/8/e^2*C*d^2*x*(-e^2*x^2+d^2)^(3/2)+3/16/e
^2*C*d^4*x*(-e^2*x^2+d^2)^(1/2)+3/16/e^2*C*d^6/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-e^2*x^2+d^2)^(1/2))-1/5*x^2*(-e^2*x^2+d^2)^(3/2)*B-2/5*x^2*(-e^2*x^2
+d^2)^(3/2)/e*d*C-7/15*d^2/e^2*(-e^2*x^2+d^2)^(3/2)*B-4/15*d^3/e^3*(-e^2*x^
2+d^2)^(3/2)*C-1/4*x*(-e^2*x^2+d^2)^(3/2)*A-1/2*x*(-e^2*x^2+d^2)^(3/2)/e*B*
d+5/8*d^2*x*(-e^2*x^2+d^2)^(1/2)*A+1/4*d^3/e*x*(-e^2*x^2+d^2)^(1/2)*B+5/8*d
^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*A+1/4*d^5/e/(e^2)
^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*B-2/3*(-e^2*x^2+d^2)^(3/2
)/e*A*d
```

maxima [A] time = 1.00, size = 338, normalized size = 1.43

$$-\frac{1}{6}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cx^3+\frac{Cd^6\arcsin\left(\frac{ex}{d}\right)}{16\,e^3}+\frac{Ad^4\arcsin\left(\frac{ex}{d}\right)}{2\,e}+\frac{1}{2}\sqrt{-e^2x^2+d^2}\,Ad^2x+\frac{\sqrt{-e^2x^2+d^2}\,Cd^4x}{16\,e^2}-\frac{\left(-e^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/6*(-e^2*x^2 + d^2)^(3/2)*C*x^3 + 1/16*C*d^6*arcsin(e*x/d)/e^3 + 1/2*A*d^
4*arcsin(e*x/d)/e + 1/2*sqrt(-e^2*x^2 + d^2)*A*d^2*x + 1/16*sqrt(-e^2*x^2 +
 d^2)*C*d^4*x/e^2 - 1/8*(-e^2*x^2 + d^2)^(3/2)*C*d^2*x/e^2 + 1/8*(C*d^2 + 2
*B*d*e + A*e^2)*d^4*arcsin(e*x/d)/e^3 - 1/3*(-e^2*x^2 + d^2)^(3/2)*B*d^2/e^
2 - 2/3*(-e^2*x^2 + d^2)^(3/2)*A*d/e + 1/8*sqrt(-e^2*x^2 + d^2)*(C*d^2 + 2*
B*d*e + A*e^2)*d^2*x/e^2 - 1/5*(-e^2*x^2 + d^2)^(3/2)*(2*C*d*e + B*e^2)*x^2
/e^2 - 1/4*(-e^2*x^2 + d^2)^(3/2)*(C*d^2 + 2*B*d*e + A*e^2)*x/e^2 - 2/15*(-
e^2*x^2 + d^2)^(3/2)*(2*C*d*e + B*e^2)*d^2/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d^2 - e^2 x^2}\,(d + e\,x)^2\,\left(C\,x^2 + B\,x + A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2*(A + B*x + C*x^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2*(A + B*x + C*x^2), x)
```

sympy [C] time = 22.93, size = 1231, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
> 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + 2*
A*d*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/
2)/(3*e**2), True)) + A*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d
**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**
2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)
) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x*
*2/d**2)), True)) + B*d**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d
**2 - e**2*x**2)**(3/2)/(3*e**2), True)) + 2*B*d*e*Piecewise((-I*d**4*acosh
(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3
/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)
), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sq
rt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5
/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + B*e**2*Piecewise((-2*d**4*sqrt(d*
*2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x*
*4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) + C*d**2
*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2
*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*
sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*
e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**
2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + 2*C*d*e*
Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 -
 e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqr
t(d**2)/4, True)) + C*e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d*
*5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e
**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(
6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)
/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2
*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*
x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.2 $\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$

Optimal. Leaf size=186

$$\frac{dx\sqrt{d^2-e^2x^2}\left(e(4Ae+Bd)+Cd^2\right)}{8e^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(5e(Ae+Bd)+2Cd^2\right)}{15e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(4Ae+Bd)\right.}{8e^3}$$

```
[Out] -1/15*(2*C*d^2+5*e*(A*e+B*d))*(-e^2*x^2+d^2)^(3/2)/e^3-1/4*(B*e+C*d)*x*(-e^
2*x^2+d^2)^(3/2)/e^2-1/5*C*x^2*(-e^2*x^2+d^2)^(3/2)/e+1/8*d^3*(C*d^2+e*(4*A
*e+B*d))*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*d*(C*d^2+e*(4*A*e+B*d))*x
*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] time = 0.23, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1815, 641, 195, 217, 203}

$$\frac{dx\sqrt{d^2-e^2x^2}\left(e(4Ae+Bd)+Cd^2\right)}{8e^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(5e(Ae+Bd)+2Cd^2\right)}{15e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(4Ae+Bd)\right.}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (d*(C*d^2 + e*(B*d + 4*A*e))*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((2*C*d^2 + 5
*e*(B*d + A*e))*(d^2 - e^2*x^2)^(3/2))/(15*e^3) - ((C*d + B*e)*x*(d^2 - e^2
*x^2)^(3/2))/(4*e^2) - (C*x^2*(d^2 - e^2*x^2)^(3/2))/(5*e) + (d^3*(C*d^2 +
e*(B*d + 4*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
```

```
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx = -\frac{Cx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{\int \sqrt{d^2-e^2x^2}\left(-5Ade^2-e\left(2Cd^2+5e(Bd+\right.\right.}{5e^2}$$
$$= -\frac{(Cd+Be)x\left(d^2-e^2x^2\right)^{3/2}}{4e^2} - \frac{Cx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} + \frac{\int \left(5de^2\left(Cd^2+e\right.\right.}{}$$
$$= -\frac{\left(2Cd^2+5e(Bd+Ae)\right)\left(d^2-e^2x^2\right)^{3/2}}{15e^3} - \frac{(Cd+Be)x\left(d^2-e^2x^2\right)^{3/2}}{4e^2} -$$
$$= \frac{d\left(Cd^2+e(Bd+4Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{\left(2Cd^2+5e(Bd+Ae)\right)\left(d^2-\right.}{15e^3}$$
$$= \frac{d\left(Cd^2+e(Bd+4Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{\left(2Cd^2+5e(Bd+Ae)\right)\left(d^2-\right.}{15e^3}$$
$$= \frac{d\left(Cd^2+e(Bd+4Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{\left(2Cd^2+5e(Bd+Ae)\right)\left(d^2-\right.}{15e^3}$$

Mathematica [A] time = 0.30, size = 190, normalized size = 1.02

$$\frac{\sqrt{d^2-e^2x^2}\left(15\sin^{-1}\left(\frac{ex}{d}\right)\left(d^2e(4Ae+Bd)+Cd^4\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(5e\left(4Ae\left(-2d^2+3dex+2e^2x^2\right)+B\left(-8d^3-3d^2e\right.\right.\right.\right.}{120e^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(C*(-16*d^4 - 15*d^3*e*x - 8*
d^2*e^2*x^2 + 30*d*e^3*x^3 + 24*e^4*x^4) + 5*e*(4*A*e*(-2*d^2 + 3*d*e*x + 2
*e^2*x^2) + B*(-8*d^3 - 3*d^2*e*x + 8*d*e^2*x^2 + 6*e^3*x^3))) + 15*(C*d^4
+ d^2*e*(B*d + 4*A*e))*ArcSin[(e*x)/d]))/(120*e^3*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.87, size = 173, normalized size = 0.93

$$-\frac{30\left(Cd^5+Bd^4e+4\,Ad^3e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(24\,Ce^4x^4-16\,Cd^4-40\,Bd^3e-40\,Ad^2e^2+30\left(Cde^3+Be^4\right.\right.}{120\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(30*(C*d^5 + B*d^4*e + 4*A*d^3*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2
))/(e*x)) - (24*C*e^4*x^4 - 16*C*d^4 - 40*B*d^3*e - 40*A*d^2*e^2 + 30*(C*d*
e^3 + B*e^4)*x^3 - 8*(C*d^2*e^2 - 5*B*d*e^3 - 5*A*e^4)*x^2 - 15*(C*d^3*e +
B*d^2*e^2 - 4*A*d*e^3)*x)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [A] time = 0.21, size = 160, normalized size = 0.86

$$\frac{1}{8}\left(Cd^5+Bd^4e+4\,Ad^3e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)+\frac{1}{120}\sqrt{-x^2e^2+d^2}\left(\left(2\left(3\left(4\,Cxe+5\left(Cde^6+Be^7\right)e^{(-6)}\right)x-4\left(C\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(C*d^5 + B*d^4*e + 4*A*d^3*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) + 1/120*sqr
t(-x^2*e^2 + d^2)*((2*(3*(4*C*x*e + 5*(C*d*e^6 + B*e^7)*e^(-6))*x - 4*(C*d^
2*e^5 - 5*B*d*e^6 - 5*A*e^7)*e^(-6))*x - 15*(C*d^3*e^4 + B*d^2*e^5 - 4*A*d*
e^6)*e^(-6))*x - 8*(2*C*d^4*e^3 + 5*B*d^3*e^4 + 5*A*d^2*e^5)*e^(-6))
```

maple [A] time = 0.02, size = 304, normalized size = 1.63

$$\frac{A\,d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}+\frac{B\,d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e}+\frac{C\,d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e^2}+\frac{\sqrt{-e^2x^2+d^2}\,Adx}{2}+\frac{\sqrt{-e^2x^2+d}}{8e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/5*C*x^2*(-e^2*x^2+d^2)^(3/2)/e-2/15/e^3*C*d^2*(-e^2*x^2+d^2)^(3/2)-1/4*x
*(-e^2*x^2+d^2)^(3/2)/e*B-1/4*x*(-e^2*x^2+d^2)^(3/2)/e^2*C*d+1/8*d^2/e*x*(-
e^2*x^2+d^2)^(1/2)*B+1/8*d^3/e^2*x*(-e^2*x^2+d^2)^(1/2)*C+1/8*d^4/e/(e^2)^(
1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)*B+1/8*d^5/e^2/(e^2)^(1/2)*a
rctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)*C-1/3*(-e^2*x^2+d^2)^(3/2)/e*A-1/
3*(-e^2*x^2+d^2)^(3/2)/e^2*B*d+1/2*d*A*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^3*A/(e^
2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.98, size = 202, normalized size = 1.09

$$\frac{Ad^3\arcsin\left(\frac{ex}{d}\right)}{2\,e}+\frac{1}{2}\sqrt{-e^2x^2+d^2}\,Adx-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cx^2}{5\,e}+\frac{(Cd+Be)d^4\arcsin\left(\frac{ex}{d}\right)}{8\,e^3}+\frac{\sqrt{-e^2x^2+d^2}\,(Cd+Be)d}{8\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*A*d^3*arcsin(e*x/d)/e + 1/2*sqrt(-e^2*x^2 + d^2)*A*d*x - 1/5*(-e^2*x^2
+ d^2)^(3/2)*C*x^2/e + 1/8*(C*d + B*e)*d^4*arcsin(e*x/d)/e^3 + 1/8*sqrt(-e^
2*x^2 + d^2)*(C*d + B*e)*d^2*x/e^2 - 2/15*(-e^2*x^2 + d^2)^(3/2)*C*d^2/e^3
- 1/3*(-e^2*x^2 + d^2)^(3/2)*B*d/e^2 - 1/3*(-e^2*x^2 + d^2)^(3/2)*A/e - 1/4
*(-e^2*x^2 + d^2)^(3/2)*(C*d + B*e)*x/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)\,\left(C\,x^2+B\,x+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)*(d + e*x)*(A + B*x + C*x^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)*(d + e*x)*(A + B*x + C*x^2), x)
```

sympy [C] time = 12.77, size = 670, normalized size = 3.60

$$Ad\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)+Ae\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)+Bd\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2}\\ -\frac{\left(d^2-e^2\right.}{3e}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d*
*2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1
), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + A*e*P
iecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e
**2), True)) + B*d*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e*
*2*x**2)**(3/2)/(3*e**2), True)) + B*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e
**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1
+ e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*
x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*
x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1
 - e**2*x**2/d**2)), True)) + C*d*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3)
+ I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**
2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/
d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/
d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e*
*2*x**2/d**2)), True)) + C*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*
e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*
x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True))
```

3.3 $\int \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$

Optimal. Leaf size=125

$$\frac{1}{8}x\sqrt{d^2-e^2x^2}\left(4A+\frac{Cd^2}{e^2}\right)+\frac{d^2\left(4Ae^2+Cd^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}-\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}$$

```
[Out] -1/3*B*(-e^2*x^2+d^2)^(3/2)/e^2-1/4*C*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/8*d^2*(4
*A*e^2+C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*(4*A+C*d^2/e^2)*x*(-
e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1815, 641, 195, 217, 203}

$$\frac{1}{8}x\sqrt{d^2-e^2x^2}\left(4A+\frac{Cd^2}{e^2}\right)+\frac{d^2\left(4Ae^2+Cd^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}-\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] ((4*A + (C*d^2)/e^2)*x*Sqrt[d^2 - e^2*x^2])/8 - (B*(d^2 - e^2*x^2)^(3/2))/(
3*e^2) - (C*x*(d^2 - e^2*x^2)^(3/2))/(4*e^2) + (d^2*(C*d^2 + 4*A*e^2)*ArcTa
n[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx = -\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2} - \frac{\int\left(-Cd^2-4Ae^2-4Be^2x\right)\sqrt{d^2-e^2x^2}\,dx}{4e^2}$$
$$= -\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2} - \frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2} - \frac{\left(-Cd^2-4Ae^2\right)\int\sqrt{d^2-e^2x^2}\,dx}{4e^2}$$
$$= \frac{\left(Cd^2+4Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2} - \frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2} - \frac{\left(d^2\left(-C\right.\right.}{}$$
$$= \frac{\left(Cd^2+4Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2} - \frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2} - \frac{\left(d^2\left(-C\right.\right.}{}$$
$$= \frac{\left(Cd^2+4Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2} - \frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2} + \frac{d^2\left(Cd\right.}{}$$

Mathematica [A] time = 0.13, size = 121, normalized size = 0.97

$$\frac{\sqrt{d^2-e^2x^2}\left(e\sqrt{1-\frac{e^2x^2}{d^2}}\left(12Ae^2x-8Bd^2+8Be^2x^2-3Cd^2x+6Ce^2x^3\right)+3\left(4Ade^2+Cd^3\right)\sin^{-1}\left(\frac{ex}{d}\right)\right)}{24e^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(e*Sqrt[1 - (e^2*x^2)/d^2]*(-8*B*d^2 - 3*C*d^2*x + 12*
A*e^2*x + 8*B*e^2*x^2 + 6*C*e^2*x^3) + 3*(C*d^3 + 4*A*d*e^2)*ArcSin[(e*x)/d
]))/(24*e^3*Sqrt[1 - (e^2*x^2)/d^2])
```

fricas [A] time = 0.97, size = 108, normalized size = 0.86

$$-\frac{6\left(Cd^4+4\,Ad^2e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(6\,Ce^3x^3+8\,Be^3x^2-8\,Bd^2e-3\left(Cd^2e-4\,Ae^3\right)x\right)\sqrt{-e^2x^2+d^2}}{24\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(6*(C*d^4 + 4*A*d^2*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) -
(6*C*e^3*x^3 + 8*B*e^3*x^2 - 8*B*d^2*e - 3*(C*d^2*e - 4*A*e^3)*x)*sqrt(-e^2
*x^2 + d^2))/e^3
```

giac [A] time = 0.20, size = 85, normalized size = 0.68

$$\frac{1}{8}\left(Cd^4+4\,Ad^2e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{24}\left(8\,Bd^2e^{(-2)}-\left(2\,(3\,Cx+4\,B)x-3\left(Cd^2e^2-4\,Ae^4\right)e^{(-4)}\right)x\right)\sqrt{-x^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(C*d^4 + 4*A*d^2*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/24*(8*B*d^2*e^(-2
) - (2*(3*C*x + 4*B)*x - 3*(C*d^2*e^2 - 4*A*e^4)*e^(-4))*x)*sqrt(-x^2*e^2 +
 d^2)
```

maple [A] time = 0.01, size = 154, normalized size = 1.23

$$\frac{A\,d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}+\frac{C\,d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e^2}+\frac{\sqrt{-e^2x^2+d^2}\,Ax}{2}+\frac{\sqrt{-e^2x^2+d^2}\,C\,d^2x}{8e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cx}{4e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/4*C*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/8*C*d^2/e^2*x*(-e^2*x^2+d^2)^(1/2)+1/8*
C*d^4/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/3*B*(-e^
2*x^2+d^2)^(3/2)/e^2+1/2*A*x*(-e^2*x^2+d^2)^(1/2)+1/2*A*d^2/(e^2)^(1/2)*arc
tan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.96, size = 116, normalized size = 0.93

$$\frac{Cd^4\arcsin\left(\frac{ex}{d}\right)}{8\,e^3}+\frac{Ad^2\arcsin\left(\frac{ex}{d}\right)}{2\,e}+\frac{1}{2}\sqrt{-e^2x^2+d^2}\,Ax+\frac{\sqrt{-e^2x^2+d^2}\,Cd^2x}{8\,e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cx}{4\,e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}B}{3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*C*d^4*arcsin(e*x/d)/e^3 + 1/2*A*d^2*arcsin(e*x/d)/e + 1/2*sqrt(-e^2*x^2
 + d^2)*A*x + 1/8*sqrt(-e^2*x^2 + d^2)*C*d^2*x/e^2 - 1/4*(-e^2*x^2 + d^2)^(
3/2)*C*x/e^2 - 1/3*(-e^2*x^2 + d^2)^(3/2)*B/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2), x)
```

sympy [C] time = 7.11, size = 343, normalized size = 2.74

$$A\left(\begin{cases} -\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases}\right)+B\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for } e^2=0 \\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right)+C\left(\begin{cases} -\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} \ldots \\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} \ldots \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2
)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1),
 (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + B*Piece
wise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2)
, True)) + C*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sq
rt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**
2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asi
n(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*
sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)
)
```

3.4 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{d^2-e^2x^2}\left(Cd^2-e(Bd-2Ae)\right)}{2e^3}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(Cd^2-e(Bd-2Ae)\right)}{2e^3}+\frac{\left(d^2-e^2x^2\right)^{3/2}(Cd-Be)}{2e^3(d+ex)}-\frac{C\left(d^2-e^2x\right.}{3e^3}$$

```
[Out] -1/3*C*(-e^2*x^2+d^2)^(3/2)/e^3+1/2*(-B*e+C*d)*(-e^2*x^2+d^2)^(3/2)/e^3/(e*
x+d)+1/2*d*(C*d^2-e*(-2*A*e+B*d))*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/2*
(C*d^2-e*(-2*A*e+B*d))*(-e^2*x^2+d^2)^(1/2)/e^3
```

Rubi [A] time = 0.18, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1639, 795, 665, 217, 203}

$$\frac{\sqrt{d^2-e^2x^2}\left(Cd^2-e(Bd-2Ae)\right)}{2e^3}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(Cd^2-e(Bd-2Ae)\right)}{2e^3}+\frac{\left(d^2-e^2x^2\right)^{3/2}(Cd-Be)}{2e^3(d+ex)}-\frac{C\left(d^2-e^2x\right.}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] ((C*d^2 - e*(B*d - 2*A*e))*Sqrt[d^2 - e^2*x^2])/(2*e^3) - (C*(d^2 - e^2*x^2
)^(3/2))/(3*e^3) + ((C*d - B*e)*(d^2 - e^2*x^2)^(3/2))/(2*e^3*(d + e*x)) +
(d*(C*d^2 - e*(B*d - 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
```

```
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3} - \frac{\int \frac{\left(-3Ae^4+3e^3(Cd-Be)x\right)\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{3e^4}$$

$$= -\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3} + \frac{(Cd-Be)\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)} + \frac{\left(Cd^2-e(Bd-2Ae)\right)\int \frac{\sqrt{\cdots}}{\cdots}}{2e^2}$$

$$= \frac{\left(Cd^2-e(Bd-2Ae)\right)\sqrt{d^2-e^2x^2}}{2e^3} - \frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3} + \frac{(Cd-Be)\left(d^2-e^2\cdots\right.}{2e^3(d+ex)}$$

$$= \frac{\left(Cd^2-e(Bd-2Ae)\right)\sqrt{d^2-e^2x^2}}{2e^3} - \frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3} + \frac{(Cd-Be)\left(d^2-e^2\cdots\right.}{2e^3(d+ex)}$$

$$= \frac{\left(Cd^2-e(Bd-2Ae)\right)\sqrt{d^2-e^2x^2}}{2e^3} - \frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3} + \frac{(Cd-Be)\left(d^2-e^2\cdots\right.}{2e^3(d+ex)}$$

Mathematica [A] time = 0.22, size = 103, normalized size = 0.70

$$\frac{\sqrt{d^2-e^2x^2}\left(3e(2Ae-2Bd+Bex)+C\left(4d^2-3dex+2e^2x^2\right)\right)+3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(2Ae-Bd)+Cd^2\right)}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(3*e*(-2*B*d + 2*A*e + B*e*x) + C*(4*d^2 - 3*d*e*x + 2
*e^2*x^2)) + 3*d*(C*d^2 + e*(-(B*d) + 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x
^2]])/(6*e^3)
```

fricas [A] time = 0.89, size = 112, normalized size = 0.76

$$-\frac{6\left(Cd^3-Bd^2e+2\,Ade^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(2\,Ce^2x^2+4\,Cd^2-6\,Bde+6\,Ae^2-3\left(Cde-Be^2\right)x\right)\sqrt{-e^2\cdots}}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(6*(C*d^3 - B*d^2*e + 2*A*d*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e
*x)) - (2*C*e^2*x^2 + 4*C*d^2 - 6*B*d*e + 6*A*e^2 - 3*(C*d*e - B*e^2)*x)*sq
rt(-e^2*x^2 + d^2))/e^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
4*A*d*exp(2)^3-4*A*d*exp(1)^4*exp(2)+4*B*d^2*exp(1)^3*exp(2)-4*B*d^2*exp(1)
*exp(2)^2)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))
/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^4/exp(1)-1/4*(-2
*C*d^3-4*A*d*exp(2)+2*B*d^2*exp(1))*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)/
exp(2)+2*((16*exp(1)^4*C*1/96/exp(1)^5*x-(-24*exp(1)^4*B+24*exp(1)^3*C*d)*1
/96/exp(1)^5)*x-(-48*exp(1)^4*A+48*exp(1)^3*d*B-32*exp(1)^2*C*d^2)*1/96/exp
(1)^5)*sqrt(d^2-x^2*exp(2))
```

maple [B] time = 0.02, size = 384, normalized size = 2.59

$$\frac{Ad\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}}-\frac{B\,d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,e}+\frac{B\,d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e}+\frac{C\,d^3\arctan\left(\frac{}{\sqrt{2(x-}}\right)}{\sqrt{e^2}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
```

```
[Out] -1/3*C*(-e^2*x^2+d^2)^(3/2)/e^3+1/2/e*B*x*(-e^2*x^2+d^2)^(1/2)+1/2/e*B*d^2/
(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/2/e^2*C*d*x*(-e^2*
x^2+d^2)^(1/2)-1/2/e^2*C*d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^
(1/2)*x)+1/e*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)*A-1/e^2*(-(x+d/e)^2*e^2+2
*d*e*(x+d/e))^(1/2)*B*d+1/e^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)*C*d^2+d/
(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))*A-1/
e*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
)*B+1/e^2*d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(1/2))*C
```

maxima [A] time = 1.09, size = 171, normalized size = 1.16

$$\frac{Cd^3\arcsin\left(\frac{ex}{d}\right)}{2\,e^3}-\frac{Bd^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^2}+\frac{Ad\arcsin\left(\frac{ex}{d}\right)}{e}-\frac{\sqrt{-e^2x^2+d^2}\,Cdx}{2\,e^2}+\frac{\sqrt{-e^2x^2+d^2}\,Bx}{2\,e}+\frac{\sqrt{-e^2x^2+d^2}\,Cd^2}{e^3}-\frac{\sqrt{-e}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*C*d^3*arcsin(e*x/d)/e^3 - 1/2*B*d^2*arcsin(e*x/d)/e^2 + A*d*arcsin(e*x/
d)/e - 1/2*sqrt(-e^2*x^2 + d^2)*C*d*x/e^2 + 1/2*sqrt(-e^2*x^2 + d^2)*B*x/e
+ sqrt(-e^2*x^2 + d^2)*C*d^2/e^3 - sqrt(-e^2*x^2 + d^2)*B*d/e^2 + sqrt(-e^2
*x^2 + d^2)*A/e - 1/3*(-e^2*x^2 + d^2)^(3/2)*C/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x),x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(A+Bx+Cx^2\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x), x)
```

3.5 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=170

$$-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{de^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\,(5Cd^2-2e(2Bd-Ae))}{2de^3}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(5Cd^2-2e(2Bd-Ae))}{2e^3}$$

```
[Out] -(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^2-1/2*C*(-e^2*x^2+d
^2)^(3/2)/e^3/(e*x+d)-1/2*(5*C*d^2-2*e*(-A*e+2*B*d))*arctan(e*x/(-e^2*x^2+d
^2)^(1/2))/e^3-1/2*(5*C*d^2-2*e*(-A*e+2*B*d))*(-e^2*x^2+d^2)^(1/2)/d/e^3
```

Rubi [A] time = 0.20, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1639, 793, 665, 217, 203}

$$-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{de^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\,(5Cd^2-2e(2Bd-Ae))}{2de^3}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(5Cd^2-2e(2Bd-Ae))}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^2,x]
```

```
[Out] -((5*C*d^2 - 2*e*(2*B*d - A*e))*Sqrt[d^2 - e^2*x^2])/(2*d*e^3) - ((C*d^2 -
B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(d*e^3*(d + e*x)^2) - (C*(d^2 - e^2*x
^2)^(3/2))/(2*e^3*(d + e*x)) - ((5*C*d^2 - 2*e*(2*B*d - A*e))*ArcTan[(e*x)/
Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx = -\frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)} - \frac{\int \frac{\left(e^2\left(Cd^2-2Ae^2\right)+e^3(3Cd-2Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{2e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{de^3(d+ex)^2} - \frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)} - \frac{\left(-3e^5\left(Cd^2-2A\right.\right.}{}$$

$$= -\frac{\left(5Cd^2-2e(2Bd-Ae)\right)\sqrt{d^2-e^2x^2}}{2de^3} - \frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{de^3(d+ex)^2}$$

$$= -\frac{\left(5Cd^2-2e(2Bd-Ae)\right)\sqrt{d^2-e^2x^2}}{2de^3} - \frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{de^3(d+ex)^2}$$

$$= -\frac{\left(5Cd^2-2e(2Bd-Ae)\right)\sqrt{d^2-e^2x^2}}{2de^3} - \frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{de^3(d+ex)^2}$$

Mathematica [A] time = 0.23, size = 109, normalized size = 0.64

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(2e(-2Ae+3Bd+Bex)+C\left(-8d^2-3dex+e^2x^2\right)\right)}{d+ex} - \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(2e(Ae-2Bd)+5Cd^2\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^2,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(2*e*(3*B*d - 2*A*e + B*e*x) + C*(-8*d^2 - 3*d*e*x +
e^2*x^2)))/(d + e*x) - (5*C*d^2 + 2*e*(-2*B*d + A*e))*ArcTan[(e*x)/Sqrt[d^2
 - e^2*x^2]])/(2*e^3)
```

fricas [A] time = 0.68, size = 190, normalized size = 1.12

$$-\frac{8\,Cd^3-6\,Bd^2e+4\,Ade^2+2\left(4\,Cd^2e-3\,Bde^2+2\,Ae^3\right)x-2\left(5\,Cd^3-4\,Bd^2e+2\,Ade^2+\left(5\,Cd^2e-4\,Bde^2+2\right.\right.}{2\left(e^4x+de\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(8*C*d^3 - 6*B*d^2*e + 4*A*d*e^2 + 2*(4*C*d^2*e - 3*B*d*e^2 + 2*A*e^3)
*x - 2*(5*C*d^3 - 4*B*d^2*e + 2*A*d*e^2 + (5*C*d^2*e - 4*B*d*e^2 + 2*A*e^3)
*x)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (C*e^2*x^2 - 8*C*d^2 + 6*B*
d*e - 4*A*e^2 - (3*C*d*e - 2*B*e^2)*x)*sqrt(-e^2*x^2 + d^2))/(e^4*x + d*e^3
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.03, size = 439, normalized size = 2.58

$$-\frac{A\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}}+\frac{2Bd\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,e}-\frac{3C\,d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,e^2}+\frac{C\,d^2\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x)
```

```
[Out] 1/2*C*x*(-e^2*x^2+d^2)^(1/2)/e^2+1/2*C/e^2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/
2)/(-e^2*x^2+d^2)^(1/2)*x)-1/e^3/d/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(3/2)*A+1/e^4/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*B-1/e^5*d/(x+d/
e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*C-1/e/d*(2*(x+d/e)*d*e-(x+d/e)^2*e
^2)^(1/2)*A+2/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*B-3/e^3*d*(2*(x+d/e)*
d*e-(x+d/e)^2*e^2)^(1/2)*C-1/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-
(x+d/e)^2*e^2)^(1/2)*x)*A+2/e/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e
-(x+d/e)^2*e^2)^(1/2)*x)*B*d-3/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e
)*d*e-(x+d/e)^2*e^2)^(1/2)*x)*C*d^2
```

maxima [A] time = 1.01, size = 197, normalized size = 1.16

$$-\frac{2\sqrt{-e^2x^2+d^2}\,Cd^2}{e^4x+de^3}+\frac{2\sqrt{-e^2x^2+d^2}\,Bd}{e^3x+de^2}-\frac{5\,Cd^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^3}+\frac{2\,Bd\arcsin\left(\frac{ex}{d}\right)}{e^2}-\frac{A\arcsin\left(\frac{ex}{d}\right)}{e}-\frac{2\sqrt{-e^2x^2+d^2}\,A}{e^2x+de}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="maxima
")
```

```
[Out] -2*sqrt(-e^2*x^2 + d^2)*C*d^2/(e^4*x + d*e^3) + 2*sqrt(-e^2*x^2 + d^2)*B*d/
(e^3*x + d*e^2) - 5/2*C*d^2*arcsin(e*x/d)/e^3 + 2*B*d*arcsin(e*x/d)/e^2 - A
*arcsin(e*x/d)/e - 2*sqrt(-e^2*x^2 + d^2)*A/(e^2*x + d*e) + 1/2*sqrt(-e^2*x
^2 + d^2)*C*x/e^2 - 2*sqrt(-e^2*x^2 + d^2)*C*d/e^3 + sqrt(-e^2*x^2 + d^2)*B
/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(A+Bx+Cx^2\right)}{\left(d+ex\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**2, x)
```

3.6 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx$

Optimal. Leaf size=149

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{3de^3(d+ex)^3}+\frac{2\sqrt{d^2-e^2x^2}\,(3Cd-Be)}{e^3(d+ex)}+\frac{(3Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2}$$

```
[Out] -1/3*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^3-C*(-e^2*x^2+d
^2)^(3/2)/e^3/(e*x+d)^2+(-B*e+3*C*d)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+2
*(-B*e+3*C*d)*(-e^2*x^2+d^2)^(1/2)/e^3/(e*x+d)
```

Rubi [A] time = 0.19, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1639, 793, 663, 217, 203}

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{3de^3(d+ex)^3}+\frac{2\sqrt{d^2-e^2x^2}\,(3Cd-Be)}{e^3(d+ex)}+\frac{(3Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^3,x]
```

```
[Out] (2*(3*C*d - B*e)*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - ((C*d^2 - B*d*e + A
*e^2)*(d^2 - e^2*x^2)^(3/2))/(3*d*e^3*(d + e*x)^3) - (C*(d^2 - e^2*x^2)^(3/
2))/(e^3*(d + e*x)^2) + ((3*C*d - B*e)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
^3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx = -\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2} - \frac{\int \frac{\left(e^2\left(2Cd^2-Ae^2\right)+e^3(3Cd-Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{3de^3(d+ex)^3} - \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2} - \frac{(3Cd-Be)\int \frac{\sqrt{\cdots}}{(\cdots}}{e^2}$$

$$= \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{3de^3(d+ex)^3} - \frac{C\left(d^2-e\cdots\right.}{e^3(d+\cdots}$$

$$= \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{3de^3(d+ex)^3} - \frac{C\left(d^2-e\cdots\right.}{e^3(d+\cdots}$$

$$= \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{3de^3(d+ex)^3} - \frac{C\left(d^2-e\cdots\right.}{e^3(d+\cdots}$$

Mathematica [A] time = 0.22, size = 114, normalized size = 0.77

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(e(Ae(ex-d)-Bd(5d+7ex))+Cd\left(14d^2+19dex+3e^2x^2\right)\right)}{d(d+ex)^2}+3(3Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^3,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(C*d*(14*d^2 + 19*d*e*x + 3*e^2*x^2) + e*(A*e*(-d + e
*x) - B*d*(5*d + 7*e*x))))/(d*(d + e*x)^2) + 3*(3*C*d - B*e)*ArcTan[(e*x)/S
qrt[d^2 - e^2*x^2]])/(3*e^3)
```

fricas [A] time = 0.85, size = 258, normalized size = 1.73

$$14\,Cd^4-5\,Bd^3e-Ad^2e^2+\left(14\,Cd^2e^2-5\,Bde^3-Ae^4\right)x^2+2\left(14\,Cd^3e-5\,Bd^2e^2-Ade^3\right)x-6\left(3\,Cd^4-Bd^3e+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="fricas
")
```

```
[Out] 1/3*(14*C*d^4 - 5*B*d^3*e - A*d^2*e^2 + (14*C*d^2*e^2 - 5*B*d*e^3 - A*e^4)*
x^2 + 2*(14*C*d^3*e - 5*B*d^2*e^2 - A*d*e^3)*x - 6*(3*C*d^4 - B*d^3*e + (3*
C*d^2*e^2 - B*d*e^3)*x^2 + 2*(3*C*d^3*e - B*d^2*e^2)*x)*arctan(-(d - sqrt(-
e^2*x^2 + d^2))/(e*x)) + (3*C*d*e^2*x^2 + 14*C*d^3 - 5*B*d^2*e - A*d*e^2 +
(19*C*d^2*e - 7*B*d*e^2 + A*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d*e^5*x^2 + 2*d^
2*e^4*x + d^3*e^3)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-8*A
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp
(2)^2-2*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp
(1)^10*exp(2)-2*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^3*exp(1)^8*exp(2)^2-4*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^3*exp(1)^6*exp(2)^3-A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^3+A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^4-4*A*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^4-A*exp(1)^6*exp(2)^3-
B*d*exp(1)^5*exp(2)^3-4*A*exp(1)^4*exp(2)^4+4*B*d*exp(1)^3*exp(2)^4-8*C*d^2
*exp(2)^5+2*B*d*exp(1)*exp(2)^5+2*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^2+8*B*d*(-1/2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^7*exp(2)^2+4*B*d*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^5*exp(2)^3-8*C*d^2*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(
2)^2-4*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*
exp(1)^4*exp(2)^3+3*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^2*exp(1)^5*exp(2)^3+B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^3*exp(1)^3*exp(2)^4-5*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^3-3*C*d^2*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(2)^5+4*B*d*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^3*exp(2)^4+3*C*d^2*e
xp(1)^4*exp(2)^3-8*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^2*exp(2)^5+1/2*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(
1)^4*exp(2)^4/x/exp(2)+6*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(
1)^6*exp(2)^3/x/exp(2)+A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)
^8*exp(2)^2/x/exp(2)-2*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^2*exp(1)^9*exp(2)+2*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^2*exp(1)*exp(2)^5+6*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)+13/2*C*d^2*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^5/x/exp(2)-7/2*B*d*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))*exp(1)^3*exp(2)^4/x/exp(2)-6*B*d*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))*exp(1)^5*exp(2)^3/x/exp(2)+2*B*d*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*exp(2)^2/x/exp(2)+6*C*d^2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^3/x/exp(2)-5*C*d^2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^2/x/exp(2))/((-1/2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^2/(-d*exp(1)^9+d*exp(1)^5*exp(2)^2-d
*exp(1)^7*exp(2)+d*exp(1)*exp(2)^4)+1/2*(4*A*exp(1)^8*exp(2)^2-12*B*d*exp(1
)^7*exp(2)^2+20*C*d^2*exp(1)^6*exp(2)^2+2*A*exp(1)^6*exp(2)^3-6*B*d*exp(1)^
5*exp(2)^3+18*C*d^2*exp(1)^4*exp(2)^3+4*A*exp(1)^4*exp(2)^4+4*B*d*exp(1)^3*
exp(2)^4-24*C*d^2*exp(2)^5+4*B*d*exp(1)*exp(2)^5-4*C*d^2*exp(1)^8*exp(2))*a
tan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1
)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(d*exp(1)^11-d*exp(1)^7*exp(2)^2-d*
exp(1)^5*exp(2)^3+d*exp(1)^9*exp(2))-1/4*(4*B*exp(1)-12*C*d)*sign(d)*asin(x
*exp(2)/d/exp(1))/exp(1)/exp(2)+4*exp(1)^2*C*1/4/exp(1)^5*sqrt(d^2-x^2*exp(
2))
```

maple [B] time = 0.02, size = 318, normalized size = 2.13

$$-\frac{B \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}\, e}+\frac{3Cd \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}\, e^2}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}\, B}{d\, e^2}+\frac{3\sqrt{2\left(x-\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x)
```

```
[Out] -1/e^4/d/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*B+2/e^5/(x+d/e)^2*(2
*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*C-1/e^2/d*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(
1/2)*B+3/e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*C-1/e/(e^2)^(1/2)*arctan((
e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)*B+3/e^2/(e^2)^(1/2)*arcta
n((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)*C*d-1/3*(A*e^2-B*d*e+C
*d^2)/e^6/d/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="maxima
")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)}{{\left(d+e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(A+Bx+Cx^2\right)}{\left(d+ex\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**3,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**3, x)
```

3.7 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx$

Optimal. Leaf size=196

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{15d^2e^3(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{5de^3(d+ex)^4}+\frac{\left(d^2-e^2x^2\right)^{3/2}(2Cd-Be)}{3de^3(d+ex)^3}-\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

```
[Out] -1/5*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^4+1/3*(-B*e+2*C
*d)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^3-1/15*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2
+d^2)^(3/2)/d^2/e^3/(e*x+d)^3-C*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-2*C*(-
e^2*x^2+d^2)^(1/2)/e^3/(e*x+d)
```

Rubi [A] time = 0.18, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1637, 659, 651, 663, 217, 203}

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{15d^2e^3(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{5de^3(d+ex)^4}+\frac{\left(d^2-e^2x^2\right)^{3/2}(2Cd-Be)}{3de^3(d+ex)^3}-\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (-2*C*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - ((C*d^2 - B*d*e + A*e^2)*(d^2
- e^2*x^2)^(3/2))/(5*d*e^3*(d + e*x)^4) + ((2*C*d - B*e)*(d^2 - e^2*x^2)^(3
/2))/(3*d*e^3*(d + e*x)^3) - ((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2)
)/(15*d^2*e^3*(d + e*x)^3) - (C*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
```

```
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \left(\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{e^2(d+ex)^4}+\frac{(-2Cd+Be)\sqrt{d^2-e^2x^2}}{e^2(d+ex)^3}+\frac{C\sqrt{d^2\cdots}}{e^2(d+\cdots}\right.$$

$$=\frac{C\int\frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{e^2}-\frac{(2Cd-Be)\int\frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^2}+\frac{\left(Cd^2-Bde+Ae^2\right)\int\frac{\sqrt{d^2\cdots}}{(d+\cdots}}{e^2}$$

$$=-\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{5de^3(d+ex)^4}+\frac{(2Cd-Be)\left(d^2-\cdots\right.}{3de^3(d+ex\cdots}$$

$$=-\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{5de^3(d+ex)^4}+\frac{(2Cd-Be)\left(d^2-\cdots\right.}{3de^3(d+ex\cdots}$$

$$=-\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{5de^3(d+ex)^4}+\frac{(2Cd-Be)\left(d^2-\cdots\right.}{3de^3(d+ex\cdots}$$

Mathematica [A] time = 0.30, size = 112, normalized size = 0.57

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(e(d-ex)(Ae(4d+ex)+Bd(d+4ex))+3Cd^2\left(8d^2+19dex+13e^2x^2\right)\right)}{d^2(d+ex)^3}+15C\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{15e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -1/15*((Sqrt[d^2 - e^2*x^2]*(3*C*d^2*(8*d^2 + 19*d*e*x + 13*e^2*x^2) + e*(d
 - e*x)*(A*e*(4*d + e*x) + B*d*(d + 4*e*x))))/(d^2*(d + e*x)^3) + 15*C*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

fricas [A] time = 0.83, size = 304, normalized size = 1.55

$$-\frac{24\,Cd^5+Bd^4e+4\,Ad^3e^2+\left(24\,Cd^2e^3+Bde^4+4\,Ae^5\right)x^3+3\left(24\,Cd^3e^2+Bd^2e^3+4\,Ade^4\right)x^2+3\left(24\,Cd^4e+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas
")
```

```
[Out] -1/15*(24*C*d^5 + B*d^4*e + 4*A*d^3*e^2 + (24*C*d^2*e^3 + B*d*e^4 + 4*A*e^5
)*x^3 + 3*(24*C*d^3*e^2 + B*d^2*e^3 + 4*A*d*e^4)*x^2 + 3*(24*C*d^4*e + B*d^
3*e^2 + 4*A*d^2*e^3)*x - 30*(C*d^2*e^3*x^3 + 3*C*d^3*e^2*x^2 + 3*C*d^4*e*x
```

```
+ C*d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (24*C*d^4 + B*d^3*e +
4*A*d^2*e^2 + (39*C*d^2*e^2 - 4*B*d*e^3 - A*e^4)*x^2 + 3*(19*C*d^3*e + B*d^
2*e^2 - A*d*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^2*e^6*x^3 + 3*d^3*e^5*x^2 + 3*
d^4*e^4*x + d^5*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (8*A*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^16*exp
(2)+12*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(
1)^14*exp(2)^2+6*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^5*exp(1)^12*exp(2)^3+3/2*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*
exp(1)^4*exp(2)^7/x/exp(2)+42*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
*exp(1)^6*exp(2)^6/x/exp(2)+9*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
*exp(1)^8*exp(2)^5/x/exp(2)+3*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
*exp(1)^10*exp(2)^4/x/exp(2)-3*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)*exp(1)^12*exp(2)^3/x/exp(2)+6*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^4*exp(1)^13*exp(2)^2+12*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*exp(2)^3+6*A*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^10*exp(2)^4+48*B*d*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^13*exp(2)^2
+36*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1
)^11*exp(2)^3+6*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^5*exp(1)^9*exp(2)^4-96*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^3*exp(1)^12*exp(2)^2-84*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3-18*C*d^2*(-1/2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4+12*A*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^14*exp(
2)^2-8*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(
1)^12*exp(2)^3-24*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^4*exp(1)^10*exp(2)^4-12*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^5*exp(1)^8*exp(2)^5+6*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2
*exp(2))*exp(1))/x/exp(2))^2*exp(1)^13*exp(2)^2+14*B*d*(-1/2*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^11*exp(2)^3+3*B*d*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^9*exp(2)^4+3*B
*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^7*e
xp(2)^5+24*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^2*exp(1)^12*exp(2)^2-32*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3-24*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^8*exp(2)^4-6*C*d^2*(-1/2*(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5-12*A*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^3
-72*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^
10*exp(2)^4-84*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^4*exp(1)^8*exp(2)^5-24*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^5*exp(1)^6*exp(2)^6+108*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^2*exp(1)^11*exp(2)^3+96*B*d*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^9*exp(2)^4+48*B*d*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^7*exp(2)^5+12*B*
d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^5*ex
p(2)^6-204*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^2*exp(1)^10*exp(2)^3-120*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4-12*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^5-30*A*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^4-30*A*(-1/2*(
```

```
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^5-3*
A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*ex
p(2)^6+3*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*ex
p(1)^4*exp(2)^7+24*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^2*exp(1)^9*exp(2)^4+12*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^3*exp(1)^7*exp(2)^5+6*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^5*exp(2)^6-102*C*d^2*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)^4-42*C*d^2*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(
2)^5+3*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*
exp(1)^4*exp(2)^6-132*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^2*exp(1)^8*exp(2)^5-108*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^6-18*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^7+3*C*d^2*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8+60*B*d*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^7*exp(2)^5+36*B*d*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^5*exp(2)
^6+6*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(
1)^3*exp(2)^7+12*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^2*exp(1)^6*exp(2)^5+36*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6+2*A*exp(1)^10*exp(2)^4-12*A*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^6+
B*d*exp(1)^9*exp(2)^4+2*C*d^2*exp(1)^8*exp(2)^4+12*C*d^2*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^8+36*C*d^2*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6-36*A*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^
7+12*B*d*exp(1)^7*exp(2)^5-24*C*d^2*exp(1)^6*exp(2)^5+36*C*d^2*(-1/2*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(2)^8+12*B*d*(-1/2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^3*exp(2)^7-5*A*
exp(1)^6*exp(2)^6+2*B*d*exp(1)^5*exp(2)^6-11*C*d^2*exp(1)^4*exp(2)^6+24*C*d
^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^8-1
8*A*exp(1)^4*exp(2)^7+6*B*d*exp(1)^3*exp(2)^7+12*C*d^2*exp(2)^8+4*B*d*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^15*exp(2)+8
*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)
^14*exp(2)-33/2*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/
x/exp(2)-12*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^5*exp(2)
^6/x/exp(2)-9/2*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*ex
p(2)^5/x/exp(2)-33*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^9
*exp(2)^4/x/exp(2)-3*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)
^11*exp(2)^3/x/exp(2)-18*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*
exp(1)^4*exp(2)^6/x/exp(2)+30*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))*exp(1)^6*exp(2)^5/x/exp(2)+63*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)-6*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x+exp(2))^3/(3*d^2*exp(1)^13-6*d^2*exp(1)^9*exp(2)^2-6*d^2*exp(1)^7*
exp(2)^3+3*d^2*exp(1)^5*exp(2)^4+3*d^2*exp(1)^11*exp(2)+3*d^2*exp(1)*exp(2)
^6)+1/2*(-4*B*d*exp(1)^11*exp(2)^2+16*C*d^2*exp(1)^10*exp(2)^2-2*B*d*exp(1)
^9*exp(2)^3+8*C*d^2*exp(1)^8*exp(2)^3+8*A*exp(1)^8*exp(2)^4-8*B*d*exp(1)^7*
exp(2)^4-8*C*d^2*exp(1)^6*exp(2)^4+2*A*exp(1)^6*exp(2)^5-10*C*d^2*exp(1)^4*
exp(2)^5+4*A*exp(1)^4*exp(2)^6+8*C*d^2*exp(2)^7)*atan((-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(
1)^4+exp(2)^2)/(-d^2*exp(1)^15+2*d^2*exp(1)^11*exp(2)^2+2*d^2*exp(1)^9*exp(
2)^3-d^2*exp(1)^7*exp(2)^4-d^2*exp(1)^5*exp(2)^5-d^2*exp(1)^13*exp(2))-C*si
gn(d)*asin(x*exp(2)/d/exp(1))/exp(1)/exp(2)
```

maple [B] time = 0.02, size = 453, normalized size = 2.31

$$-\frac{C\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}\,e^2}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}\,C}{d\,e^3}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}} A}{5\left(x+\frac{d}{e}\right)^4 d\,e^5}-\frac{\left(2\left(x+\frac{d}{e}\right)de\right.}{15\left(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -C/e^5/d/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-C/e^3/d*(2*(x+d/e)*d
*e-(x+d/e)^2*e^2)^(1/2)-C/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e
-(x+d/e)^2*e^2)^(1/2)*x)-1/5/e^5/d/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(3/2)*A+1/5/e^6/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*B-1/5/e^7*d/(
x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*C-1/15/e^4/d^2/(x+d/e)^3*(2*(x
+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*A+1/15/e^5/d/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e
)^2*e^2)^(3/2)*B-1/15/e^6/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*C-1
/3*(B*e-2*C*d)/e^6/d/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^2x^2+d^2}\left(Cx^2+Bx+A\right)}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)*(C*x^2 + B*x + A)/(e*x + d)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)}{(d+e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^4,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}\,\left(A+Bx+Cx^2\right)}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**4, x)
```

3.8 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx$

Optimal. Leaf size=180

$$-\frac{(d^2-e^2x^2)^{3/2}(e(2Ae+5Bd)+23Cd^2)}{35d^2e^3(d+ex)^4}-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{7de^3(d+ex)^5}-\frac{(d^2-e^2x^2)^{3/2}(e(2Ae+5Bd)+2$$

```
[Out] -1/7*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^5+C*(-e^2*x^2+d
^2)^(3/2)/e^3/(e*x+d)^4-1/35*(23*C*d^2+e*(2*A*e+5*B*d))*(-e^2*x^2+d^2)^(3/2
)/d^2/e^3/(e*x+d)^4-1/105*(23*C*d^2+e*(2*A*e+5*B*d))*(-e^2*x^2+d^2)^(3/2)/d
^3/e^3/(e*x+d)^3
```

Rubi [A] time = 0.21, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1639, 793, 659, 651}

$$-\frac{(d^2-e^2x^2)^{3/2}(e(2Ae+5Bd)+23Cd^2)}{105d^3e^3(d+ex)^3}-\frac{(d^2-e^2x^2)^{3/2}(e(2Ae+5Bd)+23Cd^2)}{35d^2e^3(d+ex)^4}-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+}{7de^3(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^5,x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(7*d*e^3*(d + e*x)^5) + (C
*(d^2 - e^2*x^2)^(3/2))/(e^3*(d + e*x)^4) - ((23*C*d^2 + e*(5*B*d + 2*A*e))
*(d^2 - e^2*x^2)^(3/2))/(35*d^2*e^3*(d + e*x)^4) - ((23*C*d^2 + e*(5*B*d +
2*A*e))*(d^2 - e^2*x^2)^(3/2))/(105*d^3*e^3*(d + e*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
```

```
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx = \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4} + \frac{\int \frac{\left(e^2\left(4Cd^2+Ae^2\right)+e^3(3Cd+Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx}{e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{7de^3(d+ex)^5} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4} + \frac{\left(23Cd^2+e(5Bd+\ldots\right.}{7\ldots}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{7de^3(d+ex)^5} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4} - \frac{\left(23Cd^2+e(5Bd+\ldots\right.}{35d^2e^3\ldots}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{7de^3(d+ex)^5} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4} - \frac{\left(23Cd^2+e(5Bd+\ldots\right.}{35d^2e^3\ldots}$$

Mathematica [A] time = 0.20, size = 109, normalized size = 0.61

$$-\frac{(d-ex)\sqrt{d^2-e^2x^2}\left(e\left(Ae\left(23d^2+10dex+2e^2x^2\right)+5Bd\left(d^2+5dex+e^2x^2\right)\right)+Cd^2\left(2d^2+10dex+23e^2x^2\right)\right)}{105d^3e^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^5,x]
```

```
[Out] -1/105*((d - e*x)*Sqrt[d^2 - e^2*x^2]*(C*d^2*(2*d^2 + 10*d*e*x + 23*e^2*x^2
) + e*(5*B*d*(d^2 + 5*d*e*x + e^2*x^2) + A*e*(23*d^2 + 10*d*e*x + 2*e^2*x^2
))))/(d^3*e^3*(d + e*x)^4)
```

fricas [A] time = 0.92, size = 320, normalized size = 1.78

$$-\frac{2\,Cd^6+5\,Bd^5e+23\,Ad^4e^2+\left(2\,Cd^2e^4+5\,Bde^5+23\,Ae^6\right)x^4+4\left(2\,Cd^3e^3+5\,Bd^2e^4+23\,Ade^5\right)x^3+6\left(2\,Cd^4e^2\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="fricas
")
```

```
[Out] -1/105*(2*C*d^6 + 5*B*d^5*e + 23*A*d^4*e^2 + (2*C*d^2*e^4 + 5*B*d*e^5 + 23*
A*e^6)*x^4 + 4*(2*C*d^3*e^3 + 5*B*d^2*e^4 + 23*A*d*e^5)*x^3 + 6*(2*C*d^4*e^
2 + 5*B*d^3*e^3 + 23*A*d^2*e^4)*x^2 + 4*(2*C*d^5*e + 5*B*d^4*e^2 + 23*A*d^3
*e^3)*x + (2*C*d^5 + 5*B*d^4*e + 23*A*d^3*e^2 - (23*C*d^2*e^3 + 5*B*d*e^4 +
 2*A*e^5)*x^3 + (13*C*d^3*e^2 - 20*B*d^2*e^3 - 8*A*d*e^4)*x^2 + (8*C*d^4*e
+ 20*B*d^3*e^2 - 13*A*d^2*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^7*x^4 + 4*d^
4*e^6*x^3 + 6*d^5*e^5*x^2 + 4*d^6*e^4*x + d^7*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, choosing root of [1,0,%%%{2,[2,0]%%%},0,%%%{1,[4,0]%%%}+%%%
{-4,[2,1]%%%}+%%%{4,[0,2]%%%}] at parameters values [86,-97]Limit: Max orde
r reached or unable to make series expansion Error: Bad Argument Value
```

maple [A] time = 0.01, size = 116, normalized size = 0.64

$$-\frac{(-ex+d)\left(2A\,e^4x^2+5Bd\,e^3x^2+23C\,d^2e^2x^2+10Ad\,e^3x+25B\,d^2e^2x+10C\,d^3ex+23A\,d^2e^2+5B\,d^3e+2C\,d\right.}{105\,(ex+d)^4\,d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x)
```

```
[Out] -1/105*(-e*x+d)*(2*A*e^4*x^2+5*B*d*e^3*x^2+23*C*d^2*e^2*x^2+10*A*d*e^3*x+25
*B*d^2*e^2*x+10*C*d^3*e*x+23*A*d^2*e^2+5*B*d^3*e+2*C*d^4)*(-e^2*x^2+d^2)^(1
/2)/(e*x+d)^4/d^3/e^3
```

maxima [B] time = 0.54, size = 945, normalized size = 5.25

$$-\frac{2\sqrt{-e^2x^2+d^2}\,Cd^2}{7\left(e^7x^4+4\,de^6x^3+6\,d^2e^5x^2+4\,d^3e^4x+d^4e^3\right)}+\frac{\sqrt{-e^2x^2+d^2}\,Cd^2}{35\left(de^6x^3+3\,d^2e^5x^2+3\,d^3e^4x+d^4e^3\right)}+\frac{2\sqrt{-e^2x^2+d^2}}{105\left(d^2e^5x^2+2\,d^3e^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="maxima
")
```

```
[Out] -2/7*sqrt(-e^2*x^2 + d^2)*C*d^2/(e^7*x^4 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*
d^3*e^4*x + d^4*e^3) + 1/35*sqrt(-e^2*x^2 + d^2)*C*d^2/(d*e^6*x^3 + 3*d^2*e
^5*x^2 + 3*d^3*e^4*x + d^4*e^3) + 2/105*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^5
*x^2 + 2*d^3*e^4*x + d^4*e^3) + 2/105*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^3*e^4*x
 + d^4*e^3) + 2/7*sqrt(-e^2*x^2 + d^2)*B*d/(e^6*x^4 + 4*d*e^5*x^3 + 6*d^2*e
^4*x^2 + 4*d^3*e^3*x + d^4*e^2) - 1/35*sqrt(-e^2*x^2 + d^2)*B*d/(d*e^5*x^3
+ 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4*e^2) - 2/105*sqrt(-e^2*x^2 + d^2)*B*d/(
d^2*e^4*x^2 + 2*d^3*e^3*x + d^4*e^2) - 2/105*sqrt(-e^2*x^2 + d^2)*B*d/(d^3*
e^3*x + d^4*e^2) + 4/5*sqrt(-e^2*x^2 + d^2)*C*d/(e^6*x^3 + 3*d*e^5*x^2 + 3*
d^2*e^4*x + d^3*e^3) - 2/15*sqrt(-e^2*x^2 + d^2)*C*d/(d*e^5*x^2 + 2*d^2*e^4
*x + d^3*e^3) - 2/15*sqrt(-e^2*x^2 + d^2)*C*d/(d^2*e^4*x + d^3*e^3) - 2/7*s
qrt(-e^2*x^2 + d^2)*A/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^3*x^2 + 4*d^3*e^2*x
+ d^4*e) + 1/35*sqrt(-e^2*x^2 + d^2)*A/(d*e^4*x^3 + 3*d^2*e^3*x^2 + 3*d^3*e
^2*x + d^4*e) + 2/105*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^3*x^2 + 2*d^3*e^2*x + d
^4*e) + 2/105*sqrt(-e^2*x^2 + d^2)*A/(d^3*e^2*x + d^4*e) - 2/5*sqrt(-e^2*x^
2 + d^2)*B/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2) + 1/15*sqrt(-e^2
*x^2 + d^2)*B/(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) + 1/15*sqrt(-e^2*x^2 + d^
2)*B/(d^2*e^3*x + d^3*e^2) - 2/3*sqrt(-e^2*x^2 + d^2)*C/(e^5*x^2 + 2*d*e^4*
x + d^2*e^3) + 1/3*sqrt(-e^2*x^2 + d^2)*C/(d*e^4*x + d^2*e^3)
```

mupad [B] time = 4.67, size = 601, normalized size = 3.34

$$\frac{B\sqrt{d^2-e^2\,x^2}}{21\,\left(d^3\,e^2+x\,d^2\,e^3\right)}-\frac{3\,B\sqrt{d^2-e^2\,x^2}}{7\,\left(d^3\,e^2+3\,d^2\,e^3\,x+3\,d\,e^4\,x^2+e^5\,x^3\right)}+\frac{2\,A\sqrt{d^2-e^2\,x^2}}{105\,\left(d^4\,e+2\,d^3\,e^2\,x+d^2\,e^3\,x^2\right)}+\frac{B\sqrt{d^2-}}{21\,\left(d^3\,e^2+2\,d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^5,x)
```

```
[Out] (B*(d^2 - e^2*x^2)^(1/2))/(21*(d^3*e^2 + d^2*e^3*x)) - (3*B*(d^2 - e^2*x^2)
^(1/2))/(7*(d^3*e^2 + e^5*x^3 + 3*d^2*e^3*x + 3*d*e^4*x^2)) + (2*A*(d^2 - e
^2*x^2)^(1/2))/(105*(d^4*e + 2*d^3*e^2*x + d^2*e^3*x^2)) + (B*(d^2 - e^2*x^
2)^(1/2))/(21*(d^3*e^2 + 2*d^2*e^3*x + d*e^4*x^2)) - (82*C*(d^2 - e^2*x^2)^
(1/2))/(105*(d^2*e^3 + e^5*x^2 + 2*d*e^4*x)) + (2*A*(d^2 - e^2*x^2)^(1/2))/
(105*(d^4*e + d^3*e^2*x)) + (23*C*(d^2 - e^2*x^2)^(1/2))/(105*(d^2*e^3 + d*
e^4*x)) - (2*A*(d^2 - e^2*x^2)^(1/2))/(7*(d^4*e + e^5*x^4 + 4*d^3*e^2*x + 4
*d*e^4*x^3 + 6*d^2*e^3*x^2)) + (A*(d^2 - e^2*x^2)^(1/2))/(35*(d^4*e + 3*d^3
*e^2*x + d*e^4*x^3 + 3*d^2*e^3*x^2)) - (2*C*d^2*(d^2 - e^2*x^2)^(1/2))/(7*(
d^4*e^3 + e^7*x^4 + 4*d^3*e^4*x + 4*d*e^6*x^3 + 6*d^2*e^5*x^2)) + (2*B*d*(d
^2 - e^2*x^2)^(1/2))/(7*(d^4*e^2 + e^6*x^4 + 4*d^3*e^3*x + 4*d*e^5*x^3 + 6*
d^2*e^4*x^2)) + (29*C*d*(d^2 - e^2*x^2)^(1/2))/(35*(d^3*e^3 + e^6*x^3 + 3*d
^2*e^4*x + 3*d*e^5*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}\left(A+Bx+Cx^2\right)}{(d+ex)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**5,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**5, x)
```

3.9 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6} \, dx$

Optimal. Leaf size=234

$$-\frac{(d^2-e^2x^2)^{3/2}(2e(Ae+2Bd)+11Cd^2)}{42d^2e^3(d+ex)^5}-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{9de^3(d+ex)^6}-\frac{(d^2-e^2x^2)^{3/2}(2e(Ae+2Bd)+1}{315d^4e^3(d+ex)^3}$$

```
[Out] -1/9*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^6+1/2*C*(-e^2*x
^2+d^2)^(3/2)/e^3/(e*x+d)^5-1/42*(11*C*d^2+2*e*(A*e+2*B*d))*(-e^2*x^2+d^2)^
(3/2)/d^2/e^3/(e*x+d)^5-1/105*(11*C*d^2+2*e*(A*e+2*B*d))*(-e^2*x^2+d^2)^(3/
2)/d^3/e^3/(e*x+d)^4-1/315*(11*C*d^2+2*e*(A*e+2*B*d))*(-e^2*x^2+d^2)^(3/2)/
d^4/e^3/(e*x+d)^3
```

Rubi [A] time = 0.26, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1639, 793, 659, 651}

$$-\frac{(d^2-e^2x^2)^{3/2}(2e(Ae+2Bd)+11Cd^2)}{315d^4e^3(d+ex)^3}-\frac{(d^2-e^2x^2)^{3/2}(2e(Ae+2Bd)+11Cd^2)}{105d^3e^3(d+ex)^4}-\frac{(d^2-e^2x^2)^{3/2}(2e(Ae+2Bd}{42d^2e^3(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^6,x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(9*d*e^3*(d + e*x)^6) + (C
*(d^2 - e^2*x^2)^(3/2))/(2*e^3*(d + e*x)^5) - ((11*C*d^2 + 2*e*(2*B*d + A*e
))*(d^2 - e^2*x^2)^(3/2))/(42*d^2*e^3*(d + e*x)^5) - ((11*C*d^2 + 2*e*(2*B*
d + A*e))*(d^2 - e^2*x^2)^(3/2))/(105*d^3*e^3*(d + e*x)^4) - ((11*C*d^2 + 2
*e*(2*B*d + A*e))*(d^2 - e^2*x^2)^(3/2))/(315*d^4*e^3*(d + e*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
```

```
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx = \frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5}+\frac{\int \frac{\left(e^2\left(5Cd^2+2Ae^2\right)+e^3(3Cd+2Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx}{2e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{9de^3(d+ex)^6}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5}+\frac{\left(11Cd^2+2e(2Bd+\right.}{6}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{9de^3(d+ex)^6}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5}-\frac{\left(11Cd^2+2e(2Bd+\right.}{42d^2e^3(}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{9de^3(d+ex)^6}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5}-\frac{\left(11Cd^2+2e(2Bd+\right.}{42d^2e^3(}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{9de^3(d+ex)^6}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5}-\frac{\left(11Cd^2+2e(2Bd+\right.}{42d^2e^3(}$$

Mathematica [A] time = 0.22, size = 144, normalized size = 0.62

$$-\frac{(d-ex)\sqrt{d^2-e^2x^2}\left(e\left(Ae\left(58d^3+33d^2ex+12de^2x^2+2e^3x^3\right)+Bd\left(11d^3+66d^2ex+24de^2x^2+4e^3x^3\right)\right)+Cd^2\right.}{315d^4e^3(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^6,x]
```

```
[Out] -1/315*((d - e*x)*Sqrt[d^2 - e^2*x^2]*(C*d^2*(4*d^3 + 24*d^2*e*x + 66*d*e^2
*x^2 + 11*e^3*x^3) + e*(A*e*(58*d^3 + 33*d^2*e*x + 12*d*e^2*x^2 + 2*e^3*x^3
) + B*d*(11*d^3 + 66*d^2*e*x + 24*d*e^2*x^2 + 4*e^3*x^3))))/(d^4*e^3*(d + e
*x)^5)
```

fricas [A] time = 1.07, size = 399, normalized size = 1.71

$$-\frac{4\,Cd^7+11\,Bd^6e+58\,Ad^5e^2+\left(4\,Cd^2e^5+11\,Bde^6+58\,Ae^7\right)x^5+5\left(4\,Cd^3e^4+11\,Bd^2e^5+58\,Ade^6\right)x^4+10\left(4\,Cd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="fricas
")
```

```
[Out] -1/315*(4*C*d^7 + 11*B*d^6*e + 58*A*d^5*e^2 + (4*C*d^2*e^5 + 11*B*d*e^6 + 5
8*A*e^7)*x^5 + 5*(4*C*d^3*e^4 + 11*B*d^2*e^5 + 58*A*d*e^6)*x^4 + 10*(4*C*d^
4*e^3 + 11*B*d^3*e^4 + 58*A*d^2*e^5)*x^3 + 10*(4*C*d^5*e^2 + 11*B*d^4*e^3 +
 58*A*d^3*e^4)*x^2 + 5*(4*C*d^6*e + 11*B*d^5*e^2 + 58*A*d^4*e^3)*x + (4*C*d
^6 + 11*B*d^5*e + 58*A*d^4*e^2 - (11*C*d^2*e^4 + 4*B*d*e^5 + 2*A*e^6)*x^4 -
 5*(11*C*d^3*e^3 + 4*B*d^2*e^4 + 2*A*d*e^5)*x^3 + 21*(2*C*d^4*e^2 - 2*B*d^3
*e^3 - A*d^2*e^4)*x^2 + 5*(4*C*d^5*e + 11*B*d^4*e^2 - 5*A*d^3*e^3)*x)*sqrt(
```

```
-e^2*x^2 + d^2))/(d^4*e^8*x^5 + 5*d^5*e^7*x^4 + 10*d^6*e^6*x^3 + 10*d^7*e^5
*x^2 + 5*d^8*e^4*x + d^9*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-960
*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^24*
exp(2)^2-320*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
5*exp(1)^24*exp(2)^2-384*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^5*exp(1)^26*exp(2)-960*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^7*exp(1)^22*exp(2)^3-480*A*(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^20*exp(2)^4-120*A*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(1)^18*exp(2)^5-800*A*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^22*exp(2
)^3-800*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp
(1)^20*exp(2)^4-480*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^8*exp(1)^18*exp(2)^5-120*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^9*exp(1)^16*exp(2)^6-960*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^24*exp(2)^2-352*A*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^22*exp(2)^3+2480*A*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^20*exp(2)^
4+3440*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(
1)^18*exp(2)^5+1920*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^8*exp(1)^16*exp(2)^6+480*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^9*exp(1)^14*exp(2)^7-800*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^22*exp(2)^3-160*A*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^20*exp(2)^4+1680*A*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^18*exp(2)^
5+2640*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(
1)^16*exp(2)^6+1920*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^8*exp(1)^14*exp(2)^7+480*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^9*exp(1)^12*exp(2)^8-960*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^22*exp(2)^3+2480*A*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^20*exp(2)^4+4736*A*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^18*exp(2)
^5-320*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(
1)^16*exp(2)^6-4160*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^7*exp(1)^14*exp(2)^7-2880*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^8*exp(1)^12*exp(2)^8-720*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(1)^10*exp(2)^9-800*A*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^20*exp(2)^4+3120*A*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^18*exp(2)
^5+12680*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*ex
p(1)^16*exp(2)^6+15140*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^6*exp(1)^14*exp(2)^7+5900*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x/exp(2))^7*exp(1)^12*exp(2)^8-450*A*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^10*exp(2)^9-450*A*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(1)^8*exp(2)^10-480*A*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^20*exp(
2)^4+3440*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*e
xp(1)^18*exp(2)^5-80*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^4*exp(1)^16*exp(2)^6-6100*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^5*exp(1)^14*exp(2)^7-2010*A*(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^12*exp(2)^8+2850*A*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^10*exp(2)^9+2325*A*
```

```
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^8*exp(
2)^10+525*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*e
xp(1)^6*exp(2)^11-320*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^2*exp(1)^18*exp(2)^5+3760*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^3*exp(1)^16*exp(2)^6+25860*A*(-1/2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^14*exp(2)^7+42700*A*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^12*exp(2)^8+3678
0*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^10
*exp(2)^9+21780*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^7*exp(1)^8*exp(2)^10+7860*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^8*exp(1)^6*exp(2)^11+1140*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^9*exp(1)^4*exp(2)^12+1880*A*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^16*exp(2)^6-3840*A*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)^7
-490*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)
^12*exp(2)^8+9930*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^5*exp(1)^10*exp(2)^9+9030*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^6*exp(1)^8*exp(2)^10+2730*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^6*exp(2)^11+60*A*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^4*exp(2)^12-60*A*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(2)^14+1580*A*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^14*exp(2)
^7+28100*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*ex
p(1)^12*exp(2)^8+57720*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^4*exp(1)^10*exp(2)^9+59400*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^10+33000*A*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^6*exp(2)^11+8280*A*(-1/2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^4*exp(2)^12-24*
A*exp(1)^16*exp(2)^6+600*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^8*exp(2)^14-2590*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^2*exp(1)^12*exp(2)^8+5590*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^9+11080*A*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^8*exp(2)^10+5400*A*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^1
1+600*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1
)^4*exp(2)^12-6*B*d*exp(1)^15*exp(2)^6-20*A*exp(1)^14*exp(2)^7-120*A*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(2)^14+16100*A*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(
2)^9+41420*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*
exp(1)^8*exp(2)^10+42800*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^4*exp(1)^6*exp(2)^11+18000*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^5*exp(1)^4*exp(2)^12+98*A*exp(1)^12*exp(2)^8+2400
*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(2)^14+
4130*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)
^8*exp(2)^10+4470*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^3*exp(1)^6*exp(2)^11+1200*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^12+27*B*d*exp(1)^11*exp(2)^8+90*A*exp(1
)^10*exp(2)^9+18040*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^2*exp(1)^6*exp(2)^11+15720*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^12-170*B*d*exp(1)^9*exp(2)^9-149*A*e
xp(1)^8*exp(2)^10+3600*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^4*exp(2)^14+840*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*exp(1)^4*exp(2)^12-86*B*d*exp(1)^7*exp(2)^10+380*A*exp(1)^6*
exp(2)^11+120*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^3*exp(2)^14-30*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^14/x/e
xp(2)-760*B*d*exp(1)^5*exp(2)^11+180*A*exp(1)^4*exp(2)^12+2400*A*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^14-40*B*d*exp(1)
^3*exp(2)^12+600*A*exp(2)^14-120*B*d*exp(1)*exp(2)^13-2430*A*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^12/x/exp(2)-1275/2*A*(-2*d*ex
```

```
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^11/x/exp(2)-2125*A*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^10/x/exp(2)+385*A*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^9/x/exp(2)-210*
A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^12*exp(2)^8/x/exp(2)-2
50*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^14*exp(2)^7/x/exp(2
)+40*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^16*exp(2)^6/x/exp
(2)+60*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^18*exp(2)^5/x/e
xp(2)-480*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2
*exp(1)*exp(2)^13-240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^6*exp(1)^23*exp(2)^2-240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^7*exp(1)^21*exp(2)^3-120*B*d*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^19*exp(2)^4-160*C*d^2*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^22*exp(2)^2
-160*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*ex
p(1)^20*exp(2)^3-240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^7*exp(1)^19*exp(2)^4-120*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^8*exp(1)^17*exp(2)^5+320*C*d^2*(-1/2*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^22*exp(2)^2-160*C*d^2*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^20*exp(2)^
3-160*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*e
xp(1)^18*exp(2)^4-240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^4*exp(1)^23*exp(2)^2-48*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^5*exp(1)^21*exp(2)^3+720*B*d*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^19*exp(2)^4+960*B*d*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^17*exp(2)^5+48
0*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^
15*exp(2)^6-160*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^4*exp(1)^22*exp(2)^2+128*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x/exp(2))^5*exp(1)^20*exp(2)^3+640*C*d^2*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^18*exp(2)^4+640*C*d^2*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^16*exp(2)^5
-2720*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp
(1)^19*exp(2)^4-3560*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^6*exp(1)^17*exp(2)^5-920*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^7*exp(1)^15*exp(2)^6-60*B*d*(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^13*exp(2)^7-60*B*d*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(1)^11*exp(2)^8+1760
*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)
^20*exp(2)^3+11360*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^5*exp(1)^18*exp(2)^4+19120*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^16*exp(2)^5+11920*C*d^2*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^14*exp(2)^6+3240*C*d
^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^12*
exp(2)^7+360*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^9*exp(1)^10*exp(2)^8-240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^3*exp(1)^21*exp(2)^3+720*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^19*exp(2)^4+604*B*d*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^17*exp(2)^5-1250*B*d
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^15*ex
p(2)^6-1910*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^7*exp(1)^13*exp(2)^7-855*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^8*exp(1)^11*exp(2)^8-15*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^9*exp(1)^9*exp(2)^9-160*C*d^2*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^20*exp(2)^3+1120*C*d^2*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^18*exp
(2)^4+3416*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^5*exp(1)^16*exp(2)^5+3660*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^6*exp(1)^14*exp(2)^6+1860*C*d^2*(-1/2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^12*exp(2)^7+810*C*d^2*(-1/2*(-
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2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^10*exp(2)^8+90
*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(1)
^8*exp(2)^9+240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^3*exp(1)^19*exp(2)^4-10040*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^4*exp(1)^17*exp(2)^5-25760*B*d*(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^15*exp(2)^6-32380*B*d*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^13*exp(2)^7-2
1460*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(
1)^11*exp(2)^8-6390*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^8*exp(1)^9*exp(2)^9-630*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^9*exp(1)^7*exp(2)^10+1760*C*d^2*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^18*exp(2)^4+32080*C*d^2*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^16*exp(2)
^5+56120*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
5*exp(1)^14*exp(2)^6+43140*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^6*exp(1)^12*exp(2)^7+20460*C*d^2*(-1/2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^10*exp(2)^8+5670*C*d^2*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^8*exp(2)^9+63
0*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(1
)^6*exp(2)^10-120*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^2*exp(1)^19*exp(2)^4+960*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^3*exp(1)^17*exp(2)^5-2450*B*d*(-1/2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^15*exp(2)^6-720*B*d*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)*exp(2)^13-6710*B
*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^13*
exp(2)^7-96*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^5*exp(1)^25*exp(2)-5590*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^6*exp(1)^11*exp(2)^8-1930*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^9*exp(2)^9-330*B*d*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^7*exp(2)^10-90*B*d*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(1)^5*exp(2)^11
-160*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*ex
p(1)^18*exp(2)^4+1440*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^3*exp(1)^16*exp(2)^5+6140*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^14*exp(2)^6+7320*C*d^2*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^12*exp(2)^7+4350*C*
d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^10
*exp(2)^8+1530*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/ex
p(2))^7*exp(1)^8*exp(2)^9+135*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^8*exp(1)^6*exp(2)^10+15*C*d^2*(-1/2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(1)^4*exp(2)^11+120*B*d*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^17*exp(2)^5-1268
0*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^
15*exp(2)^6-48820*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^4*exp(1)^13*exp(2)^7-67820*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x/exp(2))^5*exp(1)^11*exp(2)^8-41940*B*d*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^9*exp(2)^9-13260*B*d*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^7*exp(2)^10-
2760*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(
1)^5*exp(2)^11-360*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^9*exp(1)^3*exp(2)^12+800*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^2*exp(1)^16*exp(2)^5+37680*C*d^2*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)^6+63860*C*d^2*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*exp
(2)^7+51900*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^5*exp(1)^10*exp(2)^8+24300*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^6*exp(1)^8*exp(2)^9+5460*C*d^2*(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^6*exp(2)^10+540*C*d^2*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^4*exp(2)^11+6
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0*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(2
)^13+570*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*
exp(1)^15*exp(2)^6-3890*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^3*exp(1)^13*exp(2)^7-7660*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^11*exp(2)^8-5780*B*d*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^9*exp(2)^9-2220*B*d*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^7*exp(2)^
10-480*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*ex
p(1)*exp(2)^13-660*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^7*exp(1)^5*exp(2)^11-120*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^8*exp(1)^3*exp(2)^12-6740*B*d*(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^13*exp(2)^7-46140*B*d*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^11*exp(2)^8-6
0560*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(
1)^9*exp(2)^9-32400*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^5*exp(1)^7*exp(2)^10-9840*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^6*exp(1)^5*exp(2)^11-1920*B*d*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^3*exp(2)^12-4*C*d^2*exp(1)^
14*exp(2)^6+120*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^7*exp(2)^13-2450*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*exp(1)^11*exp(2)^8-4710*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^9*exp(2)^9-3440*B*d*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^7*exp(2)^10-1200*B*d*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^5*exp(2)^
11-240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*ex
p(1)^3*exp(2)^12+20*C*d^2*exp(1)^12*exp(2)^7-24860*B*d*(-1/2*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^9*exp(2)^9-26740*B*d*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^7*exp(2)^10
-12160*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*ex
p(1)^5*exp(2)^11-3600*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^5*exp(1)^3*exp(2)^12+28*C*d^2*exp(1)^10*exp(2)^8-1700*B*d*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^7*exp(2)^10-
940*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1
)^5*exp(2)^11-160*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^4*exp(1)^3*exp(2)^12+610*C*d^2*exp(1)^8*exp(2)^9-5840*B*d*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^5*exp(2)^11-2880
*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^3
*exp(2)^12+81*C*d^2*exp(1)^6*exp(2)^10-120*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(2)^13-80*B*d*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^3*exp(2)^12+420*C*d^2*exp(1)
^4*exp(2)^11-120*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/ex
p(2))^8*exp(1)*exp(2)^13+1100*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^2*exp(1)^14*exp(2)^6+5740*C*d^2*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^12*exp(2)^7+5650*C*d^2*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^8
+2430*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*e
xp(1)^8*exp(2)^9+330*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^6*exp(1)^6*exp(2)^10-90*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2
*exp(2))*exp(1))/x/exp(2))^7*exp(1)^4*exp(2)^11+21260*C*d^2*(-1/2*(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^7+37540*C*d
^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10*
exp(2)^8+32200*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/ex
p(2))^4*exp(1)^8*exp(2)^9+12600*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^10+1800*C*d^2*(-1/2*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^4*exp(2)^11+2570*C*d^2*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)
^8+1710*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3
*exp(1)^8*exp(2)^9+480*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^4*exp(1)^6*exp(2)^10+14180*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^

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2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)^9+11340*C*d^2*(-1/2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^10+2400*
C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^
4*exp(2)^11+270*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^2*exp(1)^6*exp(2)^10+90*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^11+1560*C*d^2*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^11-64*C*d^2*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^24*exp(2)+30
*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^13/x/exp(2)+420*B
*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^3*exp(2)^12/x/exp(2)+
155*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^5*exp(2)^11/x/ex
p(2)+3485*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*exp(2)^1
0/x/exp(2)+845/2*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^9*e
xp(2)^9/x/exp(2)+820*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)
^11*exp(2)^8/x/exp(2)-135*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*e
xp(1)^13*exp(2)^7/x/exp(2)+30*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))*exp(1)^17*exp(2)^5/x/exp(2)+15/2*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))*exp(1)^4*exp(2)^11/x/exp(2)-1785*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))*exp(1)^6*exp(2)^10/x/exp(2)-360*C*d^2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^9/x/exp(2)-2870*C*d^2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^8/x/exp(2)-140*C*d^2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^12*exp(2)^7/x/exp(2)-100*C*d
^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^14*exp(2)^6/x/exp(2)+
20*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^16*exp(2)^5/x/e
xp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2
)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^5/(-60*d^4*exp(1)^1
9+240*d^4*exp(1)^15*exp(2)^2+240*d^4*exp(1)^13*exp(2)^3-360*d^4*exp(1)^11*e
xp(2)^4-360*d^4*exp(1)^9*exp(2)^5+240*d^4*exp(1)^7*exp(2)^6+240*d^4*exp(1)^
5*exp(2)^7-60*d^4*exp(1)^17*exp(2)-120*d^4*exp(1)*exp(2)^9)+1/2*(-4*B*d*exp
(1)^11*exp(2)^4+24*C*d^2*exp(1)^10*exp(2)^4-B*d*exp(1)^9*exp(2)^5+6*C*d^2*e
xp(1)^8*exp(2)^5+18*A*exp(1)^8*exp(2)^6-42*B*d*exp(1)^7*exp(2)^6+42*C*d^2*e
xp(1)^6*exp(2)^6+3*A*exp(1)^6*exp(2)^7-6*B*d*exp(1)^5*exp(2)^7+C*d^2*exp(1)
^4*exp(2)^7+44*A*exp(1)^4*exp(2)^8-24*B*d*exp(1)^3*exp(2)^8+4*C*d^2*exp(2)^
9+12*A*exp(2)^10)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+
exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-2*d^4*exp(1)^1
9+8*d^4*exp(1)^15*exp(2)^2+8*d^4*exp(1)^13*exp(2)^3-12*d^4*exp(1)^11*exp(2)
^4-12*d^4*exp(1)^9*exp(2)^5+8*d^4*exp(1)^7*exp(2)^6+8*d^4*exp(1)^5*exp(2)^7
-2*d^4*exp(1)^17*exp(2)-4*d^4*exp(1)*exp(2)^9)
```

maple [A] time = 0.01, size = 152, normalized size = 0.65

$$-\frac{(-ex+d)\left(2A\,e^5x^3+4Bd\,e^4x^3+11C\,d^2e^3x^3+12Ad\,e^4x^2+24B\,d^2e^3x^2+66C\,d^3e^2x^2+33A\,d^2e^3x+66B\,d^3e^2x+\cdots\right.}{315\,(ex+d)^5\,d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x)
```

```
[Out] -1/315*(-e*x+d)*(2*A*e^5*x^3+4*B*d*e^4*x^3+11*C*d^2*e^3*x^3+12*A*d*e^4*x^2+
24*B*d^2*e^3*x^2+66*C*d^3*e^2*x^2+33*A*d^2*e^3*x+66*B*d^3*e^2*x+24*C*d^4*e*
x+58*A*d^3*e^2+11*B*d^4*e+4*C*d^5)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5/d^4/e^3
```

maxima [B] time = 0.58, size = 1378, normalized size = 5.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="maxima
")
```

```
[Out] -2/9*sqrt(-e^2*x^2 + d^2)*C*d^2/(e^8*x^5 + 5*d*e^7*x^4 + 10*d^2*e^6*x^3 + 1
0*d^3*e^5*x^2 + 5*d^4*e^4*x + d^5*e^3) + 1/63*sqrt(-e^2*x^2 + d^2)*C*d^2/(d
*e^7*x^4 + 4*d^2*e^6*x^3 + 6*d^3*e^5*x^2 + 4*d^4*e^4*x + d^5*e^3) + 1/105*s
qrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^6*x^3 + 3*d^3*e^5*x^2 + 3*d^4*e^4*x + d^5*
e^3) + 2/315*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^3*e^5*x^2 + 2*d^4*e^4*x + d^5*e^
3) + 2/315*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^4*e^4*x + d^5*e^3) + 2/9*sqrt(-e^2
*x^2 + d^2)*B*d/(e^7*x^5 + 5*d*e^6*x^4 + 10*d^2*e^5*x^3 + 10*d^3*e^4*x^2 +
5*d^4*e^3*x + d^5*e^2) - 1/63*sqrt(-e^2*x^2 + d^2)*B*d/(d*e^6*x^4 + 4*d^2*e
^5*x^3 + 6*d^3*e^4*x^2 + 4*d^4*e^3*x + d^5*e^2) - 1/105*sqrt(-e^2*x^2 + d^2
)*B*d/(d^2*e^5*x^3 + 3*d^3*e^4*x^2 + 3*d^4*e^3*x + d^5*e^2) - 2/315*sqrt(-e
^2*x^2 + d^2)*B*d/(d^3*e^4*x^2 + 2*d^4*e^3*x + d^5*e^2) - 2/315*sqrt(-e^2*x
^2 + d^2)*B*d/(d^4*e^3*x + d^5*e^2) + 4/7*sqrt(-e^2*x^2 + d^2)*C*d/(e^7*x^4
 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3) - 2/35*sqrt(-e^2*x^
2 + d^2)*C*d/(d*e^6*x^3 + 3*d^2*e^5*x^2 + 3*d^3*e^4*x + d^4*e^3) - 4/105*sq
rt(-e^2*x^2 + d^2)*C*d/(d^2*e^5*x^2 + 2*d^3*e^4*x + d^4*e^3) - 4/105*sqrt(-
e^2*x^2 + d^2)*C*d/(d^3*e^4*x + d^4*e^3) - 2/9*sqrt(-e^2*x^2 + d^2)*A/(e^6*
x^5 + 5*d*e^5*x^4 + 10*d^2*e^4*x^3 + 10*d^3*e^3*x^2 + 5*d^4*e^2*x + d^5*e)
+ 1/63*sqrt(-e^2*x^2 + d^2)*A/(d*e^5*x^4 + 4*d^2*e^4*x^3 + 6*d^3*e^3*x^2 +
4*d^4*e^2*x + d^5*e) + 1/105*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^4*x^3 + 3*d^3*e^
3*x^2 + 3*d^4*e^2*x + d^5*e) + 2/315*sqrt(-e^2*x^2 + d^2)*A/(d^3*e^3*x^2 +
2*d^4*e^2*x + d^5*e) + 2/315*sqrt(-e^2*x^2 + d^2)*A/(d^4*e^2*x + d^5*e) - 2
/7*sqrt(-e^2*x^2 + d^2)*B/(e^6*x^4 + 4*d*e^5*x^3 + 6*d^2*e^4*x^2 + 4*d^3*e^
3*x + d^4*e^2) + 1/35*sqrt(-e^2*x^2 + d^2)*B/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3
*d^3*e^3*x + d^4*e^2) + 2/105*sqrt(-e^2*x^2 + d^2)*B/(d^2*e^4*x^2 + 2*d^3*e
^3*x + d^4*e^2) + 2/105*sqrt(-e^2*x^2 + d^2)*B/(d^3*e^3*x + d^4*e^2) - 2/5*
sqrt(-e^2*x^2 + d^2)*C/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3) + 1/
15*sqrt(-e^2*x^2 + d^2)*C/(d*e^5*x^2 + 2*d^2*e^4*x + d^3*e^3) + 1/15*sqrt(-
e^2*x^2 + d^2)*C/(d^2*e^4*x + d^3*e^3)
```

mupad [B] time = 5.24, size = 960, normalized size = 4.10

$$\frac{B\sqrt{d^2-e^2\,x^2}}{63\,\left(d^4\,e^2+x\,d^3\,e^3\right)}+\frac{C\sqrt{d^2-e^2\,x^2}}{135\,\left(d^3\,e^3+x\,d^2\,e^4\right)}-\frac{19\,B\sqrt{d^2-e^2\,x^2}}{63\,\left(d^4\,e^2+4\,d^3\,e^3\,x+6\,d^2\,e^4\,x^2+4\,d\,e^5\,x^3+e^6\,x^4\right)}+\frac{}{105\,\left(d^5\,e+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^6,x)
```

```
[Out] (B*(d^2 - e^2*x^2)^(1/2))/(63*(d^4*e^2 + d^3*e^3*x)) + (C*(d^2 - e^2*x^2)^(
1/2))/(135*(d^3*e^3 + d^2*e^4*x)) - (19*B*(d^2 - e^2*x^2)^(1/2))/(63*(d^4*e
^2 + e^6*x^4 + 4*d^3*e^3*x + 4*d*e^5*x^3 + 6*d^2*e^4*x^2)) + (A*(d^2 - e^2*
x^2)^(1/2))/(105*(d^5*e + 3*d^4*e^2*x + 3*d^3*e^3*x^2 + d^2*e^4*x^3)) + (2*
B*(d^2 - e^2*x^2)^(1/2))/(105*(d^4*e^2 + 3*d^3*e^3*x + d*e^5*x^3 + 3*d^2*e^
4*x^2)) - (47*C*(d^2 - e^2*x^2)^(1/2))/(105*(d^3*e^3 + e^6*x^3 + 3*d^2*e^4*
x + 3*d*e^5*x^2)) + (2*A*(d^2 - e^2*x^2)^(1/2))/(315*(d^5*e + 2*d^4*e^2*x +
 d^3*e^3*x^2)) + (11*C*(d^2 - e^2*x^2)^(1/2))/(315*(d^3*e^3 + 2*d^2*e^4*x +
 d*e^5*x^2)) - (2*A*(d^2 - e^2*x^2)^(1/2))/(9*(d^5*e + e^6*x^5 + 5*d^4*e^2*
x + 5*d*e^5*x^4 + 10*d^3*e^3*x^2 + 10*d^2*e^4*x^3)) + (A*(d^2 - e^2*x^2)^(1
/2))/(63*(d^5*e + 4*d^4*e^2*x + d*e^5*x^4 + 6*d^3*e^3*x^2 + 4*d^2*e^4*x^3))
 - (2*A*(d^2 - e^2*x^2)^(1/2))/(945*(d^5*e + d^4*e^2*x)) + (4*B*(d^2 - e^2*
x^2)^(1/2))/(315*(d^4*e^2 + 2*d^3*e^3*x + d^2*e^4*x^2)) + (2*B*d*(d^2 - e^2
*x^2)^(1/2))/(9*(d^5*e^2 + e^7*x^5 + 5*d^4*e^3*x + 5*d*e^6*x^4 + 10*d^3*e^4
*x^2 + 10*d^2*e^5*x^3)) + (37*C*d*(d^2 - e^2*x^2)^(1/2))/(63*(d^4*e^3 + e^7
*x^4 + 4*d^3*e^4*x + 4*d*e^6*x^3 + 6*d^2*e^5*x^2)) - (2*C*d^2*(d^2 - e^2*x^
2)^(1/2))/(9*(d^5*e^3 + e^8*x^5 + 5*d^4*e^4*x + 5*d*e^7*x^4 + 10*d^3*e^5*x^
2 + 10*d^2*e^6*x^3)) + (8*A*e^2*(d^2 - e^2*x^2)^(1/2))/(945*(d^5*e^3 + d^4*
e^4*x)) + (26*C*d^2*(d^2 - e^2*x^2)^(1/2))/(945*(d^5*e^3 + d^4*e^4*x)) - (B
*d*e*(d^2 - e^2*x^2)^(1/2))/(315*(d^5*e^3 + d^4*e^4*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**6,x)
```

```
[Out] Timed out
```

3.10 $\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=236

$$-\frac{x^2\sqrt{d^2-e^2x^2}\left(5e(Ae+3Bd)+19Cd^2\right)}{15e}-\frac{dx\sqrt{d^2-e^2x^2}\left(12Ae^2+15Bde+13Cd^2\right)}{8e^2}-\frac{d^2\sqrt{d^2-e^2x^2}\left(55Ae^2+4\right.}{15e^3}$$

```
[Out] 1/8*d^3*(20*A*e^2+15*B*d*e+13*C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1
/15*d^2*(55*A*e^2+45*B*d*e+38*C*d^2)*(-e^2*x^2+d^2)^(1/2)/e^3-1/8*d*(12*A*e
^2+15*B*d*e+13*C*d^2)*x*(-e^2*x^2+d^2)^(1/2)/e^2-1/15*(19*C*d^2+5*e*(A*e+3*
B*d))*x^2*(-e^2*x^2+d^2)^(1/2)/e-1/4*(B*e+3*C*d)*x^3*(-e^2*x^2+d^2)^(1/2)-1
/5*C*e*x^4*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1815, 641, 217, 203}

$$-\frac{x^2\sqrt{d^2-e^2x^2}\left(5e(Ae+3Bd)+19Cd^2\right)}{15e}-\frac{dx\sqrt{d^2-e^2x^2}\left(12Ae^2+15Bde+13Cd^2\right)}{8e^2}-\frac{d^2\sqrt{d^2-e^2x^2}\left(55Ae^2+4\right.}{15e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -(d^2*(38*C*d^2 + 45*B*d*e + 55*A*e^2)*Sqrt[d^2 - e^2*x^2])/(15*e^3) - (d*(
13*C*d^2 + 15*B*d*e + 12*A*e^2)*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((19*C*d^2
 + 5*e*(3*B*d + A*e))*x^2*Sqrt[d^2 - e^2*x^2])/(15*e) - ((3*C*d + B*e)*x^3*
Sqrt[d^2 - e^2*x^2])/4 - (C*e*x^4*Sqrt[d^2 - e^2*x^2])/5 + (d^3*(13*C*d^2 +
 15*B*d*e + 20*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2} - \frac{\int \frac{-5Ad^3e^2-5d^2e^2(Bd+3Ae)x-5de^2\left(Cd^2+3e(Bd+Ae)\right)x^2-e^3\left(19Cd^2+5e(3\right.}{\sqrt{d^2-e^2x^2}}}{5e^2}$$

$$= -\frac{1}{4}(3Cd+Be)x^3\sqrt{d^2-e^2x^2} - \frac{1}{5}Cex^4\sqrt{d^2-e^2x^2} + \frac{\int \frac{20Ad^3e^4+20d^2e^4(Bd+3Ae)x+5d}{}}{}$$

$$= -\frac{\left(19Cd^2+5e(3Bd+Ae)\right)x^2\sqrt{d^2-e^2x^2}}{15e} - \frac{1}{4}(3Cd+Be)x^3\sqrt{d^2-e^2x^2} - \frac{1}{5}Cex$$

$$= -\frac{d\left(13Cd^2+15Bde+12Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{\left(19Cd^2+5e(3Bd+Ae)\right)x^2\sqrt{d^2}}{15e}$$

$$= -\frac{d^2\left(38Cd^2+45Bde+55Ae^2\right)\sqrt{d^2-e^2x^2}}{15e^3} - \frac{d\left(13Cd^2+15Bde+12Ae^2\right)x\sqrt{d}}{8e^2}$$

$$= -\frac{d^2\left(38Cd^2+45Bde+55Ae^2\right)\sqrt{d^2-e^2x^2}}{15e^3} - \frac{d\left(13Cd^2+15Bde+12Ae^2\right)x\sqrt{d}}{8e^2}$$

$$= -\frac{d^2\left(38Cd^2+45Bde+55Ae^2\right)\sqrt{d^2-e^2x^2}}{15e^3} - \frac{d\left(13Cd^2+15Bde+12Ae^2\right)x\sqrt{d}}{8e^2}$$

Mathematica [A] time = 0.45, size = 174, normalized size = 0.74

$$\frac{15d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(5e(4Ae+3Bd)+13Cd^2\right) - \sqrt{d^2-e^2x^2}\left(5e\left(4Ae\left(22d^2+9dex+2e^2x^2\right)+3B\left(24d^3+15d^2\right.\right.\right.}{120e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(C*(304*d^4 + 195*d^3*e*x + 152*d^2*e^2*x^2 + 90*d*e
^3*x^3 + 24*e^4*x^4) + 5*e*(4*A*e*(22*d^2 + 9*d*e*x + 2*e^2*x^2) + 3*B*(24*
d^3 + 15*d^2*e*x + 8*d*e^2*x^2 + 2*e^3*x^3)))) + 15*d^3*(13*C*d^2 + 5*e*(3*
B*d + 4*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(120*e^3)
```

fricas [A] time = 0.61, size = 178, normalized size = 0.75

$$-\frac{30\left(13\,Cd^5+15\,Bd^4e+20\,Ad^3e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(24\,Ce^4x^4+304\,Cd^4+360\,Bd^3e+440\,Ad^2e^2+30\,\right.}{120\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/120*(30*(13*C*d^5 + 15*B*d^4*e + 20*A*d^3*e^2)*arctan(-(d - sqrt(-e^2*x^
2 + d^2))/(e*x)) + (24*C*e^4*x^4 + 304*C*d^4 + 360*B*d^3*e + 440*A*d^2*e^2
+ 30*(3*C*d*e^3 + B*e^4)*x^3 + 8*(19*C*d^2*e^2 + 15*B*d*e^3 + 5*A*e^4)*x^2
+ 15*(13*C*d^3*e + 15*B*d^2*e^2 + 12*A*d*e^3)*x)*sqrt(-e^2*x^2 + d^2))/e^3
```

giac [A] time = 0.37, size = 166, normalized size = 0.70

$$\frac{1}{8}\left(13\,Cd^5+15\,Bd^4e+20\,Ad^3e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d) - \frac{1}{120}\sqrt{-x^2e^2+d^2}\left(\left(2\left(3\left(4\,Cxe+5\left(3\,Cde^6+Be^7\right)e^{(-6)}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(13*C*d^5 + 15*B*d^4*e + 20*A*d^3*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/
120*sqrt(-x^2*e^2 + d^2)*((2*(3*(4*C*x*e + 5*(3*C*d*e^6 + B*e^7)*e^(-6))*x
+ 4*(19*C*d^2*e^5 + 15*B*d*e^6 + 5*A*e^7)*e^(-6))*x + 15*(13*C*d^3*e^4 + 15
*B*d^2*e^5 + 12*A*d*e^6)*e^(-6))*x + 8*(38*C*d^4*e^3 + 45*B*d^3*e^4 + 55*A*
d^2*e^5)*e^(-6))
```

maple [A] time = 0.03, size = 374, normalized size = 1.58

$$-\frac{\sqrt{-e^2x^2+d^2}\; Ce\,x^4}{5}+\frac{5A\,d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}+\frac{15B\,d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\;e}-\frac{\sqrt{-e^2x^2+d^2}\;Be\,x^3}{4}+\frac{13C\,d^5\,\text{arc}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/5*C*e*x^4*(-e^2*x^2+d^2)^(1/2)-19/15/e*C*d^2*x^2*(-e^2*x^2+d^2)^(1/2)-38
/15/e^3*C*d^4*(-e^2*x^2+d^2)^(1/2)-1/4*x^3*e*(-e^2*x^2+d^2)^(1/2)*B-3/4*x^3
*(-e^2*x^2+d^2)^(1/2)*d*C-15/8*(-e^2*x^2+d^2)^(1/2)*B*d^2/e*x-13/8*(-e^2*x^
2+d^2)^(1/2)*C*d^3/e^2*x+15/8/(e^2)^(1/2)*B*d^4/e*arctan((e^2)^(1/2)/(-e^2*
x^2+d^2)^(1/2)*x)+13/8/(e^2)^(1/2)*C*d^5/e^2*arctan((e^2)^(1/2)/(-e^2*x^2+d
^2)^(1/2)*x)-1/3*x^2*e*(-e^2*x^2+d^2)^(1/2)*A-x^2*(-e^2*x^2+d^2)^(1/2)*d*B-
11/3*d^2/e*(-e^2*x^2+d^2)^(1/2)*A-3*d^3/e^2*(-e^2*x^2+d^2)^(1/2)*B-3/2*(-e^
2*x^2+d^2)^(1/2)*A*d*x+5/2/(e^2)^(1/2)*A*d^3*arctan((e^2)^(1/2)/(-e^2*x^2+d
^2)^(1/2)*x)
```

maxima [A] time = 0.98, size = 390, normalized size = 1.65

$$-\frac{1}{5}\sqrt{-e^2x^2+d^2}\,Cex^4-\frac{4\sqrt{-e^2x^2+d^2}\,Cd^2x^2}{15\,e}+\frac{Ad^3\arcsin\left(\frac{ex}{d}\right)}{e}-\frac{8\sqrt{-e^2x^2+d^2}\,Cd^4}{15\,e^3}-\frac{\sqrt{-e^2x^2+d^2}\,Bd^3}{e^2}-\frac{3\sqrt{-e}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*C*e*x^4 - 4/15*sqrt(-e^2*x^2 + d^2)*C*d^2*x^2/e +
 A*d^3*arcsin(e*x/d)/e - 8/15*sqrt(-e^2*x^2 + d^2)*C*d^4/e^3 - sqrt(-e^2*x^
2 + d^2)*B*d^3/e^2 - 3*sqrt(-e^2*x^2 + d^2)*A*d^2/e - 1/4*(3*C*d*e^2 + B*e^
3)*sqrt(-e^2*x^2 + d^2)*x^3/e^2 - 1/3*(3*C*d^2*e + 3*B*d*e^2 + A*e^3)*sqrt(
-e^2*x^2 + d^2)*x^2/e^2 + 3/8*(3*C*d*e^2 + B*e^3)*d^4*arcsin(e*x/d)/e^5 + 1
/2*(C*d^3 + 3*B*d^2*e + 3*A*d*e^2)*d^2*arcsin(e*x/d)/e^3 - 3/8*(3*C*d*e^2 +
 B*e^3)*sqrt(-e^2*x^2 + d^2)*d^2*x/e^4 - 1/2*(C*d^3 + 3*B*d^2*e + 3*A*d*e^2
)*sqrt(-e^2*x^2 + d^2)*x/e^2 - 2/3*(3*C*d^2*e + 3*B*d*e^2 + A*e^3)*sqrt(-e^
2*x^2 + d^2)*d^2/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3\,\left(C\,x^2+B\,x+A\right)}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int(((d + e*x)^3*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2), x)
```

sympy [A] time = 24.44, size = 1268, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d**3*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2
 > 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2)
, (d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(
-d**2), (d**2 < 0) & (e**2 < 0))) + 3*A*d**2*e*Piecewise((x**2/(2*sqrt(d**2
)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + 3*A*d*e**2*Piecew
ise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**
2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt
(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True)) + A*e**
3*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 - e**
2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)) + B*d**3*Piecewis
e((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True))
 + 3*B*d**2*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e*
*2*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3
) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d*
*2)), True)) + 3*B*d*e**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4
) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)),
True)) + B*e**3*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*
e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2
)) - I*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (3*d
**4*asin(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x
**3/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2))
, True)) + C*d**3*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1
+ e**2*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*
e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**
2/d**2)), True)) + 3*C*d**2*e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*
e**4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2
)), True)) + 3*C*d*e**2*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d*
*3*x/(8*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x
**2/d**2)) - I*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
1), (3*d**4*asin(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2
)) + d*x**3/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**
2/d**2)), True)) + C*e**3*Piecewise((-8*d**4*sqrt(d**2 - e**2*x**2)/(15*e**
6) - 4*d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**4) - x**4*sqrt(d**2 - e**2*x
**2)/(5*e**2), Ne(e, 0)), (x**6/(6*sqrt(d**2)), True))
```

3.11 $\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=191

$$-\frac{x\sqrt{d^2-e^2x^2}\left(4e(Ae+2Bd)+7Cd^2\right)}{8e^2}-\frac{d\sqrt{d^2-e^2x^2}\left(e(6Ae+5Bd)+4Cd^2\right)}{3e^3}+\frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(12Ae^2+8\right.}{8e^3}$$

```
[Out] 1/8*d^2*(12*A*e^2+8*B*d*e+7*C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1/3
*d*(4*C*d^2+e*(6*A*e+5*B*d))*(-e^2*x^2+d^2)^(1/2)/e^3-1/8*(7*C*d^2+4*e*(A*e
+2*B*d))*x*(-e^2*x^2+d^2)^(1/2)/e^2-1/3*(B*e+2*C*d)*x^2*(-e^2*x^2+d^2)^(1/2
)/e-1/4*C*x^3*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1815, 641, 217, 203}

$$-\frac{x\sqrt{d^2-e^2x^2}\left(4e(Ae+2Bd)+7Cd^2\right)}{8e^2}-\frac{d\sqrt{d^2-e^2x^2}\left(e(6Ae+5Bd)+4Cd^2\right)}{3e^3}+\frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(12Ae^2+8\right.}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -(d*(4*C*d^2 + e*(5*B*d + 6*A*e))*Sqrt[d^2 - e^2*x^2])/(3*e^3) - ((7*C*d^2
+ 4*e*(2*B*d + A*e))*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((2*C*d + B*e)*x^2*Sq
rt[d^2 - e^2*x^2])/(3*e) - (C*x^3*Sqrt[d^2 - e^2*x^2])/4 + (d^2*(7*C*d^2 +
8*B*d*e + 12*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2} - \frac{\int \frac{-4Ad^2e^2-4de^2(Bd+2Ae)x-e^2\left(7Cd^2+4e(2Bd+Ae)\right)x^2-4e^3(2Cd+Be)x^3}{\sqrt{d^2-e^2x^2}}\,d}{4e^2}$$

$$= -\frac{(2Cd+Be)x^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}Cx^3\sqrt{d^2-e^2x^2} + \frac{\int \frac{12Ad^2e^4+4de^3\left(4Cd^2+e(5Bd+6Ae)\right.}{\sqrt{d^2-e^2}}}{12e}$$

$$= -\frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{(2Cd+Be)x^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}Cx^3\sqrt{d}$$

$$= -\frac{d\left(4Cd^2+e(5Bd+6Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2}$$

$$= -\frac{d\left(4Cd^2+e(5Bd+6Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2}$$

$$= -\frac{d\left(4Cd^2+e(5Bd+6Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2}$$

Mathematica [A] time = 0.18, size = 139, normalized size = 0.73

$$\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(4e(3Ae+2Bd)+7Cd^2\right)-\sqrt{d^2-e^2x^2}\left(4e\left(3Ae(4d+ex)+2B\left(5d^2+3dex+e^2x^2\right)\right)+C\left(3\right.\right.}{24e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(C*(32*d^3 + 21*d^2*e*x + 16*d*e^2*x^2 + 6*e^3*x^3)
+ 4*e*(3*A*e*(4*d + e*x) + 2*B*(5*d^2 + 3*d*e*x + e^2*x^2)))) + 3*d^2*(7*C*
d^2 + 4*e*(2*B*d + 3*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(24*e^3)
```

fricas [A] time = 0.96, size = 145, normalized size = 0.76

$$-\frac{6\left(7\,Cd^4+8\,Bd^3e+12\,Ad^2e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(6\,Ce^3x^3+32\,Cd^3+40\,Bd^2e+48\,Ade^2+8\left(2\,Cde^2+B\right.\right.}{24\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/24*(6*(7*C*d^4 + 8*B*d^3*e + 12*A*d^2*e^2)*arctan(-(d - sqrt(-e^2*x^2 +
d^2))/(e*x)) + (6*C*e^3*x^3 + 32*C*d^3 + 40*B*d^2*e + 48*A*d*e^2 + 8*(2*C*d
*e^2 + B*e^3)*x^2 + 3*(7*C*d^2*e + 8*B*d*e^2 + 4*A*e^3)*x)*sqrt(-e^2*x^2 +
d^2))/e^3
```

giac [A] time = 0.32, size = 131, normalized size = 0.69

$$\frac{1}{8}\left(7\,Cd^4+8\,Bd^3e+12\,Ad^2e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{24}\sqrt{-x^2e^2+d^2}\left(\left(2\left(3\,Cx+4\left(2\,Cde^4+Be^5\right)e^{(-5)}\right)x+3\right(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(7*C*d^4 + 8*B*d^3*e + 12*A*d^2*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/24
*sqrt(-x^2*e^2 + d^2)*((2*(3*C*x + 4*(2*C*d*e^4 + B*e^5)*e^(-5))*x + 3*(7*C
```

```
*d^2*e^3 + 8*B*d*e^4 + 4*A*e^5)*e^(-5))*x + 8*(4*C*d^3*e^2 + 5*B*d^2*e^3 +
6*A*d*e^4)*e^(-5))
```

maple [A] time = 0.01, size = 301, normalized size = 1.58

$$\frac{3A\,d^2 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}+\frac{B\,d^3 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e}+\frac{7C\,d^4 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e^2}-\frac{\sqrt{-e^2x^2+d^2}\,C\,x^3}{4}-\frac{\sqrt{-e^2x^2+}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/4*C*x^3*(-e^2*x^2+d^2)^(1/2)-7/8*(-e^2*x^2+d^2)^(1/2)*C*d^2/e^2*x+7/8/(e
^2)^(1/2)*C*d^4/e^2*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/3*x^2*(-e^
2*x^2+d^2)^(1/2)*B-2/3*x^2/e*(-e^2*x^2+d^2)^(1/2)*d*C-5/3*d^2/e^2*(-e^2*x^2
+d^2)^(1/2)*B-4/3*d^3/e^3*(-e^2*x^2+d^2)^(1/2)*C-1/2*(-e^2*x^2+d^2)^(1/2)*A
*x-x/e*(-e^2*x^2+d^2)^(1/2)*B*d+3/2/(e^2)^(1/2)*A*d^2*arctan((e^2)^(1/2)/(-
e^2*x^2+d^2)^(1/2)*x)+d^3/e/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(
1/2)*x)*B-2/e*(-e^2*x^2+d^2)^(1/2)*A*d
```

maxima [A] time = 0.98, size = 253, normalized size = 1.32

$$-\frac{1}{4}\sqrt{-e^2x^2+d^2}\,Cx^3+\frac{3\,Cd^4 \arcsin\left(\frac{ex}{d}\right)}{8\,e^3}+\frac{Ad^2 \arcsin\left(\frac{ex}{d}\right)}{e}-\frac{3\sqrt{-e^2x^2+d^2}\,Cd^2x}{8\,e^2}-\frac{\sqrt{-e^2x^2+d^2}\,Bd^2}{e^2}-\frac{2\sqrt{-e^2x^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/4*sqrt(-e^2*x^2 + d^2)*C*x^3 + 3/8*C*d^4*arcsin(e*x/d)/e^3 + A*d^2*arcsi
n(e*x/d)/e - 3/8*sqrt(-e^2*x^2 + d^2)*C*d^2*x/e^2 - sqrt(-e^2*x^2 + d^2)*B*
d^2/e^2 - 2*sqrt(-e^2*x^2 + d^2)*A*d/e - 1/3*sqrt(-e^2*x^2 + d^2)*(2*C*d*e
+ B*e^2)*x^2/e^2 + 1/2*(C*d^2 + 2*B*d*e + A*e^2)*d^2*arcsin(e*x/d)/e^3 - 1/
2*sqrt(-e^2*x^2 + d^2)*(C*d^2 + 2*B*d*e + A*e^2)*x/e^2 - 2/3*sqrt(-e^2*x^2
+ d^2)*(2*C*d*e + B*e^2)*d^2/e^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d+e\,x\right)}^2\,\left(C\,x^2+B\,x+A\right)}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int(((d + e*x)^2*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2), x)
```

sympy [A] time = 18.03, size = 891, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d**2*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2
 > 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2)
, (d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(
-d**2), (d**2 < 0) & (e**2 < 0))) + 2*A*d*e*Piecewise((x**2/(2*sqrt(d**2)),
 Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + A*e**2*Piecewise((-I
```

```
*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2), Abs
(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1 - e*
*2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True)) + B*d**2*Piece
wise((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, Tru
e)) + 2*B*d*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e*
*2*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3
) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d*
*2)), True)) + B*e**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) -
x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True
)) + C*d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2
*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3)
- d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2
)), True)) + 2*C*d*e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x
**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)
) + C*e**2*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*
sqrt(-1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) -
I*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (3*d**4*a
sin(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(
8*e**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), Tru
e))
```

3.12 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=143

$$-\frac{\sqrt{d^2-e^2x^2}\,\left(3e(Ae+Bd)+2Cd^2\right)}{3e^3}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(2Ae+Bd)+Cd^2\right)}{2e^3}-\frac{x\sqrt{d^2-e^2x^2}\,(Be+Cd)}{2e^2}-\frac{Cx^2\sqrt{}}{}$$

```
[Out] 1/2*d*(C*d^2+e*(2*A*e+B*d))*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1/3*(2*C*d
^2+3*e*(A*e+B*d))*(-e^2*x^2+d^2)^(1/2)/e^3-1/2*(B*e+C*d)*x*(-e^2*x^2+d^2)^(
1/2)/e^2-1/3*C*x^2*(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A] time = 0.20, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1815, 641, 217, 203}

$$-\frac{\sqrt{d^2-e^2x^2}\,\left(3e(Ae+Bd)+2Cd^2\right)}{3e^3}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(2Ae+Bd)+Cd^2\right)}{2e^3}-\frac{x\sqrt{d^2-e^2x^2}\,(Be+Cd)}{2e^2}-\frac{Cx^2\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -((2*C*d^2 + 3*e*(B*d + A*e))*Sqrt[d^2 - e^2*x^2])/(3*e^3) - ((C*d + B*e)*x
*Sqrt[d^2 - e^2*x^2])/(2*e^2) - (C*x^2*Sqrt[d^2 - e^2*x^2])/(3*e) + (d*(C*d
^2 + e*(B*d + 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{\int \frac{-3Ade^2-e\left(2Cd^2+3e(Bd+Ae)\right)x-3e^2(Cd+Be)x^2}{\sqrt{d^2-e^2x^2}}dx}{3e^2}$$

$$= -\frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2x^2}}{3e} + \frac{\int \frac{3de^2\left(Cd^2+e(Bd+2Ae)\right)+2e^3\left(2Cd^2+3e(Bc\right.}{\sqrt{d^2-e^2x^2}}}{6e^4}$$

$$= -\frac{\left(2Cd^2+3e(Bd+Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}$$

$$= -\frac{\left(2Cd^2+3e(Bd+Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}$$

$$= -\frac{\left(2Cd^2+3e(Bd+Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}$$

Mathematica [A] time = 0.11, size = 103, normalized size = 0.72

$$\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(2Ae+Bd)+Cd^2\right)-\sqrt{d^2-e^2x^2}\left(3e(2Ae+2Bd+Bex)+C\left(4d^2+3dex+2e^2x^2\right)\right)}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(3*e*(2*B*d + 2*A*e + B*e*x) + C*(4*d^2 + 3*d*e*x +
2*e^2*x^2))) + 3*d*(C*d^2 + e*(B*d + 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(6*e^3)
```

fricas [A] time = 0.97, size = 109, normalized size = 0.76

$$-\frac{6\left(Cd^3+Bd^2e+2\,Ade^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(2\,Ce^2x^2+4\,Cd^2+6\,Bde+6\,Ae^2+3\left(Cde+Be^2\right)x\right)\sqrt{-e^2x^2}}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(6*(C*d^3 + B*d^2*e + 2*A*d*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e
*x)) + (2*C*e^2*x^2 + 4*C*d^2 + 6*B*d*e + 6*A*e^2 + 3*(C*d*e + B*e^2)*x)*sq
rt(-e^2*x^2 + d^2))/e^3
```

giac [A] time = 0.29, size = 97, normalized size = 0.68

$$\frac{1}{2}\left(Cd^3+Bd^2e+2\,Ade^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{6}\sqrt{-x^2e^2+d^2}\left(\left(2\,Cxe^{(-1)}+3\left(Cde^3+Be^4\right)e^{(-5)}\right)x+2\left(2\,Cd^2e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(C*d^3 + B*d^2*e + 2*A*d*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/6*sqrt(-x
^2*e^2 + d^2)*((2*C*x*e^(-1) + 3*(C*d*e^3 + B*e^4)*e^(-5))*x + 2*(2*C*d^2*e
^2 + 3*B*d*e^3 + 3*A*e^4)*e^(-5))
```

maple [A] time = 0.01, size = 234, normalized size = 1.64

$$\frac{Ad\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+\frac{B\,d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e}+\frac{C\,d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e^2}-\frac{\sqrt{-e^2x^2+d^2}\,C\,x^2}{3e}-\frac{\sqrt{-e^2x^2+d^2}\,Bx}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/3*C*x^2*(-e^2*x^2+d^2)^(1/2)/e-2/3/e^3*C*d^2*(-e^2*x^2+d^2)^(1/2)-1/2*(-
e^2*x^2+d^2)^(1/2)*B/e*x-1/2*(-e^2*x^2+d^2)^(1/2)*C*d/e^2*x+1/2/(e^2)^(1/2)
*B*d^2/e*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/2/(e^2)^(1/2)*C*d^3/e
^2*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/e*(-e^2*x^2+d^2)^(1/2)*A-1/
e^2*(-e^2*x^2+d^2)^(1/2)*B*d+A*d/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d
^2)^(1/2)*x)
```

maxima [A] time = 0.98, size = 150, normalized size = 1.05

$$-\frac{\sqrt{-e^2x^2+d^2}\,Cx^2}{3\,e}+\frac{Ad\arcsin\left(\frac{ex}{d}\right)}{e}+\frac{(Cd+Be)d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^3}-\frac{2\,\sqrt{-e^2x^2+d^2}\,Cd^2}{3\,e^3}-\frac{\sqrt{-e^2x^2+d^2}\,Bd}{e^2}-\frac{\sqrt{-e^2x^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*C*x^2/e + A*d*arcsin(e*x/d)/e + 1/2*(C*d + B*e)*d
^2*arcsin(e*x/d)/e^3 - 2/3*sqrt(-e^2*x^2 + d^2)*C*d^2/e^3 - sqrt(-e^2*x^2 +
 d^2)*B*d/e^2 - sqrt(-e^2*x^2 + d^2)*A/e - 1/2*sqrt(-e^2*x^2 + d^2)*(C*d +
B*e)*x/e^2
```

mupad [B] time = 5.01, size = 270, normalized size = 1.89

$$\begin{cases} \frac{2\,C\,d\,x^3+3\,B\,d\,x^2+6\,A\,d\,x}{6\,\sqrt{d^2}} \\ \frac{A\,d\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{\sqrt{-e^2}}-\frac{A\,\sqrt{d^2-e^2\,x^2}}{e}-\frac{B\,d\,\sqrt{d^2-e^2\,x^2}}{e^2}-\frac{B\,x\,\sqrt{d^2-e^2\,x^2}}{2\,e}-\frac{C\,\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+e^2\,x^2\right)}{3\,e^3}-\frac{C\,d^3\,\ln\left(2\,x\,\sqrt{-e^2}+2\,\sqrt{d^2-}\right.}{2\,\left(-e^2\right)^{3/2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] piecewise(e == 0, (6*A*d*x + 3*B*d*x^2 + 2*C*d*x^3)/(6*(d^2)^(1/2)), e ~= 0
, - (A*(d^2 - e^2*x^2)^(1/2))/e + (A*d*log(x*(-e^2)^(1/2) + (d^2 - e^2*x^2)
^(1/2)))/(-e^2)^(1/2) - (B*d*(d^2 - e^2*x^2)^(1/2))/e^2 - (B*x*(d^2 - e^2*x
^2)^(1/2))/(2*e) - (C*(d^2 - e^2*x^2)^(1/2)*(2*d^2 + e^2*x^2))/(3*e^3) - (C
*d^3*log(2*x*(-e^2)^(1/2) + 2*(d^2 - e^2*x^2)^(1/2)))/(2*(-e^2)^(3/2)) - (B
*d^2*e*log(2*x*(-e^2)^(1/2) + 2*(d^2 - e^2*x^2)^(1/2)))/(2*(-e^2)^(3/2)) -
(C*d*x*(d^2 - e^2*x^2)^(1/2))/(2*e^2))
```

sympy [A] time = 10.17, size = 484, normalized size = 3.38

$$Ad\left(\begin{cases} \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2>0\wedge e^2>0 \\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2>0\wedge e^2<0 \\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for}\ d^2<0\wedge e^2<0 \end{cases}\right)+Ae\left(\begin{cases} \frac{x^2}{2\sqrt{d^2}} & \text{for}\ e^2=0 \\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise} \end{cases}\right)+Bd\left(\begin{cases} \frac{x^2}{2\sqrt{d^2}} & \text{for}\ e^2=0 \\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwis} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 >
0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2), (
d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d*
*2), (d**2 < 0) & (e**2 < 0))) + A*e*Piecewise((x**2/(2*sqrt(d**2)), Eq(e**
2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + B*d*Piecewise((x**2/(2*sqrt
(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + B*e*Piecewis
e((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2)
, Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1
 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True)) + C*d*Pie
cewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*
e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*s
qrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True)) + C*
e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 - e**
2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True))
```

3.13 $\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=87

$$\frac{(2Ae^2+Cd^2)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}-\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}$$

```
[Out] 1/2*(2*A*e^2+C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-B*(-e^2*x^2+d^2)^(
1/2)/e^2-1/2*C*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1815, 641, 217, 203}

$$\frac{(2Ae^2+Cd^2)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}-\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -((B*Sqrt[d^2 - e^2*x^2])/e^2) - (C*x*Sqrt[d^2 - e^2*x^2])/(2*e^2) + ((C*d^
2 + 2*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2} - \frac{\int \frac{-Cd^2-2Ae^2-2Be^2x}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}$$
$$= -\frac{B\sqrt{d^2-e^2x^2}}{e^2} - \frac{Cx\sqrt{d^2-e^2x^2}}{2e^2} - \frac{\left(-Cd^2-2Ae^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}$$
$$= -\frac{B\sqrt{d^2-e^2x^2}}{e^2} - \frac{Cx\sqrt{d^2-e^2x^2}}{2e^2} - \frac{\left(-Cd^2-2Ae^2\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$
$$= -\frac{B\sqrt{d^2-e^2x^2}}{e^2} - \frac{Cx\sqrt{d^2-e^2x^2}}{2e^2} + \frac{\left(Cd^2+2Ae^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.77

$$\frac{\left(2Ae^2+Cd^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - e(2B+Cx)\sqrt{d^2-e^2x^2}}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(e*(2*B + C*x)*Sqrt[d^2 - e^2*x^2]) + (C*d^2 + 2*A*e^2)*ArcTan[(e*x)/Sqrt
[d^2 - e^2*x^2]])/(2*e^3)
```

fricas [A] time = 0.64, size = 71, normalized size = 0.82

$$-\frac{2\left(Cd^2+2\,Ae^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \sqrt{-e^2x^2+d^2}\,(Cex+2\,Be)}{2\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(C*d^2 + 2*A*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(
-e^2*x^2 + d^2)*(C*e*x + 2*B*e))/e^3
```

giac [A] time = 0.34, size = 52, normalized size = 0.60

$$\frac{1}{2}\left(Cd^2+2\,Ae^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\text{sgn}(d) - \frac{1}{2}\sqrt{-x^2e^2+d^2}\left(Cxe^{(-2)}+2\,Be^{(-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(C*d^2 + 2*A*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/2*sqrt(-x^2*e^2 + d^2
)*(C*x*e^(-2) + 2*B*e^(-2))
```

maple [A] time = 0.01, size = 108, normalized size = 1.24

$$\frac{A\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} + \frac{C\,d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e^2} - \frac{\sqrt{-e^2x^2+d^2}\,Cx}{2e^2} - \frac{\sqrt{-e^2x^2+d^2}\,B}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/2*(-e^2*x^2+d^2)^(1/2)*C/e^2*x+1/2/(e^2)^(1/2)*C*d^2/e^2*arctan((e^2)^(1
/2)/(-e^2*x^2+d^2)^(1/2)*x)-B*(-e^2*x^2+d^2)^(1/2)/e^2+A/(e^2)^(1/2)*arctan
((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

maxima [A] time = 0.99, size = 70, normalized size = 0.80

$$\frac{Cd^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^3}+\frac{A\arcsin\left(\frac{ex}{d}\right)}{e}-\frac{\sqrt{-e^2x^2+d^2}\,Cx}{2\,e^2}-\frac{\sqrt{-e^2x^2+d^2}\,B}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*C*d^2*arcsin(e*x/d)/e^3 + A*arcsin(e*x/d)/e - 1/2*sqrt(-e^2*x^2 + d^2)*
C*x/e^2 - sqrt(-e^2*x^2 + d^2)*B/e^2
```

mupad [B] time = 4.40, size = 148, normalized size = 1.70

$$\begin{cases}\frac{2\,C\,x^3+3\,B\,x^2+6\,A\,x}{6\,\sqrt{d^2}} & \text{if } e=0\\ \frac{A\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{\sqrt{-e^2}}-\frac{B\,\sqrt{d^2-e^2\,x^2}}{e^2}-\frac{C\,x\,\sqrt{d^2-e^2\,x^2}}{2\,e^2}-\frac{C\,d^2\,\ln\left(2\,x\,\sqrt{-e^2}+2\,\sqrt{d^2-e^2\,x^2}\right)}{2\,{\left(-e^2\right)}^{3/2}} & \text{if } e\neq 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] piecewise(e == 0, (6*A*x + 3*B*x^2 + 2*C*x^3)/(6*(d^2)^(1/2)), e ~= 0, (A*l
og(x*(-e^2)^(1/2) + (d^2 - e^2*x^2)^(1/2)))/(-e^2)^(1/2) - (B*(d^2 - e^2*x^
2)^(1/2))/e^2 - (C*x*(d^2 - e^2*x^2)^(1/2))/(2*e^2) - (C*d^2*log(2*x*(-e^2)
^(1/2) + 2*(d^2 - e^2*x^2)^(1/2)))/(2*(-e^2)^(3/2)))
```

sympy [A] time = 4.56, size = 262, normalized size = 3.01

$$A\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for } d^2<0\wedge e^2<0\end{cases}\right)+B\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for } e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+C\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}-\frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 > 0)
 & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2), (d*
*2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d**2
), (d**2 < 0) & (e**2 < 0))) + B*Piecewise((x**2/(2*sqrt(d**2)), Eq(e**2, 0
)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + C*Piecewise((-I*d**2*acosh(e*x/
d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2)
 > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) +
 x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True))
```

3.14 $\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{d^2-e^2x^2}\,(Ae^2-Bde+Cd^2)}{de^3(d+ex)}-\frac{(Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}-\frac{C\sqrt{d^2-e^2x^2}}{e^3}$$

```
[Out] -(-B*e+C*d)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-C*(-e^2*x^2+d^2)^(1/2)/e^3
-(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)
```

Rubi [A] time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1639, 793, 217, 203}

$$-\frac{\sqrt{d^2-e^2x^2}\,(Ae^2-Bde+Cd^2)}{de^3(d+ex)}-\frac{(Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}-\frac{C\sqrt{d^2-e^2x^2}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((C*Sqrt[d^2 - e^2*x^2])/e^3) - ((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^
2])/(d*e^3*(d + e*x)) - ((C*d - B*e)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{C\sqrt{d^2-e^2x^2}}{e^3} - \frac{\int \frac{-Ae^4+e^3(Cd-Be)x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= -\frac{C\sqrt{d^2-e^2x^2}}{e^3} - \frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{de^3(d+ex)} - \frac{(Cd-Be)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}$$

$$= -\frac{C\sqrt{d^2-e^2x^2}}{e^3} - \frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{de^3(d+ex)} - \frac{(Cd-Be)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,\right.}{e^2}$$

$$= -\frac{C\sqrt{d^2-e^2x^2}}{e^3} - \frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{de^3(d+ex)} - \frac{(Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Mathematica [A] time = 0.16, size = 83, normalized size = 0.81

$$\frac{(Be-Cd)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{\sqrt{d^2-e^2x^2}\,(e(Ae-Bd)+Cd(2d+ex))}{d(d+ex)}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
[Out] (-((Sqrt[d^2 - e^2*x^2]*(e*(-(B*d) + A*e) + C*d*(2*d + e*x)))/(d*(d + e*x))
) + (-(C*d) + B*e)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

fricas [A] time = 0.85, size = 155, normalized size = 1.50

$$-\frac{2\,Cd^3 - Bd^2e + Ade^2 + \left(2\,Cd^2e - Bde^2 + Ae^3\right)x - 2\left(Cd^3 - Bd^2e + \left(Cd^2e - Bde^2\right)x\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) +}{de^4x + d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
[Out] -(2*C*d^3 - B*d^2*e + A*d*e^2 + (2*C*d^2*e - B*d*e^2 + A*e^3)*x - 2*(C*d^3
- B*d^2*e + (C*d^2*e - B*d*e^2)*x)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)
) + (C*d*e*x + 2*C*d^2 - B*d*e + A*e^2)*sqrt(-e^2*x^2 + d^2))/(d*e^4*x + d^
2*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
-4*A*exp(2)^2-4*C*d^2*exp(2)+4*B*d*exp(1)*exp(2))*atan((-1/2*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp
(1)^4+exp(2)^2)/d/exp(1)/exp(2)-1/4*(-4*B*exp(1)+4*C*d)*sign(d)*asin(x*exp(
2)/d/exp(1))/exp(1)/exp(2)-4*exp(1)^2*C*1/4/exp(1)^5*sqrt(-exp(2)*x^2+d^2)
```

maple [A] time = 0.01, size = 149, normalized size = 1.45

$$\frac{B\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e} - \frac{Cd\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e^2} - \frac{\sqrt{-e^2x^2+d^2}\,C}{e^3} - \frac{\left(A\,e^2-Bde+C\,d^2\right)\sqrt{2\left(x+\frac{d}{e}\right)de - \left(x+\frac{d}{e}\right.}}{\left(x+\frac{d}{e}\right)d\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -C*(-e^2*x^2+d^2)^(1/2)/e^3+1/e*B/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+
d^2)^(1/2)*x)-1/e^2*C*d/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)
*x)-(A*e^2-B*d*e+C*d^2)/e^4/d/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

maxima [A] time = 0.99, size = 138, normalized size = 1.34

$$-\frac{\sqrt{-e^2x^2+d^2}\,Cd}{e^4x+de^3}-\frac{\sqrt{-e^2x^2+d^2}\,A}{de^2x+d^2e}+\frac{\sqrt{-e^2x^2+d^2}\,B}{e^3x+de^2}-\frac{Cd\arcsin\left(\frac{ex}{d}\right)}{e^3}+\frac{B\arcsin\left(\frac{ex}{d}\right)}{e^2}-\frac{\sqrt{-e^2x^2+d^2}\,C}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-e^2*x^2 + d^2)*C*d/(e^4*x + d*e^3) - sqrt(-e^2*x^2 + d^2)*A/(d*e^2*x
 + d^2*e) + sqrt(-e^2*x^2 + d^2)*B/(e^3*x + d*e^2) - C*d*arcsin(e*x/d)/e^3
+ B*arcsin(e*x/d)/e^2 - sqrt(-e^2*x^2 + d^2)*C/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\,x^2+B\,x+A}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.15 $\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=163

$$-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{3d^2e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{3de^3(d+ex)^2}+\frac{\sqrt{d^2-e^2x^2}\,(2Cd-Be)}{de^3(d+ex)}+\frac{C\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] C*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1/3*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^
2)^(1/2)/d/e^3/(e*x+d)^2+(-B*e+2*C*d)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)-1/
3*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d^2/e^3/(e*x+d)
```

Rubi [A] time = 0.17, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1637, 217, 203, 659, 651}

$$-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{3d^2e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{3de^3(d+ex)^2}+\frac{\sqrt{d^2-e^2x^2}\,(2Cd-Be)}{de^3(d+ex)}+\frac{C\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(3*d*e^3*(d + e*x)^2) + ((2*
C*d - B*e)*Sqrt[d^2 - e^2*x^2])/(d*e^3*(d + e*x)) - ((C*d^2 - B*d*e + A*e^2
)*Sqrt[d^2 - e^2*x^2])/(3*d^2*e^3*(d + e*x)) + (C*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]])/e^3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
```

```
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = \int \left(\frac{C}{e^2\sqrt{d^2-e^2x^2}}+\frac{Cd^2-Bde+Ae^2}{e^2(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{-2Cd+Be}{e^2(d+ex)\sqrt{d^2-e^2x^2}}\right)dx$$
$$= \frac{C\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}-\frac{(2Cd-Be)\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e^2}+\frac{\left(Cd^2-Bde+Ae^2\right)\int \frac{1}{(d+ex)^2\sqrt{d^2\ldots}}}{e^2}$$
$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{3de^3(d+ex)^2}+\frac{(2Cd-Be)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}+\frac{C\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,\ldots\right.}{e^2}$$
$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{3de^3(d+ex)^2}+\frac{(2Cd-Be)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}-\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{\ldots}}{3d^2e^3(d+ex\ldots}$$

Mathematica [A] time = 0.22, size = 95, normalized size = 0.58

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(Cd^2(4d+5ex)-e(Ae(2d+ex)+Bd(d+2ex))\right)}{d^2(d+ex)^2}+3C\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(C*d^2*(4*d + 5*e*x) - e*(A*e*(2*d + e*x) + B*d*(d +
2*e*x))))/(d^2*(d + e*x)^2) + 3*C*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(3*e^3
)
```

fricas [A] time = 0.92, size = 221, normalized size = 1.36

$$\frac{4\,Cd^4-Bd^3e-2\,Ad^2e^2+\left(4\,Cd^2e^2-Bde^3-2\,Ae^4\right)x^2+2\left(4\,Cd^3e-Bd^2e^2-2\,Ade^3\right)x-6\left(Cd^2e^2x^2+2\,Cd^3ex+\ldots\right.}{3\left(d^2e^5x^2+2\,d^3e^4x+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/3*(4*C*d^4 - B*d^3*e - 2*A*d^2*e^2 + (4*C*d^2*e^2 - B*d*e^3 - 2*A*e^4)*x^
2 + 2*(4*C*d^3*e - B*d^2*e^2 - 2*A*d*e^3)*x - 6*(C*d^2*e^2*x^2 + 2*C*d^3*e*
x + C*d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4*C*d^3 - B*d^2*e -
 2*A*d*e^2 + (5*C*d^2*e - 2*B*d*e^2 - A*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^2*
e^5*x^2 + 2*d^3*e^4*x + d^4*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
```

```
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]index.cc index_m i_lex_is_greater Error: Bad Argument Value
```

maple [B] time = 0.02, size = 355, normalized size = 2.18

$$\frac{C\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e^2}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,A}{3\left(x+\frac{d}{e}\right)^2d\,e^3}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,A}{3\left(x+\frac{d}{e}\right)d^2e^2}+\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\right.}}{3\left(x+\frac{d}{e}\right)d\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] C/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/3/e^3/d/(x+d
/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*A+1/3/e^4/(x+d/e)^2*(2*(x+d/e)*d*
e-(x+d/e)^2*e^2)^(1/2)*B-1/3/e^5*d/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(1/2)*C-1/3/e^2/d^2/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*A+1/3/e^3/d
/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*B-1/3/e^4/(x+d/e)*(2*(x+d/e)*d
*e-(x+d/e)^2*e^2)^(1/2)*C-1/e^4*(B*e-2*C*d)/d/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e
)^2*e^2)^(1/2)
```

maxima [B] time = 1.00, size = 317, normalized size = 1.94

$$-\frac{\sqrt{-e^2x^2+d^2}\,Cd^2}{3\left(de^5x^2+2\,d^2e^4x+d^3e^3\right)}-\frac{\sqrt{-e^2x^2+d^2}\,Cd^2}{3\left(d^2e^4x+d^3e^3\right)}+\frac{\sqrt{-e^2x^2+d^2}\,Bd}{3\left(de^4x^2+2\,d^2e^3x+d^3e^2\right)}+\frac{\sqrt{-e^2x^2+d^2}\,Bd}{3\left(d^2e^3x+d^3e^2\right)}-\frac{\sqrt{-e^2x^2+}}{3\left(de^3x^2+2\,d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*C*d^2/(d*e^5*x^2 + 2*d^2*e^4*x + d^3*e^3) - 1/3*s
qrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^4*x + d^3*e^3) + 1/3*sqrt(-e^2*x^2 + d^2)*
B*d/(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) + 1/3*sqrt(-e^2*x^2 + d^2)*B*d/(d^2
*e^3*x + d^3*e^2) - 1/3*sqrt(-e^2*x^2 + d^2)*A/(d*e^3*x^2 + 2*d^2*e^2*x + d
^3*e) - 1/3*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^2*x + d^3*e) - sqrt(-e^2*x^2 + d^
2)*B/(d*e^3*x + d^2*e^2) + 2*sqrt(-e^2*x^2 + d^2)*C/(e^4*x + d*e^3) + C*arc
sin(e*x/d)/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\,x^2+B\,x+A}{\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**2), x)
```

3.16 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=180

$$-\frac{\sqrt{d^2-e^2x^2}\,\left(e(2Ae+3Bd)+7Cd^2\right)}{15d^2e^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\,\left(Ae^2-Bde+Cd^2\right)}{5de^3(d+ex)^3}-\frac{\sqrt{d^2-e^2x^2}\,\left(e(2Ae+3Bd)+7Cd^2\right)}{15d^3e^3(d+ex)}+\frac{C\sqrt{}}{e^3(}$$

```
[Out] -1/5*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)^3+C*(-e^2*x^2+d
^2)^(1/2)/e^3/(e*x+d)^2-1/15*(7*C*d^2+e*(2*A*e+3*B*d))*(-e^2*x^2+d^2)^(1/2)
/d^2/e^3/(e*x+d)^2-1/15*(7*C*d^2+e*(2*A*e+3*B*d))*(-e^2*x^2+d^2)^(1/2)/d^3/
e^3/(e*x+d)
```

Rubi [A] time = 0.21, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1639, 793, 659, 651}

$$-\frac{\sqrt{d^2-e^2x^2}\,\left(e(2Ae+3Bd)+7Cd^2\right)}{15d^3e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\,\left(e(2Ae+3Bd)+7Cd^2\right)}{15d^2e^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\,\left(Ae^2-Bde+Cd^2\right)}{5de^3(d+ex)^3}+\frac{C\sqrt{}}{e^3(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(5*d*e^3*(d + e*x)^3) + (C*S
qrt[d^2 - e^2*x^2])/(e^3*(d + e*x)^2) - ((7*C*d^2 + e*(3*B*d + 2*A*e))*Sqrt
[d^2 - e^2*x^2])/(15*d^2*e^3*(d + e*x)^2) - ((7*C*d^2 + e*(3*B*d + 2*A*e))*
Sqrt[d^2 - e^2*x^2])/(15*d^3*e^3*(d + e*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
```

```
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2} + \frac{\int \frac{e^2\left(2Cd^2+Ae^2\right)+e^3(Cd+Be)x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3} + \frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2} + \frac{\left(7Cd^2+e(3Bd+2Ae)\right)\int \frac{}{(d}}{5de^2}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3} + \frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2} - \frac{\left(7Cd^2+e(3Bd+2Ae)\right)\sqrt{d^2}}{15d^2e^3(d+ex)^2}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3} + \frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2} - \frac{\left(7Cd^2+e(3Bd+2Ae)\right)\sqrt{d^2}}{15d^2e^3(d+ex)^2}$$

Mathematica [A] time = 0.20, size = 103, normalized size = 0.57

$$-\frac{\sqrt{d^2-e^2x^2}\left(e\left(Ae\left(7d^2+6dex+2e^2x^2\right)+3Bd\left(d^2+3dex+e^2x^2\right)\right)+Cd^2\left(2d^2+6dex+7e^2x^2\right)\right)}{15d^3e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(C*d^2*(2*d^2 + 6*d*e*x + 7*e^2*x^2) + e*(3*B*d*
(d^2 + 3*d*e*x + e^2*x^2) + A*e*(7*d^2 + 6*d*e*x + 2*e^2*x^2))))/(d^3*e^3*(
d + e*x)^3)
```

fricas [A] time = 0.79, size = 244, normalized size = 1.36

$$-\frac{2\,Cd^5+3\,Bd^4e+7\,Ad^3e^2+\left(2\,Cd^2e^3+3\,Bde^4+7\,Ae^5\right)x^3+3\left(2\,Cd^3e^2+3\,Bd^2e^3+7\,Ade^4\right)x^2+3\left(2\,Cd^4e+\right.}{15\left(d^3e^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/15*(2*C*d^5 + 3*B*d^4*e + 7*A*d^3*e^2 + (2*C*d^2*e^3 + 3*B*d*e^4 + 7*A*e
^5)*x^3 + 3*(2*C*d^3*e^2 + 3*B*d^2*e^3 + 7*A*d*e^4)*x^2 + 3*(2*C*d^4*e + 3*
B*d^3*e^2 + 7*A*d^2*e^3)*x + (2*C*d^4 + 3*B*d^3*e + 7*A*d^2*e^2 + (7*C*d^2*
e^2 + 3*B*d*e^3 + 2*A*e^4)*x^2 + 3*(2*C*d^3*e + 3*B*d^2*e^2 + 2*A*d*e^3)*x)
*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^3 + 3*d^4*e^5*x^2 + 3*d^5*e^4*x + d^6*e^3
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-2*A
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp
(2)^2+7*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp
(1)^6*exp(2)^3-2*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^2*exp(1)^10*exp(2)-2*B*d*exp(1)*exp(2)^5+2*C*d^2*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^2+5*A*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^4-11/2*A*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^4/x/exp(2)+A*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^2/x/exp(2)-5*B*d*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^5*exp(2
)^3-2*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp
(1)^9*exp(2)-2*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^2*exp(1)*exp(2)^5-3*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^3*exp(1)^3*exp(2)^4+3*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^3+C*d^2*(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(2)^5-A*exp(1)^6*exp(2)^3-B*d*exp
(1)^5*exp(2)^3+3*C*d^2*exp(1)^4*exp(2)^3+4*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^6+4*A*exp(2)^6+6*C*d^2*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)+1/2*C*d^2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^5/x/exp(2)+5/2*B*d*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^3*exp(2)^4/x/exp(2)+2*B*d*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*exp(2)^2/x/exp(2)-5*C*d^2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^2/x/exp(2))/((
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^2/(-d^3*exp(1)^9+2*d^3*exp(1
)^5*exp(2)^2-d^3*exp(1)*exp(2)^4)+1/2*(-2*A*exp(1)^4*exp(2)^3+6*B*d*exp(1)^
3*exp(2)^3-2*C*d^2*exp(2)^4-4*A*exp(2)^5-4*C*d^2*exp(1)^6*exp(2))*atan((-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(
2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(d^3*exp(1)^9-2*d^3*exp(1)^5*exp(2)^2+d^3*e
xp(1)*exp(2)^4)
```

maple [A] time = 0.01, size = 116, normalized size = 0.64

$$-\frac{(-ex+d)\left(2A\,e^4x^2+3Bd\,e^3x^2+7C\,d^2e^2x^2+6Ad\,e^3x+9B\,d^2e^2x+6C\,d^3ex+7A\,d^2e^2+3B\,d^3e+2C\,d^4\right)}{15\,(ex+d)^2\sqrt{-e^2x^2+d^2}\;d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/15*(-e*x+d)*(2*A*e^4*x^2+3*B*d*e^3*x^2+7*C*d^2*e^2*x^2+6*A*d*e^3*x+9*B*d
^2*e^2*x+6*C*d^3*e*x+7*A*d^2*e^2+3*B*d^3*e+2*C*d^4)/(e*x+d)^2/d^3/e^3/(-e^2
*x^2+d^2)^(1/2)
```

maxima [B] time = 1.02, size = 608, normalized size = 3.38

$$-\frac{\sqrt{-e^2x^2+d^2}\,Cd^2}{5\left(de^6x^3+3\,d^2e^5x^2+3\,d^3e^4x+d^4e^3\right)}-\frac{2\sqrt{-e^2x^2+d^2}\,Cd^2}{15\left(d^2e^5x^2+2\,d^3e^4x+d^4e^3\right)}-\frac{2\sqrt{-e^2x^2+d^2}\,Cd^2}{15\left(d^3e^4x+d^4e^3\right)}+\frac{\sqrt{-e^2x^2+}}{5\left(de^5x^3+3\,d^2e^4x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*C*d^2/(d*e^6*x^3 + 3*d^2*e^5*x^2 + 3*d^3*e^4*x +
d^4*e^3) - 2/15*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^5*x^2 + 2*d^3*e^4*x + d^4
*e^3) - 2/15*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^3*e^4*x + d^4*e^3) + 1/5*sqrt(-e
^2*x^2 + d^2)*B*d/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4*e^2) + 2/1
5*sqrt(-e^2*x^2 + d^2)*B*d/(d^2*e^4*x^2 + 2*d^3*e^3*x + d^4*e^2) + 2/15*sqr
t(-e^2*x^2 + d^2)*B*d/(d^3*e^3*x + d^4*e^2) + 2/3*sqrt(-e^2*x^2 + d^2)*C*d/
```

```
(d*e^5*x^2 + 2*d^2*e^4*x + d^3*e^3) + 2/3*sqrt(-e^2*x^2 + d^2)*C*d/(d^2*e^4
*x + d^3*e^3) - 1/5*sqrt(-e^2*x^2 + d^2)*A/(d*e^4*x^3 + 3*d^2*e^3*x^2 + 3*d
^3*e^2*x + d^4*e) - 2/15*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^3*x^2 + 2*d^3*e^2*x
+ d^4*e) - 2/15*sqrt(-e^2*x^2 + d^2)*A/(d^3*e^2*x + d^4*e) - 1/3*sqrt(-e^2*
x^2 + d^2)*B/(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) - 1/3*sqrt(-e^2*x^2 + d^2)
*B/(d^2*e^3*x + d^3*e^2) - sqrt(-e^2*x^2 + d^2)*C/(d*e^4*x + d^2*e^3)
```

mupad [B] time = 3.80, size = 109, normalized size = 0.61

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,C\,d^4+6\,C\,d^3\,e\,x+3\,B\,d^3\,e+7\,C\,d^2\,e^2\,x^2+9\,B\,d^2\,e^2\,x+7\,A\,d^2\,e^2+3\,B\,d\,e^3\,x^2+6\,A\,d\,e^3\,x+2}{15\,d^3\,e^3\,{\left(d+e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(2*C*d^4 + 7*A*d^2*e^2 + 2*A*e^4*x^2 + 3*B*d^3*e +
7*C*d^2*e^2*x^2 + 6*A*d*e^3*x + 6*C*d^3*e*x + 9*B*d^2*e^2*x + 3*B*d*e^3*x^2
))/(15*d^3*e^3*(d + e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.17 $\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=234

$$-\frac{\sqrt{d^2-e^2x^2}\left(6Ae^2+8Bde+13Cd^2\right)}{70d^2e^3(d+ex)^3}-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{7de^3(d+ex)^4}-\frac{\sqrt{d^2-e^2x^2}\left(6Ae^2+8Bde+13Cd^2\right)}{105d^4e^3(d+ex)}-\frac{\sqrt{d}}{}$$

```
[Out] -1/7*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)^4+1/2*C*(-e^2*x
^2+d^2)^(1/2)/e^3/(e*x+d)^3-1/70*(6*A*e^2+8*B*d*e+13*C*d^2)*(-e^2*x^2+d^2)^
(1/2)/d^2/e^3/(e*x+d)^3-1/105*(6*A*e^2+8*B*d*e+13*C*d^2)*(-e^2*x^2+d^2)^(1/
2)/d^3/e^3/(e*x+d)^2-1/105*(6*A*e^2+8*B*d*e+13*C*d^2)*(-e^2*x^2+d^2)^(1/2)/
d^4/e^3/(e*x+d)
```

Rubi [A] time = 0.25, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1639, 793, 659, 651}

$$-\frac{\sqrt{d^2-e^2x^2}\left(6Ae^2+8Bde+13Cd^2\right)}{105d^4e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\left(6Ae^2+8Bde+13Cd^2\right)}{105d^3e^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\left(6Ae^2+8Bde+13Cd^2\right)}{70d^2e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(7*d*e^3*(d + e*x)^4) + (C*S
qrt[d^2 - e^2*x^2])/(2*e^3*(d + e*x)^3) - ((13*C*d^2 + 8*B*d*e + 6*A*e^2)*S
qrt[d^2 - e^2*x^2])/(70*d^2*e^3*(d + e*x)^3) - ((13*C*d^2 + 8*B*d*e + 6*A*e
^2)*Sqrt[d^2 - e^2*x^2])/(105*d^3*e^3*(d + e*x)^2) - ((13*C*d^2 + 8*B*d*e +
 6*A*e^2)*Sqrt[d^2 - e^2*x^2])/(105*d^4*e^3*(d + e*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
```

```
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} + \frac{\int \frac{e^2(3Cd^2+2Ae^2)+e^3(Cd+2Be)x}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx}{2e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} + \frac{\left(13Cd^2+8Bde+6Ae^2\right)\int}{14de^2}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} - \frac{\left(13Cd^2+8Bde+6Ae^2\right)\sqrt{d^2}}{70d^2e^3(d+ex)^3}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} - \frac{\left(13Cd^2+8Bde+6Ae^2\right)\sqrt{d^2}}{70d^2e^3(d+ex)^3}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} - \frac{\left(13Cd^2+8Bde+6Ae^2\right)\sqrt{d^2}}{70d^2e^3(d+ex)^3}$$

Mathematica [A] time = 0.22, size = 139, normalized size = 0.59

$$-\frac{\sqrt{d^2-e^2x^2}\left(e\left(3Ae\left(12d^3+13d^2ex+8de^2x^2+2e^3x^3\right)+Bd\left(13d^3+52d^2ex+32de^2x^2+8e^3x^3\right)\right)+Cd^2\left(8d^3\right.\right.}{105d^4e^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/105*(Sqrt[d^2 - e^2*x^2]*(C*d^2*(8*d^3 + 32*d^2*e*x + 52*d*e^2*x^2 + 13*
e^3*x^3) + e*(3*A*e*(12*d^3 + 13*d^2*e*x + 8*d*e^2*x^2 + 2*e^3*x^3) + B*d*(
13*d^3 + 52*d^2*e*x + 32*d*e^2*x^2 + 8*e^3*x^3))))/(d^4*e^3*(d + e*x)^4)
```

fricas [A] time = 0.92, size = 320, normalized size = 1.37

$$-\frac{8\,Cd^6+13\,Bd^5e+36\,Ad^4e^2+\left(8\,Cd^2e^4+13\,Bde^5+36\,Ae^6\right)x^4+4\left(8\,Cd^3e^3+13\,Bd^2e^4+36\,Ade^5\right)x^3+6\left(8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/105*(8*C*d^6 + 13*B*d^5*e + 36*A*d^4*e^2 + (8*C*d^2*e^4 + 13*B*d*e^5 + 3
6*A*e^6)*x^4 + 4*(8*C*d^3*e^3 + 13*B*d^2*e^4 + 36*A*d*e^5)*x^3 + 6*(8*C*d^4
*e^2 + 13*B*d^3*e^3 + 36*A*d^2*e^4)*x^2 + 4*(8*C*d^5*e + 13*B*d^4*e^2 + 36*
A*d^3*e^3)*x + (8*C*d^5 + 13*B*d^4*e + 36*A*d^3*e^2 + (13*C*d^2*e^3 + 8*B*d
*e^4 + 6*A*e^5)*x^3 + 4*(13*C*d^3*e^2 + 8*B*d^2*e^3 + 6*A*d*e^4)*x^2 + (32*
C*d^4*e + 52*B*d^3*e^2 + 39*A*d^2*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^4*e^7*x^
4 + 4*d^5*e^6*x^3 + 6*d^6*e^5*x^2 + 4*d^7*e^4*x + d^8*e^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (64*C
*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^1
0*exp(2)^3-18*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^2*exp(1)^10*exp(2)^4+8*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^3*exp(1)^16*exp(2)+12*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^4*exp(1)^14*exp(2)^2+6*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^12*exp(2)^3+6*B*d*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^13*exp(2)^2+12*A*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^14*exp(2)^2-8
*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^12*
exp(2)^3-36*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4
*exp(1)^10*exp(2)^4-18*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^5*exp(1)^8*exp(2)^5+6*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^2*exp(1)^13*exp(2)^2-34*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^11*exp(2)^3-33*B*d*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^9*exp(2)^4-3*B*d*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^7*exp(2)
^5+24*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*e
xp(1)^12*exp(2)^2+60*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^4*exp(1)^8*exp(2)^4+12*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5+42*A*(-1/2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^5+81*A*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^6+27*A*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^4*exp(2)^7
-84*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1
)^9*exp(2)^4-84*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^3*exp(1)^7*exp(2)^5-42*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^4*exp(1)^5*exp(2)^6-12*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^3*exp(2)^7+102*C*d^2*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)^4+78*C*d^2*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)
^5+15*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*e
xp(1)^4*exp(2)^6+3*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^5*exp(2)^8+2*A*exp(1)^10*exp(2)^4+120*A*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^6+108*A*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^7+B*d*exp(1
)^9*exp(2)^4+2*C*d^2*exp(1)^8*exp(2)^4-60*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^5*exp(2)^6+18*A*(-1/2*(-2*d*exp(1)-
2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^9-10*B*d*exp(1)^5*exp(2)^
6-6*B*d*exp(1)*exp(2)^8-36*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^3*exp(1)^3*exp(2)^7+24*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6-5*A*exp(1)^6*exp(2)^6+13*
C*d^2*exp(1)^4*exp(2)^6+36*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^2*exp(2)^9+18*A*exp(2)^9-81/2*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))*exp(1)^4*exp(2)^7/x/exp(2)+6*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))*exp(1)^8*exp(2)^5/x/exp(2)-3*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))*exp(1)^12*exp(2)^3/x/exp(2)-12*B*d*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)*exp(2)^8+4*B*d*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^15*exp(2)-6*B*d*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)*exp(2)^8+8*C
*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^1
4*exp(2)+3/2*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/e
xp(2)+12*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^3*exp(2)^7/
x/exp(2)+57/2*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*exp(
2)^5/x/exp(2)-3*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^11*e
xp(2)^3/x/exp(2)-33*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1
```

```
)^6*exp(2)^5/x/exp(2)-6*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*e
xp(1)^10*exp(2)^3/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2)
)^3/(3*d^4*exp(1)^13-9*d^4*exp(1)^9*exp(2)^2+9*d^4*exp(1)^5*exp(2)^4-3*d^4*
exp(1)*exp(2)^6)+1/2*(2*B*d*exp(1)^7*exp(2)^3-8*C*d^2*exp(1)^6*exp(2)^3-6*A
*exp(1)^4*exp(2)^5+8*B*d*exp(1)^3*exp(2)^5-2*C*d^2*exp(2)^6-4*A*exp(2)^7)*a
tan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1
)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-d^4*exp(1)^13+3*d^4*exp(1)^9*exp(
2)^2-3*d^4*exp(1)^5*exp(2)^4+d^4*exp(1)*exp(2)^6)
```

maple [A] time = 0.01, size = 152, normalized size = 0.65

$$-\frac{(-ex+d)\left(6A e^5x^3+8Bd e^4x^3+13C d^2e^3x^3+24Ad e^4x^2+32B d^2e^3x^2+52C d^3e^2x^2+39A d^2e^3x+52B d^3e\right.}{105\,(ex+d)^3\sqrt{-e^2x^2+d^2}\;d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/105*(-e*x+d)*(6*A*e^5*x^3+8*B*d*e^4*x^3+13*C*d^2*e^3*x^3+24*A*d*e^4*x^2+
32*B*d^2*e^3*x^2+52*C*d^3*e^2*x^2+39*A*d^2*e^3*x+52*B*d^3*e^2*x+32*C*d^4*e*
x+36*A*d^3*e^2+13*B*d^4*e+8*C*d^5)/(e*x+d)^3/d^4/e^3/(-e^2*x^2+d^2)^(1/2)
```

maxima [B] time = 1.06, size = 975, normalized size = 4.17

$$-\frac{\sqrt{-e^2x^2+d^2}\,Cd^2}{7\left(de^7x^4+4\,d^2e^6x^3+6\,d^3e^5x^2+4\,d^4e^4x+d^5e^3\right)}-\frac{3\,\sqrt{-e^2x^2+d^2}\,Cd^2}{35\left(d^2e^6x^3+3\,d^3e^5x^2+3\,d^4e^4x+d^5e^3\right)}-\frac{2\,\sqrt{-e^2x^2+}}{35\left(d^3e^5x^2+2\,d^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/7*sqrt(-e^2*x^2 + d^2)*C*d^2/(d*e^7*x^4 + 4*d^2*e^6*x^3 + 6*d^3*e^5*x^2
+ 4*d^4*e^4*x + d^5*e^3) - 3/35*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^6*x^3 + 3
*d^3*e^5*x^2 + 3*d^4*e^4*x + d^5*e^3) - 2/35*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^
3*e^5*x^2 + 2*d^4*e^4*x + d^5*e^3) - 2/35*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^4*e
^4*x + d^5*e^3) + 1/7*sqrt(-e^2*x^2 + d^2)*B*d/(d*e^6*x^4 + 4*d^2*e^5*x^3 +
 6*d^3*e^4*x^2 + 4*d^4*e^3*x + d^5*e^2) + 3/35*sqrt(-e^2*x^2 + d^2)*B*d/(d^
2*e^5*x^3 + 3*d^3*e^4*x^2 + 3*d^4*e^3*x + d^5*e^2) + 2/35*sqrt(-e^2*x^2 + d
^2)*B*d/(d^3*e^4*x^2 + 2*d^4*e^3*x + d^5*e^2) + 2/35*sqrt(-e^2*x^2 + d^2)*B
*d/(d^4*e^3*x + d^5*e^2) + 2/5*sqrt(-e^2*x^2 + d^2)*C*d/(d*e^6*x^3 + 3*d^2*
e^5*x^2 + 3*d^3*e^4*x + d^4*e^3) + 4/15*sqrt(-e^2*x^2 + d^2)*C*d/(d^2*e^5*x
^2 + 2*d^3*e^4*x + d^4*e^3) + 4/15*sqrt(-e^2*x^2 + d^2)*C*d/(d^3*e^4*x + d^
4*e^3) - 1/7*sqrt(-e^2*x^2 + d^2)*A/(d*e^5*x^4 + 4*d^2*e^4*x^3 + 6*d^3*e^3*
x^2 + 4*d^4*e^2*x + d^5*e) - 3/35*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^4*x^3 + 3*d
^3*e^3*x^2 + 3*d^4*e^2*x + d^5*e) - 2/35*sqrt(-e^2*x^2 + d^2)*A/(d^3*e^3*x^
2 + 2*d^4*e^2*x + d^5*e) - 2/35*sqrt(-e^2*x^2 + d^2)*A/(d^4*e^2*x + d^5*e)
- 1/5*sqrt(-e^2*x^2 + d^2)*B/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4
*e^2) - 2/15*sqrt(-e^2*x^2 + d^2)*B/(d^2*e^4*x^2 + 2*d^3*e^3*x + d^4*e^2) -
 2/15*sqrt(-e^2*x^2 + d^2)*B/(d^3*e^3*x + d^4*e^2) - 1/3*sqrt(-e^2*x^2 + d^
2)*C/(d*e^5*x^2 + 2*d^2*e^4*x + d^3*e^3) - 1/3*sqrt(-e^2*x^2 + d^2)*C/(d^2*
e^4*x + d^3*e^3)
```

mupad [B] time = 3.78, size = 204, normalized size = 0.87

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{C}{5\,e^3}-\frac{-4\,C\,d^2+4\,B\,d\,e+3\,A\,e^2}{35\,d^2\,e^3}\right)}{{\left(d+e\,x\right)}^3}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{A}{7\,d\,e}+\frac{d\,\left(\frac{C}{7\,e^2}-\frac{B}{7\,d\,e}\right)}{e}\right)}{{\left(d+e\,x\right)}^4}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(13\,C\,d^2+8\,B\,d\,e+6\,A\right.}{105\,d^3\,e^3\,{\left(d+e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(C/(5*e^3) - (3*A*e^2 - 4*C*d^2 + 4*B*d*e)/(35*d^2*e
^3)))/(d + e*x)^3 - ((d^2 - e^2*x^2)^(1/2)*(A/(7*d*e) + (d*(C/(7*e^2) - B/(
7*d*e)))/e))/(d + e*x)^4 - ((d^2 - e^2*x^2)^(1/2)*(6*A*e^2 + 13*C*d^2 + 8*B
*d*e))/(105*d^3*e^3*(d + e*x)^2) - ((d^2 - e^2*x^2)^(1/2)*(6*A*e^2 + 13*C*d
^2 + 8*B*d*e))/(105*d^4*e^3*(d + e*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{\sqrt{-(-d + ex)(d + ex)}\,(d + ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**4/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**4), x)
```

3.18 $\int (d+ex)^3 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=175

$$\frac{(d+ex)^6 \left(aCe^2+c\left(6Cd^2-e(3Bd-Ae)\right)\right)}{6e^5}-\frac{(d+ex)^5 \left(ae^2(2Cd-Be)+cd\left(4Cd^2-e(3Bd-2Ae)\right)\right)}{5e^5}+\frac{(d+ex)^4}{}$$

```
[Out] 1/4*(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)*(e*x+d)^4/e^5-1/5*(a*e^2*(-B*e+2*C*d)
+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*(e*x+d)^5/e^5+1/6*(a*C*e^2+c*(6*C*d^2-e*(-
A*e+3*B*d)))*(e*x+d)^6/e^5-1/7*c*(-B*e+4*C*d)*(e*x+d)^7/e^5+1/8*c*C*(e*x+d)
^8/e^5
```

Rubi [A] time = 0.31, antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1628}

$$\frac{(d+ex)^6 \left(aCe^2-ce(3Bd-Ae)+6cCd^2\right)}{6e^5}-\frac{(d+ex)^5 \left(ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3\right)}{5e^5}+\frac{(d+ex)^4 \left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*(d + e*x)^4)/(4*e^5) - ((4*c*C*d^3
 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*(d + e*x)^5)/(5*e^5) + ((6*
c*C*d^2 + a*C*e^2 - c*e*(3*B*d - A*e))*(d + e*x)^6)/(6*e^5) - (c*(4*C*d - B
*e)*(d + e*x)^7)/(7*e^5) + (c*C*(d + e*x)^8)/(8*e^5)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) dx = \int \left(\frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde+Ae^2\right)(d+ex)^3}{e^4}+\frac{\left(-4cCd^3+cde(3B\right.}{}\right.$$

$$=\frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde+Ae^2\right)(d+ex)^4}{4e^5}-\frac{\left(4cCd^3-cde(3Bd-2\right.}{}$$

Mathematica [A] time = 0.09, size = 208, normalized size = 1.19

$$\frac{1}{5}x^5 \left(ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3\right)+\frac{1}{6}ex^6 \left(aCe^2+ce(Ae+3Bd)+3cCd^2\right)+\frac{1}{3}dx^3 \left(A\left(3ae^2+cd^2\right)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d^3*x + (a*d^2*(B*d + 3*A*e)*x^2)/2 + (d*(a*d*(C*d + 3*B*e) + A*(c*d^2
+ 3*a*e^2))*x^3)/3 + ((B*c*d^3 + 3*A*c*d^2*e + 3*a*C*d^2*e + 3*a*B*d*e^2 +
a*A*e^3)*x^4)/4 + ((c*C*d^3 + 3*c*d*e*(B*d + A*e) + a*e^2*(3*C*d + B*e))*x^
5)/5 + (e*(3*c*C*d^2 + a*C*e^2 + c*e*(3*B*d + A*e))*x^6)/6 + (c*e^2*(3*C*d
+ B*e)*x^7)/7 + (c*C*e^3*x^8)/8
```

fricas [A] time = 0.75, size = 248, normalized size = 1.42

$$\frac{1}{8}x^8e^3cC+\frac{3}{7}x^7e^2dcC+\frac{1}{7}x^7e^3cB+\frac{1}{2}x^6ed^2cC+\frac{1}{6}x^6e^3aC+\frac{1}{2}x^6e^2dcB+\frac{1}{6}x^6e^3cA+\frac{1}{5}x^5d^3cC+\frac{3}{5}x^5e^2daC+\frac{3}{5}x^5ed^2cB+\frac{1}{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e^3*c*C + 3/7*x^7*e^2*d*c*C + 1/7*x^7*e^3*c*B + 1/2*x^6*e*d^2*c*C +
 1/6*x^6*e^3*a*C + 1/2*x^6*e^2*d*c*B + 1/6*x^6*e^3*c*A + 1/5*x^5*d^3*c*C +
3/5*x^5*e^2*d*a*C + 3/5*x^5*e*d^2*c*B + 1/5*x^5*e^3*a*B + 3/5*x^5*e^2*d*c*A
 + 3/4*x^4*e*d^2*a*C + 1/4*x^4*d^3*c*B + 3/4*x^4*e^2*d*a*B + 3/4*x^4*e*d^2*
c*A + 1/4*x^4*e^3*a*A + 1/3*x^3*d^3*a*C + x^3*e*d^2*a*B + 1/3*x^3*d^3*c*A +
 x^3*e^2*d*a*A + 1/2*x^2*d^3*a*B + 3/2*x^2*e*d^2*a*A + x*d^3*a*A
```

giac [A] time = 0.17, size = 242, normalized size = 1.38

$$\frac{1}{8}Ccx^8e^3+\frac{3}{7}Ccdx^7e^2+\frac{1}{2}Ccd^2x^6e+\frac{1}{5}Ccd^3x^5+\frac{1}{7}Bcx^7e^3+\frac{1}{2}Bcdx^6e^2+\frac{3}{5}Bcd^2x^5e+\frac{1}{4}Bcd^3x^4+\frac{1}{6}Cax^6e^3+\frac{1}{6}Acx^6e^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*C*c*x^8*e^3 + 3/7*C*c*d*x^7*e^2 + 1/2*C*c*d^2*x^6*e + 1/5*C*c*d^3*x^5 +
 1/7*B*c*x^7*e^3 + 1/2*B*c*d*x^6*e^2 + 3/5*B*c*d^2*x^5*e + 1/4*B*c*d^3*x^4
+ 1/6*C*a*x^6*e^3 + 1/6*A*c*x^6*e^3 + 3/5*C*a*d*x^5*e^2 + 3/5*A*c*d*x^5*e^2
 + 3/4*C*a*d^2*x^4*e + 3/4*A*c*d^2*x^4*e + 1/3*C*a*d^3*x^3 + 1/3*A*c*d^3*x^
3 + 1/5*B*a*x^5*e^3 + 3/4*B*a*d*x^4*e^2 + B*a*d^2*x^3*e + 1/2*B*a*d^3*x^2 +
 1/4*A*a*x^4*e^3 + A*a*d*x^3*e^2 + 3/2*A*a*d^2*x^2*e + A*a*d^3*x
```

maple [A] time = 0.00, size = 217, normalized size = 1.24

$$\frac{Cc\,e^3x^8}{8}+\frac{\left(e^3cB+3d\,e^2cC\right)x^7}{7}+Aa\,d^3x+\frac{\left(Ac\,e^3+3Bcd\,e^2+\left(e^3a+3d^2ec\right)C\right)x^6}{6}+\frac{\left(3Acd\,e^2+\left(e^3a+3d^2ec\right)B+\left(3\right.\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x)
```

```
[Out] 1/8*e^3*c*C*x^8+1/7*(B*c*e^3+3*C*c*d*e^2)*x^7+1/6*((a*e^3+3*c*d^2*e)*C+3*d*
e^2*c*B+e^3*c*A)*x^6+1/5*((3*a*d*e^2+c*d^3)*C+(a*e^3+3*c*d^2*e)*B+3*d*e^2*c
*A)*x^5+1/4*(3*d^2*e*a*C+(3*a*d*e^2+c*d^3)*B+(a*e^3+3*c*d^2*e)*A)*x^4+1/3*(
d^3*a*C+3*d^2*e*a*B+(3*a*d*e^2+c*d^3)*A)*x^3+1/2*(3*A*a*d^2*e+B*a*d^3)*x^2+
d^3*a*A*x
```

maxima [A] time = 0.45, size = 202, normalized size = 1.15

$$\frac{1}{8}Cce^3x^8+\frac{1}{7}\left(3\,Ccde^2+Bce^3\right)x^7+\frac{1}{6}\left(3\,Ccd^2e+3\,Bcde^2+(Ca+Ac)e^3\right)x^6+Aad^3x+\frac{1}{5}\left(Ccd^3+3\,Bcd^2e+Bae^3+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*C*c*e^3*x^8 + 1/7*(3*C*c*d*e^2 + B*c*e^3)*x^7 + 1/6*(3*C*c*d^2*e + 3*B*
c*d*e^2 + (C*a + A*c)*e^3)*x^6 + A*a*d^3*x + 1/5*(C*c*d^3 + 3*B*c*d^2*e + B
*a*e^3 + 3*(C*a + A*c)*d*e^2)*x^5 + 1/4*(B*c*d^3 + 3*B*a*d*e^2 + A*a*e^3 +
3*(C*a + A*c)*d^2*e)*x^4 + 1/3*(3*B*a*d^2*e + 3*A*a*d*e^2 + (C*a + A*c)*d^3
)*x^3 + 1/2*(B*a*d^3 + 3*A*a*d^2*e)*x^2
```

mupad [B] time = 0.09, size = 206, normalized size = 1.18

$$x^3\left(\frac{A\,c\,d^3}{3}+\frac{C\,a\,d^3}{3}+A\,a\,d\,e^2+B\,a\,d^2\,e\right)+x^6\left(\frac{A\,c\,e^3}{6}+\frac{C\,a\,e^3}{6}+\frac{B\,c\,d\,e^2}{2}+\frac{C\,c\,d^2\,e}{2}\right)+x^4\left(\frac{A\,a\,e^3}{4}+\frac{B\,c\,d^3}{4}+\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*(d + e*x)^3*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*c*d^3)/3 + (C*a*d^3)/3 + A*a*d*e^2 + B*a*d^2*e) + x^6*((A*c*e^3)/6
+ (C*a*e^3)/6 + (B*c*d*e^2)/2 + (C*c*d^2*e)/2) + x^4*((A*a*e^3)/4 + (B*c*d^
3)/4 + (3*B*a*d*e^2)/4 + (3*A*c*d^2*e)/4 + (3*C*a*d^2*e)/4) + x^5*((B*a*e^3
)/5 + (C*c*d^3)/5 + (3*A*c*d*e^2)/5 + (3*C*a*d*e^2)/5 + (3*B*c*d^2*e)/5) +
A*a*d^3*x + (C*c*e^3*x^8)/8 + (a*d^2*x^2*(3*A*e + B*d))/2 + (c*e^2*x^7*(B*e
 + 3*C*d))/7
```

sympy [A] time = 0.12, size = 257, normalized size = 1.47

$$Aad^3x+\frac{Cce^3x^8}{8}+x^7\left(\frac{Bce^3}{7}+\frac{3Ccde^2}{7}\right)+x^6\left(\frac{Ace^3}{6}+\frac{Bcde^2}{2}+\frac{Cae^3}{6}+\frac{Ccd^2e}{2}\right)+x^5\left(\frac{3Acde^2}{5}+\frac{Bae^3}{5}+\frac{3Bcd^2e}{5}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*d**3*x + C*c*e**3*x**8/8 + x**7*(B*c*e**3/7 + 3*C*c*d*e**2/7) + x**6*(A
*c*e**3/6 + B*c*d*e**2/2 + C*a*e**3/6 + C*c*d**2*e/2) + x**5*(3*A*c*d*e**2/
5 + B*a*e**3/5 + 3*B*c*d**2*e/5 + 3*C*a*d*e**2/5 + C*c*d**3/5) + x**4*(A*a*
e**3/4 + 3*A*c*d**2*e/4 + 3*B*a*d*e**2/4 + B*c*d**3/4 + 3*C*a*d**2*e/4) + x
**3*(A*a*d*e**2 + A*c*d**3/3 + B*a*d**2*e + C*a*d**3/3) + x**2*(3*A*a*d**2*
e/2 + B*a*d**3/2)
```

3.19 $\int (d+ex)^2 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=175

$$\frac{(d+ex)^5 \left(aCe^2+c\left(6Cd^2-e(3Bd-Ae)\right)\right)}{5e^5}-\frac{(d+ex)^4 \left(ae^2(2Cd-Be)+cd\left(4Cd^2-e(3Bd-2Ae)\right)\right)}{4e^5}+\frac{(d+ex)^3 \left(a\right.}{}$$

```
[Out] 1/3*(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)*(e*x+d)^3/e^5-1/4*(a*e^2*(-B*e+2*C*d)
+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*(e*x+d)^4/e^5+1/5*(a*C*e^2+c*(6*C*d^2-e*(-
A*e+3*B*d)))*(e*x+d)^5/e^5-1/6*c*(-B*e+4*C*d)*(e*x+d)^6/e^5+1/7*c*C*(e*x+d)
^7/e^5
```

Rubi [A] time = 0.22, antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1628}

$$\frac{(d+ex)^5 \left(aCe^2-ce(3Bd-Ae)+6cCd^2\right)}{5e^5}-\frac{(d+ex)^4 \left(ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3\right)}{4e^5}+\frac{(d+ex)^3 \left(ae^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*(d + e*x)^3)/(3*e^5) - ((4*c*C*d^3
 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*(d + e*x)^4)/(4*e^5) + ((6*
c*C*d^2 + a*C*e^2 - c*e*(3*B*d - A*e))*(d + e*x)^5)/(5*e^5) - (c*(4*C*d - B
*e)*(d + e*x)^6)/(6*e^5) + (c*C*(d + e*x)^7)/(7*e^5)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) dx &= \int \left(\frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde+Ae^2\right)(d+ex)^2}{e^4}+\frac{\left(-4cCd^3+cde(3Bd-\right.}{}\right.\\ &= \frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde+Ae^2\right)(d+ex)^3}{3e^5}-\frac{\left(4cCd^3-cde(3Bd-2Ae)\right.}{4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 150, normalized size = 0.86

$$\frac{1}{5}x^5\left(aCe^2+Ace^2+2Bcde+cCd^2\right)+\frac{1}{4}x^4\left(aBe^2+2aCde+2Acde+Bcd^2\right)+\frac{1}{3}x^3\left(aAe^2+2aBde+aCd^2+Acd^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d^2*x + (a*d*(B*d + 2*A*e)*x^2)/2 + ((A*c*d^2 + a*C*d^2 + 2*a*B*d*e + a
*A*e^2)*x^3)/3 + ((B*c*d^2 + 2*A*c*d*e + 2*a*C*d*e + a*B*e^2)*x^4)/4 + ((c*
C*d^2 + 2*B*c*d*e + A*c*e^2 + a*C*e^2)*x^5)/5 + (c*e*(2*C*d + B*e)*x^6)/6 +
 (c*C*e^2*x^7)/7
```

fricas [A] time = 0.74, size = 171, normalized size = 0.98

$$\frac{1}{7}x^7e^2cC+\frac{1}{3}x^6edcC+\frac{1}{6}x^6e^2cB+\frac{1}{5}x^5d^2cC+\frac{1}{5}x^5e^2aC+\frac{2}{5}x^5edcB+\frac{1}{5}x^5e^2cA+\frac{1}{2}x^4edaC+\frac{1}{4}x^4d^2cB+\frac{1}{4}x^4e^2aB+\frac{1}{2}x^4edc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e^2*c*C + 1/3*x^6*e*d*c*C + 1/6*x^6*e^2*c*B + 1/5*x^5*d^2*c*C + 1/5
*x^5*e^2*a*C + 2/5*x^5*e*d*c*B + 1/5*x^5*e^2*c*A + 1/2*x^4*e*d*a*C + 1/4*x^
4*d^2*c*B + 1/4*x^4*e^2*a*B + 1/2*x^4*e*d*c*A + 1/3*x^3*d^2*a*C + 2/3*x^3*e
*d*a*B + 1/3*x^3*d^2*c*A + 1/3*x^3*e^2*a*A + 1/2*x^2*d^2*a*B + x^2*e*d*a*A
+ x*d^2*a*A
```

giac [A] time = 0.15, size = 171, normalized size = 0.98

$$\frac{1}{7}Ccx^7e^2+\frac{1}{3}Ccdx^6e+\frac{1}{5}Ccd^2x^5+\frac{1}{6}Bcx^6e^2+\frac{2}{5}Bcdx^5e+\frac{1}{4}Bcd^2x^4+\frac{1}{5}Cax^5e^2+\frac{1}{5}Acx^5e^2+\frac{1}{2}Cadx^4e+\frac{1}{2}Acdx^4e+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/7*C*c*x^7*e^2 + 1/3*C*c*d*x^6*e + 1/5*C*c*d^2*x^5 + 1/6*B*c*x^6*e^2 + 2/5
*B*c*d*x^5*e + 1/4*B*c*d^2*x^4 + 1/5*C*a*x^5*e^2 + 1/5*A*c*x^5*e^2 + 1/2*C*
a*d*x^4*e + 1/2*A*c*d*x^4*e + 1/3*C*a*d^2*x^3 + 1/3*A*c*d^2*x^3 + 1/4*B*a*x
^4*e^2 + 2/3*B*a*d*x^3*e + 1/2*B*a*d^2*x^2 + 1/3*A*a*x^3*e^2 + A*a*d*x^2*e
+ A*a*d^2*x
```

maple [A] time = 0.00, size = 148, normalized size = 0.85

$$\frac{Cc\,e^2x^7}{7}+\frac{\left(c\,e^2B+2decC\right)x^6}{6}+Aa\,d^2x+\frac{\left(Ac\,e^2+2Bcde+\left(a\,e^2+c\,d^2\right)C\right)x^5}{5}+\frac{\left(2Acde+2Cade+\left(a\,e^2+c\,d^2\right)B\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x)
```

```
[Out] 1/7*c*e^2*C*x^7+1/6*(B*c*e^2+2*C*c*d*e)*x^6+1/5*((a*e^2+c*d^2)*C+2*d*e*c*B+
c*e^2*A)*x^5+1/4*(2*d*e*a*C+(a*e^2+c*d^2)*B+2*d*e*c*A)*x^4+1/3*(d^2*a*C+2*d
*e*a*B+A*(a*e^2+c*d^2))*x^3+1/2*(2*A*a*d*e+B*a*d^2)*x^2+d^2*a*A*x
```

maxima [A] time = 0.45, size = 141, normalized size = 0.81

$$\frac{1}{7}Cce^2x^7+\frac{1}{6}\left(2\,Ccde+Bce^2\right)x^6+\frac{1}{5}\left(Ccd^2+2\,Bcde+(Ca+Ac)e^2\right)x^5+Aad^2x+\frac{1}{4}\left(Bcd^2+Bae^2+2\,(Ca+Ac)d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/7*C*c*e^2*x^7 + 1/6*(2*C*c*d*e + B*c*e^2)*x^6 + 1/5*(C*c*d^2 + 2*B*c*d*e
+ (C*a + A*c)*e^2)*x^5 + A*a*d^2*x + 1/4*(B*c*d^2 + B*a*e^2 + 2*(C*a + A*c)
*d*e)*x^4 + 1/3*(2*B*a*d*e + A*a*e^2 + (C*a + A*c)*d^2)*x^3 + 1/2*(B*a*d^2
+ 2*A*a*d*e)*x^2
```

mupad [B] time = 3.61, size = 143, normalized size = 0.82

$$x^3\left(\frac{A\,a\,e^2}{3}+\frac{A\,c\,d^2}{3}+\frac{C\,a\,d^2}{3}+\frac{2\,B\,a\,d\,e}{3}\right)+x^5\left(\frac{A\,c\,e^2}{5}+\frac{C\,a\,e^2}{5}+\frac{C\,c\,d^2}{5}+\frac{2\,B\,c\,d\,e}{5}\right)+x^4\left(\frac{B\,a\,e^2}{4}+\frac{B\,c\,d^2}{4}+\frac{A}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*(d + e*x)^2*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*a*e^2)/3 + (A*c*d^2)/3 + (C*a*d^2)/3 + (2*B*a*d*e)/3) + x^5*((A*c*e
^2)/5 + (C*a*e^2)/5 + (C*c*d^2)/5 + (2*B*c*d*e)/5) + x^4*((B*a*e^2)/4 + (B*
```

```
c*d^2)/4 + (A*c*d*e)/2 + (C*a*d*e)/2) + A*a*d^2*x + (a*d*x^2*(2*A*e + B*d))
/2 + (c*e*x^6*(B*e + 2*C*d))/6 + (C*c*e^2*x^7)/7
```

sympy [A] time = 0.10, size = 173, normalized size = 0.99

$$Aad^2x+\frac{Cce^2x^7}{7}+x^6\left(\frac{Bce^2}{6}+\frac{Ccde}{3}\right)+x^5\left(\frac{Ace^2}{5}+\frac{2Bcde}{5}+\frac{Cae^2}{5}+\frac{Ccd^2}{5}\right)+x^4\left(\frac{Acde}{2}+\frac{Bae^2}{4}+\frac{Bcd^2}{4}+\frac{Cade}{2}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*d**2*x + C*c*e**2*x**7/7 + x**6*(B*c*e**2/6 + C*c*d*e/3) + x**5*(A*c*e*
*2/5 + 2*B*c*d*e/5 + C*a*e**2/5 + C*c*d**2/5) + x**4*(A*c*d*e/2 + B*a*e**2/
4 + B*c*d**2/4 + C*a*d*e/2) + x**3*(A*a*e**2/3 + A*c*d**2/3 + 2*B*a*d*e/3 +
 C*a*d**2/3) + x**2*(A*a*d*e + B*a*d**2/2)
```

3.20 $\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=86

$$\frac{1}{4}x^4(aCe+Ace+Bcd)+\frac{1}{3}x^3(aBe+aCd+Acd)+\frac{1}{2}ax^2(Ae+Bd)+aAdx+\frac{1}{5}cx^5(Be+Cd)+\frac{1}{6}cCex^6$$

```
[Out] a*A*d*x+1/2*a*(A*e+B*d)*x^2+1/3*(A*c*d+B*a*e+C*a*d)*x^3+1/4*(A*c*e+B*c*d+C*
a*e)*x^4+1/5*c*(B*e+C*d)*x^5+1/6*c*C*e*x^6
```

Rubi [A] time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$\frac{1}{4}x^4(aCe+Ace+Bcd)+\frac{1}{3}x^3(aBe+aCd+Acd)+\frac{1}{2}ax^2(Ae+Bd)+aAdx+\frac{1}{5}cx^5(Be+Cd)+\frac{1}{6}cCex^6$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d*x + (a*(B*d + A*e)*x^2)/2 + ((A*c*d + a*C*d + a*B*e)*x^3)/3 + ((B*c*d
 + A*c*e + a*C*e)*x^4)/4 + (c*(C*d + B*e)*x^5)/5 + (c*C*e*x^6)/6
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)dx=\int \left(aAd+a(Bd+Ae)x+(Acd+aCd+aBe)x^2+(Bcd+Ace+aC\right.$$
$$=aAdx+\frac{1}{2}a(Bd+Ae)x^2+\frac{1}{3}(Acd+aCd+aBe)x^3+\frac{1}{4}(Bcd+Ace+$$

Mathematica [A] time = 0.03, size = 86, normalized size = 1.00

$$\frac{1}{4}x^4(aCe+Ace+Bcd)+\frac{1}{3}x^3(aBe+aCd+Acd)+\frac{1}{2}ax^2(Ae+Bd)+aAdx+\frac{1}{5}cx^5(Be+Cd)+\frac{1}{6}cCex^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d*x + (a*(B*d + A*e)*x^2)/2 + ((A*c*d + a*C*d + a*B*e)*x^3)/3 + ((B*c*d
 + A*c*e + a*C*e)*x^4)/4 + (c*(C*d + B*e)*x^5)/5 + (c*C*e*x^6)/6
```

fricas [A] time = 0.75, size = 94, normalized size = 1.09

$$\frac{1}{6}x^6ecC+\frac{1}{5}x^5dcC+\frac{1}{5}x^5ecB+\frac{1}{4}x^4eaC+\frac{1}{4}x^4dcB+\frac{1}{4}x^4ecA+\frac{1}{3}x^3daC+\frac{1}{3}x^3eaB+\frac{1}{3}x^3dcA+\frac{1}{2}x^2daB+\frac{1}{2}x^2eaA+xdaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e*c*C + 1/5*x^5*d*c*C + 1/5*x^5*e*c*B + 1/4*x^4*e*a*C + 1/4*x^4*d*c
*B + 1/4*x^4*e*c*A + 1/3*x^3*d*a*C + 1/3*x^3*e*a*B + 1/3*x^3*d*c*A + 1/2*x^
2*d*a*B + 1/2*x^2*e*a*A + x*d*a*A
```

giac [**A**] time = 0.15, size = 100, normalized size = 1.16

$$\frac{1}{6}\,Ccx^6e+\frac{1}{5}\,Ccdx^5+\frac{1}{5}\,Bcx^5e+\frac{1}{4}\,Bcdx^4+\frac{1}{4}\,Cax^4e+\frac{1}{4}\,Acx^4e+\frac{1}{3}\,Cadx^3+\frac{1}{3}\,Acdx^3+\frac{1}{3}\,Bax^3e+\frac{1}{2}\,Badx^2+\frac{1}{2}\,Aax^2e+Aa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/6*C*c*x^6*e + 1/5*C*c*d*x^5 + 1/5*B*c*x^5*e + 1/4*B*c*d*x^4 + 1/4*C*a*x^4
*e + 1/4*A*c*x^4*e + 1/3*C*a*d*x^3 + 1/3*A*c*d*x^3 + 1/3*B*a*x^3*e + 1/2*B*
a*d*x^2 + 1/2*A*a*x^2*e + A*a*d*x
```

maple [**A**] time = 0.00, size = 79, normalized size = 0.92

$$\frac{Cce\,x^6}{6}+\frac{(ecB+cdC)\,x^5}{5}+Aadx+\frac{(Ace+Bcd+aCe)\,x^4}{4}+\frac{(Acd+Bae+Cad)\,x^3}{3}+\frac{(aeA+adB)\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x)
```

```
[Out] 1/6*c*C*e*x^6+1/5*(B*c*e+C*c*d)*x^5+1/4*(A*c*e+B*c*d+C*a*e)*x^4+1/3*(A*c*d+
B*a*e+C*a*d)*x^3+1/2*(A*a*e+B*a*d)*x^2+a*A*d*x
```

maxima [**A**] time = 0.45, size = 80, normalized size = 0.93

$$\frac{1}{6}\,Ccex^6+\frac{1}{5}\,(Ccd+Bce)x^5+\frac{1}{4}\,(Bcd+(Ca+Ac)e)x^4+Aadx+\frac{1}{3}\,(Bae+(Ca+Ac)d)x^3+\frac{1}{2}\,(Bad+Aae)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/6*C*c*e*x^6 + 1/5*(C*c*d + B*c*e)*x^5 + 1/4*(B*c*d + (C*a + A*c)*e)*x^4 +
 A*a*d*x + 1/3*(B*a*e + (C*a + A*c)*d)*x^3 + 1/2*(B*a*d + A*a*e)*x^2
```

mupad [**B**] time = 3.56, size = 80, normalized size = 0.93

$$\frac{C\,c\,e\,x^6}{6}+\frac{c\,(B\,e+C\,d)\,x^5}{5}+\left(\frac{A\,c\,e}{4}+\frac{B\,c\,d}{4}+\frac{C\,a\,e}{4}\right)\,x^4+\left(\frac{A\,c\,d}{3}+\frac{B\,a\,e}{3}+\frac{C\,a\,d}{3}\right)\,x^3+\frac{a\,(A\,e+B\,d)\,x^2}{2}+A\,a\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*(d + e*x)*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*c*d)/3 + (B*a*e)/3 + (C*a*d)/3) + x^4*((A*c*e)/4 + (B*c*d)/4 + (C*a
*e)/4) + (a*x^2*(A*e + B*d))/2 + (c*x^5*(B*e + C*d))/5 + (C*c*e*x^6)/6 + A*
a*d*x
```

sympy [**A**] time = 0.08, size = 97, normalized size = 1.13

$$Aadx+\frac{Ccex^6}{6}+x^5\left(\frac{Bce}{5}+\frac{Ccd}{5}\right)+x^4\left(\frac{Ace}{4}+\frac{Bcd}{4}+\frac{Cae}{4}\right)+x^3\left(\frac{Acd}{3}+\frac{Bae}{3}+\frac{Cad}{3}\right)+x^2\left(\frac{Aae}{2}+\frac{Bad}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*d*x + C*c*e*x**6/6 + x**5*(B*c*e/5 + C*c*d/5) + x**4*(A*c*e/4 + B*c*d/4
 + C*a*e/4) + x**3*(A*c*d/3 + B*a*e/3 + C*a*d/3) + x**2*(A*a*e/2 + B*a*d/2)
```

3.21 $\int \left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=46

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}aBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

```
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*c+C*a)*x^3+1/4*B*c*x^4+1/5*c*C*x^5
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1657}

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}aBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*c + a*C)*x^3)/3 + (B*c*x^4)/4 + (c*C*x^5)/5
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx &= \int \left(aA+aBx+(Ac+aC)x^2+Bcx^3+cCx^4\right)\,dx\\ &= aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.00

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}aBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*c + a*C)*x^3)/3 + (B*c*x^4)/4 + (c*C*x^5)/5
```

fricas [A] time = 0.84, size = 40, normalized size = 0.87

$$\frac{1}{5}x^5cC+\frac{1}{4}x^4cB+\frac{1}{3}x^3aC+\frac{1}{3}x^3cA+\frac{1}{2}x^2aB+xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c*C + 1/4*x^4*c*B + 1/3*x^3*a*C + 1/3*x^3*c*A + 1/2*x^2*a*B + x*a*A
```

giac [A] time = 0.16, size = 40, normalized size = 0.87

$$\frac{1}{5}\,Ccx^5+\frac{1}{4}\,Bcx^4+\frac{1}{3}\,Cax^3+\frac{1}{3}\,Acx^3+\frac{1}{2}\,Bax^2+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/3*C*a*x^3 + 1/3*A*c*x^3 + 1/2*B*a*x^2 + A*a*x
```

maple [A] time = 0.00, size = 39, normalized size = 0.85

$$\frac{Cc\,x^5}{5}+\frac{Bc\,x^4}{4}+\frac{Ba\,x^2}{2}+Aax+\frac{(Ac+aC)\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(C*x^2+B*x+A),x)
```

```
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*c+C*a)*x^3+1/4*B*c*x^4+1/5*c*C*x^5
```

maxima [A] time = 0.44, size = 38, normalized size = 0.83

$$\frac{1}{5}\,Ccx^5+\frac{1}{4}\,Bcx^4+\frac{1}{2}\,Bax^2+\frac{1}{3}\,(Ca+Ac)x^3+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*a*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

mupad [B] time = 0.03, size = 39, normalized size = 0.85

$$\frac{C\,c\,x^5}{5}+\frac{B\,c\,x^4}{4}+\left(\frac{A\,c}{3}+\frac{C\,a}{3}\right)\,x^3+\frac{B\,a\,x^2}{2}+A\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*c)/3 + (C*a)/3) + A*a*x + (B*a*x^2)/2 + (B*c*x^4)/4 + (C*c*x^5)/5
```

sympy [A] time = 0.07, size = 42, normalized size = 0.91

$$Aax+\frac{Bax^2}{2}+\frac{Bcx^4}{4}+\frac{Ccx^5}{5}+x^3\left(\frac{Ac}{3}+\frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*x + B*a*x**2/2 + B*c*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 + C*a/3)
```

3.22 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx$

Optimal. Leaf size=145

$$\frac{(ae^2+cd^2)\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{e^5}-\frac{x\left(ae^2(Cd-Be)+cd\left(Cd^2-e(Bd-Ae)\right)\right)}{e^4}+\frac{x^2\left(aCe^2+c\left(Cd^2-e\right.\right.}{2e^3}$$

```
[Out] -(a*e^2*(-B*e+C*d)+c*d*(C*d^2-e*(-A*e+B*d)))*x/e^4+1/2*(a*C*e^2+c*(C*d^2-e*
(-A*e+B*d)))*x^2/e^3-1/3*c*(-B*e+C*d)*x^3/e^2+1/4*c*C*x^4/e+(a*e^2+c*d^2)*(
A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e^5
```

Rubi [A] time = 0.25, antiderivative size = 143, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1628}

$$\frac{x^2\left(aCe^2-ce(Bd-Ae)+cCd^2\right)}{2e^3}-\frac{x\left(ae^2(Cd-Be)-cde(Bd-Ae)+cCd^3\right)}{e^4}+\frac{(ae^2+cd^2)\log(d+ex)\left(Ae^2-Bd\right.}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] -(((c*C*d^3 - c*d*e*(B*d - A*e) + a*e^2*(C*d - B*e))*x)/e^4) + ((c*C*d^2 +
a*C*e^2 - c*e*(B*d - A*e))*x^2)/(2*e^3) - (c*(C*d - B*e)*x^3)/(3*e^2) + (c*
C*x^4)/(4*e) + ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/e^5
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx=\int\left(\frac{-ae^2(Cd-Be)-c\left(Cd^3-de(Bd-Ae)\right)}{e^4}+\frac{\left(cCd^2+aCe^2-ce(Bd-Ae)\right.}{e^3}\right.$$
$$=-\frac{\left(cCd^3-cde(Bd-Ae)+ae^2(Cd-Be)\right)x}{e^4}+\frac{\left(cCd^2+aCe^2-ce(Bd-Ae)\right)}{2e^3}$$

Mathematica [A] time = 0.07, size = 136, normalized size = 0.94

$$\frac{12(ae^2+cd^2)\log(d+ex)\left(e(Ae-Bd)+Cd^2\right)+ex\left(6ae^2(2Be-2Cd+Cex)+2ce\left(3Ae(ex-2d)+B\left(6d^2-3\right.\right.\right.}{12e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] (e*x*(6*a*e^2*(-2*C*d + 2*B*e + C*e*x) + c*C*(-12*d^3 + 6*d^2*e*x - 4*d*e^2
*x^2 + 3*e^3*x^3) + 2*c*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d*e*x + 2*e^2*
x^2))) + 12*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e))*Log[d + e*x])/(12*e^
5)
```

fricas [A] time = 1.61, size = 161, normalized size = 1.11

$$\frac{3Cce^4x^4-4\left(Ccde^3-Bce^4\right)x^3+6\left(Ccd^2e^2-Bcde^3+(Ca+Ac)e^4\right)x^2-12\left(Ccd^3e-Bcd^2e^2-Bae^4+(Ca+A\right.}{12e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*c*e^4*x^4 - 4*(C*c*d*e^3 - B*c*e^4)*x^3 + 6*(C*c*d^2*e^2 - B*c*d*
e^3 + (C*a + A*c)*e^4)*x^2 - 12*(C*c*d^3*e - B*c*d^2*e^2 - B*a*e^4 + (C*a +
 A*c)*d*e^3)*x + 12*(C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + A*a*e^4 + (C*a + A*c
)*d^2*e^2)*log(e*x + d))/e^5
```

giac [A] time = 0.15, size = 170, normalized size = 1.17

$$\left(Ccd^4 - Bcd^3e + Cad^2e^2 + Acd^2e^2 - Bade^3 + Aae^4\right)e^{(-5)}\log\left(|xe + d|\right)+\frac{1}{12}\left(3\,Ccx^4e^3 - 4\,Ccdx^3e^2 + 6\,Ccd^2x^2e - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

```
[Out] (C*c*d^4 - B*c*d^3*e + C*a*d^2*e^2 + A*c*d^2*e^2 - B*a*d*e^3 + A*a*e^4)*e^(
-5)*log(abs(x*e + d)) + 1/12*(3*C*c*x^4*e^3 - 4*C*c*d*x^3*e^2 + 6*C*c*d^2*x
^2*e - 12*C*c*d^3*x + 4*B*c*x^3*e^3 - 6*B*c*d*x^2*e^2 + 12*B*c*d^2*x*e + 6*
C*a*x^2*e^3 + 6*A*c*x^2*e^3 - 12*C*a*d*x*e^2 - 12*A*c*d*x*e^2 + 12*B*a*x*e^
3)*e^(-4)
```

maple [A] time = 0.01, size = 210, normalized size = 1.45

$$\frac{Cc\,x^4}{4e}+\frac{Bc\,x^3}{3e}-\frac{Ccd\,x^3}{3e^2}+\frac{Ac\,x^2}{2e}-\frac{Bcd\,x^2}{2e^2}+\frac{Ca\,x^2}{2e}+\frac{Cc\,d^2x^2}{2e^3}+\frac{Aa\ln\left(ex+d\right)}{e}+\frac{Ac\,d^2\ln\left(ex+d\right)}{e^3}-\frac{Acdx}{e^2}-\frac{Bad\ln\left(ex+d\right.}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x)
```

```
[Out] 1/4*c*C*x^4/e+1/3/e*B*x^3*c-1/3/e^2*C*x^3*c*d+1/2/e*A*x^2*c-1/2/e^2*B*x^2*c
*d+1/2/e*C*x^2*a+1/2/e^3*C*x^2*c*d^2-1/e^2*A*x*c*d+1/e*B*x*a+1/e^3*B*x*c*d^
2-1/e^2*C*x*a*d-1/e^4*C*x*c*d^3+1/e*ln(e*x+d)*A*a+1/e^3*ln(e*x+d)*A*c*d^2-1
/e^2*ln(e*x+d)*B*a*d-1/e^4*ln(e*x+d)*B*c*d^3+1/e^3*ln(e*x+d)*C*a*d^2+1/e^5*
ln(e*x+d)*C*c*d^4
```

maxima [A] time = 0.45, size = 159, normalized size = 1.10

$$\frac{3\,Cce^3x^4 - 4\left(Ccde^2 - Bce^3\right)x^3 + 6\left(Ccd^2e - Bcde^2 + (Ca + Ac)e^3\right)x^2 - 12\left(Ccd^3 - Bcd^2e - Bae^3 + (Ca + Ac)de^2\right)}{12\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/12*(3*C*c*e^3*x^4 - 4*(C*c*d*e^2 - B*c*e^3)*x^3 + 6*(C*c*d^2*e - B*c*d*e^
2 + (C*a + A*c)*e^3)*x^2 - 12*(C*c*d^3 - B*c*d^2*e - B*a*e^3 + (C*a + A*c)*
d*e^2)*x)/e^4 + (C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + A*a*e^4 + (C*a + A*c)*d^
2*e^2)*log(e*x + d)/e^5
```

mupad [B] time = 3.62, size = 175, normalized size = 1.21

$$x^3\left(\frac{B\,c}{3\,e}-\frac{C\,c\,d}{3\,e^2}\right)-x\left(\frac{d\left(\frac{A\,c+C\,a}{e}-\frac{d\left(\frac{B\,c}{e}-\frac{C\,c\,d}{e^2}\right)}{e}\right)}{e}-\frac{B\,a}{e}\right)+x^2\left(\frac{A\,c+C\,a}{2\,e}-\frac{d\left(\frac{B\,c}{e}-\frac{C\,c\,d}{e^2}\right)}{2\,e}\right)+\frac{\ln\left(d+e\,x\right)\left(A\,a\,e^4+C\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x)
```

```
[Out] x^3*((B*c)/(3*e) - (C*c*d)/(3*e^2)) - x*((d*((A*c + C*a)/e - (d*((B*c)/e -
(C*c*d)/e^2))/e))/e - (B*a)/e) + x^2*((A*c + C*a)/(2*e) - (d*((B*c)/e - (C*
c*d)/e^2))/(2*e)) + (log(d + e*x)*(A*a*e^4 + C*c*d^4 - B*a*d*e^3 - B*c*d^3*
e + A*c*d^2*e^2 + C*a*d^2*e^2))/e^5 + (C*c*x^4)/(4*e)
```

sympy [A] time = 0.64, size = 148, normalized size = 1.02

$$\frac{Ccx^4}{4e}+x^3\left(\frac{Bc}{3e}-\frac{Ccd}{3e^2}\right)+x^2\left(\frac{Ac}{2e}-\frac{Bcd}{2e^2}+\frac{Ca}{2e}+\frac{Ccd^2}{2e^3}\right)+x\left(-\frac{Acd}{e^2}+\frac{Ba}{e}+\frac{Bcd^2}{e^3}-\frac{Cad}{e^2}-\frac{Ccd^3}{e^4}\right)+\frac{\left(ae^2+cd^2\right)\left(\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d),x)
```

```
[Out] C*c*x**4/(4*e) + x**3*(B*c/(3*e) - C*c*d/(3*e**2)) + x**2*(A*c/(2*e) - B*c*
d/(2*e**2) + C*a/(2*e) + C*c*d**2/(2*e**3)) + x*(-A*c*d/e**2 + B*a/e + B*c*
d**2/e**3 - C*a*d/e**2 - C*c*d**3/e**4) + (a*e**2 + c*d**2)*(A*e**2 - B*d*e
 + C*d**2)*log(d + e*x)/e**5
```

3.23 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx$

Optimal. Leaf size=153

$$-\frac{(ae^2+cd^2)(Ae^2-Bde+Cd^2)}{e^5(d+ex)}-\frac{\log(d+ex)\left(ae^2(2Cd-Be)+cd\left(4Cd^2-e(3Bd-2Ae)\right)\right)}{e^5}+\frac{x\left(aCe^2+c\left(3Cd^2-\right.\right.}{e^4}$$

```
[Out] (a*C*e^2+c*(3*C*d^2-e*(-A*e+2*B*d)))*x/e^4-1/2*c*(-B*e+2*C*d)*x^2/e^3+1/3*c
*C*x^3/e^2-(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)/e^5/(e*x+d)-(a*e^2*(-B*e+2*C*d
)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*ln(e*x+d)/e^5
```

Rubi [A] time = 0.20, antiderivative size = 151, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1628}

$$\frac{x\left(aCe^2-ce(2Bd-Ae)+3cCd^2\right)}{e^4}-\frac{(ae^2+cd^2)(Ae^2-Bde+Cd^2)}{e^5(d+ex)}-\frac{\log(d+ex)\left(ae^2(2Cd-Be)-cde(3Bd-2Ae\right.}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] ((3*c*C*d^2 + a*C*e^2 - c*e*(2*B*d - A*e))*x)/e^4 - (c*(2*C*d - B*e)*x^2)/(
2*e^3) + (c*C*x^3)/(3*e^2) - ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2))/(e^5
*(d + e*x)) - ((4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*Lo
g[d + e*x])/e^5
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx=\int \left(\frac{3cCd^2+aCe^2-ce(2Bd-Ae)}{e^4}+\frac{c(-2Cd+Be)x}{e^3}+\frac{cCx^2}{e^2}+\frac{(cd^2+ae^2)(C}{e^4(d}\right.$$

$$=\frac{\left(3cCd^2+aCe^2-ce(2Bd-Ae)\right)x}{e^4}-\frac{c(2Cd-Be)x^2}{2e^3}+\frac{cCx^3}{3e^2}-\frac{(cd^2+ae^2)(Cd}{e^5(d}$$

Mathematica [A] time = 0.15, size = 142, normalized size = 0.93

$$\frac{6\log(d+ex)\left(ae^2(Be-2Cd)+cde(3Bd-2Ae)-4cCd^3\right)+6ex\left(aCe^2+ce(Ae-2Bd)+3cCd^2\right)-\frac{6(ae^2+cd^2)(e(Ae-B}{d+ex}}{6e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] (6*e*(3*c*C*d^2 + a*C*e^2 + c*e*(-2*B*d + A*e))*x + 3*c*e^2*(-2*C*d + B*e)*
x^2 + 2*c*C*e^3*x^3 - (6*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e
*x) + 6*(-4*c*C*d^3 + c*d*e*(3*B*d - 2*A*e) + a*e^2*(-2*C*d + B*e))*Log[d +
 e*x])/(6*e^5)
```

fricas [A] time = 0.87, size = 250, normalized size = 1.63

$$\frac{2\,Cce^4x^4 - 6\,Ccd^4 + 6\,Bcd^3e + 6\,Bade^3 - 6\,Aae^4 - 6\,(Ca + Ac)d^2e^2 - \left(4\,Ccde^3 - 3\,Bce^4\right)x^3 + 3\left(4\,Ccd^2e^2 - 3\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*C*c*e^4*x^4 - 6*C*c*d^4 + 6*B*c*d^3*e + 6*B*a*d*e^3 - 6*A*a*e^4 - 6*
(C*a + A*c)*d^2*e^2 - (4*C*c*d*e^3 - 3*B*c*e^4)*x^3 + 3*(4*C*c*d^2*e^2 - 3*
B*c*d*e^3 + 2*(C*a + A*c)*e^4)*x^2 + 6*(3*C*c*d^3*e - 2*B*c*d^2*e^2 + (C*a
+ A*c)*d*e^3)*x - 6*(4*C*c*d^4 - 3*B*c*d^3*e - B*a*d*e^3 + 2*(C*a + A*c)*d^
2*e^2 + (4*C*c*d^3*e - 3*B*c*d^2*e^2 - B*a*e^4 + 2*(C*a + A*c)*d*e^3)*x)*lo
g(e*x + d))/(e^6*x + d*e^5)
```

giac [A] time = 0.16, size = 240, normalized size = 1.57

$$\frac{1}{6}\left(2\,Cc - \frac{3\left(4\,Ccde - Bce^2\right)e^{(-1)}}{xe + d} + \frac{6\left(6\,Ccd^2e^2 - 3\,Bcde^3 + Cae^4 + Ace^4\right)e^{(-2)}}{(xe + d)^2}\right)(xe + d)^3e^{(-5)} + \left(4\,Ccd^3 - 3\,Bcd^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/6*(2*C*c - 3*(4*C*c*d*e - B*c*e^2)*e^(-1)/(x*e + d) + 6*(6*C*c*d^2*e^2 -
3*B*c*d*e^3 + C*a*e^4 + A*c*e^4)*e^(-2)/(x*e + d)^2)*(x*e + d)^3*e^(-5) + (
4*C*c*d^3 - 3*B*c*d^2*e + 2*C*a*d*e^2 + 2*A*c*d*e^2 - B*a*e^3)*e^(-5)*log(a
bs(x*e + d)*e^(-1)/(x*e + d)^2) - (C*c*d^4*e^3/(x*e + d) - B*c*d^3*e^4/(x*e
 + d) + C*a*d^2*e^5/(x*e + d) + A*c*d^2*e^5/(x*e + d) - B*a*d*e^6/(x*e + d)
 + A*a*e^7/(x*e + d))*e^(-8)
```

maple [A] time = 0.01, size = 234, normalized size = 1.53

$$\frac{Cc\,x^3}{3e^2} + \frac{Bc\,x^2}{2e^2} - \frac{Ccd\,x^2}{e^3} - \frac{Aa}{(ex + d)\,e} - \frac{Ac\,d^2}{(ex + d)\,e^3} - \frac{2Acd\ln(ex + d)}{e^3} + \frac{Acx}{e^2} + \frac{Bad}{(ex + d)\,e^2} + \frac{Ba\ln(ex + d)}{e^2} + \frac{Bc\,d^3}{(ex + d)\,e^{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x)
```

```
[Out] 1/3*c*C*x^3/e^2+1/2/e^2*B*x^2*c-1/e^3*C*x^2*c*d+1/e^2*A*c*x-2/e^3*B*c*d*x+1
/e^2*a*C*x+3/e^4*C*c*d^2*x-1/e/(e*x+d)*A*a-1/e^3/(e*x+d)*A*c*d^2+1/e^2/(e*x
+d)*B*d*a+1/e^4/(e*x+d)*B*c*d^3-1/e^3/(e*x+d)*C*a*d^2-1/e^5/(e*x+d)*C*c*d^4
-2/e^3*ln(e*x+d)*A*c*d+1/e^2*ln(e*x+d)*B*a+3/e^4*ln(e*x+d)*B*c*d^2-2/e^3*ln
(e*x+d)*C*a*d-4/e^5*ln(e*x+d)*C*c*d^3
```

maxima [A] time = 0.45, size = 169, normalized size = 1.10

$$-\frac{Ccd^4 - Bcd^3e - Bade^3 + Aae^4 + (Ca + Ac)d^2e^2}{e^6x + de^5} + \frac{2\,Cce^2x^3 - 3\left(2\,Ccde - Bce^2\right)x^2 + 6\left(3\,Ccd^2 - 2\,Bcde + (Ca\right.}{6\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + A*a*e^4 + (C*a + A*c)*d^2*e^2)/(e^6*x +
 d*e^5) + 1/6*(2*C*c*e^2*x^3 - 3*(2*C*c*d*e - B*c*e^2)*x^2 + 6*(3*C*c*d^2 -
 2*B*c*d*e + (C*a + A*c)*e^2)*x)/e^4 - (4*C*c*d^3 - 3*B*c*d^2*e - B*a*e^3 +
 2*(C*a + A*c)*d*e^2)*log(e*x + d)/e^5
```

mupad [B] time = 0.09, size = 192, normalized size = 1.25

$$x^2\left(\frac{B\,c}{2\,e^2}-\frac{C\,c\,d}{e^3}\right)-x\left(\frac{2\,d\,\left(\frac{B\,c}{e^2}-\frac{2\,C\,c\,d}{e^3}\right)}{e}-\frac{A\,c+C\,a}{e^2}+\frac{C\,c\,d^2}{e^4}\right)-\frac{\ln\left(d+e\,x\right)\,\left(4\,C\,c\,d^3-B\,a\,e^3+2\,A\,c\,d\,e^2+2\,C\,a\,d\right.}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

```
[Out] x^2*((B*c)/(2*e^2) - (C*c*d)/e^3) - x*((2*d*((B*c)/e^2 - (2*C*c*d)/e^3))/e
- (A*c + C*a)/e^2 + (C*c*d^2)/e^4) - (log(d + e*x)*(4*C*c*d^3 - B*a*e^3 + 2
*A*c*d*e^2 + 2*C*a*d*e^2 - 3*B*c*d^2*e))/e^5 - (A*a*e^4 + C*c*d^4 - B*a*d*e
^3 - B*c*d^3*e + A*c*d^2*e^2 + C*a*d^2*e^2)/(e*(d*e^4 + e^5*x)) + (C*c*x^3)
/(3*e^2)
```

sympy [A] time = 1.26, size = 185, normalized size = 1.21

$$\frac{Ccx^3}{3e^2}+x^2\left(\frac{Bc}{2e^2}-\frac{Ccd}{e^3}\right)+x\left(\frac{Ac}{e^2}-\frac{2Bcd}{e^3}+\frac{Ca}{e^2}+\frac{3Ccd^2}{e^4}\right)+\frac{-Aae^4-Acd^2e^2+Bade^3+Bcd^3e-Cad^2e^2-Ccd^4}{de^5+e^6x}-\frac{(2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

```
[Out] C*c*x**3/(3*e**2) + x**2*(B*c/(2*e**2) - C*c*d/e**3) + x*(A*c/e**2 - 2*B*c*
d/e**3 + C*a/e**2 + 3*C*c*d**2/e**4) + (-A*a*e**4 - A*c*d**2*e**2 + B*a*d*e
**3 + B*c*d**3*e - C*a*d**2*e**2 - C*c*d**4)/(d*e**5 + e**6*x) - (2*A*c*d*e
**2 - B*a*e**3 - 3*B*c*d**2*e + 2*C*a*d*e**2 + 4*C*c*d**3)*log(d + e*x)/e**
5
```

3.24 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3} dx$

Optimal. Leaf size=156

$$\frac{ae^2(2Cd-Be)+cd\left(4Cd^2-e(3Bd-2Ae)\right)}{e^5(d+ex)}-\frac{\left(ae^2+cd^2\right)\left(Ae^2-Bde+Cd^2\right)}{2e^5(d+ex)^2}+\frac{\log(d+ex)\left(aCe^2+c\left(6Cd^2-e\right.\right.}{e^5}$$

```
[Out] -c*(-B*e+3*C*d)*x/e^4+1/2*c*C*x^2/e^3-1/2*(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)
/e^5/(e*x+d)^2+(a*e^2*(-B*e+2*C*d)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))/e^5/(e*x
+d)+(a*C*e^2+c*(6*C*d^2-e*(-A*e+3*B*d)))*ln(e*x+d)/e^5
```

Rubi [A] time = 0.20, antiderivative size = 154, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1628}

$$\frac{ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3}{e^5(d+ex)}-\frac{\left(ae^2+cd^2\right)\left(Ae^2-Bde+Cd^2\right)}{2e^5(d+ex)^2}+\frac{\log(d+ex)\left(aCe^2-ce(3Bd-Ae)\right.}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] -((c*(3*C*d - B*e)*x)/e^4) + (c*C*x^2)/(2*e^3) - ((c*d^2 + a*e^2)*(C*d^2 -
B*d*e + A*e^2))/(2*e^5*(d + e*x)^2) + (4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) +
a*e^2*(2*C*d - B*e))/(e^5*(d + e*x)) + ((6*c*C*d^2 + a*C*e^2 - c*e*(3*B*d -
 A*e))*Log[d + e*x])/e^5
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)\left(A+Bx+Cx^2\right)}{(d+ex)^3}dx=\int\left(\frac{c(-3Cd+Be)}{e^4}+\frac{cCx}{e^3}+\frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde+Ae^2\right)}{e^4(d+ex)^3}+\frac{-4cCd^3+cd\right.$$

$$=-\frac{c(3Cd-Be)x}{e^4}+\frac{cCx^2}{2e^3}-\frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde+Ae^2\right)}{2e^5(d+ex)^2}+\frac{4cCd^3-cde(3}{}$$

Mathematica [A] time = 0.10, size = 176, normalized size = 1.13

$$\frac{\log(d+ex)\left(aCe^2+Ace^2-3Bcde+6cCd^2\right)}{e^5}+\frac{-aBe^3+2aCde^2+2Acde^2-3Bcd^2e+4cCd^3}{e^5(d+ex)}+\frac{-aAe^4+aBde^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] (c*(-3*C*d + B*e)*x)/e^4 + (c*C*x^2)/(2*e^3) + (-(c*C*d^4) + B*c*d^3*e - A*
c*d^2*e^2 - a*C*d^2*e^2 + a*B*d*e^3 - a*A*e^4)/(2*e^5*(d + e*x)^2) + (4*c*C
*d^3 - 3*B*c*d^2*e + 2*A*c*d*e^2 + 2*a*C*d*e^2 - a*B*e^3)/(e^5*(d + e*x)) +
 ((6*c*C*d^2 - 3*B*c*d*e + A*c*e^2 + a*C*e^2)*Log[d + e*x])/e^5
```

fricas [A] time = 0.79, size = 273, normalized size = 1.75

$$\frac{Cce^4x^4 + 7\,Ccd^4 - 5\,Bcd^3e - Bade^3 - Aae^4 + 3\,(Ca + Ac)d^2e^2 - 2\left(2\,Ccde^3 - Bce^4\right)x^3 - \left(11\,Ccd^2e^2 - 4\,Bcde^3\right)x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(C*c*e^4*x^4 + 7*C*c*d^4 - 5*B*c*d^3*e - B*a*d*e^3 - A*a*e^4 + 3*(C*a +
 A*c)*d^2*e^2 - 2*(2*C*c*d*e^3 - B*c*e^4)*x^3 - (11*C*c*d^2*e^2 - 4*B*c*d*e
^3)*x^2 + 2*(C*c*d^3*e - 2*B*c*d^2*e^2 - B*a*e^4 + 2*(C*a + A*c)*d*e^3)*x +
 2*(6*C*c*d^4 - 3*B*c*d^3*e + (C*a + A*c)*d^2*e^2 + (6*C*c*d^2*e^2 - 3*B*c*
d*e^3 + (C*a + A*c)*e^4)*x^2 + 2*(6*C*c*d^3*e - 3*B*c*d^2*e^2 + (C*a + A*c)
*d*e^3)*x)*log(e*x + d))/(e^7*x^2 + 2*d*e^6*x + d^2*e^5)
```

giac [A] time = 0.15, size = 167, normalized size = 1.07

$$\left(6\,Ccd^2 - 3\,Bcde + Cae^2 + Ace^2\right)e^{(-5)}\log\left(|xe + d|\right) + \frac{1}{2}\left(Ccx^2e^3 - 6\,Ccdxe^2 + 2\,Bcxe^3\right)e^{(-6)} + \frac{\left(7\,Ccd^4 - 5\,Bcd^3e + 3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (6*C*c*d^2 - 3*B*c*d*e + C*a*e^2 + A*c*e^2)*e^(-5)*log(abs(x*e + d)) + 1/2*
(C*c*x^2*e^3 - 6*C*c*d*x*e^2 + 2*B*c*x*e^3)*e^(-6) + 1/2*(7*C*c*d^4 - 5*B*c
*d^3*e + 3*C*a*d^2*e^2 + 3*A*c*d^2*e^2 - B*a*d*e^3 - A*a*e^4 + 2*(4*C*c*d^3
*e - 3*B*c*d^2*e^2 + 2*C*a*d*e^3 + 2*A*c*d*e^3 - B*a*e^4)*x)*e^(-5)/(x*e +
d)^2
```

maple [A] time = 0.01, size = 257, normalized size = 1.65

$$-\frac{Aa}{2\left(ex+d\right)^{2}e}-\frac{Ac\,d^{2}}{2\left(ex+d\right)^{2}e^{3}}+\frac{Bad}{2\left(ex+d\right)^{2}e^{2}}+\frac{Bc\,d^{3}}{2\left(ex+d\right)^{2}e^{4}}-\frac{Ca\,d^{2}}{2\left(ex+d\right)^{2}e^{3}}-\frac{Cc\,d^{4}}{2\left(ex+d\right)^{2}e^{5}}+\frac{Cc\,x^{2}}{2e^{3}}+\frac{2Acd}{\left(ex+d\right)e^{3}}+\frac{A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x)
```

```
[Out] 1/2*c*C*x^2/e^3+c/e^3*B*x-3*c/e^4*C*d*x+2/e^3/(e*x+d)*A*c*d-1/e^2/(e*x+d)*B
*a-3/e^4/(e*x+d)*B*c*d^2+2/e^3/(e*x+d)*C*a*d+4/e^5/(e*x+d)*C*c*d^3+1/e^3*ln
(e*x+d)*A*c-3/e^4*ln(e*x+d)*B*c*d+1/e^3*ln(e*x+d)*a*C+6/e^5*ln(e*x+d)*C*c*d
^2-1/2/e/(e*x+d)^2*A*a-1/2/e^3/(e*x+d)^2*A*d^2*c+1/2/e^2/(e*x+d)^2*B*d*a+1/
2/e^4/(e*x+d)^2*B*c*d^3-1/2/e^3/(e*x+d)^2*C*d^2*a-1/2/e^5/(e*x+d)^2*C*c*d^4
```

maxima [A] time = 0.47, size = 177, normalized size = 1.13

$$\frac{7\,Ccd^4 - 5\,Bcd^3e - Bade^3 - Aae^4 + 3\,(Ca + Ac)d^2e^2 + 2\left(4\,Ccd^3e - 3\,Bcd^2e^2 - Bae^4 + 2\,(Ca + Ac)de^3\right)x}{2\left(e^7x^2 + 2\,de^6x + d^2e^5\right)} + \frac{Ccex^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(7*C*c*d^4 - 5*B*c*d^3*e - B*a*d*e^3 - A*a*e^4 + 3*(C*a + A*c)*d^2*e^2
+ 2*(4*C*c*d^3*e - 3*B*c*d^2*e^2 - B*a*e^4 + 2*(C*a + A*c)*d*e^3)*x)/(e^7*x
^2 + 2*d*e^6*x + d^2*e^5) + 1/2*(C*c*e*x^2 - 2*(3*C*c*d - B*c*e)*x)/e^4 + (
6*C*c*d^2 - 3*B*c*d*e + (C*a + A*c)*e^2)*log(e*x + d)/e^5
```

mupad [B] time = 0.09, size = 185, normalized size = 1.19

$$\frac{x\left(4\,C\,c\,d^3-B\,a\,e^3+2\,A\,c\,d\,e^2+2\,C\,a\,d\,e^2-3\,B\,c\,d^2\,e\right)-\frac{A\,a\,e^4-7\,C\,c\,d^4+B\,a\,d\,e^3+5\,B\,c\,d^3\,e-3\,A\,c\,d^2\,e^2-3\,C\,a\,d^2\,e^2}{2\,e}}{d^2\,e^4+2\,d\,e^5\,x+e^6\,x^2}+x\left(\frac{B\,c}{e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

```
[Out] (x*(4*C*c*d^3 - B*a*e^3 + 2*A*c*d*e^2 + 2*C*a*d*e^2 - 3*B*c*d^2*e) - (A*a*e
^4 - 7*C*c*d^4 + B*a*d*e^3 + 5*B*c*d^3*e - 3*A*c*d^2*e^2 - 3*C*a*d^2*e^2)/(
2*e))/(d^2*e^4 + e^6*x^2 + 2*d*e^5*x) + x*((B*c)/e^3 - (3*C*c*d)/e^4) + (lo
g(d + e*x)*(A*c*e^2 + C*a*e^2 + 6*C*c*d^2 - 3*B*c*d*e))/e^5 + (C*c*x^2)/(2*
e^3)
```

sympy [A] time = 5.29, size = 206, normalized size = 1.32

$$\frac{Ccx^2}{2e^3}+x\left(\frac{Bc}{e^3}-\frac{3Ccd}{e^4}\right)+\frac{-Aae^4+3Acd^2e^2-Bade^3-5Bcd^3e+3Cad^2e^2+7Ccd^4+x\left(4Acde^3-2Bae^4-6Bcd\right.}{2d^2e^5+4de^6x+2e^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

```
[Out] C*c*x**2/(2*e**3) + x*(B*c/e**3 - 3*C*c*d/e**4) + (-A*a*e**4 + 3*A*c*d**2*e
**2 - B*a*d*e**3 - 5*B*c*d**3*e + 3*C*a*d**2*e**2 + 7*C*c*d**4 + x*(4*A*c*d
*e**3 - 2*B*a*e**4 - 6*B*c*d**2*e**2 + 4*C*a*d*e**3 + 8*C*c*d**3*e))/(2*d**
2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + (A*c*e**2 - 3*B*c*d*e + C*a*e**2 + 6*C
*c*d**2)*log(d + e*x)/e**5
```

3.25 $\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=304

$$\frac{1}{4}a^2ex^4\left(e(Ae+3Bd)+3Cd^2\right)+a^2Ad^3x+\frac{1}{8}cex^8\left(2aCe^2+c\left(e(Ae+3Bd)+3Cd^2\right)\right)+\frac{1}{7}cx^7\left(2ae^2(Be+3Cd)+cd\left(3\right.\right.$$

```
[Out] a^2*A*d^3*x+1/3*a*d*(a*d*(3*B*e+C*d)+A*(3*a*e^2+2*c*d^2))*x^3+1/4*a^2*e*(3*
C*d^2+e*(A*e+3*B*d))*x^4+1/5*(A*c*d*(6*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d)+2*
c*d^2*(3*B*e+C*d)))*x^5+1/6*a*e*(a*C*e^2+2*c*(3*C*d^2+e*(A*e+3*B*d)))*x^6+1
/7*c*(2*a*e^2*(B*e+3*C*d)+c*d*(C*d^2+3*e*(A*e+B*d)))*x^7+1/8*c*e*(2*a*C*e^2
+c*(3*C*d^2+e*(A*e+3*B*d)))*x^8+1/9*c^2*e^2*(B*e+3*C*d)*x^9+1/10*c^2*C*e^3*
x^10+1/6*d^2*(3*A*e+B*d)*(c*x^2+a)^3/c
```

Rubi [A] time = 0.53, antiderivative size = 301, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1582, 1810}

$$\frac{1}{4}a^2ex^4\left(e(Ae+3Bd)+3Cd^2\right)+a^2Ad^3x+\frac{1}{8}cex^8\left(2aCe^2+ce(Ae+3Bd)+3cCd^2\right)+\frac{1}{7}cx^7\left(2ae^2(Be+3Cd)+3cde(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^3*x + (a*d*(a*d*(C*d + 3*B*e) + A*(2*c*d^2 + 3*a*e^2))*x^3)/3 + (a^
2*e*(3*C*d^2 + e*(3*B*d + A*e))*x^4)/4 + ((A*c*d*(c*d^2 + 6*a*e^2) + a*(a*e
^2*(3*C*d + B*e) + 2*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a*e*(6*c*C*d^2 + a*C*e
^2 + 2*c*e*(3*B*d + A*e))*x^6)/6 + (c*(c*C*d^3 + 3*c*d*e*(B*d + A*e) + 2*a*
e^2*(3*C*d + B*e))*x^7)/7 + (c*e*(3*c*C*d^2 + 2*a*C*e^2 + c*e*(3*B*d + A*e)
)*x^8)/8 + (c^2*e^2*(3*C*d + B*e)*x^9)/9 + (c^2*C*e^3*x^10)/10 + (d^2*(B*d
+ 3*A*e)*(a + c*x^2)^3)/(6*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)dx &= \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^3}{6c}+\int \left(a+cx^2\right)^2\left(-\left(Bd^3+3Ad^2e\right)x+(d+\right.\\ &= \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^3}{6c}+\int \left(a^2Ad^3+ad\left(ad(Cd+3Be)+A\left(2cd^2\right.\right.\right.\\ &= a^2Ad^3x+\frac{1}{3}ad\left(ad(Cd+3Be)+A\left(2cd^2+3ae^2\right)\right)x^3+\frac{1}{4}a^2e\left(3Cd^2+\right.\end{aligned}$$

Mathematica [A] time = 0.13, size = 335, normalized size = 1.10

$$\frac{1}{2}a^2d^2x^2(3Ae+Bd)+a^2Ad^3x+\frac{1}{7}cx^7\left(2ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3\right)+\frac{1}{8}cex^8\left(2aCe^2+ce(Ae+3Bd)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^3*x + (a^2*d^2*(B*d + 3*A*e)*x^2)/2 + (a*d*(a*d*(C*d + 3*B*e) + A*(
2*c*d^2 + 3*a*e^2))*x^3)/3 + (a*(2*B*c*d^3 + 6*A*c*d^2*e + 3*a*C*d^2*e + 3*
a*B*d*e^2 + a*A*e^3)*x^4)/4 + ((A*c*d*(c*d^2 + 6*a*e^2) + a*(a*e^2*(3*C*d +
 B*e) + 2*c*d^2*(C*d + 3*B*e)))*x^5)/5 + ((a*C*e*(6*c*d^2 + a*e^2) + A*c*e*
(3*c*d^2 + 2*a*e^2) + B*c*d*(c*d^2 + 6*a*e^2))*x^6)/6 + (c*(c*C*d^3 + 3*c*d
*e*(B*d + A*e) + 2*a*e^2*(3*C*d + B*e))*x^7)/7 + (c*e*(3*c*C*d^2 + 2*a*C*e^
2 + c*e*(3*B*d + A*e))*x^8)/8 + (c^2*e^2*(3*C*d + B*e)*x^9)/9 + (c^2*C*e^3*
x^10)/10
```

fricas [A] time = 0.76, size = 432, normalized size = 1.42

$$\frac{1}{10}x^{10}e^3c^2C+\frac{1}{3}x^9e^2dc^2C+\frac{1}{9}x^9e^3c^2B+\frac{3}{8}x^8ed^2c^2C+\frac{1}{4}x^8e^3caC+\frac{3}{8}x^8e^2dc^2B+\frac{1}{8}x^8e^3c^2A+\frac{1}{7}x^7d^3c^2C+\frac{6}{7}x^7e^2dcaC+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e^3*c^2*C + 1/3*x^9*e^2*d*c^2*C + 1/9*x^9*e^3*c^2*B + 3/8*x^8*e*d
^2*c^2*C + 1/4*x^8*e^3*c*a*C + 3/8*x^8*e^2*d*c^2*B + 1/8*x^8*e^3*c^2*A + 1/
7*x^7*d^3*c^2*C + 6/7*x^7*e^2*d*c*a*C + 3/7*x^7*e*d^2*c^2*B + 2/7*x^7*e^3*c
*a*B + 3/7*x^7*e^2*d*c^2*A + x^6*e*d^2*c*a*C + 1/6*x^6*e^3*a^2*C + 1/6*x^6*
d^3*c^2*B + x^6*e^2*d*c*a*B + 1/2*x^6*e*d^2*c^2*A + 1/3*x^6*e^3*c*a*A + 2/5
*x^5*d^3*c*a*C + 3/5*x^5*e^2*d*a^2*C + 6/5*x^5*e*d^2*c*a*B + 1/5*x^5*e^3*a^
2*B + 1/5*x^5*d^3*c^2*A + 6/5*x^5*e^2*d*c*a*A + 3/4*x^4*e*d^2*a^2*C + 1/2*x
^4*d^3*c*a*B + 3/4*x^4*e^2*d*a^2*B + 3/2*x^4*e*d^2*c*a*A + 1/4*x^4*e^3*a^2*
A + 1/3*x^3*d^3*a^2*C + x^3*e*d^2*a^2*B + 2/3*x^3*d^3*c*a*A + x^3*e^2*d*a^2
*A + 1/2*x^2*d^3*a^2*B + 3/2*x^2*e*d^2*a^2*A + x*d^3*a^2*A
```

giac [A] time = 0.16, size = 423, normalized size = 1.39

$$\frac{1}{10}Cc^2x^{10}e^3+\frac{1}{3}Cc^2dx^9e^2+\frac{3}{8}Cc^2d^2x^8e+\frac{1}{7}Cc^2d^3x^7+\frac{1}{9}Bc^2x^9e^3+\frac{3}{8}Bc^2dx^8e^2+\frac{3}{7}Bc^2d^2x^7e+\frac{1}{6}Bc^2d^3x^6+\frac{1}{4}Cacx^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*C*c^2*x^10*e^3 + 1/3*C*c^2*d*x^9*e^2 + 3/8*C*c^2*d^2*x^8*e + 1/7*C*c^2
*d^3*x^7 + 1/9*B*c^2*x^9*e^3 + 3/8*B*c^2*d*x^8*e^2 + 3/7*B*c^2*d^2*x^7*e +
1/6*B*c^2*d^3*x^6 + 1/4*C*a*c*x^8*e^3 + 1/8*A*c^2*x^8*e^3 + 6/7*C*a*c*d*x^7
*e^2 + 3/7*A*c^2*d*x^7*e^2 + C*a*c*d^2*x^6*e + 1/2*A*c^2*d^2*x^6*e + 2/5*C*
a*c*d^3*x^5 + 1/5*A*c^2*d^3*x^5 + 2/7*B*a*c*x^7*e^3 + B*a*c*d*x^6*e^2 + 6/5
*B*a*c*d^2*x^5*e + 1/2*B*a*c*d^3*x^4 + 1/6*C*a^2*x^6*e^3 + 1/3*A*a*c*x^6*e^
3 + 3/5*C*a^2*d*x^5*e^2 + 6/5*A*a*c*d*x^5*e^2 + 3/4*C*a^2*d^2*x^4*e + 3/2*A
*a*c*d^2*x^4*e + 1/3*C*a^2*d^3*x^3 + 2/3*A*a*c*d^3*x^3 + 1/5*B*a^2*x^5*e^3
+ 3/4*B*a^2*d*x^4*e^2 + B*a^2*d^2*x^3*e + 1/2*B*a^2*d^3*x^2 + 1/4*A*a^2*x^4
*e^3 + A*a^2*d*x^3*e^2 + 3/2*A*a^2*d^2*x^2*e + A*a^2*d^3*x
```

maple [A] time = 0.00, size = 385, normalized size = 1.27

$$\frac{Cc^2e^3x^{10}}{10}+\frac{\left(e^3c^2B+3de^2c^2C\right)x^9}{9}+\frac{\left(Ac^2e^3+3Bc^2de^2+\left(2e^3ac+3d^2ec^2\right)C\right)x^8}{8}+Aa^2d^3x+\frac{\left(3Ac^2de^2+\left(2e^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x)
```

```
[Out] 1/10*c^2*C*e^3*x^10+1/9*(B*c^2*e^3+3*C*c^2*d*e^2)*x^9+1/8*((2*a*c*e^3+3*c^2
*d^2*e)*C+3*d*e^2*c^2*B+e^3*c^2*A)*x^8+1/7*((6*a*c*d*e^2+c^2*d^3)*C+(2*a*c*
e^3+3*c^2*d^2*e)*B+3*d*e^2*c^2*A)*x^7+1/6*((a^2*e^3+6*a*c*d^2*e)*C+(6*a*c*d
*e^2+c^2*d^3)*B+(2*a*c*e^3+3*c^2*d^2*e)*A)*x^6+1/5*((3*a^2*d*e^2+2*a*c*d^3)
*C+(a^2*e^3+6*a*c*d^2*e)*B+(6*a*c*d*e^2+c^2*d^3)*A)*x^5+1/4*(3*d^2*e*a^2*C+
(3*a^2*d*e^2+2*a*c*d^3)*B+(a^2*e^3+6*a*c*d^2*e)*A)*x^4+1/3*(d^3*a^2*C+3*d^2
*e*a^2*B+(3*a^2*d*e^2+2*a*c*d^3)*A)*x^3+1/2*(3*A*a^2*d^2*e+B*a^2*d^3)*x^2+a
^2*A*d^3*x
```

maxima [A] time = 0.45, size = 360, normalized size = 1.18

$$\frac{1}{10}\,Cc^2e^3x^{10}+\frac{1}{9}\left(3\,Cc^2de^2+Bc^2e^3\right)x^9+\frac{1}{8}\left(3\,Cc^2d^2e+3\,Bc^2de^2+\left(2\,Cac+Ac^2\right)e^3\right)x^8+\frac{1}{7}\left(Cc^2d^3+3\,Bc^2d^2e+2\,Ba\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*C*c^2*e^3*x^10 + 1/9*(3*C*c^2*d*e^2 + B*c^2*e^3)*x^9 + 1/8*(3*C*c^2*d^
2*e + 3*B*c^2*d*e^2 + (2*C*a*c + A*c^2)*e^3)*x^8 + 1/7*(C*c^2*d^3 + 3*B*c^2
*d^2*e + 2*B*a*c*e^3 + 3*(2*C*a*c + A*c^2)*d*e^2)*x^7 + A*a^2*d^3*x + 1/6*(
B*c^2*d^3 + 6*B*a*c*d*e^2 + 3*(2*C*a*c + A*c^2)*d^2*e + (C*a^2 + 2*A*a*c)*e
^3)*x^6 + 1/5*(6*B*a*c*d^2*e + B*a^2*e^3 + (2*C*a*c + A*c^2)*d^3 + 3*(C*a^2
 + 2*A*a*c)*d*e^2)*x^5 + 1/4*(2*B*a*c*d^3 + 3*B*a^2*d*e^2 + A*a^2*e^3 + 3*(
C*a^2 + 2*A*a*c)*d^2*e)*x^4 + 1/3*(3*B*a^2*d^2*e + 3*A*a^2*d*e^2 + (C*a^2 +
 2*A*a*c)*d^3)*x^3 + 1/2*(B*a^2*d^3 + 3*A*a^2*d^2*e)*x^2
```

mupad [B] time = 0.14, size = 332, normalized size = 1.09

$$x^5\left(\frac{3\,C\,a^2\,d\,e^2}{5}+\frac{B\,a^2\,e^3}{5}+\frac{2\,C\,a\,c\,d^3}{5}+\frac{6\,B\,a\,c\,d^2\,e}{5}+\frac{6\,A\,a\,c\,d\,e^2}{5}+\frac{A\,c^2\,d^3}{5}\right)+x^6\left(\frac{C\,a^2\,e^3}{6}+C\,a\,c\,d^2\,e+B\,a\,c\,d\,e^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^2*(d + e*x)^3*(A + B*x + C*x^2),x)
```

```
[Out] x^5*((A*c^2*d^3)/5 + (B*a^2*e^3)/5 + (2*C*a*c*d^3)/5 + (3*C*a^2*d*e^2)/5 +
(6*A*a*c*d*e^2)/5 + (6*B*a*c*d^2*e)/5) + x^6*((B*c^2*d^3)/6 + (C*a^2*e^3)/6
 + (A*a*c*e^3)/3 + (A*c^2*d^2*e)/2 + B*a*c*d*e^2 + C*a*c*d^2*e) + (a*x^4*(A
*a*e^3 + 2*B*c*d^3 + 3*B*a*d*e^2 + 6*A*c*d^2*e + 3*C*a*d^2*e))/4 + (c*x^7*(
2*B*a*e^3 + C*c*d^3 + 3*A*c*d*e^2 + 6*C*a*d*e^2 + 3*B*c*d^2*e))/7 + (C*c^2*
e^3*x^10)/10 + (a^2*d^2*x^2*(3*A*e + B*d))/2 + (c^2*e^2*x^9*(B*e + 3*C*d))/
9 + (a*d*x^3*(3*A*a*e^2 + 2*A*c*d^2 + C*a*d^2 + 3*B*a*d*e))/3 + (c*e*x^8*(A
*c*e^2 + 2*C*a*e^2 + 3*C*c*d^2 + 3*B*c*d*e))/8 + A*a^2*d^3*x
```

sympy [A] time = 0.13, size = 445, normalized size = 1.46

$$Aa^2d^3x+\frac{Cc^2e^3x^{10}}{10}+x^9\left(\frac{Bc^2e^3}{9}+\frac{Cc^2de^2}{3}\right)+x^8\left(\frac{Ac^2e^3}{8}+\frac{3Bc^2de^2}{8}+\frac{Cace^3}{4}+\frac{3Cc^2d^2e}{8}\right)+x^7\left(\frac{3Ac^2de^2}{7}+\frac{2Bace^3}{7}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*d**3*x + C*c**2*e**3*x**10/10 + x**9*(B*c**2*e**3/9 + C*c**2*d*e**2/
3) + x**8*(A*c**2*e**3/8 + 3*B*c**2*d*e**2/8 + C*a*c*e**3/4 + 3*C*c**2*d**2
*e/8) + x**7*(3*A*c**2*d*e**2/7 + 2*B*a*c*e**3/7 + 3*B*c**2*d**2*e/7 + 6*C*
a*c*d*e**2/7 + C*c**2*d**3/7) + x**6*(A*a*c*e**3/3 + A*c**2*d**2*e/2 + B*a*
```

```
c*d*e**2 + B*c**2*d**3/6 + C*a**2*e**3/6 + C*a*c*d**2*e) + x**5*(6*A*a*c*d*
e**2/5 + A*c**2*d**3/5 + B*a**2*e**3/5 + 6*B*a*c*d**2*e/5 + 3*C*a**2*d*e**2
/5 + 2*C*a*c*d**3/5) + x**4*(A*a**2*e**3/4 + 3*A*a*c*d**2*e/2 + 3*B*a**2*d*
e**2/4 + B*a*c*d**3/2 + 3*C*a**2*d**2*e/4) + x**3*(A*a**2*d*e**2 + 2*A*a*c*
d**3/3 + B*a**2*d**2*e + C*a**2*d**3/3) + x**2*(3*A*a**2*d**2*e/2 + B*a**2*
d**3/2)
```

3.26 $\int (d+ex)^2 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=217

$$a^2Ad^2x+\frac{1}{4}a^2ex^4(Be+2Cd)+\frac{1}{7}cx^7\left(2aCe^2+c\left(e(Ae+2Bd)+Cd^2\right)\right)+\frac{1}{5}x^5\left(Ac\left(2ae^2+cd^2\right)+a\left(aCe^2+2cd(2Be+\right.\right.$$

```
[Out] a^2*A*d^2*x+1/3*a*(a*d*(2*B*e+C*d)+A*(a*e^2+2*c*d^2))*x^3+1/4*a^2*e*(B*e+2*
C*d)*x^4+1/5*(A*c*(2*a*e^2+c*d^2)+a*(a*C*e^2+2*c*d*(2*B*e+C*d)))*x^5+1/3*a*
c*e*(B*e+2*C*d)*x^6+1/7*c*(2*a*C*e^2+c*(C*d^2+e*(A*e+2*B*d)))*x^7+1/8*c^2*e
*(B*e+2*C*d)*x^8+1/9*c^2*C*e^2*x^9+1/6*d*(2*A*e+B*d)*(c*x^2+a)^3/c
```

Rubi [A] time = 0.31, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1582, 1810}

$$a^2Ad^2x+\frac{1}{4}a^2ex^4(Be+2Cd)+\frac{1}{7}cx^7\left(2aCe^2+ce(Ae+2Bd)+cCd^2\right)+\frac{1}{5}x^5\left(Ac\left(2ae^2+cd^2\right)+a\left(aCe^2+2cd(2Be+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^2*x + (a*(a*d*(C*d + 2*B*e) + A*(2*c*d^2 + a*e^2))*x^3)/3 + (a^2*e*
(2*C*d + B*e)*x^4)/4 + ((A*c*(c*d^2 + 2*a*e^2) + a*(a*C*e^2 + 2*c*d*(C*d +
2*B*e)))*x^5)/5 + (a*c*e*(2*C*d + B*e)*x^6)/3 + (c*(c*C*d^2 + 2*a*C*e^2 + c
*e*(2*B*d + A*e))*x^7)/7 + (c^2*e*(2*C*d + B*e)*x^8)/8 + (c^2*C*e^2*x^9)/9
+ (d*(B*d + 2*A*e)*(a + c*x^2)^3)/(6*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)dx &= \frac{d(Bd+2Ae)\left(a+cx^2\right)^3}{6c}+\int \left(a+cx^2\right)^2\left(-\left(Bd^2+2Ade\right)x+(d+ex\right.\\ &= \frac{d(Bd+2Ae)\left(a+cx^2\right)^3}{6c}+\int \left(a^2Ad^2+a\left(ad(Cd+2Be)+A\left(2cd^2+\right.\right.\right.\\ &= a^2Ad^2x+\frac{1}{3}a\left(ad(Cd+2Be)+A\left(2cd^2+ae^2\right)\right)x^3+\frac{1}{4}a^2e(2Cd+Be\end{aligned}$$

Mathematica [A] time = 0.09, size = 241, normalized size = 1.11

$$\frac{1}{2}a^2dx^2(2Ae+Bd)+a^2Ad^2x+\frac{1}{7}cx^7\left(2aCe^2+ce(Ae+2Bd)+cCd^2\right)+\frac{1}{6}cx^6\left(2aBe^2+4aCde+2Acde+Bcd^2\right)+\frac{1}{5}x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^2*x + (a^2*d*(B*d + 2*A*e)*x^2)/2 + (a*(a*d*(C*d + 2*B*e) + A*(2*c*
d^2 + a*e^2))*x^3)/3 + (a*(2*B*c*d^2 + 4*A*c*d*e + 2*a*C*d*e + a*B*e^2)*x^4
)/4 + ((A*c*(c*d^2 + 2*a*e^2) + a*(a*C*e^2 + 2*c*d*(C*d + 2*B*e)))*x^5)/5 +
 (c*(B*c*d^2 + 2*A*c*d*e + 4*a*C*d*e + 2*a*B*e^2)*x^6)/6 + (c*(c*C*d^2 + 2*
a*C*e^2 + c*e*(2*B*d + A*e))*x^7)/7 + (c^2*e*(2*C*d + B*e)*x^8)/8 + (c^2*C*
e^2*x^9)/9
```

fricas [A] time = 0.44, size = 302, normalized size = 1.39

$$\frac{1}{9}x^9e^2c^2C+\frac{1}{4}x^8edc^2C+\frac{1}{8}x^8e^2c^2B+\frac{1}{7}x^7d^2c^2C+\frac{2}{7}x^7e^2caC+\frac{2}{7}x^7edc^2B+\frac{1}{7}x^7e^2c^2A+\frac{2}{3}x^6edcaC+\frac{1}{6}x^6d^2c^2B+\frac{1}{3}x^6e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*c^2*C + 1/4*x^8*e*d*c^2*C + 1/8*x^8*e^2*c^2*B + 1/7*x^7*d^2*c^2
*C + 2/7*x^7*e^2*c*a*C + 2/7*x^7*e*d*c^2*B + 1/7*x^7*e^2*c^2*A + 2/3*x^6*e*
d*c*a*C + 1/6*x^6*d^2*c^2*B + 1/3*x^6*e^2*c*a*B + 1/3*x^6*e*d*c^2*A + 2/5*x
^5*d^2*c*a*C + 1/5*x^5*e^2*a^2*C + 4/5*x^5*e*d*c*a*B + 1/5*x^5*d^2*c^2*A +
2/5*x^5*e^2*c*a*A + 1/2*x^4*e*d*a^2*C + 1/2*x^4*d^2*c*a*B + 1/4*x^4*e^2*a^2
*B + x^4*e*d*c*a*A + 1/3*x^3*d^2*a^2*C + 2/3*x^3*e*d*a^2*B + 2/3*x^3*d^2*c*
a*A + 1/3*x^3*e^2*a^2*A + 1/2*x^2*d^2*a^2*B + x^2*e*d*a^2*A + x*d^2*a^2*A
```

giac [A] time = 0.17, size = 302, normalized size = 1.39

$$\frac{1}{9}Cc^2x^9e^2+\frac{1}{4}Cc^2dx^8e+\frac{1}{7}Cc^2d^2x^7+\frac{1}{8}Bc^2x^8e^2+\frac{2}{7}Bc^2dx^7e+\frac{1}{6}Bc^2d^2x^6+\frac{2}{7}Cacx^7e^2+\frac{1}{7}Ac^2x^7e^2+\frac{2}{3}Cacdx^6e+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*C*c^2*x^9*e^2 + 1/4*C*c^2*d*x^8*e + 1/7*C*c^2*d^2*x^7 + 1/8*B*c^2*x^8*e
^2 + 2/7*B*c^2*d*x^7*e + 1/6*B*c^2*d^2*x^6 + 2/7*C*a*c*x^7*e^2 + 1/7*A*c^2*
x^7*e^2 + 2/3*C*a*c*d*x^6*e + 1/3*A*c^2*d*x^6*e + 2/5*C*a*c*d^2*x^5 + 1/5*A
*c^2*d^2*x^5 + 1/3*B*a*c*x^6*e^2 + 4/5*B*a*c*d*x^5*e + 1/2*B*a*c*d^2*x^4 +
1/5*C*a^2*x^5*e^2 + 2/5*A*a*c*x^5*e^2 + 1/2*C*a^2*d*x^4*e + A*a*c*d*x^4*e +
 1/3*C*a^2*d^2*x^3 + 2/3*A*a*c*d^2*x^3 + 1/4*B*a^2*x^4*e^2 + 2/3*B*a^2*d*x^
3*e + 1/2*B*a^2*d^2*x^2 + 1/3*A*a^2*x^3*e^2 + A*a^2*d*x^2*e + A*a^2*d^2*x
```

maple [A] time = 0.00, size = 268, normalized size = 1.24

$$\frac{C\,c^2e^2x^9}{9}+\frac{\left(e^2c^2B+2de\,c^2C\right)x^8}{8}+\frac{\left(A\,c^2e^2+2B\,c^2de+\left(2e^2ac+c^2d^2\right)C\right)x^7}{7}+A\,a^2d^2x+\frac{\left(2A\,c^2de+4Cacde+\left(2\right.\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x)
```

```
[Out] 1/9*c^2*C*e^2*x^9+1/8*(B*c^2*e^2+2*C*c^2*d*e)*x^8+1/7*((2*a*c*e^2+c^2*d^2)*
C+2*d*e*c^2*B+e^2*c^2*A)*x^7+1/6*(4*d*e*a*c*C+(2*a*c*e^2+c^2*d^2)*B+2*d*e*c
^2*A)*x^6+1/5*((a^2*e^2+2*a*c*d^2)*C+4*d*e*a*c*B+(2*a*c*e^2+c^2*d^2)*A)*x^5
+1/4*(2*d*e*a^2*C+(a^2*e^2+2*a*c*d^2)*B+4*d*e*a*c*A)*x^4+1/3*(d^2*a^2*C+2*d
*e*a^2*B+(a^2*e^2+2*a*c*d^2)*A)*x^3+1/2*(2*A*a^2*d*e+B*a^2*d^2)*x^2+a^2*A*d
^2*x
```

maxima [A] time = 0.44, size = 257, normalized size = 1.18

$$\frac{1}{9}Cc^2e^2x^9+\frac{1}{8}\left(2Cc^2de+Bc^2e^2\right)x^8+\frac{1}{7}\left(Cc^2d^2+2Bc^2de+\left(2Cac+Ac^2\right)e^2\right)x^7+\frac{1}{6}\left(Bc^2d^2+2Bace^2+2\left(2Cac\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*C*c^2*e^2*x^9 + 1/8*(2*C*c^2*d*e + B*c^2*e^2)*x^8 + 1/7*(C*c^2*d^2 + 2*
B*c^2*d*e + (2*C*a*c + A*c^2)*e^2)*x^7 + 1/6*(B*c^2*d^2 + 2*B*a*c*e^2 + 2*(
2*C*a*c + A*c^2)*d*e)*x^6 + A*a^2*d^2*x + 1/5*(4*B*a*c*d*e + (2*C*a*c + A*c
^2)*d^2 + (C*a^2 + 2*A*a*c)*e^2)*x^5 + 1/4*(2*B*a*c*d^2 + B*a^2*e^2 + 2*(C*
a^2 + 2*A*a*c)*d*e)*x^4 + 1/3*(2*B*a^2*d*e + A*a^2*e^2 + (C*a^2 + 2*A*a*c)*
d^2)*x^3 + 1/2*(B*a^2*d^2 + 2*A*a^2*d*e)*x^2
```

mupad [B] time = 3.72, size = 244, normalized size = 1.12

$$x^3\left(\frac{C\,a^2\,d^2}{3}+\frac{2\,B\,a^2\,d\,e}{3}+\frac{A\,a^2\,e^2}{3}+\frac{2\,A\,c\,a\,d^2}{3}\right)+x^7\left(\frac{C\,c^2\,d^2}{7}+\frac{2\,B\,c^2\,d\,e}{7}+\frac{A\,c^2\,e^2}{7}+\frac{2\,C\,a\,c\,e^2}{7}\right)+x^5\left(\frac{C\,a^2\,e^2}{5}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^2*(d + e*x)^2*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*a^2*e^2)/3 + (C*a^2*d^2)/3 + (2*A*a*c*d^2)/3 + (2*B*a^2*d*e)/3) + x
^7*((A*c^2*e^2)/7 + (C*c^2*d^2)/7 + (2*C*a*c*e^2)/7 + (2*B*c^2*d*e)/7) + x^
5*((A*c^2*d^2)/5 + (C*a^2*e^2)/5 + (2*A*a*c*e^2)/5 + (2*C*a*c*d^2)/5 + (4*B
*a*c*d*e)/5) + (a*x^4*(B*a*e^2 + 2*B*c*d^2 + 4*A*c*d*e + 2*C*a*d*e))/4 + (c
*x^6*(2*B*a*e^2 + B*c*d^2 + 2*A*c*d*e + 4*C*a*d*e))/6 + (C*c^2*e^2*x^9)/9 +
 A*a^2*d^2*x + (a^2*d*x^2*(2*A*e + B*d))/2 + (c^2*e*x^8*(B*e + 2*C*d))/8
```

sympy [A] time = 0.12, size = 311, normalized size = 1.43

$$Aa^2d^2x+\frac{Cc^2e^2x^9}{9}+x^8\left(\frac{Bc^2e^2}{8}+\frac{Cc^2de}{4}\right)+x^7\left(\frac{Ac^2e^2}{7}+\frac{2Bc^2de}{7}+\frac{2Cace^2}{7}+\frac{Cc^2d^2}{7}\right)+x^6\left(\frac{Ac^2de}{3}+\frac{Bace^2}{3}+\frac{Bc^2d^2}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*d**2*x + C*c**2*e**2*x**9/9 + x**8*(B*c**2*e**2/8 + C*c**2*d*e/4) +
x**7*(A*c**2*e**2/7 + 2*B*c**2*d*e/7 + 2*C*a*c*e**2/7 + C*c**2*d**2/7) + x*
*6*(A*c**2*d*e/3 + B*a*c*e**2/3 + B*c**2*d**2/6 + 2*C*a*c*d*e/3) + x**5*(2*
A*a*c*e**2/5 + A*c**2*d**2/5 + 4*B*a*c*d*e/5 + C*a**2*e**2/5 + 2*C*a*c*d**2
/5) + x**4*(A*a*c*d*e + B*a**2*e**2/4 + B*a*c*d**2/2 + C*a**2*d*e/2) + x**3
*(A*a**2*e**2/3 + 2*A*a*c*d**2/3 + 2*B*a**2*d*e/3 + C*a**2*d**2/3) + x**2*(
A*a**2*d*e + B*a**2*d**2/2)
```

3.27 $\int (d+ex) \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=128

$$a^2Adx+\frac{1}{4}a^2Cex^4+\frac{1}{5}cx^5(2a(Be+Cd)+Acd)+\frac{1}{3}ax^3(aBe+aCd+2Acd)+\frac{\left(a+cx^2\right)^3(Ae+Bd)}{6c}+\frac{1}{3}acCex^6+\frac{1}{7}c^2x^7($$

```
[Out] a^2*A*d*x+1/3*a*(2*A*c*d+B*a*e+C*a*d)*x^3+1/4*a^2*C*e*x^4+1/5*c*(A*c*d+2*a*
(B*e+C*d))*x^5+1/3*a*c*C*e*x^6+1/7*c^2*(B*e+C*d)*x^7+1/8*c^2*C*e*x^8+1/6*(A
*e+B*d)*(c*x^2+a)^3/c
```

Rubi [A] time = 0.16, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1582, 1810}

$$a^2Adx+\frac{1}{4}a^2Cex^4+\frac{1}{5}cx^5(2a(Be+Cd)+Acd)+\frac{1}{3}ax^3(aBe+aCd+2Acd)+\frac{\left(a+cx^2\right)^3(Ae+Bd)}{6c}+\frac{1}{3}acCex^6+\frac{1}{7}c^2x^7($$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d*x + (a*(2*A*c*d + a*C*d + a*B*e)*x^3)/3 + (a^2*C*e*x^4)/4 + (c*(A*c
*d + 2*a*(C*d + B*e))*x^5)/5 + (a*c*C*e*x^6)/3 + (c^2*(C*d + B*e)*x^7)/7 +
(c^2*C*e*x^8)/8 + ((B*d + A*e)*(a + c*x^2)^3)/(6*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)dx &= \frac{(Bd+Ae)\left(a+cx^2\right)^3}{6c}+\int \left(a+cx^2\right)^2\left(-(Bd+Ae)x+(d+ex)\left(A\right.\right.\\
&= \frac{(Bd+Ae)\left(a+cx^2\right)^3}{6c}+\int \left(a^2Ad+a(2Acd+aCd+aBe)x^2+a^2C\right.\\
&= a^2Adx+\frac{1}{3}a(2Acd+aCd+aBe)x^3+\frac{1}{4}a^2Cex^4+\frac{1}{5}c(Acd+2a(Cd
\end{aligned}$$

Mathematica [A] time = 0.05, size = 144, normalized size = 1.12

$$\frac{1}{2}a^2x^2(Ae+Bd)+a^2Adx+\frac{1}{6}cx^6(2aCe+Ace+Bcd)+\frac{1}{5}cx^5(2aBe+2aCd+Acd)+\frac{1}{4}ax^4(aCe+2Ace+2Bcd)+\frac{1}{3}ax^3(aB$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d*x + (a^2*(B*d + A*e)*x^2)/2 + (a*(2*A*c*d + a*C*d + a*B*e)*x^3)/3 +
 (a*(2*B*c*d + 2*A*c*e + a*C*e)*x^4)/4 + (c*(A*c*d + 2*a*C*d + 2*a*B*e)*x^5
)/5 + (c*(B*c*d + A*c*e + 2*a*C*e)*x^6)/6 + (c^2*(C*d + B*e)*x^7)/7 + (c^2*
C*e*x^8)/8
```

fricas [A] time = 0.81, size = 172, normalized size = 1.34

$$\frac{1}{8}x^8ec^2C+\frac{1}{7}x^7dc^2C+\frac{1}{7}x^7ec^2B+\frac{1}{3}x^6ecaC+\frac{1}{6}x^6dc^2B+\frac{1}{6}x^6ec^2A+\frac{2}{5}x^5dcaC+\frac{2}{5}x^5ecaB+\frac{1}{5}x^5dc^2A+\frac{1}{4}x^4ea^2C+\frac{1}{2}x^4dca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e*c^2*C + 1/7*x^7*d*c^2*C + 1/7*x^7*e*c^2*B + 1/3*x^6*e*c*a*C + 1/6
*x^6*d*c^2*B + 1/6*x^6*e*c^2*A + 2/5*x^5*d*c*a*C + 2/5*x^5*e*c*a*B + 1/5*x^
5*d*c^2*A + 1/4*x^4*e*a^2*C + 1/2*x^4*d*c*a*B + 1/2*x^4*e*c*a*A + 1/3*x^3*d
*a^2*C + 1/3*x^3*e*a^2*B + 2/3*x^3*d*c*a*A + 1/2*x^2*d*a^2*B + 1/2*x^2*e*a^
2*A + x*d*a^2*A
```

giac [A] time = 0.15, size = 181, normalized size = 1.41

$$\frac{1}{8}Cc^2x^8e+\frac{1}{7}Cc^2dx^7+\frac{1}{7}Bc^2x^7e+\frac{1}{6}Bc^2dx^6+\frac{1}{3}Cacx^6e+\frac{1}{6}Ac^2x^6e+\frac{2}{5}Cacdx^5+\frac{1}{5}Ac^2dx^5+\frac{2}{5}Bacx^5e+\frac{1}{2}Bacdx^4+\frac{1}{4}C$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*C*c^2*x^8*e + 1/7*C*c^2*d*x^7 + 1/7*B*c^2*x^7*e + 1/6*B*c^2*d*x^6 + 1/3
*C*a*c*x^6*e + 1/6*A*c^2*x^6*e + 2/5*C*a*c*d*x^5 + 1/5*A*c^2*d*x^5 + 2/5*B*
a*c*x^5*e + 1/2*B*a*c*d*x^4 + 1/4*C*a^2*x^4*e + 1/2*A*a*c*x^4*e + 1/3*C*a^2
*d*x^3 + 2/3*A*a*c*d*x^3 + 1/3*B*a^2*x^3*e + 1/2*B*a^2*d*x^2 + 1/2*A*a^2*x^
2*e + A*a^2*d*x
```

maple [A] time = 0.00, size = 151, normalized size = 1.18

$$\frac{C\,c^2e\,x^8}{8}+\frac{\left(c^2eB+c^2dC\right)x^7}{7}+\frac{\left(c^2eA+c^2dB+2eacC\right)x^6}{6}+A\,a^2dx+\frac{\left(c^2dA+2eacB+2dacC\right)x^5}{5}+\frac{\left(2eacA+2dacB\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x)
```

```
[Out] 1/8*c^2*C*e*x^8+1/7*(B*c^2*e+C*c^2*d)*x^7+1/6*(A*c^2*e+B*c^2*d+2*C*a*c*e)*x
^6+1/5*(A*c^2*d+2*B*a*c*e+2*C*a*c*d)*x^5+1/4*(2*A*a*c*e+2*B*a*c*d+C*a^2*e)*
x^4+1/3*(2*A*a*c*d+B*a^2*e+C*a^2*d)*x^3+1/2*(A*a^2*e+B*a^2*d)*x^2+a^2*A*d*x
```

maxima [A] time = 0.45, size = 154, normalized size = 1.20

$$\frac{1}{8}Cc^2ex^8+\frac{1}{7}\left(Cc^2d+Bc^2e\right)x^7+\frac{1}{6}\left(Bc^2d+\left(2\,Cac+Ac^2\right)e\right)x^6+\frac{1}{5}\left(2\,Bace+\left(2\,Cac+Ac^2\right)d\right)x^5+Aa^2dx+\frac{1}{4}\left(2\,Bacd\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*C*c^2*e*x^8 + 1/7*(C*c^2*d + B*c^2*e)*x^7 + 1/6*(B*c^2*d + (2*C*a*c + A
*c^2)*e)*x^6 + 1/5*(2*B*a*c*e + (2*C*a*c + A*c^2)*d)*x^5 + A*a^2*d*x + 1/4*
```

```
(2*B*a*c*d + (C*a^2 + 2*A*a*c)*e)*x^4 + 1/3*(B*a^2*e + (C*a^2 + 2*A*a*c)*d)
*x^3 + 1/2*(B*a^2*d + A*a^2*e)*x^2
```

mupad [B] time = 3.69, size = 140, normalized size = 1.09

$$x^3\left(\frac{B\,a^2\,e}{3}+\frac{C\,a^2\,d}{3}+\frac{2\,A\,a\,c\,d}{3}\right)+x^6\left(\frac{A\,c^2\,e}{6}+\frac{B\,c^2\,d}{6}+\frac{C\,a\,c\,e}{3}\right)+\frac{c\,x^5\,\left(A\,c\,d+2\,B\,a\,e+2\,C\,a\,d\right)}{5}+\frac{a\,x^4\,\left(2\,A\,c\,e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^2*(d + e*x)*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((B*a^2*e)/3 + (C*a^2*d)/3 + (2*A*a*c*d)/3) + x^6*((A*c^2*e)/6 + (B*c^2
*d)/6 + (C*a*c*e)/3) + (c*x^5*(A*c*d + 2*B*a*e + 2*C*a*d))/5 + (a*x^4*(2*A*
c*e + 2*B*c*d + C*a*e))/4 + (a^2*x^2*(A*e + B*d))/2 + (c^2*x^7*(B*e + C*d))
/7 + A*a^2*d*x + (C*c^2*e*x^8)/8
```

sympy [A] time = 0.10, size = 180, normalized size = 1.41

$$Aa^2dx+\frac{Cc^2ex^8}{8}+x^7\left(\frac{Bc^2e}{7}+\frac{Cc^2d}{7}\right)+x^6\left(\frac{Ac^2e}{6}+\frac{Bc^2d}{6}+\frac{Cace}{3}\right)+x^5\left(\frac{Ac^2d}{5}+\frac{2Bace}{5}+\frac{2Cacd}{5}\right)+x^4\left(\frac{Aace}{2}+\frac{B}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*d*x + C*c**2*e*x**8/8 + x**7*(B*c**2*e/7 + C*c**2*d/7) + x**6*(A*c**
2*e/6 + B*c**2*d/6 + C*a*c*e/3) + x**5*(A*c**2*d/5 + 2*B*a*c*e/5 + 2*C*a*c*
d/5) + x**4*(A*a*c*e/2 + B*a*c*d/2 + C*a**2*e/4) + x**3*(2*A*a*c*d/3 + B*a*
*2*e/3 + C*a**2*d/3) + x**2*(A*a**2*e/2 + B*a**2*d/2)
```

3.28 $\int \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=67

$$a^2Ax+\frac{1}{5}cx^5(2aC+Ac)+\frac{1}{3}ax^3(aC+2Ac)+\frac{B\left(a+cx^2\right)^3}{6c}+\frac{1}{7}c^2Cx^7$$

```
[Out] a^2*A*x+1/3*a*(2*A*c+C*a)*x^3+1/5*c*(A*c+2*C*a)*x^5+1/7*c^2*C*x^7+1/6*B*(c*
x^2+a)^3/c
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1582, 373}

$$a^2Ax+\frac{1}{5}cx^5(2aC+Ac)+\frac{1}{3}ax^3(aC+2Ac)+\frac{B\left(a+cx^2\right)^3}{6c}+\frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*x + (a*(2*A*c + a*C)*x^3)/3 + (c*(A*c + 2*a*C)*x^5)/5 + (c^2*C*x^7)/7
 + (B*(a + c*x^2)^3)/(6*c)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\begin{aligned}
\int \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx &= \frac{B\left(a+cx^2\right)^3}{6c}+\int \left(a+cx^2\right)^2 \left(A+Cx^2\right) dx\\
&= \frac{B\left(a+cx^2\right)^3}{6c}+\int \left(a^2A+a(2Ac+aC)x^2+c(Ac+2aC)x^4+c^2Cx^6\right) dx\\
&= a^2Ax+\frac{1}{3}a(2Ac+aC)x^3+\frac{1}{5}c(Ac+2aC)x^5+\frac{1}{7}c^2Cx^7+\frac{B\left(a+cx^2\right)^3}{6c}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 69, normalized size = 1.03

$$\frac{1}{210}x\left(35a^2(6A+x(3B+2Cx))+7acx^2(20A+3x(5B+4Cx))+c^2x^4(42A+5x(7B+6Cx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] (x*(35*a^2*(6*A + x*(3*B + 2*C*x)) + 7*a*c*x^2*(20*A + 3*x*(5*B + 4*C*x)) +
 c^2*x^4*(42*A + 5*x*(7*B + 6*C*x))))/210
```

fricas [A] time = 0.90, size = 76, normalized size = 1.13

$$\frac{1}{7}x^7c^2C + \frac{1}{6}x^6c^2B + \frac{2}{5}x^5caC + \frac{1}{5}x^5c^2A + \frac{1}{2}x^4caB + \frac{1}{3}x^3a^2C + \frac{2}{3}x^3caA + \frac{1}{2}x^2a^2B + xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c^2*C + 1/6*x^6*c^2*B + 2/5*x^5*c*a*C + 1/5*x^5*c^2*A + 1/2*x^4*c*a
*B + 1/3*x^3*a^2*C + 2/3*x^3*c*a*A + 1/2*x^2*a^2*B + x*a^2*A
```

giac [A] time = 0.15, size = 76, normalized size = 1.13

$$\frac{1}{7}Cc^2x^7 + \frac{1}{6}Bc^2x^6 + \frac{2}{5}Cacx^5 + \frac{1}{5}Ac^2x^5 + \frac{1}{2}Bacx^4 + \frac{1}{3}Ca^2x^3 + \frac{2}{3}Aacx^3 + \frac{1}{2}Ba^2x^2 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 2/5*C*a*c*x^5 + 1/5*A*c^2*x^5 + 1/2*B*a*c*x
^4 + 1/3*C*a^2*x^3 + 2/3*A*a*c*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

maple [A] time = 0.00, size = 75, normalized size = 1.12

$$\frac{C\,c^2x^7}{7} + \frac{B\,c^2x^6}{6} + \frac{Bac\,x^4}{2} + \frac{B\,a^2x^2}{2} + \frac{\left(A\,c^2 + 2acC\right)x^5}{5} + A\,a^2x + \frac{\left(2acA + a^2C\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A),x)
```

```
[Out] 1/7*c^2*C*x^7+1/6*c^2*B*x^6+1/5*(A*c^2+2*C*a*c)*x^5+1/2*a*c*B*x^4+1/3*(2*A*
a*c+C*a^2)*x^3+1/2*a^2*B*x^2+a^2*A*x
```

maxima [A] time = 0.44, size = 74, normalized size = 1.10

$$\frac{1}{7}Cc^2x^7 + \frac{1}{6}Bc^2x^6 + \frac{1}{2}Bacx^4 + \frac{1}{5}\left(2\,Cac + Ac^2\right)x^5 + \frac{1}{2}Ba^2x^2 + Aa^2x + \frac{1}{3}\left(Ca^2 + 2\,Aac\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*a*c*x^4 + 1/5*(2*C*a*c + A*c^2)*x^5 +
 1/2*B*a^2*x^2 + A*a^2*x + 1/3*(C*a^2 + 2*A*a*c)*x^3
```

mupad [B] time = 0.04, size = 74, normalized size = 1.10

$$x^3\,\left(\frac{C\,a^2}{3} + \frac{2\,A\,c\,a}{3}\right) + x^5\,\left(\frac{A\,c^2}{5} + \frac{2\,C\,a\,c}{5}\right) + \frac{B\,a^2\,x^2}{2} + \frac{B\,c^2\,x^6}{6} + \frac{C\,c^2\,x^7}{7} + A\,a^2\,x + \frac{B\,a\,c\,x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^2*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((C*a^2)/3 + (2*A*a*c)/3) + x^5*((A*c^2)/5 + (2*C*a*c)/5) + (B*a^2*x^2)
/2 + (B*c^2*x^6)/6 + (C*c^2*x^7)/7 + A*a^2*x + (B*a*c*x^4)/2
```

sympy [A] time = 0.08, size = 83, normalized size = 1.24

$$Aa^2x + \frac{Ba^2x^2}{2} + \frac{Bacx^4}{2} + \frac{Bc^2x^6}{6} + \frac{Cc^2x^7}{7} + x^5\left(\frac{Ac^2}{5} + \frac{2Cac}{5}\right) + x^3\left(\frac{2Aac}{3} + \frac{Ca^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x + B*a**2*x**2/2 + B*a*c*x**4/2 + B*c**2*x**6/6 + C*c**2*x**7/7 + x
**5*(A*c**2/5 + 2*C*a*c/5) + x**3*(2*A*a*c/3 + C*a**2/3)
```

3.29 $\int \frac{\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)}{d+ex}\,dx$

Optimal. Leaf size=297

$$\frac{x^2\left(a^2Ce^4+2ace^2\left(Cd^2-e(Bd-Ae)\right)+c^2d^2\left(Cd^2-e(Bd-Ae)\right)\right)}{2e^5}-\frac{x\left(a^2e^4(Cd-Be)+2acde^2\left(Cd^2-e(Bd-\right.\right.}{e^6}$$

```
[Out] -(a^2*e^4*(-B*e+C*d)+c^2*d^3*(C*d^2-e*(-A*e+B*d))+2*a*c*d*e^2*(C*d^2-e*(-A*
e+B*d)))*x/e^6+1/2*(a^2*C*e^4+c^2*d^2*(C*d^2-e*(-A*e+B*d))+2*a*c*e^2*(C*d^2
-e*(-A*e+B*d)))*x^2/e^5-1/3*c*(2*a*e^2*(-B*e+C*d)+c*d*(C*d^2-e*(-A*e+B*d)))
*x^3/e^4+1/4*c*(2*a*C*e^2+c*(C*d^2-e*(-A*e+B*d)))*x^4/e^3-1/5*c^2*(-B*e+C*d
)*x^5/e^2+1/6*c^2*C*x^6/e+(a*e^2+c*d^2)^2*(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e^7
```

Rubi [A] time = 0.64, antiderivative size = 295, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1628}

$$\frac{x^2\left(a^2Ce^4+2ace^2\left(Cd^2-e(Bd-Ae)\right)+c^2\left(Cd^4-d^2e(Bd-Ae)\right)\right)}{2e^5}-\frac{x\left(a^2e^4(Cd-Be)+2acde^2\left(Cd^2-e(Bd-\right.\right.}{e^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] -(((a^2*e^4*(C*d - B*e) + 2*a*c*d*e^2*(C*d^2 - e*(B*d - A*e)) + c^2*(C*d^5
- d^3*e*(B*d - A*e)))*x)/e^6) + ((a^2*C*e^4 + 2*a*c*e^2*(C*d^2 - e*(B*d - A
*e)) + c^2*(C*d^4 - d^2*e*(B*d - A*e)))*x^2)/(2*e^5) - (c*(c*C*d^3 - c*d*e*
(B*d - A*e) + 2*a*e^2*(C*d - B*e))*x^3)/(3*e^4) + (c*(c*C*d^2 + 2*a*C*e^2 -
 c*e*(B*d - A*e))*x^4)/(4*e^3) - (c^2*(C*d - B*e)*x^5)/(5*e^2) + (c^2*C*x^6
)/(6*e) + ((c*d^2 + a*e^2)^2*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/e^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)}{d+ex}\,dx = \int \left(\frac{-a^2e^4(Cd-Be)-2acde^2\left(Cd^2-e(Bd-Ae)\right)-c^2\left(Cd^5-d^3e(Bd-Ae\right.}{e^6}\right.$$

$$= -\frac{\left(a^2e^4(Cd-Be)+2acde^2\left(Cd^2-e(Bd-Ae)\right)+c^2\left(Cd^5-d^3e(Bd-Ae)\right)\right)}{e^6}$$

Mathematica [A] time = 0.17, size = 285, normalized size = 0.96

$$ex\left(30a^2e^4(2Be-2Cd+Cex)+10ace^2\left(2e\left(3Ae(ex-2d)+B\left(6d^2-3dex+2e^2x^2\right)\right)+C\left(-12d^3+6d^2ex-4d\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] (e*x*(30*a^2*e^4*(-2*C*d + 2*B*e + C*e*x) + 10*a*c*e^2*(C*(-12*d^3 + 6*d^2*
e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 2*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d*e
```

```
*x + 2*e^2*x^2))) + c^2*(C*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d^2*
e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + e*(5*A*e*(-12*d^3 + 6*d^2*e*x - 4*d*
e^2*x^2 + 3*e^3*x^3) + B*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x
^3 + 12*e^4*x^4)))) + 60*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e))*Log[d
 + e*x])/(60*e^7)
```

fricas [A] time = 0.62, size = 379, normalized size = 1.28

$$\frac{10\,Cc^2e^6x^6 - 12\left(Cc^2de^5 - Bc^2e^6\right)x^5 + 15\left(Cc^2d^2e^4 - Bc^2de^5 + \left(2\,Cac + Ac^2\right)e^6\right)x^4 - 20\left(Cc^2d^3e^3 - Bc^2d^2e^4 - 2\,B\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*c^2*e^6*x^6 - 12*(C*c^2*d*e^5 - B*c^2*e^6)*x^5 + 15*(C*c^2*d^2*e
^4 - B*c^2*d*e^5 + (2*C*a*c + A*c^2)*e^6)*x^4 - 20*(C*c^2*d^3*e^3 - B*c^2*d
^2*e^4 - 2*B*a*c*e^6 + (2*C*a*c + A*c^2)*d*e^5)*x^3 + 30*(C*c^2*d^4*e^2 - B
*c^2*d^3*e^3 - 2*B*a*c*d*e^5 + (2*C*a*c + A*c^2)*d^2*e^4 + (C*a^2 + 2*A*a*c
)*e^6)*x^2 - 60*(C*c^2*d^5*e - B*c^2*d^4*e^2 - 2*B*a*c*d^2*e^4 - B*a^2*e^6
+ (2*C*a*c + A*c^2)*d^3*e^3 + (C*a^2 + 2*A*a*c)*d*e^5)*x + 60*(C*c^2*d^6 -
B*c^2*d^5*e - 2*B*a*c*d^3*e^3 - B*a^2*d*e^5 + A*a^2*e^6 + (2*C*a*c + A*c^2)
*d^4*e^2 + (C*a^2 + 2*A*a*c)*d^2*e^4)*log(e*x + d))/e^7
```

giac [A] time = 0.16, size = 416, normalized size = 1.40

$$\left(Cc^2d^6 - Bc^2d^5e + 2\,Cacd^4e^2 + Ac^2d^4e^2 - 2\,Bacd^3e^3 + Ca^2d^2e^4 + 2\,Aacd^2e^4 - Ba^2de^5 + Aa^2e^6\right)e^{(-7)}\log\left(|xe + d|\right)\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

```
[Out] (C*c^2*d^6 - B*c^2*d^5*e + 2*C*a*c*d^4*e^2 + A*c^2*d^4*e^2 - 2*B*a*c*d^3*e^
3 + C*a^2*d^2*e^4 + 2*A*a*c*d^2*e^4 - B*a^2*d*e^5 + A*a^2*e^6)*e^(-7)*log(a
bs(x*e + d)) + 1/60*(10*C*c^2*x^6*e^5 - 12*C*c^2*d*x^5*e^4 + 15*C*c^2*d^2*x
^4*e^3 - 20*C*c^2*d^3*x^3*e^2 + 30*C*c^2*d^4*x^2*e - 60*C*c^2*d^5*x + 12*B*
c^2*x^5*e^5 - 15*B*c^2*d*x^4*e^4 + 20*B*c^2*d^2*x^3*e^3 - 30*B*c^2*d^3*x^2*
e^2 + 60*B*c^2*d^4*x*e + 30*C*a*c*x^4*e^5 + 15*A*c^2*x^4*e^5 - 40*C*a*c*d*x
^3*e^4 - 20*A*c^2*d*x^3*e^4 + 60*C*a*c*d^2*x^2*e^3 + 30*A*c^2*d^2*x^2*e^3 -
 120*C*a*c*d^3*x*e^2 - 60*A*c^2*d^3*x*e^2 + 40*B*a*c*x^3*e^5 - 60*B*a*c*d*x
^2*e^4 + 120*B*a*c*d^2*x*e^3 + 30*C*a^2*x^2*e^5 + 60*A*a*c*x^2*e^5 - 60*C*a
^2*d*x*e^4 - 120*A*a*c*d*x*e^4 + 60*B*a^2*x*e^5)*e^(-6)
```

maple [A] time = 0.01, size = 490, normalized size = 1.65

$$\frac{C\,c^2x^6}{6e} + \frac{B\,c^2x^5}{5e} - \frac{C\,c^2d\,x^5}{5e^2} + \frac{A\,c^2x^4}{4e} - \frac{B\,c^2d\,x^4}{4e^2} + \frac{Cac\,x^4}{2e} + \frac{C\,c^2d^2x^4}{4e^3} - \frac{A\,c^2d\,x^3}{3e^2} + \frac{2Bac\,x^3}{3e} + \frac{B\,c^2d^2x^3}{3e^3} - \frac{2Cacd\,x^3}{3e^2} - \frac{C\,c^2d\cdots}{3e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x)
```

```
[Out] 1/3/e^3*B*x^3*c^2*d^2-1/3/e^4*C*x^3*c^2*d^3-1/5/e^2*C*x^5*c^2*d-1/4/e^2*B*x
^4*c^2*d-1/e^2*ln(e*x+d)*B*a^2*d+1/e^5*B*x*c^2*d^4-1/e^2*C*x*a^2*d-1/e^4*A*
x*c^2*d^3+1/e*A*x^2*a*c-1/e^6*C*x*c^2*d^5+1/2/e^3*A*x^2*c^2*d^2-1/2/e^4*B*x
^2*c^2*d^3+1/2/e^5*C*x^2*c^2*d^4+1/e^7*ln(e*x+d)*C*c^2*d^6+1/e^5*ln(e*x+d)*
A*c^2*d^4+1/6*c^2*C*x^6/e-2/e^4*ln(e*x+d)*B*a*c*d^3+2/e^5*ln(e*x+d)*C*a*c*d
^4+1/e^3*C*x^2*a*c*d^2-1/e^2*B*x^2*a*c*d-2/3/e^2*C*x^3*a*c*d-2/e^4*C*x*a*c*
d^3-2/e^2*A*x*a*c*d+2/e^3*B*x*a*c*d^2+2/e^3*ln(e*x+d)*A*a*c*d^2+1/2/e*C*x^2
*a^2+1/5/e*B*x^5*c^2+1/4/e*A*x^4*c^2+1/e*B*x*a^2+1/e*ln(e*x+d)*A*a^2+2/3/e*
B*x^3*a*c-1/3/e^2*A*x^3*c^2*d+1/2/e*C*x^4*a*c+1/4/e^3*C*x^4*c^2*d^2+1/e^3*l
n(e*x+d)*C*a^2*d^2-1/e^6*ln(e*x+d)*B*c^2*d^5
```

maxima [A] time = 0.48, size = 377, normalized size = 1.27

$$10\,Cc^2e^5x^6 - 12\left(Cc^2de^4 - Bc^2e^5\right)x^5 + 15\left(Cc^2d^2e^3 - Bc^2de^4 + \left(2\,Cac + Ac^2\right)e^5\right)x^4 - 20\left(Cc^2d^3e^2 - Bc^2d^2e^3 - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/60*(10*C*c^2*e^5*x^6 - 12*(C*c^2*d*e^4 - B*c^2*e^5)*x^5 + 15*(C*c^2*d^2*e
^3 - B*c^2*d*e^4 + (2*C*a*c + A*c^2)*e^5)*x^4 - 20*(C*c^2*d^3*e^2 - B*c^2*d
^2*e^3 - 2*B*a*c*e^5 + (2*C*a*c + A*c^2)*d*e^4)*x^3 + 30*(C*c^2*d^4*e - B*c
^2*d^3*e^2 - 2*B*a*c*d*e^4 + (2*C*a*c + A*c^2)*d^2*e^3 + (C*a^2 + 2*A*a*c)*
e^5)*x^2 - 60*(C*c^2*d^5 - B*c^2*d^4*e - 2*B*a*c*d^2*e^3 - B*a^2*e^5 + (2*C
*a*c + A*c^2)*d^3*e^2 + (C*a^2 + 2*A*a*c)*d*e^4)*x)/e^6 + (C*c^2*d^6 - B*c^
2*d^5*e - 2*B*a*c*d^3*e^3 - B*a^2*d*e^5 + A*a^2*e^6 + (2*C*a*c + A*c^2)*d^4
*e^2 + (C*a^2 + 2*A*a*c)*d^2*e^4)*log(e*x + d)/e^7
```

mupad [B] time = 3.68, size = 422, normalized size = 1.42

$$x^5\left(\frac{B\,c^2}{5\,e}-\frac{C\,c^2\,d}{5\,e^2}\right)-x\left(\frac{d\left(\frac{C\,a^2+2\,A\,c\,a}{e}+\frac{d\left(\frac{d\left(\frac{A\,c^2+2\,C\,a\,c}{e}-\frac{d\left(\frac{B\,c^2}{e}-\frac{C\,c^2\,d}{e^2}\right)}{e}\right)}{e}-\frac{2\,B\,a\,c}{e}\right)}{e}\right)}{e}-\frac{B\,a^2}{e}\right)+x^4\left(\frac{A\,c^2+2\,C\,a\,c}{4\,e}-\frac{d\left(\frac{B\,c^2}{e}\right.}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x)
```

```
[Out] x^5*((B*c^2)/(5*e) - (C*c^2*d)/(5*e^2)) - x*((d*((C*a^2 + 2*A*a*c)/e + (d*(
(d*((A*c^2 + 2*C*a*c)/e - (d*((B*c^2)/e - (C*c^2*d)/e^2))/e))/e - (2*B*a*c)
/e))/e))/e - (B*a^2)/e) + x^4*((A*c^2 + 2*C*a*c)/(4*e) - (d*((B*c^2)/e - (C
*c^2*d)/e^2))/(4*e)) - x^3*((d*((A*c^2 + 2*C*a*c)/e - (d*((B*c^2)/e - (C*c^
2*d)/e^2))/e))/(3*e) - (2*B*a*c)/(3*e)) + x^2*((C*a^2 + 2*A*a*c)/(2*e) + (d
*((d*((A*c^2 + 2*C*a*c)/e - (d*((B*c^2)/e - (C*c^2*d)/e^2))/e))/e - (2*B*a*
c)/e))/(2*e)) + (log(d + e*x)*(A*a^2*e^6 + C*c^2*d^6 - B*a^2*d*e^5 - B*c^2*
d^5*e + A*c^2*d^4*e^2 + C*a^2*d^2*e^4 + 2*A*a*c*d^2*e^4 - 2*B*a*c*d^3*e^3 +
 2*C*a*c*d^4*e^2))/e^7 + (C*c^2*x^6)/(6*e)
```

sympy [A] time = 0.94, size = 359, normalized size = 1.21

$$\frac{Cc^2x^6}{6e}+x^5\left(\frac{Bc^2}{5e}-\frac{Cc^2d}{5e^2}\right)+x^4\left(\frac{Ac^2}{4e}-\frac{Bc^2d}{4e^2}+\frac{Cac}{2e}+\frac{Cc^2d^2}{4e^3}\right)+x^3\left(-\frac{Ac^2d}{3e^2}+\frac{2Bac}{3e}+\frac{Bc^2d^2}{3e^3}-\frac{2Cacd}{3e^2}-\frac{Cc^2d^3}{3e^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d),x)
```

```
[Out] C*c**2*x**6/(6*e) + x**5*(B*c**2/(5*e) - C*c**2*d/(5*e**2)) + x**4*(A*c**2/
(4*e) - B*c**2*d/(4*e**2) + C*a*c/(2*e) + C*c**2*d**2/(4*e**3)) + x**3*(-A*
c**2*d/(3*e**2) + 2*B*a*c/(3*e) + B*c**2*d**2/(3*e**3) - 2*C*a*c*d/(3*e**2)
 - C*c**2*d**3/(3*e**4)) + x**2*(A*a*c/e + A*c**2*d**2/(2*e**3) - B*a*c*d/e
**2 - B*c**2*d**3/(2*e**4) + C*a**2/(2*e) + C*a*c*d**2/e**3 + C*c**2*d**4/(
2*e**5)) + x*(-2*A*a*c*d/e**2 - A*c**2*d**3/e**4 + B*a**2/e + 2*B*a*c*d**2/
e**3 + B*c**2*d**4/e**5 - C*a**2*d/e**2 - 2*C*a*c*d**3/e**4 - C*c**2*d**5/e
**6) + (a*e**2 + c*d**2)**2*(A*e**2 - B*d*e + C*d**2)*log(d + e*x)/e**7
```

3.30 $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^2}\,dx$

Optimal. Leaf size=292

$$\frac{x\left(a^2Ce^4+2ace^2\left(3Cd^2-e(2Bd-Ae)\right)+c^2d^2\left(5Cd^2-e(4Bd-3Ae)\right)\right)}{e^6}-\frac{\left(ae^2+cd^2\right)^2\left(Ae^2-Bde+Cd^2\right)}{e^7(d+ex)}-\frac{(ae$$

```
[Out] (a^2*C*e^4+c^2*d^2*(5*C*d^2-e*(-3*A*e+4*B*d))+2*a*c*e^2*(3*C*d^2-e*(-A*e+2*
B*d)))*x/e^6-1/2*c*(2*a*e^2*(-B*e+2*C*d)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*x^
2/e^5+1/3*c*(2*a*C*e^2+c*(3*C*d^2-e*(-A*e+2*B*d)))*x^3/e^4-1/4*c^2*(-B*e+2*
C*d)*x^4/e^3+1/5*c^2*C*x^5/e^2-(a*e^2+c*d^2)^2*(A*e^2-B*d*e+C*d^2)/e^7/(e*x
+d)-(a*e^2+c*d^2)*(a*e^2*(-B*e+2*C*d)+c*d*(6*C*d^2-e*(-4*A*e+5*B*d)))*ln(e*
x+d)/e^7
```

Rubi [A] time = 0.53, antiderivative size = 289, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1628}

$$\frac{x\left(a^2Ce^4+2ace^2\left(3Cd^2-e(2Bd-Ae)\right)+c^2\left(5Cd^4-d^2e(4Bd-3Ae)\right)\right)}{e^6}+\frac{cx^3\left(2aCe^2-ce(2Bd-Ae)+3cCd^2\right)}{3e^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] ((a^2*C*e^4 + c^2*(5*C*d^4 - d^2*e*(4*B*d - 3*A*e)) + 2*a*c*e^2*(3*C*d^2 -
e*(2*B*d - A*e)))*x)/e^6 - (c*(4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + 2*a*e^2*
(2*C*d - B*e))*x^2)/(2*e^5) + (c*(3*c*C*d^2 + 2*a*C*e^2 - c*e*(2*B*d - A*e)
)*x^3)/(3*e^4) - (c^2*(2*C*d - B*e)*x^4)/(4*e^3) + (c^2*C*x^5)/(5*e^2) - ((
c*d^2 + a*e^2)^2*(C*d^2 - B*d*e + A*e^2))/(e^7*(d + e*x)) - ((c*d^2 + a*e^2
)*(6*c*C*d^3 - c*d*e*(5*B*d - 4*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e*x])/e
^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)}{(d+ex)^2}\,dx=\int \left(\frac{a^2Ce^4+c^2\left(5Cd^4-d^2e(4Bd-3Ae)\right)+2ace^2\left(3Cd^2-e(2Bd-Ae)\right)}{e^6}+\right.$$

$$=\frac{\left(a^2Ce^4+c^2\left(5Cd^4-d^2e(4Bd-3Ae)\right)+2ace^2\left(3Cd^2-e(2Bd-Ae)\right)\right)x}{e^6}-$$

Mathematica [A] time = 0.28, size = 272, normalized size = 0.93

$$\frac{60ex\left(a^2Ce^4+2ace^2\left(e(Ae-2Bd)+3Cd^2\right)+c^2\left(d^2e(3Ae-4Bd)+5Cd^4\right)\right)-30ce^2x^2\left(-2ae^2(Be-2Cd)+cde\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] (60*e*(a^2*C*e^4 + 2*a*c*e^2*(3*C*d^2 + e*(-2*B*d + A*e)) + c^2*(5*C*d^4 +
d^2*e*(-4*B*d + 3*A*e)))*x - 30*c*e^2*(4*c*C*d^3 + c*d*e*(-3*B*d + 2*A*e) -
 2*a*e^2*(-2*C*d + B*e))*x^2 + 20*c*e^3*(3*c*C*d^2 + 2*a*C*e^2 + c*e*(-2*B*
d + A*e))*x^3 + 15*c^2*e^4*(-2*C*d + B*e)*x^4 + 12*c^2*C*e^5*x^5 - (60*(c*d
^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) - 60*(c*d^2 + a*e^2)*(6
*c*C*d^3 + c*d*e*(-5*B*d + 4*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e*x])/(60*
e^7)
```

fricas [A] time = 0.97, size = 553, normalized size = 1.89

$$\frac{12\,Cc^2e^6x^6 - 60\,Cc^2d^6 + 60\,Bc^2d^5e + 120\,Bacd^3e^3 + 60\,Ba^2de^5 - 60\,Aa^2e^6 - 60\left(2\,Cac + Ac^2\right)d^4e^2 - 60\left(Ca^2 + 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*c^2*e^6*x^6 - 60*C*c^2*d^6 + 60*B*c^2*d^5*e + 120*B*a*c*d^3*e^3
+ 60*B*a^2*d*e^5 - 60*A*a^2*e^6 - 60*(2*C*a*c + A*c^2)*d^4*e^2 - 60*(C*a^2
+ 2*A*a*c)*d^2*e^4 - 3*(6*C*c^2*d*e^5 - 5*B*c^2*e^6)*x^5 + 5*(6*C*c^2*d^2*e
^4 - 5*B*c^2*d*e^5 + 4*(2*C*a*c + A*c^2)*e^6)*x^4 - 10*(6*C*c^2*d^3*e^3 - 5
*B*c^2*d^2*e^4 - 6*B*a*c*e^6 + 4*(2*C*a*c + A*c^2)*d*e^5)*x^3 + 30*(6*C*c^2
*d^4*e^2 - 5*B*c^2*d^3*e^3 - 6*B*a*c*d*e^5 + 4*(2*C*a*c + A*c^2)*d^2*e^4 +
2*(C*a^2 + 2*A*a*c)*e^6)*x^2 + 60*(5*C*c^2*d^5*e - 4*B*c^2*d^4*e^2 - 4*B*a*
c*d^2*e^4 + 3*(2*C*a*c + A*c^2)*d^3*e^3 + (C*a^2 + 2*A*a*c)*d*e^5)*x - 60*(
6*C*c^2*d^6 - 5*B*c^2*d^5*e - 6*B*a*c*d^3*e^3 - B*a^2*d*e^5 + 4*(2*C*a*c +
A*c^2)*d^4*e^2 + 2*(C*a^2 + 2*A*a*c)*d^2*e^4 + (6*C*c^2*d^5*e - 5*B*c^2*d^4
*e^2 - 6*B*a*c*d^2*e^4 - B*a^2*e^6 + 4*(2*C*a*c + A*c^2)*d^3*e^3 + 2*(C*a^2
 + 2*A*a*c)*d*e^5)*x)*log(e*x + d))/(e^8*x + d*e^7)
```

giac [A] time = 0.18, size = 497, normalized size = 1.70

$$\frac{1}{60}\left(12\,Cc^2 - \frac{15\left(6\,Cc^2de - Bc^2e^2\right)e^{(-1)}}{xe + d} + \frac{20\left(15\,Cc^2d^2e^2 - 5\,Bc^2de^3 + 2\,Cace^4 + Ac^2e^4\right)e^{(-2)}}{(xe + d)^2} - \frac{60\left(10\,Cc^2d^3e^3 - 5\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/60*(12*C*c^2 - 15*(6*C*c^2*d*e - B*c^2*e^2)*e^(-1)/(x*e + d) + 20*(15*C*c
^2*d^2*e^2 - 5*B*c^2*d*e^3 + 2*C*a*c*e^4 + A*c^2*e^4)*e^(-2)/(x*e + d)^2 -
60*(10*C*c^2*d^3*e^3 - 5*B*c^2*d^2*e^4 + 4*C*a*c*d*e^5 + 2*A*c^2*d*e^5 - B*
a*c*e^6)*e^(-3)/(x*e + d)^3 + 60*(15*C*c^2*d^4*e^4 - 10*B*c^2*d^3*e^5 + 12*
C*a*c*d^2*e^6 + 6*A*c^2*d^2*e^6 - 6*B*a*c*d*e^7 + C*a^2*e^8 + 2*A*a*c*e^8)*
e^(-4)/(x*e + d)^4)*(x*e + d)^5*e^(-7) + (6*C*c^2*d^5 - 5*B*c^2*d^4*e + 8*C
*a*c*d^3*e^2 + 4*A*c^2*d^3*e^2 - 6*B*a*c*d^2*e^3 + 2*C*a^2*d*e^4 + 4*A*a*c*
d*e^4 - B*a^2*e^5)*e^(-7)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) - (C*c^2*d^6
*e^5/(x*e + d) - B*c^2*d^5*e^6/(x*e + d) + 2*C*a*c*d^4*e^7/(x*e + d) + A*c^
2*d^4*e^7/(x*e + d) - 2*B*a*c*d^3*e^8/(x*e + d) + C*a^2*d^2*e^9/(x*e + d) +
 2*A*a*c*d^2*e^9/(x*e + d) - B*a^2*d*e^10/(x*e + d) + A*a^2*e^11/(x*e + d))
*e^(-12)
```

maple [A] time = 0.01, size = 527, normalized size = 1.80

$$\frac{C\,c^2x^5}{5e^2} + \frac{B\,c^2x^4}{4e^2} - \frac{C\,c^2d\,x^4}{2e^3} + \frac{A\,c^2x^3}{3e^2} - \frac{2B\,c^2d\,x^3}{3e^3} + \frac{2Cac\,x^3}{3e^2} + \frac{C\,c^2d^2x^3}{e^4} - \frac{A\,c^2d\,x^2}{e^3} + \frac{Bac\,x^2}{e^2} + \frac{3B\,c^2d^2x^2}{2e^4} - \frac{2Cacd\,x^2}{e^3} - \frac{2C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x)
```

```
[Out] -2/e^3*C*x^2*a*c*d-4/e^3*d*a*c*B*x+6/e^4*C*a*c*d^2*x-2/e^3/(e*x+d)*A*a*c*d^
2+2/e^4/(e*x+d)*B*a*c*d^3-2/e^5/(e*x+d)*C*a*c*d^4-4/e^3*ln(e*x+d)*A*a*c*d+6
/e^4*ln(e*x+d)*B*a*c*d^2-8/e^5*ln(e*x+d)*C*a*c*d^3+1/5*c^2*C*x^5/e^2-1/e/(e
*x+d)*A*a^2+1/e^2*ln(e*x+d)*B*a^2+1/4/e^2*B*x^4*c^2+1/3/e^2*A*x^3*c^2+1/e^2
*a^2*C*x-4/e^5*ln(e*x+d)*A*c^2*d^3+5/e^6*ln(e*x+d)*B*c^2*d^4-4/e^5*B*c^2*d^
3*x+5/e^6*C*c^2*d^4*x-1/e^5/(e*x+d)*A*c^2*d^4+1/e^2/(e*x+d)*B*d*a^2+1/e^6/(
e*x+d)*B*c^2*d^5-1/e^3/(e*x+d)*C*a^2*d^2-1/e^7/(e*x+d)*C*c^2*d^6+2/3/e^2*C*
x^3*a*c+1/e^4*C*x^3*c^2*d^2-1/e^3*A*x^2*c^2*d+1/e^2*B*x^2*a*c+3/2/e^4*B*x^2
*c^2*d^2-2/e^5*C*x^2*c^2*d^3+2/e^2*A*a*c*x+3/e^4*A*c^2*d^2*x-1/2/e^3*C*x^4*
c^2*d-2/3/e^3*B*x^3*c^2*d-2/e^3*ln(e*x+d)*C*a^2*d-6/e^7*ln(e*x+d)*C*c^2*d^5
```

maxima [A] time = 0.48, size = 392, normalized size = 1.34

$$-\frac{Cc^2d^6 - Bc^2d^5e - 2\,Bacd^3e^3 - Ba^2de^5 + Aa^2e^6 + \left(2\,Cac + Ac^2\right)d^4e^2 + \left(Ca^2 + 2\,Aac\right)d^2e^4}{e^8x + de^7} + \frac{12\,Cc^2e^4x^5 - 15\,(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(C*c^2*d^6 - B*c^2*d^5*e - 2*B*a*c*d^3*e^3 - B*a^2*d*e^5 + A*a^2*e^6 + (2*
C*a*c + A*c^2)*d^4*e^2 + (C*a^2 + 2*A*a*c)*d^2*e^4)/(e^8*x + d*e^7) + 1/60*
(12*C*c^2*e^4*x^5 - 15*(2*C*c^2*d*e^3 - B*c^2*e^4)*x^4 + 20*(3*C*c^2*d^2*e^
2 - 2*B*c^2*d*e^3 + (2*C*a*c + A*c^2)*e^4)*x^3 - 30*(4*C*c^2*d^3*e - 3*B*c^
2*d^2*e^2 - 2*B*a*c*e^4 + 2*(2*C*a*c + A*c^2)*d*e^3)*x^2 + 60*(5*C*c^2*d^4
- 4*B*c^2*d^3*e - 4*B*a*c*d*e^3 + 3*(2*C*a*c + A*c^2)*d^2*e^2 + (C*a^2 + 2*
A*a*c)*e^4)*x)/e^6 - (6*C*c^2*d^5 - 5*B*c^2*d^4*e - 6*B*a*c*d^2*e^3 - B*a^2
*e^5 + 4*(2*C*a*c + A*c^2)*d^3*e^2 + 2*(C*a^2 + 2*A*a*c)*d*e^4)*log(e*x + d
)/e^7
```

mupad [B] time = 0.12, size = 575, normalized size = 1.97

$$x^4\left(\frac{B\,c^2}{4\,e^2} - \frac{C\,c^2\,d}{2\,e^3}\right) + x\left(\frac{C\,a^2 + 2\,A\,c\,a}{e^2} + \frac{d^2\left(\frac{2\,d\left(\frac{B\,c^2}{e^2} - \frac{2\,C\,c^2\,d}{e^3}\right)}{e} - \frac{A\,c^2 + 2\,C\,a\,c}{e^2} + \frac{C\,c^2\,d^2}{e^4}\right)}{e^2} - \frac{2\,d\left(\frac{2\,d\left(\frac{2\,d\left(\frac{B\,c^2}{e^2} - \frac{2\,C\,c^2\,d}{e^3}\right)}{e} - \frac{A\,c^2 + \cdots}{e\cdots}\right)}{e}\cdots\right)}{\cdots}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

```
[Out] x^4*((B*c^2)/(4*e^2) - (C*c^2*d)/(2*e^3)) + x*((C*a^2 + 2*A*a*c)/e^2 + (d^2
*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c^2 + 2*C*a*c)/e^2 + (C*c^2*
d^2)/e^4))/e^2 - (2*d*((2*d*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c
^2 + 2*C*a*c)/e^2 + (C*c^2*d^2)/e^4))/e - (d^2*((B*c^2)/e^2 - (2*C*c^2*d)/e
^3))/e^2 + (2*B*a*c)/e^2))/e) - x^3*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/
(3*e) - (A*c^2 + 2*C*a*c)/(3*e^2) + (C*c^2*d^2)/(3*e^4)) + x^2*((d*((2*d*((
B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c^2 + 2*C*a*c)/e^2 + (C*c^2*d^2)/e^4)
)/e - (d^2*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/(2*e^2) + (B*a*c)/e^2) - (A*a^2
*e^6 + C*c^2*d^6 - B*a^2*d*e^5 - B*c^2*d^5*e + A*c^2*d^4*e^2 + C*a^2*d^2*e^
4 + 2*A*a*c*d^2*e^4 - 2*B*a*c*d^3*e^3 + 2*C*a*c*d^4*e^2)/(e*(d*e^6 + e^7*x)
) - (log(d + e*x)*(6*C*c^2*d^5 - B*a^2*e^5 + 2*C*a^2*d*e^4 - 5*B*c^2*d^4*e
```

```
+ 4*A*c^2*d^3*e^2 + 4*A*a*c*d*e^4 - 6*B*a*c*d^2*e^3 + 8*C*a*c*d^3*e^2))/e^7
 + (C*c^2*x^5)/(5*e^2)
```

sympy [A] time = 2.79, size = 416, normalized size = 1.42

$$\frac{Cc^2x^5}{5e^2}+x^4\left(\frac{Bc^2}{4e^2}-\frac{Cc^2d}{2e^3}\right)+x^3\left(\frac{Ac^2}{3e^2}-\frac{2Bc^2d}{3e^3}+\frac{2Cac}{3e^2}+\frac{Cc^2d^2}{e^4}\right)+x^2\left(-\frac{Ac^2d}{e^3}+\frac{Bac}{e^2}+\frac{3Bc^2d^2}{2e^4}-\frac{2Cacd}{e^3}-\frac{2Cc^2d^3}{e^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

```
[Out] C*c**2*x**5/(5*e**2) + x**4*(B*c**2/(4*e**2) - C*c**2*d/(2*e**3)) + x**3*(A
*c**2/(3*e**2) - 2*B*c**2*d/(3*e**3) + 2*C*a*c/(3*e**2) + C*c**2*d**2/e**4)
 + x**2*(-A*c**2*d/e**3 + B*a*c/e**2 + 3*B*c**2*d**2/(2*e**4) - 2*C*a*c*d/e
**3 - 2*C*c**2*d**3/e**5) + x*(2*A*a*c/e**2 + 3*A*c**2*d**2/e**4 - 4*B*a*c*
d/e**3 - 4*B*c**2*d**3/e**5 + C*a**2/e**2 + 6*C*a*c*d**2/e**4 + 5*C*c**2*d*
*4/e**6) + (-A*a**2*e**6 - 2*A*a*c*d**2*e**4 - A*c**2*d**4*e**2 + B*a**2*d*
e**5 + 2*B*a*c*d**3*e**3 + B*c**2*d**5*e - C*a**2*d**2*e**4 - 2*C*a*c*d**4*
e**2 - C*c**2*d**6)/(d*e**7 + e**8*x) - (a*e**2 + c*d**2)*(4*A*c*d*e**2 - B
*a*e**3 - 5*B*c*d**2*e + 2*C*a*d*e**2 + 6*C*c*d**3)*log(d + e*x)/e**7
```

3.31 $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^3}\,dx$

Optimal. Leaf size=295

$$\frac{\log(d+ex)\left(a^2Ce^4+2ace^2\left(6Cd^2-e(3Bd-Ae)\right)+c^2d^2\left(15Cd^2-2e(5Bd-3Ae)\right)\right)}{e^7}+\frac{\left(ae^2+cd^2\right)\left(ae^2(2Cd-\right.}{e}$$

```
[Out] -c*(2*a*e^2*(-B*e+3*C*d)+c*d*(10*C*d^2-3*e*(-A*e+2*B*d)))*x/e^6+1/2*c*(2*a*
C*e^2+c*(6*C*d^2-e*(-A*e+3*B*d)))*x^2/e^5-1/3*c^2*(-B*e+3*C*d)*x^3/e^4+1/4*
c^2*C*x^4/e^3-1/2*(a*e^2+c*d^2)^2*(A*e^2-B*d*e+C*d^2)/e^7/(e*x+d)^2+(a*e^2+
c*d^2)*(a*e^2*(-B*e+2*C*d)+c*d*(6*C*d^2-e*(-4*A*e+5*B*d)))/e^7/(e*x+d)+(a^2
*C*e^4+c^2*d^2*(15*C*d^2-2*e*(-3*A*e+5*B*d))+2*a*c*e^2*(6*C*d^2-e*(-A*e+3*B
*d)))*ln(e*x+d)/e^7
```

Rubi [A] time = 0.49, antiderivative size = 292, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1628}

$$\frac{\log(d+ex)\left(a^2Ce^4+2ace^2\left(6Cd^2-e(3Bd-Ae)\right)+c^2\left(15Cd^4-2d^2e(5Bd-3Ae)\right)\right)}{e^7}+\frac{cx^2\left(2aCe^2-ce(3Bd-\right.}{2e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] -((c*(10*c*C*d^3 - 3*c*d*e*(2*B*d - A*e) + 2*a*e^2*(3*C*d - B*e))*x)/e^6) +
 (c*(6*c*C*d^2 + 2*a*C*e^2 - c*e*(3*B*d - A*e))*x^2)/(2*e^5) - (c^2*(3*C*d
- B*e)*x^3)/(3*e^4) + (c^2*C*x^4)/(4*e^3) - ((c*d^2 + a*e^2)^2*(C*d^2 - B*d
*e + A*e^2))/(2*e^7*(d + e*x)^2) + ((c*d^2 + a*e^2)*(6*c*C*d^3 - c*d*e*(5*B
*d - 4*A*e) + a*e^2*(2*C*d - B*e)))/(e^7*(d + e*x)) + ((a^2*C*e^4 + c^2*(15
*C*d^4 - 2*d^2*e*(5*B*d - 3*A*e)) + 2*a*c*e^2*(6*C*d^2 - e*(3*B*d - A*e)))*
Log[d + e*x])/e^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^3}\,dx = \int \left(\frac{c\left(-10cCd^3+3cde(2Bd-Ae)-2ae^2(3Cd-Be)\right)}{e^6}+\frac{c\left(6cCd^2+2aCe^2\right.}{e}\right.$$

$$= -\frac{c\left(10cCd^3-3cde(2Bd-Ae)+2ae^2(3Cd-Be)\right)x}{e^6}+\frac{c\left(6cCd^2+2aCe^2-\right.}{2e^5}$$

Mathematica [A] time = 0.12, size = 274, normalized size = 0.93

$$\frac{12\log(d+ex)\left(a^2Ce^4+2ace^2\left(e(Ae-3Bd)+6Cd^2\right)+c^2\left(2d^2e(3Ae-5Bd)+15Cd^4\right)\right)-12cex\left(-2ae^2(Be-3C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] (-12*c*e*(10*c*C*d^3 + 3*c*d*e*(-2*B*d + A*e) - 2*a*e^2*(-3*C*d + B*e))*x +
 6*c*e^2*(6*c*C*d^2 + 2*a*C*e^2 + c*e*(-3*B*d + A*e))*x^2 + 4*c^2*e^3*(-3*C
*d + B*e)*x^3 + 3*c^2*C*e^4*x^4 - (6*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) +
 A*e)))/(d + e*x)^2 + (12*(c*d^2 + a*e^2)*(6*c*C*d^3 + c*d*e*(-5*B*d + 4*A*
e) + a*e^2*(2*C*d - B*e)))/(d + e*x) + 12*(a^2*C*e^4 + 2*a*c*e^2*(6*C*d^2 +
 e*(-3*B*d + A*e)) + c^2*(15*C*d^4 + 2*d^2*e*(-5*B*d + 3*A*e)))*Log[d + e*x
])/(12*e^7)
```

fricas [B] time = 0.91, size = 608, normalized size = 2.06

$$\frac{3\,Cc^2e^6x^6 + 66\,Cc^2d^6 - 54\,Bc^2d^5e - 60\,Bacd^3e^3 - 6\,Ba^2de^5 - 6\,Aa^2e^6 + 42\left(2\,Cac + Ac^2\right)d^4e^2 + 18\left(Ca^2 + 2\,Aac\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*c^2*e^6*x^6 + 66*C*c^2*d^6 - 54*B*c^2*d^5*e - 60*B*a*c*d^3*e^3 -
6*B*a^2*d*e^5 - 6*A*a^2*e^6 + 42*(2*C*a*c + A*c^2)*d^4*e^2 + 18*(C*a^2 + 2*
A*a*c)*d^2*e^4 - 2*(3*C*c^2*d*e^5 - 2*B*c^2*e^6)*x^5 + (15*C*c^2*d^2*e^4 -
10*B*c^2*d*e^5 + 6*(2*C*a*c + A*c^2)*e^6)*x^4 - 4*(15*C*c^2*d^3*e^3 - 10*B*
c^2*d^2*e^4 - 6*B*a*c*e^6 + 6*(2*C*a*c + A*c^2)*d*e^5)*x^3 - 6*(34*C*c^2*d^
4*e^2 - 21*B*c^2*d^3*e^3 - 8*B*a*c*d*e^5 + 11*(2*C*a*c + A*c^2)*d^2*e^4)*x^
2 - 12*(4*C*c^2*d^5*e - B*c^2*d^4*e^2 + 4*B*a*c*d^2*e^4 + B*a^2*e^6 - (2*C*
a*c + A*c^2)*d^3*e^3 - 2*(C*a^2 + 2*A*a*c)*d*e^5)*x + 12*(15*C*c^2*d^6 - 10
*B*c^2*d^5*e - 6*B*a*c*d^3*e^3 + 6*(2*C*a*c + A*c^2)*d^4*e^2 + (C*a^2 + 2*A
*a*c)*d^2*e^4 + (15*C*c^2*d^4*e^2 - 10*B*c^2*d^3*e^3 - 6*B*a*c*d*e^5 + 6*(2
*C*a*c + A*c^2)*d^2*e^4 + (C*a^2 + 2*A*a*c)*e^6)*x^2 + 2*(15*C*c^2*d^5*e -
10*B*c^2*d^4*e^2 - 6*B*a*c*d^2*e^4 + 6*(2*C*a*c + A*c^2)*d^3*e^3 + (C*a^2 +
 2*A*a*c)*d*e^5)*x)*log(e*x + d))/(e^9*x^2 + 2*d*e^8*x + d^2*e^7)
```

giac [A] time = 0.16, size = 397, normalized size = 1.35

$$\left(15\,Cc^2d^4 - 10\,Bc^2d^3e + 12\,Cacd^2e^2 + 6\,Ac^2d^2e^2 - 6\,Bacde^3 + Ca^2e^4 + 2\,Aace^4\right)e^{(-7)}\log\left(|xe + d|\right) + \frac{1}{12}\left(3\,Cc^2x^4e^9\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (15*C*c^2*d^4 - 10*B*c^2*d^3*e + 12*C*a*c*d^2*e^2 + 6*A*c^2*d^2*e^2 - 6*B*a
*c*d*e^3 + C*a^2*e^4 + 2*A*a*c*e^4)*e^(-7)*log(abs(x*e + d)) + 1/12*(3*C*c^
2*x^4*e^9 - 12*C*c^2*d*x^3*e^8 + 36*C*c^2*d^2*x^2*e^7 - 120*C*c^2*d^3*x*e^6
 + 4*B*c^2*x^3*e^9 - 18*B*c^2*d*x^2*e^8 + 72*B*c^2*d^2*x*e^7 + 12*C*a*c*x^2
*e^9 + 6*A*c^2*x^2*e^9 - 72*C*a*c*d*x*e^8 - 36*A*c^2*d*x*e^8 + 24*B*a*c*x*e
^9)*e^(-12) + 1/2*(11*C*c^2*d^6 - 9*B*c^2*d^5*e + 14*C*a*c*d^4*e^2 + 7*A*c^
2*d^4*e^2 - 10*B*a*c*d^3*e^3 + 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 - B*a^2*d*
e^5 - A*a^2*e^6 + 2*(6*C*c^2*d^5*e - 5*B*c^2*d^4*e^2 + 8*C*a*c*d^3*e^3 + 4*
A*c^2*d^3*e^3 - 6*B*a*c*d^2*e^4 + 2*C*a^2*d*e^5 + 4*A*a*c*d*e^5 - B*a^2*e^6
)*x)*e^(-7)/(x*e + d)^2
```

maple [A] time = 0.01, size = 563, normalized size = 1.91

$$\frac{C\,c^2x^4}{4e^3} + \frac{B\,c^2x^3}{3e^3} - \frac{C\,c^2d\,x^3}{e^4} - \frac{A\,a^2}{2\,(ex + d)^2\,e} - \frac{Aac\,d^2}{(ex + d)^2\,e^3} - \frac{A\,c^2d^4}{2\,(ex + d)^2\,e^5} + \frac{A\,c^2x^2}{2e^3} + \frac{B\,a^2d}{2\,(ex + d)^2\,e^2} + \frac{Bac\,d^3}{(ex + d)^2\,e^4} + \frac{B\,c}{2\,(ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x)
```

```
[Out] -6*c/e^4*C*x*a*d+4/e^3/(e*x+d)*A*a*c*d-6/e^4/(e*x+d)*B*a*c*d^2+8/e^5/(e*x+d
)*C*a*c*d^3-6/e^4*ln(e*x+d)*B*a*c*d+1/4*c^2*C*x^4/e^3+12/e^5*ln(e*x+d)*C*a*
c*d^2-1/e^3/(e*x+d)^2*A*d^2*a*c+1/e^4/(e*x+d)^2*B*a*c*d^3-1/e^5/(e*x+d)^2*C
*a*c*d^4+1/e^3*ln(e*x+d)*a^2*C-1/2/e/(e*x+d)^2*A*a^2+1/3*c^2/e^3*B*x^3+1/2*
c^2/e^3*A*x^2-1/e^2/(e*x+d)*B*a^2-c^2/e^4*C*x^3*d-3/2*c^2/e^4*B*x^2*d-10/e^
6*ln(e*x+d)*B*c^2*d^3+15/e^7*ln(e*x+d)*C*c^2*d^4-1/2/e^5/(e*x+d)^2*A*c^2*d^
4+1/2/e^2/(e*x+d)^2*B*d*a^2+1/2/e^6/(e*x+d)^2*B*c^2*d^5-1/2/e^3/(e*x+d)^2*C
*d^2*a^2-1/2/e^7/(e*x+d)^2*C*c^2*d^6+c/e^3*C*x^2*a+3*c^2/e^5*C*x^2*d^2-3*c^
2/e^4*A*x*d+2*c/e^3*B*x*a+6*c^2/e^5*B*x*d^2-10*c^2/e^6*C*x*d^3+4/e^5/(e*x+d
)*A*c^2*d^3-5/e^6/(e*x+d)*B*c^2*d^4+2/e^3/(e*x+d)*C*a^2*d+6/e^7/(e*x+d)*C*c
^2*d^5+2/e^3*ln(e*x+d)*A*a*c+6/e^5*ln(e*x+d)*A*c^2*d^2
```

maxima [A] time = 0.49, size = 402, normalized size = 1.36

$$\frac{11\,Cc^2d^6 - 9\,Bc^2d^5e - 10\,Bacd^3e^3 - Ba^2de^5 - Aa^2e^6 + 7\left(2\,Cac + Ac^2\right)d^4e^2 + 3\left(Ca^2 + 2\,Aac\right)d^2e^4 + 2\left(6\,Cc^2d\right.}{2\left(e^9x^2 + 2\,de^8x + d^2e^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(11*C*c^2*d^6 - 9*B*c^2*d^5*e - 10*B*a*c*d^3*e^3 - B*a^2*d*e^5 - A*a^2*
e^6 + 7*(2*C*a*c + A*c^2)*d^4*e^2 + 3*(C*a^2 + 2*A*a*c)*d^2*e^4 + 2*(6*C*c^
2*d^5*e - 5*B*c^2*d^4*e^2 - 6*B*a*c*d^2*e^4 - B*a^2*e^6 + 4*(2*C*a*c + A*c^
2)*d^3*e^3 + 2*(C*a^2 + 2*A*a*c)*d*e^5)*x)/(e^9*x^2 + 2*d*e^8*x + d^2*e^7)
+ 1/12*(3*C*c^2*e^3*x^4 - 4*(3*C*c^2*d*e^2 - B*c^2*e^3)*x^3 + 6*(6*C*c^2*d^
2*e - 3*B*c^2*d*e^2 + (2*C*a*c + A*c^2)*e^3)*x^2 - 12*(10*C*c^2*d^3 - 6*B*c
^2*d^2*e - 2*B*a*c*e^3 + 3*(2*C*a*c + A*c^2)*d*e^2)*x)/e^6 + (15*C*c^2*d^4
- 10*B*c^2*d^3*e - 6*B*a*c*d*e^3 + 6*(2*C*a*c + A*c^2)*d^2*e^2 + (C*a^2 + 2
*A*a*c)*e^4)*log(e*x + d)/e^7
```

mupad [B] time = 3.82, size = 495, normalized size = 1.68

$$x\left(\frac{3\,d\left(\frac{3\,d\left(\frac{B\,c^2}{e^3}-\frac{3\,C\,c^2\,d}{e^4}\right)}{e}-\frac{A\,c^2+2\,C\,a\,c}{e^3}+\frac{3\,C\,c^2\,d^2}{e^5}\right)}{e}-\frac{3\,d^2\,\left(\frac{B\,c^2}{e^3}-\frac{3\,C\,c^2\,d}{e^4}\right)}{e^2}+\frac{2\,B\,a\,c}{e^3}-\frac{C\,c^2\,d^3}{e^6}\right)+x^3\,\left(\frac{B\,c^2}{3\,e^3}-\frac{C\,c^2\,d}{e^4}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

```
[Out] x*((3*d*((3*d*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/e - (A*c^2 + 2*C*a*c)/e^3 +
(3*C*c^2*d^2)/e^5))/e - (3*d^2*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/e^2 + (2*B*
a*c)/e^3 - (C*c^2*d^3)/e^6) + x^3*((B*c^2)/(3*e^3) - (C*c^2*d)/e^4) - x^2*(
(3*d*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/(2*e) - (A*c^2 + 2*C*a*c)/(2*e^3) + (
3*C*c^2*d^2)/(2*e^5)) + ((11*C*c^2*d^6 - A*a^2*e^6 - B*a^2*d*e^5 - 9*B*c^2*
d^5*e + 7*A*c^2*d^4*e^2 + 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 - 10*B*a*c*d^3*
e^3 + 14*C*a*c*d^4*e^2)/(2*e) + x*(6*C*c^2*d^5 - B*a^2*e^5 + 2*C*a^2*d*e^4
- 5*B*c^2*d^4*e + 4*A*c^2*d^3*e^2 + 4*A*a*c*d*e^4 - 6*B*a*c*d^2*e^3 + 8*C*a
*c*d^3*e^2))/(d^2*e^6 + e^8*x^2 + 2*d*e^7*x) + (log(d + e*x)*(C*a^2*e^4 + 1
5*C*c^2*d^4 + 2*A*a*c*e^4 - 10*B*c^2*d^3*e + 6*A*c^2*d^2*e^2 - 6*B*a*c*d*e^
3 + 12*C*a*c*d^2*e^2))/e^7 + (C*c^2*x^4)/(4*e^3)
```

sympy [A] time = 14.20, size = 474, normalized size = 1.61

$$\frac{Cc^2x^4}{4e^3}+x^3\left(\frac{Bc^2}{3e^3}-\frac{Cc^2d}{e^4}\right)+x^2\left(\frac{Ac^2}{2e^3}-\frac{3Bc^2d}{2e^4}+\frac{Cac}{e^3}+\frac{3Cc^2d^2}{e^5}\right)+x\left(-\frac{3Ac^2d}{e^4}+\frac{2Bac}{e^3}+\frac{6Bc^2d^2}{e^5}-\frac{6Cacd}{e^4}-10\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

```
[Out] C*c**2*x**4/(4*e**3) + x**3*(B*c**2/(3*e**3) - C*c**2*d/e**4) + x**2*(A*c**
2/(2*e**3) - 3*B*c**2*d/(2*e**4) + C*a*c/e**3 + 3*C*c**2*d**2/e**5) + x*(-3
*A*c**2*d/e**4 + 2*B*a*c/e**3 + 6*B*c**2*d**2/e**5 - 6*C*a*c*d/e**4 - 10*C*
c**2*d**3/e**6) + (-A*a**2*e**6 + 6*A*a*c*d**2*e**4 + 7*A*c**2*d**4*e**2 -
B*a**2*d*e**5 - 10*B*a*c*d**3*e**3 - 9*B*c**2*d**5*e + 3*C*a**2*d**2*e**4 +
 14*C*a*c*d**4*e**2 + 11*C*c**2*d**6 + x*(8*A*a*c*d*e**5 + 8*A*c**2*d**3*e*
*3 - 2*B*a**2*e**6 - 12*B*a*c*d**2*e**4 - 10*B*c**2*d**4*e**2 + 4*C*a**2*d*
e**5 + 16*C*a*c*d**3*e**3 + 12*C*c**2*d**5*e))/(2*d**2*e**7 + 4*d*e**8*x +
2*e**9*x**2) + (2*A*a*c*e**4 + 6*A*c**2*d**2*e**2 - 6*B*a*c*d*e**3 - 10*B*c
**2*d**3*e + C*a**2*e**4 + 12*C*a*c*d**2*e**2 + 15*C*c**2*d**4)*log(d + e*x
)/e**7
```

3.32 $\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=404

$$\frac{1}{4}a^3ex^4\left(e(Ae+3Bd)+3Cd^2\right)+a^3Ad^3x+\frac{1}{6}a^2ex^6\left(aCe^2+3c\left(e(Ae+3Bd)+3Cd^2\right)\right)+\frac{1}{3}a^2dx^3\left(3A\left(ae^2+cd^2\right)+\right.$$

```
[Out] a^3*A*d^3*x+1/3*a^2*d*(a*d*(3*B*e+C*d)+3*A*(a*e^2+c*d^2))*x^3+1/4*a^3*e*(3*
C*d^2+e*(A*e+3*B*d))*x^4+1/5*a*(3*A*c*d*(3*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d
)+3*c*d^2*(3*B*e+C*d)))*x^5+1/6*a^2*e*(a*C*e^2+3*c*(3*C*d^2+e*(A*e+3*B*d)))
*x^6+1/7*c*(A*c*d*(9*a*e^2+c*d^2)+3*a*(a*e^2*(B*e+3*C*d)+c*d^2*(3*B*e+C*d))
)*x^7+3/8*a*c*e*(a*C*e^2+c*(3*C*d^2+e*(A*e+3*B*d)))*x^8+1/9*c^2*(3*a*e^2*(B
*e+3*C*d)+c*d*(C*d^2+3*e*(A*e+B*d)))*x^9+1/10*c^2*e*(3*a*C*e^2+c*(3*C*d^2+e
*(A*e+3*B*d)))*x^10+1/11*c^3*e^2*(B*e+3*C*d)*x^11+1/12*c^3*C*e^3*x^12+1/8*d
^2*(3*A*e+B*d)*(c*x^2+a)^4/c
```

Rubi [A] time = 0.69, antiderivative size = 400, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1582, 1810}

$$\frac{1}{6}a^2ex^6\left(aCe^2+3ce(Ae+3Bd)+9cCd^2\right)+\frac{1}{3}a^2dx^3\left(3A\left(ae^2+cd^2\right)+ad(3Be+Cd)\right)+\frac{1}{4}a^3ex^4\left(e(Ae+3Bd)+3\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^3*x + (a^2*d*(a*d*(C*d + 3*B*e) + 3*A*(c*d^2 + a*e^2))*x^3)/3 + (a^
3*e*(3*C*d^2 + e*(3*B*d + A*e))*x^4)/4 + (a*(3*A*c*d*(c*d^2 + 3*a*e^2) + a*
(a*e^2*(3*C*d + B*e) + 3*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a^2*e*(9*c*C*d^2 +
 a*C*e^2 + 3*c*e*(3*B*d + A*e))*x^6)/6 + (c*(A*c*d*(c*d^2 + 9*a*e^2) + 3*a*
(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*x^7)/7 + (3*a*c*e*(3*c*C*d^2 +
 a*C*e^2 + c*e*(3*B*d + A*e))*x^8)/8 + (c^2*(c*C*d^3 + 3*c*d*e*(B*d + A*e)
+ 3*a*e^2*(3*C*d + B*e))*x^9)/9 + (c^2*e*(3*c*C*d^2 + 3*a*C*e^2 + c*e*(3*B*
d + A*e))*x^10)/10 + (c^3*e^2*(3*C*d + B*e)*x^11)/11 + (c^3*C*e^3*x^12)/12
+ (d^2*(B*d + 3*A*e)*(a + c*x^2)^4)/(8*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx = \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a+cx^2\right)^3 \left(-\left(Bd^3+3Ad^2e\right)x+(d+$$

$$= \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a^3Ad^3 + a^2d\left(ad(Cd+3Be)+3A\left(c\right.\right.\right.$$

$$= a^3Ad^3x + \frac{1}{3}a^2d\left(ad(Cd+3Be)+3A\left(cd^2+ae^2\right)\right)x^3 + \frac{1}{4}a^3e\left(3Cd^2+\right.$$

Mathematica [**A**] time = 0.21, size = 459, normalized size = 1.14

$$\frac{1}{2}a^3d^2x^2(3Ae+Bd)+a^3Ad^3x+\frac{1}{3}a^2dx^3\left(3A\left(ae^2+cd^2\right)+ad(3Be+Cd)\right)+\frac{1}{4}a^2x^4\left(aAe^3+3aBde^2+3aCd^2e+9Acd\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^3*x + (a^3*d^2*(B*d + 3*A*e)*x^2)/2 + (a^2*d*(a*d*(C*d + 3*B*e) + 3
*A*(c*d^2 + a*e^2))*x^3)/3 + (a^2*(3*B*c*d^3 + 9*A*c*d^2*e + 3*a*C*d^2*e +
3*a*B*d*e^2 + a*A*e^3)*x^4)/4 + (a*(3*A*c*d*(c*d^2 + 3*a*e^2) + a*(a*e^2*(3
*C*d + B*e) + 3*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a*(3*A*c*e*(3*c*d^2 + a*e^2
) + a*C*e*(9*c*d^2 + a*e^2) + 3*B*c*d*(c*d^2 + 3*a*e^2))*x^6)/6 + (c*(A*c*d
*(c*d^2 + 9*a*e^2) + 3*a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*x^7)/
7 + (c*(B*c*d*(c*d^2 + 9*a*e^2) + 3*e*(A*c*(c*d^2 + a*e^2) + a*C*(3*c*d^2 +
 a*e^2)))*x^8)/8 + (c^2*(c*C*d^3 + 3*c*d*e*(B*d + A*e) + 3*a*e^2*(3*C*d + B
*e))*x^9)/9 + (c^2*e*(3*c*C*d^2 + 3*a*C*e^2 + c*e*(3*B*d + A*e))*x^10)/10 +
 (c^3*e^2*(3*C*d + B*e)*x^11)/11 + (c^3*C*e^3*x^12)/12
```

fricas [**A**] time = 0.74, size = 618, normalized size = 1.53

$$\frac{1}{12}x^{12}e^3c^3C+\frac{3}{11}x^{11}e^2dc^3C+\frac{1}{11}x^{11}e^3c^3B+\frac{3}{10}x^{10}ed^2c^3C+\frac{3}{10}x^{10}e^3c^2aC+\frac{3}{10}x^{10}e^2dc^3B+\frac{1}{10}x^{10}e^3c^3A+\frac{1}{9}x^9d^3c^3C+x^9e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e^3*c^3*C + 3/11*x^11*e^2*d*c^3*C + 1/11*x^11*e^3*c^3*B + 3/10*x^
10*e*d^2*c^3*C + 3/10*x^10*e^3*c^2*a*C + 3/10*x^10*e^2*d*c^3*B + 1/10*x^10*
e^3*c^3*A + 1/9*x^9*d^3*c^3*C + x^9*e^2*d*c^2*a*C + 1/3*x^9*e*d^2*c^3*B + 1
/3*x^9*e^3*c^2*a*B + 1/3*x^9*e^2*d*c^3*A + 9/8*x^8*e*d^2*c^2*a*C + 3/8*x^8*
e^3*c*a^2*C + 1/8*x^8*d^3*c^3*B + 9/8*x^8*e^2*d*c^2*a*B + 3/8*x^8*e*d^2*c^3
*A + 3/8*x^8*e^3*c^2*a*A + 3/7*x^7*d^3*c^2*a*C + 9/7*x^7*e^2*d*c*a^2*C + 9/
7*x^7*e*d^2*c^2*a*B + 3/7*x^7*e^3*c*a^2*B + 1/7*x^7*d^3*c^3*A + 9/7*x^7*e^2
*d*c^2*a*A + 3/2*x^6*e*d^2*c*a^2*C + 1/6*x^6*e^3*a^3*C + 1/2*x^6*d^3*c^2*a*
B + 3/2*x^6*e^2*d*c*a^2*B + 3/2*x^6*e*d^2*c^2*a*A + 1/2*x^6*e^3*c*a^2*A + 3
/5*x^5*d^3*c*a^2*C + 3/5*x^5*e^2*d*a^3*C + 9/5*x^5*e*d^2*c*a^2*B + 1/5*x^5*
e^3*a^3*B + 3/5*x^5*d^3*c^2*a*A + 9/5*x^5*e^2*d*c*a^2*A + 3/4*x^4*e*d^2*a^3
*C + 3/4*x^4*d^3*c*a^2*B + 3/4*x^4*e^2*d*a^3*B + 9/4*x^4*e*d^2*c*a^2*A + 1/
4*x^4*e^3*a^3*A + 1/3*x^3*d^3*a^3*C + x^3*e*d^2*a^3*B + x^3*d^3*c*a^2*A + x
^3*e^2*d*a^3*A + 1/2*x^2*d^3*a^3*B + 3/2*x^2*e*d^2*a^3*A + x*d^3*a^3*A
```

giac [**A**] time = 0.20, size = 606, normalized size = 1.50

$$\frac{1}{12}Cc^3x^{12}e^3+\frac{3}{11}Cc^3dx^{11}e^2+\frac{3}{10}Cc^3d^2x^{10}e+\frac{1}{9}Cc^3d^3x^9+\frac{1}{11}Bc^3x^{11}e^3+\frac{3}{10}Bc^3dx^{10}e^2+\frac{1}{3}Bc^3d^2x^9e+\frac{1}{8}Bc^3d^3x^8+\frac{3}{10}C$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/12*C*c^3*x^12*e^3 + 3/11*C*c^3*d*x^11*e^2 + 3/10*C*c^3*d^2*x^10*e + 1/9*C
*c^3*d^3*x^9 + 1/11*B*c^3*x^11*e^3 + 3/10*B*c^3*d*x^10*e^2 + 1/3*B*c^3*d^2*
x^9*e + 1/8*B*c^3*d^3*x^8 + 3/10*C*a*c^2*x^10*e^3 + 1/10*A*c^3*x^10*e^3 + C
*a*c^2*d*x^9*e^2 + 1/3*A*c^3*d*x^9*e^2 + 9/8*C*a*c^2*d^2*x^8*e + 3/8*A*c^3*
d^2*x^8*e + 3/7*C*a*c^2*d^3*x^7 + 1/7*A*c^3*d^3*x^7 + 1/3*B*a*c^2*x^9*e^3 +
 9/8*B*a*c^2*d*x^8*e^2 + 9/7*B*a*c^2*d^2*x^7*e + 1/2*B*a*c^2*d^3*x^6 + 3/8*
C*a^2*c*x^8*e^3 + 3/8*A*a*c^2*x^8*e^3 + 9/7*C*a^2*c*d*x^7*e^2 + 9/7*A*a*c^2
*d*x^7*e^2 + 3/2*C*a^2*c*d^2*x^6*e + 3/2*A*a*c^2*d^2*x^6*e + 3/5*C*a^2*c*d^
3*x^5 + 3/5*A*a*c^2*d^3*x^5 + 3/7*B*a^2*c*x^7*e^3 + 3/2*B*a^2*c*d*x^6*e^2 +
 9/5*B*a^2*c*d^2*x^5*e + 3/4*B*a^2*c*d^3*x^4 + 1/6*C*a^3*x^6*e^3 + 1/2*A*a^
2*c*x^6*e^3 + 3/5*C*a^3*d*x^5*e^2 + 9/5*A*a^2*c*d*x^5*e^2 + 3/4*C*a^3*d^2*x
^4*e + 9/4*A*a^2*c*d^2*x^4*e + 1/3*C*a^3*d^3*x^3 + A*a^2*c*d^3*x^3 + 1/5*B*
a^3*x^5*e^3 + 3/4*B*a^3*d*x^4*e^2 + B*a^3*d^2*x^3*e + 1/2*B*a^3*d^3*x^2 + 1
/4*A*a^3*x^4*e^3 + A*a^3*d*x^3*e^2 + 3/2*A*a^3*d^2*x^2*e + A*a^3*d^3*x
```

maple [A] time = 0.00, size = 553, normalized size = 1.37

$$\frac{C c^3 e^3 x^{12}}{12}+\frac{\left(e^3 c^3 B+3 d e^2 c^3 C\right) x^{11}}{11}+\frac{\left(A c^3 e^3+3 B c^3 d e^2+\left(3 e^3 a c^2+3 d^2 e c^3\right) C\right) x^{10}}{10}+\frac{\left(3 A c^3 d e^2+\left(3 e^3 a c^2+3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x)
```

```
[Out] 1/12*c^3*C*e^3*x^12+1/11*(B*c^3*e^3+3*C*c^3*d*e^2)*x^11+1/10*((3*a*c^2*e^3+
3*c^3*d^2*e)*C+3*d*e^2*c^3*B+e^3*c^3*A)*x^10+1/9*((9*a*c^2*d*e^2+c^3*d^3)*C
+(3*a*c^2*e^3+3*c^3*d^2*e)*B+3*d*e^2*c^3*A)*x^9+1/8*((3*a^2*c*e^3+9*a*c^2*d
^2*e)*C+(9*a*c^2*d*e^2+c^3*d^3)*B+(3*a*c^2*e^3+3*c^3*d^2*e)*A)*x^8+1/7*((9*
a^2*c*d*e^2+3*a*c^2*d^3)*C+(3*a^2*c*e^3+9*a*c^2*d^2*e)*B+(9*a*c^2*d*e^2+c^3
*d^3)*A)*x^7+1/6*((a^3*e^3+9*a^2*c*d^2*e)*C+(9*a^2*c*d*e^2+3*a*c^2*d^3)*B+(
3*a^2*c*e^3+9*a*c^2*d^2*e)*A)*x^6+1/5*((3*a^3*d*e^2+3*a^2*c*d^3)*C+(a^3*e^3
+9*a^2*c*d^2*e)*B+(9*a^2*c*d*e^2+3*a*c^2*d^3)*A)*x^5+1/4*(3*d^2*e*a^3*C+(3*
a^3*d*e^2+3*a^2*c*d^3)*B+(a^3*e^3+9*a^2*c*d^2*e)*A)*x^4+1/3*(d^3*a^3*C+3*d^
2*e*a^3*B+(3*a^3*d*e^2+3*a^2*c*d^3)*A)*x^3+1/2*(3*A*a^3*d^2*e+B*a^3*d^3)*x^
2+a^3*A*d^3*x
```

maxima [A] time = 0.47, size = 512, normalized size = 1.27

$$\frac{1}{12}\,Cc^3e^3x^{12}+\frac{1}{11}\left(3\,Cc^3de^2+Bc^3e^3\right)x^{11}+\frac{1}{10}\left(3\,Cc^3d^2e+3\,Bc^3de^2+\left(3\,Cac^2+Ac^3\right)e^3\right)x^{10}+\frac{1}{9}\left(Cc^3d^3+3\,Bc^3d^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/12*C*c^3*e^3*x^12 + 1/11*(3*C*c^3*d*e^2 + B*c^3*e^3)*x^11 + 1/10*(3*C*c^3
*d^2*e + 3*B*c^3*d*e^2 + (3*C*a*c^2 + A*c^3)*e^3)*x^10 + 1/9*(C*c^3*d^3 + 3
*B*c^3*d^2*e + 3*B*a*c^2*e^3 + 3*(3*C*a*c^2 + A*c^3)*d*e^2)*x^9 + 1/8*(B*c^
3*d^3 + 9*B*a*c^2*d*e^2 + 3*(3*C*a*c^2 + A*c^3)*d^2*e + 3*(C*a^2*c + A*a*c^
2)*e^3)*x^8 + A*a^3*d^3*x + 1/7*(9*B*a*c^2*d^2*e + 3*B*a^2*c*e^3 + (3*C*a*c
^2 + A*c^3)*d^3 + 9*(C*a^2*c + A*a*c^2)*d*e^2)*x^7 + 1/6*(3*B*a*c^2*d^3 + 9
*B*a^2*c*d*e^2 + 9*(C*a^2*c + A*a*c^2)*d^2*e + (C*a^3 + 3*A*a^2*c)*e^3)*x^6
 + 1/5*(9*B*a^2*c*d^2*e + B*a^3*e^3 + 3*(C*a^2*c + A*a*c^2)*d^3 + 3*(C*a^3
+ 3*A*a^2*c)*d*e^2)*x^5 + 1/4*(3*B*a^2*c*d^3 + 3*B*a^3*d*e^2 + A*a^3*e^3 +
3*(C*a^3 + 3*A*a^2*c)*d^2*e)*x^4 + 1/3*(3*B*a^3*d^2*e + 3*A*a^3*d*e^2 + (C*
a^3 + 3*A*a^2*c)*d^3)*x^3 + 1/2*(B*a^3*d^3 + 3*A*a^3*d^2*e)*x^2
```

mupad [B] time = 4.05, size = 490, normalized size = 1.21

$$x^5\left(\frac{3\,C\,a^3\,d\,e^2}{5}+\frac{B\,a^3\,e^3}{5}+\frac{3\,C\,a^2\,c\,d^3}{5}+\frac{9\,B\,a^2\,c\,d^2\,e}{5}+\frac{9\,A\,a^2\,c\,d\,e^2}{5}+\frac{3\,A\,a\,c^2\,d^3}{5}\right)+x^8\left(\frac{3\,C\,a^2\,c\,e^3}{8}+\frac{9\,C\,a\,c^2\,\cdots}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^3*(d + e*x)^3*(A + B*x + C*x^2),x)
```

```
[Out] x^5*((B*a^3*e^3)/5 + (3*A*a*c^2*d^3)/5 + (3*C*a^2*c*d^3)/5 + (3*C*a^3*d*e^2
)/5 + (9*A*a^2*c*d*e^2)/5 + (9*B*a^2*c*d^2*e)/5) + x^8*((B*c^3*d^3)/8 + (3*
A*a*c^2*e^3)/8 + (3*C*a^2*c*e^3)/8 + (3*A*c^3*d^2*e)/8 + (9*B*a*c^2*d*e^2)/
8 + (9*C*a*c^2*d^2*e)/8) + x^6*((C*a^3*e^3)/6 + (A*a^2*c*e^3)/2 + (B*a*c^2*
d^3)/2 + (3*A*a*c^2*d^2*e)/2 + (3*B*a^2*c*d*e^2)/2 + (3*C*a^2*c*d^2*e)/2) +
 x^7*((A*c^3*d^3)/7 + (3*B*a^2*c*e^3)/7 + (3*C*a*c^2*d^3)/7 + (9*A*a*c^2*d*
e^2)/7 + (9*B*a*c^2*d^2*e)/7 + (9*C*a^2*c*d*e^2)/7) + (a^2*x^4*(A*a*e^3 + 3
*B*c*d^3 + 3*B*a*d*e^2 + 9*A*c*d^2*e + 3*C*a*d^2*e))/4 + (c^2*x^9*(3*B*a*e^
3 + C*c*d^3 + 3*A*c*d*e^2 + 9*C*a*d*e^2 + 3*B*c*d^2*e))/9 + (C*c^3*e^3*x^12
)/12 + (a^3*d^2*x^2*(3*A*e + B*d))/2 + (c^3*e^2*x^11*(B*e + 3*C*d))/11 + A*
a^3*d^3*x + (a^2*d*x^3*(3*A*a*e^2 + 3*A*c*d^2 + C*a*d^2 + 3*B*a*d*e))/3 + (
c^2*e*x^10*(A*c*e^2 + 3*C*a*e^2 + 3*C*c*d^2 + 3*B*c*d*e))/10
```

sympy [A] time = 0.16, size = 646, normalized size = 1.60

$$Aa^3d^3x+\frac{Cc^3e^3x^{12}}{12}+x^{11}\left(\frac{Bc^3e^3}{11}+\frac{3Cc^3de^2}{11}\right)+x^{10}\left(\frac{Ac^3e^3}{10}+\frac{3Bc^3de^2}{10}+\frac{3Cac^2e^3}{10}+\frac{3Cc^3d^2e}{10}\right)+x^9\left(\frac{Ac^3de^2}{3}+\frac{Bac^2e^3}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*d**3*x + C*c**3*e**3*x**12/12 + x**11*(B*c**3*e**3/11 + 3*C*c**3*d*e
**2/11) + x**10*(A*c**3*e**3/10 + 3*B*c**3*d*e**2/10 + 3*C*a*c**2*e**3/10 +
 3*C*c**3*d**2*e/10) + x**9*(A*c**3*d*e**2/3 + B*a*c**2*e**3/3 + B*c**3*d**
2*e/3 + C*a*c**2*d*e**2 + C*c**3*d**3/9) + x**8*(3*A*a*c**2*e**3/8 + 3*A*c*
*3*d**2*e/8 + 9*B*a*c**2*d*e**2/8 + B*c**3*d**3/8 + 3*C*a**2*c*e**3/8 + 9*C
*a*c**2*d**2*e/8) + x**7*(9*A*a*c**2*d*e**2/7 + A*c**3*d**3/7 + 3*B*a**2*c*
e**3/7 + 9*B*a*c**2*d**2*e/7 + 9*C*a**2*c*d*e**2/7 + 3*C*a*c**2*d**3/7) + x
**6*(A*a**2*c*e**3/2 + 3*A*a*c**2*d**2*e/2 + 3*B*a**2*c*d*e**2/2 + B*a*c**2
*d**3/2 + C*a**3*e**3/6 + 3*C*a**2*c*d**2*e/2) + x**5*(9*A*a**2*c*d*e**2/5
+ 3*A*a*c**2*d**3/5 + B*a**3*e**3/5 + 9*B*a**2*c*d**2*e/5 + 3*C*a**3*d*e**2
/5 + 3*C*a**2*c*d**3/5) + x**4*(A*a**3*e**3/4 + 9*A*a**2*c*d**2*e/4 + 3*B*a
**3*d*e**2/4 + 3*B*a**2*c*d**3/4 + 3*C*a**3*d**2*e/4) + x**3*(A*a**3*d*e**2
 + A*a**2*c*d**3 + B*a**3*d**2*e + C*a**3*d**3/3) + x**2*(3*A*a**3*d**2*e/2
 + B*a**3*d**3/2)
```

3.33 $\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=289

$$a^3Ad^2x+\frac{1}{4}a^3ex^4(Be+2Cd)+\frac{1}{3}a^2x^3\left(A\left(ae^2+3cd^2\right)+ad(2Be+Cd)\right)+\frac{1}{2}a^2cex^6(Be+2Cd)+\frac{1}{9}c^2x^9\left(3aCe^2+c\left(e\right.\right.$$

```
[Out] a^3*A*d^2*x+1/3*a^2*(a*d*(2*B*e+C*d)+A*(a*e^2+3*c*d^2))*x^3+1/4*a^3*e*(B*e+
2*C*d)*x^4+1/5*a*(3*A*c*(a*e^2+c*d^2)+a*(a*C*e^2+3*c*d*(2*B*e+C*d)))*x^5+1/
2*a^2*c*e*(B*e+2*C*d)*x^6+1/7*c*(A*c*(3*a*e^2+c*d^2)+3*a*(a*C*e^2+c*d*(2*B*
e+C*d)))*x^7+3/8*a*c^2*e*(B*e+2*C*d)*x^8+1/9*c^2*(3*a*C*e^2+c*(C*d^2+e*(A*e
+2*B*d)))*x^9+1/10*c^3*e*(B*e+2*C*d)*x^10+1/11*c^3*C*e^2*x^11+1/8*d*(2*A*e+
B*d)*(c*x^2+a)^4/c
```

Rubi [A] time = 0.42, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1582, 1810}

$$\frac{1}{3}a^2x^3\left(A\left(ae^2+3cd^2\right)+ad(2Be+Cd)\right)+a^3Ad^2x+\frac{1}{2}a^2cex^6(Be+2Cd)+\frac{1}{4}a^3ex^4(Be+2Cd)+\frac{1}{9}c^2x^9\left(3aCe^2+ce(A\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^2*x + (a^2*(a*d*(C*d + 2*B*e) + A*(3*c*d^2 + a*e^2))*x^3)/3 + (a^3*
e*(2*C*d + B*e)*x^4)/4 + (a*(3*A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + 3*c*d*(C*
d + 2*B*e)))*x^5)/5 + (a^2*c*e*(2*C*d + B*e)*x^6)/2 + (c*(A*c*(c*d^2 + 3*a*
e^2) + 3*a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x^7)/7 + (3*a*c^2*e*(2*C*d + B*e)
*x^8)/8 + (c^2*(c*C*d^2 + 3*a*C*e^2 + c*e*(2*B*d + A*e))*x^9)/9 + (c^3*e*(2
*C*d + B*e)*x^10)/10 + (c^3*C*e^2*x^11)/11 + (d*(B*d + 2*A*e)*(a + c*x^2)^4
)/(8*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)dx &= \frac{d(Bd+2Ae)\left(a+cx^2\right)^4}{8c}+\int \left(a+cx^2\right)^3\left(-\left(Bd^2+2Ade\right)x+(d\right.\\ &= \frac{d(Bd+2Ae)\left(a+cx^2\right)^4}{8c}+\int \left(a^3Ad^2+a^2\left(ad(Cd+2Be)+A\left(3c\right.\right.\right.\\ &= a^3Ad^2x+\frac{1}{3}a^2\left(ad(Cd+2Be)+A\left(3cd^2+ae^2\right)\right)x^3+\frac{1}{4}a^3e(2Cd+\end{aligned}$$

Mathematica [A] time = 0.13, size = 329, normalized size = 1.14

$$\frac{1}{2}a^3dx^2(2Ae+Bd)+a^3Ad^2x+\frac{1}{4}a^2x^4\left(aBe^2+2aCde+6Acde+3Bcd^2\right)+\frac{1}{3}a^2x^3\left(A\left(ae^2+3cd^2\right)+ad(2Be+Cd)\right)+\frac{1}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^2*x + (a^3*d*(B*d + 2*A*e)*x^2)/2 + (a^2*(a*d*(C*d + 2*B*e) + A*(3*
c*d^2 + a*e^2))*x^3)/3 + (a^2*(3*B*c*d^2 + 6*A*c*d*e + 2*a*C*d*e + a*B*e^2)
*x^4)/4 + (a*(3*A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + 3*c*d*(C*d + 2*B*e)))*x^
5)/5 + (a*c*(2*(A*c + a*C)*d*e + B*(c*d^2 + a*e^2))*x^6)/2 + (c*(A*c*(c*d^2
 + 3*a*e^2) + 3*a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x^7)/7 + (c^2*(B*c*d^2 + 2
*A*c*d*e + 6*a*C*d*e + 3*a*B*e^2)*x^8)/8 + (c^2*(c*C*d^2 + 3*a*C*e^2 + c*e*
(2*B*d + A*e))*x^9)/9 + (c^3*e*(2*C*d + B*e)*x^10)/10 + (c^3*C*e^2*x^11)/11
```

fricas [A] time = 0.75, size = 432, normalized size = 1.49

$$\frac{1}{11}x^{11}e^2c^3C+\frac{1}{5}x^{10}edc^3C+\frac{1}{10}x^{10}e^2c^3B+\frac{1}{9}x^9d^2c^3C+\frac{1}{3}x^9e^2c^2aC+\frac{2}{9}x^9edc^3B+\frac{1}{9}x^9e^2c^3A+\frac{3}{4}x^8edc^2aC+\frac{1}{8}x^8d^2c^3B+\frac{3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^2*c^3*C + 1/5*x^10*e*d*c^3*C + 1/10*x^10*e^2*c^3*B + 1/9*x^9*d^
2*c^3*C + 1/3*x^9*e^2*c^2*a*C + 2/9*x^9*e*d*c^3*B + 1/9*x^9*e^2*c^3*A + 3/4
*x^8*e*d*c^2*a*C + 1/8*x^8*d^2*c^3*B + 3/8*x^8*e^2*c^2*a*B + 1/4*x^8*e*d*c^
3*A + 3/7*x^7*d^2*c^2*a*C + 3/7*x^7*e^2*c*a^2*C + 6/7*x^7*e*d*c^2*a*B + 1/7
*x^7*d^2*c^3*A + 3/7*x^7*e^2*c^2*a*A + x^6*e*d*c*a^2*C + 1/2*x^6*d^2*c^2*a*
B + 1/2*x^6*e^2*c*a^2*B + x^6*e*d*c^2*a*A + 3/5*x^5*d^2*c*a^2*C + 1/5*x^5*e
^2*a^3*C + 6/5*x^5*e*d*c*a^2*B + 3/5*x^5*d^2*c^2*a*A + 3/5*x^5*e^2*c*a^2*A
+ 1/2*x^4*e*d*a^3*C + 3/4*x^4*d^2*c*a^2*B + 1/4*x^4*e^2*a^3*B + 3/2*x^4*e*d
*c*a^2*A + 1/3*x^3*d^2*a^3*C + 2/3*x^3*e*d*a^3*B + x^3*d^2*c*a^2*A + 1/3*x^
3*e^2*a^3*A + 1/2*x^2*d^2*a^3*B + x^2*e*d*a^3*A + x*d^2*a^3*A
```

giac [A] time = 0.15, size = 432, normalized size = 1.49

$$\frac{1}{11}Cc^3x^{11}e^2+\frac{1}{5}Cc^3dx^{10}e+\frac{1}{9}Cc^3d^2x^9+\frac{1}{10}Bc^3x^{10}e^2+\frac{2}{9}Bc^3dx^9e+\frac{1}{8}Bc^3d^2x^8+\frac{1}{3}Cac^2x^9e^2+\frac{1}{9}Ac^3x^9e^2+\frac{3}{4}Cac^2dx^8e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/11*C*c^3*x^11*e^2 + 1/5*C*c^3*d*x^10*e + 1/9*C*c^3*d^2*x^9 + 1/10*B*c^3*x
^10*e^2 + 2/9*B*c^3*d*x^9*e + 1/8*B*c^3*d^2*x^8 + 1/3*C*a*c^2*x^9*e^2 + 1/9
*A*c^3*x^9*e^2 + 3/4*C*a*c^2*d*x^8*e + 1/4*A*c^3*d*x^8*e + 3/7*C*a*c^2*d^2*
x^7 + 1/7*A*c^3*d^2*x^7 + 3/8*B*a*c^2*x^8*e^2 + 6/7*B*a*c^2*d*x^7*e + 1/2*B
*a*c^2*d^2*x^6 + 3/7*C*a^2*c*x^7*e^2 + 3/7*A*a*c^2*x^7*e^2 + C*a^2*c*d*x^6*
e + A*a*c^2*d*x^6*e + 3/5*C*a^2*c*d^2*x^5 + 3/5*A*a*c^2*d^2*x^5 + 1/2*B*a^2
*c*x^6*e^2 + 6/5*B*a^2*c*d*x^5*e + 3/4*B*a^2*c*d^2*x^4 + 1/5*C*a^3*x^5*e^2
+ 3/5*A*a^2*c*x^5*e^2 + 1/2*C*a^3*d*x^4*e + 3/2*A*a^2*c*d*x^4*e + 1/3*C*a^3
*d^2*x^3 + A*a^2*c*d^2*x^3 + 1/4*B*a^3*x^4*e^2 + 2/3*B*a^3*d*x^3*e + 1/2*B*
a^3*d^2*x^2 + 1/3*A*a^3*x^3*e^2 + A*a^3*d*x^2*e + A*a^3*d^2*x
```

maple [A] time = 0.00, size = 388, normalized size = 1.34

$$\frac{C\,c^3e^2x^{11}}{11}+\frac{\left(e^2c^3B+2de\,c^3C\right)x^{10}}{10}+\frac{\left(A\,c^3e^2+2B\,c^3de+\left(3e^2a\,c^2+d^2c^3\right)C\right)x^9}{9}+\frac{\left(2A\,c^3de+6Ca\,c^2de+\left(3e^2a\,c^2+\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x)
```

```
[Out] 1/11*c^3*C*e^2*x^11+1/10*(B*c^3*e^2+2*C*c^3*d*e)*x^10+1/9*((3*a*c^2*e^2+c^3
*d^2)*C+2*d*e*c^3*B+e^2*c^3*A)*x^9+1/8*(6*d*e*a*c^2*C+(3*a*c^2*e^2+c^3*d^2)
*B+2*d*e*c^3*A)*x^8+1/7*((3*a^2*c*e^2+3*a*c^2*d^2)*C+6*d*e*a*c^2*B+(3*a*c^2
*e^2+c^3*d^2)*A)*x^7+1/6*(6*d*e*a^2*c*C+(3*a^2*c*e^2+3*a*c^2*d^2)*B+6*d*e*a
*c^2*A)*x^6+1/5*((a^3*e^2+3*a^2*c*d^2)*C+6*d*e*a^2*c*B+(3*a^2*c*e^2+3*a*c^2
*d^2)*A)*x^5+1/4*(2*d*e*a^3*C+(a^3*e^2+3*a^2*c*d^2)*B+6*d*e*a^2*c*A)*x^4+1/
3*(d^2*a^3*C+2*d*e*a^3*B+(a^3*e^2+3*a^2*c*d^2)*A)*x^3+1/2*(2*A*a^3*d*e+B*a^
3*d^2)*x^2+a^3*A*d^2*x
```

maxima [A] time = 0.45, size = 367, normalized size = 1.27

$$\frac{1}{11}Cc^3e^2x^{11}+\frac{1}{10}\left(2Cc^3de+Bc^3e^2\right)x^{10}+\frac{1}{9}\left(Cc^3d^2+2Bc^3de+\left(3Cac^2+Ac^3\right)e^2\right)x^9+\frac{1}{8}\left(Bc^3d^2+3Bac^2e^2+2\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/11*C*c^3*e^2*x^11 + 1/10*(2*C*c^3*d*e + B*c^3*e^2)*x^10 + 1/9*(C*c^3*d^2
+ 2*B*c^3*d*e + (3*C*a*c^2 + A*c^3)*e^2)*x^9 + 1/8*(B*c^3*d^2 + 3*B*a*c^2*e
^2 + 2*(3*C*a*c^2 + A*c^3)*d*e)*x^8 + 1/7*(6*B*a*c^2*d*e + (3*C*a*c^2 + A*c
^3)*d^2 + 3*(C*a^2*c + A*a*c^2)*e^2)*x^7 + A*a^3*d^2*x + 1/2*(B*a*c^2*d^2 +
 B*a^2*c*e^2 + 2*(C*a^2*c + A*a*c^2)*d*e)*x^6 + 1/5*(6*B*a^2*c*d*e + 3*(C*a
^2*c + A*a*c^2)*d^2 + (C*a^3 + 3*A*a^2*c)*e^2)*x^5 + 1/4*(3*B*a^2*c*d^2 + B
*a^3*e^2 + 2*(C*a^3 + 3*A*a^2*c)*d*e)*x^4 + 1/3*(2*B*a^3*d*e + A*a^3*e^2 +
(C*a^3 + 3*A*a^2*c)*d^2)*x^3 + 1/2*(B*a^3*d^2 + 2*A*a^3*d*e)*x^2
```

mupad [B] time = 3.94, size = 343, normalized size = 1.19

$$x^3\left(\frac{C\,a^3\,d^2}{3}+\frac{2\,B\,a^3\,d\,e}{3}+\frac{A\,a^3\,e^2}{3}+A\,c\,a^2\,d^2\right)+x^9\left(\frac{C\,c^3\,d^2}{9}+\frac{2\,B\,c^3\,d\,e}{9}+\frac{A\,c^3\,e^2}{9}+\frac{C\,a\,c^2\,e^2}{3}\right)+x^5\left(\frac{C\,a^3\,e^2}{5}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^3*(d + e*x)^2*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*a^3*e^2)/3 + (C*a^3*d^2)/3 + (2*B*a^3*d*e)/3 + A*a^2*c*d^2) + x^9*(
(A*c^3*e^2)/9 + (C*c^3*d^2)/9 + (2*B*c^3*d*e)/9 + (C*a*c^2*e^2)/3) + x^5*((
C*a^3*e^2)/5 + (3*A*a*c^2*d^2)/5 + (3*A*a^2*c*e^2)/5 + (3*C*a^2*c*d^2)/5 +
(6*B*a^2*c*d*e)/5) + x^7*((A*c^3*d^2)/7 + (3*A*a*c^2*e^2)/7 + (3*C*a*c^2*d^
2)/7 + (3*C*a^2*c*e^2)/7 + (6*B*a*c^2*d*e)/7) + (a^2*x^4*(B*a*e^2 + 3*B*c*d
^2 + 6*A*c*d*e + 2*C*a*d*e))/4 + (c^2*x^8*(3*B*a*e^2 + B*c*d^2 + 2*A*c*d*e
+ 6*C*a*d*e))/8 + (C*c^3*e^2*x^11)/11 + (a*c*x^6*(B*a*e^2 + B*c*d^2 + 2*A*c
*d*e + 2*C*a*d*e))/2 + A*a^3*d^2*x + (a^3*d*x^2*(2*A*e + B*d))/2 + (c^3*e*x
^10*(B*e + 2*C*d))/10
```

sympy [A] time = 0.15, size = 447, normalized size = 1.55

$$Aa^3d^2x+\frac{Cc^3e^2x^{11}}{11}+x^{10}\left(\frac{Bc^3e^2}{10}+\frac{Cc^3de}{5}\right)+x^9\left(\frac{Ac^3e^2}{9}+\frac{2Bc^3de}{9}+\frac{Cac^2e^2}{3}+\frac{Cc^3d^2}{9}\right)+x^8\left(\frac{Ac^3de}{4}+\frac{3Bac^2e^2}{8}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*d**2*x + C*c**3*e**2*x**11/11 + x**10*(B*c**3*e**2/10 + C*c**3*d*e/5
) + x**9*(A*c**3*e**2/9 + 2*B*c**3*d*e/9 + C*a*c**2*e**2/3 + C*c**3*d**2/9)
 + x**8*(A*c**3*d*e/4 + 3*B*a*c**2*e**2/8 + B*c**3*d**2/8 + 3*C*a*c**2*d*e/
4) + x**7*(3*A*a*c**2*e**2/7 + A*c**3*d**2/7 + 6*B*a*c**2*d*e/7 + 3*C*a**2*
c*e**2/7 + 3*C*a*c**2*d**2/7) + x**6*(A*a*c**2*d*e + B*a**2*c*e**2/2 + B*a*
```

```
c**2*d**2/2 + C*a**2*c*d*e) + x**5*(3*A*a**2*c*e**2/5 + 3*A*a*c**2*d**2/5 +
 6*B*a**2*c*d*e/5 + C*a**3*e**2/5 + 3*C*a**2*c*d**2/5) + x**4*(3*A*a**2*c*d
*e/2 + B*a**3*e**2/4 + 3*B*a**2*c*d**2/4 + C*a**3*d*e/2) + x**3*(A*a**3*e**
2/3 + A*a**2*c*d**2 + 2*B*a**3*d*e/3 + C*a**3*d**2/3) + x**2*(A*a**3*d*e +
B*a**3*d**2/2)
```

3.34 $\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=169

$$a^3Adx+\frac{1}{4}a^3Cex^4+\frac{1}{3}a^2x^3(aBe+aCd+3Acd)+\frac{1}{2}a^2cCex^6+\frac{1}{7}c^2x^7(3a(Be+Cd)+Acd)+\frac{3}{5}acx^5(aBe+aCd+Acd)+\frac{(a$$

```
[Out] a^3*A*d*x+1/3*a^2*(3*A*c*d+B*a*e+C*a*d)*x^3+1/4*a^3*C*e*x^4+3/5*a*c*(A*c*d+
B*a*e+C*a*d)*x^5+1/2*a^2*c*C*e*x^6+1/7*c^2*(A*c*d+3*a*(B*e+C*d))*x^7+3/8*a*
c^2*C*e*x^8+1/9*c^3*(B*e+C*d)*x^9+1/10*c^3*C*e*x^10+1/8*(A*e+B*d)*(c*x^2+a)
^4/c
```

Rubi [A] time = 0.19, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1582, 1810}

$$\frac{1}{3}a^2x^3(aBe+aCd+3Acd)+a^3Adx+\frac{1}{2}a^2cCex^6+\frac{1}{4}a^3Cex^4+\frac{1}{7}c^2x^7(3a(Be+Cd)+Acd)+\frac{3}{5}acx^5(aBe+aCd+Acd)+\frac{(a$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d*x + (a^2*(3*A*c*d + a*C*d + a*B*e)*x^3)/3 + (a^3*C*e*x^4)/4 + (3*a*
c*(A*c*d + a*C*d + a*B*e)*x^5)/5 + (a^2*c*C*e*x^6)/2 + (c^2*(A*c*d + 3*a*(C
*d + B*e))*x^7)/7 + (3*a*c^2*C*e*x^8)/8 + (c^3*(C*d + B*e)*x^9)/9 + (c^3*C*
e*x^10)/10 + ((B*d + A*e)*(a + c*x^2)^4)/(8*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)dx &= \frac{(Bd+Ae)\left(a+cx^2\right)^4}{8c}+\int \left(a+cx^2\right)^3\left(-(Bd+Ae)x+(d+ex)\left(A\right.\right.\\ &= \frac{(Bd+Ae)\left(a+cx^2\right)^4}{8c}+\int \left(a^3Ad+a^2(3Acd+aCd+aBe)x^2+a^3\right.\\ &= a^3Adx+\frac{1}{3}a^2(3Acd+aCd+aBe)x^3+\frac{1}{4}a^3Cex^4+\frac{3}{5}ac(Acd+aCd\end{aligned}$$

Mathematica [A] time = 0.07, size = 196, normalized size = 1.16

$$\frac{1}{2}a^3x^2(Ae+Bd)+a^3Adx+\frac{1}{4}a^2x^4(aCe+3Ace+3Bcd)+\frac{1}{3}a^2x^3(aBe+aCd+3Acd)+\frac{1}{8}c^2x^8(3aCe+Ace+Bcd)+\frac{1}{7}c^2x^7($$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d*x + (a^3*(B*d + A*e)*x^2)/2 + (a^2*(3*A*c*d + a*C*d + a*B*e)*x^3)/3
 + (a^2*(3*B*c*d + 3*A*c*e + a*C*e)*x^4)/4 + (3*a*c*(A*c*d + a*C*d + a*B*e)
*x^5)/5 + (a*c*(B*c*d + A*c*e + a*C*e)*x^6)/2 + (c^2*(A*c*d + 3*a*C*d + 3*a
*B*e)*x^7)/7 + (c^2*(B*c*d + A*c*e + 3*a*C*e)*x^8)/8 + (c^3*(C*d + B*e)*x^9
)/9 + (c^3*C*e*x^10)/10
```

fricas [A] time = 0.77, size = 249, normalized size = 1.47

$$\frac{1}{10}x^{10}ec^3C+\frac{1}{9}x^9dc^3C+\frac{1}{9}x^9ec^3B+\frac{3}{8}x^8ec^2aC+\frac{1}{8}x^8dc^3B+\frac{1}{8}x^8ec^3A+\frac{3}{7}x^7dc^2aC+\frac{3}{7}x^7ec^2aB+\frac{1}{7}x^7dc^3A+\frac{1}{2}x^6eca^2C+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e*c^3*C + 1/9*x^9*d*c^3*C + 1/9*x^9*e*c^3*B + 3/8*x^8*e*c^2*a*C +
 1/8*x^8*d*c^3*B + 1/8*x^8*e*c^3*A + 3/7*x^7*d*c^2*a*C + 3/7*x^7*e*c^2*a*B
+ 1/7*x^7*d*c^3*A + 1/2*x^6*e*c*a^2*C + 1/2*x^6*d*c^2*a*B + 1/2*x^6*e*c^2*a
*A + 3/5*x^5*d*c*a^2*C + 3/5*x^5*e*c*a^2*B + 3/5*x^5*d*c^2*a*A + 1/4*x^4*e*
a^3*C + 3/4*x^4*d*c*a^2*B + 3/4*x^4*e*c*a^2*A + 1/3*x^3*d*a^3*C + 1/3*x^3*e
*a^3*B + x^3*d*c*a^2*A + 1/2*x^2*d*a^3*B + 1/2*x^2*e*a^3*A + x*d*a^3*A
```

giac [A] time = 0.19, size = 261, normalized size = 1.54

$$\frac{1}{10}Cc^3x^{10}e+\frac{1}{9}Cc^3dx^9+\frac{1}{9}Bc^3x^9e+\frac{1}{8}Bc^3dx^8+\frac{3}{8}Cac^2x^8e+\frac{1}{8}Ac^3x^8e+\frac{3}{7}Cac^2dx^7+\frac{1}{7}Ac^3dx^7+\frac{3}{7}Bac^2x^7e+\frac{1}{2}Bac^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*C*c^3*x^10*e + 1/9*C*c^3*d*x^9 + 1/9*B*c^3*x^9*e + 1/8*B*c^3*d*x^8 + 3
/8*C*a*c^2*x^8*e + 1/8*A*c^3*x^8*e + 3/7*C*a*c^2*d*x^7 + 1/7*A*c^3*d*x^7 +
3/7*B*a*c^2*x^7*e + 1/2*B*a*c^2*d*x^6 + 1/2*C*a^2*c*x^6*e + 1/2*A*a*c^2*x^6
*e + 3/5*C*a^2*c*d*x^5 + 3/5*A*a*c^2*d*x^5 + 3/5*B*a^2*c*x^5*e + 3/4*B*a^2*
c*d*x^4 + 1/4*C*a^3*x^4*e + 3/4*A*a^2*c*x^4*e + 1/3*C*a^3*d*x^3 + A*a^2*c*d
*x^3 + 1/3*B*a^3*x^3*e + 1/2*B*a^3*d*x^2 + 1/2*A*a^3*x^2*e + A*a^3*d*x
```

maple [A] time = 0.00, size = 223, normalized size = 1.32

$$\frac{C c^3 e x^{10}}{10}+\frac{\left(e c^3 B + c^3 d C\right) x^9}{9}+\frac{\left(e c^3 A + c^3 d B + 3 e a c^2 C\right) x^8}{8}+\frac{\left(c^3 d A + 3 e a c^2 B + 3 d a c^2 C\right) x^7}{7}+A a^3 d x+\frac{\left(3 e a c^2 A + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x)
```

```
[Out] 1/10*c^3*C*e*x^10+1/9*(B*c^3*e+C*c^3*d)*x^9+1/8*(A*c^3*e+B*c^3*d+3*C*a*c^2*
e)*x^8+1/7*(A*c^3*d+3*B*a*c^2*e+3*C*a*c^2*d)*x^7+1/6*(3*A*a*c^2*e+3*B*a*c^2
*d+3*C*a^2*c*e)*x^6+1/5*(3*A*a*c^2*d+3*B*a^2*c*e+3*C*a^2*c*d)*x^5+1/4*(3*A*
a^2*c*e+3*B*a^2*c*d+C*a^3*e)*x^4+1/3*(3*A*a^2*c*d+B*a^3*e+C*a^3*d)*x^3+1/2*
(A*a^3*e+B*a^3*d)*x^2+a^3*A*d*x
```

maxima [A] time = 0.44, size = 222, normalized size = 1.31

$$\frac{1}{10}Cc^3ex^{10}+\frac{1}{9}\left(Cc^3d+Bc^3e\right)x^9+\frac{1}{8}\left(Bc^3d+\left(3\,Cac^2+Ac^3\right)e\right)x^8+\frac{1}{7}\left(3\,Bac^2e+\left(3\,Cac^2+Ac^3\right)d\right)x^7+\frac{1}{2}\left(Bac^2d+\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*C*c^3*e*x^10 + 1/9*(C*c^3*d + B*c^3*e)*x^9 + 1/8*(B*c^3*d + (3*C*a*c^2
 + A*c^3)*e)*x^8 + 1/7*(3*B*a*c^2*e + (3*C*a*c^2 + A*c^3)*d)*x^7 + 1/2*(B*a
*c^2*d + (C*a^2*c + A*a*c^2)*e)*x^6 + A*a^3*d*x + 3/5*(B*a^2*c*e + (C*a^2*c
 + A*a*c^2)*d)*x^5 + 1/4*(3*B*a^2*c*d + (C*a^3 + 3*A*a^2*c)*e)*x^4 + 1/3*(B
*a^3*e + (C*a^3 + 3*A*a^2*c)*d)*x^3 + 1/2*(B*a^3*d + A*a^3*e)*x^2
```

mupad [B] time = 0.10, size = 187, normalized size = 1.11

$$x^3\left(\frac{B\,a^3\,e}{3}+\frac{C\,a^3\,d}{3}+A\,a^2\,c\,d\right)+x^8\left(\frac{A\,c^3\,e}{8}+\frac{B\,c^3\,d}{8}+\frac{3\,C\,a\,c^2\,e}{8}\right)+\frac{a^3\,x^2\,\left(A\,e+B\,d\right)}{2}+\frac{c^3\,x^9\,\left(B\,e+C\,d\right)}{9}+\frac{c^2\,x^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^3*(d + e*x)*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((B*a^3*e)/3 + (C*a^3*d)/3 + A*a^2*c*d) + x^8*((A*c^3*e)/8 + (B*c^3*d)/
8 + (3*C*a*c^2*e)/8) + (a^3*x^2*(A*e + B*d))/2 + (c^3*x^9*(B*e + C*d))/9 +
(c^2*x^7*(A*c*d + 3*B*a*e + 3*C*a*d))/7 + (a^2*x^4*(3*A*c*e + 3*B*c*d + C*a
*e))/4 + A*a^3*d*x + (3*a*c*x^5*(A*c*d + B*a*e + C*a*d))/5 + (a*c*x^6*(A*c*
e + B*c*d + C*a*e))/2 + (C*c^3*e*x^10)/10
```

sympy [A] time = 0.11, size = 265, normalized size = 1.57

$$Aa^3dx+\frac{Cc^3ex^{10}}{10}+x^9\left(\frac{Bc^3e}{9}+\frac{Cc^3d}{9}\right)+x^8\left(\frac{Ac^3e}{8}+\frac{Bc^3d}{8}+\frac{3Cac^2e}{8}\right)+x^7\left(\frac{Ac^3d}{7}+\frac{3Bac^2e}{7}+\frac{3Cac^2d}{7}\right)+x^6\left(\frac{Aac}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*d*x + C*c**3*e*x**10/10 + x**9*(B*c**3*e/9 + C*c**3*d/9) + x**8*(A*c
**3*e/8 + B*c**3*d/8 + 3*C*a*c**2*e/8) + x**7*(A*c**3*d/7 + 3*B*a*c**2*e/7
+ 3*C*a*c**2*d/7) + x**6*(A*a*c**2*e/2 + B*a*c**2*d/2 + C*a**2*c*e/2) + x**
5*(3*A*a*c**2*d/5 + 3*B*a**2*c*e/5 + 3*C*a**2*c*d/5) + x**4*(3*A*a**2*c*e/4
 + 3*B*a**2*c*d/4 + C*a**3*e/4) + x**3*(A*a**2*c*d + B*a**3*e/3 + C*a**3*d/
3) + x**2*(A*a**3*e/2 + B*a**3*d/2)
```

3.35 $\int \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=87

$$a^3Ax+\frac{1}{3}a^2x^3(aC+3Ac)+\frac{1}{7}c^2x^7(3aC+Ac)+\frac{3}{5}acx^5(aC+Ac)+\frac{B\left(a+cx^2\right)^4}{8c}+\frac{1}{9}c^3Cx^9$$

```
[Out] a^3*A*x+1/3*a^2*(3*A*c+C*a)*x^3+3/5*a*c*(A*c+C*a)*x^5+1/7*c^2*(A*c+3*C*a)*x
^7+1/9*c^3*C*x^9+1/8*B*(c*x^2+a)^4/c
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1582, 373}

$$\frac{1}{3}a^2x^3(aC+3Ac)+a^3Ax+\frac{1}{7}c^2x^7(3aC+Ac)+\frac{3}{5}acx^5(aC+Ac)+\frac{B\left(a+cx^2\right)^4}{8c}+\frac{1}{9}c^3Cx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*x + (a^2*(3*A*c + a*C)*x^3)/3 + (3*a*c*(A*c + a*C)*x^5)/5 + (c^2*(A*c
 + 3*a*C)*x^7)/7 + (c^3*C*x^9)/9 + (B*(a + c*x^2)^4)/(8*c)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx &= \frac{B\left(a+cx^2\right)^4}{8c}+\int \left(a+cx^2\right)^3 \left(A+Cx^2\right) dx\\ &= \frac{B\left(a+cx^2\right)^4}{8c}+\int \left(a^3A+a^2(3Ac+aC)x^2+3ac(Ac+aC)x^4+c^2(Ac+3aC\right.\\ &= a^3Ax+\frac{1}{3}a^2(3Ac+aC)x^3+\frac{3}{5}ac(Ac+aC)x^5+\frac{1}{7}c^2(Ac+3aC)x^7+\frac{1}{9}c^3Cx^9\end{aligned}$$

Mathematica [A] time = 0.03, size = 100, normalized size = 1.15

$$\frac{1}{6}a^3x(6A+x(3B+2Cx))+\frac{1}{20}a^2cx^3(20A+3x(5B+4Cx))+\frac{1}{70}ac^2x^5(42A+5x(7B+6Cx))+\frac{1}{504}c^3x^7(72A+7x(9B+8C$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] (a^3*x*(6*A + x*(3*B + 2*C*x)))/6 + (a^2*c*x^3*(20*A + 3*x*(5*B + 4*C*x)))/
20 + (a*c^2*x^5*(42*A + 5*x*(7*B + 6*C*x)))/70 + (c^3*x^7*(72*A + 7*x*(9*B
+ 8*C*x)))/504
```

fricas [A] time = 0.77, size = 111, normalized size = 1.28

$$\frac{1}{9}x^9c^3C+\frac{1}{8}x^8c^3B+\frac{3}{7}x^7c^2aC+\frac{1}{7}x^7c^3A+\frac{1}{2}x^6c^2aB+\frac{3}{5}x^5ca^2C+\frac{3}{5}x^5c^2aA+\frac{3}{4}x^4ca^2B+\frac{1}{3}x^3a^3C+x^3ca^2A+\frac{1}{2}x^2a^3B+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^3*C + 1/8*x^8*c^3*B + 3/7*x^7*c^2*a*C + 1/7*x^7*c^3*A + 1/2*x^6*c
^2*a*B + 3/5*x^5*c*a^2*C + 3/5*x^5*c^2*a*A + 3/4*x^4*c*a^2*B + 1/3*x^3*a^3*
C + x^3*c*a^2*A + 1/2*x^2*a^3*B + x*a^3*A
```

giac [A] time = 0.19, size = 111, normalized size = 1.28

$$\frac{1}{9}Cc^3x^9+\frac{1}{8}Bc^3x^8+\frac{3}{7}Cac^2x^7+\frac{1}{7}Ac^3x^7+\frac{1}{2}Bac^2x^6+\frac{3}{5}Ca^2cx^5+\frac{3}{5}Aac^2x^5+\frac{3}{4}Ba^2cx^4+\frac{1}{3}Ca^3x^3+Aa^2cx^3+\frac{1}{2}Ba^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*C*c^3*x^9 + 1/8*B*c^3*x^8 + 3/7*C*a*c^2*x^7 + 1/7*A*c^3*x^7 + 1/2*B*a*c
^2*x^6 + 3/5*C*a^2*c*x^5 + 3/5*A*a*c^2*x^5 + 3/4*B*a^2*c*x^4 + 1/3*C*a^3*x^
3 + A*a^2*c*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

maple [A] time = 0.00, size = 111, normalized size = 1.28

$$\frac{C\,c^3x^9}{9}+\frac{B\,c^3x^8}{8}+\frac{Ba\,c^2x^6}{2}+\frac{3B\,a^2c\,x^4}{4}+\frac{\left(c^3A+3a\,c^2C\right)x^7}{7}+\frac{B\,a^3x^2}{2}+A\,a^3x+\frac{\left(3a\,c^2A+3a^2cC\right)x^5}{5}+\frac{\left(3a^2cA+a^3C\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A),x)
```

```
[Out] 1/9*c^3*C*x^9+1/8*c^3*B*x^8+1/7*(A*c^3+3*C*a*c^2)*x^7+1/2*a*c^2*B*x^6+1/5*(
3*A*a*c^2+3*C*a^2*c)*x^5+3/4*a^2*c*B*x^4+1/3*(3*A*a^2*c+C*a^3)*x^3+1/2*a^3*
B*x^2+a^3*A*x
```

maxima [A] time = 0.43, size = 108, normalized size = 1.24

$$\frac{1}{9}Cc^3x^9+\frac{1}{8}Bc^3x^8+\frac{1}{2}Bac^2x^6+\frac{3}{4}Ba^2cx^4+\frac{1}{7}\left(3\,Cac^2+Ac^3\right)x^7+\frac{1}{2}Ba^3x^2+\frac{3}{5}\left(Ca^2c+Aac^2\right)x^5+Aa^3x+\frac{1}{3}\left(Ca^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*C*c^3*x^9 + 1/8*B*c^3*x^8 + 1/2*B*a*c^2*x^6 + 3/4*B*a^2*c*x^4 + 1/7*(3*
C*a*c^2 + A*c^3)*x^7 + 1/2*B*a^3*x^2 + 3/5*(C*a^2*c + A*a*c^2)*x^5 + A*a^3*
x + 1/3*(C*a^3 + 3*A*a^2*c)*x^3
```

mupad [B] time = 0.06, size = 103, normalized size = 1.18

$$x^3\,\left(\frac{C\,a^3}{3}+A\,c\,a^2\right)+x^7\,\left(\frac{A\,c^3}{7}+\frac{3\,C\,a\,c^2}{7}\right)+\frac{B\,a^3\,x^2}{2}+\frac{B\,c^3\,x^8}{8}+\frac{C\,c^3\,x^9}{9}+A\,a^3\,x+\frac{3\,a\,c\,x^5\,\left(A\,c+C\,a\right)}{5}+\frac{3\,B\,a^2\,c\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^3*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((C*a^3)/3 + A*a^2*c) + x^7*((A*c^3)/7 + (3*C*a*c^2)/7) + (B*a^3*x^2)/2
 + (B*c^3*x^8)/8 + (C*c^3*x^9)/9 + A*a^3*x + (3*a*c*x^5*(A*c + C*a))/5 + (3
*B*a^2*c*x^4)/4 + (B*a*c^2*x^6)/2
```

sympy [A] time = 0.09, size = 122, normalized size = 1.40

$$Aa^3x+\frac{Ba^3x^2}{2}+\frac{3Ba^2cx^4}{4}+\frac{Bac^2x^6}{2}+\frac{Bc^3x^8}{8}+\frac{Cc^3x^9}{9}+x^7\left(\frac{Ac^3}{7}+\frac{3Cac^2}{7}\right)+x^5\left(\frac{3Aac^2}{5}+\frac{3Ca^2c}{5}\right)+x^3\left(Aa^2c+\frac{Ca^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x + B*a**3*x**2/2 + 3*B*a**2*c*x**4/4 + B*a*c**2*x**6/2 + B*c**3*x**
8/8 + C*c**3*x**9/9 + x**7*(A*c**3/7 + 3*C*a*c**2/7) + x**5*(3*A*a*c**2/5 +
 3*C*a**2*c/5) + x**3*(A*a**2*c + C*a**3/3)
```

3.36 $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{d+ex}\,dx$

Optimal. Leaf size=490

$$\frac{c(d+ex)^4\left(3a^2Ce^4+3ace^2\left(15Cd^2-e(5Bd-Ae)\right)+5c^2d^2\left(14Cd^2-e(7Bd-3Ae)\right)\right)}{4e^9}+\frac{(d+ex)^2\left(ae^2+cd^2\right)\left(a\right.}{}$$

```
[Out] -(a*e^2+c*d^2)^2*(a*e^2*(-B*e+2*C*d)+c*d*(8*C*d^2-e*(-6*A*e+7*B*d)))*x/e^8+
1/2*(a*e^2+c*d^2)*(a^2*C*e^4+c^2*d^2*(28*C*d^2-3*e*(-5*A*e+7*B*d))+a*c*e^2*
(17*C*d^2-3*e*(-A*e+3*B*d)))*(e*x+d)^2/e^9-1/3*c*(3*a^2*e^4*(-B*e+4*C*d)+c^
2*d^3*(56*C*d^2-5*e*(-4*A*e+7*B*d))+6*a*c*d*e^2*(10*C*d^2-e*(-2*A*e+5*B*d))
)*(e*x+d)^3/e^9+1/4*c*(3*a^2*C*e^4+5*c^2*d^2*(14*C*d^2-e*(-3*A*e+7*B*d))+3*
a*c*e^2*(15*C*d^2-e*(-A*e+5*B*d)))*(e*x+d)^4/e^9-1/5*c^2*(3*a*e^2*(-B*e+6*C
*d)+c*d*(56*C*d^2-3*e*(-2*A*e+7*B*d)))*(e*x+d)^5/e^9+1/6*c^2*(3*a*C*e^2+c*(
28*C*d^2-e*(-A*e+7*B*d)))*(e*x+d)^6/e^9-1/7*c^3*(-B*e+8*C*d)*(e*x+d)^7/e^9+
1/8*c^3*C*(e*x+d)^8/e^9+(a*e^2+c*d^2)^3*(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e^9
```

Rubi [A] time = 1.10, antiderivative size = 487, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1628}

$$\frac{c(d+ex)^4\left(3a^2Ce^4+3ace^2\left(15Cd^2-e(5Bd-Ae)\right)+5c^2\left(14Cd^4-d^2e(7Bd-3Ae)\right)\right)}{4e^9}-\frac{c(d+ex)^3\left(3a^2e^4(4Cd-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] -(((c*d^2 + a*e^2)^2*(8*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*
e))*x)/e^8) + ((c*d^2 + a*e^2)*(a^2*C*e^4 + c^2*(28*C*d^4 - 3*d^2*e*(7*B*d
- 5*A*e)) + a*c*e^2*(17*C*d^2 - 3*e*(3*B*d - A*e)))*(d + e*x)^2)/(2*e^9) -
(c*(3*a^2*e^4*(4*C*d - B*e) + c^2*(56*C*d^5 - 5*d^3*e*(7*B*d - 4*A*e)) + 6*
a*c*d*e^2*(10*C*d^2 - e*(5*B*d - 2*A*e)))*(d + e*x)^3)/(3*e^9) + (c*(3*a^2*
C*e^4 + 5*c^2*(14*C*d^4 - d^2*e*(7*B*d - 3*A*e)) + 3*a*c*e^2*(15*C*d^2 - e*
(5*B*d - A*e)))*(d + e*x)^4)/(4*e^9) - (c^2*(56*c*C*d^3 - 3*c*d*e*(7*B*d -
2*A*e) + 3*a*e^2*(6*C*d - B*e))*(d + e*x)^5)/(5*e^9) + (c^2*(28*c*C*d^2 + 3
*a*C*e^2 - c*e*(7*B*d - A*e))*(d + e*x)^6)/(6*e^9) - (c^3*(8*C*d - B*e)*(d
+ e*x)^7)/(7*e^9) + (c^3*C*(d + e*x)^8)/(8*e^9) + ((c*d^2 + a*e^2)^3*(C*d^2
 - B*d*e + A*e^2)*Log[d + e*x])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{d+ex}\,dx = \int \left(\frac{\left(cd^2+ae^2\right)^2\left(-8cCd^3+cde(7Bd-6Ae)-ae^2(2Cd-Be)\right)}{e^8}+\frac{\left(cd^2+a\right.}{}\right.$$

$$= -\frac{\left(cd^2+ae^2\right)^2\left(8cCd^3-cde(7Bd-6Ae)+ae^2(2Cd-Be)\right)x}{e^8}+\frac{\left(cd^2+ae^2\right)}{}$$

Mathematica [A] time = 0.47, size = 498, normalized size = 1.02

$$\frac{x\left(420a^3e^6(2Be-2Cd+Cex)+210a^2ce^4\left(2e\left(3Ae(ex-2d)+B\left(6d^2-3dex+2e^2x^2\right)\right)+C\left(-12d^3+6d^2ex-4de\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] (x*(420*a^3*e^6*(-2*C*d + 2*B*e + C*e*x) + 210*a^2*c*e^4*(C*(-12*d^3 + 6*d^
2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 2*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d
*e*x + 2*e^2*x^2))) + 42*a*c^2*e^2*(C*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^
2 + 15*d^2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + e*(5*A*e*(-12*d^3 + 6*d^2
*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + B*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 -
 15*d*e^3*x^3 + 12*e^4*x^4))) + c^3*(C*(-840*d^7 + 420*d^6*e*x - 280*d^5*e^
2*x^2 + 210*d^4*e^3*x^3 - 168*d^3*e^4*x^4 + 140*d^2*e^5*x^5 - 120*d*e^6*x^6
 + 105*e^7*x^7) + 2*e*(7*A*e*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d^
2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + B*(420*d^6 - 210*d^5*e*x + 140*d^4
*e^2*x^2 - 105*d^3*e^3*x^3 + 84*d^2*e^4*x^4 - 70*d*e^5*x^5 + 60*e^6*x^6))))
)/(840*e^8) + ((c*d^2 + a*e^2)^3*(C*d^2 + e*(-(B*d) + A*e))*Log[d + e*x])/e
^9
```

fricas [A] time = 0.87, size = 674, normalized size = 1.38

$$\frac{105\,Cc^3e^8x^8-120\left(Cc^3de^7-Bc^3e^8\right)x^7+140\left(Cc^3d^2e^6-Bc^3de^7+\left(3\,Cac^2+Ac^3\right)e^8\right)x^6-168\left(Cc^3d^3e^5-Bc^3d^2e^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/840*(105*C*c^3*e^8*x^8 - 120*(C*c^3*d*e^7 - B*c^3*e^8)*x^7 + 140*(C*c^3*d
^2*e^6 - B*c^3*d*e^7 + (3*C*a*c^2 + A*c^3)*e^8)*x^6 - 168*(C*c^3*d^3*e^5 -
B*c^3*d^2*e^6 - 3*B*a*c^2*e^8 + (3*C*a*c^2 + A*c^3)*d*e^7)*x^5 + 210*(C*c^3
*d^4*e^4 - B*c^3*d^3*e^5 - 3*B*a*c^2*d*e^7 + (3*C*a*c^2 + A*c^3)*d^2*e^6 +
3*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 280*(C*c^3*d^5*e^3 - B*c^3*d^4*e^4 - 3*B*a
*c^2*d^2*e^6 - 3*B*a^2*c*e^8 + (3*C*a*c^2 + A*c^3)*d^3*e^5 + 3*(C*a^2*c + A
*a*c^2)*d*e^7)*x^3 + 420*(C*c^3*d^6*e^2 - B*c^3*d^5*e^3 - 3*B*a*c^2*d^3*e^5
 - 3*B*a^2*c*d*e^7 + (3*C*a*c^2 + A*c^3)*d^4*e^4 + 3*(C*a^2*c + A*a*c^2)*d^
2*e^6 + (C*a^3 + 3*A*a^2*c)*e^8)*x^2 - 840*(C*c^3*d^7*e - B*c^3*d^6*e^2 - 3
*B*a*c^2*d^4*e^4 - 3*B*a^2*c*d^2*e^6 - B*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^5*
e^3 + 3*(C*a^2*c + A*a*c^2)*d^3*e^5 + (C*a^3 + 3*A*a^2*c)*d*e^7)*x + 840*(C
*c^3*d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 - B*a^3*d*e^
7 + A*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a*c^2)*d^4*e^4
 + (C*a^3 + 3*A*a^2*c)*d^2*e^6)*log(e*x + d))/e^9
```

giac [A] time = 0.17, size = 764, normalized size = 1.56

$$\left(Cc^3d^8-Bc^3d^7e+3\,Cac^2d^6e^2+Ac^3d^6e^2-3\,Bac^2d^5e^3+3\,Ca^2cd^4e^4+3\,Aac^2d^4e^4-3\,Ba^2cd^3e^5+Ca^3d^2e^6+3\,Aa\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

```
[Out] (C*c^3*d^8 - B*c^3*d^7*e + 3*C*a*c^2*d^6*e^2 + A*c^3*d^6*e^2 - 3*B*a*c^2*d^
5*e^3 + 3*C*a^2*c*d^4*e^4 + 3*A*a*c^2*d^4*e^4 - 3*B*a^2*c*d^3*e^5 + C*a^3*d
^2*e^6 + 3*A*a^2*c*d^2*e^6 - B*a^3*d*e^7 + A*a^3*e^8)*e^(-9)*log(abs(x*e +
d)) + 1/840*(105*C*c^3*x^8*e^7 - 120*C*c^3*d*x^7*e^6 + 140*C*c^3*d^2*x^6*e^
5 - 168*C*c^3*d^3*x^5*e^4 + 210*C*c^3*d^4*x^4*e^3 - 280*C*c^3*d^5*x^3*e^2 +
 420*C*c^3*d^6*x^2*e - 840*C*c^3*d^7*x + 120*B*c^3*x^7*e^7 - 140*B*c^3*d*x^
```

```
6*e^6 + 168*B*c^3*d^2*x^5*e^5 - 210*B*c^3*d^3*x^4*e^4 + 280*B*c^3*d^4*x^3*e
^3 - 420*B*c^3*d^5*x^2*e^2 + 840*B*c^3*d^6*x*e + 420*C*a*c^2*x^6*e^7 + 140*
A*c^3*x^6*e^7 - 504*C*a*c^2*d*x^5*e^6 - 168*A*c^3*d*x^5*e^6 + 630*C*a*c^2*d
^2*x^4*e^5 + 210*A*c^3*d^2*x^4*e^5 - 840*C*a*c^2*d^3*x^3*e^4 - 280*A*c^3*d^
3*x^3*e^4 + 1260*C*a*c^2*d^4*x^2*e^3 + 420*A*c^3*d^4*x^2*e^3 - 2520*C*a*c^2
*d^5*x*e^2 - 840*A*c^3*d^5*x*e^2 + 504*B*a*c^2*x^5*e^7 - 630*B*a*c^2*d*x^4*
e^6 + 840*B*a*c^2*d^2*x^3*e^5 - 1260*B*a*c^2*d^3*x^2*e^4 + 2520*B*a*c^2*d^4
*x*e^3 + 630*C*a^2*c*x^4*e^7 + 630*A*a*c^2*x^4*e^7 - 840*C*a^2*c*d*x^3*e^6
- 840*A*a*c^2*d*x^3*e^6 + 1260*C*a^2*c*d^2*x^2*e^5 + 1260*A*a*c^2*d^2*x^2*e
^5 - 2520*C*a^2*c*d^3*x*e^4 - 2520*A*a*c^2*d^3*x*e^4 + 840*B*a^2*c*x^3*e^7
- 1260*B*a^2*c*d*x^2*e^6 + 2520*B*a^2*c*d^2*x*e^5 + 420*C*a^3*x^2*e^7 + 126
0*A*a^2*c*x^2*e^7 - 840*C*a^3*d*x*e^6 - 2520*A*a^2*c*d*x*e^6 + 840*B*a^3*x*
e^7)*e^(-8)
```

maple [A] time = 0.01, size = 880, normalized size = 1.80

$$\frac{Cc^3x^8}{8e}+\frac{Bc^3x^7}{7e}-\frac{Cc^3dx^7}{7e^2}+\frac{Ac^3x^6}{6e}-\frac{Bc^3dx^6}{6e^2}+\frac{Cac^2x^6}{2e}+\frac{Cc^3d^2x^6}{6e^3}-\frac{Ac^3dx^5}{5e^2}+\frac{3Bac^2x^5}{5e}+\frac{Bc^3d^2x^5}{5e^3}-\frac{3Cac^2dx^5}{5e^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x)
```

```
[Out] 3/e^3*ln(e*x+d)*A*a^2*c*d^2+3/e^5*ln(e*x+d)*A*a*c^2*d^4-3/e^4*ln(e*x+d)*B*a
^2*c*d^3-3/e^4*A*x*a*c^2*d^3-3/e^6*ln(e*x+d)*B*a*c^2*d^5+3/e^5*ln(e*x+d)*C*
a^2*c*d^4+3/e^7*ln(e*x+d)*C*a*c^2*d^6+3/e^3*B*x*a^2*c*d^2+3/e^5*B*x*a*c^2*d
^4-3/e^2*A*x*a^2*c*d-3/e^4*C*x*a^2*c*d^3-3/e^6*C*x*a*c^2*d^5-3/5/e^2*C*x^5*
a*c^2*d-3/4/e^2*B*x^4*a*c^2*d+3/4/e^3*C*x^4*a*c^2*d^2-1/e^2*A*x^3*a*c^2*d-1
/e^2*C*x^3*a^2*c*d-1/e^4*C*x^3*a*c^2*d^3+3/2/e^3*A*x^2*a*c^2*d^2-3/2/e^2*B*
x^2*a^2*c*d-3/2/e^4*B*x^2*a*c^2*d^3+3/2/e^3*C*x^2*a^2*c*d^2+3/2/e^5*C*x^2*a
*c^2*d^4+1/e^3*B*x^3*a*c^2*d^2+1/8/e*C*c^3*x^8+1/7/e*B*x^7*c^3+1/6/e*A*x^6*
c^3+1/e*B*x*a^3+1/e*ln(e*x+d)*A*a^3+1/2/e*C*x^2*a^3+1/2/e*C*x^6*a*c^2+1/4/e
^5*C*x^4*c^3*d^4-1/2/e^6*B*x^2*c^3*d^5+1/2/e^7*C*x^2*c^3*d^6+1/e*B*x^3*a^2*
c+3/5/e*B*x^5*a*c^2+1/5/e^3*B*x^5*c^3*d^2-1/5/e^4*C*x^5*c^3*d^3+3/4/e*A*x^4
*a*c^2+1/4/e^3*A*x^4*c^3*d^2-1/4/e^4*B*x^4*c^3*d^3+3/4/e*C*x^4*a^2*c+1/e^7*
ln(e*x+d)*A*c^3*d^6-1/e^2*ln(e*x+d)*B*a^3*d-1/e^8*ln(e*x+d)*B*c^3*d^7+1/e^3
*ln(e*x+d)*C*a^3*d^2+1/e^9*ln(e*x+d)*C*c^3*d^8-1/e^6*A*x*c^3*d^5+1/e^7*B*x*
c^3*d^6-1/6/e^2*B*x^6*c^3*d+1/6/e^3*C*x^6*c^3*d^2-1/5/e^2*A*x^5*c^3*d-1/7/e
^2*C*x^7*c^3*d-1/3/e^4*A*x^3*c^3*d^3+1/3/e^5*B*x^3*c^3*d^4-1/3/e^6*C*x^3*c^
3*d^5+3/2/e*A*x^2*a^2*c+1/2/e^5*A*x^2*c^3*d^4-1/e^2*C*x*a^3*d-1/e^8*C*x*c^3
*d^7
```

maxima [A] time = 0.49, size = 672, normalized size = 1.37

$$\frac{105\,Cc^3e^7x^8-120\left(Cc^3de^6-Bc^3e^7\right)x^7+140\left(Cc^3d^2e^5-Bc^3de^6+\left(3\,Cac^2+Ac^3\right)e^7\right)x^6-168\left(Cc^3d^3e^4-Bc^3d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/840*(105*C*c^3*e^7*x^8 - 120*(C*c^3*d*e^6 - B*c^3*e^7)*x^7 + 140*(C*c^3*d
^2*e^5 - B*c^3*d*e^6 + (3*C*a*c^2 + A*c^3)*e^7)*x^6 - 168*(C*c^3*d^3*e^4 -
B*c^3*d^2*e^5 - 3*B*a*c^2*e^7 + (3*C*a*c^2 + A*c^3)*d*e^6)*x^5 + 210*(C*c^3
*d^4*e^3 - B*c^3*d^3*e^4 - 3*B*a*c^2*d*e^6 + (3*C*a*c^2 + A*c^3)*d^2*e^5 +
3*(C*a^2*c + A*a*c^2)*e^7)*x^4 - 280*(C*c^3*d^5*e^2 - B*c^3*d^4*e^3 - 3*B*a
*c^2*d^2*e^5 - 3*B*a^2*c*e^7 + (3*C*a*c^2 + A*c^3)*d^3*e^4 + 3*(C*a^2*c + A
*a*c^2)*d*e^6)*x^3 + 420*(C*c^3*d^6*e - B*c^3*d^5*e^2 - 3*B*a*c^2*d^3*e^4 -
 3*B*a^2*c*d*e^6 + (3*C*a*c^2 + A*c^3)*d^4*e^3 + 3*(C*a^2*c + A*a*c^2)*d^2*
e^5 + (C*a^3 + 3*A*a^2*c)*e^7)*x^2 - 840*(C*c^3*d^7 - B*c^3*d^6*e - 3*B*a*c
^2*d^4*e^3 - 3*B*a^2*c*d^2*e^5 - B*a^3*e^7 + (3*C*a*c^2 + A*c^3)*d^5*e^2 +
```

```
3*(C*a^2*c + A*a*c^2)*d^3*e^4 + (C*a^3 + 3*A*a^2*c)*d*e^6)*x)/e^8 + (C*c^3*
d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 - B*a^3*d*e^7 + A
*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a*c^2)*d^4*e^4 + (C
*a^3 + 3*A*a^2*c)*d^2*e^6)*log(e*x + d)/e^9
```

mupad [B] time = 3.88, size = 741, normalized size = 1.51

$$x\left(\frac{B\,a^3}{e}-\frac{d\left(\frac{C\,a^3+3\,A\,c\,a^2}{e}+\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{A\,c^3+3\,C\,a\,c^2}{e}-\frac{d\left(\frac{B\,c^3}{e}-\frac{C\,c^3\,d}{e^2}\right)}{e}\right)}{e}-\frac{3\,B\,a\,c^2}{e}\right)}{e}+\frac{3\,a\,c\,(A\,c+C\,a)}{e}\right)}{e}-\frac{3\,B\,a^2\,c}{e}\right)}{e}\right)}{e}\right)+x^7\,\left(\frac{B\,c^3}{7\,e}-\frac{C\,c^3\,d}{7\,e^2}\right)-x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x)
```

```
[Out] x*((B*a^3)/e - (d*((C*a^3 + 3*A*a^2*c)/e + (d*((d*((d*((d*((A*c^3 + 3*C*a*c
^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/e + (3*a*c*
(A*c + C*a))/e))/e - (3*B*a^2*c)/e))/e))/e) + x^7*((B*c^3)/(7*e) - (C*c^3*d
)/(7*e^2)) - x^5*((d*((A*c^3 + 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2
))/e))/(5*e) - (3*B*a*c^2)/(5*e)) + x^4*((d*((d*((A*c^3 + 3*C*a*c^2)/e - (d
*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/(4*e) + (3*a*c*(A*c +
 C*a))/(4*e)) + x^2*((C*a^3 + 3*A*a^2*c)/(2*e) + (d*((d*((d*((d*((A*c^3 + 3
*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/e + (
3*a*c*(A*c + C*a))/e))/e - (3*B*a^2*c)/e))/(2*e)) + x^6*((A*c^3 + 3*C*a*c^2
)/(6*e) - (d*((B*c^3)/e - (C*c^3*d)/e^2))/(6*e)) - x^3*((d*((d*((d*((A*c^3
+ 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/e
+ (3*a*c*(A*c + C*a))/e))/(3*e) - (B*a^2*c)/e) + (log(d + e*x)*(A*a^3*e^8 +
 C*c^3*d^8 - B*a^3*d*e^7 - B*c^3*d^7*e + A*c^3*d^6*e^2 + C*a^3*d^2*e^6 + 3*
A*a*c^2*d^4*e^4 + 3*A*a^2*c*d^2*e^6 - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5
 + 3*C*a*c^2*d^6*e^2 + 3*C*a^2*c*d^4*e^4))/e^9 + (C*c^3*x^8)/(8*e)
```

sympy [A] time = 1.47, size = 685, normalized size = 1.40

$$\frac{Cc^3x^8}{8e}+x^7\left(\frac{Bc^3}{7e}-\frac{Cc^3d}{7e^2}\right)+x^6\left(\frac{Ac^3}{6e}-\frac{Bc^3d}{6e^2}+\frac{Cac^2}{2e}+\frac{Cc^3d^2}{6e^3}\right)+x^5\left(-\frac{Ac^3d}{5e^2}+\frac{3Bac^2}{5e}+\frac{Bc^3d^2}{5e^3}-\frac{3Cac^2d}{5e^2}-\frac{Cc^3d^3}{5e^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d),x)
```

```
[Out] C*c**3*x**8/(8*e) + x**7*(B*c**3/(7*e) - C*c**3*d/(7*e**2)) + x**6*(A*c**3/
(6*e) - B*c**3*d/(6*e**2) + C*a*c**2/(2*e) + C*c**3*d**2/(6*e**3)) + x**5*(
-A*c**3*d/(5*e**2) + 3*B*a*c**2/(5*e) + B*c**3*d**2/(5*e**3) - 3*C*a*c**2*d
/(5*e**2) - C*c**3*d**3/(5*e**4)) + x**4*(3*A*a*c**2/(4*e) + A*c**3*d**2/(4
*e**3) - 3*B*a*c**2*d/(4*e**2) - B*c**3*d**3/(4*e**4) + 3*C*a**2*c/(4*e) +
3*C*a*c**2*d**2/(4*e**3) + C*c**3*d**4/(4*e**5)) + x**3*(-A*a*c**2*d/e**2 -
 A*c**3*d**3/(3*e**4) + B*a**2*c/e + B*a*c**2*d**2/e**3 + B*c**3*d**4/(3*e*
*5) - C*a**2*c*d/e**2 - C*a*c**2*d**3/e**4 - C*c**3*d**5/(3*e**6)) + x**2*(
3*A*a**2*c/(2*e) + 3*A*a*c**2*d**2/(2*e**3) + A*c**3*d**4/(2*e**5) - 3*B*a*
*2*c*d/(2*e**2) - 3*B*a*c**2*d**3/(2*e**4) - B*c**3*d**5/(2*e**6) + C*a**3/
(2*e) + 3*C*a**2*c*d**2/(2*e**3) + 3*C*a*c**2*d**4/(2*e**5) + C*c**3*d**6/(
2*e**7)) + x*(-3*A*a**2*c*d/e**2 - 3*A*a*c**2*d**3/e**4 - A*c**3*d**5/e**6
+ B*a**3/e + 3*B*a**2*c*d**2/e**3 + 3*B*a*c**2*d**4/e**5 + B*c**3*d**6/e**7
 - C*a**3*d/e**2 - 3*C*a**2*c*d**3/e**4 - 3*C*a*c**2*d**5/e**6 - C*c**3*d**
7/e**8) + (a*e**2 + c*d**2)**3*(A*e**2 - B*d*e + C*d**2)*log(d + e*x)/e**9
```

3.37 $\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^2} dx$

Optimal. Leaf size=486

$$\frac{cx^3 \left(3a^2Ce^4 + 3ace^2 \left(3Cd^2 - e(2Bd - Ae)\right) + c^2d^2 \left(5Cd^2 - e(4Bd - 3Ae)\right)\right)}{3e^6} - \frac{cx^2 \left(3a^2e^4(2Cd - Be) + 3acde^2 (4\ldots\right.}{}$$

```
[Out] (a^3*C*e^6+c^3*d^4*(7*C*d^2-e*(-5*A*e+6*B*d))+3*a*c^2*d^2*e^2*(5*C*d^2-e*(-
3*A*e+4*B*d))+3*a^2*c*e^4*(3*C*d^2-e*(-A*e+2*B*d)))*x/e^8-1/2*c*(3*a^2*e^4*
(-B*e+2*C*d)+c^2*d^3*(6*C*d^2-e*(-4*A*e+5*B*d))+3*a*c*d*e^2*(4*C*d^2-e*(-2*
A*e+3*B*d)))*x^2/e^7+1/3*c*(3*a^2*C*e^4+c^2*d^2*(5*C*d^2-e*(-3*A*e+4*B*d))+
3*a*c*e^2*(3*C*d^2-e*(-A*e+2*B*d)))*x^3/e^6-1/4*c^2*(3*a*e^2*(-B*e+2*C*d)+c
*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*x^4/e^5+1/5*c^2*(3*a*C*e^2+c*(3*C*d^2-e*(-A*
e+2*B*d)))*x^5/e^4-1/6*c^3*(-B*e+2*C*d)*x^6/e^3+1/7*c^3*C*x^7/e^2-(a*e^2+c*
d^2)^3*(A*e^2-B*d*e+C*d^2)/e^9/(e*x+d)-(a*e^2+c*d^2)^2*(a*e^2*(-B*e+2*C*d)+
c*d*(8*C*d^2-e*(-6*A*e+7*B*d)))*ln(e*x+d)/e^9
```

Rubi [A] time = 0.98, antiderivative size = 483, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1628}

$$\frac{cx^3 \left(3a^2Ce^4 + 3ace^2 \left(3Cd^2 - e(2Bd - Ae)\right) + c^2 \left(5Cd^4 - d^2e(4Bd - 3Ae)\right)\right)}{3e^6} - \frac{cx^2 \left(3a^2e^4(2Cd - Be) + 3acde^2 (4\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] ((a^3*C*e^6 + c^3*(7*C*d^6 - d^4*e*(6*B*d - 5*A*e)) + 3*a*c^2*d^2*e^2*(5*C*
d^2 - e*(4*B*d - 3*A*e)) + 3*a^2*c*e^4*(3*C*d^2 - e*(2*B*d - A*e)))*x)/e^8
- (c*(3*a^2*e^4*(2*C*d - B*e) + c^2*(6*C*d^5 - d^3*e*(5*B*d - 4*A*e)) + 3*a
*c*d*e^2*(4*C*d^2 - e*(3*B*d - 2*A*e)))*x^2)/(2*e^7) + (c*(3*a^2*C*e^4 + c^
2*(5*C*d^4 - d^2*e*(4*B*d - 3*A*e)) + 3*a*c*e^2*(3*C*d^2 - e*(2*B*d - A*e))
)*x^3)/(3*e^6) - (c^2*(4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + 3*a*e^2*(2*C*d -
 B*e))*x^4)/(4*e^5) + (c^2*(3*c*C*d^2 + 3*a*C*e^2 - c*e*(2*B*d - A*e))*x^5)
/(5*e^4) - (c^3*(2*C*d - B*e)*x^6)/(6*e^3) + (c^3*C*x^7)/(7*e^2) - ((c*d^2
+ a*e^2)^3*(C*d^2 - B*d*e + A*e^2))/(e^9*(d + e*x)) - ((c*d^2 + a*e^2)^2*(8
*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e*x])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a + cx^2)^3 (A + Bx + Cx^2)}{(d + ex)^2} dx = \int \left(\frac{a^3Ce^6 + c^3 \left(7Cd^6 - d^4e(6Bd - 5Ae)\right) + 3ac^2d^2e^2 \left(5Cd^2 - e(4Bd - 3A\ldots\right.}{e^8} \right.$$

$$= \frac{\left(a^3Ce^6 + c^3 \left(7Cd^6 - d^4e(6Bd - 5Ae)\right) + 3ac^2d^2e^2 \left(5Cd^2 - e(4Bd - 3Ae)\right)\ldots\right.}{e^8}$$

Mathematica [A] time = 0.40, size = 641, normalized size = 1.32

$$\frac{420a^3e^6 \left(e(Bd - Ae) + C \left(-d^2 + dex + e^2x^2\right)\right) + 210a^2ce^4 \left(3e \left(2Ae \left(-d^2 + dex + e^2x^2\right) + B \left(2d^3 - 4d^2ex - 3de^2\ldots\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] (420*a^3*e^6*(e*(B*d - A*e) + C*(-d^2 + d*e*x + e^2*x^2)) + 210*a^2*c*e^4*(
2*C*(-3*d^4 + 9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + 3*e*(2*A
*e*(-d^2 + d*e*x + e^2*x^2) + B*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x^3)
)) + 21*a*c^2*e^2*(-6*C*(10*d^6 - 50*d^5*e*x - 30*d^4*e^2*x^2 + 10*d^3*e^3*
x^3 - 5*d^2*e^4*x^4 + 3*d*e^5*x^5 - 2*e^6*x^6) + 5*e*(4*A*e*(-3*d^4 + 9*d^3
*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + B*(12*d^5 - 48*d^4*e*x - 30
*d^3*e^2*x^2 + 10*d^2*e^3*x^3 - 5*d*e^4*x^4 + 3*e^5*x^5))) + c^3*(-4*C*(105
*d^8 - 735*d^7*e*x - 420*d^6*e^2*x^2 + 140*d^5*e^3*x^3 - 70*d^4*e^4*x^4 + 4
2*d^3*e^5*x^5 - 28*d^2*e^6*x^6 + 20*d*e^7*x^7 - 15*e^8*x^8) + 7*e*(6*A*e*(-
10*d^6 + 50*d^5*e*x + 30*d^4*e^2*x^2 - 10*d^3*e^3*x^3 + 5*d^2*e^4*x^4 - 3*d
*e^5*x^5 + 2*e^6*x^6) + B*(60*d^7 - 360*d^6*e*x - 210*d^5*e^2*x^2 + 70*d^4*
e^3*x^3 - 35*d^3*e^4*x^4 + 21*d^2*e^5*x^5 - 14*d*e^6*x^6 + 10*e^7*x^7))) -
420*(c*d^2 + a*e^2)^2*(8*c*C*d^3 + c*d*e*(-7*B*d + 6*A*e) + a*e^2*(2*C*d -
B*e))*(d + e*x)*Log[d + e*x])/(420*e^9*(d + e*x))
```

fricas [A] time = 0.93, size = 932, normalized size = 1.92

$$\frac{60\,Cc^3e^8x^8 - 420\,Cc^3d^8 + 420\,Bc^3d^7e + 1260\,Bac^2d^5e^3 + 1260\,Ba^2cd^3e^5 + 420\,Ba^3de^7 - 420\,Aa^3e^8 - 420\left(3\,Cac^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/420*(60*C*c^3*e^8*x^8 - 420*C*c^3*d^8 + 420*B*c^3*d^7*e + 1260*B*a*c^2*d^
5*e^3 + 1260*B*a^2*c*d^3*e^5 + 420*B*a^3*d*e^7 - 420*A*a^3*e^8 - 420*(3*C*a
*c^2 + A*c^3)*d^6*e^2 - 1260*(C*a^2*c + A*a*c^2)*d^4*e^4 - 420*(C*a^3 + 3*A
*a^2*c)*d^2*e^6 - 10*(8*C*c^3*d*e^7 - 7*B*c^3*e^8)*x^7 + 14*(8*C*c^3*d^2*e^
6 - 7*B*c^3*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*e^8)*x^6 - 21*(8*C*c^3*d^3*e^5 -
7*B*c^3*d^2*e^6 - 15*B*a*c^2*e^8 + 6*(3*C*a*c^2 + A*c^3)*d*e^7)*x^5 + 35*(8
*C*c^3*d^4*e^4 - 7*B*c^3*d^3*e^5 - 15*B*a*c^2*d*e^7 + 6*(3*C*a*c^2 + A*c^3)
*d^2*e^6 + 12*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 70*(8*C*c^3*d^5*e^3 - 7*B*c^3*
d^4*e^4 - 15*B*a*c^2*d^2*e^6 - 9*B*a^2*c*e^8 + 6*(3*C*a*c^2 + A*c^3)*d^3*e^
5 + 12*(C*a^2*c + A*a*c^2)*d*e^7)*x^3 + 210*(8*C*c^3*d^6*e^2 - 7*B*c^3*d^5*
e^3 - 15*B*a*c^2*d^3*e^5 - 9*B*a^2*c*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^4*e^4
+ 12*(C*a^2*c + A*a*c^2)*d^2*e^6 + 2*(C*a^3 + 3*A*a^2*c)*e^8)*x^2 + 420*(7*
C*c^3*d^7*e - 6*B*c^3*d^6*e^2 - 12*B*a*c^2*d^4*e^4 - 6*B*a^2*c*d^2*e^6 + 5*
(3*C*a*c^2 + A*c^3)*d^5*e^3 + 9*(C*a^2*c + A*a*c^2)*d^3*e^5 + (C*a^3 + 3*A*
a^2*c)*d*e^7)*x - 420*(8*C*c^3*d^8 - 7*B*c^3*d^7*e - 15*B*a*c^2*d^5*e^3 - 9
*B*a^2*c*d^3*e^5 - B*a^3*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 12*(C*a^2*
c + A*a*c^2)*d^4*e^4 + 2*(C*a^3 + 3*A*a^2*c)*d^2*e^6 + (8*C*c^3*d^7*e - 7*B
*c^3*d^6*e^2 - 15*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*e^6 - B*a^3*e^8 + 6*(3*C*
a*c^2 + A*c^3)*d^5*e^3 + 12*(C*a^2*c + A*a*c^2)*d^3*e^5 + 2*(C*a^3 + 3*A*a^
2*c)*d*e^7)*x)*log(e*x + d))/(e^10*x + d*e^9)
```

giac [A] time = 0.20, size = 838, normalized size = 1.72

$$\frac{1}{420}\left(60\,Cc^3 - \frac{70\left(8\,Cc^3de - Bc^3e^2\right)e^{(-1)}}{xe+d} + \frac{84\left(28\,Cc^3d^2e^2 - 7\,Bc^3de^3 + 3\,Cac^2e^4 + Ac^3e^4\right)e^{(-2)}}{(xe+d)^2} - \frac{105\left(56\,Cc^3d^3e\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/420*(60*C*c^3 - 70*(8*C*c^3*d*e - B*c^3*e^2)*e^(-1)/(x*e + d) + 84*(28*C*
c^3*d^2*e^2 - 7*B*c^3*d*e^3 + 3*C*a*c^2*e^4 + A*c^3*e^4)*e^(-2)/(x*e + d)^2
 - 105*(56*C*c^3*d^3*e^3 - 21*B*c^3*d^2*e^4 + 18*C*a*c^2*d*e^5 + 6*A*c^3*d*
```

```
e^5 - 3*B*a*c^2*e^6)*e^(-3)/(x*e + d)^3 + 140*(70*C*c^3*d^4*e^4 - 35*B*c^3*
d^3*e^5 + 45*C*a*c^2*d^2*e^6 + 15*A*c^3*d^2*e^6 - 15*B*a*c^2*d*e^7 + 3*C*a^
2*c*e^8 + 3*A*a*c^2*e^8)*e^(-4)/(x*e + d)^4 - 210*(56*C*c^3*d^5*e^5 - 35*B*
c^3*d^4*e^6 + 60*C*a*c^2*d^3*e^7 + 20*A*c^3*d^3*e^7 - 30*B*a*c^2*d^2*e^8 +
12*C*a^2*c*d*e^9 + 12*A*a*c^2*d*e^9 - 3*B*a^2*c*e^10)*e^(-5)/(x*e + d)^5 +
420*(28*C*c^3*d^6*e^6 - 21*B*c^3*d^5*e^7 + 45*C*a*c^2*d^4*e^8 + 15*A*c^3*d^
4*e^8 - 30*B*a*c^2*d^3*e^9 + 18*C*a^2*c*d^2*e^10 + 18*A*a*c^2*d^2*e^10 - 9*
B*a^2*c*d*e^11 + C*a^3*e^12 + 3*A*a^2*c*e^12)*e^(-6)/(x*e + d)^6)*(x*e + d)
^7*e^(-9) + (8*C*c^3*d^7 - 7*B*c^3*d^6*e + 18*C*a*c^2*d^5*e^2 + 6*A*c^3*d^5
*e^2 - 15*B*a*c^2*d^4*e^3 + 12*C*a^2*c*d^3*e^4 + 12*A*a*c^2*d^3*e^4 - 9*B*a
^2*c*d^2*e^5 + 2*C*a^3*d*e^6 + 6*A*a^2*c*d*e^6 - B*a^3*e^7)*e^(-9)*log(abs(
x*e + d)*e^(-1)/(x*e + d)^2) - (C*c^3*d^8*e^7/(x*e + d) - B*c^3*d^7*e^8/(x*
e + d) + 3*C*a*c^2*d^6*e^9/(x*e + d) + A*c^3*d^6*e^9/(x*e + d) - 3*B*a*c^2*
d^5*e^10/(x*e + d) + 3*C*a^2*c*d^4*e^11/(x*e + d) + 3*A*a*c^2*d^4*e^11/(x*e
 + d) - 3*B*a^2*c*d^3*e^12/(x*e + d) + C*a^3*d^2*e^13/(x*e + d) + 3*A*a^2*c
*d^2*e^13/(x*e + d) - B*a^3*d*e^14/(x*e + d) + A*a^3*e^15/(x*e + d))*e^(-16
)
```

maple [A] time = 0.02, size = 928, normalized size = 1.91

$$\frac{C\,c^3x^7}{7e^2}+\frac{B\,c^3x^6}{6e^2}-\frac{C\,c^3d\,x^6}{3e^3}+\frac{A\,c^3x^5}{5e^2}-\frac{2B\,c^3d\,x^5}{5e^3}+\frac{3Ca\,c^2x^5}{5e^2}+\frac{3C\,c^3d^2x^5}{5e^4}-\frac{A\,c^3d\,x^4}{2e^3}+\frac{3Ba\,c^2x^4}{4e^2}+\frac{3B\,c^3d^2x^4}{4e^4}-\frac{3Ca\,c^2}{2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x)
```

```
[Out] 1/7*c^3*C*x^7/e^2-1/e/(e*x+d)*A*a^3+1/e^2*ln(e*x+d)*B*a^3+1/e^2*a^3*C*x+1/5
/e^2*A*x^5*c^3+1/6/e^2*B*x^6*c^3-3/e^3*A*x^2*a*c^2*d+9/2/e^4*B*x^2*a*c^2*d^
2-3/e^3*C*x^2*a^2*c*d-3/e^3/(e*x+d)*A*a^2*c*d^2-3/e^5/(e*x+d)*A*a*c^2*d^4+3
/e^4/(e*x+d)*B*a^2*c*d^3+3/e^6/(e*x+d)*B*a*c^2*d^5-3/e^5/(e*x+d)*C*a^2*c*d^
4-3/e^7/(e*x+d)*C*a*c^2*d^6-6/e^3*ln(e*x+d)*A*a^2*c*d-12/e^5*ln(e*x+d)*A*a*
c^2*d^3+9/e^4*ln(e*x+d)*B*a^2*c*d^2+15/e^6*ln(e*x+d)*B*a*c^2*d^4-12/e^5*ln(
e*x+d)*C*a^2*c*d^3-18/e^7*ln(e*x+d)*C*a*c^2*d^5-3/2/e^3*C*x^4*a*c^2*d-6/e^5
*C*x^2*a*c^2*d^3+9/e^4*A*a*c^2*d^2*x-6/e^3*d*a^2*c*B*x-12/e^5*B*a*c^2*d^3*x
+9/e^4*C*a^2*c*d^2*x+15/e^6*C*a*c^2*d^4*x-2/e^3*B*x^3*a*c^2*d+3/e^4*C*x^3*a
*c^2*d^2+3/5/e^2*C*x^5*a*c^2+3/5/e^4*C*x^5*c^3*d^2-1/2/e^3*A*x^4*c^3*d+3/4/
e^2*B*x^4*a*c^2+3/4/e^4*B*x^4*c^3*d^2-1/e^5*C*x^4*c^3*d^3-4/3/e^5*B*x^3*c^3
*d^3+5/3/e^6*C*x^3*c^3*d^4-2/e^5*A*x^2*c^3*d^3+3/2/e^2*B*x^2*a^2*c+5/2/e^6*
B*x^2*c^3*d^4-3/e^7*C*x^2*c^3*d^5+3/e^2*A*a^2*c*x+5/e^6*A*c^3*d^4*x-6/e^7*B
*c^3*d^5*x-1/3/e^3*C*x^6*c^3*d+1/e^2*C*x^3*a^2*c+1/e^2*A*x^3*a*c^2+1/e^4*A*
x^3*c^3*d^2-1/e^7/(e*x+d)*A*c^3*d^6+1/e^2/(e*x+d)*B*d*a^3+7/e^8*C*c^3*d^6*x
-2/5/e^3*B*x^5*c^3*d+1/e^8/(e*x+d)*B*c^3*d^7-1/e^3/(e*x+d)*C*a^3*d^2-1/e^9/
(e*x+d)*C*c^3*d^8-6/e^7*ln(e*x+d)*A*c^3*d^5+7/e^8*ln(e*x+d)*B*c^3*d^6-2/e^3
*ln(e*x+d)*C*a^3*d-8/e^9*ln(e*x+d)*C*c^3*d^7
```

maxima [A] time = 0.49, size = 691, normalized size = 1.42

$$-\frac{Cc^3d^8-Bc^3d^7e-3\,Bac^2d^5e^3-3\,Ba^2cd^3e^5-Ba^3de^7+Aa^3e^8+\left(3\,Cac^2+Ac^3\right)d^6e^2+3\left(Ca^2c+Aac^2\right)d^4e^4+}{e^{10}x+de^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(C*c^3*d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 - B*a^3*d
*e^7 + A*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a*c^2)*d^4*
e^4 + (C*a^3 + 3*A*a^2*c)*d^2*e^6)/(e^10*x + d*e^9) + 1/420*(60*C*c^3*e^6*x
^7 - 70*(2*C*c^3*d*e^5 - B*c^3*e^6)*x^6 + 84*(3*C*c^3*d^2*e^4 - 2*B*c^3*d*e
^5 + (3*C*a*c^2 + A*c^3)*e^6)*x^5 - 105*(4*C*c^3*d^3*e^3 - 3*B*c^3*d^2*e^4
- 3*B*a*c^2*e^6 + 2*(3*C*a*c^2 + A*c^3)*d*e^5)*x^4 + 140*(5*C*c^3*d^4*e^2 -
```

```
 4*B*c^3*d^3*e^3 - 6*B*a*c^2*d*e^5 + 3*(3*C*a*c^2 + A*c^3)*d^2*e^4 + 3*(C*a
^2*c + A*a*c^2)*e^6)*x^3 - 210*(6*C*c^3*d^5*e - 5*B*c^3*d^4*e^2 - 9*B*a*c^2
*d^2*e^4 - 3*B*a^2*c*e^6 + 4*(3*C*a*c^2 + A*c^3)*d^3*e^3 + 6*(C*a^2*c + A*a
*c^2)*d*e^5)*x^2 + 420*(7*C*c^3*d^6 - 6*B*c^3*d^5*e - 12*B*a*c^2*d^3*e^3 -
6*B*a^2*c*d*e^5 + 5*(3*C*a*c^2 + A*c^3)*d^4*e^2 + 9*(C*a^2*c + A*a*c^2)*d^2
*e^4 + (C*a^3 + 3*A*a^2*c)*e^6)*x)/e^8 - (8*C*c^3*d^7 - 7*B*c^3*d^6*e - 15*
B*a*c^2*d^4*e^3 - 9*B*a^2*c*d^2*e^5 - B*a^3*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^5
*e^2 + 12*(C*a^2*c + A*a*c^2)*d^3*e^4 + 2*(C*a^3 + 3*A*a^2*c)*d*e^6)*log(e*
x + d)/e^9
```

mupad [B] time = 3.99, size = 1511, normalized size = 3.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

```
[Out] x*((C*a^3 + 3*A*a^2*c)/e^2 + (2*d*((2*d*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3
*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*d*
((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3
*d^2)/e^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2
))/e + (3*a*c*(A*c + C*a))/e^2))/e + (d^2*((2*d*((2*d*((B*c^3)/e^2 - (2*C*c
^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c^
3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/e^2 - (3*B*a^2*c)/e^2))/
e - (d^2*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^
2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d
)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c^3)/e
^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/e + (3*a*c*(A*c + C*a))/e^2)
)/e^2) + x^4*((d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*
c^2)/e^2 + (C*c^3*d^2)/e^4))/(2*e) - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/
(4*e^2) + (3*B*a*c^2)/(4*e^2)) - x^2*((d*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^
3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*d
*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^
3*d^2)/e^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^
2))/e + (3*a*c*(A*c + C*a))/e^2))/e + (d^2*((2*d*((2*d*((B*c^3)/e^2 - (2*C*
c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c
^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/(2*e^2) - (3*B*a^2*c)/(
2*e^2)) + x^6*((B*c^3)/(6*e^2) - (C*c^3*d)/(3*e^3)) - x^5*((2*d*((B*c^3)/e^
2 - (2*C*c^3*d)/e^3))/(5*e) - (A*c^3 + 3*C*a*c^2)/(5*e^2) + (C*c^3*d^2)/(5*
e^4)) + x^3*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a
*c^2)/e^2 + (C*c^3*d^2)/e^4))/(3*e^2) - (2*d*((2*d*((2*d*((B*c^3)/e^2 - (2*
C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B
*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/(3*e) + (a*c*(A*c + C
*a))/e^2) - (A*a^3*e^8 + C*c^3*d^8 - B*a^3*d*e^7 - B*c^3*d^7*e + A*c^3*d^6*
e^2 + C*a^3*d^2*e^6 + 3*A*a*c^2*d^4*e^4 + 3*A*a^2*c*d^2*e^6 - 3*B*a*c^2*d^5
*e^3 - 3*B*a^2*c*d^3*e^5 + 3*C*a*c^2*d^6*e^2 + 3*C*a^2*c*d^4*e^4)/(e*(d*e^8
 + e^9*x)) - (log(d + e*x)*(8*C*c^3*d^7 - B*a^3*e^7 + 2*C*a^3*d*e^6 - 7*B*c
^3*d^6*e + 6*A*c^3*d^5*e^2 + 12*A*a*c^2*d^3*e^4 - 15*B*a*c^2*d^4*e^3 - 9*B*
a^2*c*d^2*e^5 + 18*C*a*c^2*d^5*e^2 + 12*C*a^2*c*d^3*e^4 + 6*A*a^2*c*d*e^6))
/e^9 + (C*c^3*x^7)/(7*e^2)
```

sympy [A] time = 4.95, size = 748, normalized size = 1.54

$$\frac{Cc^3x^7}{7e^2}+x^6\left(\frac{Bc^3}{6e^2}-\frac{Cc^3d}{3e^3}\right)+x^5\left(\frac{Ac^3}{5e^2}-\frac{2Bc^3d}{5e^3}+\frac{3Cac^2}{5e^2}+\frac{3Cc^3d^2}{5e^4}\right)+x^4\left(-\frac{Ac^3d}{2e^3}+\frac{3Bac^2}{4e^2}+\frac{3Bc^3d^2}{4e^4}-\frac{3Cac^2d}{2e^3}-\frac{C\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

```
[Out] C*c**3*x**7/(7*e**2) + x**6*(B*c**3/(6*e**2) - C*c**3*d/(3*e**3)) + x**5*(A
*c**3/(5*e**2) - 2*B*c**3*d/(5*e**3) + 3*C*a*c**2/(5*e**2) + 3*C*c**3*d**2/
(5*e**4)) + x**4*(-A*c**3*d/(2*e**3) + 3*B*a*c**2/(4*e**2) + 3*B*c**3*d**2/
(4*e**4) - 3*C*a*c**2*d/(2*e**3) - C*c**3*d**3/e**5) + x**3*(A*a*c**2/e**2
+ A*c**3*d**2/e**4 - 2*B*a*c**2*d/e**3 - 4*B*c**3*d**3/(3*e**5) + C*a**2*c/
e**2 + 3*C*a*c**2*d**2/e**4 + 5*C*c**3*d**4/(3*e**6)) + x**2*(-3*A*a*c**2*d
/e**3 - 2*A*c**3*d**3/e**5 + 3*B*a**2*c/(2*e**2) + 9*B*a*c**2*d**2/(2*e**4)
 + 5*B*c**3*d**4/(2*e**6) - 3*C*a**2*c*d/e**3 - 6*C*a*c**2*d**3/e**5 - 3*C*
c**3*d**5/e**7) + x*(3*A*a**2*c/e**2 + 9*A*a*c**2*d**2/e**4 + 5*A*c**3*d**4
/e**6 - 6*B*a**2*c*d/e**3 - 12*B*a*c**2*d**3/e**5 - 6*B*c**3*d**5/e**7 + C*
a**3/e**2 + 9*C*a**2*c*d**2/e**4 + 15*C*a*c**2*d**4/e**6 + 7*C*c**3*d**6/e*
*8) + (-A*a**3*e**8 - 3*A*a**2*c*d**2*e**6 - 3*A*a*c**2*d**4*e**4 - A*c**3*
d**6*e**2 + B*a**3*d*e**7 + 3*B*a**2*c*d**3*e**5 + 3*B*a*c**2*d**5*e**3 + B
*c**3*d**7*e - C*a**3*d**2*e**6 - 3*C*a**2*c*d**4*e**4 - 3*C*a*c**2*d**6*e*
*2 - C*c**3*d**8)/(d*e**9 + e**10*x) - (a*e**2 + c*d**2)**2*(6*A*c*d*e**2 -
 B*a*e**3 - 7*B*c*d**2*e + 2*C*a*d*e**2 + 8*C*c*d**3)*log(d + e*x)/e**9
```

3.38 $\int \frac{\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)}{(d+ex)^3}\,dx$

Optimal. Leaf size=466

$$\frac{\left(ae^2+cd^2\right)\log(d+ex)\left(a^2Ce^4+ace^2\left(17Cd^2-3e(3Bd-Ae)\right)+c^2d^2\left(28Cd^2-3e(7Bd-5Ae)\right)\right)}{e^9}+\frac{cx^2\left(3a^2Ce^4+\right.}{}$$

[Out] -c*(3*a^2*e^4*(-B*e+3*C*d)+c^2*d^3*(21*C*d^2-5*e*(-2*A*e+3*B*d))+3*a*c*d*e^2*(10*C*d^2-3*e*(-A*e+2*B*d)))*x/e^8+1/2*c*(3*a^2*C*e^4+c^2*d^2*(15*C*d^2-2*e*(-3*A*e+5*B*d))+3*a*c*e^2*(6*C*d^2-e*(-A*e+3*B*d)))*x^2/e^7-1/3*c^2*(3*a*e^2*(-B*e+3*C*d)+c*d*(10*C*d^2-3*e*(-A*e+2*B*d)))*x^3/e^6+1/4*c^2*(3*a*C*e^2+c*(6*C*d^2-e*(-A*e+3*B*d)))*x^4/e^5-1/5*c^3*(-B*e+3*C*d)*x^5/e^4+1/6*c^3*C*x^6/e^3-1/2*(a*e^2+c*d^2)^3*(A*e^2-B*d*e+C*d^2)/e^9/(e*x+d)^2+(a*e^2+c*d^2)^2*(a*e^2*(-B*e+2*C*d)+c*d*(8*C*d^2-e*(-6*A*e+7*B*d)))/e^9/(e*x+d)+(a*e^2+c*d^2)*(a^2*C*e^4+c^2*d^2*(28*C*d^2-3*e*(-5*A*e+7*B*d))+a*c*e^2*(17*C*d^2-3*e*(-A*e+3*B*d)))*ln(e*x+d)/e^9

Rubi [A] time = 0.97, antiderivative size = 463, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1628}

$$\frac{cx^2\left(3a^2Ce^4+3ace^2\left(6Cd^2-e(3Bd-Ae)\right)+c^2\left(15Cd^4-2d^2e(5Bd-3Ae)\right)\right)}{2e^7}-\frac{cx\left(3a^2e^4(3Cd-Be)+3acde^2\left(10C\right.\right.}{}$$

Antiderivative was successfully verified.

[In] Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x]

[Out] -((c*(3*a^2*e^4*(3*C*d - B*e) + c^2*(21*C*d^5 - 5*d^3*e*(3*B*d - 2*A*e)) + 3*a*c*d*e^2*(10*C*d^2 - 3*e*(2*B*d - A*e)))*x)/e^8) + (c*(3*a^2*C*e^4 + c^2*(15*C*d^4 - 2*d^2*e*(5*B*d - 3*A*e)) + 3*a*c*e^2*(6*C*d^2 - e*(3*B*d - A*e)))*x^2)/(2*e^7) - (c^2*(10*c*C*d^3 - 3*c*d*e*(2*B*d - A*e) + 3*a*e^2*(3*C*d - B*e))*x^3)/(3*e^6) + (c^2*(6*c*C*d^2 + 3*a*C*e^2 - c*e*(3*B*d - A*e))*x^4)/(4*e^5) - (c^3*(3*C*d - B*e)*x^5)/(5*e^4) + (c^3*C*x^6)/(6*e^3) - ((c*d^2 + a*e^2)^3*(C*d^2 - B*d*e + A*e^2))/(2*e^9*(d + e*x)^2) + ((c*d^2 + a*e^2)^2*(8*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*e)))/(e^9*(d + e*x)) + ((c*d^2 + a*e^2)*(a^2*C*e^4 + c^2*(28*C*d^4 - 3*d^2*e*(7*B*d - 5*A*e)) + a*c*e^2*(17*C*d^2 - 3*e*(3*B*d - A*e)))*Log[d + e*x])/e^9

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)}{(d+ex)^3}\,dx = \int \left(\frac{c\left(-3a^2e^4(3Cd-Be)-c^2\left(21Cd^5-5d^3e(3Bd-2Ae)\right)-3acde^2\left(10Cd^2-\right.\right.}{e^8}\right.$$

$$= -\frac{c\left(3a^2e^4(3Cd-Be)+c^2\left(21Cd^5-5d^3e(3Bd-2Ae)\right)+3acde^2\left(10Cd^2-3e(\right.\right.}{e^8}$$

Mathematica [A] time = 0.23, size = 438, normalized size = 0.94

$$\frac{30ce^2x^2\left(3a^2Ce^4+3ace^2\left(e(Ae-3Bd)+6Cd^2\right)+c^2\left(2d^2e(3Ae-5Bd)+15Cd^4\right)\right)+60\left(ae^2+cd^2\right)\log(d+ex)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] (-60*c*e*(-3*a^2*e^4*(-3*C*d + B*e) + 3*a*c*d*e^2*(10*C*d^2 + 3*e*(-2*B*d +
 A*e)) + c^2*(21*C*d^5 + 5*d^3*e*(-3*B*d + 2*A*e)))*x + 30*c*e^2*(3*a^2*C*e
^4 + 3*a*c*e^2*(6*C*d^2 + e*(-3*B*d + A*e)) + c^2*(15*C*d^4 + 2*d^2*e*(-5*B
*d + 3*A*e)))*x^2 - 20*c^2*e^3*(10*c*C*d^3 + 3*c*d*e*(-2*B*d + A*e) - 3*a*e
^2*(-3*C*d + B*e))*x^3 + 15*c^2*e^4*(6*c*C*d^2 + 3*a*C*e^2 + c*e*(-3*B*d +
A*e))*x^4 + 12*c^3*e^5*(-3*C*d + B*e)*x^5 + 10*c^3*C*e^6*x^6 - (30*(c*d^2 +
 a*e^2)^3*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2 + (60*(c*d^2 + a*e^2)^2*(
8*c*C*d^3 + c*d*e*(-7*B*d + 6*A*e) + a*e^2*(2*C*d - B*e)))/(d + e*x) + 60*(
c*d^2 + a*e^2)*(a^2*C*e^4 + a*c*e^2*(17*C*d^2 + 3*e*(-3*B*d + A*e)) + c^2*(
28*C*d^4 + 3*d^2*e*(-7*B*d + 5*A*e)))*Log[d + e*x])/(60*e^9)
```

fricas [B] time = 0.93, size = 1025, normalized size = 2.20

$$\frac{10\,Cc^3e^8x^8+450\,Cc^3d^8-390\,Bc^3d^7e-810\,Bac^2d^5e^3-450\,Ba^2cd^3e^5-30\,Ba^3de^7-30\,Aa^3e^8+330\left(3\,Cac^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*c^3*e^8*x^8 + 450*C*c^3*d^8 - 390*B*c^3*d^7*e - 810*B*a*c^2*d^5*
e^3 - 450*B*a^2*c*d^3*e^5 - 30*B*a^3*d*e^7 - 30*A*a^3*e^8 + 330*(3*C*a*c^2
+ A*c^3)*d^6*e^2 + 630*(C*a^2*c + A*a*c^2)*d^4*e^4 + 90*(C*a^3 + 3*A*a^2*c)
*d^2*e^6 - 4*(4*C*c^3*d*e^7 - 3*B*c^3*e^8)*x^7 + (28*C*c^3*d^2*e^6 - 21*B*c
^3*d*e^7 + 15*(3*C*a*c^2 + A*c^3)*e^8)*x^6 - 2*(28*C*c^3*d^3*e^5 - 21*B*c^3
*d^2*e^6 - 30*B*a*c^2*e^8 + 15*(3*C*a*c^2 + A*c^3)*d*e^7)*x^5 + 5*(28*C*c^3
*d^4*e^4 - 21*B*c^3*d^3*e^5 - 30*B*a*c^2*d*e^7 + 15*(3*C*a*c^2 + A*c^3)*d^2
*e^6 + 18*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 20*(28*C*c^3*d^5*e^3 - 21*B*c^3*d^
4*e^4 - 30*B*a*c^2*d^2*e^6 - 9*B*a^2*c*e^8 + 15*(3*C*a*c^2 + A*c^3)*d^3*e^5
 + 18*(C*a^2*c + A*a*c^2)*d*e^7)*x^3 - 30*(69*C*c^3*d^6*e^2 - 50*B*c^3*d^5*
e^3 - 63*B*a*c^2*d^3*e^5 - 12*B*a^2*c*d*e^7 + 34*(3*C*a*c^2 + A*c^3)*d^4*e^
4 + 33*(C*a^2*c + A*a*c^2)*d^2*e^6)*x^2 - 60*(13*C*c^3*d^7*e - 8*B*c^3*d^6*
e^2 - 3*B*a*c^2*d^4*e^4 + 6*B*a^2*c*d^2*e^6 + B*a^3*e^8 + 4*(3*C*a*c^2 + A*
c^3)*d^5*e^3 - 3*(C*a^2*c + A*a*c^2)*d^3*e^5 - 2*(C*a^3 + 3*A*a^2*c)*d*e^7)
*x + 60*(28*C*c^3*d^8 - 21*B*c^3*d^7*e - 30*B*a*c^2*d^5*e^3 - 9*B*a^2*c*d^3
*e^5 + 15*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 18*(C*a^2*c + A*a*c^2)*d^4*e^4 + (C
*a^3 + 3*A*a^2*c)*d^2*e^6 + (28*C*c^3*d^6*e^2 - 21*B*c^3*d^5*e^3 - 30*B*a*c
^2*d^3*e^5 - 9*B*a^2*c*d*e^7 + 15*(3*C*a*c^2 + A*c^3)*d^4*e^4 + 18*(C*a^2*c
 + A*a*c^2)*d^2*e^6 + (C*a^3 + 3*A*a^2*c)*e^8)*x^2 + 2*(28*C*c^3*d^7*e - 21
*B*c^3*d^6*e^2 - 30*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*e^6 + 15*(3*C*a*c^2 + A
*c^3)*d^5*e^3 + 18*(C*a^2*c + A*a*c^2)*d^3*e^5 + (C*a^3 + 3*A*a^2*c)*d*e^7)
*x)*log(e*x + d))/(e^11*x^2 + 2*d*e^10*x + d^2*e^9)
```

giac [A] time = 0.17, size = 727, normalized size = 1.56

$$\left(28\,Cc^3d^6-21\,Bc^3d^5e+45\,Cac^2d^4e^2+15\,Ac^3d^4e^2-30\,Bac^2d^3e^3+18\,Ca^2cd^2e^4+18\,Aac^2d^2e^4-9\,Ba^2cde^5+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (28*C*c^3*d^6 - 21*B*c^3*d^5*e + 45*C*a*c^2*d^4*e^2 + 15*A*c^3*d^4*e^2 - 30
*B*a*c^2*d^3*e^3 + 18*C*a^2*c*d^2*e^4 + 18*A*a*c^2*d^2*e^4 - 9*B*a^2*c*d*e^
5 + C*a^3*e^6 + 3*A*a^2*c*e^6)*e^(-9)*log(abs(x*e + d)) + 1/60*(10*C*c^3*x^
6*e^15 - 36*C*c^3*d*x^5*e^14 + 90*C*c^3*d^2*x^4*e^13 - 200*C*c^3*d^3*x^3*e^
12 + 450*C*c^3*d^4*x^2*e^11 - 1260*C*c^3*d^5*x*e^10 + 12*B*c^3*x^5*e^15 - 4
5*B*c^3*d*x^4*e^14 + 120*B*c^3*d^2*x^3*e^13 - 300*B*c^3*d^3*x^2*e^12 + 900*
B*c^3*d^4*x*e^11 + 45*C*a*c^2*x^4*e^15 + 15*A*c^3*x^4*e^15 - 180*C*a*c^2*d*
x^3*e^14 - 60*A*c^3*d*x^3*e^14 + 540*C*a*c^2*d^2*x^2*e^13 + 180*A*c^3*d^2*x
^2*e^13 - 1800*C*a*c^2*d^3*x*e^12 - 600*A*c^3*d^3*x*e^12 + 60*B*a*c^2*x^3*e
^15 - 270*B*a*c^2*d*x^2*e^14 + 1080*B*a*c^2*d^2*x*e^13 + 90*C*a^2*c*x^2*e^1
5 + 90*A*a*c^2*x^2*e^15 - 540*C*a^2*c*d*x*e^14 - 540*A*a*c^2*d*x*e^14 + 180
*B*a^2*c*x*e^15)*e^(-18) + 1/2*(15*C*c^3*d^8 - 13*B*c^3*d^7*e + 33*C*a*c^2*
d^6*e^2 + 11*A*c^3*d^6*e^2 - 27*B*a*c^2*d^5*e^3 + 21*C*a^2*c*d^4*e^4 + 21*A
*a*c^2*d^4*e^4 - 15*B*a^2*c*d^3*e^5 + 3*C*a^3*d^2*e^6 + 9*A*a^2*c*d^2*e^6 -
 B*a^3*d*e^7 - A*a^3*e^8 + 2*(8*C*c^3*d^7*e - 7*B*c^3*d^6*e^2 + 18*C*a*c^2*
d^5*e^3 + 6*A*c^3*d^5*e^3 - 15*B*a*c^2*d^4*e^4 + 12*C*a^2*c*d^3*e^5 + 12*A*
a*c^2*d^3*e^5 - 9*B*a^2*c*d^2*e^6 + 2*C*a^3*d*e^7 + 6*A*a^2*c*d*e^7 - B*a^3
*e^8)*x)*e^(-9)/(x*e + d)^2
```

maple [B] time = 0.02, size = 978, normalized size = 2.10

$$\frac{C c^3x^6}{6e^3}+\frac{B c^3x^5}{5e^3}-\frac{3C c^3d x^5}{5e^4}+\frac{A c^3x^4}{4e^3}-\frac{3B c^3d x^4}{4e^4}+\frac{3Ca c^2x^4}{4e^3}+\frac{3C c^3d^2x^4}{2e^5}-\frac{A c^3d x^3}{e^4}+\frac{Ba c^2x^3}{e^3}+\frac{2B c^3d^2x^3}{e^5}-\frac{3Ca c^2d x^3}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x)
```

```
[Out] 1/6*c^3*C*x^6/e^3-1/e^2/(e*x+d)*B*a^3+1/e^3*ln(e*x+d)*a^3*C-1/2/e/(e*x+d)^2
*A*a^3+1/5*c^3/e^3*B*x^5+1/4*c^3/e^3*A*x^4-3/2/e^7/(e*x+d)^2*C*a*c^2*d^6-9*
c^2/e^4*A*x*a*d+18*c^2/e^5*B*x*a*d^2-30*c^2/e^6*C*x*a*d^3-3*c^2/e^4*C*x^3*a
*d-9/2*c^2/e^4*B*x^2*a*d-9*c/e^4*C*x*a^2*d+9*c^2/e^5*C*x^2*a*d^2+6/e^3/(e*x
+d)*A*a^2*c*d+12/e^5/(e*x+d)*A*a*c^2*d^3-9/e^4/(e*x+d)*B*a^2*c*d^2-15/e^6/(
e*x+d)*B*a*c^2*d^4+12/e^5/(e*x+d)*C*a^2*c*d^3+18/e^7/(e*x+d)*C*a*c^2*d^5+18
/e^5*ln(e*x+d)*A*a*c^2*d^2-9/e^4*ln(e*x+d)*B*a^2*c*d-30/e^6*ln(e*x+d)*B*a*c
^2*d^3+18/e^5*ln(e*x+d)*C*a^2*c*d^2+45/e^7*ln(e*x+d)*C*a*c^2*d^4-3/2/e^3/(e
*x+d)^2*A*d^2*a^2*c-3/2/e^5/(e*x+d)^2*A*a*c^2*d^4+3/2/e^4/(e*x+d)^2*B*a^2*c
*d^3+3/2/e^6/(e*x+d)^2*B*a*c^2*d^5-3/2/e^5/(e*x+d)^2*C*a^2*c*d^4-21/e^8*ln(
e*x+d)*B*c^3*d^5+28/e^9*ln(e*x+d)*C*c^3*d^6-1/2/e^7/(e*x+d)^2*A*c^3*d^6+1/2
/e^2/(e*x+d)^2*B*d*a^3+1/2/e^8/(e*x+d)^2*B*c^3*d^7-1/2/e^3/(e*x+d)^2*C*d^2*
a^3-1/2/e^9/(e*x+d)^2*C*c^3*d^8+3/2*c/e^3*C*x^2*a^2+15*c^3/e^7*B*x*d^4-10*c
^3/e^6*A*x*d^3+3/2*c^2/e^3*A*x^2*a+3*c/e^3*B*x*a^2-21*c^3/e^8*C*x*d^5+3*c^3
/e^5*A*x^2*d^2-5*c^3/e^6*B*x^2*d^3+15/2*c^3/e^7*C*x^2*d^4+3/4*c^2/e^3*C*x^4
*a+3/2*c^3/e^5*C*x^4*d^2-c^3/e^4*A*x^3*d+c^2/e^3*B*x^3*a+2*c^3/e^5*B*x^3*d^
2-10/3*c^3/e^6*C*x^3*d^3+6/e^7/(e*x+d)*A*c^3*d^5-7/e^8/(e*x+d)*B*c^3*d^6+2/
e^3/(e*x+d)*C*a^3*d+8/e^9/(e*x+d)*C*c^3*d^7-3/5*c^3/e^4*C*x^5*d-3/4*c^3/e^4
*B*x^4*d+3/e^3*ln(e*x+d)*A*a^2*c+15/e^7*ln(e*x+d)*A*c^3*d^4
```

maxima [A] time = 0.53, size = 701, normalized size = 1.50

$$\frac{15\, Cc^3d^8 - 13\, Bc^3d^7e - 27\, Bac^2d^5e^3 - 15\, Ba^2cd^3e^5 - Ba^3de^7 - Aa^3e^8 + 11\left(3\, Cac^2 + Ac^3\right)d^6e^2 + 21\left(Ca^2c + Aac^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(15*C*c^3*d^8 - 13*B*c^3*d^7*e - 27*B*a*c^2*d^5*e^3 - 15*B*a^2*c*d^3*e^
5 - B*a^3*d*e^7 - A*a^3*e^8 + 11*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 21*(C*a^2*c
+ A*a*c^2)*d^4*e^4 + 3*(C*a^3 + 3*A*a^2*c)*d^2*e^6 + 2*(8*C*c^3*d^7*e - 7*B
*c^3*d^6*e^2 - 15*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*e^6 - B*a^3*e^8 + 6*(3*C*
```

```
a*c^2 + A*c^3)*d^5*e^3 + 12*(C*a^2*c + A*a*c^2)*d^3*e^5 + 2*(C*a^3 + 3*A*a^
2*c)*d*e^7)*x)/(e^11*x^2 + 2*d*e^10*x + d^2*e^9) + 1/60*(10*C*c^3*e^5*x^6 -
 12*(3*C*c^3*d*e^4 - B*c^3*e^5)*x^5 + 15*(6*C*c^3*d^2*e^3 - 3*B*c^3*d*e^4 +
 (3*C*a*c^2 + A*c^3)*e^5)*x^4 - 20*(10*C*c^3*d^3*e^2 - 6*B*c^3*d^2*e^3 - 3*
B*a*c^2*e^5 + 3*(3*C*a*c^2 + A*c^3)*d*e^4)*x^3 + 30*(15*C*c^3*d^4*e - 10*B*
c^3*d^3*e^2 - 9*B*a*c^2*d*e^4 + 6*(3*C*a*c^2 + A*c^3)*d^2*e^3 + 3*(C*a^2*c
+ A*a*c^2)*e^5)*x^2 - 60*(21*C*c^3*d^5 - 15*B*c^3*d^4*e - 18*B*a*c^2*d^2*e^
3 - 3*B*a^2*c*e^5 + 10*(3*C*a*c^2 + A*c^3)*d^3*e^2 + 9*(C*a^2*c + A*a*c^2)*
d*e^4)*x)/e^8 + (28*C*c^3*d^6 - 21*B*c^3*d^5*e - 30*B*a*c^2*d^3*e^3 - 9*B*a
^2*c*d*e^5 + 15*(3*C*a*c^2 + A*c^3)*d^4*e^2 + 18*(C*a^2*c + A*a*c^2)*d^2*e^
4 + (C*a^3 + 3*A*a^2*c)*e^6)*log(e*x + d)/e^9
```

mupad [B] time = 3.94, size = 1290, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

```
[Out] x^3*((d*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3
+ (3*C*c^3*d^2)/e^5))/e - (d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (B*a*
c^2)/e^3 - (C*c^3*d^3)/(3*e^6)) + x*((3*d*((3*d*((3*d*((3*d*((B*c^3)/e^3 -
(3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e - (3*
d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*a*c^2)/e^3 - (C*c^3*d^3)/e^
6))/e - (3*d^2*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^
2)/e^3 + (3*C*c^3*d^2)/e^5))/e^2 + (d^3*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^
3 - (3*a*c*(A*c + C*a))/e^3))/e + (d^3*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4
))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e^3 - (3*d^2*((3*d*((3
*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*
d^2)/e^5))/e - (3*d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*a*c^2)/e^
3 - (C*c^3*d^3)/e^6))/e^2 + (3*B*a^2*c)/e^3) + x^5*((B*c^3)/(5*e^3) - (3*C*
c^3*d)/(5*e^4)) - x^4*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/(4*e) - (A*c^3
 + 3*C*a*c^2)/(4*e^3) + (3*C*c^3*d^2)/(4*e^5)) - x^2*((3*d*((3*d*((3*d*((B*
c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^
5))/e - (3*d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*a*c^2)/e^3 - (C*
c^3*d^3)/e^6))/(2*e) - (3*d^2*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A
*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/(2*e^2) + (d^3*((B*c^3)/e^3 - (
3*C*c^3*d)/e^4))/(2*e^3) - (3*a*c*(A*c + C*a))/(2*e^3)) + ((15*C*c^3*d^8 -
A*a^3*e^8 - B*a^3*d*e^7 - 13*B*c^3*d^7*e + 11*A*c^3*d^6*e^2 + 3*C*a^3*d^2*e
^6 + 21*A*a*c^2*d^4*e^4 + 9*A*a^2*c*d^2*e^6 - 27*B*a*c^2*d^5*e^3 - 15*B*a^2
*c*d^3*e^5 + 33*C*a*c^2*d^6*e^2 + 21*C*a^2*c*d^4*e^4)/(2*e) + x*(8*C*c^3*d^
7 - B*a^3*e^7 + 2*C*a^3*d*e^6 - 7*B*c^3*d^6*e + 6*A*c^3*d^5*e^2 + 12*A*a*c^
2*d^3*e^4 - 15*B*a*c^2*d^4*e^3 - 9*B*a^2*c*d^2*e^5 + 18*C*a*c^2*d^5*e^2 + 1
2*C*a^2*c*d^3*e^4 + 6*A*a^2*c*d*e^6))/(d^2*e^8 + e^10*x^2 + 2*d*e^9*x) + (l
og(d + e*x)*(C*a^3*e^6 + 28*C*c^3*d^6 + 3*A*a^2*c*e^6 - 21*B*c^3*d^5*e + 15
*A*c^3*d^4*e^2 + 18*A*a*c^2*d^2*e^4 - 30*B*a*c^2*d^3*e^3 + 45*C*a*c^2*d^4*e
^2 + 18*C*a^2*c*d^2*e^4 - 9*B*a^2*c*d*e^5))/e^9 + (C*c^3*x^6)/(6*e^3)
```

sympy [A] time = 25.28, size = 816, normalized size = 1.75

$$\frac{Cc^3x^6}{6e^3}+x^5\left(\frac{Bc^3}{5e^3}-\frac{3Cc^3d}{5e^4}\right)+x^4\left(\frac{Ac^3}{4e^3}-\frac{3Bc^3d}{4e^4}+\frac{3Cac^2}{4e^3}+\frac{3Cc^3d^2}{2e^5}\right)+x^3\left(-\frac{Ac^3d}{e^4}+\frac{Bac^2}{e^3}+\frac{2Bc^3d^2}{e^5}-\frac{3Cac^2d}{e^4}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

```
[Out] C*c**3*x**6/(6*e**3) + x**5*(B*c**3/(5*e**3) - 3*C*c**3*d/(5*e**4)) + x**4*
(A*c**3/(4*e**3) - 3*B*c**3*d/(4*e**4) + 3*C*a*c**2/(4*e**3) + 3*C*c**3*d**
2/(2*e**5)) + x**3*(-A*c**3*d/e**4 + B*a*c**2/e**3 + 2*B*c**3*d**2/e**5 - 3
```

```
*C*a*c**2*d/e**4 - 10*C*c**3*d**3/(3*e**6)) + x**2*(3*A*a*c**2/(2*e**3) + 3
*A*c**3*d**2/e**5 - 9*B*a*c**2*d/(2*e**4) - 5*B*c**3*d**3/e**6 + 3*C*a**2*c
/(2*e**3) + 9*C*a*c**2*d**2/e**5 + 15*C*c**3*d**4/(2*e**7)) + x*(-9*A*a*c**
2*d/e**4 - 10*A*c**3*d**3/e**6 + 3*B*a**2*c/e**3 + 18*B*a*c**2*d**2/e**5 +
15*B*c**3*d**4/e**7 - 9*C*a**2*c*d/e**4 - 30*C*a*c**2*d**3/e**6 - 21*C*c**3
*d**5/e**8) + (-A*a**3*e**8 + 9*A*a**2*c*d**2*e**6 + 21*A*a*c**2*d**4*e**4
+ 11*A*c**3*d**6*e**2 - B*a**3*d*e**7 - 15*B*a**2*c*d**3*e**5 - 27*B*a*c**2
*d**5*e**3 - 13*B*c**3*d**7*e + 3*C*a**3*d**2*e**6 + 21*C*a**2*c*d**4*e**4
+ 33*C*a*c**2*d**6*e**2 + 15*C*c**3*d**8 + x*(12*A*a**2*c*d*e**7 + 24*A*a*c
**2*d**3*e**5 + 12*A*c**3*d**5*e**3 - 2*B*a**3*e**8 - 18*B*a**2*c*d**2*e**6
 - 30*B*a*c**2*d**4*e**4 - 14*B*c**3*d**6*e**2 + 4*C*a**3*d*e**7 + 24*C*a**
2*c*d**3*e**5 + 36*C*a*c**2*d**5*e**3 + 16*C*c**3*d**7*e))/(2*d**2*e**9 + 4
*d*e**10*x + 2*e**11*x**2) + (a*e**2 + c*d**2)*(3*A*a*c*e**4 + 15*A*c**2*d*
*2*e**2 - 9*B*a*c*d*e**3 - 21*B*c**2*d**3*e + C*a**2*e**4 + 17*C*a*c*d**2*e
**2 + 28*C*c**2*d**4)*log(d + e*x)/e**9
```

3.39 $\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx$

Optimal. Leaf size=17

$$\frac{(a+bx^2)^2}{c+dx}$$

```
[Out] (b*x^2+a)^2/(d*x+c)
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1590}

$$\frac{(a+bx^2)^2}{c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(-(a*d) + 4*b*c*x + 3*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^2/(c + d*x)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx = \frac{(a+bx^2)^2}{c+dx}$$

Mathematica [B] time = 0.04, size = 62, normalized size = 3.65

$$\frac{a^2d^4+2abd^2(c^2+cdx+d^2x^2)+b^2(c^4+c^3dx+d^4x^4)}{d^4(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(-(a*d) + 4*b*c*x + 3*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a^2*d^4 + 2*a*b*d^2*(c^2 + c*d*x + d^2*x^2) + b^2*(c^4 + c^3*d*x + d^4*x^4
))/(d^4*(c + d*x))
```

fricas [B] time = 0.95, size = 78, normalized size = 4.59

$$\frac{b^2d^4x^4+2\,abd^4x^2+b^2c^4+2\,abc^2d^2+a^2d^4+(b^2c^3d+2\,abcd^3)x}{d^5x+cd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] (b^2*d^4*x^4 + 2*a*b*d^4*x^2 + b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4 + (b^2*c^3
*d + 2*a*b*c*d^3)*x)/(d^5*x + c*d^4)
```

giac [B] time = 0.17, size = 111, normalized size = 6.53

$$\frac{\left(b^2 - \frac{4\,b^2c}{dx+c} + \frac{6\,b^2c^2}{(dx+c)^2} + \frac{2\,abd^2}{(dx+c)^2}\right)(dx+c)^3}{d^4} + \frac{\frac{b^2c^4d^3}{dx+c} + \frac{2\,abc^2d^5}{dx+c} + \frac{a^2d^7}{dx+c}}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b^2 - 4*b^2*c/(d*x + c) + 6*b^2*c^2/(d*x + c)^2 + 2*a*b*d^2/(d*x + c)^2)*(
d*x + c)^3/d^4 + (b^2*c^4*d^3/(d*x + c) + 2*a*b*c^2*d^5/(d*x + c) + a^2*d^7
/(d*x + c))/d^7
```

maple [B] time = 0.01, size = 76, normalized size = 4.47

$$\frac{\left(b\,d^2x^3 - bcd\,x^2 + 2a\,d^2x + b\,c^2x\right)b}{d^3} - \frac{-a^2d^4 - 2ab\,c^2d^2 - b^2c^4}{(dx+c)\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x)
```

```
[Out] b/d^3*(b*d^2*x^3-b*c*d*x^2+2*a*d^2*x+b*c^2*x)-(-a^2*d^4-2*a*b*c^2*d^2-b^2*c
^4)/d^4/(d*x+c)
```

maxima [B] time = 0.46, size = 82, normalized size = 4.82

$$\frac{b^2c^4 + 2\,abc^2d^2 + a^2d^4}{d^5x + cd^4} + \frac{b^2d^2x^3 - b^2cdx^2 + \left(b^2c^2 + 2\,abd^2\right)x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(d^5*x + c*d^4) + (b^2*d^2*x^3 - b^2*c*
d*x^2 + (b^2*c^2 + 2*a*b*d^2)*x)/d^3
```

mupad [B] time = 0.08, size = 85, normalized size = 5.00

$$x\left(\frac{b^2\,c^2}{d^3} + \frac{2\,a\,b}{d}\right) + \frac{b^2\,x^3}{d} + \frac{a^2\,d^4 + 2\,a\,b\,c^2\,d^2 + b^2\,c^4}{d\,\left(x\,d^4 + c\,d^3\right)} - \frac{b^2\,c\,x^2}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(4*b*c*x - a*d + 3*b*d*x^2))/(c + d*x)^2,x)
```

```
[Out] x*((b^2*c^2)/d^3 + (2*a*b)/d) + (b^2*x^3)/d + (a^2*d^4 + b^2*c^4 + 2*a*b*c^
2*d^2)/(d*(c*d^3 + d^4*x)) - (b^2*c*x^2)/d^2
```

sympy [B] time = 0.37, size = 73, normalized size = 4.29

$$-\frac{b^2cx^2}{d^2} + \frac{b^2x^3}{d} + x\left(\frac{2ab}{d} + \frac{b^2c^2}{d^3}\right) + \frac{a^2d^4 + 2abc^2d^2 + b^2c^4}{cd^4 + d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(3*b*d*x**2+4*b*c*x-a*d)/(d*x+c)**2,x)
```

```
[Out] -b**2*c*x**2/d**2 + b**2*x**3/d + x*(2*a*b/d + b**2*c**2/d**3) + (a**2*d**4
 + 2*a*b*c**2*d**2 + b**2*c**4)/(c*d**4 + d**5*x)
```

3.40 $\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2}\,dx$

Optimal. Leaf size=17

$$\frac{(a+bx^2)^2}{c+dx}$$

```
[Out] (b*x^2+a)^2/(d*x+c)
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1590}

$$\frac{(a+bx^2)^2}{c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(-(a*d) + b*x*(4*c + 3*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^2/(c + d*x)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2}\,dx = \frac{(a+bx^2)^2}{c+dx}$$

Mathematica [B] time = 0.02, size = 62, normalized size = 3.65

$$\frac{a^2d^4+2abd^2(c^2+cdx+d^2x^2)+b^2(c^4+c^3dx+d^4x^4)}{d^4(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(-(a*d) + b*x*(4*c + 3*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a^2*d^4 + 2*a*b*d^2*(c^2 + c*d*x + d^2*x^2) + b^2*(c^4 + c^3*d*x + d^4*x^4
))/(d^4*(c + d*x))
```

fricas [B] time = 0.84, size = 78, normalized size = 4.59

$$\frac{b^2d^4x^4+2\,abd^4x^2+b^2c^4+2\,abc^2d^2+a^2d^4+(b^2c^3d+2\,abcd^3)x}{d^5x+cd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^4*x^4 + 2*a*b*d^4*x^2 + b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4 + (b^2*c^3
*d + 2*a*b*c*d^3)*x)/(d^5*x + c*d^4)
```

giac [B] time = 0.15, size = 111, normalized size = 6.53

$$\frac{\left(b^2 - \frac{4\,b^2c}{dx+c} + \frac{6\,b^2c^2}{(dx+c)^2} + \frac{2\,abd^2}{(dx+c)^2}\right)(dx+c)^3}{d^4} + \frac{\frac{b^2c^4d^3}{dx+c} + \frac{2\,abc^2d^5}{dx+c} + \frac{a^2d^7}{dx+c}}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b^2 - 4*b^2*c/(d*x + c) + 6*b^2*c^2/(d*x + c)^2 + 2*a*b*d^2/(d*x + c)^2)*(
d*x + c)^3/d^4 + (b^2*c^4*d^3/(d*x + c) + 2*a*b*c^2*d^5/(d*x + c) + a^2*d^7
/(d*x + c))/d^7
```

maple [B] time = 0.00, size = 76, normalized size = 4.47

$$\frac{\left(b\,d^2x^3 - bcd\,x^2 + 2a\,d^2x + b\,c^2x\right)b}{d^3} - \frac{-a^2d^4 - 2ab\,c^2d^2 - b^2c^4}{(dx+c)\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x)
```

```
[Out] (b*d^2*x^3-b*c*d*x^2+2*a*d^2*x+b*c^2*x)*b/d^3-(-a^2*d^4-2*a*b*c^2*d^2-b^2*c
^4)/(d*x+c)/d^4
```

maxima [B] time = 0.47, size = 82, normalized size = 4.82

$$\frac{b^2c^4 + 2\,abc^2d^2 + a^2d^4}{d^5x + cd^4} + \frac{b^2d^2x^3 - b^2cdx^2 + \left(b^2c^2 + 2\,abd^2\right)x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(d^5*x + c*d^4) + (b^2*d^2*x^3 - b^2*c*
d*x^2 + (b^2*c^2 + 2*a*b*d^2)*x)/d^3
```

mupad [B] time = 3.84, size = 85, normalized size = 5.00

$$x\left(\frac{b^2\,c^2}{d^3} + \frac{2\,a\,b}{d}\right) + \frac{b^2\,x^3}{d} + \frac{a^2\,d^4 + 2\,a\,b\,c^2\,d^2 + b^2\,c^4}{d\left(x\,d^4 + c\,d^3\right)} - \frac{b^2\,c\,x^2}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*d - b*x*(4*c + 3*d*x))*(a + b*x^2))/(c + d*x)^2,x)
```

```
[Out] x*((b^2*c^2)/d^3 + (2*a*b)/d) + (b^2*x^3)/d + (a^2*d^4 + b^2*c^4 + 2*a*b*c^
2*d^2)/(d*(c*d^3 + d^4*x)) - (b^2*c*x^2)/d^2
```

sympy [B] time = 0.37, size = 73, normalized size = 4.29

$$-\frac{b^2cx^2}{d^2} + \frac{b^2x^3}{d} + x\left(\frac{2ab}{d} + \frac{b^2c^2}{d^3}\right) + \frac{a^2d^4 + 2abc^2d^2 + b^2c^4}{cd^4 + d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)**2,x)
```

```
[Out] -b**2*c*x**2/d**2 + b**2*x**3/d + x*(2*a*b/d + b**2*c**2/d**3) + (a**2*d**4
 + 2*a*b*c**2*d**2 + b**2*c**4)/(c*d**4 + d**5*x)
```

3.41 $\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2} dx$

Optimal. Leaf size=17

$$\frac{(a+bx^2)^3}{c+dx}$$

```
[Out] (b*x^2+a)^3/(d*x+c)
```

Rubi [A] time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1590}

$$\frac{(a+bx^2)^3}{c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(-(a*d) + 6*b*c*x + 5*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^3/(c + d*x)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2} dx = \frac{(a+bx^2)^3}{c+dx}$$

Mathematica [B] time = 0.04, size = 90, normalized size = 5.29

$$\frac{a^3d^6+3a^2bd^4(c^2+cdx+d^2x^2)+3ab^2d^2(c^4+c^3dx+d^4x^4)+b^3(c^6+c^5dx+d^6x^6)}{d^6(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(-(a*d) + 6*b*c*x + 5*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a^3*d^6 + 3*a^2*b*d^4*(c^2 + c*d*x + d^2*x^2) + 3*a*b^2*d^2*(c^4 + c^3*d*x
 + d^4*x^4) + b^3*(c^6 + c^5*d*x + d^6*x^6))/(d^6*(c + d*x))
```

fricas [B] time = 0.87, size = 120, normalized size = 7.06

$$\frac{b^3d^6x^6+3\,ab^2d^6x^4+3\,a^2bd^6x^2+b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6+(b^3c^5d+3\,ab^2c^3d^3+3\,a^2bcd^5)x}{d^7x+cd^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="frica
s")
```

```
[Out] (b^3*d^6*x^6 + 3*a*b^2*d^6*x^4 + 3*a^2*b*d^6*x^2 + b^3*c^6 + 3*a*b^2*c^4*d^
2 + 3*a^2*b*c^2*d^4 + a^3*d^6 + (b^3*c^5*d + 3*a*b^2*c^3*d^3 + 3*a^2*b*c*d^
5)*x)/(d^7*x + c*d^6)
```

giac [B] time = 0.18, size = 216, normalized size = 12.71

$$\frac{\left(b^{3} - \frac{6\,b^{3}c}{dx+c} + \frac{15\,b^{3}c^{2}}{(dx+c)^{2}} - \frac{20\,b^{3}c^{3}}{(dx+c)^{3}} + \frac{15\,b^{3}c^{4}}{(dx+c)^{4}} + \frac{3\,ab^{2}d^{2}}{(dx+c)^{2}} - \frac{12\,ab^{2}cd^{2}}{(dx+c)^{3}} + \frac{18\,ab^{2}c^{2}d^{2}}{(dx+c)^{4}} + \frac{3\,a^{2}bd^{4}}{(dx+c)^{4}}\right)(dx+c)^{5}}{d^{6}} + \frac{\frac{b^{3}c^{6}d^{5}}{dx+c} + \frac{3\,ab^{2}c^{4}d^{7}}{dx+c} + \frac{3\,a^{2}bc^{\cdots}}{dx+\cdots}}{d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="giac"
)
```

```
[Out] (b^3 - 6*b^3*c/(d*x + c) + 15*b^3*c^2/(d*x + c)^2 - 20*b^3*c^3/(d*x + c)^3
+ 15*b^3*c^4/(d*x + c)^4 + 3*a*b^2*d^2/(d*x + c)^2 - 12*a*b^2*c*d^2/(d*x +
c)^3 + 18*a*b^2*c^2*d^2/(d*x + c)^4 + 3*a^2*b*d^4/(d*x + c)^4)*(d*x + c)^5/
d^6 + (b^3*c^6*d^5/(d*x + c) + 3*a*b^2*c^4*d^7/(d*x + c) + 3*a^2*b*c^2*d^9/
(d*x + c) + a^3*d^11/(d*x + c))/d^11
```

maple [B] time = 0.01, size = 157, normalized size = 9.24

$$\frac{\left(b^{2}d^{4}x^{5} - b^{2}c\,d^{3}x^{4} + 3ab\,d^{4}x^{3} + b^{2}c^{2}d^{2}x^{3} - 3abc\,d^{3}x^{2} - b^{2}c^{3}d\,x^{2} + 3a^{2}d^{4}x + 3ab\,c^{2}d^{2}x + b^{2}c^{4}x\right)b}{d^{5}} - \frac{-a^{3}d^{6} - 3a^{2}b\cdots}{(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x)
```

```
[Out] b/d^5*(b^2*d^4*x^5-b^2*c*d^3*x^4+3*a*b*d^4*x^3+b^2*c^2*d^2*x^3-3*a*b*c*d^3*
x^2-b^2*c^3*d*x^2+3*a^2*d^4*x+3*a*b*c^2*d^2*x+b^2*c^4*x)-(-a^3*d^6-3*a^2*b*
c^2*d^4-3*a*b^2*c^4*d^2-b^3*c^6)/d^6/(d*x+c)
```

maxima [B] time = 0.45, size = 160, normalized size = 9.41

$$\frac{b^{3}c^{6} + 3\,ab^{2}c^{4}d^{2} + 3\,a^{2}bc^{2}d^{4} + a^{3}d^{6}}{d^{7}x + cd^{6}} + \frac{b^{3}d^{4}x^{5} - b^{3}cd^{3}x^{4} + \left(b^{3}c^{2}d^{2} + 3\,ab^{2}d^{4}\right)x^{3} - \left(b^{3}c^{3}d + 3\,ab^{2}cd^{3}\right)x^{2} + \left(b^{3}c^{4} + \cdots\right.}{d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="maxim
a")
```

```
[Out] (b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)/(d^7*x + c*d^6) + (
b^3*d^4*x^5 - b^3*c*d^3*x^4 + (b^3*c^2*d^2 + 3*a*b^2*d^4)*x^3 - (b^3*c^3*d
+ 3*a*b^2*c*d^3)*x^2 + (b^3*c^4 + 3*a*b^2*c^2*d^2 + 3*a^2*b*d^4)*x)/d^5
```

mupad [B] time = 3.78, size = 252, normalized size = 14.82

$$x^{3}\left(\frac{3\,a\,b^{2}}{d} + \frac{b^{3}\,c^{2}}{d^{3}}\right) - x\left(\frac{2\,c\left(\frac{4\,b^{3}\,c^{3}}{d^{4}} - \frac{2\,c\left(\frac{9\,a\,b^{2}}{d} + \frac{3\,b^{3}\,c^{2}}{d^{3}}\right)}{d} + \frac{12\,a\,b^{2}\,c}{d^{2}}\right)}{d} + \frac{c^{2}\left(\frac{9\,a\,b^{2}}{d} + \frac{3\,b^{3}\,c^{2}}{d^{3}}\right)}{d^{2}} - \frac{3\,a^{2}\,b}{d}\right) + x^{2}\left(\frac{2\,b^{3}\,c^{3}}{d^{4}} - \frac{c\left(\frac{9\,a\cdots}{d}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(6*b*c*x - a*d + 5*b*d*x^2))/(c + d*x)^2,x)
```

```
[Out] x^3*((3*a*b^2)/d + (b^3*c^2)/d^3) - x*((2*c*((4*b^3*c^3)/d^4 - (2*c*((9*a*b
^2)/d + (3*b^3*c^2)/d^3))/d + (12*a*b^2*c)/d^2))/d + (c^2*((9*a*b^2)/d + (3
*b^3*c^2)/d^3))/d^2 - (3*a^2*b)/d) + x^2*((2*b^3*c^3)/d^4 - (c*((9*a*b^2)/d
 + (3*b^3*c^2)/d^3))/d + (6*a*b^2*c)/d^2) + (a^3*d^6 + b^3*c^6 + 3*a*b^2*c^
4*d^2 + 3*a^2*b*c^2*d^4)/(d*(c*d^5 + d^6*x)) + (b^3*x^5)/d - (b^3*c*x^4)/d^
2
```

sympy [**B**] time = 0.59, size = 153, normalized size = 9.00

$$-\frac{b^3cx^4}{d^2}+\frac{b^3x^5}{d}+x^3\left(\frac{3ab^2}{d}+\frac{b^3c^2}{d^3}\right)+x^2\left(-\frac{3ab^2c}{d^2}-\frac{b^3c^3}{d^4}\right)+x\left(\frac{3a^2b}{d}+\frac{3ab^2c^2}{d^3}+\frac{b^3c^4}{d^5}\right)+\frac{a^3d^6+3a^2bc^2d^4+3ab^2c^4}{cd^6+d^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(5*b*d*x**2+6*b*c*x-a*d)/(d*x+c)**2,x)
```

```
[Out] -b**3*c*x**4/d**2 + b**3*x**5/d + x**3*(3*a*b**2/d + b**3*c**2/d**3) + x**2
*(-3*a*b**2*c/d**2 - b**3*c**3/d**4) + x*(3*a**2*b/d + 3*a*b**2*c**2/d**3 +
 b**3*c**4/d**5) + (a**3*d**6 + 3*a**2*b*c**2*d**4 + 3*a*b**2*c**4*d**2 + b
**3*c**6)/(c*d**6 + d**7*x)
```

3.42 $\int \frac{\left(a+bx^2\right)^2(-ad+bx(6c+5dx))}{(c+dx)^2}\,dx$

Optimal. Leaf size=17

$$\frac{\left(a+bx^2\right)^3}{c+dx}$$

```
[Out] (b*x^2+a)^3/(d*x+c)
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {1590}

$$\frac{\left(a+bx^2\right)^3}{c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(-(a*d) + b*x*(6*c + 5*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^3/(c + d*x)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2(-ad+bx(6c+5dx))}{(c+dx)^2}\,dx=\frac{\left(a+bx^2\right)^3}{c+dx}$$

Mathematica [B] time = 0.02, size = 90, normalized size = 5.29

$$\frac{a^3d^6+3a^2bd^4\left(c^2+cdx+d^2x^2\right)+3ab^2d^2\left(c^4+c^3dx+d^4x^4\right)+b^3\left(c^6+c^5dx+d^6x^6\right)}{d^6(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(-(a*d) + b*x*(6*c + 5*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a^3*d^6 + 3*a^2*b*d^4*(c^2 + c*d*x + d^2*x^2) + 3*a*b^2*d^2*(c^4 + c^3*d*x
 + d^4*x^4) + b^3*(c^6 + c^5*d*x + d^6*x^6))/(d^6*(c + d*x))
```

fricas [B] time = 0.89, size = 120, normalized size = 7.06

$$\frac{b^3d^6x^6+3\,ab^2d^6x^4+3\,a^2bd^6x^2+b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6+\left(b^3c^5d+3\,ab^2c^3d^3+3\,a^2bcd^5\right)x}{d^7x+cd^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="fricas
")
```

```
[Out] (b^3*d^6*x^6 + 3*a*b^2*d^6*x^4 + 3*a^2*b*d^6*x^2 + b^3*c^6 + 3*a*b^2*c^4*d^
2 + 3*a^2*b*c^2*d^4 + a^3*d^6 + (b^3*c^5*d + 3*a*b^2*c^3*d^3 + 3*a^2*b*c*d^
5)*x)/(d^7*x + c*d^6)
```

giac [B] time = 0.20, size = 216, normalized size = 12.71

$$\frac{\left(b^3 - \frac{6 b^3 c}{dx+c} + \frac{15 b^3 c^2}{(dx+c)^2} - \frac{20 b^3 c^3}{(dx+c)^3} + \frac{15 b^3 c^4}{(dx+c)^4} + \frac{3 ab^2 d^2}{(dx+c)^2} - \frac{12 ab^2 cd^2}{(dx+c)^3} + \frac{18 ab^2 c^2 d^2}{(dx+c)^4} + \frac{3 a^2 bd^4}{(dx+c)^4}\right)(dx+c)^5}{d^6} + \frac{\frac{b^3 c^6 d^5}{dx+c} + \frac{3 ab^2 c^4 d^7}{dx+c} + \frac{3 a}{}}{d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b^3 - 6*b^3*c/(d*x + c) + 15*b^3*c^2/(d*x + c)^2 - 20*b^3*c^3/(d*x + c)^3
+ 15*b^3*c^4/(d*x + c)^4 + 3*a*b^2*d^2/(d*x + c)^2 - 12*a*b^2*c*d^2/(d*x +
c)^3 + 18*a*b^2*c^2*d^2/(d*x + c)^4 + 3*a^2*b*d^4/(d*x + c)^4)*(d*x + c)^5/
d^6 + (b^3*c^6*d^5/(d*x + c) + 3*a*b^2*c^4*d^7/(d*x + c) + 3*a^2*b*c^2*d^9/
(d*x + c) + a^3*d^11/(d*x + c))/d^11
```

maple [B] time = 0.01, size = 157, normalized size = 9.24

$$\frac{\left(b^2 d^4 x^5 - b^2 c\, d^3 x^4 + 3ab\, d^4 x^3 + b^2 c^2 d^2 x^3 - 3abc\, d^3 x^2 - b^2 c^3 d\, x^2 + 3a^2 d^4 x + 3ab\, c^2 d^2 x + b^2 c^4 x\right) b}{d^5} - \frac{-a^3 d^6 - 3a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x)
```

```
[Out] (b^2*d^4*x^5-b^2*c*d^3*x^4+3*a*b*d^4*x^3+b^2*c^2*d^2*x^3-3*a*b*c*d^3*x^2-b^
2*c^3*d*x^2+3*a^2*d^4*x+3*a*b*c^2*d^2*x+b^2*c^4*x)*b/d^5-(-a^3*d^6-3*a^2*b*
c^2*d^4-3*a*b^2*c^4*d^2-b^3*c^6)/(d*x+c)/d^6
```

maxima [B] time = 0.44, size = 160, normalized size = 9.41

$$\frac{b^3 c^6 + 3\, ab^2 c^4 d^2 + 3\, a^2 bc^2 d^4 + a^3 d^6}{d^7 x + cd^6} + \frac{b^3 d^4 x^5 - b^3 cd^3 x^4 + \left(b^3 c^2 d^2 + 3\, ab^2 d^4\right) x^3 - \left(b^3 c^3 d + 3\, ab^2 cd^3\right) x^2 + \left(b^3 c^4\right.}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="maxima
")
```

```
[Out] (b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)/(d^7*x + c*d^6) + (
b^3*d^4*x^5 - b^3*c*d^3*x^4 + (b^3*c^2*d^2 + 3*a*b^2*d^4)*x^3 - (b^3*c^3*d
+ 3*a*b^2*c*d^3)*x^2 + (b^3*c^4 + 3*a*b^2*c^2*d^2 + 3*a^2*b*d^4)*x)/d^5
```

mupad [B] time = 0.05, size = 252, normalized size = 14.82

$$x^3 \left(\frac{3\, a\, b^2}{d} + \frac{b^3\, c^2}{d^3}\right) - x \left(\frac{2\, c \left(\frac{4\, b^3\, c^3}{d^4} - \frac{2\, c \left(\frac{9\, a\, b^2}{d} + \frac{3\, b^3\, c^2}{d^3}\right)}{d} + \frac{12\, a\, b^2\, c}{d^2}\right)}{d} + \frac{c^2 \left(\frac{9\, a\, b^2}{d} + \frac{3\, b^3\, c^2}{d^3}\right)}{d^2} - \frac{3\, a^2\, b}{d}\right) + x^2 \left(\frac{2\, b^3\, c^3}{d^4} - \frac{c\, (}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*d - b*x*(6*c + 5*d*x))*(a + b*x^2)^2)/(c + d*x)^2,x)
```

```
[Out] x^3*((3*a*b^2)/d + (b^3*c^2)/d^3) - x*((2*c*((4*b^3*c^3)/d^4 - (2*c*((9*a*b
^2)/d + (3*b^3*c^2)/d^3))/d + (12*a*b^2*c)/d^2))/d + (c^2*((9*a*b^2)/d + (3
*b^3*c^2)/d^3))/d^2 - (3*a^2*b)/d) + x^2*((2*b^3*c^3)/d^4 - (c*((9*a*b^2)/d
 + (3*b^3*c^2)/d^3))/d + (6*a*b^2*c)/d^2) + (a^3*d^6 + b^3*c^6 + 3*a*b^2*c^
4*d^2 + 3*a^2*b*c^2*d^4)/(d*(c*d^5 + d^6*x)) + (b^3*x^5)/d - (b^3*c*x^4)/d^
2
```

sympy [B] time = 0.61, size = 153, normalized size = 9.00

$$-\frac{b^3cx^4}{d^2}+\frac{b^3x^5}{d}+x^3\left(\frac{3ab^2}{d}+\frac{b^3c^2}{d^3}\right)+x^2\left(-\frac{3ab^2c}{d^2}-\frac{b^3c^3}{d^4}\right)+x\left(\frac{3a^2b}{d}+\frac{3ab^2c^2}{d^3}+\frac{b^3c^4}{d^5}\right)+\frac{a^3d^6+3a^2bc^2d^4+3ab^2c^4d^2}{cd^6+d^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)**2,x)
```

```
[Out] -b**3*c*x**4/d**2 + b**3*x**5/d + x**3*(3*a*b**2/d + b**3*c**2/d**3) + x**2
*(-3*a*b**2*c/d**2 - b**3*c**3/d**4) + x*(3*a**2*b/d + 3*a*b**2*c**2/d**3 +
 b**3*c**4/d**5) + (a**3*d**6 + 3*a**2*b*c**2*d**4 + 3*a*b**2*c**4*d**2 + b
**3*c**6)/(c*d**6 + d**7*x)
```

3.43 $\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{a+cx^2} dx$

Optimal. Leaf size=240

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(cd^2-3ae^2\right)+a\left(ae^2(Be+3Cd)-cd^2(3Be+Cd)\right)\right)}{\sqrt{a}\,c^{5/2}}+\frac{\log\left(a+cx^2\right)\left(e(Ac-aC)\left(3cd^2-ae^2\right)\right.}{2c^3}$$

```
[Out] -(a*e^2*(B*e+3*C*d)-c*d*(C*d^2+3*e*(A*e+B*d)))*x/c^2-1/2*e*(a*C*e^2-c*(3*C*
d^2+e*(A*e+3*B*d)))*x^2/c^2+1/3*e^2*(B*e+3*C*d)*x^3/c+1/4*C*e^3*x^4/c+1/2*(
B*c*d*(-3*a*e^2+c*d^2)+(A*c-C*a)*e*(-a*e^2+3*c*d^2))*ln(c*x^2+a)/c^3+(A*c*d
*(-3*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d)-c*d^2*(3*B*e+C*d)))*arctan(x*c^(1/2)
/a^(1/2))/c^(5/2)/a^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 237, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1629, 635, 205, 260}

$$\frac{ex^2\left(-aCe^2+ce(Ae+3Bd)+3cCd^2\right)}{2c^2}+\frac{\log\left(a+cx^2\right)\left(e(Ac-aC)\left(3cd^2-ae^2\right)+Bcd\left(cd^2-3ae^2\right)\right)}{2c^3}+\frac{x\left(-ae^2(B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((c*C*d^3 + 3*c*d*e*(B*d + A*e) - a*e^2*(3*C*d + B*e))*x)/c^2 + (e*(3*c*C*d
^2 - a*C*e^2 + c*e*(3*B*d + A*e))*x^2)/(2*c^2) + (e^2*(3*C*d + B*e)*x^3)/(3
*c) + (C*e^3*x^4)/(4*c) + ((A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d + B*e
) - c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) +
((B*c*d*(c*d^2 - 3*a*e^2) + (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2]
)/(2*c^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx = \int\left(\frac{cCd^3+3cde(Bd+Ae)-ae^2(3Cd+Be)}{c^2}+\frac{e\left(3cCd^2-aCe^2+ce(3Bd+Ae\right.}{c^2}\right.$$

$$= \frac{\left(cCd^3+3cde(Bd+Ae)-ae^2(3Cd+Be)\right)x}{c^2}+\frac{e\left(3cCd^2-aCe^2+ce(3Bd+Ae\right.}{2c^2}$$

$$= \frac{\left(cCd^3+3cde(Bd+Ae)-ae^2(3Cd+Be)\right)x}{c^2}+\frac{e\left(3cCd^2-aCe^2+ce(3Bd+Ae\right.}{2c^2}$$

$$= \frac{\left(cCd^3+3cde(Bd+Ae)-ae^2(3Cd+Be)\right)x}{c^2}+\frac{e\left(3cCd^2-aCe^2+ce(3Bd+Ae\right.}{2c^2}$$

Mathematica [A] time = 0.24, size = 223, normalized size = 0.93

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(cd^2-3ae^2\right)+a\left(ae^2(Be+3Cd)-cd^2(3Be+Cd)\right)\right)}{\sqrt{a}\,c^{5/2}}+\frac{6\log\left(a+cx^2\right)\left(e(Ac-aC)\left(3cd^2-ae^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) - c*d^2*(C*d + 3*B*e)))*
ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + (c*x*(-6*a*e^2*(6*C*d + 2*
B*e + C*e*x) + 3*c*C*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 2*c*e*(3
*A*e*(6*d + e*x) + B*(18*d^2 + 9*d*e*x + 2*e^2*x^2))) + 6*(B*c*d*(c*d^2 - 3
*a*e^2) + (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2])/(12*c^3)
```

fricas [A] time = 0.90, size = 592, normalized size = 2.47

$$\left[\frac{3\,Cac^2e^3x^4+4\left(3\,Cac^2de^2+Bac^2e^3\right)x^3+6\left(3\,Cac^2d^2e+3\,Bac^2de^2-\left(Ca^2c-Aac^2\right)e^3\right)x^2+6\left(3\,Bacd^2e-Ba^2e^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*C*a*c^2*e^3*x^4 + 4*(3*C*a*c^2*d*e^2 + B*a*c^2*e^3)*x^3 + 6*(3*C*a
*c^2*d^2*e + 3*B*a*c^2*d*e^2 - (C*a^2*c - A*a*c^2)*e^3)*x^2 + 6*(3*B*a*c*d^
2*e - B*a^2*e^3 + (C*a*c - A*c^2)*d^3 - 3*(C*a^2 - A*a*c)*d*e^2)*sqrt(-a*c)
*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 12*(C*a*c^2*d^3 + 3*B*a*c^
2*d^2*e - B*a^2*c*e^3 - 3*(C*a^2*c - A*a*c^2)*d*e^2)*x + 6*(B*a*c^2*d^3 - 3
*B*a^2*c*d*e^2 - 3*(C*a^2*c - A*a*c^2)*d^2*e + (C*a^3 - A*a^2*c)*e^3)*log(c
*x^2 + a))/(a*c^3), 1/12*(3*C*a*c^2*e^3*x^4 + 4*(3*C*a*c^2*d*e^2 + B*a*c^2*
e^3)*x^3 + 6*(3*C*a*c^2*d^2*e + 3*B*a*c^2*d*e^2 - (C*a^2*c - A*a*c^2)*e^3)*
x^2 - 12*(3*B*a*c*d^2*e - B*a^2*e^3 + (C*a*c - A*c^2)*d^3 - 3*(C*a^2 - A*a*
c)*d*e^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 12*(C*a*c^2*d^3 + 3*B*a*c^2*d^2
*e - B*a^2*c*e^3 - 3*(C*a^2*c - A*a*c^2)*d*e^2)*x + 6*(B*a*c^2*d^3 - 3*B*a^
2*c*d*e^2 - 3*(C*a^2*c - A*a*c^2)*d^2*e + (C*a^3 - A*a^2*c)*e^3)*log(c*x^2
+ a))/(a*c^3)]
```

giac [A] time = 0.17, size = 279, normalized size = 1.16

$$-\frac{\left(Cacd^3-Ac^2d^3+3\,Bacd^2e-3\,Ca^2de^2+3\,Aacde^2-Ba^2e^3\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c^2}+\frac{\left(Bc^2d^3-3\,Cacd^2e+3\,Ac^2d^2e-3\,B\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a*c*d^3 - A*c^2*d^3 + 3*B*a*c*d^2*e - 3*C*a^2*d*e^2 + 3*A*a*c*d*e^2 - B
*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/2*(B*c^2*d^3 - 3*C*a*c*
d^2*e + 3*A*c^2*d^2*e - 3*B*a*c*d*e^2 + C*a^2*e^3 - A*a*c*e^3)*log(c*x^2 +
a)/c^3 + 1/12*(3*C*c^3*x^4*e^3 + 12*C*c^3*d*x^3*e^2 + 18*C*c^3*d^2*x^2*e +
12*C*c^3*d^3*x + 4*B*c^3*x^3*e^3 + 18*B*c^3*d*x^2*e^2 + 36*B*c^3*d^2*x*e -
6*C*a*c^2*x^2*e^3 + 6*A*c^3*x^2*e^3 - 36*C*a*c^2*d*x*e^2 + 36*A*c^3*d*x*e^2
 - 12*B*a*c^2*x*e^3)/c^4
```

maple [A] time = 0.01, size = 399, normalized size = 1.66

$$\frac{C e^3x^4}{4c}+\frac{B e^3x^3}{3c}+\frac{Cd e^2x^3}{c}-\frac{3Aad e^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\, c}+\frac{A e^3x^2}{2c}+\frac{A d^3 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}+\frac{B a^2e^3 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\, c^2}-\frac{3Ba d^2e \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x)
```

```
[Out] 1/4*C*e^3*x^4/c+1/3/c*B*x^3*e^3+1/c*C*x^3*d*e^2+1/2/c*A*x^2*e^3+3/2/c*B*x^2
*d*e^2-1/2/c^2*C*x^2*a*e^3+3/2/c*C*x^2*d^2*e+3/c*A*x*d*e^2-1/c^2*B*x*a*e^3+
3/c*B*x*d^2*e-3/c^2*C*x*a*d*e^2+1/c*C*x*d^3-1/2/c^2*ln(c*x^2+a)*A*a*e^3+3/2
/c*ln(c*x^2+a)*A*d^2*e-3/2/c^2*ln(c*x^2+a)*B*a*d*e^2+1/2/c*ln(c*x^2+a)*B*d^
3+1/2/c^3*ln(c*x^2+a)*C*a^2*e^3-3/2/c^2*ln(c*x^2+a)*C*a*d^2*e-3/c/(a*c)^(1/
2)*arctan(x*c/(a*c)^(1/2))*A*a*d*e^2+1/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*
A*d^3+1/c^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*a^2*e^3-3/c/(a*c)^(1/2)*a
rctan(x*c/(a*c)^(1/2))*B*a*d^2*e+3/c^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*
C*a^2*d*e^2-1/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*a*d^3
```

maxima [A] time = 0.98, size = 244, normalized size = 1.02

$$-\frac{\left(3\, Bacd^2e - Ba^2e^3 + \left(Cac - Ac^2\right)d^3 - 3\left(Ca^2 - Aac\right)de^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\, c^2}+\frac{3\, Cce^3x^4 + 4\left(3\, Ccde^2 + Bce^3\right)x^3 + 6 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -(3*B*a*c*d^2*e - B*a^2*e^3 + (C*a*c - A*c^2)*d^3 - 3*(C*a^2 - A*a*c)*d*e^2
)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/12*(3*C*c*e^3*x^4 + 4*(3*C*c*d*
e^2 + B*c*e^3)*x^3 + 6*(3*C*c*d^2*e + 3*B*c*d*e^2 - (C*a - A*c)*e^3)*x^2 +
12*(C*c*d^3 + 3*B*c*d^2*e - B*a*e^3 - 3*(C*a - A*c)*d*e^2)*x)/c^2 + 1/2*(B*
c^2*d^3 - 3*B*a*c*d*e^2 - 3*(C*a*c - A*c^2)*d^2*e + (C*a^2 - A*a*c)*e^3)*lo
g(c*x^2 + a)/c^3
```

mupad [B] time = 3.99, size = 277, normalized size = 1.15

$$x^2\left(\frac{3\, C\, d^2\, e + 3\, B\, d\, e^2 + A\, e^3}{2\, c} - \frac{C\, a\, e^3}{2\, c^2}\right) + x\left(\frac{C\, d^3 + 3\, B\, d^2\, e + 3\, A\, d\, e^2}{c} - \frac{a\,\left(B\, e^3 + 3\, C\, d\, e^2\right)}{c^2}\right) + \frac{x^3\,\left(B\, e^3 + 3\, C\, d \cdots\right.}{3\, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x)
```

```
[Out] x^2*((A*e^3 + 3*B*d*e^2 + 3*C*d^2*e)/(2*c) - (C*a*e^3)/(2*c^2)) + x*((C*d^3
 + 3*A*d*e^2 + 3*B*d^2*e)/c - (a*(B*e^3 + 3*C*d*e^2))/c^2) + (x^3*(B*e^3 +
3*C*d*e^2))/(3*c) + (C*e^3*x^4)/(4*c) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^2*d
^3 + B*a^2*e^3 - C*a*c*d^3 + 3*C*a^2*d*e^2 - 3*A*a*c*d*e^2 - 3*B*a*c*d^2*e)
```

```
)/(a^(1/2)*c^(5/2)) + (log(a + c*x^2)*(4*B*a*c^5*d^3 - 4*A*a^2*c^4*e^3 + 4*
C*a^3*c^3*e^3 - 12*B*a^2*c^4*d*e^2 - 12*C*a^2*c^4*d^2*e + 12*A*a*c^5*d^2*e)
)/(8*a*c^6)
```

sympy [B] time = 5.46, size = 1008, normalized size = 4.20

$$\frac{Ce^3x^4}{4c}+x^3\left(\frac{Be^3}{3c}+\frac{Cde^2}{c}\right)+x^2\left(\frac{Ae^3}{2c}+\frac{3Bde^2}{2c}-\frac{Cae^3}{2c^2}+\frac{3Cd^2e}{2c}\right)+x\left(\frac{3Ade^2}{c}-\frac{Bae^3}{c^2}+\frac{3Bd^2e}{c}-\frac{3Cade^2}{c^2}+\frac{Cd^3}{c}\right)+($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*e**3*x**4/(4*c) + x**3*(B*e**3/(3*c) + C*d*e**2/c) + x**2*(A*e**3/(2*c) +
 3*B*d*e**2/(2*c) - C*a*e**3/(2*c**2) + 3*C*d**2*e/(2*c)) + x*(3*A*d*e**2/c
 - B*a*e**3/c**2 + 3*B*d**2*e/c - 3*C*a*d*e**2/c**2 + C*d**3/c) + ((-A*a*c*
e**3 + 3*A*c**2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*e**3 - 3*C*a
*c*d**2*e)/(2*c**3) - sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B*a**2
*e**3 - 3*B*a*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**6))*log(x +
(A*a**2*c*e**3 - 3*A*a*c**2*d**2*e + 3*B*a**2*c*d*e**2 - B*a*c**2*d**3 - C*
a**3*e**3 + 3*C*a**2*c*d**2*e + 2*a*c**3*((-A*a*c*e**3 + 3*A*c**2*d**2*e -
3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*e**3 - 3*C*a*c*d**2*e)/(2*c**3) - sqr
t(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B*a**2*e**3 - 3*B*a*c*d**2*e +
3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**6)))/(-3*A*a*c**2*d*e**2 + A*c**3*d**
3 + B*a**2*c*e**3 - 3*B*a*c**2*d**2*e + 3*C*a**2*c*d*e**2 - C*a*c**2*d**3))
 + ((-A*a*c*e**3 + 3*A*c**2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*
e**3 - 3*C*a*c*d**2*e)/(2*c**3) + sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d
**3 + B*a**2*e**3 - 3*B*a*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**
6))*log(x + (A*a**2*c*e**3 - 3*A*a*c**2*d**2*e + 3*B*a**2*c*d*e**2 - B*a*c*
*2*d**3 - C*a**3*e**3 + 3*C*a**2*c*d**2*e + 2*a*c**3*((-A*a*c*e**3 + 3*A*c*
*2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*e**3 - 3*C*a*c*d**2*e)/(2
*c**3) + sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B*a**2*e**3 - 3*B*a
*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**6)))/(-3*A*a*c**2*d*e**2
+ A*c**3*d**3 + B*a**2*c*e**3 - 3*B*a*c**2*d**2*e + 3*C*a**2*c*d*e**2 - C*a
*c**2*d**3))
```

3.44 $\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{a+cx^2} \, dx$

Optimal. Leaf size=168

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(2Be+Cd)\right)\right)}{\sqrt{a}\,c^{5/2}}+\frac{\log\left(a+cx^2\right)\left(-aBe^2-2aCde+2Acde+Bcd^2\right)}{2c^2}-\frac{x}{}$$

```
[Out] -(a*C*e^2-c*(C*d^2+e*(A*e+2*B*d)))*x/c^2+1/2*e*(B*e+2*C*d)*x^2/c+1/3*C*e^2*
x^3/c+1/2*(2*A*c*d*e-B*a*e^2+B*c*d^2-2*C*a*d*e)*ln(c*x^2+a)/c^2+(A*c*(-a*e^
2+c*d^2)+a*(a*C*e^2-c*d*(2*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/c^(5/2)/a^(
1/2)
```

Rubi [A] time = 0.26, antiderivative size = 166, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1629, 635, 205, 260}

$$\frac{\log\left(a+cx^2\right)\left(-aBe^2-2aCde+2Acde+Bcd^2\right)}{2c^2}+\frac{x\left(-aCe^2+ce(Ae+2Bd)+cCd^2\right)}{c^2}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(cd^2-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((c*C*d^2 - a*C*e^2 + c*e*(2*B*d + A*e))*x)/c^2 + (e*(2*C*d + B*e)*x^2)/(2*
c) + (C*e^2*x^3)/(3*c) + ((A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*d*(C*d + 2*
B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + ((B*c*d^2 + 2*A*c*d
*e - 2*a*C*d*e - a*B*e^2)*Log[a + c*x^2])/(2*c^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx = \int \left(\frac{cCd^2-aCe^2+ce(2Bd+Ae)}{c^2} + \frac{e(2Cd+Be)x}{c} + \frac{Ce^2x^2}{c} + \frac{Ac\left(cd^2-ae^2\right)+\ldots}{\ldots}\right.$$

$$= \frac{\left(cCd^2-aCe^2+ce(2Bd+Ae)\right)x}{c^2} + \frac{e(2Cd+Be)x^2}{2c} + \frac{Ce^2x^3}{3c} + \frac{\int \frac{Ac\left(cd^2-ae^2\right)+a(\ldots}{\ldots}}{\ldots}$$

$$= \frac{\left(cCd^2-aCe^2+ce(2Bd+Ae)\right)x}{c^2} + \frac{e(2Cd+Be)x^2}{2c} + \frac{Ce^2x^3}{3c} + \frac{\left(Bcd^2+2Acde\ldots\right.}{\ldots}$$

$$= \frac{\left(cCd^2-aCe^2+ce(2Bd+Ae)\right)x}{c^2} + \frac{e(2Cd+Be)x^2}{2c} + \frac{Ce^2x^3}{3c} + \frac{\left(Ac\left(cd^2-ae^2\right)\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.17, size = 155, normalized size = 0.92

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(2Be+Cd)\right)\right)}{\sqrt{a}\,c^{5/2}} + \frac{x\left(-6aCe^2+3ce(2Ae+4Bd+Bex)+2cC\left(3d^2+3dex\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*d*(C*d + 2*B*e)))*ArcTan[(Sqrt[c]*x)
/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + (x*(-6*a*C*e^2 + 3*c*e*(4*B*d + 2*A*e + B*e*
x) + 2*c*C*(3*d^2 + 3*d*e*x + e^2*x^2)) + 3*(B*c*d^2 + 2*A*c*d*e - 2*a*C*d*
e - a*B*e^2)*Log[a + c*x^2])/(6*c^2)
```

fricas [A] time = 0.79, size = 404, normalized size = 2.40

$$\left[\frac{2\,Cac^2e^2x^3+3\left(2\,Cac^2de+Bac^2e^2\right)x^2-3\left(2\,Bacde+\left(Cac-Ac^2\right)d^2-\left(Ca^2-Aac\right)e^2\right)\sqrt{-ac}\,\log\left(\frac{cx^2+2\,\sqrt{-ac}\,x-a}{cx^2+a}\right)\ldots}{6\,ac^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*C*a*c^2*e^2*x^3 + 3*(2*C*a*c^2*d*e + B*a*c^2*e^2)*x^2 - 3*(2*B*a*c*
d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)*sqrt(-a*c)*log((c*x^2 + 2*
sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(C*a*c^2*d^2 + 2*B*a*c^2*d*e - (C*a^2*c
- A*a*c^2)*e^2)*x + 3*(B*a*c^2*d^2 - B*a^2*c*e^2 - 2*(C*a^2*c - A*a*c^2)*d*
e)*log(c*x^2 + a))/(a*c^3), 1/6*(2*C*a*c^2*e^2*x^3 + 3*(2*C*a*c^2*d*e + B*a
*c^2*e^2)*x^2 - 6*(2*B*a*c*d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)
*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 6*(C*a*c^2*d^2 + 2*B*a*c^2*d*e - (C*a^2*
c - A*a*c^2)*e^2)*x + 3*(B*a*c^2*d^2 - B*a^2*c*e^2 - 2*(C*a^2*c - A*a*c^2)*
d*e)*log(c*x^2 + a))/(a*c^3)]
```

giac [A] time = 0.16, size = 176, normalized size = 1.05

$$\frac{\left(Bcd^2-2\,Cade+2\,Acde-Bae^2\right)\log\left(cx^2+a\right)}{2\,c^2} - \frac{\left(Cacd^2-Ac^2d^2+2\,Bacde-Ca^2e^2+Aace^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c^2} + \frac{2\,C\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d^2 - 2*C*a*d*e + 2*A*c*d*e - B*a*e^2)*log(c*x^2 + a)/c^2 - (C*a*c
*d^2 - A*c^2*d^2 + 2*B*a*c*d*e - C*a^2*e^2 + A*a*c*e^2)*arctan(c*x/sqrt(a*c
```

```
))/(sqrt(a*c)*c^2) + 1/6*(2*C*c^2*x^3*e^2 + 6*C*c^2*d*x^2*e + 6*C*c^2*d^2*x
 + 3*B*c^2*x^2*e^2 + 12*B*c^2*d*x*e - 6*C*a*c*x*e^2 + 6*A*c^2*x*e^2)/c^3
```

maple [A] time = 0.01, size = 256, normalized size = 1.52

$$\frac{C e^2 x^3}{3c} - \frac{A a e^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\ c} + \frac{A d^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} - \frac{2 B a d e \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\ c} + \frac{B e^2 x^2}{2c} + \frac{C a^2 e^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\ c^2} - \frac{C a d^2 a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x)
```

```
[Out] 1/3*C*e^2*x^3/c+1/2/c*B*x^2*e^2+1/c*C*x^2*d*e+1/c*A*e^2*x+2/c*B*d*e*x-1/c^2
*a*C*e^2*x+1/c*C*d^2*x+1/c*ln(c*x^2+a)*A*d*e-1/2/c^2*ln(c*x^2+a)*B*a*e^2+1/
2/c*ln(c*x^2+a)*B*d^2-1/c^2*ln(c*x^2+a)*C*a*d*e-1/c/(a*c)^(1/2)*arctan(1/(a
*c)^(1/2)*c*x)*A*a*e^2+1/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d^2-2/c/(a
*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*a*d*e+1/c^2/(a*c)^(1/2)*arctan(1/(a*c
)^(1/2)*c*x)*a^2*C*e^2-1/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*a*d^2
```

maxima [A] time = 0.99, size = 161, normalized size = 0.96

$$\frac{\left(Bcd^2 - Bae^2 - 2\,(Ca - Ac)de\right)\log\left(cx^2 + a\right)}{2\,c^2} - \frac{\left(2\,Bacde + \left(Cac - Ac^2\right)d^2 - \left(Ca^2 - Aac\right)e^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c^2} + \frac{2\,C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*c*d^2 - B*a*e^2 - 2*(C*a - A*c)*d*e)*log(c*x^2 + a)/c^2 - (2*B*a*c*d
*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)*arctan(c*x/sqrt(a*c))/(sqrt
(a*c)*c^2) + 1/6*(2*C*c*e^2*x^3 + 3*(2*C*c*d*e + B*c*e^2)*x^2 + 6*(C*c*d^2
+ 2*B*c*d*e - (C*a - A*c)*e^2)*x)/c^2
```

mupad [B] time = 3.90, size = 181, normalized size = 1.08

$$x\left(\frac{C\,d^2 + 2\,B\,d\,e + A\,e^2}{c} - \frac{C\,a\,e^2}{c^2}\right) + \frac{x^2\,\left(B\,e^2 + 2\,C\,d\,e\right)}{2\,c} + \frac{C\,e^2\,x^3}{3\,c} - \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(-C\,a^2\,e^2 + C\,a\,c\,d^2 + 2\,B\,a\,c\,d\,e\right.}{\sqrt{a}\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x)
```

```
[Out] x*((A*e^2 + C*d^2 + 2*B*d*e)/c - (C*a*e^2)/c^2) + (x^2*(B*e^2 + 2*C*d*e))/(
2*c) + (C*e^2*x^3)/(3*c) - (atan((c^(1/2)*x)/a^(1/2))*(A*a*c*e^2 - C*a^2*e^
2 - A*c^2*d^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(a^(1/2)*c^(5/2)) + (log(a + c*x^
2)*(4*B*a*c^4*d^2 - 4*B*a^2*c^3*e^2 + 8*A*a*c^4*d*e - 8*C*a^2*c^3*d*e))/(8*
a*c^5)
```

sympy [B] time = 3.15, size = 638, normalized size = 3.80

$$\frac{Ce^2x^3}{3c} + x^2\left(\frac{Be^2}{2c} + \frac{Cde}{c}\right) + x\left(\frac{Ae^2}{c} + \frac{2Bde}{c} - \frac{Cae^2}{c^2} + \frac{Cd^2}{c}\right) + \left(-\frac{-2Acde + Bae^2 - Bcd^2 + 2Cade}{2c^2} - \frac{\sqrt{-ac^5}\,\left(-Aa\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*e**2*x**3/(3*c) + x**2*(B*e**2/(2*c) + C*d*e/c) + x*(A*e**2/c + 2*B*d*e/c
 - C*a*e**2/c**2 + C*d**2/c) + (-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2*C*a*
d*e)/(2*c**2) - sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*
a**2*e**2 - C*a*c*d**2)/(2*a*c**5))*log(x + (-2*A*a*c*d*e + B*a**2*e**2 - B
*a*c*d**2 + 2*C*a**2*d*e + 2*a*c**2*(-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2
*C*a*d*e)/(2*c**2) - sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e
 + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5)))/(-A*a*c*e**2 + A*c**2*d**2 - 2*B*
a*c*d*e + C*a**2*e**2 - C*a*c*d**2)) + (-(-2*A*c*d*e + B*a*e**2 - B*c*d**2
+ 2*C*a*d*e)/(2*c**2) + sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*
d*e + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5))*log(x + (-2*A*a*c*d*e + B*a**2*
e**2 - B*a*c*d**2 + 2*C*a**2*d*e + 2*a*c**2*(-(-2*A*c*d*e + B*a*e**2 - B*c*
d**2 + 2*C*a*d*e)/(2*c**2) + sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B
*a*c*d*e + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5)))/(-A*a*c*e**2 + A*c**2*d**
2 - 2*B*a*c*d*e + C*a**2*e**2 - C*a*c*d**2))
```

3.45 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx$

Optimal. Leaf size=93

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(Acd-a(Be+Cd))}{\sqrt{a}\,c^{3/2}}+\frac{\log\left(a+cx^2\right)(-aCe+Ace+Bcd)}{2c^2}+\frac{x(Be+Cd)}{c}+\frac{Cex^2}{2c}$$

```
[Out] (B*e+C*d)*x/c+1/2*C*e*x^2/c+1/2*(A*c*e+B*c*d-C*a*e)*ln(c*x^2+a)/c^2+(A*c*d-
a*(B*e+C*d))*arctan(x*c^(1/2)/a^(1/2))/c^(3/2)/a^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1629, 635, 205, 260}

$$\frac{\log\left(a+cx^2\right)(-aCe+Ace+Bcd)}{2c^2}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(Acd-a(Be+Cd))}{\sqrt{a}\,c^{3/2}}+\frac{x(Be+Cd)}{c}+\frac{Cex^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((C*d + B*e)*x)/c + (C*e*x^2)/(2*c) + ((A*c*d - a*(C*d + B*e))*ArcTan[(Sqrt
[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + ((B*c*d + A*c*e - a*C*e)*Log[a + c*x^2
])/(2*c^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx = \int \left(\frac{Cd+Be}{c}+\frac{Cex}{c}+\frac{Acd-a(Cd+Be)+(Bcd+Ace-aCe)x}{c\left(a+cx^2\right)}\right)dx$$
$$= \frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{\int \frac{Acd-a(Cd+Be)+(Bcd+Ace-aCe)x}{a+cx^2}\,dx}{c}$$
$$= \frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{(Bcd+Ace-aCe)\int \frac{x}{a+cx^2}\,dx}{c}+\frac{(Acd-a(Cd+Be))\int \frac{}{a}$$
$$= \frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{(Acd-a(Cd+Be))\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{(Bcd+Ace-aCe)\,\mathrm{lo}}{2c^2}$$

Mathematica [A] time = 0.09, size = 86, normalized size = 0.92

$$\frac{\log\left(a+cx^2\right)(-aCe+Ace+Bcd)-\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd-Acd)}{\sqrt{a}}+cx(2Be+2Cd+Cex)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] (c*x*(2*C*d + 2*B*e + C*e*x) - (2*Sqrt[c]*(-(A*c*d) + a*C*d + a*B*e)*ArcTan
[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] + (B*c*d + A*c*e - a*C*e)*Log[a + c*x^2])/(2
*c^2)
```

fricas [A] time = 0.58, size = 206, normalized size = 2.22

$$\left[\frac{Cacex^2-(Bae+(Ca-Ac)d)\sqrt{-ac}\,\log\left(\frac{cx^2+2\,\sqrt{-ac}\,x-a}{cx^2+a}\right)+2\,(Cacd+Bace)x+\left(Bacd-\left(Ca^2-Aac\right)e\right)\log\left(cx^2+\right.}{2\,ac^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(C*a*c*e*x^2 - (B*a*e + (C*a - A*c)*d)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(
-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a*c*d + B*a*c*e)*x + (B*a*c*d - (C*a^2 - A
*a*c)*e)*log(c*x^2 + a))/(a*c^2), 1/2*(C*a*c*e*x^2 - 2*(B*a*e + (C*a - A*c)
*d)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 2*(C*a*c*d + B*a*c*e)*x + (B*a*c*d -
(C*a^2 - A*a*c)*e)*log(c*x^2 + a))/(a*c^2)]
```

giac [A] time = 0.16, size = 91, normalized size = 0.98

$$-\frac{(Cad-Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c}+\frac{(Bcd-Cae+Ace)\log\left(cx^2+a\right)}{2\,c^2}+\frac{Ccx^2e+2\,Ccdx+2\,Bcxe}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a*d - A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + 1/2*(B*c*d -
 C*a*e + A*c*e)*log(c*x^2 + a)/c^2 + 1/2*(C*c*x^2*e + 2*C*c*d*x + 2*B*c*x*e
)/c^2
```

maple [A] time = 0.01, size = 133, normalized size = 1.43

$$\frac{Ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}-\frac{Bae\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\;c}-\frac{Cad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\;c}+\frac{Ce\,x^2}{2c}+\frac{Ae\ln\left(c\,x^2+a\right)}{2c}+\frac{Bd\ln\left(c\,x^2+a\right)}{2c}+\frac{Bex}{c}-\frac{Cae\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x)
```

```
[Out] 1/2*C*e*x^2/c+1/c*B*e*x+1/c*C*d*x+1/2/c*ln(c*x^2+a)*A*e+1/2/c*ln(c*x^2+a)*B
*d-1/2/c^2*ln(c*x^2+a)*a*C*e+1/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d-1/
c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*a*e-1/c/(a*c)^(1/2)*arctan(1/(a*c
)^(1/2)*c*x)*C*a*d
```

maxima [A] time = 0.97, size = 86, normalized size = 0.92

$$-\frac{(Bae+(Ca-Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c}+\frac{Cex^2+2\,(Cd+Be)x}{2\,c}+\frac{(Bcd-(Ca-Ac)e)\log\left(cx^2+a\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -(B*a*e + (C*a - A*c)*d)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + 1/2*(C*e*x^2
 + 2*(C*d + B*e)*x)/c + 1/2*(B*c*d - (C*a - A*c)*e)*log(c*x^2 + a)/c^2
```

mupad [B] time = 3.78, size = 97, normalized size = 1.04

$$\frac{x\,\left(B\,e+C\,d\right)}{c}-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(B\,a\,e-A\,c\,d+C\,a\,d\right)}{\sqrt{a}\,c^{3/2}}+\frac{C\,e\,x^2}{2\,c}+\frac{\ln\left(c\,x^2+a\right)\,\left(4\,A\,a\,c^3\,e+4\,B\,a\,c^3\,d-4\,C\,a^2\,c^2\,e\right)}{8\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x)
```

```
[Out] (x*(B*e + C*d))/c - (atan((c^(1/2)*x)/a^(1/2))*(B*a*e - A*c*d + C*a*d))/(a^
(1/2)*c^(3/2)) + (C*e*x^2)/(2*c) + (log(a + c*x^2)*(4*A*a*c^3*e + 4*B*a*c^3
*d - 4*C*a^2*c^2*e))/(8*a*c^4)
```

sympy [B] time = 1.66, size = 337, normalized size = 3.62

$$\frac{Cex^2}{2c}+x\left(\frac{Be}{c}+\frac{Cd}{c}\right)+\left(-\frac{-Ace-Bcd+Cae}{2c^2}-\frac{\sqrt{-ac^5}\,(-Acd+Bae+Cad)}{2ac^4}\right)\log\left(x+\frac{Aace+Bacd-Ca^2e-2a\cdots}{-A\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*e*x**2/(2*c) + x*(B*e/c + C*d/c) + (-(-A*c*e - B*c*d + C*a*e)/(2*c**2) -
sqrt(-a*c**5)*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4))*log(x + (A*a*c*e + B*a*c
*d - C*a**2*e - 2*a*c**2*(-(-A*c*e - B*c*d + C*a*e)/(2*c**2) - sqrt(-a*c**5
)*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4)))/(-A*c**2*d + B*a*c*e + C*a*c*d)) +
(-(-A*c*e - B*c*d + C*a*e)/(2*c**2) + sqrt(-a*c**5)*(-A*c*d + B*a*e + C*a*d
)/(2*a*c**4))*log(x + (A*a*c*e + B*a*c*d - C*a**2*e - 2*a*c**2*(-(-A*c*e -
B*c*d + C*a*e)/(2*c**2) + sqrt(-a*c**5)*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4)
))/(-A*c**2*d + B*a*c*e + C*a*c*d))
```

3.46 $\int \frac{A+Bx+Cx^2}{a+cx^2} dx$

Optimal. Leaf size=55

$$\frac{(Ac-aC)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}+\frac{Cx}{c}$$

```
[Out] C*x/c+1/2*B*ln(c*x^2+a)/c+(A*c-C*a)*arctan(x*c^(1/2)/a^(1/2))/c^(3/2)/a^(1/
2)
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1810, 635, 205, 260}

$$\frac{(Ac-aC)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2),x]
```

```
[Out] (C*x)/c + ((A*c - a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (B*
Log[a + c*x^2])/(2*c)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx = \int \left(\frac{C}{c}+\frac{Ac-aC+Bcx}{c\left(a+cx^2\right)}\right)dx$$
$$= \frac{Cx}{c}+\frac{\int \frac{Ac-aC+Bcx}{a+cx^2}\,dx}{c}$$
$$= \frac{Cx}{c}+B\int \frac{x}{a+cx^2}\,dx+\frac{(Ac-aC)\int \frac{1}{a+cx^2}\,dx}{c}$$
$$= \frac{Cx}{c}+\frac{(Ac-aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 1.02

$$-\frac{(aC-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2),x]
```

```
[Out] (C*x)/c - ((-(A*c) + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) +
(B*Log[a + c*x^2])/(2*c)
```

fricas [A] time = 0.80, size = 125, normalized size = 2.27

$$\left[\frac{2\,Cacx+Bac\log\left(cx^2+a\right)+(Ca-Ac)\sqrt{-ac}\,\log\left(\frac{cx^2-2\,\sqrt{-ac}\,x-a}{cx^2+a}\right)}{2\,ac^2},\ \frac{2\,Cacx+Bac\log\left(cx^2+a\right)-2\,(Ca-Ac)\sqrt{\cdots}}{2\,ac^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*C*a*c*x + B*a*c*log(c*x^2 + a) + (C*a - A*c)*sqrt(-a*c)*log((c*x^2
- 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a*c^2), 1/2*(2*C*a*c*x + B*a*c*log(c*x
^2 + a) - 2*(C*a - A*c)*sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(a*c^2)]
```

giac [A] time = 0.16, size = 48, normalized size = 0.87

$$\frac{Cx}{c}+\frac{B\log\left(cx^2+a\right)}{2\,c}-\frac{(Ca-Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] C*x/c + 1/2*B*log(c*x^2 + a)/c - (C*a - A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*
c)*c)
```

maple [A] time = 0.00, size = 59, normalized size = 1.07

$$\frac{A\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}-\frac{Ca\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c}+\frac{B\ln\left(c\,x^2+a\right)}{2c}+\frac{Cx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a),x)
```

```
[Out] C*x/c+1/2*B*ln(c*x^2+a)/c+1/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A-1/c/(a*
c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*a*C
```

maxima [A] time = 0.97, size = 48, normalized size = 0.87

$$\frac{Cx}{c} + \frac{B\log\left(cx^2 + a\right)}{2\,c} - \frac{(Ca - Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] C*x/c + 1/2*B*log(c*x^2 + a)/c - (C*a - A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*
c)*c)
```

mupad [B] time = 3.73, size = 56, normalized size = 1.02

$$\frac{B\,\ln\left(c\,x^2 + a\right)}{2\,c} + \frac{C\,x}{c} + \frac{A\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}} - \frac{C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2),x)
```

```
[Out] (B*log(a + c*x^2))/(2*c) + (C*x)/c + (A*atan((c^(1/2)*x)/a^(1/2)))/(a^(1/2)
*c^(1/2)) - (C*a^(1/2)*atan((c^(1/2)*x)/a^(1/2)))/c^(3/2)
```

sympy [B] time = 0.49, size = 156, normalized size = 2.84

$$\frac{Cx}{c} + \left(\frac{B}{2c} - \frac{\sqrt{-ac^3}\,(-Ac + Ca)}{2ac^3}\right)\log\left(x + \frac{Ba - 2ac\left(\frac{B}{2c} - \frac{\sqrt{-ac^3}\,(-Ac+Ca)}{2ac^3}\right)}{-Ac + Ca}\right) + \left(\frac{B}{2c} + \frac{\sqrt{-ac^3}\,(-Ac + Ca)}{2ac^3}\right)\log\left(x + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*x/c + (B/(2*c) - sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3))*log(x + (B*a - 2*
a*c*(B/(2*c) - sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3)))/(-A*c + C*a)) + (B/(
2*c) + sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3))*log(x + (B*a - 2*a*c*(B/(2*c)
 + sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3)))/(-A*c + C*a))
```

3.47 $\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)}\,dx$

Optimal. Leaf size=133

$$\frac{\log\left(a+cx^2\right)(aCe-Ace+Bcd)}{2c\left(ae^2+cd^2\right)}+\frac{\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{e\left(ae^2+cd^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe-aCd+Acd)}{\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)}$$

```
[Out] (A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e/(a*e^2+c*d^2)+1/2*(-A*c*e+B*c*d+C*a*e)*ln(c
*x^2+a)/c/(a*e^2+c*d^2)+(A*c*d+B*a*e-C*a*d)*arctan(x*c^(1/2)/a^(1/2))/(a*e^
2+c*d^2)/a^(1/2)/c^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1629, 635, 205, 260}

$$\frac{\log\left(a+cx^2\right)(aCe-Ace+Bcd)}{2c\left(ae^2+cd^2\right)}+\frac{\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{e\left(ae^2+cd^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe-aCd+Acd)}{\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)),x]
```

```
[Out] ((A*c*d - a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]*(c*d
^2 + a*e^2)) + ((C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/(e*(c*d^2 + a*e^2)) +
 ((B*c*d - A*c*e + a*C*e)*Log[a + c*x^2])/(2*c*(c*d^2 + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)}\,dx = \int \left(\frac{Cd^2-Bde+Ae^2}{\left(cd^2+ae^2\right)(d+ex)} + \frac{Acd-aCd+aBe+(Bcd-Ace+aCe)x}{\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)dx$$

$$= \frac{\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{e\left(cd^2+ae^2\right)} + \frac{\int \frac{Acd-aCd+aBe+(Bcd-Ace+aCe)x}{a+cx^2}\,dx}{cd^2+ae^2}$$

$$= \frac{\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{e\left(cd^2+ae^2\right)} + \frac{(Acd-aCd+aBe)\int \frac{1}{a+cx^2}\,dx}{cd^2+ae^2} + \frac{(Bcd-Ace+aCe)}{cd^2+ae^2}$$

$$= \frac{(Acd-aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}\,\left(cd^2+ae^2\right)} + \frac{\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{e\left(cd^2+ae^2\right)} + \frac{(Bcd-Ace+aC}{2c\left(cd^2\right.}$$

Mathematica [A] time = 0.10, size = 120, normalized size = 0.90

$$\frac{\sqrt{a}\,\left(e\log\left(a+cx^2\right)(aCe-Ace+Bcd)+2c\log(d+ex)\left(Ae^2-Bde+Cd^2\right)\right)+2\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe-aCd+A}{2\sqrt{a}\,ce\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)),x]
```

```
[Out] (2*Sqrt[c]*e*(A*c*d - a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]] + Sqrt[a]*
(2*c*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x] + e*(B*c*d - A*c*e + a*C*e)*Log[a
 + c*x^2]))/(2*Sqrt[a]*c*e*(c*d^2 + a*e^2))
```

fricas [A] time = 14.21, size = 262, normalized size = 1.97

$$\left[-\frac{\left(Bae^2-(Ca-Ac)de\right)\sqrt{-ac}\,\log\left(\frac{cx^2-2\sqrt{-ac}\,x-a}{cx^2+a}\right)-\left(Bacde+\left(Ca^2-Aac\right)e^2\right)\log\left(cx^2+a\right)-2\left(Cacd^2-Bacde+\right.}{2\left(ac^2d^2e+a^2ce^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*((B*a*e^2 - (C*a - A*c)*d*e)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x -
 a)/(c*x^2 + a)) - (B*a*c*d*e + (C*a^2 - A*a*c)*e^2)*log(c*x^2 + a) - 2*(C*
a*c*d^2 - B*a*c*d*e + A*a*c*e^2)*log(e*x + d))/(a*c^2*d^2*e + a^2*c*e^3), 1
/2*(2*(B*a*e^2 - (C*a - A*c)*d*e)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (B*a*c*
d*e + (C*a^2 - A*a*c)*e^2)*log(c*x^2 + a) + 2*(C*a*c*d^2 - B*a*c*d*e + A*a*
c*e^2)*log(e*x + d))/(a*c^2*d^2*e + a^2*c*e^3)]
```

giac [A] time = 0.16, size = 125, normalized size = 0.94

$$\frac{(Bcd+Cae-Ace)\log\left(cx^2+a\right)}{2\left(c^2d^2+ace^2\right)}+\frac{\left(Cd^2-Bde+Ae^2\right)\log\left(|xe+d|\right)}{cd^2e+ae^3}-\frac{(Cad-Acd-Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(cd^2+ae^2\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d + C*a*e - A*c*e)*log(c*x^2 + a)/(c^2*d^2 + a*c*e^2) + (C*d^2 - B
*d*e + A*e^2)*log(abs(x*e + d))/(c*d^2*e + a*e^3) - (C*a*d - A*c*d - B*a*e)
*arctan(c*x/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))
```

maple [A] time = 0.01, size = 247, normalized size = 1.86

$$\frac{Acd\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{ac}}+\frac{Bae\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{ac}}-\frac{Cad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{ac}}-\frac{Ae\ln\left(c\,x^{2}+a\right)}{2\left(a\,e^{2}+c\,d^{2}\right)}+\frac{Ae\ln\left(ex+d\right)}{a\,e^{2}+c\,d^{2}}+\frac{Bd\ln\left(c\,x^{2}+a\right)}{2a\,e^{2}+2c\,d^{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x)
```

```
[Out] 1/(a*e^2+c*d^2)*e*ln(e*x+d)*A-1/(a*e^2+c*d^2)*ln(e*x+d)*B*d+1/(a*e^2+c*d^2)
/e*ln(e*x+d)*C*d^2-1/2/(a*e^2+c*d^2)*ln(c*x^2+a)*A*e+1/2/(a*e^2+c*d^2)*ln(c
*x^2+a)*B*d+1/2/(a*e^2+c*d^2)/c*ln(c*x^2+a)*a*C*e+1/(a*e^2+c*d^2)/(a*c)^(1/
2)*arctan(1/(a*c)^(1/2)*c*x)*A*c*d+1/(a*e^2+c*d^2)/(a*c)^(1/2)*arctan(1/(a*
c)^(1/2)*c*x)*B*a*e-1/(a*e^2+c*d^2)/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C
*a*d
```

maxima [A] time = 0.97, size = 123, normalized size = 0.92

$$\frac{(Bcd+(Ca-Ac)e)\log\left(cx^{2}+a\right)}{2\left(c^{2}d^{2}+ace^{2}\right)}+\frac{\left(Cd^{2}-Bde+Ae^{2}\right)\log\left(ex+d\right)}{cd^{2}e+ae^{3}}+\frac{(Bae-(Ca-Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(cd^{2}+ae^{2}\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*c*d + (C*a - A*c)*e)*log(c*x^2 + a)/(c^2*d^2 + a*c*e^2) + (C*d^2 - B
*d*e + A*e^2)*log(e*x + d)/(c*d^2*e + a*e^3) + (B*a*e - (C*a - A*c)*d)*arct
an(c*x/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))
```

mupad [B] time = 6.49, size = 840, normalized size = 6.32

$$\frac{\ln\left(d+e\,x\right)\left(C\,d^{2}-B\,d\,e+A\,e^{2}\right)}{c\,d^{2}\,e+a\,e^{3}}-\frac{\ln\left(x\,\left(c\,e\,B^{2}-c\,d\,B\,C+a\,e\,C^{2}-A\,c\,e\,C\right)+C^{2}\,a\,d+\frac{\left(c^{2}\left(\frac{A\,a\,e}{2}-\frac{B\,a\,d}{2}\right)-c\left(\frac{C\,a^{2}\,e}{2}-\frac{A\,d}{}\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)*(d + e*x)),x)
```

```
[Out] (log(d + e*x)*(A*e^2 + C*d^2 - B*d*e))/(a*e^3 + c*d^2*e) - (log(x*(C^2*a*e
+ B^2*c*e - A*C*c*e - B*C*c*d) + C^2*a*d + ((c^2*((A*a*e)/2 - (B*a*d)/2) -
c*((C*a^2*e)/2 - (A*d*(-a*c^3)^(1/2))/2) + (B*a*e*(-a*c^3)^(1/2))/2 - (C*a*
d*(-a*c^3)^(1/2))/2)*(((x*(6*a*c^2*e^3 - 2*c^3*d^2*e) + 8*a*c^2*d*e^2)*(c^2
*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 - (A*d*(-a*c^3)^(1/2))/2) + (B*a*
e*(-a*c^3)^(1/2))/2 - (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^2 + a^2*c^2*e^2)
- x*(3*A*c^2*e^2 + 2*C*c^2*d^2 - 5*C*a*c*e^2 - B*c^2*d*e) + B*a*c*e^2 - A*c
^2*d*e + 5*C*a*c*d*e))/(a*c^3*d^2 + a^2*c^2*e^2) + A*B*c*e - A*C*c*d)*(c^2*
((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 - (A*d*(-a*c^3)^(1/2))/2) + (B*a*e
*(-a*c^3)^(1/2))/2 - (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^2 + a^2*c^2*e^2) -
 (log(x*(C^2*a*e + B^2*c*e - A*C*c*e - B*C*c*d) + C^2*a*d + ((c^2*((A*a*e)/
2 - (B*a*d)/2) - c*((C*a^2*e)/2 + (A*d*(-a*c^3)^(1/2))/2) - (B*a*e*(-a*c^3)
^(1/2))/2 + (C*a*d*(-a*c^3)^(1/2))/2)*(((x*(6*a*c^2*e^3 - 2*c^3*d^2*e) + 8*
a*c^2*d*e^2)*(c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 + (A*d*(-a*c^3)^
(1/2))/2) - (B*a*e*(-a*c^3)^(1/2))/2 + (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^
2 + a^2*c^2*e^2) - x*(3*A*c^2*e^2 + 2*C*c^2*d^2 - 5*C*a*c*e^2 - B*c^2*d*e)
```

```
+ B*a*c*e^2 - A*c^2*d*e + 5*C*a*c*d*e))/(a*c^3*d^2 + a^2*c^2*e^2) + A*B*c*e
 - A*C*c*d)*(c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 + (A*d*(-a*c^3)^(
1/2))/2) - (B*a*e*(-a*c^3)^(1/2))/2 + (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^2
 + a^2*c^2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a),x)
```

```
[Out] Timed out
```

3.48 $\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=214

$$\frac{\log\left(a+cx^2\right)\left(-aBe^2+2aCde-2Acde+Bcd^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{Ae^2-Bde+Cd^2}{e(d+ex)\left(ae^2+cd^2\right)}-\frac{\log(d+ex)\left(-aBe^2+2aCde-2Acde\right.}{\left(ae^2+cd^2\right)^2}$$

```
[Out] (-A*e^2+B*d*e-C*d^2)/e/(a*e^2+c*d^2)/(e*x+d)-(-2*A*c*d*e-B*a*e^2+B*c*d^2+2*
C*a*d*e)*ln(e*x+d)/(a*e^2+c*d^2)^2+1/2*(-2*A*c*d*e-B*a*e^2+B*c*d^2+2*C*a*d*
e)*ln(c*x^2+a)/(a*e^2+c*d^2)^2+(A*c*(-a*e^2+c*d^2)+a*(a*C*e^2-c*d*(-2*B*e+C
*d)))*arctan(x*c^(1/2)/a^(1/2))/(a*e^2+c*d^2)^2/a^(1/2)/c^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1629, 635, 205, 260}

$$\frac{\log\left(a+cx^2\right)\left(-aBe^2+2aCde-2Acde+Bcd^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{Ae^2-Bde+Cd^2}{e(d+ex)\left(ae^2+cd^2\right)}-\frac{\log(d+ex)\left(-aBe^2+2aCde-2Acde\right.}{\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)),x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)/(e*(c*d^2 + a*e^2)*(d + e*x))) + ((A*c*(c*d^2 - a
*e^2) + a*(a*C*e^2 - c*d*(C*d - 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt
[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) - ((B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^
2)*Log[d + e*x])/(c*d^2 + a*e^2)^2 + ((B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*
B*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)}\,dx = \int \left(\frac{Cd^2-Bde+Ae^2}{\left(cd^2+ae^2\right)(d+ex)^2} + \frac{e\left(-Bcd^2+2Acde-2aCde+aBe^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)} + \frac{Ac\left(cd^2-ae^2\right)+a}{}\right.$$

$$= -\frac{Cd^2-Bde+Ae^2}{e\left(cd^2+ae^2\right)(d+ex)} - \frac{\left(Bcd^2-2Acde+2aCde-aBe^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^2} + \int \frac{Ac\left(cd^2-ae^2\right)}{}$$

$$= -\frac{Cd^2-Bde+Ae^2}{e\left(cd^2+ae^2\right)(d+ex)} - \frac{\left(Bcd^2-2Acde+2aCde-aBe^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^2} + \frac{\left(c\left(Bcd^2-2\right.\right.}{}$$

$$= -\frac{Cd^2-Bde+Ae^2}{e\left(cd^2+ae^2\right)(d+ex)} + \frac{\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}\left(cd^2+ae^2\right)^2} -$$

Mathematica [A] time = 0.31, size = 188, normalized size = 0.88

$$\frac{\log\left(a+cx^2\right)\left(-aBe^2+2aCde-2Acde+Bcd^2\right)-\frac{2\left(ae^2+cd^2\right)\left(e(Ae-Bd)+Cd^2\right)}{e(d+ex)}+\log(d+ex)\left(2aBe^2-4aCde+4Acde\right.}{2\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)),x]
```

```
[Out] ((-2*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(e*(d + e*x)) + (2*(A*c*(c
*d^2 - a*e^2) + a*(a*C*e^2 + c*d*(-(C*d) + 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt
[a]])/(Sqrt[a]*Sqrt[c]) + (-2*B*c*d^2 + 4*A*c*d*e - 4*a*C*d*e + 2*a*B*e^2)*
Log[d + e*x] + (B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2)*Log[a + c*x^2])/
(2*(c*d^2 + a*e^2)^2)
```

fricas [B] time = 69.91, size = 904, normalized size = 4.22

$$\left[-\frac{2\,Cac^2d^4-2\,Bac^2d^3e-2\,Ba^2cde^3+2\,Aa^2ce^4+2\left(Ca^2c+Aac^2\right)d^2e^2-\left(2\,Bacd^2e^2-\left(Cac-Ac^2\right)d^3e+\left(Ca^2-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*C*a*c^2*d^4 - 2*B*a*c^2*d^3*e - 2*B*a^2*c*d*e^3 + 2*A*a^2*c*e^4 +
2*(C*a^2*c + A*a*c^2)*d^2*e^2 - (2*B*a*c*d^2*e^2 - (C*a*c - A*c^2)*d^3*e +
(C*a^2 - A*a*c)*d*e^3 + (2*B*a*c*d*e^3 - (C*a*c - A*c^2)*d^2*e^2 + (C*a^2 -
 A*a*c)*e^4)*x)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) -
(B*a*c^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*c^2*d
^2*e^2 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*log(c*x^2 + a) + 2*(
B*a*c^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*c^2*d^
2*e^2 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*log(e*x + d))/(a*c^3*
d^5*e + 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5 + (a*c^3*d^4*e^2 + 2*a^2*c^2*d^2*e^
4 + a^3*c*e^6)*x), -1/2*(2*C*a*c^2*d^4 - 2*B*a*c^2*d^3*e - 2*B*a^2*c*d*e^3
+ 2*A*a^2*c*e^4 + 2*(C*a^2*c + A*a*c^2)*d^2*e^2 - 2*(2*B*a*c*d^2*e^2 - (C*a
*c - A*c^2)*d^3*e + (C*a^2 - A*a*c)*d*e^3 + (2*B*a*c*d*e^3 - (C*a*c - A*c^2
)*d^2*e^2 + (C*a^2 - A*a*c)*e^4)*x)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (B*a*
c^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*c^2*d^2*e^
2 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*log(c*x^2 + a) + 2*(B*a*c
^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*c^2*d^2*e^2
```

```
 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*log(e*x + d))/(a*c^3*d^5*e
 + 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5 + (a*c^3*d^4*e^2 + 2*a^2*c^2*d^2*e^4 + a
^3*c*e^6)*x)]
```

giac [A] time = 0.17, size = 270, normalized size = 1.26

$$-\frac{\left(Cacd^2e^2 - Ac^2d^2e^2 - 2\,Bacde^3 - Ca^2e^4 + Aace^4\right)\arctan\left(\frac{\left(cd - \frac{cd^2}{xe+d} - \frac{ae^2}{xe+d}\right)e^{(-1)}}{\sqrt{ac}}\right)e^{(-2)}}{\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)\sqrt{ac}} + \frac{\left(Bcd^2 + 2\,Cade - 2\,Acde\right.}{2\left(c^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a*c*d^2*e^2 - A*c^2*d^2*e^2 - 2*B*a*c*d*e^3 - C*a^2*e^4 + A*a*c*e^4)*ar
ctan((c*d - c*d^2/(x*e + d) - a*e^2/(x*e + d))*e^(-1)/sqrt(a*c))*e^(-2)/((c
^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) + 1/2*(B*c*d^2 + 2*C*a*d*e - 2
*A*c*d*e - B*a*e^2)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*
e + d)^2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - (C*d^2*e/(x*e + d) - B*d*e^
2/(x*e + d) + A*e^3/(x*e + d))/(c*d^2*e^2 + a*e^4)
```

maple [B] time = 0.01, size = 462, normalized size = 2.16

$$-\frac{Aac\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}} + \frac{A\,c^2d^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}} + \frac{2Bacde\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}} + \frac{C\,a^2e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}} - \frac{Cac\,d^2\arctan\left(\frac{cx}{\sqrt{a}}\right.}{\left(a\,e^2+c\,d^2\right)^2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x)
```

```
[Out] -1/(a*e^2+c*d^2)*e/(e*x+d)*A+1/(a*e^2+c*d^2)/(e*x+d)*B*d-1/(a*e^2+c*d^2)/e/
(e*x+d)*C*d^2+2/(a*e^2+c*d^2)^2*ln(e*x+d)*A*c*d*e+1/(a*e^2+c*d^2)^2*ln(e*x+
d)*B*a*e^2-1/(a*e^2+c*d^2)^2*ln(e*x+d)*B*c*d^2-2/(a*e^2+c*d^2)^2*ln(e*x+d)*
C*a*d*e-1/(a*e^2+c*d^2)^2*c*ln(c*x^2+a)*A*d*e-1/2/(a*e^2+c*d^2)^2*ln(c*x^2+
a)*e^2*B*a+1/2/(a*e^2+c*d^2)^2*c*ln(c*x^2+a)*d^2*B+1/(a*e^2+c*d^2)^2*ln(c*x
^2+a)*C*a*d*e-1/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*a*c
*e^2+1/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c^2*d^2+2/(a
*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*a*c*d*e+1/(a*e^2+c*d^
2)^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*a^2*C*e^2-1/(a*e^2+c*d^2)^2/(a*c
)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*a*c*d^2
```

maxima [A] time = 1.04, size = 255, normalized size = 1.19

$$\frac{\left(Bcd^2 - Bae^2 + 2\,(Ca - Ac)de\right)\log\left(cx^2+a\right)}{2\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)} - \frac{\left(Bcd^2 - Bae^2 + 2\,(Ca - Ac)de\right)\log\left(ex+d\right)}{c^2d^4 + 2\,acd^2e^2 + a^2e^4} + \frac{\left(2\,Bacde - \left(Cac - \right.\right.}{\left(c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*c*d^2 - B*a*e^2 + 2*(C*a - A*c)*d*e)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c
*d^2*e^2 + a^2*e^4) - (B*c*d^2 - B*a*e^2 + 2*(C*a - A*c)*d*e)*log(e*x + d)/
(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + (2*B*a*c*d*e - (C*a*c - A*c^2)*d^2 +
(C*a^2 - A*a*c)*e^2)*arctan(c*x/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*
e^4)*sqrt(a*c)) - (C*d^2 - B*d*e + A*e^2)/(c*d^3*e + a*d*e^3 + (c*d^2*e^2 +
 a*e^4)*x)
```

mupad [B] time = 6.77, size = 1199, normalized size = 5.60

$$\ln\left(C\,c\,d^4\,(-a\,c)^{3/2}-A\,a\,e^4\,(-a\,c)^{3/2}+3\,B\,a\,c^3\,d^4+3\,B\,a^3\,c\,e^4+A\,c^4\,d^4\,x+A\,c^3\,d^4\,\sqrt{-a\,c}-C\,a^3\,e^4\,\sqrt{-a\,c}-C\,a\,c^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)*(d + e*x)^2),x)
```

```
[Out] (log(C*c*d^4*(-a*c)^(3/2) - A*a*e^4*(-a*c)^(3/2) + 3*B*a*c^3*d^4 + 3*B*a^3*
c*e^4 + A*c^4*d^4*x + A*c^3*d^4*(-a*c)^(1/2) - C*a^3*e^4*(-a*c)^(1/2) - C*a
*c^3*d^4*x - C*a^3*c*e^4*x + 14*A*c*d^2*e^2*(-a*c)^(3/2) - 14*C*a*d^2*e^2*(
-a*c)^(3/2) - 3*B*c^3*d^4*x*(-a*c)^(1/2) + 8*A*a^2*c^2*d*e^3 + 8*C*a^2*c^2*
d^3*e + A*a^2*c^2*e^4*x - 10*B*a^2*c^2*d^2*e^2 + 8*B*a*d*e^3*(-a*c)^(3/2) -
 8*B*c*d^3*e*(-a*c)^(3/2) + 3*B*a*e^4*x*(-a*c)^(3/2) - 8*A*a*c^3*d^3*e - 8*
C*a^3*c*d*e^3 + 14*C*a^2*c^2*d^2*e^2*x + 8*A*c*d*e^3*x*(-a*c)^(3/2) - 8*C*a
*d*e^3*x*(-a*c)^(3/2) + 8*C*c*d^3*e*x*(-a*c)^(3/2) + 8*B*a*c^3*d^3*e*x + 8*
A*c^3*d^3*e*x*(-a*c)^(1/2) - 10*B*c*d^2*e^2*x*(-a*c)^(3/2) - 14*A*a*c^3*d^2
*e^2*x - 8*B*a^2*c^2*d*e^3*x)*(c^2*(a*((B*d^2)/2 - A*d*e) + (A*d^2*(-a*c)^(
1/2))/2) - c*(a^2*((B*e^2)/2 - C*d*e) + a*((A*e^2*(-a*c)^(1/2))/2 + (C*d^2*
(-a*c)^(1/2))/2 - B*d*e*(-a*c)^(1/2))) + (C*a^2*e^2*(-a*c)^(1/2))/2))/(a*c^
3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - (log(d + e*x)*(c*(B*d^2 - 2*A*d*e)
 - a*(B*e^2 - 2*C*d*e)))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) - (log(A*a*e^4
*(-a*c)^(3/2) - C*c*d^4*(-a*c)^(3/2) + 3*B*a*c^3*d^4 + 3*B*a^3*c*e^4 + A*c^
4*d^4*x - A*c^3*d^4*(-a*c)^(1/2) + C*a^3*e^4*(-a*c)^(1/2) - C*a*c^3*d^4*x -
 C*a^3*c*e^4*x - 14*A*c*d^2*e^2*(-a*c)^(3/2) + 14*C*a*d^2*e^2*(-a*c)^(3/2)
+ 3*B*c^3*d^4*x*(-a*c)^(1/2) + 8*A*a^2*c^2*d*e^3 + 8*C*a^2*c^2*d^3*e + A*a^
2*c^2*e^4*x - 10*B*a^2*c^2*d^2*e^2 - 8*B*a*d*e^3*(-a*c)^(3/2) + 8*B*c*d^3*e
*(-a*c)^(3/2) - 3*B*a*e^4*x*(-a*c)^(3/2) - 8*A*a*c^3*d^3*e - 8*C*a^3*c*d*e^
3 + 14*C*a^2*c^2*d^2*e^2*x - 8*A*c*d*e^3*x*(-a*c)^(3/2) + 8*C*a*d*e^3*x*(-a
*c)^(3/2) - 8*C*c*d^3*e*x*(-a*c)^(3/2) + 8*B*a*c^3*d^3*e*x - 8*A*c^3*d^3*e*
x*(-a*c)^(1/2) + 10*B*c*d^2*e^2*x*(-a*c)^(3/2) - 14*A*a*c^3*d^2*e^2*x - 8*B
*a^2*c^2*d*e^3*x)*(c*(a^2*((B*e^2)/2 - C*d*e) - a*((A*e^2*(-a*c)^(1/2))/2 +
 (C*d^2*(-a*c)^(1/2))/2 - B*d*e*(-a*c)^(1/2))) - c^2*(a*((B*d^2)/2 - A*d*e)
 - (A*d^2*(-a*c)^(1/2))/2) + (C*a^2*e^2*(-a*c)^(1/2))/2))/(a*c^3*d^4 + a^3*
c*e^4 + 2*a^2*c^2*d^2*e^2) - (A*e^2 + C*d^2 - B*d*e)/(e*(a*e^2 + c*d^2)*(d
+ e*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a),x)
```

```
[Out] Timed out
```

3.49 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=305

$$\frac{\log\left(a+cx^2\right)\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)}{2\left(ae^2+cd^2\right)^3}-\frac{Ae^2-Bde+Cd^2}{2e(d+ex)^2\left(ae^2+cd^2\right)}+\frac{-aBe^2+2aCde-2Acde}{(d+ex)\left(ae^2+cd^2\right.}$$

```
[Out] 1/2*(-A*e^2+B*d*e-C*d^2)/e/(a*e^2+c*d^2)/(e*x+d)^2+(-2*A*c*d*e-B*a*e^2+B*c*
d^2+2*C*a*d*e)/(a*e^2+c*d^2)^2/(e*x+d)-(B*c*d*(-3*a*e^2+c*d^2)-(A*c-C*a)*e*
(-a*e^2+3*c*d^2))*ln(e*x+d)/(a*e^2+c*d^2)^3+1/2*(B*c*d*(-3*a*e^2+c*d^2)-(A*
c-C*a)*e*(-a*e^2+3*c*d^2))*ln(c*x^2+a)/(a*e^2+c*d^2)^3+(A*c*d*(-3*a*e^2+c*d
^2)-a*(c*d^2*(-3*B*e+C*d)-a*e^2*(-B*e+3*C*d)))*arctan(x*c^(1/2)/a^(1/2))*c^
(1/2)/(a*e^2+c*d^2)^3/a^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1629, 635, 205, 260}

$$\frac{\log\left(a+cx^2\right)\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)}{2\left(ae^2+cd^2\right)^3}-\frac{Ae^2-Bde+Cd^2}{2e(d+ex)^2\left(ae^2+cd^2\right)}+\frac{-aBe^2+2aCde-2Acde}{(d+ex)\left(ae^2+cd^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)),x]
```

```
[Out] -(C*d^2 - B*d*e + A*e^2)/(2*e*(c*d^2 + a*e^2)*(d + e*x)^2) + (B*c*d^2 - 2*A
*c*d*e + 2*a*C*d*e - a*B*e^2)/((c*d^2 + a*e^2)^2*(d + e*x)) + (Sqrt[c]*(A*c
*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e)))*ArcTa
n[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^3) - ((B*c*d*(c*d^2 - 3*a*
e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[d + e*x])/(c*d^2 + a*e^2)^3 + (
(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2])
/(2*(c*d^2 + a*e^2)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)}\,dx = \int \left(\frac{Cd^2-Bde+Ae^2}{\left(cd^2+ae^2\right)(d+ex)^3} + \frac{e\left(-Bcd^2+2Acde-2aCde+aBe^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)^2} + \frac{e\left(-Bcd\left(cd^2-3ae\right.\right.}{\left(cd^2\right.}\right.$$

$$= -\frac{Cd^2-Bde+Ae^2}{2e\left(cd^2+ae^2\right)(d+ex)^2} + \frac{Bcd^2-2Acde+2aCde-aBe^2}{\left(cd^2+ae^2\right)^2(d+ex)} - \frac{\left(Bcd\left(cd^2-3ae^2\right)-(Ac\right.}{(}$$

$$= -\frac{Cd^2-Bde+Ae^2}{2e\left(cd^2+ae^2\right)(d+ex)^2} + \frac{Bcd^2-2Acde+2aCde-aBe^2}{\left(cd^2+ae^2\right)^2(d+ex)} - \frac{\left(Bcd\left(cd^2-3ae^2\right)-(Ac\right.}{(}$$

$$= -\frac{Cd^2-Bde+Ae^2}{2e\left(cd^2+ae^2\right)(d+ex)^2} + \frac{Bcd^2-2Acde+2aCde-aBe^2}{\left(cd^2+ae^2\right)^2(d+ex)} + \frac{\sqrt{c}\left(Acd\left(cd^2-3ae^2\right)-\right.}{}$$

Mathematica [**A**] time = 0.31, size = 277, normalized size = 0.91

$$\frac{\log\left(a+cx^2\right)\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right) - \frac{\left(ae^2+cd^2\right)^2\left(e(Ae-Bd)+Cd^2\right)}{e(d+ex)^2} + \frac{2\left(ae^2+cd^2\right)\left(-aBe^2+2aCde-2Acde+\right.}{d+ex}}{2\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)),x]
```

```
[Out] (-(((c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(e*(d + e*x)^2)) + (2*(c*
d^2 + a*e^2)*(B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2))/(d + e*x) + (2*Sq
rt[c]*(A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d - B*e) + c*d^2*(-(C*d) + 3
*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] - 2*(B*c*d*(c*d^2 - 3*a*e^2) -
 (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[d + e*x] + (B*c*d*(c*d^2 - 3*a*e^2) -
 (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**A**] time = 0.20, size = 489, normalized size = 1.60

$$\frac{\left(Bc^2d^3+3\,Cacd^2e-3\,Ac^2d^2e-3\,Bacde^2-Ca^2e^3+Aace^3\right)\log\left(cx^2+a\right)}{2\left(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6\right)} - \frac{\left(Bc^2d^3e+3\,Cacd^2e^2-3\,Ac^2d^2e^2-3\right.}{c^3d^6e+3\,ac^2d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*c^2*d^3 + 3*C*a*c*d^2*e - 3*A*c^2*d^2*e - 3*B*a*c*d*e^2 - C*a^2*e^3
+ A*a*c*e^3)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 +
a^3*e^6) - (B*c^2*d^3*e + 3*C*a*c*d^2*e^2 - 3*A*c^2*d^2*e^2 - 3*B*a*c*d*e^3
 - C*a^2*e^4 + A*a*c*e^4)*log(abs(x*e + d))/(c^3*d^6*e + 3*a*c^2*d^4*e^3 +
3*a^2*c*d^2*e^5 + a^3*e^7) - (C*a*c^2*d^3 - A*c^3*d^3 - 3*B*a*c^2*d^2*e - 3
*C*a^2*c*d*e^2 + 3*A*a*c^2*d*e^2 + B*a^2*c*e^3)*arctan(c*x/sqrt(a*c))/((c^3
```

```
*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*sqrt(a*c)) - 1/2*(C*c^2
*d^6 - 3*B*c^2*d^5*e - 2*C*a*c*d^4*e^2 + 5*A*c^2*d^4*e^2 - 2*B*a*c*d^3*e^3
- 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 + B*a^2*d*e^5 + A*a^2*e^6 - 2*(B*c^2*d^
4*e^2 + 2*C*a*c*d^3*e^3 - 2*A*c^2*d^3*e^3 + 2*C*a^2*d*e^5 - 2*A*a*c*d*e^5 -
 B*a^2*e^6)*x)*e^(-1)/((c*d^2 + a*e^2)^3*(x*e + d)^2)
```

maple [B] time = 0.02, size = 754, normalized size = 2.47

$$-\frac{3Aa\,c^2d\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^3\sqrt{ac}}+\frac{A\,c^3d^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^3\sqrt{ac}}-\frac{B\,a^2c\,e^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^3\sqrt{ac}}+\frac{3Ba\,c^2d^2e\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^3\sqrt{ac}}+\frac{3C\,a^2cd\,e^2}{\left(a\,e^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x)
```

```
[Out] -1/2/(a*e^2+c*d^2)*e/(e*x+d)^2*A+1/2/(a*e^2+c*d^2)/(e*x+d)^2*B*d+2/(a*e^2+c
*d^2)^2/(e*x+d)*C*a*d*e+1/2*c/(a*e^2+c*d^2)^3*ln(c*x^2+a)*A*e^3*a-3/2*c^2/(
a*e^2+c*d^2)^3*ln(c*x^2+a)*A*d^2*e+c^3/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(1
/(a*c)^(1/2)*c*x)*A*d^3-1/(a*e^2+c*d^2)^3*ln(e*x+d)*A*c*e^3*a+3/(a*e^2+c*d^
2)^3*ln(e*x+d)*A*c^2*d^2*e-2/(a*e^2+c*d^2)^2/(e*x+d)*A*c*d*e-3/2*c/(a*e^2+c
*d^2)^3*ln(c*x^2+a)*B*d*e^2*a+3/2*c/(a*e^2+c*d^2)^3*ln(c*x^2+a)*C*a*d^2*e-c
/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*a^2*e^3-c^2/(a*e^2
+c*d^2)^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*a*d^3+3/(a*e^2+c*d^2)^3*l
n(e*x+d)*B*c*d*e^2*a-3/(a*e^2+c*d^2)^3*ln(e*x+d)*C*a*c*d^2*e-3*c^2/(a*e^2+c
*d^2)^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*a*d*e^2+3*c^2/(a*e^2+c*d^2)
^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*a*d^2*e+3*c/(a*e^2+c*d^2)^3/(a*c
)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*a^2*d*e^2+1/2*c^2/(a*e^2+c*d^2)^3*ln(c*
x^2+a)*d^3*B-1/(a*e^2+c*d^2)^3*ln(e*x+d)*d^3*c^2*B+1/(a*e^2+c*d^2)^3*ln(e*x
+d)*C*a^2*e^3-1/2/(a*e^2+c*d^2)/e/(e*x+d)^2*C*d^2-1/(a*e^2+c*d^2)^2/(e*x+d)
*B*a*e^2+1/(a*e^2+c*d^2)^2/(e*x+d)*B*c*d^2-1/2/(a*e^2+c*d^2)^3*ln(c*x^2+a)*
C*a^2*e^3
```

maxima [A] time = 1.05, size = 495, normalized size = 1.62

$$\frac{\left(Bc^2d^3-3\,Bacde^2+3\left(Cac-Ac^2\right)d^2e-\left(Ca^2-Aac\right)e^3\right)\log\left(cx^2+a\right)}{2\left(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6\right)}-\frac{\left(Bc^2d^3-3\,Bacde^2+3\left(Cac-Ac^2\right)d^2e\right.}{c^3d^6+3\,ac^2d^4e^2+3\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*c^2*d^3 - 3*B*a*c*d*e^2 + 3*(C*a*c - A*c^2)*d^2*e - (C*a^2 - A*a*c)*
e^3)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)
 - (B*c^2*d^3 - 3*B*a*c*d*e^2 + 3*(C*a*c - A*c^2)*d^2*e - (C*a^2 - A*a*c)*e
^3)*log(e*x + d)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) +
(3*B*a*c^2*d^2*e - B*a^2*c*e^3 - (C*a*c^2 - A*c^3)*d^3 + 3*(C*a^2*c - A*a*c
^2)*d*e^2)*arctan(c*x/sqrt(a*c))/((c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*
e^4 + a^3*e^6)*sqrt(a*c)) - 1/2*(C*c*d^4 - 3*B*c*d^3*e + B*a*d*e^3 + A*a*e^
4 - (3*C*a - 5*A*c)*d^2*e^2 - 2*(B*c*d^2*e^2 - B*a*e^4 + 2*(C*a - A*c)*d*e^
3)*x)/(c^2*d^6*e + 2*a*c*d^4*e^3 + a^2*d^2*e^5 + (c^2*d^4*e^3 + 2*a*c*d^2*e
^5 + a^2*e^7)*x^2 + 2*(c^2*d^5*e^2 + 2*a*c*d^3*e^4 + a^2*d*e^6)*x)
```

mupad [B] time = 9.19, size = 2980, normalized size = 9.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)*(d + e*x)^3),x)
```

```
[Out] (log(d + e*x)*(e^3*(C*a^2 - A*a*c) - B*c^2*d^3 + d^2*e*(3*A*c^2 - 3*C*a*c)
+ 3*B*a*c*d*e^2))/(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4) -
 (log(9*A^2*a^5*e^10*(-a*c)^(5/2) + A^2*c^5*d^10*(-a*c)^(5/2) - B^2*a^7*e^1
0*(-a*c)^(3/2) - 9*B^2*c^3*d^10*(-a*c)^(7/2) + 9*C^2*a^9*e^10*(-a*c)^(1/2)
+ C^2*c*d^10*(-a*c)^(9/2) + 9*C^2*a^9*c*e^10*x - 6*A^2*a*d^4*e^6*(-a*c)^(9/
2) - 6*B^2*a*d^6*e^4*(-a*c)^(9/2) + 106*A^2*c*d^6*e^4*(-a*c)^(9/2) + 77*C^2
*a*d^8*e^2*(-a*c)^(9/2) - 27*B^2*c*d^8*e^2*(-a*c)^(9/2) + A^2*a^2*c^8*d^10*
x + 9*A^2*a^7*c^3*e^10*x + 9*B^2*a^3*c^7*d^10*x + B^2*a^8*c^2*e^10*x + C^2*
a^4*c^6*d^10*x + 27*A^2*a^3*d^2*e^8*(-a*c)^(7/2) - 106*B^2*a^3*d^4*e^6*(-a*
c)^(7/2) + 77*B^2*a^5*d^2*e^8*(-a*c)^(5/2) - 77*A^2*c^3*d^8*e^2*(-a*c)^(7/2
) - 106*C^2*a^3*d^6*e^4*(-a*c)^(7/2) - 6*C^2*a^5*d^4*e^6*(-a*c)^(5/2) + 27*
C^2*a^7*d^2*e^8*(-a*c)^(3/2) + 18*A*C*a^7*e^10*(-a*c)^(3/2) + 2*A*C*c^3*d^1
0*(-a*c)^(7/2) + 224*A*B*a*d^5*e^5*(-a*c)^(9/2) - 48*A*B*a^5*d*e^9*(-a*c)^(
5/2) - 212*A*C*a*d^6*e^4*(-a*c)^(9/2) + 64*A*B*c*d^7*e^3*(-a*c)^(9/2) + 48*
A*B*c^3*d^9*e*(-a*c)^(7/2) - 64*B*C*a*d^7*e^3*(-a*c)^(9/2) - 48*B*C*a^7*d*e
^9*(-a*c)^(3/2) - 154*A*C*c*d^8*e^2*(-a*c)^(9/2) + 77*A^2*a^3*c^7*d^8*e^2*x
 + 106*A^2*a^4*c^6*d^6*e^4*x - 6*A^2*a^5*c^5*d^4*e^6*x - 27*A^2*a^6*c^4*d^2
*e^8*x - 27*B^2*a^4*c^6*d^8*e^2*x - 6*B^2*a^5*c^5*d^6*e^4*x + 106*B^2*a^6*c
^4*d^4*e^6*x + 77*B^2*a^7*c^3*d^2*e^8*x + 77*C^2*a^5*c^5*d^8*e^2*x + 106*C^
2*a^6*c^4*d^6*e^4*x - 6*C^2*a^7*c^3*d^4*e^6*x - 27*C^2*a^8*c^2*d^2*e^8*x -
2*A*C*a^3*c^7*d^10*x - 18*A*C*a^8*c^2*e^10*x - 64*A*B*a^3*d^3*e^7*(-a*c)^(7
/2) - 12*A*C*a^3*d^4*e^6*(-a*c)^(7/2) + 54*A*C*a^5*d^2*e^8*(-a*c)^(5/2) + 2
24*B*C*a^3*d^5*e^5*(-a*c)^(7/2) - 64*B*C*a^5*d^3*e^7*(-a*c)^(5/2) + 48*B*C*
c*d^9*e*(-a*c)^(9/2) - 48*A*B*a^3*c^7*d^9*e*x - 48*A*B*a^7*c^3*d*e^9*x + 48
*B*C*a^4*c^6*d^9*e*x + 48*B*C*a^8*c^2*d*e^9*x + 64*A*B*a^4*c^6*d^7*e^3*x +
224*A*B*a^5*c^5*d^5*e^5*x + 64*A*B*a^6*c^4*d^3*e^7*x - 154*A*C*a^4*c^6*d^8*
e^2*x - 212*A*C*a^5*c^5*d^6*e^4*x + 12*A*C*a^6*c^4*d^4*e^6*x + 54*A*C*a^7*c
^3*d^2*e^8*x - 64*B*C*a^5*c^5*d^7*e^3*x - 224*B*C*a^6*c^4*d^5*e^5*x - 64*B*
C*a^7*c^3*d^3*e^7*x)*(e^2*((3*B*a^2*c*d)/2 - (3*C*a^2*d*(-a*c)^(1/2))/2 + (
3*A*a*c*d*(-a*c)^(1/2))/2) + e^3*((C*a^3)/2 - (A*a^2*c)/2 + (B*a^2*(-a*c)^(
1/2))/2) - e*((3*C*a^2*c*d^2)/2 - (3*A*a*c^2*d^2)/2 + (3*B*a*c*d^2*(-a*c)^(
1/2))/2) - (B*a*c^2*d^3)/2 - (A*c^2*d^3*(-a*c)^(1/2))/2 + (C*a*c*d^3*(-a*c)
^(1/2))/2))/(a^4*e^6 + a*c^3*d^6 + 3*a^3*c*d^2*e^4 + 3*a^2*c^2*d^4*e^2) - (
log(B^2*a^7*e^10*(-a*c)^(3/2) - A^2*c^5*d^10*(-a*c)^(5/2) - 9*A^2*a^5*e^10*
(-a*c)^(5/2) + 9*B^2*c^3*d^10*(-a*c)^(7/2) - 9*C^2*a^9*e^10*(-a*c)^(1/2) -
C^2*c*d^10*(-a*c)^(9/2) + 9*C^2*a^9*c*e^10*x + 6*A^2*a*d^4*e^6*(-a*c)^(9/2)
 + 6*B^2*a*d^6*e^4*(-a*c)^(9/2) - 106*A^2*c*d^6*e^4*(-a*c)^(9/2) - 77*C^2*a
*d^8*e^2*(-a*c)^(9/2) + 27*B^2*c*d^8*e^2*(-a*c)^(9/2) + A^2*a^2*c^8*d^10*x
+ 9*A^2*a^7*c^3*e^10*x + 9*B^2*a^3*c^7*d^10*x + B^2*a^8*c^2*e^10*x + C^2*a^
4*c^6*d^10*x - 27*A^2*a^3*d^2*e^8*(-a*c)^(7/2) + 106*B^2*a^3*d^4*e^6*(-a*c)
^(7/2) - 77*B^2*a^5*d^2*e^8*(-a*c)^(5/2) + 77*A^2*c^3*d^8*e^2*(-a*c)^(7/2)
+ 106*C^2*a^3*d^6*e^4*(-a*c)^(7/2) + 6*C^2*a^5*d^4*e^6*(-a*c)^(5/2) - 27*C^
2*a^7*d^2*e^8*(-a*c)^(3/2) - 18*A*C*a^7*e^10*(-a*c)^(3/2) - 2*A*C*c^3*d^10*
(-a*c)^(7/2) - 224*A*B*a*d^5*e^5*(-a*c)^(9/2) + 48*A*B*a^5*d*e^9*(-a*c)^(5/
2) + 212*A*C*a*d^6*e^4*(-a*c)^(9/2) - 64*A*B*c*d^7*e^3*(-a*c)^(9/2) - 48*A*
B*c^3*d^9*e*(-a*c)^(7/2) + 64*B*C*a*d^7*e^3*(-a*c)^(9/2) + 48*B*C*a^7*d*e^9
*(-a*c)^(3/2) + 154*A*C*c*d^8*e^2*(-a*c)^(9/2) + 77*A^2*a^3*c^7*d^8*e^2*x +
 106*A^2*a^4*c^6*d^6*e^4*x - 6*A^2*a^5*c^5*d^4*e^6*x - 27*A^2*a^6*c^4*d^2*e
^8*x - 27*B^2*a^4*c^6*d^8*e^2*x - 6*B^2*a^5*c^5*d^6*e^4*x + 106*B^2*a^6*c^4
*d^4*e^6*x + 77*B^2*a^7*c^3*d^2*e^8*x + 77*C^2*a^5*c^5*d^8*e^2*x + 106*C^2*
a^6*c^4*d^6*e^4*x - 6*C^2*a^7*c^3*d^4*e^6*x - 27*C^2*a^8*c^2*d^2*e^8*x - 2*
A*C*a^3*c^7*d^10*x - 18*A*C*a^8*c^2*e^10*x + 64*A*B*a^3*d^3*e^7*(-a*c)^(7/2
) + 12*A*C*a^3*d^4*e^6*(-a*c)^(7/2) - 54*A*C*a^5*d^2*e^8*(-a*c)^(5/2) - 224
*B*C*a^3*d^5*e^5*(-a*c)^(7/2) + 64*B*C*a^5*d^3*e^7*(-a*c)^(5/2) - 48*B*C*c*
d^9*e*(-a*c)^(9/2) - 48*A*B*a^3*c^7*d^9*e*x - 48*A*B*a^7*c^3*d*e^9*x + 48*B
*C*a^4*c^6*d^9*e*x + 48*B*C*a^8*c^2*d*e^9*x + 64*A*B*a^4*c^6*d^7*e^3*x + 22
4*A*B*a^5*c^5*d^5*e^5*x + 64*A*B*a^6*c^4*d^3*e^7*x - 154*A*C*a^4*c^6*d^8*e^
2*x - 212*A*C*a^5*c^5*d^6*e^4*x + 12*A*C*a^6*c^4*d^4*e^6*x + 54*A*C*a^7*c^3
*d^2*e^8*x - 64*B*C*a^5*c^5*d^7*e^3*x - 224*B*C*a^6*c^4*d^5*e^5*x - 64*B*C*
```

```
a^7*c^3*d^3*e^7*x)*(e^2*((3*B*a^2*c*d)/2 + (3*C*a^2*d*(-a*c)^(1/2))/2 - (3*
A*a*c*d*(-a*c)^(1/2))/2) - e^3*((A*a^2*c)/2 - (C*a^3)/2 + (B*a^2*(-a*c)^(1/
2))/2) + e*((3*A*a*c^2*d^2)/2 - (3*C*a^2*c*d^2)/2 + (3*B*a*c*d^2*(-a*c)^(1/
2))/2) - (B*a*c^2*d^3)/2 + (A*c^2*d^3*(-a*c)^(1/2))/2 - (C*a*c*d^3*(-a*c)^(
1/2))/2))/(a^4*e^6 + a*c^3*d^6 + 3*a^3*c*d^2*e^4 + 3*a^2*c^2*d^4*e^2) - ((A
*a*e^4 + C*c*d^4 + B*a*d*e^3 - 3*B*c*d^3*e + 5*A*c*d^2*e^2 - 3*C*a*d^2*e^2)
/(2*e*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(B*a*e^3 + 2*A*c*d*e^2 - 2*
C*a*d*e^2 - B*c*d^2*e))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))/(d^2 + e^2*x^2
 + 2*d*e*x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a),x)
[Out] Timed out
```

3.50 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^2} dx$

Optimal. Leaf size=216

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)-a\left(3ae^2(Be+3Cd)-cd^2(3Be+Cd)\right)\right)}{2a^{3/2}c^{5/2}}-\frac{e\log\left(a+cx^2\right)\left(2aCe^2-c\left(e(Ae+3Bd)\right.\right.}{2c^3}$$

[Out] -3/2*e^2*(A*c*d-a*(B*e+3*C*d))*x/a/c^2-1/2*(A*c-2*C*a)*e^3*x^2/a/c^2-1/2*(a*B-(A*c-C*a)*x)*(e*x+d)^3/a/c/(c*x^2+a)+1/2*(A*c*d*(3*a*e^2+c*d^2)-a*(3*a*e^2*(B*e+3*C*d)-c*d^2*(3*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/a^(3/2)/c^(5/2)-1/2*e*(2*a*C*e^2-c*(3*C*d^2+e*(A*e+3*B*d)))*ln(c*x^2+a)/c^3

Rubi [A] time = 0.50, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1645, 801, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)-a\left(3ae^2(Be+3Cd)-cd^2(3Be+Cd)\right)\right)}{2a^{3/2}c^{5/2}}-\frac{e\log\left(a+cx^2\right)\left(2aCe^2-c\left(e(Ae+3Bd)\right.\right.}{2c^3}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x]

[Out] (-3*e^2*(A*c*d - a*(3*C*d + B*e))*x)/(2*a*c^2) - ((A*c - 2*a*C)*e^3*x^2)/(2*a*c^2) - ((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(2*a*c*(a + c*x^2)) + ((A*c*d*(c*d^2 + 3*a*e^2) - a*(3*a*e^2*(3*C*d + B*e) - c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(5/2)) - (e*(2*a*C*e^2 - c*(3*C*d^2 + e*(3*B*d + A*e)))*Log[a + c*x^2])/(2*c^3)

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
```

```
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^2(-Acd-aCd-3aBe+2(Ac-2aC)ex)}{a+cx^2}\,dx}{2ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\left(a+cx^2\right)} - \frac{\int \left(\frac{3e^2(Acd-3aCd-aBe)}{c} + \frac{2(Ac-2aC)e^3x}{c} - \frac{Acd\left(cd^2\right.}{}\right.}{}$$

$$= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)}{2ac\left(a+cx^2\right)}$$

$$= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)}{2ac\left(a+cx^2\right)}$$

$$= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)}{2ac\left(a+cx^2\right)}$$

Mathematica [A] time = 0.20, size = 233, normalized size = 1.08

$$\frac{\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)+a\left(cd^2(3Be+Cd)-3ae^2(Be+3Cd)\right)\right)}{a^{3/2}} + \frac{-a^3Ce^3+a^2ce(e(Ae+3Bd+Bex)+3Cd(d+ex))-ac^2d\left(3Ae(d+ex)+Bd(d+3ex)+\right.}{a\left(a+cx^2\right)}}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] (2*c*e^2*(3*C*d + B*e)*x + c*C*e^3*x^2 + (-(a^3*C*e^3) + A*c^3*d^3*x - a*c^
2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d*(d + 3*e*x)) + a^2*c*e*(3*C*d*(d + e*x
) + e*(3*B*d + A*e + B*e*x))))/(a*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c*d^2 + 3*
a*e^2) + a*(-3*a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*
x)/Sqrt[a]])/a^(3/2) + e*(3*c*C*d^2 - 2*a*C*e^2 + c*e*(3*B*d + A*e))*Log[a
+ c*x^2])/(2*c^3)
```

fricas [B] time = 0.92, size = 931, normalized size = 4.31

$$\left[\frac{2\,Ca^2c^2e^3x^4 + 2\,Ca^3ce^3x^2 - 2\,Ba^2c^2d^3 + 6\,Ba^3cde^2 + 6\left(Ca^3c - Aa^2c^2\right)d^2e - 2\left(Ca^4 - Aa^3c\right)e^3 + 4\left(3\,Ca^2c^2de\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*C*a^2*c^2*e^3*x^4 + 2*C*a^3*c*e^3*x^2 - 2*B*a^2*c^2*d^3 + 6*B*a^3*c
*d*e^2 + 6*(C*a^3*c - A*a^2*c^2)*d^2*e - 2*(C*a^4 - A*a^3*c)*e^3 + 4*(3*C*a
^2*c^2*d*e^2 + B*a^2*c^2*e^3)*x^3 + (3*B*a^2*c*d^2*e - 3*B*a^3*e^3 + (C*a^2
*c + A*a*c^2)*d^3 - 3*(3*C*a^3 - A*a^2*c)*d*e^2 + (3*B*a*c^2*d^2*e - 3*B*a^
```

```
2*c*e^3 + (C*a*c^2 + A*c^3)*d^3 - 3*(3*C*a^2*c - A*a*c^2)*d*e^2)*x^2)*sqrt(
-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(3*B*a^2*c^2*d^2*e
- 3*B*a^3*c*e^3 + (C*a^2*c^2 - A*a*c^3)*d^3 - 3*(3*C*a^3*c - A*a^2*c^2)*d*e
^2)*x + 2*(3*C*a^3*c*d^2*e + 3*B*a^3*c*d*e^2 - (2*C*a^4 - A*a^3*c)*e^3 + (3
*C*a^2*c^2*d^2*e + 3*B*a^2*c^2*d*e^2 - (2*C*a^3*c - A*a^2*c^2)*e^3)*x^2)*lo
g(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3), 1/2*(C*a^2*c^2*e^3*x^4 + C*a^3*c*e^3
*x^2 - B*a^2*c^2*d^3 + 3*B*a^3*c*d*e^2 + 3*(C*a^3*c - A*a^2*c^2)*d^2*e - (C
*a^4 - A*a^3*c)*e^3 + 2*(3*C*a^2*c^2*d*e^2 + B*a^2*c^2*e^3)*x^3 + (3*B*a^2*
c*d^2*e - 3*B*a^3*e^3 + (C*a^2*c + A*a*c^2)*d^3 - 3*(3*C*a^3 - A*a^2*c)*d*e
^2 + (3*B*a*c^2*d^2*e - 3*B*a^2*c*e^3 + (C*a*c^2 + A*c^3)*d^3 - 3*(3*C*a^2*
c - A*a*c^2)*d*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (3*B*a^2*c^2*d^2
*e - 3*B*a^3*c*e^3 + (C*a^2*c^2 - A*a*c^3)*d^3 - 3*(3*C*a^3*c - A*a^2*c^2)*
d*e^2)*x + (3*C*a^3*c*d^2*e + 3*B*a^3*c*d*e^2 - (2*C*a^4 - A*a^3*c)*e^3 + (
3*C*a^2*c^2*d^2*e + 3*B*a^2*c^2*d*e^2 - (2*C*a^3*c - A*a^2*c^2)*e^3)*x^2)*l
og(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3)]
```

giac [A] time = 0.17, size = 289, normalized size = 1.34

$$\frac{\left(3\,Ccd^2e+3\,Bcde^2-2\,Cae^3+Ace^3\right)\log\left(cx^2+a\right)}{2\,c^3}+\frac{\left(Cacd^3+Ac^2d^3+3\,Bacd^2e-9\,Ca^2de^2+3\,Aacde^2-3\,Ba^2e^3\right.}{2\,\sqrt{ac}\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*C*c*d^2*e + 3*B*c*d*e^2 - 2*C*a*e^3 + A*c*e^3)*log(c*x^2 + a)/c^3 +
1/2*(C*a*c*d^3 + A*c^2*d^3 + 3*B*a*c*d^2*e - 9*C*a^2*d*e^2 + 3*A*a*c*d*e^2
- 3*B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2) + 1/2*(C*c^2*x^2*e^3
 + 6*C*c^2*d*x*e^2 + 2*B*c^2*x*e^3)/c^4 - 1/2*(B*a*c^2*d^3 - 3*C*a^2*c*d^2*
e + 3*A*a*c^2*d^2*e - 3*B*a^2*c*d*e^2 + C*a^3*e^3 - A*a^2*c*e^3 + (C*a*c^2*
d^3 - A*c^3*d^3 + 3*B*a*c^2*d^2*e - 3*C*a^2*c*d*e^2 + 3*A*a*c^2*d*e^2 - B*a
^2*c*e^3)*x)/((c*x^2 + a)*a*c^3)
```

maple [B] time = 0.02, size = 484, normalized size = 2.24

$$\frac{A\,d^3x}{2\left(c\,x^2+a\right)a}+\frac{A\,d^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;a}-\frac{3Ad\,e^2x}{2\left(c\,x^2+a\right)c}+\frac{3Ad\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;c}+\frac{Ba\,e^3x}{2\left(c\,x^2+a\right)c^2}-\frac{3Ba\,e^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;c^2}-\frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x)
```

```
[Out] -1/2/c/(c*x^2+a)*B*d^3+1/2/c^2*ln(c*x^2+a)*A*e^3+1/2*e^3/c^2*C*x^2+e^3/c^2*
B*x+3/2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d*e^2-3/2/c^2*a/(a*c)^(1/
2)*arctan(1/(a*c)^(1/2)*c*x)*B*e^3+3/2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c
*x)*B*d^2*e-3/2/c/(c*x^2+a)*A*x*d*e^2+1/2/c^2/(c*x^2+a)*B*x*a*e^3-3/2/c/(c*
x^2+a)*B*x*d^2*e+3/2/c^2/(c*x^2+a)*B*a*d*e^2+3/2/c^2/(c*x^2+a)*C*a*d^2*e+1/
2/(c*x^2+a)/a*x*A*d^3+3*e^2/c^2*C*d*x+3/2/c^2/(c*x^2+a)*C*x*a*d*e^2-9/2/c^2
*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d*e^2+3/2/c^2*ln(c*x^2+a)*B*d*e^
2-1/c^3*a*ln(c*x^2+a)*C*e^3+3/2/c^2*ln(c*x^2+a)*C*d^2*e+1/2/a/(a*c)^(1/2)*a
rctan(1/(a*c)^(1/2)*c*x)*A*d^3+1/2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*
C*d^3-1/2/c/(c*x^2+a)*C*x*d^3+1/2/c^2/(c*x^2+a)*A*a*e^3-3/2/c/(c*x^2+a)*A*d
^2*e-1/2/c^3/(c*x^2+a)*C*a^2*e^3
```

maxima [A] time = 0.98, size = 287, normalized size = 1.33

$$-\frac{Bac^2d^3-3\,Ba^2cde^2-3\left(Ca^2c-Aac^2\right)d^2e+\left(Ca^3-Aa^2c\right)e^3+\left(3\,Bac^2d^2e-Ba^2ce^3+\left(Cac^2-Ac^3\right)d^3-3\left(Ca^2c\right.\right.}{2\left(ac^4x^2+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a*c^2*d^3 - 3*B*a^2*c*d*e^2 - 3*(C*a^2*c - A*a*c^2)*d^2*e + (C*a^3
- A*a^2*c)*e^3 + (3*B*a*c^2*d^2*e - B*a^2*c*e^3 + (C*a*c^2 - A*c^3)*d^3 - 3
*(C*a^2*c - A*a*c^2)*d*e^2)*x)/(a*c^4*x^2 + a^2*c^3) + 1/2*(C*e^3*x^2 + 2*(
3*C*d*e^2 + B*e^3)*x)/c^2 + 1/2*(3*C*c*d^2*e + 3*B*c*d*e^2 - (2*C*a - A*c)*
e^3)*log(c*x^2 + a)/c^3 + 1/2*(3*B*a*c*d^2*e - 3*B*a^2*e^3 + (C*a*c + A*c^2
)*d^3 - 3*(3*C*a^2 - A*a*c)*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2)
```

mupad [B] time = 4.01, size = 303, normalized size = 1.40

$$\frac{x\,\left(B\,e^3+3\,C\,d\,e^2\right)}{c^2}-\frac{\frac{C\,a^2\,e^3-3\,C\,a\,c\,d^2\,e-3\,B\,a\,c\,d\,e^2-A\,a\,c\,e^3+B\,c^2\,d^3+3\,A\,c^2\,d^2\,e}{2\,c}-\frac{x\,\left(3\,C\,a^2\,d\,e^2+B\,a^2\,e^3-C\,a\,c\,d^3-3\,B\,a\,c\,d^2\,e-3\,A\,a\,c\,d\,e^2+A}{2\,a}}{c^3\,x^2+a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x)
```

```
[Out] (x*(B*e^3 + 3*C*d*e^2))/c^2 - ((B*c^2*d^3 + C*a^2*e^3 - A*a*c*e^3 + 3*A*c^2
*d^2*e - 3*B*a*c*d*e^2 - 3*C*a*c*d^2*e)/(2*c) - (x*(A*c^2*d^3 + B*a^2*e^3 -
 C*a*c*d^3 + 3*C*a^2*d*e^2 - 3*A*a*c*d*e^2 - 3*B*a*c*d^2*e))/(2*a))/(a*c^2
+ c^3*x^2) + (C*e^3*x^2)/(2*c^2) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^2*d^3 -
3*B*a^2*e^3 + C*a*c*d^3 - 9*C*a^2*d*e^2 + 3*A*a*c*d*e^2 + 3*B*a*c*d^2*e))/(
2*a^(3/2)*c^(5/2)) + (log(a + c*x^2)*(16*A*a^3*c^4*e^3 - 32*C*a^4*c^3*e^3 +
 48*B*a^3*c^4*d*e^2 + 48*C*a^3*c^4*d^2*e))/(32*a^3*c^6)
```

sympy [B] time = 34.46, size = 952, normalized size = 4.41

$$\frac{Ce^3x^2}{2c^2}+x\left(\frac{Be^3}{c^2}+\frac{3Cde^2}{c^2}\right)+\left(-\frac{e\left(-Ace^2-3Bcde+2Cae^2-3Ccd^2\right)}{2c^3}-\frac{\sqrt{-a^3c^7}\left(-3Aacde^2-Ac^2d^3+3Ba^2e^3-\right.}{4a^3c^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] C*e**3*x**2/(2*c**2) + x*(B*e**3/c**2 + 3*C*d*e**2/c**2) + (-e*(-A*c*e**2 -
 3*B*c*d*e + 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) - sqrt(-a**3*c**7)*(-3*A*a*c
*d*e**2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 -
C*a*c*d**3)/(4*a**3*c**6))*log(x + (2*A*a**2*c*e**3 + 6*B*a**2*c*d*e**2 - 4
*C*a**3*e**3 + 6*C*a**2*c*d**2*e - 4*a**2*c**3*(-e*(-A*c*e**2 - 3*B*c*d*e +
 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) - sqrt(-a**3*c**7)*(-3*A*a*c*d*e**2 - A*
c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 - C*a*c*d**3)/
(4*a**3*c**6)))/(-3*A*a*c**2*d*e**2 - A*c**3*d**3 + 3*B*a**2*c*e**3 - 3*B*a
*c**2*d**2*e + 9*C*a**2*c*d*e**2 - C*a*c**2*d**3)) + (-e*(-A*c*e**2 - 3*B*c
*d*e + 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) + sqrt(-a**3*c**7)*(-3*A*a*c*d*e**
2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 - C*a*c*
d**3)/(4*a**3*c**6))*log(x + (2*A*a**2*c*e**3 + 6*B*a**2*c*d*e**2 - 4*C*a**
3*e**3 + 6*C*a**2*c*d**2*e - 4*a**2*c**3*(-e*(-A*c*e**2 - 3*B*c*d*e + 2*C*a
*e**2 - 3*C*c*d**2)/(2*c**3) + sqrt(-a**3*c**7)*(-3*A*a*c*d*e**2 - A*c**2*d
**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 - C*a*c*d**3)/(4*a**
3*c**6)))/(-3*A*a*c**2*d*e**2 - A*c**3*d**3 + 3*B*a**2*c*e**3 - 3*B*a*c**2*
d**2*e + 9*C*a**2*c*d*e**2 - C*a*c**2*d**3)) + (A*a**2*c*e**3 - 3*A*a*c**2*
d**2*e + 3*B*a**2*c*d*e**2 - B*a*c**2*d**3 - C*a**3*e**3 + 3*C*a**2*c*d**2*
e + x*(-3*A*a*c**2*d*e**2 + A*c**3*d**3 + B*a**2*c*e**3 - 3*B*a*c**2*d**2*e
 + 3*C*a**2*c*d*e**2 - C*a*c**2*d**3))/(2*a**2*c**3 + 2*a*c**4*x**2)
```

3.51 $\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=146

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd(2aBe+aCd+Acd)+ae^2(Ac-3aC)\right)}{2a^{3/2}c^{5/2}}-\frac{(d+ex)^2(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}-\frac{e^2x(Ac-3aC)}{2ac^2}+\frac{e\log\left(a+\right.}{}$$

```
[Out] -1/2*(A*c-3*C*a)*e^2*x/a/c^2-1/2*(a*B-(A*c-C*a)*x)*(e*x+d)^2/a/c/(c*x^2+a)+
1/2*(a*(A*c-3*C*a)*e^2+c*d*(A*c*d+2*B*a*e+C*a*d))*arctan(x*c^(1/2)/a^(1/2))
/a^(3/2)/c^(5/2)+1/2*e*(B*e+2*C*d)*ln(c*x^2+a)/c^2
```

Rubi [A] time = 0.25, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1645, 774, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd(2aBe+aCd+Acd)+ae^2(Ac-3aC)\right)}{2a^{3/2}c^{5/2}}-\frac{(d+ex)^2(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}-\frac{e^2x(Ac-3aC)}{2ac^2}+\frac{e\log\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] -((A*c - 3*a*C)*e^2*x)/(2*a*c^2) - ((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(2*a
*c*(a + c*x^2)) + ((a*(A*c - 3*a*C)*e^2 + c*d*(A*c*d + a*C*d + 2*a*B*e))*Ar
cTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(5/2)) + (e*(2*C*d + B*e)*Log[a + c
*x^2])/(2*c^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
```

```
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)(-Acd-aCd-2aBe+(Ac-3aC)ex)}{a+cx^2}\,dx}{2ac}$$

$$= -\frac{(Ac-3aC)e^2x}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)} - \frac{\int \frac{-a(Ac-3aC)e^2+cd(-Acd-aCd}{}}{}$$

$$= -\frac{(Ac-3aC)e^2x}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)} + \frac{(e(2Cd+Be))\int \frac{x}{a+cx^2}\,dx}{c}$$

$$= -\frac{(Ac-3aC)e^2x}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)} + \frac{\left(a(Ac-3aC)e^2+cd(Acd\right.}{2a}$$

Mathematica [A] time = 0.14, size = 175, normalized size = 1.20

$$\frac{\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(ae^2+cd^2\right)+a\left(cd(2Be+Cd)-3aCe^2\right)\right)}{a^{3/2}}+\frac{\sqrt{c}\left(a^2e(Be+2Cd+Cex)-ac\left(Ae(2d+ex)+Bd(d+2ex)+Cd^2x\right)+Ac^2d^2x\right)}{a\left(a+cx^2\right)}+\sqrt{c}\,e\log\left(a+\right.}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] (2*Sqrt[c]*C*e^2*x + (Sqrt[c]*(A*c^2*d^2*x + a^2*e*(2*C*d + B*e + C*e*x) -
a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x))))/(a*(a + c*x^2)) + ((A*c
*(c*d^2 + a*e^2) + a*(-3*a*C*e^2 + c*d*(C*d + 2*B*e)))*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/a^(3/2) + Sqrt[c]*e*(2*C*d + B*e)*Log[a + c*x^2])/(2*c^(5/2))
```

fricas [B] time = 0.97, size = 631, normalized size = 4.32

$$\left[\frac{4\,Ca^2c^2e^2x^3-2\,Ba^2c^2d^2+2\,Ba^3ce^2+4\left(Ca^3c-Aa^2c^2\right)de-\left(2\,Ba^2cde+\left(Ca^2c+Aac^2\right)d^2-\left(3\,Ca^3-Aa^2c\right)e\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*C*a^2*c^2*e^2*x^3 - 2*B*a^2*c^2*d^2 + 2*B*a^3*c*e^2 + 4*(C*a^3*c -
A*a^2*c^2)*d*e - (2*B*a^2*c*d*e + (C*a^2*c + A*a*c^2)*d^2 - (3*C*a^3 - A*a^
2*c)*e^2 + (2*B*a*c^2*d*e + (C*a*c^2 + A*c^3)*d^2 - (3*C*a^2*c - A*a*c^2)*e
^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(2*B*
a^2*c^2*d*e + (C*a^2*c^2 - A*a*c^3)*d^2 - (3*C*a^3*c - A*a^2*c^2)*e^2)*x +
2*(2*C*a^3*c*d*e + B*a^3*c*e^2 + (2*C*a^2*c^2*d*e + B*a^2*c^2*e^2)*x^2)*log
(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3), 1/2*(2*C*a^2*c^2*e^2*x^3 - B*a^2*c^2*
d^2 + B*a^3*c*e^2 + 2*(C*a^3*c - A*a^2*c^2)*d*e + (2*B*a^2*c*d*e + (C*a^2*c
 + A*a*c^2)*d^2 - (3*C*a^3 - A*a^2*c)*e^2 + (2*B*a*c^2*d*e + (C*a*c^2 + A*c
^3)*d^2 - (3*C*a^2*c - A*a*c^2)*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) -
 (2*B*a^2*c^2*d*e + (C*a^2*c^2 - A*a*c^3)*d^2 - (3*C*a^3*c - A*a^2*c^2)*e^2
)*x + (2*C*a^3*c*d*e + B*a^3*c*e^2 + (2*C*a^2*c^2*d*e + B*a^2*c^2*e^2)*x^2)
*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3)]
```

giac [A] time = 0.17, size = 184, normalized size = 1.26

$$\frac{Cxe^2}{c^2}+\frac{\left(2\,Cde+Be^2\right)\log\left(cx^2+a\right)}{2\,c^2}+\frac{\left(Cacd^2+Ac^2d^2+2\,Bacde-3\,Ca^2e^2+Aace^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,ac^2}-\frac{Bacd^2-2\,Ca^2\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] C*x*e^2/c^2 + 1/2*(2*C*d*e + B*e^2)*log(c*x^2 + a)/c^2 + 1/2*(C*a*c*d^2 + A
*c^2*d^2 + 2*B*a*c*d*e - 3*C*a^2*e^2 + A*a*c*e^2)*arctan(c*x/sqrt(a*c))/(sq
rt(a*c)*a*c^2) - 1/2*(B*a*c*d^2 - 2*C*a^2*d*e + 2*A*a*c*d*e - B*a^2*e^2 + (
C*a*c*d^2 - A*c^2*d^2 + 2*B*a*c*d*e - C*a^2*e^2 + A*a*c*e^2)*x)/((c*x^2 + a
)*a*c^2)
```

maple [B] time = 0.01, size = 323, normalized size = 2.21

$$\frac{A\,d^2x}{2\left(c\,x^2+a\right)a}+\frac{A\,d^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;a}-\frac{A\,e^2x}{2\left(c\,x^2+a\right)c}+\frac{A\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;c}-\frac{Bdex}{\left(c\,x^2+a\right)c}+\frac{Bde\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\;c}+\frac{Ca\,e^2\cdots}{2\left(c\,x^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x)
```

```
[Out] C*e^2/c^2*x-1/2/c/(c*x^2+a)*A*e^2*x+1/2/(c*x^2+a)/a*x*A*d^2-1/c/(c*x^2+a)*B
*d*e*x+1/2/c^2/(c*x^2+a)*a*C*e^2*x-1/2/c/(c*x^2+a)*C*d^2*x-1/c/(c*x^2+a)*A*
d*e+1/2/c^2/(c*x^2+a)*B*a*e^2-1/2/c/(c*x^2+a)*B*d^2+1/c^2/(c*x^2+a)*C*a*d*e
+1/2/c^2*ln(c*x^2+a)*B*e^2+1/c^2*ln(c*x^2+a)*C*d*e+1/2/c/(a*c)^(1/2)*arctan
(1/(a*c)^(1/2)*c*x)*A*e^2+1/2/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d^2
+1/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*d*e-3/2/c^2*a/(a*c)^(1/2)*arct
an(1/(a*c)^(1/2)*c*x)*C*e^2+1/2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d
^2
```

maxima [A] time = 0.96, size = 188, normalized size = 1.29

$$\frac{Ce^2x}{c^2}-\frac{Bacd^2-Ba^2e^2-2\left(Ca^2-Aac\right)de+\left(2\,Bacde+\left(Cac-Ac^2\right)d^2-\left(Ca^2-Aac\right)e^2\right)x}{2\left(ac^3x^2+a^2c^2\right)}+\frac{\left(2\,Cde+Be^2\right)\log\left(cx^2\cdots\right.}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] C*e^2*x/c^2 - 1/2*(B*a*c*d^2 - B*a^2*e^2 - 2*(C*a^2 - A*a*c)*d*e + (2*B*a*c
*d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)*x)/(a*c^3*x^2 + a^2*c^2)
+ 1/2*(2*C*d*e + B*e^2)*log(c*x^2 + a)/c^2 + 1/2*(2*B*a*c*d*e + (C*a*c + A*
c^2)*d^2 - (3*C*a^2 - A*a*c)*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2)
```

mupad [B] time = 0.23, size = 195, normalized size = 1.34

$$\frac{C\,e^2\,x}{c^2}-\frac{\frac{x\left(-C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2-A\,c^2\,d^2\right)}{2\,a}-\frac{B\,a\,e^2}{2}+\frac{B\,c\,d^2}{2}+A\,c\,d\,e-C\,a\,d\,e}{c^3\,x^2+a\,c^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(-3\,C\,a^2\,e^2+C\,a\,c\,d\cdots\right.}{2\,a^{3/\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x)
```

```
[Out] (C*e^2*x)/c^2 - ((x*(A*a*c*e^2 - C*a^2*e^2 - A*c^2*d^2 + C*a*c*d^2 + 2*B*a*
c*d*e))/(2*a) - (B*a*e^2)/2 + (B*c*d^2)/2 + A*c*d*e - C*a*d*e)/(a*c^2 + c^3
```

```
*x^2) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^2*d^2 - 3*C*a^2*e^2 + A*a*c*e^2 + C
*a*c*d^2 + 2*B*a*c*d*e))/(2*a^(3/2)*c^(5/2)) + (log(a + c*x^2)*(16*B*a^3*c^
3*e^2 + 32*C*a^3*c^3*d*e))/(32*a^3*c^5)
```

sympy [B] time = 18.40, size = 593, normalized size = 4.06

$$\frac{Ce^2x}{c^2}+\left(\frac{e\,(Be+2Cd)}{2c^2}-\frac{\sqrt{-a^3c^5}\,\left(-Aace^2-Ac^2d^2-2Bacde+3Ca^2e^2-Cacd^2\right)}{4a^3c^5}\right)\log\left(x+\frac{2Ba^2e^2+4Ca^2de-}{-A}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] C*e**2*x/c**2 + (e*(B*e + 2*C*d)/(2*c**2) - sqrt(-a**3*c**5)*(-A*a*c*e**2 -
 A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)/(4*a**3*c**5))*log
(x + (2*B*a**2*e**2 + 4*C*a**2*d*e - 4*a**2*c**2*(e*(B*e + 2*C*d)/(2*c**2)
- sqrt(-a**3*c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2
 - C*a*c*d**2)/(4*a**3*c**5)))/(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3
*C*a**2*e**2 - C*a*c*d**2)) + (e*(B*e + 2*C*d)/(2*c**2) + sqrt(-a**3*c**5)*
(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)/(4*a
**3*c**5))*log(x + (2*B*a**2*e**2 + 4*C*a**2*d*e - 4*a**2*c**2*(e*(B*e + 2*
C*d)/(2*c**2) + sqrt(-a**3*c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e +
 3*C*a**2*e**2 - C*a*c*d**2)/(4*a**3*c**5)))/(-A*a*c*e**2 - A*c**2*d**2 - 2
*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)) + (-2*A*a*c*d*e + B*a**2*e**2 - B
*a*c*d**2 + 2*C*a**2*d*e + x*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*a
**2*e**2 - C*a*c*d**2))/(2*a**2*c**2 + 2*a*c**3*x**2)
```

3.52 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2} \, dx$

Optimal. Leaf size=97

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+Acd)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}+\frac{Ce\log\left(a+cx^2\right)}{2c^2}$$

```
[Out] -1/2*(a*B-(A*c-C*a)*x)*(e*x+d)/a/c/(c*x^2+a)+1/2*(A*c*d+B*a*e+C*a*d)*arctan
(x*c^(1/2)/a^(1/2))/a^(3/2)/c^(3/2)+1/2*C*e*ln(c*x^2+a)/c^2
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1645, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+Acd)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}+\frac{Ce\log\left(a+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x))/(2*a*c*(a + c*x^2)) + ((A*c*d + a*C*d +
a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2)) + (C*e*Log[a + c*x^
2])/(2*c^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\left(a+cx^2\right)} - \frac{\int \frac{-Acd-a(Cd+Be)-2aCex}{a+cx^2}\,dx}{2ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\left(a+cx^2\right)} + \frac{(Ce)\int \frac{x}{a+cx^2}\,dx}{c} + \frac{(Acd+aCd+aBe)\int \frac{1}{a+cx^2}}{2ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\left(a+cx^2\right)} + \frac{(Acd+aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}} + \frac{Ce\log\left(a\right.}{2c^2}$$

Mathematica [A] time = 0.09, size = 102, normalized size = 1.05

$$\frac{\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+Acd)}{a^{3/2}} + \frac{a^2Ce-ac(Ae+B(d+ex)+Cdx)+Ac^2dx}{a\left(a+cx^2\right)} + Ce\log\left(a+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] ((a^2*C*e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x)))/(a*(a + c*x^2)) +
(Sqrt[c]*(A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) + C*e
*Log[a + c*x^2])/(2*c^2)
```

fricas [A] time = 0.97, size = 337, normalized size = 3.47

$$\left[-\frac{2Ba^2cd + \left(Ba^2e + \left(Bace + \left(Cac + Ac^2\right)d\right)x^2 + \left(Ca^2 + Aac\right)d\right)\sqrt{-ac}\,\log\left(\frac{cx^2-2\sqrt{-ac}\,x-a}{cx^2+a}\right) - 2\left(Ca^3 - Aa^2c\right)e}{4\left(a^2c^3x^2 + a^3c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c*d + (B*a^2*e + (B*a*c*e + (C*a*c + A*c^2)*d)*x^2 + (C*a^2
+ A*a*c)*d)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(C
*a^3 - A*a^2*c)*e + 2*(B*a^2*c*e + (C*a^2*c - A*a*c^2)*d)*x - 2*(C*a^2*c*e*
x^2 + C*a^3*e)*log(c*x^2 + a))/(a^2*c^3*x^2 + a^3*c^2), -1/2*(B*a^2*c*d - (
B*a^2*e + (B*a*c*e + (C*a*c + A*c^2)*d)*x^2 + (C*a^2 + A*a*c)*d)*sqrt(a*c)*
arctan(sqrt(a*c)*x/a) - (C*a^3 - A*a^2*c)*e + (B*a^2*c*e + (C*a^2*c - A*a*c
^2)*d)*x - (C*a^2*c*e*x^2 + C*a^3*e)*log(c*x^2 + a))/(a^2*c^3*x^2 + a^3*c^2
)]
```

giac [A] time = 0.22, size = 112, normalized size = 1.15

$$\frac{Ce\log\left(cx^2+a\right)}{2c^2} + \frac{(Cad+Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\,ac} - \frac{(Cad-Acd+Bae)x + \frac{Bacd-Ca^2e+Aace}{c}}{2\left(cx^2+a\right)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*C*e*log(c*x^2 + a)/c^2 + 1/2*(C*a*d + A*c*d + B*a*e)*arctan(c*x/sqrt(a*
c))/(sqrt(a*c)*a*c) - 1/2*((C*a*d - A*c*d + B*a*e)*x + (B*a*c*d - C*a^2*e +
 A*a*c*e)/c)/((c*x^2 + a)*a*c)
```

maple [A] time = 0.01, size = 134, normalized size = 1.38

$$\frac{Ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;a}+\frac{Be\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;c}+\frac{Cd\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;c}+\frac{Ce\ln\left(c\,x^2+a\right)}{2c^2}+\frac{\frac{(Acd-Bae-Cad)x}{2ac}-\frac{Ace+Bcd-aCe}{2c^2}}{c\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x)
```

```
[Out] (1/2*(A*c*d-B*a*e-C*a*d)/a/c*x-1/2*(A*c*e+B*c*d-C*a*e)/c^2)/(c*x^2+a)+1/2*C
*e*ln(c*x^2+a)/c^2+1/2/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d+1/2/c/(a
*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*e+1/2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1
/2)*c*x)*C*d
```

maxima [A] time = 0.97, size = 113, normalized size = 1.16

$$\frac{Ce\log\left(cx^2+a\right)}{2\,c^2}-\frac{Bacd-\left(Ca^2-Aac\right)e+\left(Bace+\left(Cac-Ac^2\right)d\right)x}{2\left(ac^3x^2+a^2c^2\right)}+\frac{\left(Bae+(Ca+Ac)d\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*C*e*log(c*x^2 + a)/c^2 - 1/2*(B*a*c*d - (C*a^2 - A*a*c)*e + (B*a*c*e +
(C*a*c - A*c^2)*d)*x)/(a*c^3*x^2 + a^2*c^2) + 1/2*(B*a*e + (C*a + A*c)*d)*a
rctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c)
```

mupad [B] time = 0.14, size = 191, normalized size = 1.97

$$\frac{C\,e\,\ln\left(c\,x^2+a\right)}{2\,c^2}-\frac{B\,d}{2\,\left(c^2\,x^2+a\,c\right)}-\frac{B\,e\,x}{2\,\left(c^2\,x^2+a\,c\right)}-\frac{C\,d\,x}{2\,\left(c^2\,x^2+a\,c\right)}-\frac{A\,e}{2\,\left(c^2\,x^2+a\,c\right)}+\frac{C\,a\,e}{2\,\left(c^3\,x^2+a\,c^2\right)}+\frac{A\,d\,x}{2\,\left(a^2+c\,a\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x)
```

```
[Out] (C*e*log(a + c*x^2))/(2*c^2) - (B*d)/(2*(a*c + c^2*x^2)) - (B*e*x)/(2*(a*c
+ c^2*x^2)) - (C*d*x)/(2*(a*c + c^2*x^2)) - (A*e)/(2*(a*c + c^2*x^2)) + (C*
a*e)/(2*(a*c^2 + c^3*x^2)) + (A*d*x)/(2*(a^2 + a*c*x^2)) + (A*d*atan((c^(1/
2)*x)/a^(1/2)))/(2*a^(3/2)*c^(1/2)) + (B*e*atan((c^(1/2)*x)/a^(1/2)))/(2*a^
(1/2)*c^(3/2)) + (C*d*atan((c^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*c^(3/2))
```

sympy [B] time = 6.41, size = 318, normalized size = 3.28

$$\left(\frac{Ce}{2c^2}-\frac{\sqrt{-a^3c^5}\,\left(Acd+Bae+Cad\right)}{4a^3c^4}\right)\log\left(x+\frac{-2Ca^2e+4a^2c^2\left(\frac{Ce}{2c^2}-\frac{\sqrt{-a^3c^5}\,(Acd+Bae+Cad)}{4a^3c^4}\right)}{Ac^2d+Bace+Cacd}\right)+\left(\frac{Ce}{2c^2}+\frac{\sqrt{-a^3c^5}\,\left(Ac\right.}{4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] (C*e/(2*c**2) - sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*d)/(4*a**3*c**4))*log
(x + (-2*C*a**2*e + 4*a**2*c**2*(C*e/(2*c**2) - sqrt(-a**3*c**5)*(A*c*d + B
*a*e + C*a*d)/(4*a**3*c**4)))/(A*c**2*d + B*a*c*e + C*a*c*d)) + (C*e/(2*c**
2) + sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*d)/(4*a**3*c**4))*log(x + (-2*C*
a**2*e + 4*a**2*c**2*(C*e/(2*c**2) + sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*
d)/(4*a**3*c**4)))/(A*c**2*d + B*a*c*e + C*a*c*d)) + (-A*a*c*e - B*a*c*d +
C*a**2*e + x*(A*c**2*d - B*a*c*e - C*a*c*d))/(2*a**2*c**2 + 2*a*c**3*x**2)
```

3.53 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^2} \, dx$

Optimal. Leaf size=69

$$\frac{(aC+Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}-\frac{aB-x(Ac-aC)}{2ac\left(a+cx^2\right)}$$

```
[Out] 1/2*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)+1/2*(A*c+C*a)*arctan(x*c^(1/2)/a^(1/2)
)/a^(3/2)/c^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1814, 12, 205}

$$\frac{(aC+Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}-\frac{aB-x(Ac-aC)}{2ac\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^2,x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(2*a*c*(a + c*x^2)) + ((A*c + a*C)*ArcTan[(Sqrt[c]*x
)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^2} \, dx &= -\frac{aB-(Ac-aC)x}{2ac\left(a+cx^2\right)}-\frac{\int \frac{-A-\frac{aC}{c}}{a+cx^2} \, dx}{2a}\\
&= -\frac{aB-(Ac-aC)x}{2ac\left(a+cx^2\right)}+\frac{(Ac+aC)\int \frac{1}{a+cx^2} \, dx}{2ac}\\
&= -\frac{aB-(Ac-aC)x}{2ac\left(a+cx^2\right)}+\frac{(Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 0.99

$$\frac{(aC+Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}+\frac{-aB-aCx+Acx}{2ac\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^2,x]
```

```
[Out] (-(a*B) + A*c*x - a*C*x)/(2*a*c*(a + c*x^2)) + ((A*c + a*C)*ArcTan[(Sqrt[c]
*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

fricas [A] time = 0.81, size = 195, normalized size = 2.83

$$\left[-\frac{2\,Ba^2c+\left(Ca^2+Aac+\left(Cac+Ac^2\right)x^2\right)\sqrt{-ac}\,\log\left(\frac{cx^2-2\,\sqrt{-ac}\,x-a}{cx^2+a}\right)+2\left(Ca^2c-Aac^2\right)x}{4\left(a^2c^3x^2+a^3c^2\right)},-\frac{Ba^2c-\left(Ca^2+Aac+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c + (C*a^2 + A*a*c + (C*a*c + A*c^2)*x^2)*sqrt(-a*c)*log((c*
x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a^2*c - A*a*c^2)*x)/(a^2*c^3*
x^2 + a^3*c^2), -1/2*(B*a^2*c - (C*a^2 + A*a*c + (C*a*c + A*c^2)*x^2)*sqrt(
a*c)*arctan(sqrt(a*c)*x/a) + (C*a^2*c - A*a*c^2)*x)/(a^2*c^3*x^2 + a^3*c^2)
]
```

giac [A] time = 0.18, size = 60, normalized size = 0.87

$$\frac{(Ca+Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,ac}-\frac{Cax-Acx+Ba}{2\left(cx^2+a\right)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*a + A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c) - 1/2*(C*a*x - A*c*x
+ B*a)/((c*x^2 + a)*a*c)
```

maple [A] time = 0.01, size = 76, normalized size = 1.10

$$\frac{A\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;a}+\frac{C\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;c}+\frac{-\frac{B}{2c}+\frac{(Ac-aC)x}{2ac}}{c\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^2,x)
```

```
[Out] (1/2*(A*c-C*a)/a/c*x-1/2*B/c)/(c*x^2+a)+1/2/a/(a*c)^(1/2)*arctan(1/(a*c)^(1
/2)*c*x)*A+1/2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C
```

maxima [A] time = 0.96, size = 62, normalized size = 0.90

$$-\frac{Ba+(Ca-Ac)x}{2\left(ac^2x^2+a^2c\right)}+\frac{(Ca+Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a + (C*a - A*c)*x)/(a*c^2*x^2 + a^2*c) + 1/2*(C*a + A*c)*arctan(c*x
/sqrt(a*c))/(sqrt(a*c)*a*c)
```

mupad [B] time = 0.10, size = 60, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(A\,c+C\,a\right)}{2\,a^{3/2}\,c^{3/2}}-\frac{\frac{B}{2\,c}-\frac{x\,\left(A\,c-C\,a\right)}{2\,a\,c}}{c\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^2,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(A*c + C*a))/(2*a^(3/2)*c^(3/2)) - (B/(2*c) - (x
*(A*c - C*a))/(2*a*c))/(a + c*x^2)
```

sympy [A] time = 0.65, size = 116, normalized size = 1.68

$$-\frac{\sqrt{-\frac{1}{a^3c^3}}\,(Ac+Ca)\log\left(-a^2c\sqrt{-\frac{1}{a^3c^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3c^3}}\,(Ac+Ca)\log\left(a^2c\sqrt{-\frac{1}{a^3c^3}}+x\right)}{4}+\frac{-Ba+x\,(Ac-Ca)}{2a^2c+2ac^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**3*c**3))*(A*c + C*a)*log(-a**2*c*sqrt(-1/(a**3*c**3)) + x)/4 +
 sqrt(-1/(a**3*c**3))*(A*c + C*a)*log(a**2*c*sqrt(-1/(a**3*c**3)) + x)/4 +
(-B*a + x*(A*c - C*a))/(2*a**2*c + 2*a*c**2*x**2)
```

3.54 $\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2} \, dx$

Optimal. Leaf size=226

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)+a\left(cd^2-ae^2\right)(Cd-Be)\right)}{2a^{3/2}\sqrt{c}\,\left(ae^2+cd^2\right)^2}-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{2ac\left(a+cx^2\right)\left(ae^2+cd^2\right)}-\frac{e\log\left(a\right.}{}$$

```
[Out] 1/2*(-a*(-A*c*e+B*c*d+C*a*e)+c*(A*c*d+B*a*e-C*a*d)*x)/a/c/(a*e^2+c*d^2)/(c*
x^2+a)+e*(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/(a*e^2+c*d^2)^2-1/2*e*(A*e^2-B*d*e+C
*d^2)*ln(c*x^2+a)/(a*e^2+c*d^2)^2+1/2*(a*(-B*e+C*d)*(-a*e^2+c*d^2)+A*c*d*(3
*a*e^2+c*d^2))*arctan(x*c^(1/2)/a^(1/2))/a^(3/2)/(a*e^2+c*d^2)^2/c^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1647, 801, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)+a\left(cd^2-ae^2\right)(Cd-Be)\right)}{2a^{3/2}\sqrt{c}\,\left(ae^2+cd^2\right)^2}-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{2ac\left(a+cx^2\right)\left(ae^2+cd^2\right)}-\frac{e\log\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^2),x]
```

```
[Out] -(a*(B*c*d - A*c*e + a*C*e) - c*(A*c*d - a*C*d + a*B*e)*x)/(2*a*c*(c*d^2 +
a*e^2)*(a + c*x^2)) + ((a*(C*d - B*e)*(c*d^2 - a*e^2) + A*c*d*(c*d^2 + 3*a*
e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[c]*(c*d^2 + a*e^2)^2) +
(e*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^2 - (e*(C*d^2 - B*
d*e + A*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
```

```
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx = -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{-\frac{c\left(ad(Cd-Be)+A\left(cd^2+2ae^2\right)\right)}{cd^2+ae^2}-\frac{ce(Acd-aCd+aB\cdots}{cd^2+ae^2}}{(d+ex)\left(a+cx^2\right)}}{2ac}\cdots$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int\left(-\frac{2ace^2\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)}+\frac{c\left(-a(Cd-B\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^2}\cdots$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^2}\cdots$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(a(Cd-Be)\left(cd^2-ae^2\right)+Acd\left(c\cdots\right.\right.}{2a^{3/2}\sqrt{c}\left(cd^2+\cdots\right.}$$

Mathematica [A] time = 0.22, size = 195, normalized size = 0.86

$$\frac{\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)+a\left(cd^2-ae^2\right)(Cd-Be)\right)}{a^{3/2}\sqrt{c}}+\frac{\left(ae^2+cd^2\right)\left(a^2(-C)e+ac(Ae-Bd+Bex-Cdx)+Ac^2dx\right)}{ac\left(a+cx^2\right)}-e\log\left(a+cx^2\right)\left(e(Ae-Bd)\cdots\right.}{2\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^2),x]
```

```
[Out] (((c*d^2 + a*e^2)*(-(a^2*C*e) + A*c^2*d*x + a*c*(-(B*d) + A*e - C*d*x + B*e
*x)))/(a*c*(a + c*x^2)) + ((a*(C*d - B*e)*(c*d^2 - a*e^2) + A*c*d*(c*d^2 +
3*a*e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[c]) + 2*e*(C*d^2 + e*(
-(B*d) + A*e))*Log[d + e*x] - e*(C*d^2 + e*(-(B*d) + A*e))*Log[a + c*x^2])/
(2*(c*d^2 + a*e^2)^2)
```

fricas [B] time = 64.28, size = 1024, normalized size = 4.53

$$\left[-\frac{2Ba^2c^2d^3+2Ba^3cde^2+2\left(Ca^3c-Aa^2c^2\right)d^2e+2\left(Ca^4-Aa^3c\right)e^3-\left(Ba^2cd^2e-Ba^3e^3-\left(Ca^2c+Aac^2\right)d^3+\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c^2*d^3 + 2*B*a^3*c*d*e^2 + 2*(C*a^3*c - A*a^2*c^2)*d^2*e +
2*(C*a^4 - A*a^3*c)*e^3 - (B*a^2*c*d^2*e - B*a^3*e^3 - (C*a^2*c + A*a*c^2)*
d^3 + (C*a^3 - 3*A*a^2*c)*d*e^2 + (B*a*c^2*d^2*e - B*a^2*c*e^3 - (C*a*c^2 +
```

```
 A*c^3)*d^3 + (C*a^2*c - 3*A*a*c^2)*d*e^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*s
qrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(B*a^2*c^2*d^2*e + B*a^3*c*e^3 - (C*a^2*c
^2 - A*a*c^3)*d^3 - (C*a^3*c - A*a^2*c^2)*d*e^2)*x + 2*(C*a^3*c*d^2*e - B*a
^3*c*d*e^2 + A*a^3*c*e^3 + (C*a^2*c^2*d^2*e - B*a^2*c^2*d*e^2 + A*a^2*c^2*e
^3)*x^2)*log(c*x^2 + a) - 4*(C*a^3*c*d^2*e - B*a^3*c*d*e^2 + A*a^3*c*e^3 +
(C*a^2*c^2*d^2*e - B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2)*log(e*x + d))/(a^3
*c^3*d^4 + 2*a^4*c^2*d^2*e^2 + a^5*c*e^4 + (a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2
 + a^4*c^2*e^4)*x^2), -1/2*(B*a^2*c^2*d^3 + B*a^3*c*d*e^2 + (C*a^3*c - A*a^
2*c^2)*d^2*e + (C*a^4 - A*a^3*c)*e^3 + (B*a^2*c*d^2*e - B*a^3*e^3 - (C*a^2*
c + A*a*c^2)*d^3 + (C*a^3 - 3*A*a^2*c)*d*e^2 + (B*a*c^2*d^2*e - B*a^2*c*e^3
 - (C*a*c^2 + A*c^3)*d^3 + (C*a^2*c - 3*A*a*c^2)*d*e^2)*x^2)*sqrt(a*c)*arct
an(sqrt(a*c)*x/a) - (B*a^2*c^2*d^2*e + B*a^3*c*e^3 - (C*a^2*c^2 - A*a*c^3)*
d^3 - (C*a^3*c - A*a^2*c^2)*d*e^2)*x + (C*a^3*c*d^2*e - B*a^3*c*d*e^2 + A*a
^3*c*e^3 + (C*a^2*c^2*d^2*e - B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2)*log(c*x
^2 + a) - 2*(C*a^3*c*d^2*e - B*a^3*c*d*e^2 + A*a^3*c*e^3 + (C*a^2*c^2*d^2*e
 - B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2)*log(e*x + d))/(a^3*c^3*d^4 + 2*a^4
*c^2*d^2*e^2 + a^5*c*e^4 + (a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2 + a^4*c^2*e^4)*
x^2)]
```

giac [A] time = 0.19, size = 350, normalized size = 1.55

$$-\frac{\left(Cd^2e - Bde^2 + Ae^3\right)\log\left(cx^2 + a\right)}{2\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)} + \frac{\left(Cd^2e^2 - Bde^3 + Ae^4\right)\log\left(|xe + d|\right)}{c^2d^4e + 2\,acd^2e^3 + a^2e^5} + \frac{\left(Cacd^3 + Ac^2d^3 - Bacd^2e - Ca^2de^2 +\right.}{2\left(ac^2d^4 + 2\,a^2cd^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(C*d^2*e - B*d*e^2 + A*e^3)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4) + (C*d^2*e^2 - B*d*e^3 + A*e^4)*log(abs(x*e + d))/(c^2*d^4*e + 2*a
*c*d^2*e^3 + a^2*e^5) + 1/2*(C*a*c*d^3 + A*c^2*d^3 - B*a*c*d^2*e - C*a^2*d*
e^2 + 3*A*a*c*d*e^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) - 1/2*(B*a*c^2*d^3 + C*a^2*c*d^2*e - A*a*c^
2*d^2*e + B*a^2*c*d*e^2 + C*a^3*e^3 - A*a^2*c*e^3 + (C*a*c^2*d^3 - A*c^3*d^
3 - B*a*c^2*d^2*e + C*a^2*c*d*e^2 - A*a*c^2*d*e^2 - B*a^2*c*e^3)*x)/((c*d^2
 + a*e^2)^2*(c*x^2 + a)*a*c)
```

maple [B] time = 0.02, size = 742, normalized size = 3.28

$$\frac{A\,c^2d^3x}{2\left(a\,e^2 + c\,d^2\right)^2\left(c\,x^2 + a\right)a} + \frac{A\,c^2d^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\left(a\,e^2 + c\,d^2\right)^2\sqrt{ac}\;a} + \frac{Acd\,e^2x}{2\left(a\,e^2 + c\,d^2\right)^2\left(c\,x^2 + a\right)} + \frac{3Acd\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\left(a\,e^2 + c\,d^2\right)^2\sqrt{ac}} + \frac{B}{2\left(a\,e^2 + c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x)
```

```
[Out] e^3/(a*e^2+c*d^2)^2*ln(e*x+d)*A-e^2/(a*e^2+c*d^2)^2*ln(e*x+d)*B*d+e/(a*e^2+
c*d^2)^2*ln(e*x+d)*C*d^2+1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*A*x*c*d*e^2+1/2/(a*e
^2+c*d^2)^2/(c*x^2+a)/a*x*A*c^2*d^3+1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*B*x*a*e^3
+1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*B*x*c*d^2*e-1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*C*
x*a*d*e^2-1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*C*x*c*d^3+1/2/(a*e^2+c*d^2)^2/(c*x^
2+a)*A*e^3*a+1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*c*A*d^2*e-1/2/(a*e^2+c*d^2)^2/(c
*x^2+a)*B*d*e^2*a-1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*c*d^3*B-1/2/(a*e^2+c*d^2)^2
/(c*x^2+a)/c*C*a^2*e^3-1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*C*a*d^2*e-1/2/(a*e^2+c
*d^2)^2*ln(c*x^2+a)*A*e^3+1/2/(a*e^2+c*d^2)^2*ln(c*x^2+a)*B*d*e^2-1/2/(a*e^
2+c*d^2)^2*ln(c*x^2+a)*C*d^2*e+3/2/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(1/(a*
c)^(1/2)*c*x)*A*c*d*e^2+1/2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*arctan(1/(a*c)^(1
/2)*c*x)*A*c^2*d^3+1/2/(a*e^2+c*d^2)^2*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c
```

```
*x)*B*e^3-1/2/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*c*d^2
*e-1/2/(a*e^2+c*d^2)^2*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d*e^2+1/2/
(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*c*d^3
```

maxima [A] time = 1.00, size = 293, normalized size = 1.30

$$-\frac{\left(Cd^2e - Bde^2 + Ae^3\right)\log\left(cx^2 + a\right)}{2\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)} + \frac{\left(Cd^2e - Bde^2 + Ae^3\right)\log\left(ex + d\right)}{c^2d^4 + 2\,acd^2e^2 + a^2e^4} - \frac{\left(Bacd^2e - Ba^2e^3 - \left(Cac + Ac^2\right)d^3 + \right.}{2\left(ac^2d^4 + 2\,a^2cd^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(C*d^2*e - B*d*e^2 + A*e^3)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4) + (C*d^2*e - B*d*e^2 + A*e^3)*log(e*x + d)/(c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4) - 1/2*(B*a*c*d^2*e - B*a^2*e^3 - (C*a*c + A*c^2)*d^3 + (C*a^2
- 3*A*a*c)*d*e^2)*arctan(c*x/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3
*e^4)*sqrt(a*c)) - 1/2*(B*a*c*d + (C*a^2 - A*a*c)*e - (B*a*c*e - (C*a*c - A
*c^2)*d)*x)/(a^2*c^2*d^2 + a^3*c*e^2 + (a*c^3*d^2 + a^2*c^2*e^2)*x^2)
```

mupad [B] time = 7.68, size = 1493, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^2*(d + e*x)),x)
```

```
[Out] (log(A*c^3*d^5*(-a^3*c)^(1/2) - B*a^3*e^5*(-a^3*c)^(1/2) + 6*A*a^4*c*e^5 -
B*a^4*c*e^5*x - 2*A*a^2*c^3*d^4*e - 8*C*a^3*c^2*d^4*e + 8*C*a^4*c*d^2*e^3 +
 C*a^2*c^3*d^5*x + C*a*c^2*d^5*(-a^3*c)^(1/2) + C*a^3*d*e^4*(-a^3*c)^(1/2)
- 12*A*a^3*c^2*d^2*e^3 + 8*B*a^3*c^2*d^3*e^2 - 8*B*a^4*c*d*e^4 + A*a*c^4*d^
5*x + 2*A*a^2*c^3*d^3*e^2*x + 14*B*a^3*c^2*d^2*e^3*x - 14*C*a^3*c^2*d^3*e^2
*x + 2*A*a*c^2*d^3*e^2*(-a^3*c)^(1/2) + 14*B*a^2*c*d^2*e^3*(-a^3*c)^(1/2) -
 14*C*a^2*c*d^3*e^2*(-a^3*c)^(1/2) + C*a^4*c*d*e^4*x - 15*A*a^3*c^2*d*e^4*x
 - B*a^2*c^3*d^4*e*x - 15*A*a^2*c*d*e^4*(-a^3*c)^(1/2) - B*a*c^2*d^4*e*(-a^
3*c)^(1/2) - 6*A*a^2*c*e^5*x*(-a^3*c)^(1/2) + 2*A*c^3*d^4*e*x*(-a^3*c)^(1/2
) + 8*B*a^2*c*d*e^4*x*(-a^3*c)^(1/2) + 8*C*a*c^2*d^4*e*x*(-a^3*c)^(1/2) + 1
2*A*a*c^2*d^2*e^3*x*(-a^3*c)^(1/2) - 8*B*a*c^2*d^3*e^2*x*(-a^3*c)^(1/2) - 8
*C*a^2*c*d^2*e^3*x*(-a^3*c)^(1/2))*(a^2*((B*e^3*(-a^3*c)^(1/2))/4 - (C*d*e^
2*(-a^3*c)^(1/2))/4) - c*(a^3*((A*e^3)/2 - (B*d*e^2)/2 + (C*d^2*e)/2) - a*(
(C*d^3*(-a^3*c)^(1/2))/4 + (3*A*d*e^2*(-a^3*c)^(1/2))/4 - (B*d^2*e*(-a^3*c)
^(1/2))/4)) + (A*c^2*d^3*(-a^3*c)^(1/2))/4))/(a^5*c*e^4 + a^3*c^3*d^4 + 2*a
^4*c^2*d^2*e^2) - (log(A*c^3*d^5*(-a^3*c)^(1/2) - B*a^3*e^5*(-a^3*c)^(1/2)
- 6*A*a^4*c*e^5 + B*a^4*c*e^5*x + 2*A*a^2*c^3*d^4*e + 8*C*a^3*c^2*d^4*e - 8
*C*a^4*c*d^2*e^3 - C*a^2*c^3*d^5*x + C*a*c^2*d^5*(-a^3*c)^(1/2) + C*a^3*d*e
^4*(-a^3*c)^(1/2) + 12*A*a^3*c^2*d^2*e^3 - 8*B*a^3*c^2*d^3*e^2 + 8*B*a^4*c*
d*e^4 - A*a*c^4*d^5*x - 2*A*a^2*c^3*d^3*e^2*x - 14*B*a^3*c^2*d^2*e^3*x + 14
*C*a^3*c^2*d^3*e^2*x + 2*A*a*c^2*d^3*e^2*(-a^3*c)^(1/2) + 14*B*a^2*c*d^2*e^
3*(-a^3*c)^(1/2) - 14*C*a^2*c*d^3*e^2*(-a^3*c)^(1/2) - C*a^4*c*d*e^4*x + 15
*A*a^3*c^2*d*e^4*x + B*a^2*c^3*d^4*e*x - 15*A*a^2*c*d*e^4*(-a^3*c)^(1/2) -
B*a*c^2*d^4*e*(-a^3*c)^(1/2) - 6*A*a^2*c*e^5*x*(-a^3*c)^(1/2) + 2*A*c^3*d^4
*e*x*(-a^3*c)^(1/2) + 8*B*a^2*c*d*e^4*x*(-a^3*c)^(1/2) + 8*C*a*c^2*d^4*e*x*
(-a^3*c)^(1/2) + 12*A*a*c^2*d^2*e^3*x*(-a^3*c)^(1/2) - 8*B*a*c^2*d^3*e^2*x*
(-a^3*c)^(1/2) - 8*C*a^2*c*d^2*e^3*x*(-a^3*c)^(1/2))*(c*(a^3*((A*e^3)/2 - (
B*d*e^2)/2 + (C*d^2*e)/2) + a*((C*d^3*(-a^3*c)^(1/2))/4 + (3*A*d*e^2*(-a^3*
c)^(1/2))/4 - (B*d^2*e*(-a^3*c)^(1/2))/4)) + a^2*((B*e^3*(-a^3*c)^(1/2))/4
- (C*d*e^2*(-a^3*c)^(1/2))/4) + (A*c^2*d^3*(-a^3*c)^(1/2))/4))/(a^5*c*e^4 +
 a^3*c^3*d^4 + 2*a^4*c^2*d^2*e^2) - ((B*c*d - A*c*e + C*a*e)/(2*c*(a*e^2 +
c*d^2)) - (x*(A*c*d + B*a*e - C*a*d))/(2*a*(a*e^2 + c*d^2)))/(a + c*x^2) +
(e*log(d + e*x)*(A*e^2 + C*d^2 - B*d*e))/(a*e^2 + c*d^2)^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a)**2,x)
[Out] Timed out
```

3.55 $\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=374

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Ac\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)+a\left(a^2Ce^4-6acde^2(Cd-Be)+c^2d^3(Cd-2Be)\right)\right)}{2a^{3/2}\sqrt{c}\left(ae^2+cd^2\right)^3}-\frac{a\left(-aBe^2+2a\right.}{}$$

```
[Out] -e*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^2/(e*x+d)+1/2*(-a*(-2*A*c*d*e-B*a*e^2+
B*c*d^2+2*C*a*d*e)+(A*c*(-a*e^2+c*d^2)+a*(a*C*e^2-c*d*(-2*B*e+C*d)))*x)/a/(
a*e^2+c*d^2)^2/(c*x^2+a)-e*(a*e^2*(-B*e+2*C*d)-c*d*(2*C*d^2-e*(-4*A*e+3*B*d
)))*ln(e*x+d)/(a*e^2+c*d^2)^3+1/2*e*(a*e^2*(-B*e+2*C*d)-c*d*(2*C*d^2-e*(-4*
A*e+3*B*d)))*ln(c*x^2+a)/(a*e^2+c*d^2)^3+1/2*(A*c*(-3*a^2*e^4+6*a*c*d^2*e^2
+c^2*d^4)+a*(a^2*C*e^4+c^2*d^3*(-2*B*e+C*d)-6*a*c*d*e^2*(-B*e+C*d)))*arctan
(x*c^(1/2)/a^(1/2))/a^(3/2)/(a*e^2+c*d^2)^3/c^(1/2)
```

Rubi [A] time = 0.95, antiderivative size = 371, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1647, 1629, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Ac\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)+a\left(a^2Ce^4-6acde^2(Cd-Be)+c^2d^3(Cd-2Be)\right)\right)}{2a^{3/2}\sqrt{c}\left(ae^2+cd^2\right)^3}-\frac{a\left(-aBe^2+2a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^2),x]
```

```
[Out] -((e*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^2*(d + e*x))) - (a*(B*c*d^2
- 2*A*c*d*e + 2*a*C*d*e - a*B*e^2) - (A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*
d*(C*d - 2*B*e)))*x)/(2*a*(c*d^2 + a*e^2)^2*(a + c*x^2)) + ((A*c*(c^2*d^4 +
 6*a*c*d^2*e^2 - 3*a^2*e^4) + a*(a^2*C*e^4 + c^2*d^3*(C*d - 2*B*e) - 6*a*c*
d*e^2*(C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[c]*(c*d^2
+ a*e^2)^3) + (e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e))*
Log[d + e*x])/(c*d^2 + a*e^2)^3 - (e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a
*e^2*(2*C*d - B*e))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx = -\frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x}{2a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

$$= -\frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x}{2a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(a\right.\right.}{2a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(a\right.\right.}{2a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(a\right.\right.}{2a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

Mathematica [A] time = 0.41, size = 320, normalized size = 0.86

$$\frac{\frac{\left(ae^2+cd^2\right)\left(a^2e(Be-2Cd+Cex)-ac\left(Ae(ex-2d)+Bd(d-2ex)+Cd^2x\right)+Ac^2d^2x\right)}{a\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)+a\left(a^2Ce^4+6acde^2(Be-Cd)+c^2d\right.\right.}{a^{3/2}\sqrt{c}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^2),x]
```

```
[Out] ((-2*e*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) + ((c*d^2 + a*
e^2)*(A*c^2*d^2*x + a^2*e*(-2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + B*d*(d -
2*e*x) + A*e*(-2*d + e*x))))/(a*(a + c*x^2)) + ((A*c*(c^2*d^4 + 6*a*c*d^2*e
^2 - 3*a^2*e^4) + a*(a^2*C*e^4 + c^2*d^3*(C*d - 2*B*e) + 6*a*c*d*e^2*(-(C*d
) + B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[c]) + 2*e*(2*c*C*d^3
+ c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e))*Log[d + e*x] - e*(2*c*C*d^
3 + c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e))*Log[a + c*x^2])/(2*(c*d^
2 + a*e^2)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.19, size = 608, normalized size = 1.63

$$\frac{\left(Cac^2d^4e^2 + Ac^3d^4e^2 - 2\,Bac^2d^3e^3 - 6\,Ca^2cd^2e^4 + 6\,Aac^2d^2e^4 + 6\,Ba^2cde^5 + Ca^3e^6 - 3\,Aa^2ce^6\right)\arctan\left(\frac{\left(\left(cd - \frac{cd}{xe-}\right.\right.}{}\right.}{2\left(ac^3d^6 + 3\,a^2c^2d^4e^2 + 3\,a^3cd^2e^4 + a^4e^6\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*a*c^2*d^4*e^2 + A*c^3*d^4*e^2 - 2*B*a*c^2*d^3*e^3 - 6*C*a^2*c*d^2*e^
4 + 6*A*a*c^2*d^2*e^4 + 6*B*a^2*c*d*e^5 + C*a^3*e^6 - 3*A*a^2*c*e^6)*arctan
((c*d - c*d^2/(x*e + d) - a*e^2/(x*e + d))*e^(-1)/sqrt(a*c))*e^(-2)/((a*c^3
*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(a*c)) - 1/2*(2*C
*c*d^3*e - 3*B*c*d^2*e^2 - 2*C*a*d*e^3 + 4*A*c*d*e^3 + B*a*e^4)*log(c - 2*c
*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2)/(c^3*d^6 + 3*a*c^2*d^
4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - (C*d^2*e^5/(x*e + d) - B*d*e^6/(x*e +
d) + A*e^7/(x*e + d))/(c^2*d^4*e^4 + 2*a*c*d^2*e^6 + a^2*e^8) - 1/2*((C*a*c
^2*d^3*e - A*c^3*d^3*e - 3*B*a*c^2*d^2*e^2 - 3*C*a^2*c*d*e^3 + 3*A*a*c^2*d*
e^3 + B*a^2*c*e^4)/(c*d^2 + a*e^2) - (C*a*c^2*d^4*e^2 - A*c^3*d^4*e^2 - 4*B
*a*c^2*d^3*e^3 - 6*C*a^2*c*d^2*e^4 + 6*A*a*c^2*d^2*e^4 + 4*B*a^2*c*d*e^5 +
C*a^3*e^6 - A*a^2*c*e^6)*e^(-1)/((c*d^2 + a*e^2)*(x*e + d)))/((c*d^2 + a*e^
2)^2*a*(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2))
```

maple [B] time = 0.02, size = 1036, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x)
```

```
[Out] -2/(a*e^2+c*d^2)^3*c*ln(c*x^2+a)*d*A*e^3+3/2/(a*e^2+c*d^2)^3*c*ln(c*x^2+a)*
e^2*d^2*B-1/(a*e^2+c*d^2)^3*c*ln(c*x^2+a)*C*d^3*e+1/2/(a*e^2+c*d^2)^3/(a*c)
^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*c^2*d^4+1/(a*e^2+c*d^2)^3*a*ln(c*x^2+a)*
C*d*e^3+1/2/(a*e^2+c*d^2)^3*a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*e^4
+2*e/(a*e^2+c*d^2)^3*ln(e*x+d)*C*c*d^3+4*e^3/(a*e^2+c*d^2)^3*ln(e*x+d)*A*c*
d-3*e^2/(a*e^2+c*d^2)^3*ln(e*x+d)*B*c*d^2-2*e^3/(a*e^2+c*d^2)^3*ln(e*x+d)*C
*a*d+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)*a^2*C*e^4*x-1/2/(a*e^2+c*d^2)^3/(c*x^2+a
)*C*c^2*d^4*x+1/(a*e^2+c*d^2)^3/(c*x^2+a)*A*c^2*d^3*e-1/(a*e^2+c*d^2)^3/(c*
x^2+a)*C*a^2*d*e^3-e^3/(a*e^2+c*d^2)^2/(e*x+d)*A-1/(a*e^2+c*d^2)^3/(c*x^2+a
)*C*a*c*d^3*e-3/2/(a*e^2+c*d^2)^3*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A
*c*e^4+1/2/(a*e^2+c*d^2)^3/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c^3*d^
4+3/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c^2*d^2*e^2-1/(
a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*c^2*d^3*e-1/2/(a*e^2
+c*d^2)^3/(c*x^2+a)*A*a*c*e^4*x+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)/a*x*A*c^3*d^4
+1/(a*e^2+c*d^2)^3/(c*x^2+a)*B*c^2*d^3*e*x+1/(a*e^2+c*d^2)^3/(c*x^2+a)*A*a*
c*d*e^3+1/(a*e^2+c*d^2)^3/(c*x^2+a)*d*a*c*B*e^3*x-3/(a*e^2+c*d^2)^3*a/(a*c)
^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*c*d^2*e^2+3/(a*e^2+c*d^2)^3*a/(a*c)^(1/2
)*arctan(1/(a*c)^(1/2)*c*x)*B*c*d*e^3-1/2/(a*e^2+c*d^2)^3*a*ln(c*x^2+a)*e^4
*B+e^4/(a*e^2+c*d^2)^3*ln(e*x+d)*B*a+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)*B*a^2*e^
4-1/2/(a*e^2+c*d^2)^3/(c*x^2+a)*B*c^2*d^4+e^2/(a*e^2+c*d^2)^2/(e*x+d)*B*d-e
/(a*e^2+c*d^2)^2/(e*x+d)*C*d^2
```

maxima [A] time = 1.04, size = 604, normalized size = 1.61

$$-\frac{\left(2\,Ccd^{3}e-3\,Bcd^{2}e^{2}+Bae^{4}-2\,(Ca-2\,Ac)de^{3}\right)\log\left(cx^{2}+a\right)}{2\left(c^{3}d^{6}+3\,ac^{2}d^{4}e^{2}+3\,a^{2}cd^{2}e^{4}+a^{3}e^{6}\right)}+\frac{\left(2\,Ccd^{3}e-3\,Bcd^{2}e^{2}+Bae^{4}-2\,(Ca-2\,Ac)de^{3}\right)}{c^{3}d^{6}+3\,ac^{2}d^{4}e^{2}+3\,a^{2}cd^{2}e^{4}+a^{3}e^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*C*c*d^3*e - 3*B*c*d^2*e^2 + B*a*e^4 - 2*(C*a - 2*A*c)*d*e^3)*log(c*
x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (2*C*c*d
^3*e - 3*B*c*d^2*e^2 + B*a*e^4 - 2*(C*a - 2*A*c)*d*e^3)*log(e*x + d)/(c^3*d
^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - 1/2*(2*B*a*c^2*d^3*e -
6*B*a^2*c*d*e^3 - (C*a*c^2 + A*c^3)*d^4 + 6*(C*a^2*c - A*a*c^2)*d^2*e^2 - (
C*a^3 - 3*A*a^2*c)*e^4)*arctan(c*x/sqrt(a*c))/((a*c^3*d^6 + 3*a^2*c^2*d^4*e
^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(a*c)) - 1/2*(B*a*c*d^3 - 3*B*a^2*d*e^2
 + 2*A*a^2*e^3 + 2*(2*C*a^2 - A*a*c)*d^2*e - (4*B*a*c*d*e^2 - (3*C*a*c - A*
c^2)*d^2*e + (C*a^2 - 3*A*a*c)*e^3)*x^2 - (B*a*c*d^2*e + B*a^2*e^3 - (C*a*c
 - A*c^2)*d^3 - (C*a^2 - A*a*c)*d*e^2)*x)/(a^2*c^2*d^5 + 2*a^3*c*d^3*e^2 +
a^4*d*e^4 + (a*c^3*d^4*e + 2*a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^3 + (a*c^3*d^5
+ 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*x^2 + (a^2*c^2*d^4*e + 2*a^3*c*d^2*e^3 +
 a^4*e^5)*x)
```

mupad [B] time = 9.91, size = 2094, normalized size = 5.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^2*(d + e*x)^2),x)
```

```
[Out] ((x^2*(C*a^2*e^3 - 3*A*a*c*e^3 + A*c^2*d^2*e + 4*B*a*c*d*e^2 - 3*C*a*c*d^2*
e))/(2*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (2*A*a*e^3 + B*c*d^3 - 3*B*
a*d*e^2 - 2*A*c*d^2*e + 4*C*a*d^2*e)/(2*(a*e^2 + c*d^2)^2) + (x*(A*c*d + B*
a*e - C*a*d))/(2*a*(a*e^2 + c*d^2)))/(a*d + a*e*x + c*d*x^2 + c*e*x^3) - (l
og(3*A*e^6*(-a^3*c)^(3/2) - A*c^4*d^6*(-a^3*c)^(1/2) + C*a^4*e^6*(-a^3*c)^(
1/2) + 31*C*d^2*e^4*(-a^3*c)^(3/2) + 6*B*a^5*c*e^6 - 18*B*d*e^5*(-a^3*c)^(3
/2) - 6*B*e^6*x*(-a^3*c)^(3/2) - C*a^5*c*e^6*x + 14*C*d*e^5*x*(-a^3*c)^(3/2
) - 2*A*a^2*c^4*d^5*e + 30*A*a^4*c^2*d*e^5 - 14*C*a^3*c^3*d^5*e + 3*A*a^4*c
^2*e^6*x + C*a^2*c^4*d^6*x - C*a*c^3*d^6*(-a^3*c)^(1/2) - 36*A*a^3*c^3*d^3*
e^3 + 22*B*a^3*c^3*d^4*e^2 - 36*B*a^4*c^2*d^2*e^4 + 36*C*a^4*c^2*d^3*e^3 -
14*C*a^5*c*d*e^5 + A*a*c^5*d^6*x + 5*A*a^2*c^4*d^4*e^2*x - 57*A*a^3*c^3*d^2
*e^4*x + 44*B*a^3*c^3*d^3*e^3*x - 31*C*a^3*c^3*d^4*e^2*x + 31*C*a^4*c^2*d^2
*e^4*x - 5*A*a*c^3*d^4*e^2*(-a^3*c)^(1/2) + 57*A*a^2*c^2*d^2*e^4*(-a^3*c)^(
1/2) - 44*B*a^2*c^2*d^3*e^3*(-a^3*c)^(1/2) + 31*C*a^2*c^2*d^4*e^2*(-a^3*c)^
(1/2) - 2*B*a^2*c^4*d^5*e*x - 18*B*a^4*c^2*d*e^5*x + 2*B*a*c^3*d^5*e*(-a^3*
c)^(1/2) - 2*A*c^4*d^5*e*x*(-a^3*c)^(1/2) - 36*B*a^2*c^2*d^2*e^4*x*(-a^3*c)
^(1/2) + 36*C*a^2*c^2*d^3*e^3*x*(-a^3*c)^(1/2) - 14*C*a*c^3*d^5*e*x*(-a^3*c
)^(1/2) - 36*A*a*c^3*d^3*e^3*x*(-a^3*c)^(1/2) + 30*A*a^2*c^2*d*e^5*x*(-a^3*
c)^(1/2) + 22*B*a*c^3*d^4*e^2*x*(-a^3*c)^(1/2))*(c^2*(a*((C*d^4*(-a^3*c)^(1
/2))/4 + (3*A*d^2*e^2*(-a^3*c)^(1/2))/2 - (B*d^3*e*(-a^3*c)^(1/2))/2) + a^3
*(2*A*d*e^3 - (3*B*d^2*e^2)/2 + C*d^3*e)) - c*(a^2*((3*A*e^4*(-a^3*c)^(1/2)
)/4 + (3*C*d^2*e^2*(-a^3*c)^(1/2))/2 - (3*B*d*e^3*(-a^3*c)^(1/2))/2) - a^4*
((B*e^4)/2 - C*d*e^3)) + (A*c^3*d^4*(-a^3*c)^(1/2))/4 + (C*a^3*e^4*(-a^3*c)
^(1/2))/4))/(a^6*c*e^6 + a^3*c^4*d^6 + 3*a^4*c^3*d^4*e^2 + 3*a^5*c^2*d^2*e^
4) + (log(3*A*e^6*(-a^3*c)^(3/2) - A*c^4*d^6*(-a^3*c)^(1/2) + C*a^4*e^6*(-a
^3*c)^(1/2) + 31*C*d^2*e^4*(-a^3*c)^(3/2) - 6*B*a^5*c*e^6 - 18*B*d*e^5*(-a^
3*c)^(3/2) - 6*B*e^6*x*(-a^3*c)^(3/2) + C*a^5*c*e^6*x + 14*C*d*e^5*x*(-a^3*
c)^(3/2) + 2*A*a^2*c^4*d^5*e - 30*A*a^4*c^2*d*e^5 + 14*C*a^3*c^3*d^5*e - 3*
A*a^4*c^2*e^6*x - C*a^2*c^4*d^6*x - C*a*c^3*d^6*(-a^3*c)^(1/2) + 36*A*a^3*c
```

```
^3*d^3*e^3 - 22*B*a^3*c^3*d^4*e^2 + 36*B*a^4*c^2*d^2*e^4 - 36*C*a^4*c^2*d^3
*e^3 + 14*C*a^5*c*d*e^5 - A*a*c^5*d^6*x - 5*A*a^2*c^4*d^4*e^2*x + 57*A*a^3*
c^3*d^2*e^4*x - 44*B*a^3*c^3*d^3*e^3*x + 31*C*a^3*c^3*d^4*e^2*x - 31*C*a^4*
c^2*d^2*e^4*x - 5*A*a*c^3*d^4*e^2*(-a^3*c)^(1/2) + 57*A*a^2*c^2*d^2*e^4*(-a
^3*c)^(1/2) - 44*B*a^2*c^2*d^3*e^3*(-a^3*c)^(1/2) + 31*C*a^2*c^2*d^4*e^2*(-
a^3*c)^(1/2) + 2*B*a^2*c^4*d^5*e*x + 18*B*a^4*c^2*d*e^5*x + 2*B*a*c^3*d^5*e
*(-a^3*c)^(1/2) - 2*A*c^4*d^5*e*x*(-a^3*c)^(1/2) - 36*B*a^2*c^2*d^2*e^4*x*(
-a^3*c)^(1/2) + 36*C*a^2*c^2*d^3*e^3*x*(-a^3*c)^(1/2) - 14*C*a*c^3*d^5*e*x*
(-a^3*c)^(1/2) - 36*A*a*c^3*d^3*e^3*x*(-a^3*c)^(1/2) + 30*A*a^2*c^2*d*e^5*x
*(-a^3*c)^(1/2) + 22*B*a*c^3*d^4*e^2*x*(-a^3*c)^(1/2))*(c^2*(a*((C*d^4*(-a^
3*c)^(1/2))/4 + (3*A*d^2*e^2*(-a^3*c)^(1/2))/2 - (B*d^3*e*(-a^3*c)^(1/2))/2
) - a^3*(2*A*d*e^3 - (3*B*d^2*e^2)/2 + C*d^3*e)) - c*(a^2*((3*A*e^4*(-a^3*c
)^(1/2))/4 + (3*C*d^2*e^2*(-a^3*c)^(1/2))/2 - (3*B*d*e^3*(-a^3*c)^(1/2))/2)
 + a^4*((B*e^4)/2 - C*d*e^3)) + (A*c^3*d^4*(-a^3*c)^(1/2))/4 + (C*a^3*e^4*(
-a^3*c)^(1/2))/4))/(a^6*c*e^6 + a^3*c^4*d^6 + 3*a^4*c^3*d^4*e^2 + 3*a^5*c^2
*d^2*e^4) + (log(d + e*x)*(a*(B*e^4 - 2*C*d*e^3) + c*(4*A*d*e^3 - 3*B*d^2*e
^2 + 2*C*d^3*e)))/(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.56 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=524

$$-\frac{e\log\left(a+cx^2\right)\left(a^2Ce^4-2ace^2\left(4Cd^2-e(3Bd-Ae)\right)+c^2d^2\left(3Cd^2-2e(3Bd-5Ae)\right)\right)}{2\left(ae^2+cd^2\right)^4}+\frac{e\log(d+ex)\left(a^2Ce^4-2\right.}{}$$

```
[Out] -1/2*e*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^2/(e*x+d)^2+e*(a*e^2*(-B*e+2*C*d)-
c*d*(2*C*d^2-e*(-4*A*e+3*B*d)))/(a*e^2+c*d^2)^3/(e*x+d)+1/2*(-a*(B*c*d*(-3*
a*e^2+c*d^2)-(A*c-C*a)*e*(-a*e^2+3*c*d^2))+c*(A*c*d*(-3*a*e^2+c*d^2)-a*(c*d
^2*(-3*B*e+C*d)-a*e^2*(-B*e+3*C*d)))*x)/a/(a*e^2+c*d^2)^3/(c*x^2+a)+e*(a^2*
C*e^4+c^2*d^2*(3*C*d^2-2*e*(-5*A*e+3*B*d))-2*a*c*e^2*(4*C*d^2-e*(-A*e+3*B*d
)))*ln(e*x+d)/(a*e^2+c*d^2)^4-1/2*e*(a^2*C*e^4+c^2*d^2*(3*C*d^2-2*e*(-5*A*e
+3*B*d))-2*a*c*e^2*(4*C*d^2-e*(-A*e+3*B*d)))*ln(c*x^2+a)/(a*e^2+c*d^2)^4+1/
2*(A*c*d*(-15*a^2*e^4+10*a*c*d^2*e^2+c^2*d^4)-a*(2*a*c*d^2*e^2*(-9*B*e+7*C*
d)-c^2*d^4*(-3*B*e+C*d)-3*a^2*e^4*(-B*e+3*C*d)))*arctan(x*c^(1/2)/a^(1/2))*
c^(1/2)/a^(3/2)/(a*e^2+c*d^2)^4
```

Rubi [A] time = 1.55, antiderivative size = 524, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1647, 1629, 635, 205, 260}

$$-\frac{e\log\left(a+cx^2\right)\left(a^2Ce^4-2ace^2\left(4Cd^2-e(3Bd-Ae)\right)+c^2\left(3Cd^4-2d^2e(3Bd-5Ae)\right)\right)}{2\left(ae^2+cd^2\right)^4}+\frac{e\log(d+ex)\left(a^2Ce^4-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^2),x]
```

```
[Out] -(e*(C*d^2 - B*d*e + A*e^2))/(2*(c*d^2 + a*e^2)^2*(d + e*x)^2) - (e*(2*c*C*
d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e)))/((c*d^2 + a*e^2)^3*(d +
 e*x)) - (a*(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2)) - c
*(A*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e)))*
x)/(2*a*(c*d^2 + a*e^2)^3*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c^2*d^4 + 10*a*c*
d^2*e^2 - 15*a^2*e^4) - a*(2*a*c*d^2*e^2*(7*C*d - 9*B*e) - c^2*d^4*(C*d - 3
*B*e) - 3*a^2*e^4*(3*C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*(
c*d^2 + a*e^2)^4) + (e*(a^2*C*e^4 + c^2*(3*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e))
 - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e)))*Log[d + e*x])/(c*d^2 + a*e^2)^4 -
 (e*(a^2*C*e^4 + c^2*(3*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C*d
^2 - e*(3*B*d - A*e)))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^2}\,dx = -\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(C\right.\right.}{2a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)}$$

$$= -\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(C\right.\right.}{2a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{e\left(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^3(d+ex)}-\frac{a\left(Bc\right.}{}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{e\left(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^3(d+ex)}-\frac{a\left(Bc\right.}{}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{e\left(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^3(d+ex)}-\frac{a\left(Bc\right.}{}$$

Mathematica [A] time = 0.63, size = 466, normalized size = 0.89

$$\frac{-\log\left(a+cx^2\right)\left(a^2Ce^5-2ace^3\left(e(Ae-3Bd)+4Cd^2\right)+c^2d^2e\left(2e(5Ae-3Bd)+3Cd^2\right)\right)+2\log(d+ex)\left(a^2Ce^5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^2),x]
```

```
[Out] (-((e*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2) - (2*e*(c*
d^2 + a*e^2)*(2*c*C*d^3 + c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e)))/(
d + e*x) + ((c*d^2 + a*e^2)*(a^3*C*e^3 + A*c^3*d^3*x - a*c^2*d*(C*d^2*x + B
*d*(d - 3*e*x) + 3*A*e*(-d + e*x)) - a^2*c*e*(3*C*d*(d - e*x) + e*(-3*B*d +
 A*e + B*e*x))))/(a*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c^2*d^4 + 10*a*c*d^2*e^
2 - 15*a^2*e^4) + a*(-2*a*c*d^2*e^2*(7*C*d - 9*B*e) + c^2*d^4*(C*d - 3*B*e)
 - 3*a^2*e^4*(-3*C*d + B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) + 2*(a^2
```

```
*C*e^5 - 2*a*c*e^3*(4*C*d^2 + e*(-3*B*d + A*e)) + c^2*d^2*e*(3*C*d^2 + 2*e*
(-3*B*d + 5*A*e)))*Log[d + e*x] - (a^2*C*e^5 - 2*a*c*e^3*(4*C*d^2 + e*(-3*B
*d + A*e)) + c^2*d^2*e*(3*C*d^2 + 2*e*(-3*B*d + 5*A*e)))*Log[a + c*x^2])/(2
*(c*d^2 + a*e^2)^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.19, size = 957, normalized size = 1.83

$$-\frac{\left(3\,Cc^2d^4e-6\,Bc^2d^3e^2-8\,Cacd^2e^3+10\,Ac^2d^2e^3+6\,Bacde^4+Ca^2e^5-2\,Aace^5\right)\log\left(cx^2+a\right)}{2\left(c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8\right)}+\frac{\left(3\,Cc^2d^4e^2-6\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*C*c^2*d^4*e - 6*B*c^2*d^3*e^2 - 8*C*a*c*d^2*e^3 + 10*A*c^2*d^2*e^3
 + 6*B*a*c*d*e^4 + C*a^2*e^5 - 2*A*a*c*e^5)*log(c*x^2 + a)/(c^4*d^8 + 4*a*c^
3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + (3*C*c^2*d^4*e
^2 - 6*B*c^2*d^3*e^3 - 8*C*a*c*d^2*e^4 + 10*A*c^2*d^2*e^4 + 6*B*a*c*d*e^5 +
 C*a^2*e^6 - 2*A*a*c*e^6)*log(abs(x*e + d))/(c^4*d^8*e + 4*a*c^3*d^6*e^3 +
6*a^2*c^2*d^4*e^5 + 4*a^3*c*d^2*e^7 + a^4*e^9) + 1/2*(C*a*c^3*d^5 + A*c^4*d
^5 - 3*B*a*c^3*d^4*e - 14*C*a^2*c^2*d^3*e^2 + 10*A*a*c^3*d^3*e^2 + 18*B*a^2
*c^2*d^2*e^3 + 9*C*a^3*c*d*e^4 - 15*A*a^2*c^2*d*e^4 - 3*B*a^3*c*e^5)*arctan
(c*x/sqrt(a*c))/((a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4
*c*d^2*e^6 + a^5*e^8)*sqrt(a*c)) - 1/2*(B*a*c^3*d^7 + 8*C*a^2*c^2*d^6*e - 3
*A*a*c^3*d^6*e - 9*B*a^2*c^2*d^5*e^2 + 4*C*a^3*c*d^4*e^3 + 7*A*a^2*c^2*d^4*
e^3 - 9*B*a^3*c*d^3*e^4 - 4*C*a^4*d^2*e^5 + 11*A*a^3*c*d^2*e^5 + B*a^4*d*e^
6 + A*a^4*e^7 + (5*C*a*c^3*d^5*e^2 - A*c^4*d^5*e^2 - 9*B*a*c^3*d^4*e^3 - 2*
C*a^2*c^2*d^3*e^4 + 10*A*a*c^3*d^3*e^4 - 6*B*a^2*c^2*d^2*e^5 - 7*C*a^3*c*d*
e^6 + 11*A*a^2*c^2*d*e^6 + 3*B*a^3*c*e^7)*x^3 + (7*C*a*c^3*d^6*e - 2*A*c^4*
d^6*e - 12*B*a*c^3*d^5*e^2 + C*a^2*c^2*d^4*e^3 + 10*A*a*c^3*d^4*e^3 - 12*B*
a^2*c^2*d^3*e^4 - 7*C*a^3*c*d^2*e^5 + 14*A*a^2*c^2*d^2*e^5 - C*a^4*e^7 + 2*
A*a^3*c*e^7)*x^2 + (C*a*c^3*d^7 - A*c^4*d^7 - B*a*c^3*d^6*e + 8*C*a^2*c^2*d
^5*e^2 - 4*A*a*c^3*d^5*e^2 - 12*B*a^2*c^2*d^4*e^3 + C*a^3*c*d^3*e^4 + 7*A*a
^2*c^2*d^3*e^4 - 9*B*a^3*c*d^2*e^5 - 6*C*a^4*d*e^6 + 10*A*a^3*c*d*e^6 + 2*B
*a^4*e^7)*x)/((c*d^2 + a*e^2)^4*(c*x^2 + a)*(x*e + d)^2*a)
```

maple [B] time = 0.03, size = 1588, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x)
```

```
[Out] -1/2/(a*e^2+c*d^2)^4*c/(c*x^2+a)*A*e^5*a^2-1/2/(a*e^2+c*d^2)^4*c^3/(c*x^2+a
)*C*x*d^5+3/2/(a*e^2+c*d^2)^4*c^3/(c*x^2+a)*A*d^4*e-5/(a*e^2+c*d^2)^4*c^2*l
n(c*x^2+a)*A*d^2*e^3+3/(a*e^2+c*d^2)^4*c^2*ln(c*x^2+a)*d^3*e^2*B-3/2/(a*e^2
+c*d^2)^4*c^2*ln(c*x^2+a)*C*d^4*e+1/2/(a*e^2+c*d^2)^4*c^3/(a*c)^(1/2)*arcta
n(1/(a*c)^(1/2)*c*x)*C*d^5+1/(a*e^2+c*d^2)^4*c*a*ln(c*x^2+a)*A*e^5+2*e^3/(a
*e^2+c*d^2)^3/(e*x+d)*C*a*d-2*e/(a*e^2+c*d^2)^3/(e*x+d)*C*c*d^3-2*e^5/(a*e^
2+c*d^2)^4*ln(e*x+d)*A*a*c+10*e^3/(a*e^2+c*d^2)^4*ln(e*x+d)*A*c^2*d^2-6*e^2
```

```
/(a*e^2+c*d^2)^4*ln(e*x+d)*B*c^2*d^3+3*e/(a*e^2+c*d^2)^4*ln(e*x+d)*C*c^2*d^
4-4*e^3/(a*e^2+c*d^2)^3/(e*x+d)*A*c*d+3*e^2/(a*e^2+c*d^2)^3/(e*x+d)*B*c*d^2
-1/2*e^3/(a*e^2+c*d^2)^2/(e*x+d)^2*A+1/(a*e^2+c*d^2)^4*c^2/(c*x^2+a)*C*x*a*
d^3*e^2+9/2/(a*e^2+c*d^2)^4*c*a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d
*e^4-15/2/(a*e^2+c*d^2)^4*c^2*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d*e
^4+9/(a*e^2+c*d^2)^4*c^2*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*d^2*e^3-
7/(a*e^2+c*d^2)^4*c^2*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d^3*e^2+3/2
/(a*e^2+c*d^2)^4*c/(c*x^2+a)*C*x*a^2*d*e^4-3/2/(a*e^2+c*d^2)^4*c^2/(c*x^2+a
)*A*x*a*d*e^4+1/(a*e^2+c*d^2)^4*c^2/(c*x^2+a)*B*x*a*d^2*e^3-1/(a*e^2+c*d^2)
^4*c^3/(c*x^2+a)*A*x*d^3*e^2+1/2/(a*e^2+c*d^2)^4*c^4/(c*x^2+a)/a*x*A*d^5+3/
2/(a*e^2+c*d^2)^4*c^3/(c*x^2+a)*B*x*d^4*e+1/(a*e^2+c*d^2)^4*c^2/(c*x^2+a)*A
*d^2*e^3*a+1/(a*e^2+c*d^2)^4*c^2/(c*x^2+a)*d^3*e^2*B*a-3/2/(a*e^2+c*d^2)^4*
c^2/(c*x^2+a)*C*a*d^4*e+6*e^4/(a*e^2+c*d^2)^4*ln(e*x+d)*B*a*c*d-8*e^3/(a*e^
2+c*d^2)^4*ln(e*x+d)*C*a*c*d^2-1/2/(a*e^2+c*d^2)^4*c/(c*x^2+a)*B*x*a^2*e^5+
3/2/(a*e^2+c*d^2)^4*c/(c*x^2+a)*d*e^4*B*a^2-1/(a*e^2+c*d^2)^4*c/(c*x^2+a)*C
*a^2*d^2*e^3+1/2/(a*e^2+c*d^2)^4*c^4/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x
)*A*d^5+5/(a*e^2+c*d^2)^4*c^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d^3*e
^2-3/2/(a*e^2+c*d^2)^4*c^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*d^4*e-3/
(a*e^2+c*d^2)^4*c*a*ln(c*x^2+a)*d*e^4*B+4/(a*e^2+c*d^2)^4*c*a*ln(c*x^2+a)*C
*d^2*e^3-3/2/(a*e^2+c*d^2)^4*c*a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*
e^5+1/2/(a*e^2+c*d^2)^4/(c*x^2+a)*C*a^3*e^5-e^4/(a*e^2+c*d^2)^3/(e*x+d)*B*a
+1/2*e^2/(a*e^2+c*d^2)^2/(e*x+d)^2*B*d-1/2*e/(a*e^2+c*d^2)^2/(e*x+d)^2*C*d^
2+e^5/(a*e^2+c*d^2)^4*ln(e*x+d)*a^2*C-1/2/(a*e^2+c*d^2)^4*c^3/(c*x^2+a)*d^5
*B-1/2/(a*e^2+c*d^2)^4*a^2*ln(c*x^2+a)*C*e^5
```

maxima [B] time = 1.22, size = 1030, normalized size = 1.97

$$-\frac{\left(3\,Cc^2d^4e-6\,Bc^2d^3e^2+6\,Bacde^4-2\left(4\,Cac-5\,Ac^2\right)d^2e^3+\left(Ca^2-2\,Aac\right)e^5\right)\log\left(cx^2+a\right)}{2\left(c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8\right)}+\frac{\left(3\,Cc^2d^4e-6\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*C*c^2*d^4*e - 6*B*c^2*d^3*e^2 + 6*B*a*c*d*e^4 - 2*(4*C*a*c - 5*A*c^
2)*d^2*e^3 + (C*a^2 - 2*A*a*c)*e^5)*log(c*x^2 + a)/(c^4*d^8 + 4*a*c^3*d^6*e
^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + (3*C*c^2*d^4*e - 6*B*
c^2*d^3*e^2 + 6*B*a*c*d*e^4 - 2*(4*C*a*c - 5*A*c^2)*d^2*e^3 + (C*a^2 - 2*A*
a*c)*e^5)*log(e*x + d)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a
^3*c*d^2*e^6 + a^4*e^8) - 1/2*(3*B*a*c^3*d^4*e - 18*B*a^2*c^2*d^2*e^3 + 3*B
*a^3*c*e^5 - (C*a*c^3 + A*c^4)*d^5 + 2*(7*C*a^2*c^2 - 5*A*a*c^3)*d^3*e^2 -
3*(3*C*a^3*c - 5*A*a^2*c^2)*d*e^4)*arctan(c*x/sqrt(a*c))/((a*c^4*d^8 + 4*a^
2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(a*c)) -
 1/2*(B*a*c^2*d^5 - 10*B*a^2*c*d^3*e^2 + B*a^3*d*e^4 + A*a^3*e^5 + (8*C*a^2
*c - 3*A*a*c^2)*d^4*e - 2*(2*C*a^3 - 5*A*a^2*c)*d^2*e^3 - (9*B*a*c^2*d^2*e^
3 - 3*B*a^2*c*e^5 - (5*C*a*c^2 - A*c^3)*d^3*e^2 + (7*C*a^2*c - 11*A*a*c^2)*
d*e^4)*x^3 - (12*B*a*c^2*d^3*e^2 - (7*C*a*c^2 - 2*A*c^3)*d^4*e + 6*(C*a^2*c
 - 2*A*a*c^2)*d^2*e^3 + (C*a^3 - 2*A*a^2*c)*e^5)*x^2 - (B*a*c^2*d^4*e + 11*
B*a^2*c*d^2*e^3 - 2*B*a^3*e^5 - (C*a*c^2 - A*c^3)*d^5 - (7*C*a^2*c - 3*A*a*
c^2)*d^3*e^2 + 2*(3*C*a^3 - 5*A*a^2*c)*d*e^4)*x)/(a^2*c^3*d^8 + 3*a^3*c^2*d
^6*e^2 + 3*a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^6*e^2 + 3*a^2*c^3*d^4*e^4
 + 3*a^3*c^2*d^2*e^6 + a^4*c*e^8)*x^4 + 2*(a*c^4*d^7*e + 3*a^2*c^3*d^5*e^3
+ 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6
*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*x^2 + 2*(a^2*c^3*d^7*e + 3*a^
3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 + a^5*d*e^7)*x)
```

mupad [B] time = 14.48, size = 2828, normalized size = 5.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^2*(d + e*x)^3),x)
```

```
[Out] (log(C*c^2*d^7*(-a^3*c)^(3/2) - 3*B*a^6*e^7*(-a^3*c)^(1/2) - 6*C*a^8*e^7 +
12*A*a^7*c*e^7 - 3*B*a^7*c*e^7*x + 2*A*a^4*c^4*d^6*e + 20*C*a^5*c^3*d^6*e +
 72*C*a^7*c*d^2*e^5 - A*a^3*c^5*d^7*x - C*a^4*c^4*d^7*x + 39*A*a^2*d*e^6*(-
a^3*c)^(3/2) + 21*C*a^6*d*e^6*(-a^3*c)^(1/2) - 3*B*c^2*d^6*e*(-a^3*c)^(3/2)
 + 12*A*a^2*e^7*x*(-a^3*c)^(3/2) + 6*C*a^6*e^7*x*(-a^3*c)^(1/2) + 80*A*a^5*
c^3*d^4*e^3 - 102*A*a^6*c^2*d^2*e^5 - 42*B*a^5*c^3*d^5*e^2 + 108*B*a^6*c^2*
d^3*e^4 - 94*C*a^6*c^2*d^4*e^3 - A*a^2*c^4*d^7*(-a^3*c)^(1/2) - 93*B*a^2*d^
2*e^5*(-a^3*c)^(3/2) + 9*A*c^2*d^5*e^2*(-a^3*c)^(3/2) + 119*C*a^2*d^3*e^4*(
-a^3*c)^(3/2) - 42*B*a^7*c*d*e^6 - 9*A*a^4*c^4*d^5*e^2*x + 145*A*a^5*c^3*d^
3*e^4*x - 93*B*a^5*c^3*d^4*e^3*x + 93*B*a^6*c^2*d^2*e^5*x + 51*C*a^5*c^3*d^
5*e^2*x - 119*C*a^6*c^2*d^3*e^4*x + 80*A*c^2*d^4*e^3*x*(-a^3*c)^(3/2) + 72*
C*a^2*d^2*e^5*x*(-a^3*c)^(3/2) - 42*B*c^2*d^5*e^2*x*(-a^3*c)^(3/2) + 21*C*a
^7*c*d*e^6*x - 39*A*a^6*c^2*d*e^6*x + 3*B*a^4*c^4*d^6*e*x - 145*A*a*c*d^3*e
^4*(-a^3*c)^(3/2) + 93*B*a*c*d^4*e^3*(-a^3*c)^(3/2) - 51*C*a*c*d^5*e^2*(-a^
3*c)^(3/2) - 42*B*a^2*d*e^6*x*(-a^3*c)^(3/2) + 20*C*c^2*d^6*e*x*(-a^3*c)^(3
/2) - 102*A*a*c*d^2*e^5*x*(-a^3*c)^(3/2) + 108*B*a*c*d^3*e^4*x*(-a^3*c)^(3/
2) - 94*C*a*c*d^4*e^3*x*(-a^3*c)^(3/2) - 2*A*a^2*c^4*d^6*e*x*(-a^3*c)^(1/2)
)*(e^2*(3*B*a^3*c^2*d^3 + (5*A*a*c^2*d^3*(-a^3*c)^(1/2))/2 - (7*C*a^2*c*d^3
*(-a^3*c)^(1/2))/2) + e^3*(4*C*a^4*c*d^2 - 5*A*a^3*c^2*d^2 + (9*B*a^2*c*d^2
*(-a^3*c)^(1/2))/2) - e^4*(3*B*a^4*c*d - (9*C*a^3*d*(-a^3*c)^(1/2))/4 + (15
*A*a^2*c*d*(-a^3*c)^(1/2))/4) - e*((3*C*a^3*c^2*d^4)/2 + (3*B*a*c^2*d^4*(-a
^3*c)^(1/2))/4) - e^5*((C*a^5)/2 + (3*B*a^3*(-a^3*c)^(1/2))/4 - A*a^4*c) +
(A*c^3*d^5*(-a^3*c)^(1/2))/4 + (C*a*c^2*d^5*(-a^3*c)^(1/2))/4))/(a^7*e^8 +
a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4) - (l
og(3*B*a^6*e^7*(-a^3*c)^(1/2) - 6*C*a^8*e^7 - C*c^2*d^7*(-a^3*c)^(3/2) + 12
*A*a^7*c*e^7 - 3*B*a^7*c*e^7*x + 2*A*a^4*c^4*d^6*e + 20*C*a^5*c^3*d^6*e + 7
2*C*a^7*c*d^2*e^5 - A*a^3*c^5*d^7*x - C*a^4*c^4*d^7*x - 39*A*a^2*d*e^6*(-a^
3*c)^(3/2) - 21*C*a^6*d*e^6*(-a^3*c)^(1/2) + 3*B*c^2*d^6*e*(-a^3*c)^(3/2) -
 12*A*a^2*e^7*x*(-a^3*c)^(3/2) - 6*C*a^6*e^7*x*(-a^3*c)^(1/2) + 80*A*a^5*c^
3*d^4*e^3 - 102*A*a^6*c^2*d^2*e^5 - 42*B*a^5*c^3*d^5*e^2 + 108*B*a^6*c^2*d^
3*e^4 - 94*C*a^6*c^2*d^4*e^3 + A*a^2*c^4*d^7*(-a^3*c)^(1/2) + 93*B*a^2*d^2*
e^5*(-a^3*c)^(3/2) - 9*A*c^2*d^5*e^2*(-a^3*c)^(3/2) - 119*C*a^2*d^3*e^4*(-a
^3*c)^(3/2) - 42*B*a^7*c*d*e^6 - 9*A*a^4*c^4*d^5*e^2*x + 145*A*a^5*c^3*d^3*
e^4*x - 93*B*a^5*c^3*d^4*e^3*x + 93*B*a^6*c^2*d^2*e^5*x + 51*C*a^5*c^3*d^5*
e^2*x - 119*C*a^6*c^2*d^3*e^4*x - 80*A*c^2*d^4*e^3*x*(-a^3*c)^(3/2) - 72*C*
a^2*d^2*e^5*x*(-a^3*c)^(3/2) + 42*B*c^2*d^5*e^2*x*(-a^3*c)^(3/2) + 21*C*a^7
*c*d*e^6*x - 39*A*a^6*c^2*d*e^6*x + 3*B*a^4*c^4*d^6*e*x + 145*A*a*c*d^3*e^4
*(-a^3*c)^(3/2) - 93*B*a*c*d^4*e^3*(-a^3*c)^(3/2) + 51*C*a*c*d^5*e^2*(-a^3*
c)^(3/2) + 42*B*a^2*d*e^6*x*(-a^3*c)^(3/2) - 20*C*c^2*d^6*e*x*(-a^3*c)^(3/2
) + 102*A*a*c*d^2*e^5*x*(-a^3*c)^(3/2) - 108*B*a*c*d^3*e^4*x*(-a^3*c)^(3/2)
 + 94*C*a*c*d^4*e^3*x*(-a^3*c)^(3/2) + 2*A*a^2*c^4*d^6*e*x*(-a^3*c)^(1/2))*
(e^3*(5*A*a^3*c^2*d^2 - 4*C*a^4*c*d^2 + (9*B*a^2*c*d^2*(-a^3*c)^(1/2))/2) -
 e^2*(3*B*a^3*c^2*d^3 - (5*A*a*c^2*d^3*(-a^3*c)^(1/2))/2 + (7*C*a^2*c*d^3*(
-a^3*c)^(1/2))/2) + e^4*(3*B*a^4*c*d + (9*C*a^3*d*(-a^3*c)^(1/2))/4 - (15*A
*a^2*c*d*(-a^3*c)^(1/2))/4) + e*((3*C*a^3*c^2*d^4)/2 - (3*B*a*c^2*d^4*(-a^3
*c)^(1/2))/4) - e^5*((3*B*a^3*(-a^3*c)^(1/2))/4 - (C*a^5)/2 + A*a^4*c) + (A
*c^3*d^5*(-a^3*c)^(1/2))/4 + (C*a*c^2*d^5*(-a^3*c)^(1/2))/4))/(a^7*e^8 + a^
3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4) - ((A*
a^2*e^5 + B*c^2*d^5 + B*a^2*d*e^4 - 3*A*c^2*d^4*e - 4*C*a^2*d^2*e^3 + 8*C*a
*c*d^4*e + 10*A*a*c*d^2*e^3 - 10*B*a*c*d^3*e^2)/(2*(a*e^2 + c*d^2)*(a^2*e^4
 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x^3*(3*B*a^2*c*e^5 - A*c^3*d^3*e^2 - 9*B*a*
c^2*d^2*e^3 + 5*C*a*c^2*d^3*e^2 + 11*A*a*c^2*d*e^4 - 7*C*a^2*c*d*e^4))/(2*a
*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*(A*c^3*d^5 -
 2*B*a^3*e^5 - C*a*c^2*d^5 + 6*C*a^3*d*e^4 + 3*A*a*c^2*d^3*e^2 + 11*B*a^2*c
*d^2*e^3 - 7*C*a^2*c*d^3*e^2 - 10*A*a^2*c*d*e^4 + B*a*c^2*d^4*e))/(2*a*(a*e
^2 + c*d^2)*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x^2*(C*a^3*e^5 - 2*A*a^
2*c*e^5 + 2*A*c^3*d^4*e - 12*A*a*c^2*d^2*e^3 + 12*B*a*c^2*d^3*e^2 + 6*C*a^2
*c*d^2*e^3 - 7*C*a*c^2*d^4*e))/(2*a*(a*e^2 + c*d^2)*(a^2*e^4 + c^2*d^4 + 2*
```

```
a*c*d^2*e^2)))/(a*d^2 + x^2*(a*e^2 + c*d^2) + c*e^2*x^4 + 2*a*d*e*x + 2*c*d
*e*x^3) + (log(d + e*x)*(c^2*(10*A*d^2*e^3 - 6*B*d^3*e^2 + 3*C*d^4*e) - c*(
2*A*a*e^5 - 6*B*a*d*e^4 + 8*C*a*d^2*e^3) + C*a^2*e^5))/(a^4*e^8 + c^4*d^8 +
 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.57 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx$

Optimal. Leaf size=209

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd^2(3aBe+aCd+3Acd)+3ae^2(aBe+3aCd+Acd)\right)}{8a^{5/2}c^{5/2}}-\frac{(d+ex)\left(ae(3aBe+5aCd+3Acd)-x\left(3Ac\right.\right.}{8a^2c^2\left(a+c\right.}$$

```
[Out] -1/4*(a*B-(A*c-C*a)*x)*(e*x+d)^3/a/c/(c*x^2+a)^2-1/8*(e*x+d)*(a*e*(3*A*c*d+
3*B*a*e+5*C*a*d)-(3*A*c^2*d^2-a*(4*a*C*e^2-c*d*(3*B*e+C*d)))*x)/a^2/c^2/(c*
x^2+a)+1/8*(3*A*c*d*(a*e^2+c*d^2)+a*(3*a*e^2*(B*e+3*C*d)+c*d^2*(3*B*e+C*d))
)*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/c^(5/2)+1/2*C*e^3*ln(c*x^2+a)/c^3
```

Rubi [A] time = 0.30, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1645, 819, 635, 205, 260}

$$-\frac{(d+ex)\left(ae(3aBe+5aCd+3Acd)-x\left(3Ac^2d^2-a\left(4aCe^2-cd(3Be+Cd)\right)\right)\right)}{8a^2c^2\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd^2(3aBe+aCd+\right.}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(4*a*c*(a + c*x^2)^2) - ((d + e*x)*(a*
e*(3*A*c*d + 5*a*C*d + 3*a*B*e) - (3*A*c^2*d^2 - a*(4*a*C*e^2 - c*d*(C*d +
3*B*e)))*x))/(8*a^2*c^2*(a + c*x^2)) + ((3*a*e^2*(A*c*d + 3*a*C*d + a*B*e)
+ c*d^2*(3*A*c*d + a*C*d + 3*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2
)*c^(5/2)) + (C*e^3*Log[a + c*x^2])/(2*c^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} - \frac{\int \frac{(d+ex)^2(-3Acd-aCd-3aBe-4aCex)}{\left(a+cx^2\right)^2}\,dx}{4ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} - \frac{(d+ex)\left(ae(3Acd+5aCd+3aBe)-\left(3Ac^2\right.\right.}{8a^2c^2\left(a+c\right.}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} - \frac{(d+ex)\left(ae(3Acd+5aCd+3aBe)-\left(3Ac^2\right.\right.}{8a^2c^2\left(a+c\right.}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} - \frac{(d+ex)\left(ae(3Acd+5aCd+3aBe)-\left(3Ac^2\right.\right.}{8a^2c^2\left(a+c\right.}$$

Mathematica [A] time = 0.25, size = 281, normalized size = 1.34

$$\frac{\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(3Acd\left(ae^2+cd^2\right)+a\left(3ae^2(Be+3Cd)+cd^2(3Be+Cd)\right)\right)}{a^{5/2}} + \frac{-2a^3Ce^3+2a^2ce(e(Ae+3Bd+Bex)+3Cd(d+ex))-2ac^2d\left(3Ae(d+ex)+Bd(d+3e\right.}{a\left(a+cx^2\right)^2}}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] ((-2*a^3*C*e^3 + 2*A*c^3*d^3*x - 2*a*c^2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d
*(d + 3*e*x)) + 2*a^2*c*e*(3*C*d*(d + e*x) + e*(3*B*d + A*e + B*e*x)))/(a*(
a + c*x^2)^2) + (8*a^3*C*e^3 + 3*A*c^3*d^3*x + a*c^2*d*(C*d^2 + 3*e*(B*d +
A*e))*x - a^2*c*e*(3*C*d*(4*d + 5*e*x) + e*(12*B*d + 4*A*e + 5*B*e*x)))/(a^
2*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c*d^2 + a*e^2) + a*(3*a*e^2*(3*C*d + B*
e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 4*C*e^3*L
og[a + c*x^2])/(8*c^3)
```

fricas [B] time = 1.43, size = 1138, normalized size = 5.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c^2*d^3 + 12*B*a^4*c*d*e^2 + 12*(C*a^4*c + A*a^3*c^2)*d^2*e
 - 4*(3*C*a^5 - A*a^4*c)*e^3 - 2*(3*B*a^2*c^3*d^2*e - 5*B*a^3*c^2*e^3 + (C*
a^2*c^3 + 3*A*a*c^4)*d^3 - 3*(5*C*a^3*c^2 - A*a^2*c^3)*d*e^2)*x^3 + 8*(3*C*
```

```
a^3*c^2*d^2*e + 3*B*a^3*c^2*d*e^2 - (2*C*a^4*c - A*a^3*c^2)*e^3)*x^2 + (3*B
*a^3*c*d^2*e + 3*B*a^4*e^3 + (3*B*a*c^3*d^2*e + 3*B*a^2*c^2*e^3 + (C*a*c^3
+ 3*A*c^4)*d^3 + 3*(3*C*a^2*c^2 + A*a*c^3)*d*e^2)*x^4 + (C*a^3*c + 3*A*a^2*
c^2)*d^3 + 3*(3*C*a^4 + A*a^3*c)*d*e^2 + 2*(3*B*a^2*c^2*d^2*e + 3*B*a^3*c*e
^3 + (C*a^2*c^2 + 3*A*a*c^3)*d^3 + 3*(3*C*a^3*c + A*a^2*c^2)*d*e^2)*x^2)*sq
rt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(3*B*a^3*c^2*d^2
*e + 3*B*a^4*c*e^3 + (C*a^3*c^2 - 5*A*a^2*c^3)*d^3 + 3*(3*C*a^4*c + A*a^3*c
^2)*d*e^2)*x - 8*(C*a^3*c^2*e^3*x^4 + 2*C*a^4*c*e^3*x^2 + C*a^5*e^3)*log(c*
x^2 + a))/(a^3*c^5*x^4 + 2*a^4*c^4*x^2 + a^5*c^3), -1/8*(2*B*a^3*c^2*d^3 +
6*B*a^4*c*d*e^2 + 6*(C*a^4*c + A*a^3*c^2)*d^2*e - 2*(3*C*a^5 - A*a^4*c)*e^3
 - (3*B*a^2*c^3*d^2*e - 5*B*a^3*c^2*e^3 + (C*a^2*c^3 + 3*A*a*c^4)*d^3 - 3*(
5*C*a^3*c^2 - A*a^2*c^3)*d*e^2)*x^3 + 4*(3*C*a^3*c^2*d^2*e + 3*B*a^3*c^2*d*
e^2 - (2*C*a^4*c - A*a^3*c^2)*e^3)*x^2 - (3*B*a^3*c*d^2*e + 3*B*a^4*e^3 + (
3*B*a*c^3*d^2*e + 3*B*a^2*c^2*e^3 + (C*a*c^3 + 3*A*c^4)*d^3 + 3*(3*C*a^2*c^
2 + A*a*c^3)*d*e^2)*x^4 + (C*a^3*c + 3*A*a^2*c^2)*d^3 + 3*(3*C*a^4 + A*a^3*
c)*d*e^2 + 2*(3*B*a^2*c^2*d^2*e + 3*B*a^3*c*e^3 + (C*a^2*c^2 + 3*A*a*c^3)*d
^3 + 3*(3*C*a^3*c + A*a^2*c^2)*d*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a)
+ (3*B*a^3*c^2*d^2*e + 3*B*a^4*c*e^3 + (C*a^3*c^2 - 5*A*a^2*c^3)*d^3 + 3*(3
*C*a^4*c + A*a^3*c^2)*d*e^2)*x - 4*(C*a^3*c^2*e^3*x^4 + 2*C*a^4*c*e^3*x^2 +
 C*a^5*e^3)*log(c*x^2 + a))/(a^3*c^5*x^4 + 2*a^4*c^4*x^2 + a^5*c^3)]
```

giac [A] time = 0.27, size = 348, normalized size = 1.67

$$\frac{Ce^3\log\left(cx^2+a\right)}{2\,c^3}+\frac{\left(Cacd^3+3\,Ac^2d^3+3\,Bacd^2e+9\,Ca^2de^2+3\,Aacde^2+3\,Ba^2e^3\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2c^2}+\frac{\left(Cac^2d^3+3\,A\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*C*e^3*log(c*x^2 + a)/c^3 + 1/8*(C*a*c*d^3 + 3*A*c^2*d^3 + 3*B*a*c*d^2*e
 + 9*C*a^2*d*e^2 + 3*A*a*c*d*e^2 + 3*B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt
(a*c)*a^2*c^2) + 1/8*((C*a*c^2*d^3 + 3*A*c^3*d^3 + 3*B*a*c^2*d^2*e - 15*C*a
^2*c*d*e^2 + 3*A*a*c^2*d*e^2 - 5*B*a^2*c*e^3)*x^3 - 4*(3*C*a^2*c*d^2*e + 3*
B*a^2*c*d*e^2 - 2*C*a^3*e^3 + A*a^2*c*e^3)*x^2 - (C*a^2*c*d^3 - 5*A*a*c^2*d
^3 + 3*B*a^2*c*d^2*e + 9*C*a^3*d*e^2 + 3*A*a^2*c*d*e^2 + 3*B*a^3*e^3)*x - 2
*(B*a^2*c^2*d^3 + 3*C*a^3*c*d^2*e + 3*A*a^2*c^2*d^2*e + 3*B*a^3*c*d*e^2 - 3
*C*a^4*e^3 + A*a^3*c*e^3)/c)/((c*x^2 + a)^2*a^2*c^2)
```

maple [B] time = 0.01, size = 402, normalized size = 1.92

$$\frac{3Ad\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;ac}+\frac{3A\,d^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;a^2}+\frac{3B\,d^2e\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;ac}+\frac{3B\,e^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;c^2}+\frac{C\,d^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;ac}+\frac{9Cd\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x)
```

```
[Out] (1/8*(3*A*a*c*d*e^2+3*A*c^2*d^3-5*B*a^2*e^3+3*B*a*c*d^2*e-15*C*a^2*d*e^2+C*
a*c*d^3)/a^2/c*x^3-1/2*e*(A*c*e^2+3*B*c*d*e-2*C*a*e^2+3*C*c*d^2)/c^2*x^2-1/
8*(3*A*a*c*d*e^2-5*A*c^2*d^3+3*B*a^2*e^3+3*B*a*c*d^2*e+9*C*a^2*d*e^2+C*a*c*
d^3)/a/c^2*x-1/4*(A*a*c*e^3+3*A*c^2*d^2*e+3*B*a*c*d*e^2+B*c^2*d^3-3*C*a^2*e
^3+3*C*a*c*d^2*e)/c^3)/(c*x^2+a)^2+1/2*C*e^3*ln(c*x^2+a)/c^3+3/8/a/c/(a*c)^
(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d*e^2+3/8/a^2/(a*c)^(1/2)*arctan(1/(a*c)^
(1/2)*c*x)*A*d^3+3/8/c^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*e^3+3/8/a/
c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*d^2*e+9/8/c^2/(a*c)^(1/2)*arctan(
1/(a*c)^(1/2)*c*x)*C*d*e^2+1/8/a/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*
d^3
```

maxima [A] time = 1.00, size = 379, normalized size = 1.81

$$\frac{Ce^3 \log\left(cx^2+a\right)}{2\,c^3} - \frac{2\,Ba^2c^2d^3 + 6\,Ba^3cde^2 + 6\left(Ca^3c + Aa^2c^2\right)d^2e - 2\left(3\,Ca^4 - Aa^3c\right)e^3 - \left(3\,Bac^3d^2e - 5\,Ba^2c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*C*e^3*log(c*x^2 + a)/c^3 - 1/8*(2*B*a^2*c^2*d^3 + 6*B*a^3*c*d*e^2 + 6*(
C*a^3*c + A*a^2*c^2)*d^2*e - 2*(3*C*a^4 - A*a^3*c)*e^3 - (3*B*a*c^3*d^2*e -
 5*B*a^2*c^2*e^3 + (C*a*c^3 + 3*A*c^4)*d^3 - 3*(5*C*a^2*c^2 - A*a*c^3)*d*e^
2)*x^3 + 4*(3*C*a^2*c^2*d^2*e + 3*B*a^2*c^2*d*e^2 - (2*C*a^3*c - A*a^2*c^2)
*e^3)*x^2 + (3*B*a^2*c^2*d^2*e + 3*B*a^3*c*e^3 + (C*a^2*c^2 - 5*A*a*c^3)*d^
3 + 3*(3*C*a^3*c + A*a^2*c^2)*d*e^2)*x)/(a^2*c^5*x^4 + 2*a^3*c^4*x^2 + a^4*
c^3) + 1/8*(3*B*a*c*d^2*e + 3*B*a^2*e^3 + (C*a*c + 3*A*c^2)*d^3 + 3*(3*C*a^
2 + A*a*c)*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c^2)
```

mupad [B] time = 1.77, size = 920, normalized size = 4.40

$$\frac{5\,A\,d^3\,x}{8\,\left(a^3+2\,a^2\,c\,x^2+a\,c^2\,x^4\right)} - \frac{B\,d^3}{4\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)} + \frac{3\,C\,a^2\,e^3}{4\,\left(a^2\,c^3+2\,a\,c^4\,x^2+c^5\,x^4\right)} - \frac{3\,A\,d^2\,e}{4\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x)
```

```
[Out] (5*A*d^3*x)/(8*(a^3 + 2*a^2*c*x^2 + a*c^2*x^4)) - (B*d^3)/(4*(a^2*c + c^3*x
^4 + 2*a*c^2*x^2)) + (3*C*a^2*e^3)/(4*(a^2*c^3 + c^5*x^4 + 2*a*c^4*x^2)) -
(3*A*d^2*e)/(4*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) + (C*d^3*x^3)/(8*(a^3 + 2*a
^2*c*x^2 + a*c^2*x^4)) - (C*d^3*x)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) - (A
*a*e^3)/(4*(a^2*c^2 + c^4*x^4 + 2*a*c^3*x^2)) - (A*e^3*x^2)/(2*(a^2*c + c^3
*x^4 + 2*a*c^2*x^2)) - (5*B*e^3*x^3)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) +
(C*e^3*log(a + c*x^2))/(2*c^3) - (3*B*a*d*e^2)/(4*(a^2*c^2 + c^4*x^4 + 2*a*
c^3*x^2)) - (3*C*a*d^2*e)/(4*(a^2*c^2 + c^4*x^4 + 2*a*c^3*x^2)) + (3*A*c*d^
3*x^3)/(8*(a^4 + 2*a^3*c*x^2 + a^2*c^2*x^4)) - (3*B*a*e^3*x)/(8*(a^2*c^2 +
c^4*x^4 + 2*a*c^3*x^2)) - (3*B*d*e^2*x^2)/(2*(a^2*c + c^3*x^4 + 2*a*c^2*x^2
)) - (3*C*d^2*e*x^2)/(2*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) - (15*C*d*e^2*x^3)
/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) + (C*a*e^3*x^2)/(a^2*c^2 + c^4*x^4 + 2
*a*c^3*x^2) + (3*A*d^3*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*c^(1/2)) + (3*
B*e^3*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*c^(5/2)) + (C*d^3*atan((c^(1/2)
*x)/a^(1/2)))/(8*a^(3/2)*c^(3/2)) + (3*A*d*e^2*x^3)/(8*(a^3 + 2*a^2*c*x^2 +
 a*c^2*x^4)) + (3*B*d^2*e*x^3)/(8*(a^3 + 2*a^2*c*x^2 + a*c^2*x^4)) - (3*A*d
*e^2*x)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) - (3*B*d^2*e*x)/(8*(a^2*c + c^3
*x^4 + 2*a*c^2*x^2)) + (3*A*d*e^2*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(3/2)*c^(
3/2)) + (3*B*d^2*e*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(3/2)*c^(3/2)) + (9*C*d*
e^2*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*c^(5/2)) - (9*C*a*d*e^2*x)/(8*(a^
2*c^2 + c^4*x^4 + 2*a*c^3*x^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.58 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx$

Optimal. Leaf size=156

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd(2aBe+aCd+3Acd)+ae^2(3aC+Ac)\right)}{8a^{5/2}c^{5/2}}-\frac{(d+ex)(ae(3aC+Ac)-cx(2aBe+aCd+3Acd))}{8a^2c^2\left(a+cx^2\right)}-\frac{(d$$

```
[Out] -1/4*(a*B-(A*c-C*a)*x)*(e*x+d)^2/a/c/(c*x^2+a)^2-1/8*(e*x+d)*(a*(A*c+3*C*a)
*e-c*(3*A*c*d+2*B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)+1/8*(a*(A*c+3*C*a)*e^2+c*
d*(3*A*c*d+2*B*a*e+C*a*d))*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/c^(5/2)
```

Rubi [A] time = 0.23, antiderivative size = 175, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1645, 778, 205}

$$-\frac{x\left(ae^2(3aC+Ac)-cd(2aBe+aCd+3Acd)\right)+2ae(aBe+2aCd+2Acd)}{8a^2c^2\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd(2aBe+aCd+3Acd\right.}{8a^{5/2}c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(4*a*c*(a + c*x^2)^2) - (2*a*e*(2*A*c*
d + 2*a*C*d + a*B*e) + (a*(A*c + 3*a*C)*e^2 - c*d*(3*A*c*d + a*C*d + 2*a*B*
e))*x)/(8*a^2*c^2*(a + c*x^2)) + ((a*(A*c + 3*a*C)*e^2 + c*d*(3*A*c*d + a*C
*d + 2*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\left(a+cx^2\right)^2} - \frac{\int \frac{(d+ex)(-3Acd-aCd-2aBe-(Ac+3aC)ex)}{\left(a+cx^2\right)^2}dx}{4ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\left(a+cx^2\right)^2} - \frac{2ae(2Acd+2aCd+aBe)+\left(a(Ac+3aC)e^{2}\right.}{8a^2c^2\left(a+cx^2\right.}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\left(a+cx^2\right)^2} - \frac{2ae(2Acd+2aCd+aBe)+\left(a(Ac+3aC)e^{2}\right.}{8a^2c^2\left(a+cx^2\right.}$$

Mathematica [A] time = 0.14, size = 211, normalized size = 1.35

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Ac\left(ae^2+3cd^2\right)+a\left(3aCe^2+cd(2Be+Cd)\right)\right)}{8a^{5/2}c^{5/2}}+\frac{a^2(-e)(4Be+8Cd+5Cex)+acx\left(e(Ae+2Bd)+\right.}{8a^2c^2\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] (3*A*c^2*d^2*x + a*c*(C*d^2 + e*(2*B*d + A*e))*x - a^2*e*(8*C*d + 4*B*e + 5
*C*e*x))/(8*a^2*c^2*(a + c*x^2)) + (A*c^2*d^2*x + a^2*e*(2*C*d + B*e + C*e*
x) - a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x)))/(4*a*c^2*(a + c*x^2
)^2) + ((A*c*(3*c*d^2 + a*e^2) + a*(3*a*C*e^2 + c*d*(C*d + 2*B*e)))*ArcTan[
(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/2))
```

fricas [B] time = 1.22, size = 806, normalized size = 5.17

$$\left[-\frac{4\,Ba^3c^2d^2+4\,Ba^4ce^2-2\left(2\,Ba^2c^3de+\left(Ca^2c^3+3\,Aac^4\right)d^2-\left(5\,Ca^3c^2-Aa^2c^3\right)e^2\right)x^3+8\left(Ca^4c+Aa^3c^2\right)de}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c^2*d^2 + 4*B*a^4*c*e^2 - 2*(2*B*a^2*c^3*d*e + (C*a^2*c^3 +
 3*A*a*c^4)*d^2 - (5*C*a^3*c^2 - A*a^2*c^3)*e^2)*x^3 + 8*(C*a^4*c + A*a^3*c
^2)*d*e + 8*(2*C*a^3*c^2*d*e + B*a^3*c^2*e^2)*x^2 + (2*B*a^3*c*d*e + (2*B*a
*c^3*d*e + (C*a*c^3 + 3*A*c^4)*d^2 + (3*C*a^2*c^2 + A*a*c^3)*e^2)*x^4 + (C*
a^3*c + 3*A*a^2*c^2)*d^2 + (3*C*a^4 + A*a^3*c)*e^2 + 2*(2*B*a^2*c^2*d*e + (
C*a^2*c^2 + 3*A*a*c^3)*d^2 + (3*C*a^3*c + A*a^2*c^2)*e^2)*x^2)*sqrt(-a*c)*l
og((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(2*B*a^3*c^2*d*e + (C*a^3*
c^2 - 5*A*a^2*c^3)*d^2 + (3*C*a^4*c + A*a^3*c^2)*e^2)*x)/(a^3*c^5*x^4 + 2*a
^4*c^4*x^2 + a^5*c^3), -1/8*(2*B*a^3*c^2*d^2 + 2*B*a^4*c*e^2 - (2*B*a^2*c^3
*d*e + (C*a^2*c^3 + 3*A*a*c^4)*d^2 - (5*C*a^3*c^2 - A*a^2*c^3)*e^2)*x^3 + 4
*(C*a^4*c + A*a^3*c^2)*d*e + 4*(2*C*a^3*c^2*d*e + B*a^3*c^2*e^2)*x^2 - (2*B
*a^3*c*d*e + (2*B*a*c^3*d*e + (C*a*c^3 + 3*A*c^4)*d^2 + (3*C*a^2*c^2 + A*a*
c^3)*e^2)*x^4 + (C*a^3*c + 3*A*a^2*c^2)*d^2 + (3*C*a^4 + A*a^3*c)*e^2 + 2*(
2*B*a^2*c^2*d*e + (C*a^2*c^2 + 3*A*a*c^3)*d^2 + (3*C*a^3*c + A*a^2*c^2)*e^2
)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (2*B*a^3*c^2*d*e + (C*a^3*c^2 - 5*
A*a^2*c^3)*d^2 + (3*C*a^4*c + A*a^3*c^2)*e^2)*x)/(a^3*c^5*x^4 + 2*a^4*c^4*x
^2 + a^5*c^3)]
```

giac [A] time = 0.16, size = 254, normalized size = 1.63

$$\frac{\left(Cacd^2 + 3\,Ac^2d^2 + 2\,Bacde + 3\,Ca^2e^2 + Aace^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2c^2} + \frac{Cac^2d^2x^3 + 3\,Ac^3d^2x^3 + 2\,Bac^2dx^3e - 5\,Ca^2cx^3e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a*c*d^2 + 3*A*c^2*d^2 + 2*B*a*c*d*e + 3*C*a^2*e^2 + A*a*c*e^2)*arcta
n(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c^2) + 1/8*(C*a*c^2*d^2*x^3 + 3*A*c^3*d^2*x
^3 + 2*B*a*c^2*d*x^3*e - 5*C*a^2*c*x^3*e^2 + A*a*c^2*x^3*e^2 - 8*C*a^2*c*d*
x^2*e - C*a^2*c*d^2*x + 5*A*a*c^2*d^2*x - 4*B*a^2*c*x^2*e^2 - 2*B*a^2*c*d*x
*e - 2*B*a^2*c*d^2 - 3*C*a^3*x*e^2 - A*a^2*c*x*e^2 - 4*C*a^3*d*e - 4*A*a^2*
c*d*e - 2*B*a^3*e^2)/((c*x^2 + a)^2*a^2*c^2)
```

maple [A] time = 0.01, size = 283, normalized size = 1.81

$$\frac{A\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;ac} + \frac{3A\,d^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;a^2} + \frac{Bde\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{4\sqrt{ac}\;ac} + \frac{C\,d^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;ac} + \frac{3C\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;c^2} + \frac{-\frac{(Be+2Cd)e}{2c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x)
```

```
[Out] (1/8*(A*a*c*e^2+3*A*c^2*d^2+2*B*a*c*d*e-5*C*a^2*e^2+C*a*c*d^2)/a^2/c*x^3-1/
2*e*(B*e+2*C*d)*x^2/c-1/8*(A*a*c*e^2-5*A*c^2*d^2+2*B*a*c*d*e+3*C*a^2*e^2+C*
a*c*d^2)/a/c^2*x-1/4*(2*A*c*d*e+B*a*e^2+B*c*d^2+2*C*a*d*e)/c^2)/(c*x^2+a)^2
+1/8/a/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*e^2+3/8/a^2/(a*c)^(1/2)*ar
ctan(1/(a*c)^(1/2)*c*x)*A*d^2+1/4/a/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)
*B*d*e+3/8/c^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*e^2+1/8/a/c/(a*c)^(1
/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d^2
```

maxima [A] time = 0.99, size = 253, normalized size = 1.62

$$-\frac{2\,Ba^2cd^2 + 2\,Ba^3e^2 - \left(2\,Bac^2de + \left(Cac^2 + 3\,Ac^3\right)d^2 - \left(5\,Ca^2c - Aac^2\right)e^2\right)x^3 + 4\left(Ca^3 + Aa^2c\right)de + 4\left(2\,Ca^2cde\right.}{8\left(a^2c^4x^4 + 2\,a^3c^3x^2 + a^4c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(2*B*a^2*c*d^2 + 2*B*a^3*e^2 - (2*B*a*c^2*d*e + (C*a*c^2 + 3*A*c^3)*d^
2 - (5*C*a^2*c - A*a*c^2)*e^2)*x^3 + 4*(C*a^3 + A*a^2*c)*d*e + 4*(2*C*a^2*c
*d*e + B*a^2*c*e^2)*x^2 + (2*B*a^2*c*d*e + (C*a^2*c - 5*A*a*c^2)*d^2 + (3*C
*a^3 + A*a^2*c)*e^2)*x)/(a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2) + 1/8*(2*B*
a*c*d*e + (C*a*c + 3*A*c^2)*d^2 + (3*C*a^2 + A*a*c)*e^2)*arctan(c*x/sqrt(a*
c))/(sqrt(a*c)*a^2*c^2)
```

mupad [B] time = 3.96, size = 230, normalized size = 1.47

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(3\,C\,a^2\,e^2 + C\,a\,c\,d^2 + 2\,B\,a\,c\,d\,e + A\,a\,c\,e^2 + 3\,A\,c^2\,d^2\right)}{8\,a^{5/2}\,c^{5/2}} - \frac{\frac{B\,a\,e^2+B\,c\,d^2+2\,A\,c\,d\,e+2\,C\,a\,d\,e}{4\,c^2} + \frac{x^2\left(B\,e^2+2\,C\,d\,e\right)}{2\,c} + \frac{x}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x)
```

[Out] (atan((c^(1/2)*x)/a^(1/2))*(3*A*c^2*d^2 + 3*C*a^2*e^2 + A*a*c*e^2 + C*a*c*d
^2 + 2*B*a*c*d*e))/(8*a^(5/2)*c^(5/2)) - ((B*a*e^2 + B*c*d^2 + 2*A*c*d*e +
2*C*a*d*e)/(4*c^2) + (x^2*(B*e^2 + 2*C*d*e))/(2*c) + (x*(3*C*a^2*e^2 - 5*A*
c^2*d^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(8*a*c^2) - (x^3*(3*A*c^2*d
^2 - 5*C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(8*a^2*c))/(a^2 +
c^2*x^4 + 2*a*c*x^2)

sympy [B] time = 141.18, size = 391, normalized size = 2.51

$$-\frac{\sqrt{-\frac{1}{a^5c^5}}\left(Aace^2+3Ac^2d^2+2Bacde+3Ca^2e^2+Cacd^2\right)\log\left(-a^3c^2\sqrt{-\frac{1}{a^5c^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5c^5}}\left(Aace^2+3Ac^2d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**3,x)

[Out] -sqrt(-1/(a**5*c**5))*(A*a*c*e**2 + 3*A*c**2*d**2 + 2*B*a*c*d*e + 3*C*a**2*
e**2 + C*a*c*d**2)*log(-a**3*c**2*sqrt(-1/(a**5*c**5)) + x)/16 + sqrt(-1/(a
5*c5))*(A*a*c*e**2 + 3*A*c**2*d**2 + 2*B*a*c*d*e + 3*C*a**2*e**2 + C*a*
c*d**2)*log(a**3*c**2*sqrt(-1/(a**5*c**5)) + x)/16 + (-4*A*a**2*c*d*e - 2*B
*a**3*e**2 - 2*B*a**2*c*d**2 - 4*C*a**3*d*e + x**3*(A*a*c**2*e**2 + 3*A*c**
3*d**2 + 2*B*a*c**2*d*e - 5*C*a**2*c*e**2 + C*a*c**2*d**2) + x**2*(-4*B*a**
2*c*e**2 - 8*C*a**2*c*d*e) + x*(-A*a**2*c*e**2 + 5*A*a*c**2*d**2 - 2*B*a**2
*c*d*e - 3*C*a**3*e**2 - C*a**2*c*d**2))/(8*a**4*c**2 + 16*a**3*c**3*x**2 +
8*a**2*c**4*x**4)

3.59 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=130

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+3Acd)}{8a^{5/2}c^{3/2}}-\frac{2ae(aC+Ac)-cx(aBe+aCd+3Acd)}{8a^2c^2\left(a+cx^2\right)}-\frac{(d+ex)(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}$$

```
[Out] -1/4*(a*B-(A*c-C*a)*x)*(e*x+d)/a/c/(c*x^2+a)^2+1/8*(-2*a*(A*c+C*a)*e+c*(3*A
*c*d+B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)+1/8*(3*A*c*d+B*a*e+C*a*d)*arctan(x*c
^(1/2)/a^(1/2))/a^(5/2)/c^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1645, 639, 205}

$$-\frac{2ae(aC+Ac)-cx(aBe+aCd+3Acd)}{8a^2c^2\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+3Acd)}{8a^{5/2}c^{3/2}}-\frac{(d+ex)(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x))/(4*a*c*(a + c*x^2)^2) - (2*a*(A*c + a*C)
*e - c*(3*A*c*d + a*C*d + a*B*e)*x)/(8*a^2*c^2*(a + c*x^2)) + ((3*A*c*d + a
*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2} - \frac{\int \frac{-3Acd-a(Cd+Be)-2(Ac+aC)ex}{\left(a+cx^2\right)^2}\,dx}{4ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2} - \frac{2a(Ac+aC)e-c(3Acd+aCd+aBe)x}{8a^2c^2\left(a+cx^2\right)} + \frac{(3A}{}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2} - \frac{2a(Ac+aC)e-c(3Acd+aCd+aBe)x}{8a^2c^2\left(a+cx^2\right)} + \frac{(3A}{}$$

Mathematica [A] time = 0.10, size = 137, normalized size = 1.05

$$\frac{\frac{\sqrt{a}\left(-4a^2Ce+acx(Be+Cd)+3Ac^2dx\right)}{a+cx^2}+\frac{2a^{3/2}\left(a^2Ce-ac(Ae+B(d+ex)+Cdx)+Ac^2dx\right)}{\left(a+cx^2\right)^2}+\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+3Acd)}{8a^{5/2}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*(-4*a^2*C*e + 3*A*c^2*d*x + a*c*(C*d + B*e)*x))/(a + c*x^2) + (2*
a^(3/2)*(a^2*C*e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x))))/(a + c*x^2
)^2 + Sqrt[c]*(3*A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(
5/2)*c^2)
```

fricas [A] time = 1.20, size = 470, normalized size = 3.62

$$\left[-\frac{8\,Ca^3cex^2+4\,Ba^3cd-2\left(Ba^2c^2e+\left(Ca^2c^2+3\,Aac^3\right)d\right)x^3+\left(Ba^3e+\left(Bac^2e+\left(Cac^2+3\,Ac^3\right)d\right)x^4+2\left(Ba^2c\right.\right.}{16\left(a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(8*C*a^3*c*e*x^2 + 4*B*a^3*c*d - 2*(B*a^2*c^2*e + (C*a^2*c^2 + 3*A*a
*c^3)*d)*x^3 + (B*a^3*e + (B*a*c^2*e + (C*a*c^2 + 3*A*c^3)*d)*x^4 + 2*(B*a^
2*c*e + (C*a^2*c + 3*A*a*c^2)*d)*x^2 + (C*a^3 + 3*A*a^2*c)*d)*sqrt(-a*c)*lo
g((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 4*(C*a^4 + A*a^3*c)*e + 2*(B*
a^3*c*e + (C*a^3*c - 5*A*a^2*c^2)*d)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*
c^2), -1/8*(4*C*a^3*c*e*x^2 + 2*B*a^3*c*d - (B*a^2*c^2*e + (C*a^2*c^2 + 3*A
*a*c^3)*d)*x^3 - (B*a^3*e + (B*a*c^2*e + (C*a*c^2 + 3*A*c^3)*d)*x^4 + 2*(B*
a^2*c*e + (C*a^2*c + 3*A*a*c^2)*d)*x^2 + (C*a^3 + 3*A*a^2*c)*d)*sqrt(a*c)*a
rctan(sqrt(a*c)*x/a) + 2*(C*a^4 + A*a^3*c)*e + (B*a^3*c*e + (C*a^3*c - 5*A*
a^2*c^2)*d)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*c^2)]
```

giac [A] time = 0.19, size = 152, normalized size = 1.17

$$\frac{(Cad+3\,Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2c}+\frac{Cac^2dx^3+3\,Ac^3dx^3+Bac^2x^3e-4\,Ca^2cx^2e-Ca^2cdx+5\,Aac^2dx-Ba^2c}{8\left(cx^2+a\right)^2a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a*d + 3*A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/8
*(C*a*c^2*d*x^3 + 3*A*c^3*d*x^3 + B*a*c^2*x^3*e - 4*C*a^2*c*x^2*e - C*a^2*c
*d*x + 5*A*a*c^2*d*x - B*a^2*c*x*e - 2*B*a^2*c*d - 2*C*a^3*e - 2*A*a^2*c*e)
/((c*x^2 + a)^2*a^2*c^2)
```

maple [A] time = 0.01, size = 157, normalized size = 1.21

$$\frac{3Ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\,a^2}+\frac{Be\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\,ac}+\frac{Cd\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\,ac}+\frac{-\frac{Cex^2}{2c}+\frac{(3Acd+Bae+Cad)x^3}{8a^2}+\frac{(5Acd-Bae-Cad)x}{8ac}-\frac{Ace+Bcd+aCe}{4c^2}}{\left(cx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x)
```

```
[Out] (1/8*(3*A*c*d+B*a*e+C*a*d)/a^2*x^3-1/2*C/c*e*x^2+1/8*(5*A*c*d-B*a*e-C*a*d)/
a/c*x-1/4*(A*c*e+B*c*d+C*a*e)/c^2)/(c*x^2+a)^2+3/8/a^2/(a*c)^(1/2)*arctan(1
/(a*c)^(1/2)*c*x)*A*d+1/8/a/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*e+1/8
/a/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d
```

maxima [A] time = 0.98, size = 160, normalized size = 1.23

$$-\frac{4\,Ca^2cex^2+2\,Ba^2cd-\left(Bac^2e+\left(Cac^2+3\,Ac^3\right)d\right)x^3+2\left(Ca^3+Aa^2c\right)e+\left(Ba^2ce+\left(Ca^2c-5\,Aac^2\right)d\right)x}{8\left(a^2c^4x^4+2\,a^3c^3x^2+a^4c^2\right)}+\frac{(Bae+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*C*a^2*c*e*x^2 + 2*B*a^2*c*d - (B*a*c^2*e + (C*a*c^2 + 3*A*c^3)*d)*x
^3 + 2*(C*a^3 + A*a^2*c)*e + (B*a^2*c*e + (C*a^2*c - 5*A*a*c^2)*d)*x)/(a^2*
c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2) + 1/8*(B*a*e + (C*a + 3*A*c)*d)*arctan(c
*x/sqrt(a*c))/(sqrt(a*c)*a^2*c)
```

mupad [B] time = 0.15, size = 128, normalized size = 0.98

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(3\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{8\,a^{5/2}\,c^{3/2}}-\frac{\frac{A\,c\,e+B\,c\,d+C\,a\,e}{4\,c^2}-\frac{x^3\,\left(3\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{8\,a^2}+\frac{C\,e\,x^2}{2\,c}+\frac{x\,\left(B\,a\,e-5\,A\,c\,d+C\,a\,d\right)}{8\,a\,c}}{a^2+2\,a\,c\,x^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(3*A*c*d + B*a*e + C*a*d))/(8*a^(5/2)*c^(3/2)) -
 ((A*c*e + B*c*d + C*a*e)/(4*c^2) - (x^3*(3*A*c*d + B*a*e + C*a*d))/(8*a^2)
 + (C*e*x^2)/(2*c) + (x*(B*a*e - 5*A*c*d + C*a*d))/(8*a*c))/(a^2 + c^2*x^4
+ 2*a*c*x^2)
```

sympy [A] time = 32.42, size = 240, normalized size = 1.85

$$-\frac{\sqrt{-\frac{1}{a^5c^3}}\,(3Acd+Bae+Cad)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5c^3}}\,(3Acd+Bae+Cad)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}+\frac{-2A\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*c**3))*(3*A*c*d + B*a*e + C*a*d)*log(-a**3*c*sqrt(-1/(a**5*c
**3)) + x)/16 + sqrt(-1/(a**5*c**3))*(3*A*c*d + B*a*e + C*a*d)*log(a**3*c*s
qrt(-1/(a**5*c**3)) + x)/16 + (-2*A*a**2*c*e - 2*B*a**2*c*d - 2*C*a**3*e -
4*C*a**2*c*e*x**2 + x**3*(3*A*c**3*d + B*a*c**2*e + C*a*c**2*d) + x*(5*A*a*
c**2*d - B*a**2*c*e - C*a**2*c*d))/(8*a**4*c**2 + 16*a**3*c**3*x**2 + 8*a**
2*c**4*x**4)
```

3.60 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=98

$$\frac{(aC+3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}+\frac{x(aC+3Ac)}{8a^2c\left(a+cx^2\right)}-\frac{aB-x(Ac-aC)}{4ac\left(a+cx^2\right)^2}$$

```
[Out] 1/4*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^2+1/8*(3*A*c+C*a)*x/a^2/c/(c*x^2+a)+1/
8*(3*A*c+C*a)*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/c^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1814, 12, 199, 205}

$$\frac{(aC+3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}+\frac{x(aC+3Ac)}{8a^2c\left(a+cx^2\right)}-\frac{aB-x(Ac-aC)}{4ac\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^3,x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(4*a*c*(a + c*x^2)^2) + ((3*A*c + a*C)*x)/(8*a^2*c*(
a + c*x^2)) + ((3*A*c + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3/2
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^3}\,dx = -\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2} - \frac{\int \frac{-3A-\frac{aC}{c}}{\left(a+cx^2\right)^2}\,dx}{4a}$$

$$= -\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2} + \frac{(3Ac+aC)\int \frac{1}{\left(a+cx^2\right)^2}\,dx}{4ac}$$

$$= -\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2} + \frac{(3Ac+aC)x}{8a^2c\left(a+cx^2\right)} + \frac{(3Ac+aC)\int \frac{1}{a+cx^2}\,dx}{8a^2c}$$

$$= -\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2} + \frac{(3Ac+aC)x}{8a^2c\left(a+cx^2\right)} + \frac{(3Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

Mathematica [A] time = 0.07, size = 90, normalized size = 0.92

$$\frac{(aC+3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}} + \frac{-a^2(2B+Cx)+acx\left(5A+Cx^2\right)+3Ac^2x^3}{8a^2c\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^3,x]
```

```
[Out] (3*A*c^2*x^3 - a^2*(2*B + C*x) + a*c*x*(5*A + C*x^2))/(8*a^2*c*(a + c*x^2)^
2) + ((3*A*c + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3/2))
```

fricas [A] time = 1.26, size = 314, normalized size = 3.20

$$\left[-\frac{4\,Ba^3c-2\left(Ca^2c^2+3\,Aac^3\right)x^3+\left(\left(Cac^2+3\,Ac^3\right)x^4+Ca^3+3\,Aa^2c+2\left(Ca^2c+3\,Aac^2\right)x^2\right)\sqrt{-ac}\,\log\left(\frac{cx^2-2\sqrt{}}{cx^2}\right.}{16\left(a^3c^4x^4+2\,a^4c^3x^2+a^5c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c - 2*(C*a^2*c^2 + 3*A*a*c^3)*x^3 + ((C*a*c^2 + 3*A*c^3)*x^
4 + C*a^3 + 3*A*a^2*c + 2*(C*a^2*c + 3*A*a*c^2)*x^2)*sqrt(-a*c)*log((c*x^2
- 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a^3*c - 5*A*a^2*c^2)*x)/(a^3*c^4*
x^4 + 2*a^4*c^3*x^2 + a^5*c^2), -1/8*(2*B*a^3*c - (C*a^2*c^2 + 3*A*a*c^3)*x
^3 - ((C*a*c^2 + 3*A*c^3)*x^4 + C*a^3 + 3*A*a^2*c + 2*(C*a^2*c + 3*A*a*c^2)
*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (C*a^3*c - 5*A*a^2*c^2)*x)/(a^3*c^4
*x^4 + 2*a^4*c^3*x^2 + a^5*c^2)]
```

giac [A] time = 0.16, size = 84, normalized size = 0.86

$$\frac{(Ca+3\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\,a^2c} + \frac{Cacx^3+3\,Ac^2x^3-Ca^2x+5\,Aacx-2\,Ba^2}{8\left(cx^2+a\right)^2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a + 3*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/8*(C*a*c*x^3
+ 3*A*c^2*x^3 - C*a^2*x + 5*A*a*c*x - 2*B*a^2)/((c*x^2 + a)^2*a^2*c)
```

maple [A] time = 0.01, size = 96, normalized size = 0.98

$$\frac{3A\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\,a^2}+\frac{C\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\,ac}+\frac{\frac{(3Ac+aC)x^3}{8a^2}-\frac{B}{4c}+\frac{(5Ac-aC)x}{8ac}}{\left(c\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^3,x)
```

```
[Out] (1/8*(3*A*c+C*a)/a^2*x^3+1/8*(5*A*c-C*a)/a/c*x-1/4*B/c)/(c*x^2+a)^2+3/8/a^2
/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A+1/8/a/c/(a*c)^(1/2)*arctan(1/(a*c)
^(1/2)*c*x)*C
```

maxima [A] time = 0.97, size = 98, normalized size = 1.00

$$\frac{\left(Cac+3\,Ac^2\right)x^3-2\,Ba^2-\left(Ca^2-5\,Aac\right)x}{8\left(a^2c^3x^4+2\,a^3c^2x^2+a^4c\right)}+\frac{(Ca+3\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((C*a*c + 3*A*c^2)*x^3 - 2*B*a^2 - (C*a^2 - 5*A*a*c)*x)/(a^2*c^3*x^4 +
2*a^3*c^2*x^2 + a^4*c) + 1/8*(C*a + 3*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)
*a^2*c)
```

mupad [B] time = 3.84, size = 88, normalized size = 0.90

$$\frac{\frac{x^3\,(3\,A\,c+C\,a)}{8\,a^2}-\frac{B}{4\,c}+\frac{x\,(5\,A\,c-C\,a)}{8\,a\,c}}{a^2+2\,a\,c\,x^2+c^2\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(3\,A\,c+C\,a)}{8\,a^{5/2}\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^3,x)
```

```
[Out] ((x^3*(3*A*c + C*a))/(8*a^2) - B/(4*c) + (x*(5*A*c - C*a))/(8*a*c))/(a^2 +
c^2*x^4 + 2*a*c*x^2) + (atan((c^(1/2)*x)/a^(1/2))*(3*A*c + C*a))/(8*a^(5/2)
*c^(3/2))
```

sympy [A] time = 1.23, size = 156, normalized size = 1.59

$$-\frac{\sqrt{-\frac{1}{a^5c^3}}\,(3Ac+Ca)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5c^3}}\,(3Ac+Ca)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}+\frac{-2Ba^2+x^3\left(3Ac^2+\right.}{8a^4c+16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*c**3))*(3*A*c + C*a)*log(-a**3*c*sqrt(-1/(a**5*c**3)) + x)/1
6 + sqrt(-1/(a**5*c**3))*(3*A*c + C*a)*log(a**3*c*sqrt(-1/(a**5*c**3)) + x)
/16 + (-2*B*a**2 + x**3*(3*A*c**2 + C*a*c) + x*(5*A*a*c - C*a**2))/(8*a**4*
c + 16*a**3*c**2*x**2 + 8*a**2*c**3*x**4)
```

3.61 $\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=353

$$\frac{4a^2e\left(Ae^2-Bde+Cd^2\right)+x\left(Acd\left(7ae^2+3cd^2\right)+a\left(cd^2-3ae^2\right)(Cd-Be)\right)}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(15a^2e^4+10acd^2\right.\right.}{8}$$

```
[Out] 1/4*(-a*(-A*c*e+B*c*d+C*a*e)+c*(A*c*d+B*a*e-C*a*d)*x)/a/c/(a*e^2+c*d^2)/(c*
x^2+a)^2+1/8*(4*a^2*e*(A*e^2-B*d*e+C*d^2)+(a*(-B*e+C*d)*(-3*a*e^2+c*d^2)+A*
c*d*(7*a*e^2+3*c*d^2))*x)/a^2/(a*e^2+c*d^2)^2/(c*x^2+a)+e^3*(A*e^2-B*d*e+C*
d^2)*ln(e*x+d)/(a*e^2+c*d^2)^3-1/2*e^3*(A*e^2-B*d*e+C*d^2)*ln(c*x^2+a)/(a*e
^2+c*d^2)^3+1/8*(a*(-B*e+C*d)*(-3*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)+A*c*d*(15*
a^2*e^4+10*a*c*d^2*e^2+3*c^2*d^4))*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2
+c*d^2)^3/c^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1647, 823, 801, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(15a^2e^4+10acd^2e^2+3c^2d^4\right)+a\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)(Cd-Be)\right)}{8a^{5/2}\sqrt{c}\left(ae^2+cd^2\right)^3}+\frac{4a^2e\left(Ae^2-Bde+Cd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^3),x]
```

```
[Out] -(a*(B*c*d - A*c*e + a*C*e) - c*(A*c*d - a*C*d + a*B*e)*x)/(4*a*c*(c*d^2 +
a*e^2)*(a + c*x^2)^2) + (4*a^2*e*(C*d^2 - B*d*e + A*e^2) + (a*(C*d - B*e)*(
c*d^2 - 3*a*e^2) + A*c*d*(3*c*d^2 + 7*a*e^2))*x)/(8*a^2*(c*d^2 + a*e^2)^2*(
a + c*x^2)) + ((a*(C*d - B*e)*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4) + A*c*d
*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8
*a^(5/2)*Sqrt[c]*(c*d^2 + a*e^2)^3) + (e^3*(C*d^2 - B*d*e + A*e^2)*Log[d +
e*x])/(c*d^2 + a*e^2)^3 - (e^3*(C*d^2 - B*d*e + A*e^2)*Log[a + c*x^2])/(2*(
c*d^2 + a*e^2)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx = -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} - \frac{\int \frac{-\frac{c\left(ad(Cd-Be)+A\left(3cd^2+4ae^2\right)\right)}{cd^2+ae^2}-\frac{3ce(Acd-aCd+\cdots}{cd^2+ae^2}}{(d+ex)\left(a+cx^2\right)^2}}{4ac}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd\cdots\right.}{8a^2\left(cd^2\cdots\right.}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd\cdots\right.}{8a^2\left(cd^2\cdots\right.}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd\cdots\right.}{8a^2\left(cd^2\cdots\right.}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd\cdots\right.}{8a^2\left(cd^2\cdots\right.}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd\cdots\right.}{8a^2\left(cd^2\cdots\right.}$$

Mathematica [A] time = 0.42, size = 321, normalized size = 0.91

$$\frac{\frac{2\left(ae^2+cd^2\right)^2\left(a^2(-C)e+ac(Ae-Bd+Bex-Cdx)+Ac^2dx\right)}{ac\left(a+cx^2\right)^2}+\frac{\left(ae^2+cd^2\right)\left(a^2e(e(4Ae-4Bd+3Bex)+Cd(4d-3ex))+acdx\left(e(7Ae-Bd)+Cd^2\right)+3Ac^2d^3x\right)}{a^2\left(a+cx^2\right)}+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^3),x]
```

```
[Out] ((2*(c*d^2 + a*e^2)^2*(-(a^2*C*e) + A*c^2*d*x + a*c*(-(B*d) + A*e - C*d*x +
 B*e*x)))/(a*c*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(3*A*c^2*d^3*x + a*c*d*(C*
d^2 + e*(-(B*d) + 7*A*e))*x + a^2*e*(C*d*(4*d - 3*e*x) + e*(-4*B*d + 4*A*e
+ 3*B*e*x))))/(a^2*(a + c*x^2)) + ((a*(C*d - B*e)*(c^2*d^4 + 6*a*c*d^2*e^2
- 3*a^2*e^4) + A*c*d*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4))*ArcTan[(Sqr
t[c]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[c]) + 8*e^3*(C*d^2 + e*(-(B*d) + A*e))*Log[
d + e*x] - 4*e^3*(C*d^2 + e*(-(B*d) + A*e))*Log[a + c*x^2])/(8*(c*d^2 + a*e
^2)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.18, size = 715, normalized size = 2.03

$$-\frac{\left(Cd^2e^3 - Bde^4 + Ae^5\right)\log\left(cx^2 + a\right)}{2\left(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6\right)} + \frac{\left(Cd^2e^4 - Bde^5 + Ae^6\right)\log\left(|xe + d|\right)}{c^3d^6e + 3\,ac^2d^4e^3 + 3\,a^2cd^2e^5 + a^3e^7} + \frac{\left(Cac^2d^5 + 3\,Ac^3d^5 - Bac^2d^4e + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(C*d^2*e^3 - B*d*e^4 + A*e^5)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (C*d^2*e^4 - B*d*e^5 + A*e^6)*log(abs(x*e
+ d))/(c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7) + 1/8*(C*a*
c^2*d^5 + 3*A*c^3*d^5 - B*a*c^2*d^4*e + 6*C*a^2*c*d^3*e^2 + 10*A*a*c^2*d^3*
e^2 - 6*B*a^2*c*d^2*e^3 - 3*C*a^3*d*e^4 + 15*A*a^2*c*d*e^4 + 3*B*a^3*e^5)*a
rctan(c*x/sqrt(a*c))/((a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 +
a^5*e^6)*sqrt(a*c)) - 1/8*(2*B*a^2*c^3*d^5 - 2*C*a^3*c^2*d^4*e - 2*A*a^2*c^
3*d^4*e + 8*B*a^3*c^2*d^3*e^2 - 8*A*a^3*c^2*d^2*e^3 + 6*B*a^4*c*d*e^4 + 2*C
*a^5*e^5 - 6*A*a^4*c*e^5 - (C*a*c^4*d^5 + 3*A*c^5*d^5 - B*a*c^4*d^4*e - 2*C
*a^2*c^3*d^3*e^2 + 10*A*a*c^4*d^3*e^2 + 2*B*a^2*c^3*d^2*e^3 - 3*C*a^3*c^2*d
*e^4 + 7*A*a^2*c^3*d*e^4 + 3*B*a^3*c^2*e^5)*x^3 - 4*(C*a^2*c^3*d^4*e - B*a^
2*c^3*d^3*e^2 + C*a^3*c^2*d^2*e^3 + A*a^2*c^3*d^2*e^3 - B*a^3*c^2*d*e^4 + A
*a^3*c^2*e^5)*x^2 + (C*a^2*c^3*d^5 - 5*A*a*c^4*d^5 - B*a^2*c^3*d^4*e + 6*C*
a^3*c^2*d^3*e^2 - 14*A*a^2*c^3*d^3*e^2 - 6*B*a^3*c^2*d^2*e^3 + 5*C*a^4*c*d*
e^4 - 9*A*a^3*c^2*d*e^4 - 5*B*a^4*c*e^5)*x)/((c*d^2 + a*e^2)^3*(c*x^2 + a)^
2*a^2*c)
```

maple [B] time = 0.02, size = 1598, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x)
```

```
[Out] 5/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*x*a^2*e^5-1/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2
*C*x*c^2*d^5-3/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*d*e^4*B*a^2+3/8/(a*e^2+c*d^2)^
3*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*e^5+1/4/(a*e^2+c*d^2)^3/(c*x^2+
a)^2*c^2*A*d^4*e-1/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2/c*C*a^3*e^5+e^5/(a*e^2+c*d
^2)^3*ln(e*x+d)*A-1/2/(a*e^2+c*d^2)^3*ln(c*x^2+a)*A*e^5+5/4/(a*e^2+c*d^2)^3
/(c*x^2+a)^2*c^3/a*x^3*A*d^3*e^2-1/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*c^3/a*x^3*
```

```
B*d^4*e+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)^2*C*x^2*a*c*d^2*e^3+5/4/(a*e^2+c*d^2)
^3/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c^2*d^3*e^2-1/8/(a*e^2+c*d^2)^
3/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*c^2*d^4*e+9/8/(a*e^2+c*d^2)^3/(
c*x^2+a)^2*A*x*a*c*d*e^4+3/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*x*a*c*d^2*e^3-3/
4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*C*x*a*c*d^3*e^2-3/8/(a*e^2+c*d^2)^3/(c*x^2+a)
^2*C*x^3*a*c*d*e^4-1/2/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*x^2*a*c*d*e^4-3/8/(a*e
^2+c*d^2)^3*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d*e^4+1/8/(a*e^2+c*d^
2)^3/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*c^2*d^5+3/4/(a*e^2+c*d^2)^3/
(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*c*d^3*e^2+15/8/(a*e^2+c*d^2)^3/(a*c
)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c*d*e^4-3/4/(a*e^2+c*d^2)^3/(a*c)^(1/2)
*arctan(1/(a*c)^(1/2)*c*x)*B*c*d^2*e^3-1/2/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*x^
2*c^2*d^3*e^2+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)^2*C*x^2*c^2*d^4*e+7/8/(a*e^2+c*
d^2)^3/(c*x^2+a)^2*A*x^3*c^2*d*e^4+3/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*x^3*a*
c*e^5+1/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*x^3*c^2*d^2*e^3-1/4/(a*e^2+c*d^2)^3
/(c*x^2+a)^2*C*x^3*c^2*d^3*e^2+1/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*c^3/a*x^3*C*
d^5+5/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2/a*x*A*c^3*d^5+1/(a*e^2+c*d^2)^3/(c*x^2+
a)^2*c*A*d^2*e^3*a+1/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*x*c^2*d^4*e-5/8/(a*e^2
+c*d^2)^3/(c*x^2+a)^2*C*x*a^2*d*e^4+7/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*A*x*c^2
*d^3*e^2+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)^2*A*x^2*a*c*e^5+1/2/(a*e^2+c*d^2)^3/
(c*x^2+a)^2*A*x^2*c^2*d^2*e^3-1/(a*e^2+c*d^2)^3/(c*x^2+a)^2*c*d^3*e^2*B*a+1
/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*c*C*a*d^4*e+3/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*
c^4/a^2*x^3*A*d^5+3/8/(a*e^2+c*d^2)^3/a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*
c*x)*A*c^3*d^5+3/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*A*e^5*a^2-1/4/(a*e^2+c*d^2)^
3/(c*x^2+a)^2*c^2*d^5*B+1/2/(a*e^2+c*d^2)^3*ln(c*x^2+a)*d*e^4*B-1/2/(a*e^2+
c*d^2)^3*ln(c*x^2+a)*C*d^2*e^3-e^4/(a*e^2+c*d^2)^3*ln(e*x+d)*B*d+e^3/(a*e^2
+c*d^2)^3*ln(e*x+d)*C*d^2
```

maxima [A] time = 1.10, size = 655, normalized size = 1.86

$$-\frac{\left(Cd^2e^3 - Bde^4 + Ae^5\right)\log\left(cx^2 + a\right)}{2\left(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6\right)} + \frac{\left(Cd^2e^3 - Bde^4 + Ae^5\right)\log\left(ex + d\right)}{c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6} - \frac{\left(Bac^2d^4e + 6\,Ba^2cd^2e^3 - 3\,Ba^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(C*d^2*e^3 - B*d*e^4 + A*e^5)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (C*d^2*e^3 - B*d*e^4 + A*e^5)*log(e*x + d)
/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - 1/8*(B*a*c^2*d^4
*e + 6*B*a^2*c*d^2*e^3 - 3*B*a^3*e^5 - (C*a*c^2 + 3*A*c^3)*d^5 - 2*(3*C*a^2
*c + 5*A*a*c^2)*d^3*e^2 + 3*(C*a^3 - 5*A*a^2*c)*d*e^4)*arctan(c*x/sqrt(a*c)
)/((a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*sqrt(a*c))
 - 1/8*(2*B*a^2*c^2*d^3 + 6*B*a^3*c*d*e^2 - 2*(C*a^3*c + A*a^2*c^2)*d^2*e +
 2*(C*a^4 - 3*A*a^3*c)*e^3 + (B*a*c^3*d^2*e - 3*B*a^2*c^2*e^3 - (C*a*c^3 +
3*A*c^4)*d^3 + (3*C*a^2*c^2 - 7*A*a*c^3)*d*e^2)*x^3 - 4*(C*a^2*c^2*d^2*e -
B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2 - (B*a^2*c^2*d^2*e + 5*B*a^3*c*e^3 - (
C*a^2*c^2 - 5*A*a*c^3)*d^3 - (5*C*a^3*c - 9*A*a^2*c^2)*d*e^2)*x)/(a^4*c^3*d
^4 + 2*a^5*c^2*d^2*e^2 + a^6*c*e^4 + (a^2*c^5*d^4 + 2*a^3*c^4*d^2*e^2 + a^4
*c^3*e^4)*x^4 + 2*(a^3*c^4*d^4 + 2*a^4*c^3*d^2*e^2 + a^5*c^2*e^4)*x^2)
```

mupad [B] time = 9.90, size = 2392, normalized size = 6.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^3*(d + e*x)),x)
```

```
[Out] ((x^2*(A*c*e^3 - B*c*d*e^2 + C*c*d^2*e))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*
e^2)) - (B*c^2*d^3 + C*a^2*e^3 - 3*A*a*c*e^3 - A*c^2*d^2*e + 3*B*a*c*d*e^2
```

```
- C*a*c*d^2*e)/(4*c*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(5*A*c^2*d^3
+ 5*B*a^2*e^3 - C*a*c*d^3 - 5*C*a^2*d*e^2 + 9*A*a*c*d*e^2 + B*a*c*d^2*e))/(
8*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x^3*(3*A*c^3*d^3 + 3*B*a^2*c*e^
3 + C*a*c^2*d^3 + 7*A*a*c^2*d*e^2 - B*a*c^2*d^2*e - 3*C*a^2*c*d*e^2))/(8*a^
2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a^2 + c^2*x^4 + 2*a*c*x^2) - (log(
3*A*c^4*d^7*(-a^5*c)^(1/2) - 3*B*a^4*e^7*(-a^5*c)^(1/2) - 24*A*a^6*c*e^7 +
3*B*a^6*c*e^7*x + 6*A*a^3*c^4*d^6*e + 2*C*a^4*c^3*d^6*e - 30*C*a^6*c*d^2*e^
5 - 3*A*a^2*c^5*d^7*x - C*a^3*c^4*d^7*x + C*a*c^3*d^7*(-a^5*c)^(1/2) + 3*C*
a^4*d*e^6*(-a^5*c)^(1/2) + 20*A*a^4*c^3*d^4*e^3 + 54*A*a^5*c^2*d^2*e^5 - 2*
B*a^4*c^3*d^5*e^2 - 36*B*a^5*c^2*d^3*e^4 + 36*C*a^5*c^2*d^4*e^3 + 30*B*a^6*
c*d*e^6 - 7*A*a^3*c^4*d^5*e^2*x - 5*A*a^4*c^3*d^3*e^4*x + 5*B*a^4*c^3*d^4*e
^3*x - 57*B*a^5*c^2*d^2*e^5*x - 5*C*a^4*c^3*d^5*e^2*x + 57*C*a^5*c^2*d^3*e^
4*x + 7*A*a*c^3*d^5*e^2*(-a^5*c)^(1/2) + 57*B*a^3*c*d^2*e^5*(-a^5*c)^(1/2)
- 57*C*a^3*c*d^3*e^4*(-a^5*c)^(1/2) - 3*C*a^6*c*d*e^6*x + 5*A*a^2*c^2*d^3*e
^4*(-a^5*c)^(1/2) - 5*B*a^2*c^2*d^4*e^3*(-a^5*c)^(1/2) + 5*C*a^2*c^2*d^5*e^
2*(-a^5*c)^(1/2) + 63*A*a^5*c^2*d*e^6*x + B*a^3*c^4*d^6*e*x - 63*A*a^3*c*d*
e^6*(-a^5*c)^(1/2) - B*a*c^3*d^6*e*(-a^5*c)^(1/2) - 24*A*a^3*c*e^7*x*(-a^5*
c)^(1/2) + 6*A*c^4*d^6*e*x*(-a^5*c)^(1/2) + 54*A*a^2*c^2*d^2*e^5*x*(-a^5*c)
^(1/2) - 36*B*a^2*c^2*d^3*e^4*x*(-a^5*c)^(1/2) + 36*C*a^2*c^2*d^4*e^3*x*(-a
^5*c)^(1/2) + 30*B*a^3*c*d*e^6*x*(-a^5*c)^(1/2) + 2*C*a*c^3*d^6*e*x*(-a^5*c
)^(1/2) + 20*A*a*c^3*d^4*e^3*x*(-a^5*c)^(1/2) - 2*B*a*c^3*d^5*e^2*x*(-a^5*c
)^(1/2) - 30*C*a^3*c*d^2*e^5*x*(-a^5*c)^(1/2))*(c*(a^2*((3*C*d^3*e^2*(-a^5*
c)^(1/2))/8 - (3*B*d^2*e^3*(-a^5*c)^(1/2))/8 + (15*A*d*e^4*(-a^5*c)^(1/2))/
16) + a^5*((A*e^5)/2 + (C*d^2*e^3)/2 - (B*d*e^4)/2)) + a^3*((3*B*e^5*(-a^5*
c)^(1/2))/16 - (3*C*d*e^4*(-a^5*c)^(1/2))/16) + a*c^2*((C*d^5*(-a^5*c)^(1/2
))/16 + (5*A*d^3*e^2*(-a^5*c)^(1/2))/8 - (B*d^4*e*(-a^5*c)^(1/2))/16) + (3*
A*c^3*d^5*(-a^5*c)^(1/2))/16))/(a^8*c*e^6 + a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2
 + 3*a^7*c^2*d^2*e^4) + (log(3*A*c^4*d^7*(-a^5*c)^(1/2) - 3*B*a^4*e^7*(-a^5
*c)^(1/2) + 24*A*a^6*c*e^7 - 3*B*a^6*c*e^7*x - 6*A*a^3*c^4*d^6*e - 2*C*a^4*
c^3*d^6*e + 30*C*a^6*c*d^2*e^5 + 3*A*a^2*c^5*d^7*x + C*a^3*c^4*d^7*x + C*a*
c^3*d^7*(-a^5*c)^(1/2) + 3*C*a^4*d*e^6*(-a^5*c)^(1/2) - 20*A*a^4*c^3*d^4*e^
3 - 54*A*a^5*c^2*d^2*e^5 + 2*B*a^4*c^3*d^5*e^2 + 36*B*a^5*c^2*d^3*e^4 - 36*
C*a^5*c^2*d^4*e^3 - 30*B*a^6*c*d*e^6 + 7*A*a^3*c^4*d^5*e^2*x + 5*A*a^4*c^3*
d^3*e^4*x - 5*B*a^4*c^3*d^4*e^3*x + 57*B*a^5*c^2*d^2*e^5*x + 5*C*a^4*c^3*d^
5*e^2*x - 57*C*a^5*c^2*d^3*e^4*x + 7*A*a*c^3*d^5*e^2*(-a^5*c)^(1/2) + 57*B*
a^3*c*d^2*e^5*(-a^5*c)^(1/2) - 57*C*a^3*c*d^3*e^4*(-a^5*c)^(1/2) + 3*C*a^6*
c*d*e^6*x + 5*A*a^2*c^2*d^3*e^4*(-a^5*c)^(1/2) - 5*B*a^2*c^2*d^4*e^3*(-a^5*
c)^(1/2) + 5*C*a^2*c^2*d^5*e^2*(-a^5*c)^(1/2) - 63*A*a^5*c^2*d*e^6*x - B*a^
3*c^4*d^6*e*x - 63*A*a^3*c*d*e^6*(-a^5*c)^(1/2) - B*a*c^3*d^6*e*(-a^5*c)^(1
/2) - 24*A*a^3*c*e^7*x*(-a^5*c)^(1/2) + 6*A*c^4*d^6*e*x*(-a^5*c)^(1/2) + 54
*A*a^2*c^2*d^2*e^5*x*(-a^5*c)^(1/2) - 36*B*a^2*c^2*d^3*e^4*x*(-a^5*c)^(1/2)
 + 36*C*a^2*c^2*d^4*e^3*x*(-a^5*c)^(1/2) + 30*B*a^3*c*d*e^6*x*(-a^5*c)^(1/2
) + 2*C*a*c^3*d^6*e*x*(-a^5*c)^(1/2) + 20*A*a*c^3*d^4*e^3*x*(-a^5*c)^(1/2)
- 2*B*a*c^3*d^5*e^2*x*(-a^5*c)^(1/2) - 30*C*a^3*c*d^2*e^5*x*(-a^5*c)^(1/2))
*(c*(a^2*((3*C*d^3*e^2*(-a^5*c)^(1/2))/8 - (3*B*d^2*e^3*(-a^5*c)^(1/2))/8 +
 (15*A*d*e^4*(-a^5*c)^(1/2))/16) - a^5*((A*e^5)/2 + (C*d^2*e^3)/2 - (B*d*e^
4)/2)) + a^3*((3*B*e^5*(-a^5*c)^(1/2))/16 - (3*C*d*e^4*(-a^5*c)^(1/2))/16)
+ a*c^2*((C*d^5*(-a^5*c)^(1/2))/16 + (5*A*d^3*e^2*(-a^5*c)^(1/2))/8 - (B*d^
4*e*(-a^5*c)^(1/2))/16) + (3*A*c^3*d^5*(-a^5*c)^(1/2))/16))/(a^8*c*e^6 + a^
5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4) + (e^3*log(d + e*x)*(A*e
^2 + C*d^2 - B*d*e))/(a*e^2 + c*d^2)^3
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.62 $\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^3} \, dx$

Optimal. Leaf size=571

$$-\frac{4a^2e\left(ae^2(2Cd-Be)-cd\left(2Cd^2-e(3Bd-4Ae)\right)\right)-x\left(Ac\left(-7a^2e^4+12acd^2e^2+3c^2d^4\right)+a\left(3a^2Ce^4-2acde^2\right.\right.}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^3}$$

```
[Out] -e^3*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^3/(e*x+d)+1/4*(-a*(-2*A*c*d*e-B*a*e^
2+B*c*d^2+2*C*a*d*e)+(A*c*(-a*e^2+c*d^2)+a*(a*C*e^2-c*d*(-2*B*e+C*d)))*x)/a
/(a*e^2+c*d^2)^2/(c*x^2+a)^2+1/8*(-4*a^2*e*(a*e^2*(-B*e+2*C*d)-c*d*(2*C*d^2
-e*(-4*A*e+3*B*d)))+(A*c*(-7*a^2*e^4+12*a*c*d^2*e^2+3*c^2*d^4)+a*(3*a^2*C*e
^4-2*a*c*d*e^2*(-7*B*e+6*C*d)+c^2*d^3*(-2*B*e+C*d)))*x)/a^2/(a*e^2+c*d^2)^3
/(c*x^2+a)-e^3*(a*e^2*(-B*e+2*C*d)-c*d*(4*C*d^2-e*(-6*A*e+5*B*d)))*ln(e*x+d
)/(a*e^2+c*d^2)^4+1/2*e^3*(a*e^2*(-B*e+2*C*d)-c*d*(4*C*d^2-e*(-6*A*e+5*B*d)
))*ln(c*x^2+a)/(a*e^2+c*d^2)^4+1/8*(3*A*c*(-5*a^3*e^6+15*a^2*c*d^2*e^4+5*a*
c^2*d^4*e^2+c^3*d^6)+a*(3*a^3*C*e^6+a*c^2*d^3*e^2*(-20*B*e+13*C*d)-3*a^2*c*
d*e^4*(-10*B*e+11*C*d)+c^3*d^5*(-2*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/a^(
5/2)/(a*e^2+c*d^2)^4/c^(1/2)
```

Rubi [A] time = 1.92, antiderivative size = 566, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1647, 1629, 635, 205, 260}

$$\frac{x\left(Ac\left(-7a^2e^4+12acd^2e^2+3c^2d^4\right)+a\left(3a^2Ce^4-2acde^2(6Cd-7Be)+c^2d^3(Cd-2Be)\right)\right)+4a^2e\left(-ae^2(2Cd-\right.}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^3),x]
```

```
[Out] -((e^3*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^3*(d + e*x))) - (a*(B*c*d^
2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2) - (A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 -
c*d*(C*d - 2*B*e)))*x)/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^2)^2) + (4*a^2*e*(2*
c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e)) + (A*c*(3*c^2*d^4 +
12*a*c*d^2*e^2 - 7*a^2*e^4) + a*(3*a^2*C*e^4 - 2*a*c*d*e^2*(6*C*d - 7*B*e)
+ c^2*d^3*(C*d - 2*B*e)))*x)/(8*a^2*(c*d^2 + a*e^2)^3*(a + c*x^2)) + ((3*A*
c*(c^3*d^6 + 5*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - 5*a^3*e^6) + a*(3*a^3*C*e
^6 + a*c^2*d^3*e^2*(13*C*d - 20*B*e) - 3*a^2*c*d*e^4*(11*C*d - 10*B*e) + c^
3*d^5*(C*d - 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[c]*(c*d^
2 + a*e^2)^4) + (e^3*(4*c*C*d^3 - c*d*e*(5*B*d - 6*A*e) - a*e^2*(2*C*d - B*
e))*Log[d + e*x])/(c*d^2 + a*e^2)^4 - (e^3*(4*c*C*d^3 - c*d*e*(5*B*d - 6*A*
e) - a*e^2*(2*C*d - B*e))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^3}\,dx = -\frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

$$= -\frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

$$= -\frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^3(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(\right.\right.}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^3(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(\right.\right.}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^3(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(\right.\right.}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

Mathematica [A] time = 0.76, size = 498, normalized size = 0.87

$$\frac{\frac{2\left(ae^2+cd^2\right)^2\left(a^2e(Be-2Cd+Cex)-ac\left(Ae(ex-2d)+Bd(d-2ex)+Cd^2x\right)+Ac^2d^2x\right)}{a\left(a+cx^2\right)^2}+\frac{\left(ae^2+cd^2\right)\left(a^3e^3(4Be-8Cd+3Cex)+a^2ce\left(e(Ae(16d-7ex)-2Bd(6d-7ex))\right.\right.}{a^2\left(a+cx^2\right)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^3),x]
```

```
[Out] ((-8*e^3*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) + (2*(c*d^2
+ a*e^2)^2*(A*c^2*d^2*x + a^2*e*(-2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + B*d
*(d - 2*e*x) + A*e*(-2*d + e*x))))/(a*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(3*
A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 2*e*(-(B*d) + 6*A*e))*x + a^3*e^3*(-8*C*d
+ 4*B*e + 3*C*e*x) + a^2*c*e*(4*C*d^2*(2*d - 3*e*x) + e*(-2*B*d*(6*d - 7*e*
x) + A*e*(16*d - 7*e*x)))))/(a^2*(a + c*x^2)) + ((3*A*c*(c^3*d^6 + 5*a*c^2*
d^4*e^2 + 15*a^2*c*d^2*e^4 - 5*a^3*e^6) + a*(3*a^3*C*e^6 + a*c^2*d^3*e^2*(1
3*C*d - 20*B*e) + c^3*d^5*(C*d - 2*B*e) + 3*a^2*c*d*e^4*(-11*C*d + 10*B*e))
)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[c]) + 8*e^3*(4*c*C*d^3 + c*d*e
*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e))*Log[d + e*x] - 4*e^3*(4*c*C*d^3 +
 c*d*e*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e))*Log[a + c*x^2])/(8*(c*d^2 +
 a*e^2)^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.24, size = 1107, normalized size = 1.94

$$\frac{\left(Cac^3d^6e^2 + 3\,Ac^4d^6e^2 - 2\,Bac^3d^5e^3 + 13\,Ca^2c^2d^4e^4 + 15\,Aac^3d^4e^4 - 20\,Ba^2c^2d^3e^5 - 33\,Ca^3cd^2e^6 + 45\,Aa^2c^2d \cdots\right.}{8\left(a^2c^4d^8 + 4\,a^3c^3d^6e^2 + 6\,a^4c^2d^4e^4 + 4\,a^5cd^2e^6 + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a*c^3*d^6*e^2 + 3*A*c^4*d^6*e^2 - 2*B*a*c^3*d^5*e^3 + 13*C*a^2*c^2*d
^4*e^4 + 15*A*a*c^3*d^4*e^4 - 20*B*a^2*c^2*d^3*e^5 - 33*C*a^3*c*d^2*e^6 + 4
5*A*a^2*c^2*d^2*e^6 + 30*B*a^3*c*d*e^7 + 3*C*a^4*e^8 - 15*A*a^3*c*e^8)*arct
an((c*d - c*d^2/(x*e + d) - a*e^2/(x*e + d))*e^(-1)/sqrt(a*c))*e^(-2)/((a^2
*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^
8)*sqrt(a*c)) - 1/2*(4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 - 2*C*a*d*e^5 + 6*A*c*d*
e^5 + B*a*e^6)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d
)^2)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4
*e^8) - (C*d^2*e^9/(x*e + d) - B*d*e^10/(x*e + d) + A*e^11/(x*e + d))/(c^3*
d^6*e^6 + 3*a*c^2*d^4*e^8 + 3*a^2*c*d^2*e^10 + a^3*e^12) + 1/8*(C*a*c^4*d^5
*e + 3*A*c^5*d^5*e - 2*B*a*c^4*d^4*e^2 - 22*C*a^2*c^3*d^3*e^3 + 14*A*a*c^4*
d^3*e^3 + 32*B*a^2*c^3*d^2*e^4 + 17*C*a^3*c^2*d*e^5 - 29*A*a^2*c^3*d*e^5 -
6*B*a^3*c^2*e^6 - (3*C*a*c^4*d^6*e^2 + 9*A*c^5*d^6*e^2 - 6*B*a*c^4*d^5*e^3
- 77*C*a^2*c^3*d^4*e^4 + 41*A*a*c^4*d^4*e^4 + 116*B*a^2*c^3*d^3*e^5 + 77*C*
a^3*c^2*d^2*e^6 - 121*A*a^2*c^3*d^2*e^6 - 38*B*a^3*c^2*d*e^7 - 3*C*a^4*c*e^
8 + 7*A*a^3*c^2*e^8)*e^(-1)/(x*e + d) + (3*C*a*c^4*d^7*e^3 + 9*A*c^5*d^7*e^
3 - 6*B*a*c^4*d^6*e^4 - 89*C*a^2*c^3*d^5*e^5 + 45*A*a*c^4*d^5*e^5 + 140*B*a
^2*c^3*d^4*e^6 + 85*C*a^3*c^2*d^3*e^7 - 145*A*a^2*c^3*d^3*e^7 - 22*B*a^3*c^
2*d^2*e^8 + 17*C*a^4*c*d*e^9 - 21*A*a^3*c^2*d*e^9 - 8*B*a^4*c*e^10)*e^(-2)/
(x*e + d)^2 - (C*a*c^4*d^8*e^4 + 3*A*c^5*d^8*e^4 - 2*B*a*c^4*d^7*e^5 - 34*C
*a^2*c^3*d^6*e^6 + 18*A*a*c^4*d^6*e^6 + 58*B*a^2*c^3*d^5*e^7 + 20*C*a^3*c^2
*d^4*e^8 - 60*A*a^2*c^3*d^4*e^8 + 26*B*a^3*c^2*d^3*e^9 + 50*C*a^4*c*d^2*e^1
0 - 66*A*a^3*c^2*d^2*e^10 - 34*B*a^4*c*d*e^11 - 5*C*a^5*e^12 + 9*A*a^4*c*e^
12)*e^(-3)/(x*e + d)^3)/((c*d^2 + a*e^2)^4*a^2*(c - 2*c*d/(x*e + d) + c*d^2
/(x*e + d)^2 + a*e^2/(x*e + d)^2)^2)
```

maple [B] time = 0.03, size = 2159, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x)
```

```
[Out] -3/(a*e^2+c*d^2)^4*c*ln(c*x^2+a)*d*A*e^5+1/(a*e^2+c*d^2)^4*a*ln(c*x^2+a)*C*
d*e^5+3/8/(a*e^2+c*d^2)^4*a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*e^6-3
/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*a^3*d*e^5+1/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*
A*c^3*d^5*e+5/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*a^3*C*e^6*x-1/8/(a*e^2+c*d^2)^4
/(c*x^2+a)^2*C*c^3*d^6*x+5/2/(a*e^2+c*d^2)^4*c*ln(c*x^2+a)*d^2*e^4*B-2/(a*e
^2+c*d^2)^4*c*ln(c*x^2+a)*C*d^3*e^3+6*e^5/(a*e^2+c*d^2)^4*ln(e*x+d)*A*c*d-5
*e^4/(a*e^2+c*d^2)^4*ln(e*x+d)*B*c*d^2-2*e^5/(a*e^2+c*d^2)^4*ln(e*x+d)*C*a*
d+4*e^3/(a*e^2+c*d^2)^4*ln(e*x+d)*C*c*d^3-e^5/(a*e^2+c*d^2)^3/(e*x+d)*A-7/8
/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*a^2*c*d^2*e^4*x-13/8/(a*e^2+c*d^2)^4/(c*x^2+
a)^2*C*a*c^2*d^4*e^2*x+7/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*x^3*a*c^2*d*e^5-9/
8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^3*a*c^2*d^2*e^4+2/(a*e^2+c*d^2)^4/(c*x^2+
a)^2*A*x^2*a*c^2*d*e^5-1/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*x^2*a*c^2*d^2*e^4-1/
(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^2*a^2*c*d*e^5+15/8/(a*e^2+c*d^2)^4/a/(a*c)^
(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c^3*d^4*e^2+15/4/(a*e^2+c*d^2)^4*a/(a*c)^
(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*c*d*e^5-1/4/(a*e^2+c*d^2)^4/a/(a*c)^(1/2)
*arctan(1/(a*c)^(1/2)*c*x)*B*c^3*d^5*e-33/8/(a*e^2+c*d^2)^4*a/(a*c)^(1/2)*a
rctan(1/(a*c)^(1/2)*c*x)*C*c*d^2*e^4+15/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*c^4/a
*x^3*A*d^4*e^2-1/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*c^4/a*x^3*B*d^5*e+3/8/(a*e^2
+c*d^2)^4/(c*x^2+a)^2*A*a*c^2*d^2*e^4*x+9/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*d*a
^2*c*B*e^5*x+5/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*a*c^2*d^3*e^3*x+e^6/(a*e^2+c
*d^2)^4*ln(e*x+d)*B*a+e^4/(a*e^2+c*d^2)^3/(e*x+d)*B*d-e^3/(a*e^2+c*d^2)^3/(
e*x+d)*C*d^2+3/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*a^3*e^6-1/4/(a*e^2+c*d^2)^4/
(c*x^2+a)^2*B*c^3*d^6-1/2/(a*e^2+c*d^2)^4*a*ln(c*x^2+a)*e^6*B+45/8/(a*e^2+c
*d^2)^4/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c^2*d^2*e^4-5/2/(a*e^2+c*d^
2)^4/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*c^2*d^3*e^3+13/8/(a*e^2+c*d^2)
^4/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*c^2*d^4*e^2+3/2/(a*e^2+c*d^2)^4/
(c*x^2+a)^2*B*x^3*c^3*d^3*e^3-11/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^3*c^3*d^
4*e^2+2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*x^2*c^3*d^3*e^3+1/2/(a*e^2+c*d^2)^4/(
c*x^2+a)^2*B*x^2*a^2*c*e^6-3/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*x^2*c^3*d^4*e^
2+1/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^2*c^3*d^5*e-9/8/(a*e^2+c*d^2)^4/(c*x^2+
a)^2*A*a^2*c*e^6*x+17/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*c^3*d^4*e^2*x+1/4/(a*
e^2+c*d^2)^4/(c*x^2+a)^2*B*c^3*d^5*e*x+3/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^
3*a^2*c*e^6-7/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*x^3*a*c^2*e^6+5/8/(a*e^2+c*d^
2)^4/(c*x^2+a)^2*A*x^3*c^3*d^2*e^4+5/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*a^2*c*
d*e^5+3/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*a*c^2*d^3*e^3-3/4/(a*e^2+c*d^2)^4/(c*
x^2+a)^2*B*a^2*c*d^2*e^4-7/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*a*c^2*d^4*e^2-1/
(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*a^2*c*d^3*e^3+1/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2
*C*a*c^2*d^5*e+3/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*c^5/a^2*x^3*A*d^6+1/8/(a*e^2
+c*d^2)^4/(c*x^2+a)^2*c^4/a*x^3*C*d^6+5/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2/a*x*A
*c^4*d^6-15/8/(a*e^2+c*d^2)^4*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c*e
^6+3/8/(a*e^2+c*d^2)^4/a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c^4*d^6+
1/8/(a*e^2+c*d^2)^4/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*c^3*d^6
```

maxima [B] time = 1.24, size = 1196, normalized size = 2.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 + B*a*e^6 - 2*(C*a - 3*A*c)*d*e^5)*log(
c*x^2 + a)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6
```

```
 + a^4*e^8) + (4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 + B*a*e^6 - 2*(C*a - 3*A*c)*d*
e^5)*log(e*x + d)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*
d^2*e^6 + a^4*e^8) - 1/8*(2*B*a*c^3*d^5*e + 20*B*a^2*c^2*d^3*e^3 - 30*B*a^3
*c*d*e^5 - (C*a*c^3 + 3*A*c^4)*d^6 - (13*C*a^2*c^2 + 15*A*a*c^3)*d^4*e^2 +
3*(11*C*a^3*c - 15*A*a^2*c^2)*d^2*e^4 - 3*(C*a^4 - 5*A*a^3*c)*e^6)*arctan(c
*x/sqrt(a*c))/((a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5
*c*d^2*e^6 + a^6*e^8)*sqrt(a*c)) - 1/8*(2*B*a^2*c^2*d^5 + 12*B*a^3*c*d^3*e^
2 - 14*B*a^4*d*e^4 + 8*A*a^4*e^5 - 4*(C*a^3*c + A*a^2*c^2)*d^4*e + 20*(C*a^
4 - A*a^3*c)*d^2*e^3 + (2*B*a*c^3*d^3*e^2 - 22*B*a^2*c^2*d*e^4 - (C*a*c^3 +
 3*A*c^4)*d^4*e + 4*(5*C*a^2*c^2 - 3*A*a*c^3)*d^2*e^3 - 3*(C*a^3*c - 5*A*a^
2*c^2)*e^5)*x^4 + (2*B*a*c^3*d^4*e - 2*B*a^2*c^2*d^2*e^3 - 4*B*a^3*c*e^5 -
(C*a*c^3 + 3*A*c^4)*d^5 + 4*(C*a^2*c^2 - 3*A*a*c^3)*d^3*e^2 + (5*C*a^3*c -
9*A*a^2*c^2)*d*e^4)*x^3 + (10*B*a^2*c^2*d^3*e^2 - 38*B*a^3*c*d*e^4 - (7*C*a
^2*c^2 + 5*A*a*c^3)*d^4*e + 4*(9*C*a^3*c - 7*A*a^2*c^2)*d^2*e^3 - 5*(C*a^4
- 5*A*a^3*c)*e^5)*x^2 - (6*B*a^3*c*d^2*e^3 + 6*B*a^4*e^5 - (C*a^2*c^2 - 5*A
*a*c^3)*d^5 - 8*(C*a^3*c - 2*A*a^2*c^2)*d^3*e^2 - (7*C*a^4 - 11*A*a^3*c)*d*
e^4)*x)/(a^4*c^3*d^7 + 3*a^5*c^2*d^5*e^2 + 3*a^6*c*d^3*e^4 + a^7*d*e^6 + (a
^2*c^5*d^6*e + 3*a^3*c^4*d^4*e^3 + 3*a^4*c^3*d^2*e^5 + a^5*c^2*e^7)*x^5 + (
a^2*c^5*d^7 + 3*a^3*c^4*d^5*e^2 + 3*a^4*c^3*d^3*e^4 + a^5*c^2*d*e^6)*x^4 +
2*(a^3*c^4*d^6*e + 3*a^4*c^3*d^4*e^3 + 3*a^5*c^2*d^2*e^5 + a^6*c*e^7)*x^3 +
 2*(a^3*c^4*d^7 + 3*a^4*c^3*d^5*e^2 + 3*a^5*c^2*d^3*e^4 + a^6*c*d*e^6)*x^2
+ (a^4*c^3*d^6*e + 3*a^5*c^2*d^4*e^3 + 3*a^6*c*d^2*e^5 + a^7*e^7)*x)
```

mupad [B] time = 6.66, size = 6848, normalized size = 11.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^3*(d + e*x)^2),x)
```

```
[Out] symsum(log(root(17920*a^9*c^5*d^8*e^8*z^3 + 14336*a^10*c^4*d^6*e^10*z^3 + 1
4336*a^8*c^6*d^10*e^6*z^3 + 7168*a^11*c^3*d^4*e^12*z^3 + 7168*a^7*c^7*d^12*
e^4*z^3 + 2048*a^12*c^2*d^2*e^14*z^3 + 2048*a^6*c^8*d^14*e^2*z^3 + 256*a^5*
c^9*d^16*z^3 + 256*a^13*c*e^16*z^3 + 948*B*C*a^7*c*d*e^11*z - 12*A*B*a*c^7*
d^11*e*z + 9768*B*C*a^5*c^3*d^5*e^7*z - 7476*B*C*a^6*c^2*d^3*e^9*z - 328*B*
C*a^4*c^4*d^7*e^5*z - 92*B*C*a^3*c^5*d^9*e^3*z - 12486*A*C*a^5*c^3*d^4*e^8*
z + 5868*A*C*a^6*c^2*d^2*e^10*z + 282*A*C*a^3*c^5*d^8*e^4*z + 168*A*C*a^4*c
^4*d^6*e^6*z + 108*A*C*a^2*c^6*d^10*e^2*z + 14820*A*B*a^5*c^3*d^3*e^9*z - 8
40*A*B*a^4*c^4*d^5*e^7*z - 600*A*B*a^3*c^5*d^7*e^5*z - 180*A*B*a^2*c^6*d^9*
e^3*z - 4*B*C*a^2*c^6*d^11*e*z - 3204*A*B*a^6*c^2*d*e^11*z + 4239*C^2*a^6*c
^2*d^4*e^8*z - 3924*C^2*a^5*c^3*d^6*e^6*z + 103*C^2*a^4*c^4*d^8*e^4*z + 26*
C^2*a^3*c^5*d^10*e^2*z - 6000*B^2*a^5*c^3*d^4*e^8*z + 2820*B^2*a^6*c^2*d^2*
e^10*z + 280*B^2*a^4*c^4*d^6*e^6*z + 80*B^2*a^3*c^5*d^8*e^4*z + 4*B^2*a^2*c
^6*d^10*e^2*z - 8262*A^2*a^5*c^3*d^2*e^10*z + 1575*A^2*a^4*c^4*d^4*e^8*z +
1260*A^2*a^3*c^5*d^6*e^6*z + 495*A^2*a^2*c^6*d^8*e^4*z - 90*A*C*a^7*c*e^12*
z + 6*A*C*a*c^7*d^12*z - 966*C^2*a^7*c*d^2*e^10*z + 90*A^2*a*c^7*d^10*e^2*z
 + C^2*a^2*c^6*d^12*z + 225*A^2*a^6*c^2*e^12*z - 192*B^2*a^7*c*e^12*z + 9*A
^2*c^8*d^12*z + 9*C^2*a^8*e^12*z + 78*A*B*C*a*c^4*d^6*e^4 + 942*A*B*C*a^2*c
^3*d^4*e^6 - 342*A*B*C*a^3*c^2*d^2*e^8 - 129*B*C^2*a^4*c*d^2*e^8 + 990*A^2*
C*a^3*c^2*d*e^9 - 234*A^2*C*a*c^4*d^5*e^5 - 24*A*C^2*a*c^4*d^7*e^3 + 333*A^
2*B*a*c^4*d^4*e^6 - 252*A*B^2*a^3*c^2*d*e^9 - 60*A*B^2*a*c^4*d^5*e^5 + 204*
B^2*C*a^4*c*d*e^9 - 234*A*C^2*a^4*c*d*e^9 - 624*B^2*C*a^3*c^2*d^3*e^7 + 405
*B*C^2*a^3*c^2*d^4*e^6 - 36*B^2*C*a^2*c^3*d^5*e^5 + 21*B*C^2*a^2*c^3*d^6*e^
4 - 1296*A^2*C*a^2*c^3*d^3*e^7 + 396*A*C^2*a^3*c^2*d^3*e^7 - 330*A*C^2*a^2*
c^3*d^5*e^5 + 1863*A^2*B*a^2*c^3*d^2*e^8 - 672*A*B^2*a^2*c^3*d^3*e^7 + 90*A
*B*C*a^4*c*e^10 + 8*C^3*a^4*c*d^3*e^7 - 1350*A^3*a^2*c^3*d*e^9 - 324*A^3*a*
c^4*d^3*e^7 - 36*A^2*C*c^5*d^7*e^3 + 45*A^2*B*c^5*d^6*e^4 - 225*A^2*B*a^3*c
^2*e^10 - 86*C^3*a^3*c^2*d^5*e^5 - 4*C^3*a^2*c^3*d^7*e^3 + 316*B^3*a^3*c^2*
d^2*e^8 + 20*B^3*a^2*c^3*d^4*e^6 + 18*C^3*a^5*d*e^9 - 64*B^3*a^4*c*e^10 - 9
*B*C^2*a^5*e^10 - 54*A^3*c^5*d^5*e^5, z, k)*((120*A*a^8*c^2*e^13 - 24*C*a^9
```

*c*e^13 + 24*A*a^2*c^8*d^12*e - 112*B*a^8*c^2*d*e^12 + 8*C*a^3*c^7*d^12*e +
144*A*a^3*c^7*d^10*e^3 + 456*A*a^4*c^6*d^8*e^5 + 864*A*a^5*c^5*d^6*e^7 + 9
36*A*a^6*c^4*d^4*e^9 + 528*A*a^7*c^3*d^2*e^11 - 16*B*a^3*c^7*d^11*e^2 - 176
*B*a^4*c^6*d^9*e^4 - 544*B*a^5*c^5*d^7*e^6 - 736*B*a^6*c^4*d^5*e^8 - 464*B*
a^7*c^3*d^3*e^10 + 112*C*a^4*c^6*d^10*e^3 + 344*C*a^5*c^5*d^8*e^5 + 416*C*a
^6*c^4*d^6*e^7 + 184*C*a^7*c^3*d^4*e^9 - 16*C*a^8*c^2*d^2*e^11)/(64*(a^10*e
^12 + a^4*c^6*d^12 + 6*a^9*c*d^2*e^10 + 6*a^5*c^5*d^10*e^2 + 15*a^6*c^4*d^8
*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*d^4*e^8)) + root(17920*a^9*c^5*d^8*e
^8*z^3 + 14336*a^10*c^4*d^6*e^10*z^3 + 14336*a^8*c^6*d^10*e^6*z^3 + 7168*a^
11*c^3*d^4*e^12*z^3 + 7168*a^7*c^7*d^12*e^4*z^3 + 2048*a^12*c^2*d^2*e^14*z^
3 + 2048*a^6*c^8*d^14*e^2*z^3 + 256*a^5*c^9*d^16*z^3 + 256*a^13*c*e^16*z^3
+ 948*B*C*a^7*c*d*e^11*z - 12*A*B*a*c^7*d^11*e*z + 9768*B*C*a^5*c^3*d^5*e^7
*z - 7476*B*C*a^6*c^2*d^3*e^9*z - 328*B*C*a^4*c^4*d^7*e^5*z - 92*B*C*a^3*c^
5*d^9*e^3*z - 12486*A*C*a^5*c^3*d^4*e^8*z + 5868*A*C*a^6*c^2*d^2*e^10*z + 2
82*A*C*a^3*c^5*d^8*e^4*z + 168*A*C*a^4*c^4*d^6*e^6*z + 108*A*C*a^2*c^6*d^10
*e^2*z + 14820*A*B*a^5*c^3*d^3*e^9*z - 840*A*B*a^4*c^4*d^5*e^7*z - 600*A*B*
a^3*c^5*d^7*e^5*z - 180*A*B*a^2*c^6*d^9*e^3*z - 4*B*C*a^2*c^6*d^11*e*z - 32
04*A*B*a^6*c^2*d*e^11*z + 4239*C^2*a^6*c^2*d^4*e^8*z - 3924*C^2*a^5*c^3*d^6
*e^6*z + 103*C^2*a^4*c^4*d^8*e^4*z + 26*C^2*a^3*c^5*d^10*e^2*z - 6000*B^2*a
^5*c^3*d^4*e^8*z + 2820*B^2*a^6*c^2*d^2*e^10*z + 280*B^2*a^4*c^4*d^6*e^6*z
+ 80*B^2*a^3*c^5*d^8*e^4*z + 4*B^2*a^2*c^6*d^10*e^2*z - 8262*A^2*a^5*c^3*d^
2*e^10*z + 1575*A^2*a^4*c^4*d^4*e^8*z + 1260*A^2*a^3*c^5*d^6*e^6*z + 495*A^
2*a^2*c^6*d^8*e^4*z - 90*A*C*a^7*c*e^12*z + 6*A*C*a*c^7*d^12*z - 966*C^2*a^
7*c*d^2*e^10*z + 90*A^2*a*c^7*d^10*e^2*z + C^2*a^2*c^6*d^12*z + 225*A^2*a^6
*c^2*e^12*z - 192*B^2*a^7*c*e^12*z + 9*A^2*c^8*d^12*z + 9*C^2*a^8*e^12*z +
78*A*B*C*a*c^4*d^6*e^4 + 942*A*B*C*a^2*c^3*d^4*e^6 - 342*A*B*C*a^3*c^2*d^2*
e^8 - 129*B*C^2*a^4*c*d^2*e^8 + 990*A^2*C*a^3*c^2*d*e^9 - 234*A^2*C*a*c^4*d
^5*e^5 - 24*A*C^2*a*c^4*d^7*e^3 + 333*A^2*B*a*c^4*d^4*e^6 - 252*A*B^2*a^3*c
^2*d*e^9 - 60*A*B^2*a*c^4*d^5*e^5 + 204*B^2*C*a^4*c*d*e^9 - 234*A*C^2*a^4*c
*d*e^9 - 624*B^2*C*a^3*c^2*d^3*e^7 + 405*B*C^2*a^3*c^2*d^4*e^6 - 36*B^2*C*a
^2*c^3*d^5*e^5 + 21*B*C^2*a^2*c^3*d^6*e^4 - 1296*A^2*C*a^2*c^3*d^3*e^7 + 39
6*A*C^2*a^3*c^2*d^3*e^7 - 330*A*C^2*a^2*c^3*d^5*e^5 + 1863*A^2*B*a^2*c^3*d^
2*e^8 - 672*A*B^2*a^2*c^3*d^3*e^7 + 90*A*B*C*a^4*c*e^10 + 8*C^3*a^4*c*d^3*e
^7 - 1350*A^3*a^2*c^3*d*e^9 - 324*A^3*a*c^4*d^3*e^7 - 36*A^2*C*c^5*d^7*e^3
+ 45*A^2*B*c^5*d^6*e^4 - 225*A^2*B*a^3*c^2*e^10 - 86*C^3*a^3*c^2*d^5*e^5 -
4*C^3*a^2*c^3*d^7*e^3 + 316*B^3*a^3*c^2*d^2*e^8 + 20*B^3*a^2*c^3*d^4*e^6 +
18*C^3*a^5*d*e^9 - 64*B^3*a^4*c*e^10 - 9*B*C^2*a^5*e^10 - 54*A^3*c^5*d^5*e^
5, z, k)*((512*a^11*c^2*d*e^14 + 512*a^5*c^8*d^13*e^2 + 3072*a^6*c^7*d^11*e
^4 + 7680*a^7*c^6*d^9*e^6 + 10240*a^8*c^5*d^7*e^8 + 7680*a^9*c^4*d^5*e^10 +
3072*a^10*c^3*d^3*e^12)/(64*(a^10*e^12 + a^4*c^6*d^12 + 6*a^9*c*d^2*e^10 +
6*a^5*c^5*d^10*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*
d^4*e^8)) + (x*(384*a^11*c^2*e^15 - 128*a^4*c^9*d^14*e - 384*a^5*c^8*d^12*e
^3 + 384*a^6*c^7*d^10*e^5 + 3200*a^7*c^6*d^8*e^7 + 5760*a^8*c^5*d^6*e^9 + 4
992*a^9*c^4*d^4*e^11 + 2176*a^10*c^3*d^2*e^13))/(64*(a^10*e^12 + a^4*c^6*d^
12 + 6*a^9*c*d^2*e^10 + 6*a^5*c^5*d^10*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^
3*d^6*e^6 + 15*a^8*c^2*d^4*e^8))) + (x*(192*B*a^8*c^2*e^13 + 912*A*a^7*c^3*
d*e^12 - 336*C*a^8*c^2*d*e^12 + 48*A*a^2*c^8*d^11*e^2 + 336*A*a^3*c^7*d^9*e
^4 + 1632*A*a^4*c^6*d^7*e^6 + 3360*A*a^5*c^5*d^5*e^8 + 2928*A*a^6*c^4*d^3*e
^10 - 32*B*a^3*c^7*d^10*e^3 - 704*B*a^4*c^6*d^8*e^5 - 1728*B*a^5*c^5*d^6*e^
7 - 1280*B*a^6*c^4*d^4*e^9 - 32*B*a^7*c^3*d^2*e^11 + 16*C*a^3*c^7*d^11*e^2
+ 496*C*a^4*c^6*d^9*e^4 + 1056*C*a^5*c^5*d^7*e^6 + 352*C*a^6*c^4*d^5*e^8 -
560*C*a^7*c^3*d^3*e^10))/(64*(a^10*e^12 + a^4*c^6*d^12 + 6*a^9*c*d^2*e^10 +
6*a^5*c^5*d^10*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*
d^4*e^8))) + (9*A^2*c^7*d^9*e^2 + 198*A^2*a^2*c^5*d^5*e^6 + 216*A^2*a^3*c^4
*d^3*e^8 + 4*B^2*a^2*c^5*d^7*e^4 - 8*B^2*a^3*c^4*d^5*e^6 - 412*B^2*a^4*c^3*
d^3*e^8 + C^2*a^2*c^5*d^9*e^2 - 8*C^2*a^3*c^4*d^7*e^4 - 250*C^2*a^4*c^3*d^5
*e^6 + 296*C^2*a^5*c^2*d^3*e^8 - 120*A*B*a^5*c^2*e^11 - 39*C^2*a^6*c*d*e^10
+ 72*A^2*a*c^6*d^7*e^4 - 495*A^2*a^4*c^3*d*e^10 + 176*B^2*a^5*c^2*d*e^10 +
24*B*C*a^6*c*e^11 - 12*A*B*a*c^6*d^8*e^3 + 6*A*C*a*c^6*d^9*e^2 + 294*A*C*a

```
^5*c^2*d*e^10 - 36*A*B*a^2*c^5*d^6*e^5 + 36*A*B*a^3*c^4*d^4*e^7 + 1092*A*B*
a^4*c^3*d^2*e^9 - 108*A*C*a^3*c^4*d^5*e^6 - 960*A*C*a^4*c^3*d^3*e^8 - 4*B*C
*a^2*c^5*d^8*e^3 + 20*B*C*a^3*c^4*d^6*e^5 + 652*B*C*a^4*c^3*d^4*e^7 - 500*B
*C*a^5*c^2*d^2*e^9)/(64*(a^10*e^12 + a^4*c^6*d^12 + 6*a^9*c*d^2*e^10 + 6*a^
5*c^5*d^10*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*d^4*e
^8)) + (x*(225*A^2*a^4*c^3*e^11 + 9*A^2*c^7*d^8*e^3 + 9*C^2*a^6*c*e^11 + 54
*A^2*a^2*c^5*d^4*e^7 - 360*A^2*a^3*c^4*d^2*e^9 + 4*B^2*a^2*c^5*d^6*e^5 - 88
*B^2*a^3*c^4*d^4*e^7 + 484*B^2*a^4*c^3*d^2*e^9 + C^2*a^2*c^5*d^8*e^3 - 40*C
^2*a^3*c^4*d^6*e^5 + 406*C^2*a^4*c^3*d^4*e^7 - 120*C^2*a^5*c^2*d^2*e^9 - 90
*A*C*a^5*c^2*e^11 + 72*A^2*a*c^6*d^6*e^5 - 12*A*B*a*c^6*d^7*e^4 - 660*A*B*a
^4*c^3*d*e^10 + 6*A*C*a*c^6*d^8*e^3 + 132*B*C*a^5*c^2*d*e^10 + 84*A*B*a^2*c
^5*d^5*e^6 + 588*A*B*a^3*c^4*d^3*e^8 - 96*A*C*a^2*c^5*d^6*e^5 - 492*A*C*a^3
*c^4*d^4*e^7 + 672*A*C*a^4*c^3*d^2*e^9 - 4*B*C*a^2*c^5*d^7*e^4 + 124*B*C*a^
3*c^4*d^5*e^6 - 892*B*C*a^4*c^3*d^3*e^8))/(64*(a^10*e^12 + a^4*c^6*d^12 + 6
*a^9*c*d^2*e^10 + 6*a^5*c^5*d^10*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^3*d^6*
e^6 + 15*a^8*c^2*d^4*e^8)))*root(17920*a^9*c^5*d^8*e^8*z^3 + 14336*a^10*c^4
*d^6*e^10*z^3 + 14336*a^8*c^6*d^10*e^6*z^3 + 7168*a^11*c^3*d^4*e^12*z^3 + 7
168*a^7*c^7*d^12*e^4*z^3 + 2048*a^12*c^2*d^2*e^14*z^3 + 2048*a^6*c^8*d^14*e
^2*z^3 + 256*a^5*c^9*d^16*z^3 + 256*a^13*c*e^16*z^3 + 948*B*C*a^7*c*d*e^11*
z - 12*A*B*a*c^7*d^11*e*z + 9768*B*C*a^5*c^3*d^5*e^7*z - 7476*B*C*a^6*c^2*d
^3*e^9*z - 328*B*C*a^4*c^4*d^7*e^5*z - 92*B*C*a^3*c^5*d^9*e^3*z - 12486*A*C
*a^5*c^3*d^4*e^8*z + 5868*A*C*a^6*c^2*d^2*e^10*z + 282*A*C*a^3*c^5*d^8*e^4*
z + 168*A*C*a^4*c^4*d^6*e^6*z + 108*A*C*a^2*c^6*d^10*e^2*z + 14820*A*B*a^5*
c^3*d^3*e^9*z - 840*A*B*a^4*c^4*d^5*e^7*z - 600*A*B*a^3*c^5*d^7*e^5*z - 180
*A*B*a^2*c^6*d^9*e^3*z - 4*B*C*a^2*c^6*d^11*e*z - 3204*A*B*a^6*c^2*d*e^11*z
 + 4239*C^2*a^6*c^2*d^4*e^8*z - 3924*C^2*a^5*c^3*d^6*e^6*z + 103*C^2*a^4*c^
4*d^8*e^4*z + 26*C^2*a^3*c^5*d^10*e^2*z - 6000*B^2*a^5*c^3*d^4*e^8*z + 2820
*B^2*a^6*c^2*d^2*e^10*z + 280*B^2*a^4*c^4*d^6*e^6*z + 80*B^2*a^3*c^5*d^8*e^
4*z + 4*B^2*a^2*c^6*d^10*e^2*z - 8262*A^2*a^5*c^3*d^2*e^10*z + 1575*A^2*a^4
*c^4*d^4*e^8*z + 1260*A^2*a^3*c^5*d^6*e^6*z + 495*A^2*a^2*c^6*d^8*e^4*z - 9
0*A*C*a^7*c*e^12*z + 6*A*C*a*c^7*d^12*z - 966*C^2*a^7*c*d^2*e^10*z + 90*A^2
*a*c^7*d^10*e^2*z + C^2*a^2*c^6*d^12*z + 225*A^2*a^6*c^2*e^12*z - 192*B^2*a
^7*c*e^12*z + 9*A^2*c^8*d^12*z + 9*C^2*a^8*e^12*z + 78*A*B*C*a*c^4*d^6*e^4
+ 942*A*B*C*a^2*c^3*d^4*e^6 - 342*A*B*C*a^3*c^2*d^2*e^8 - 129*B*C^2*a^4*c*d
^2*e^8 + 990*A^2*C*a^3*c^2*d*e^9 - 234*A^2*C*a*c^4*d^5*e^5 - 24*A*C^2*a*c^4
*d^7*e^3 + 333*A^2*B*a*c^4*d^4*e^6 - 252*A*B^2*a^3*c^2*d*e^9 - 60*A*B^2*a*c
^4*d^5*e^5 + 204*B^2*C*a^4*c*d*e^9 - 234*A*C^2*a^4*c*d*e^9 - 624*B^2*C*a^3*
c^2*d^3*e^7 + 405*B*C^2*a^3*c^2*d^4*e^6 - 36*B^2*C*a^2*c^3*d^5*e^5 + 21*B*C
^2*a^2*c^3*d^6*e^4 - 1296*A^2*C*a^2*c^3*d^3*e^7 + 396*A*C^2*a^3*c^2*d^3*e^7
 - 330*A*C^2*a^2*c^3*d^5*e^5 + 1863*A^2*B*a^2*c^3*d^2*e^8 - 672*A*B^2*a^2*c
^3*d^3*e^7 + 90*A*B*C*a^4*c*e^10 + 8*C^3*a^4*c*d^3*e^7 - 1350*A^3*a^2*c^3*d
*e^9 - 324*A^3*a*c^4*d^3*e^7 - 36*A^2*C*c^5*d^7*e^3 + 45*A^2*B*c^5*d^6*e^4
- 225*A^2*B*a^3*c^2*e^10 - 86*C^3*a^3*c^2*d^5*e^5 - 4*C^3*a^2*c^3*d^7*e^3 +
 316*B^3*a^3*c^2*d^2*e^8 + 20*B^3*a^2*c^3*d^4*e^6 + 18*C^3*a^5*d*e^9 - 64*B
^3*a^4*c*e^10 - 9*B*C^2*a^5*e^10 - 54*A^3*c^5*d^5*e^5, z, k), k, 1, 3) + ((
x^4*(3*C*a^3*c*e^5 + 3*A*c^4*d^4*e - 15*A*a^2*c^2*e^5 + 12*A*a*c^3*d^2*e^3
- 2*B*a*c^3*d^3*e^2 + 22*B*a^2*c^2*d*e^4 - 20*C*a^2*c^2*d^2*e^3 + C*a*c^3*d
^4*e))/(8*a^2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (4
*A*a^2*e^5 + B*c^2*d^5 - 7*B*a^2*d*e^4 - 2*A*c^2*d^4*e + 10*C*a^2*d^2*e^3 -
 2*C*a*c*d^4*e - 10*A*a*c*d^2*e^3 + 6*B*a*c*d^3*e^2)/(4*(a*e^2 + c*d^2)*(a^
2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x^3*(3*A*c^3*d^3 + 4*B*a^2*c*e^3 + C*a
*c^2*d^3 + 9*A*a*c^2*d*e^2 - 2*B*a*c^2*d^2*e - 5*C*a^2*c*d*e^2))/(8*a^2*(a^
2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(5*A*c^2*d^3 + 6*B*a^2*e^3 - C*a*c*d
^3 - 7*C*a^2*d*e^2 + 11*A*a*c*d*e^2))/(8*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e
^2)) + (x^2*(5*C*a^3*e^5 - 25*A*a^2*c*e^5 + 5*A*c^3*d^4*e + 28*A*a*c^2*d^2*
e^3 - 10*B*a*c^2*d^3*e^2 - 36*C*a^2*c*d^2*e^3 + 38*B*a^2*c*d*e^4 + 7*C*a*c^
2*d^4*e))/(8*a*(a*e^2 + c*d^2)*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a^2*d
 + c^2*d*x^4 + c^2*e*x^5 + a^2*e*x + 2*a*c*d*x^2 + 2*a*c*e*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a)**3,x)
[Out] Timed out
```

3.63 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=753

$$-\frac{e^3\log\left(a+cx^2\right)\left(a^2Ce^4-ace^2\left(3Ae^2-9Bde+13Cd^2\right)+c^2d^2\left(10Cd^2-3e(5Bd-7Ae)\right)\right)}{2\left(ae^2+cd^2\right)^5}+\frac{e^3\log(d+ex)\left(a^2C\right.}{}$$

```
[Out] -1/2*e^3*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^3/(e*x+d)^2+e^3*(a*e^2*(-B*e+2*C
*d)-c*d*(4*C*d^2-e*(-6*A*e+5*B*d)))/(a*e^2+c*d^2)^4/(e*x+d)+1/4*(-a*(B*c*d*
(-3*a*e^2+c*d^2)-(A*c-C*a)*e*(-a*e^2+3*c*d^2))+c*(A*c*d*(-3*a*e^2+c*d^2)-a*
(c*d^2*(-3*B*e+C*d)-a*e^2*(-B*e+3*C*d)))*x)/a/(a*e^2+c*d^2)^3/(c*x^2+a)^2+1
/8*(4*a^2*e*(a^2*C*e^4+c^2*d^2*(3*C*d^2-2*e*(-5*A*e+3*B*d))-2*a*c*e^2*(4*C*
d^2-e*(-A*e+3*B*d)))+c*(3*A*c*d*(-11*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)-a*(2*a*
c*d^2*e^2*(-19*B*e+13*C*d)-c^2*d^4*(-3*B*e+C*d)-7*a^2*e^4*(-B*e+3*C*d)))*x)
/a^2/(a*e^2+c*d^2)^4/(c*x^2+a)+e^3*(a^2*C*e^4-a*c*e^2*(3*A*e^2-9*B*d*e+13*C
*d^2)+c^2*d^2*(10*C*d^2-3*e*(-7*A*e+5*B*d)))*ln(e*x+d)/(a*e^2+c*d^2)^5-1/2*
e^3*(a^2*C*e^4-a*c*e^2*(3*A*e^2-9*B*d*e+13*C*d^2)+c^2*d^2*(10*C*d^2-3*e*(-7
*A*e+5*B*d)))*ln(c*x^2+a)/(a*e^2+c*d^2)^5+1/8*(3*A*c*d*(-35*a^3*e^6+35*a^2*
c*d^2*e^4+7*a*c^2*d^4*e^2+c^3*d^6)+a*(a*c^2*d^4*e^2*(-45*B*e+23*C*d)-5*a^2*
c*d^2*e^4*(-27*B*e+25*C*d)+c^3*d^6*(-3*B*e+C*d)+15*a^3*e^6*(-B*e+3*C*d)))*a
rctan(x*c^(1/2)/a^(1/2))*c^(1/2)/a^(5/2)/(a*e^2+c*d^2)^5
```

Rubi [A] time = 3.14, antiderivative size = 753, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1647, 1629, 635, 205, 260}

$$\frac{cx\left(3Acd\left(-11a^2e^4+6acd^2e^2+c^2d^4\right)-a\left(-7a^2e^4(3Cd-Be)+2acd^2e^2(13Cd-19Be)-c^2d^4(Cd-3Be)\right)\right)+4a}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^3),x]
```

```
[Out] -(e^3*(C*d^2 - B*d*e + A*e^2))/(2*(c*d^2 + a*e^2)^3*(d + e*x)^2) - (e^3*(4*
c*C*d^3 - c*d*e*(5*B*d - 6*A*e) - a*e^2*(2*C*d - B*e)))/((c*d^2 + a*e^2)^4*
(d + e*x)) - (a*(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))
 - c*(A*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e
)))*x)/(4*a*(c*d^2 + a*e^2)^3*(a + c*x^2)^2) + (4*a^2*e*(a^2*C*e^4 + c^2*(3
*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e)))
+ c*(3*A*c*d*(c^2*d^4 + 6*a*c*d^2*e^2 - 11*a^2*e^4) - a*(2*a*c*d^2*e^2*(13*
C*d - 19*B*e) - c^2*d^4*(C*d - 3*B*e) - 7*a^2*e^4*(3*C*d - B*e)))*x)/(8*a^2
*(c*d^2 + a*e^2)^4*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c^3*d^6 + 7*a*c^2*d^4*
e^2 + 35*a^2*c*d^2*e^4 - 35*a^3*e^6) + a*(a*c^2*d^4*e^2*(23*C*d - 45*B*e) -
 5*a^2*c*d^2*e^4*(25*C*d - 27*B*e) + c^3*d^6*(C*d - 3*B*e) + 15*a^3*e^6*(3*
C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*(c*d^2 + a*e^2)^5) + (
e^3*(a^2*C*e^4 - a*c*e^2*(13*C*d^2 - 9*B*d*e + 3*A*e^2) + c^2*(10*C*d^4 - 3
*d^2*e*(5*B*d - 7*A*e)))*Log[d + e*x])/(c*d^2 + a*e^2)^5 - (e^3*(a^2*C*e^4
- a*c*e^2*(13*C*d^2 - 9*B*d*e + 3*A*e^2) + c^2*(10*C*d^4 - 3*d^2*e*(5*B*d -
 7*A*e)))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^5)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^3}\,dx = -\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-\right.\right.}{4a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)^2}$$

$$= -\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-\right.\right.}{4a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)^2}$$

$$= -\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-\right.\right.}{4a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)^2}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^3(d+ex)^2}-\frac{e^3\left(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^4(d+ex)}-\frac{a\left(Bcd\right.}{}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^3(d+ex)^2}-\frac{e^3\left(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^4(d+ex)}-\frac{a\left(Bcd\right.}{}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^3(d+ex)^2}-\frac{e^3\left(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^4(d+ex)}-\frac{a\left(Bcd\right.}{}$$

Mathematica [A] time = 1.08, size = 672, normalized size = 0.89

$$-4\log\left(a+cx^2\right)\left(a^2Ce^7+ace^5\left(-3Ae^2+9Bde-13Cd^2\right)+c^2d^2e^3\left(3e(7Ae-5Bd)+10Cd^2\right)\right)+8\log(d+ex)\Big($$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^3),x]
```

```
[Out] ((-4*e^3*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2 - (8*e^3
*(c*d^2 + a*e^2)*(4*c*C*d^3 + c*d*e*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e)
))/(d + e*x) + (2*(c*d^2 + a*e^2)^2*(a^3*C*e^3 + A*c^3*d^3*x - a*c^2*d*(C*d
^2*x + B*d*(d - 3*e*x) + 3*A*e*(-d + e*x)) - a^2*c*e*(3*C*d*(d - e*x) + e*(
-3*B*d + A*e + B*e*x))))/(a*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(4*a^4*C*e^5
+ 3*A*c^4*d^5*x + a*c^3*d^3*(C*d^2 + 3*e*(-(B*d) + 6*A*e))*x + a^3*c*e^3*(C
*d*(-32*d + 21*e*x) + e*(24*B*d - 8*A*e - 7*B*e*x)) + a^2*c^2*d*e*(2*C*d^2*
(6*d - 13*e*x) + e*(-24*B*d^2 + 40*A*d*e + 38*B*d*e*x - 33*A*e^2*x))))/(a^2
*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c^3*d^6 + 7*a*c^2*d^4*e^2 + 35*a^2*c*d^2
*e^4 - 35*a^3*e^6) + a*(a*c^2*d^4*e^2*(23*C*d - 45*B*e) - 5*a^2*c*d^2*e^4*(
25*C*d - 27*B*e) + c^3*d^6*(C*d - 3*B*e) - 15*a^3*e^6*(-3*C*d + B*e)))*ArcT
an[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 8*(a^2*C*e^7 + a*c*e^5*(-13*C*d^2 + 9*B*
d*e - 3*A*e^2) + c^2*d^2*e^3*(10*C*d^2 + 3*e*(-5*B*d + 7*A*e)))*Log[d + e*x
] - 4*(a^2*C*e^7 + a*c*e^5*(-13*C*d^2 + 9*B*d*e - 3*A*e^2) + c^2*d^2*e^3*(1
0*C*d^2 + 3*e*(-5*B*d + 7*A*e)))*Log[a + c*x^2])/(8*(c*d^2 + a*e^2)^5)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.20, size = 1532, normalized size = 2.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 - 13*C*a*c*d^2*e^5 + 21*A*c^2*d^2
*e^5 + 9*B*a*c*d*e^6 + C*a^2*e^7 - 3*A*a*c*e^7)*log(c*x^2 + a)/(c^5*d^10 +
5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8
 + a^5*e^10) + (10*C*c^2*d^4*e^4 - 15*B*c^2*d^3*e^5 - 13*C*a*c*d^2*e^6 + 21
*A*c^2*d^2*e^6 + 9*B*a*c*d*e^7 + C*a^2*e^8 - 3*A*a*c*e^8)*log(abs(x*e + d))
/(c^5*d^10*e + 5*a*c^4*d^8*e^3 + 10*a^2*c^3*d^6*e^5 + 10*a^3*c^2*d^4*e^7 +
5*a^4*c*d^2*e^9 + a^5*e^11) + 1/8*(C*a*c^4*d^7 + 3*A*c^5*d^7 - 3*B*a*c^4*d^
6*e + 23*C*a^2*c^3*d^5*e^2 + 21*A*a*c^4*d^5*e^2 - 45*B*a^2*c^3*d^4*e^3 - 12
5*C*a^3*c^2*d^3*e^4 + 105*A*a^2*c^3*d^3*e^4 + 135*B*a^3*c^2*d^2*e^5 + 45*C*
a^4*c*d*e^6 - 105*A*a^3*c^2*d*e^6 - 15*B*a^4*c*e^7)*arctan(c*x/sqrt(a*c))/(
(a^2*c^5*d^10 + 5*a^3*c^4*d^8*e^2 + 10*a^4*c^3*d^6*e^4 + 10*a^5*c^2*d^4*e^6
 + 5*a^6*c*d^2*e^8 + a^7*e^10)*sqrt(a*c)) + 1/8*(C*a*c^4*d^5*x^5*e^2 + 3*A*
c^5*d^5*x^5*e^2 + 2*C*a*c^4*d^6*x^4*e + 6*A*c^5*d^6*x^4*e + C*a*c^4*d^7*x^3
 + 3*A*c^5*d^7*x^3 - 3*B*a*c^4*d^4*x^5*e^3 - 6*B*a*c^4*d^5*x^4*e^2 - 3*B*a*
c^4*d^6*x^3*e - 58*C*a^2*c^3*d^3*x^5*e^4 + 18*A*a*c^4*d^3*x^5*e^4 - 76*C*a^
2*c^3*d^4*x^4*e^3 + 36*A*a*c^4*d^4*x^4*e^3 - 3*C*a^2*c^3*d^5*x^3*e^2 + 23*A
*a*c^4*d^5*x^3*e^2 + 10*C*a^2*c^3*d^6*x^2*e + 10*A*a*c^4*d^6*x^2*e - C*a^2*
```

```
c^3*d^7*x + 5*A*a*c^4*d^7*x + 78*B*a^2*c^3*d^2*x^5*e^5 + 96*B*a^2*c^3*d^3*x
^4*e^4 - 7*B*a^2*c^3*d^4*x^3*e^3 - 20*B*a^2*c^3*d^5*x^2*e^2 - B*a^2*c^3*d^6
*x*e - 2*B*a^2*c^3*d^7 + 37*C*a^3*c^2*d*x^5*e^6 - 81*A*a^2*c^3*d*x^5*e^6 +
22*C*a^3*c^2*d^2*x^4*e^5 - 78*A*a^2*c^3*d^2*x^4*e^5 - 129*C*a^3*c^2*d^3*x^3
*e^4 + 61*A*a^2*c^3*d^3*x^3*e^4 - 142*C*a^3*c^2*d^4*x^2*e^3 + 74*A*a^2*c^3*
d^4*x^2*e^3 - 10*C*a^3*c^2*d^5*x*e^2 + 26*A*a^2*c^3*d^5*x*e^2 + 6*C*a^3*c^2
*d^6*e + 6*A*a^2*c^3*d^6*e - 15*B*a^3*c^2*x^5*e^7 + 6*B*a^3*c^2*d*x^4*e^6 +
 163*B*a^3*c^2*d^2*x^3*e^5 + 176*B*a^3*c^2*d^3*x^2*e^4 + 2*B*a^3*c^2*d^4*x*
e^3 - 20*B*a^3*c^2*d^5*e^2 + 4*C*a^4*c*x^4*e^7 - 12*A*a^3*c^2*x^4*e^7 + 67*
C*a^4*c*d*x^3*e^6 - 151*A*a^3*c^2*d*x^3*e^6 + 46*C*a^4*c*d^2*x^2*e^5 - 146*
A*a^3*c^2*d^2*x^2*e^5 - 77*C*a^4*c*d^3*x*e^4 + 49*A*a^3*c^2*d^3*x*e^4 - 72*
C*a^4*c*d^4*e^3 + 44*A*a^3*c^2*d^4*e^3 - 25*B*a^4*c*x^3*e^7 + 4*B*a^4*c*d*x
^2*e^6 + 91*B*a^4*c*d^2*x*e^5 + 74*B*a^4*c*d^3*e^4 + 6*C*a^5*x^2*e^7 - 18*A
*a^4*c*x^2*e^7 + 28*C*a^5*d*x*e^6 - 68*A*a^4*c*d*x*e^6 + 18*C*a^5*d^2*e^5 -
 62*A*a^4*c*d^2*e^5 - 8*B*a^5*x*e^7 - 4*B*a^5*d*e^6 - 4*A*a^5*e^7)/((a^2*c^
4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*
(c*x^3*e + c*d*x^2 + a*x*e + a*d)^2)
```

maple [B] time = 0.04, size = 2737, normalized size = 3.63

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x)
```

```
[Out] 15/2/(a*e^2+c*d^2)^5*c^2*ln(c*x^2+a)*d^3*e^4*B-5/(a*e^2+c*d^2)^5*c^2*ln(c*x
^2+a)*C*d^4*e^3+3/2/(a*e^2+c*d^2)^5*c*a*ln(c*x^2+a)*A*e^7+2*e^5/(a*e^2+c*d^
2)^4/(e*x+d)*C*a*d-4*e^3/(a*e^2+c*d^2)^4/(e*x+d)*C*c*d^3-3*e^7/(a*e^2+c*d^2
)^5*ln(e*x+d)*A*a*c+21*e^5/(a*e^2+c*d^2)^5*ln(e*x+d)*A*c^2*d^2-15*e^4/(a*e^
2+c*d^2)^5*ln(e*x+d)*B*c^2*d^3+10*e^3/(a*e^2+c*d^2)^5*ln(e*x+d)*C*c^2*d^4-6
*e^5/(a*e^2+c*d^2)^4/(e*x+d)*A*c*d+5*e^4/(a*e^2+c*d^2)^4/(e*x+d)*B*c*d^2-5/
4/(a*e^2+c*d^2)^5*c/(c*x^2+a)^2*A*e^7*a^3+3/4/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)
^2*A*d^6*e-1/8/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*C*x*d^7-21/2/(a*e^2+c*d^2)^5
*c^2*ln(c*x^2+a)*A*d^2*e^5-1/2*e^5/(a*e^2+c*d^2)^3/(e*x+d)^2*A-7/2/(a*e^2+c
*d^2)^5*c^2/(c*x^2+a)^2*C*x^2*a^2*d^2*e^5-5/2/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)
^2*C*x^2*a*d^4*e^3+21/8/(a*e^2+c*d^2)^5*c^5/(c*x^2+a)^2/a*x^3*A*d^5*e^2-3/8
/(a*e^2+c*d^2)^5*c^5/(c*x^2+a)^2/a*x^3*B*d^6*e+31/8/(a*e^2+c*d^2)^5*c^3/(c*
x^2+a)^2*B*x^3*a*d^2*e^5-39/8/(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2*A*x*a^2*d*e^6
-25/8/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*A*x*a*d^3*e^4+33/8/(a*e^2+c*d^2)^5*c^
2/(c*x^2+a)^2*B*x*a^2*d^2*e^5+45/8/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*B*x*a*d^
4*e^3+5/8/(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2*C*x*a^2*d^3*e^4-23/8/(a*e^2+c*d^2
)^5*c^3/(c*x^2+a)^2*C*x*a*d^5*e^2-105/8/(a*e^2+c*d^2)^5*c^2*a/(a*c)^(1/2)*a
rctan(1/(a*c)^(1/2)*c*x)*A*d*e^6+21/8/(a*e^2+c*d^2)^5*c^4/a/(a*c)^(1/2)*arc
tan(1/(a*c)^(1/2)*c*x)*A*d^5*e^2+135/8/(a*e^2+c*d^2)^5*c^2*a/(a*c)^(1/2)*ar
ctan(1/(a*c)^(1/2)*c*x)*B*d^2*e^5-3/8/(a*e^2+c*d^2)^5*c^4/a/(a*c)^(1/2)*arc
tan(1/(a*c)^(1/2)*c*x)*B*d^6*e-125/8/(a*e^2+c*d^2)^5*c^2*a/(a*c)^(1/2)*arct
an(1/(a*c)^(1/2)*c*x)*C*d^3*e^4+45/8/(a*e^2+c*d^2)^5*c*a^2/(a*c)^(1/2)*arct
an(1/(a*c)^(1/2)*c*x)*C*d*e^6+27/8/(a*e^2+c*d^2)^5*c/(c*x^2+a)^2*C*x*a^3*d*
e^6-33/8/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*A*x^3*a*d*e^6+21/8/(a*e^2+c*d^2)^5
*c^2/(c*x^2+a)^2*C*x^3*a^2*d*e^6-5/8/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*C*x^3*
a*d^3*e^4+4/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*A*x^2*a*d^2*e^5+3/(a*e^2+c*d^2)
^5*c^2/(c*x^2+a)^2*B*x^2*a^2*d*e^6-1/2/(a*e^2+c*d^2)^5*a^2*ln(c*x^2+a)*C*e^
7-e^6/(a*e^2+c*d^2)^4/(e*x+d)*B*a+1/2*e^4/(a*e^2+c*d^2)^3/(e*x+d)^2*B*d-1/2
*e^3/(a*e^2+c*d^2)^3/(e*x+d)^2*C*d^2+e^7/(a*e^2+c*d^2)^5*ln(e*x+d)*a^2*C+3/
4/(a*e^2+c*d^2)^5/(c*x^2+a)^2*C*a^4*e^7-1/4/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2
*d^7*B-15/8/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*A*x^3*d^3*e^4+35/8/(a*e^2+c*d^2
)^5*c^4/(c*x^2+a)^2*B*x^3*d^4*e^3-25/8/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*C*x^
3*d^5*e^2-1/(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2*A*x^2*a^2*e^7+5/(a*e^2+c*d^2)^5
*c^4/(c*x^2+a)^2*A*x^2*d^4*e^3-3/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*B*x^2*d^5*
e^2+3/2/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*C*x^2*d^6*e-7/8/(a*e^2+c*d^2)^5*c^2
```

```
/(c*x^2+a)^2*B*x^3*a^2*e^7+3/8/(a*e^2+c*d^2)^5*c^6/(c*x^2+a)^2/a^2*x^3*A*d^
7+1/8/(a*e^2+c*d^2)^5*c^5/(c*x^2+a)^2/a*x^3*C*d^7+5/8/(a*e^2+c*d^2)^5*c^5/(
c*x^2+a)^2/a*x*A*d^7+17/4/(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2*A*d^2*e^5*a^2+25/
4/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*A*d^4*e^3*a+5/4/(a*e^2+c*d^2)^5*c^2/(c*x^
2+a)^2*d^3*e^4*B*a^2-11/4/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*d^5*e^2*B*a-15/4/
(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2*C*a^2*d^4*e^3+3/4/(a*e^2+c*d^2)^5*c^3/(c*x^
2+a)^2*C*a*d^6*e+9*e^6/(a*e^2+c*d^2)^5*ln(e*x+d)*B*a*c*d-13*e^5/(a*e^2+c*d^
2)^5*ln(e*x+d)*C*a*c*d^2+1/2/(a*e^2+c*d^2)^5*c/(c*x^2+a)^2*C*x^2*a^3*e^7-9/
8/(a*e^2+c*d^2)^5*c/(c*x^2+a)^2*B*x*a^3*e^7+15/4/(a*e^2+c*d^2)^5*c/(c*x^2+a
)^2*d*e^6*B*a^3-15/4/(a*e^2+c*d^2)^5*c/(c*x^2+a)^2*C*a^3*d^2*e^5+1/8/(a*e^2
+c*d^2)^5*c^4/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d^7-9/2/(a*e^2+c*d^
2)^5*c*a*ln(c*x^2+a)*d*e^6*B+13/2/(a*e^2+c*d^2)^5*c*a*ln(c*x^2+a)*C*d^2*e^5
-15/8/(a*e^2+c*d^2)^5*c*a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*e^7+105
/8/(a*e^2+c*d^2)^5*c^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d^3*e^4-45/8
/(a*e^2+c*d^2)^5*c^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*d^4*e^3+23/8/(
a*e^2+c*d^2)^5*c^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d^5*e^2+3/8/(a*e
^2+c*d^2)^5*c^5/a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d^7+19/8/(a*e^2
+c*d^2)^5*c^4/(c*x^2+a)^2*A*x*d^5*e^2+3/8/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*B
*x*d^6*e
```

maxima [**B**] time = 1.27, size = 1835, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 + 9*B*a*c*d*e^6 - (13*C*a*c - 21*
A*c^2)*d^2*e^5 + (C*a^2 - 3*A*a*c)*e^7)*log(c*x^2 + a)/(c^5*d^10 + 5*a*c^4*
d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 + a^5*e
^10) + (10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 + 9*B*a*c*d*e^6 - (13*C*a*c - 2
1*A*c^2)*d^2*e^5 + (C*a^2 - 3*A*a*c)*e^7)*log(e*x + d)/(c^5*d^10 + 5*a*c^4*
d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 + a^5*e
^10) - 1/8*(3*B*a*c^4*d^6*e + 45*B*a^2*c^3*d^4*e^3 - 135*B*a^3*c^2*d^2*e^5
+ 15*B*a^4*c*e^7 - (C*a*c^4 + 3*A*c^5)*d^7 - (23*C*a^2*c^3 + 21*A*a*c^4)*d^
5*e^2 + 5*(25*C*a^3*c^2 - 21*A*a^2*c^3)*d^3*e^4 - 15*(3*C*a^4*c - 7*A*a^3*c
^2)*d*e^6)*arctan(c*x/sqrt(a*c))/((a^2*c^5*d^10 + 5*a^3*c^4*d^8*e^2 + 10*a^
4*c^3*d^6*e^4 + 10*a^5*c^2*d^4*e^6 + 5*a^6*c*d^2*e^8 + a^7*e^10)*sqrt(a*c))
 - 1/8*(2*B*a^2*c^3*d^7 + 20*B*a^3*c^2*d^5*e^2 - 74*B*a^4*c*d^3*e^4 + 4*B*a
^5*d*e^6 + 4*A*a^5*e^7 - 6*(C*a^3*c^2 + A*a^2*c^3)*d^6*e + 4*(18*C*a^4*c -
11*A*a^3*c^2)*d^4*e^3 - 2*(9*C*a^5 - 31*A*a^4*c)*d^2*e^5 + (3*B*a*c^4*d^4*e
^3 - 78*B*a^2*c^3*d^2*e^5 + 15*B*a^3*c^2*e^7 - (C*a*c^4 + 3*A*c^5)*d^5*e^2
+ 2*(29*C*a^2*c^3 - 9*A*a*c^4)*d^3*e^4 - (37*C*a^3*c^2 - 81*A*a^2*c^3)*d*e^
6)*x^5 + 2*(3*B*a*c^4*d^5*e^2 - 48*B*a^2*c^3*d^3*e^4 - 3*B*a^3*c^2*d*e^6 -
(C*a*c^4 + 3*A*c^5)*d^6*e + 2*(19*C*a^2*c^3 - 9*A*a*c^4)*d^4*e^3 - (11*C*a^
3*c^2 - 39*A*a^2*c^3)*d^2*e^5 - 2*(C*a^4*c - 3*A*a^3*c^2)*e^7)*x^4 + (3*B*a
*c^4*d^6*e + 7*B*a^2*c^3*d^4*e^3 - 163*B*a^3*c^2*d^2*e^5 + 25*B*a^4*c*e^7 -
 (C*a*c^4 + 3*A*c^5)*d^7 + (3*C*a^2*c^3 - 23*A*a*c^4)*d^5*e^2 + (129*C*a^3*
c^2 - 61*A*a^2*c^3)*d^3*e^4 - (67*C*a^4*c - 151*A*a^3*c^2)*d*e^6)*x^3 + 2*(
10*B*a^2*c^3*d^5*e^2 - 88*B*a^3*c^2*d^3*e^4 - 2*B*a^4*c*d*e^6 - 5*(C*a^2*c^
3 + A*a*c^4)*d^6*e + (71*C*a^3*c^2 - 37*A*a^2*c^3)*d^4*e^3 - (23*C*a^4*c -
73*A*a^3*c^2)*d^2*e^5 - 3*(C*a^5 - 3*A*a^4*c)*e^7)*x^2 + (B*a^2*c^3*d^6*e -
 2*B*a^3*c^2*d^4*e^3 - 91*B*a^4*c*d^2*e^5 + 8*B*a^5*e^7 + (C*a^2*c^3 - 5*A*
a*c^4)*d^7 + 2*(5*C*a^3*c^2 - 13*A*a^2*c^3)*d^5*e^2 + 7*(11*C*a^4*c - 7*A*a
^3*c^2)*d^3*e^4 - 4*(7*C*a^5 - 17*A*a^4*c)*d*e^6)*x)/(a^4*c^4*d^10 + 4*a^5*
c^3*d^8*e^2 + 6*a^6*c^2*d^6*e^4 + 4*a^7*c*d^4*e^6 + a^8*d^2*e^8 + (a^2*c^6*
d^8*e^2 + 4*a^3*c^5*d^6*e^4 + 6*a^4*c^4*d^4*e^6 + 4*a^5*c^3*d^2*e^8 + a^6*c
^2*e^10)*x^6 + 2*(a^2*c^6*d^9*e + 4*a^3*c^5*d^7*e^3 + 6*a^4*c^4*d^5*e^5 + 4
*a^5*c^3*d^3*e^7 + a^6*c^2*d*e^9)*x^5 + (a^2*c^6*d^10 + 6*a^3*c^5*d^8*e^2 +
 14*a^4*c^4*d^6*e^4 + 16*a^5*c^3*d^4*e^6 + 9*a^6*c^2*d^2*e^8 + 2*a^7*c*e^10
```

```
)*x^4 + 4*(a^3*c^5*d^9*e + 4*a^4*c^4*d^7*e^3 + 6*a^5*c^3*d^5*e^5 + 4*a^6*c^
2*d^3*e^7 + a^7*c*d*e^9)*x^3 + (2*a^3*c^5*d^10 + 9*a^4*c^4*d^8*e^2 + 16*a^5
*c^3*d^6*e^4 + 14*a^6*c^2*d^4*e^6 + 6*a^7*c*d^2*e^8 + a^8*e^10)*x^2 + 2*(a^
4*c^4*d^9*e + 4*a^5*c^3*d^7*e^3 + 6*a^6*c^2*d^5*e^5 + 4*a^7*c*d^3*e^7 + a^8
*d*e^9)*x)
```

mupad [B] time = 7.24, size = 8774, normalized size = 11.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^3*(d + e*x)^3),x)
```

```
[Out] ((x^5*(3*A*c^5*d^5*e^2 - 15*B*a^3*c^2*e^7 + 18*A*a*c^4*d^3*e^4 - 81*A*a^2*c
^3*d*e^6 - 3*B*a*c^4*d^4*e^3 + C*a*c^4*d^5*e^2 + 37*C*a^3*c^2*d*e^6 + 78*B*
a^2*c^3*d^2*e^5 - 58*C*a^2*c^3*d^3*e^4))/(8*a^2*(a^4*e^8 + c^4*d^8 + 4*a*c^
3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) - (2*A*a^3*e^7 + B*c^3*d^
7 + 2*B*a^3*d*e^6 - 3*A*c^3*d^6*e - 9*C*a^3*d^2*e^5 - 22*A*a*c^2*d^4*e^3 +
31*A*a^2*c*d^2*e^5 + 10*B*a*c^2*d^5*e^2 - 37*B*a^2*c*d^3*e^4 + 36*C*a^2*c*d
^4*e^3 - 3*C*a*c^2*d^6*e)/(4*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c
*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (x*(5*A*c^4*d^7 - 8*B*a^4*e^7 - C*a*c^3*d^
7 + 28*C*a^4*d*e^6 + 26*A*a*c^3*d^5*e^2 + 91*B*a^3*c*d^2*e^5 - 77*C*a^3*c*d
^3*e^4 + 49*A*a^2*c^2*d^3*e^4 + 2*B*a^2*c^2*d^4*e^3 - 10*C*a^2*c^2*d^5*e^2
- 68*A*a^3*c*d*e^6 - B*a*c^3*d^6*e))/(8*a*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*
e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (x^2*(3*C*a^4*e^7 - 9*A*a^3*c
*e^7 + 5*A*c^4*d^6*e + 37*A*a*c^3*d^4*e^3 - 10*B*a*c^3*d^5*e^2 + 23*C*a^3*c
*d^2*e^5 - 73*A*a^2*c^2*d^2*e^5 + 88*B*a^2*c^2*d^3*e^4 - 71*C*a^2*c^2*d^4*e
^3 + 2*B*a^3*c*d*e^6 + 5*C*a*c^3*d^6*e))/(4*a*(a^4*e^8 + c^4*d^8 + 4*a*c^3*
d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (x^3*(3*A*c^5*d^7 - 25*B*
a^4*c*e^7 + C*a*c^4*d^7 + 23*A*a*c^4*d^5*e^2 - 151*A*a^3*c^2*d*e^6 + 61*A*a
^2*c^3*d^3*e^4 - 7*B*a^2*c^3*d^4*e^3 + 163*B*a^3*c^2*d^2*e^5 - 3*C*a^2*c^3*
d^5*e^2 - 129*C*a^3*c^2*d^3*e^4 - 3*B*a*c^4*d^6*e + 67*C*a^4*c*d*e^6))/(8*a
^2*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e
^4)) + (x^4*(2*C*a^4*c*e^7 + 3*A*c^5*d^6*e - 6*A*a^3*c^2*e^7 + 18*A*a*c^4*d
^4*e^3 - 3*B*a*c^4*d^5*e^2 + 3*B*a^3*c^2*d*e^6 - 39*A*a^2*c^3*d^2*e^5 + 48*
B*a^2*c^3*d^3*e^4 - 38*C*a^2*c^3*d^4*e^3 + 11*C*a^3*c^2*d^2*e^5 + C*a*c^4*d
^6*e))/(4*a^2*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^
2*c^2*d^4*e^4)))/(x^2*(a^2*e^2 + 2*a*c*d^2) + x^4*(c^2*d^2 + 2*a*c*e^2) + a
^2*d^2 + c^2*e^2*x^6 + 2*a^2*d*e*x + 2*c^2*d*e*x^5 + 4*a*c*d*e*x^3) + symsu
m(log(root(2560*a^14*c*d^2*e^18*z^3 + 64512*a^10*c^5*d^10*e^10*z^3 + 53760*
a^11*c^4*d^8*e^12*z^3 + 53760*a^9*c^6*d^12*e^8*z^3 + 30720*a^12*c^3*d^6*e^1
4*z^3 + 30720*a^8*c^7*d^14*e^6*z^3 + 11520*a^13*c^2*d^4*e^16*z^3 + 11520*a^
7*c^8*d^16*e^4*z^3 + 2560*a^6*c^9*d^18*e^2*z^3 + 256*a^5*c^10*d^20*z^3 + 25
6*a^15*e^20*z^3 - 4806*B*C*a^8*c*d*e^13*z - 18*A*B*a*c^8*d^13*e*z - 147930*
B*C*a^6*c^3*d^5*e^9*z + 74760*B*C*a^5*c^4*d^7*e^7*z + 66588*B*C*a^7*c^2*d^3
*e^11*z - 1050*B*C*a^4*c^5*d^9*e^5*z - 228*B*C*a^3*c^6*d^11*e^3*z + 152052*
A*C*a^6*c^3*d^4*e^10*z - 109830*A*C*a^5*c^4*d^6*e^8*z - 32490*A*C*a^7*c^2*d
^2*e^12*z + 426*A*C*a^3*c^6*d^10*e^4*z - 360*A*C*a^4*c^5*d^8*e^6*z + 180*A*
C*a^2*c^7*d^12*e^2*z + 158130*A*B*a^5*c^4*d^5*e^9*z - 121356*A*B*a^6*c^3*d^
3*e^11*z - 3240*A*B*a^4*c^5*d^7*e^7*z - 1710*A*B*a^3*c^6*d^9*e^5*z - 396*A*
B*a^2*c^7*d^11*e^3*z - 6*B*C*a^2*c^7*d^13*e*z + 13518*A*B*a^7*c^2*d*e^13*z
+ 67615*C^2*a^6*c^3*d^6*e^8*z - 47538*C^2*a^7*c^2*d^4*e^10*z - 24860*C^2*a^
5*c^4*d^8*e^6*z + 279*C^2*a^4*c^5*d^10*e^4*z + 46*C^2*a^3*c^6*d^12*e^2*z +
71415*B^2*a^6*c^3*d^4*e^10*z - 55260*B^2*a^5*c^4*d^6*e^8*z - 19602*B^2*a^7*
c^2*d^2*e^12*z + 1215*B^2*a^4*c^5*d^8*e^6*z + 270*B^2*a^3*c^6*d^10*e^4*z +
9*B^2*a^2*c^7*d^12*e^2*z - 106722*A^2*a^5*c^4*d^4*e^10*z + 35217*A^2*a^6*c^
3*d^2*e^12*z + 6615*A^2*a^4*c^5*d^6*e^8*z + 3780*A^2*a^3*c^6*d^8*e^6*z + 10
71*A^2*a^2*c^7*d^10*e^4*z + 1152*A*C*a^8*c*e^14*z + 6*A*C*a*c^8*d^14*z + 70
17*C^2*a^8*c*d^2*e^12*z + 126*A^2*a*c^8*d^12*e^2*z + C^2*a^2*c^7*d^14*z - 1
728*A^2*a^7*c^2*e^14*z + 225*B^2*a^8*c*e^14*z + 9*A^2*c^9*d^14*z - 192*C^2*
```

a^9*e^14*z + 3168*A*B*C*a^4*c^2*d*e^10 + 270*A*B*C*a*c^5*d^7*e^4 - 6930*A*B
*C*a^3*c^3*d^3*e^8 + 5148*A*B*C*a^2*c^4*d^5*e^6 - 819*A^2*C*a*c^5*d^6*e^5 -
60*A*C^2*a*c^5*d^8*e^3 - 6102*A^2*B*a^3*c^3*d*e^10 + 1512*A^2*B*a*c^5*d^5*
e^6 - 270*A*B^2*a*c^5*d^6*e^5 - 378*B*C^2*a^5*c*d*e^10 - 5049*B^2*C*a^3*c^3
*d^4*e^7 + 4698*B^2*C*a^4*c^2*d^2*e^9 + 2508*B*C^2*a^3*c^3*d^5*e^6 - 1977*B
*C^2*a^4*c^2*d^3*e^8 - 180*B^2*C*a^2*c^4*d^6*e^5 + 75*B*C^2*a^2*c^4*d^7*e^4
+ 15921*A^2*C*a^3*c^3*d^2*e^9 - 7848*A^2*C*a^2*c^4*d^4*e^7 - 6363*A*C^2*a^
4*c^2*d^2*e^9 + 4926*A*C^2*a^3*c^3*d^4*e^7 - 1443*A*C^2*a^2*c^4*d^6*e^5 + 1
4283*A^2*B*a^2*c^4*d^3*e^8 - 4617*A*B^2*a^2*c^4*d^4*e^7 - 1944*A*B^2*a^3*c^
3*d^2*e^9 + 791*C^3*a^5*c*d^2*e^9 - 2025*B^3*a^4*c^2*d*e^10 - 1674*A^3*a*c^
5*d^4*e^7 - 90*A^2*C*c^6*d^8*e^3 + 135*A^2*B*c^6*d^7*e^4 - 1728*A^2*C*a^4*c
^2*e^11 + 675*A*B^2*a^4*c^2*e^11 - 225*B^2*C*a^5*c*e^11 + 576*A*C^2*a^5*c*e
^11 - 397*C^3*a^3*c^3*d^6*e^5 - 108*C^3*a^4*c^2*d^4*e^7 - 10*C^3*a^2*c^4*d^
8*e^3 + 3294*B^3*a^3*c^3*d^3*e^8 + 135*B^3*a^2*c^4*d^5*e^6 - 11853*A^3*a^2*
c^4*d^2*e^9 - 189*A^3*c^6*d^6*e^5 + 1728*A^3*a^3*c^3*e^11 - 64*C^3*a^6*e^11
, z, k)*(root(2560*a^14*c*d^2*e^18*z^3 + 64512*a^10*c^5*d^10*e^10*z^3 + 537
60*a^11*c^4*d^8*e^12*z^3 + 53760*a^9*c^6*d^12*e^8*z^3 + 30720*a^12*c^3*d^6*
e^14*z^3 + 30720*a^8*c^7*d^14*e^6*z^3 + 11520*a^13*c^2*d^4*e^16*z^3 + 11520
*a^7*c^8*d^16*e^4*z^3 + 2560*a^6*c^9*d^18*e^2*z^3 + 256*a^5*c^10*d^20*z^3 +
256*a^15*e^20*z^3 - 4806*B*C*a^8*c*d*e^13*z - 18*A*B*a*c^8*d^13*e*z - 1479
30*B*C*a^6*c^3*d^5*e^9*z + 74760*B*C*a^5*c^4*d^7*e^7*z + 66588*B*C*a^7*c^2*
d^3*e^11*z - 1050*B*C*a^4*c^5*d^9*e^5*z - 228*B*C*a^3*c^6*d^11*e^3*z + 1520
52*A*C*a^6*c^3*d^4*e^10*z - 109830*A*C*a^5*c^4*d^6*e^8*z - 32490*A*C*a^7*c^
2*d^2*e^12*z + 426*A*C*a^3*c^6*d^10*e^4*z - 360*A*C*a^4*c^5*d^8*e^6*z + 180
*A*C*a^2*c^7*d^12*e^2*z + 158130*A*B*a^5*c^4*d^5*e^9*z - 121356*A*B*a^6*c^3
*d^3*e^11*z - 3240*A*B*a^4*c^5*d^7*e^7*z - 1710*A*B*a^3*c^6*d^9*e^5*z - 396
*A*B*a^2*c^7*d^11*e^3*z - 6*B*C*a^2*c^7*d^13*e*z + 13518*A*B*a^7*c^2*d*e^13
*z + 67615*C^2*a^6*c^3*d^6*e^8*z - 47538*C^2*a^7*c^2*d^4*e^10*z - 24860*C^2
*a^5*c^4*d^8*e^6*z + 279*C^2*a^4*c^5*d^10*e^4*z + 46*C^2*a^3*c^6*d^12*e^2*z
+ 71415*B^2*a^6*c^3*d^4*e^10*z - 55260*B^2*a^5*c^4*d^6*e^8*z - 19602*B^2*a
^7*c^2*d^2*e^12*z + 1215*B^2*a^4*c^5*d^8*e^6*z + 270*B^2*a^3*c^6*d^10*e^4*z
+ 9*B^2*a^2*c^7*d^12*e^2*z - 106722*A^2*a^5*c^4*d^4*e^10*z + 35217*A^2*a^6
*c^3*d^2*e^12*z + 6615*A^2*a^4*c^5*d^6*e^8*z + 3780*A^2*a^3*c^6*d^8*e^6*z +
1071*A^2*a^2*c^7*d^10*e^4*z + 1152*A*C*a^8*c*e^14*z + 6*A*C*a*c^8*d^14*z +
7017*C^2*a^8*c*d^2*e^12*z + 126*A^2*a*c^8*d^12*e^2*z + C^2*a^2*c^7*d^14*z
- 1728*A^2*a^7*c^2*e^14*z + 225*B^2*a^8*c*e^14*z + 9*A^2*c^9*d^14*z - 192*C
^2*a^9*e^14*z + 3168*A*B*C*a^4*c^2*d*e^10 + 270*A*B*C*a*c^5*d^7*e^4 - 6930*
A*B*C*a^3*c^3*d^3*e^8 + 5148*A*B*C*a^2*c^4*d^5*e^6 - 819*A^2*C*a*c^5*d^6*e^
5 - 60*A*C^2*a*c^5*d^8*e^3 - 6102*A^2*B*a^3*c^3*d*e^10 + 1512*A^2*B*a*c^5*d
^5*e^6 - 270*A*B^2*a*c^5*d^6*e^5 - 378*B*C^2*a^5*c*d*e^10 - 5049*B^2*C*a^3*
c^3*d^4*e^7 + 4698*B^2*C*a^4*c^2*d^2*e^9 + 2508*B*C^2*a^3*c^3*d^5*e^6 - 197
7*B*C^2*a^4*c^2*d^3*e^8 - 180*B^2*C*a^2*c^4*d^6*e^5 + 75*B*C^2*a^2*c^4*d^7*
e^4 + 15921*A^2*C*a^3*c^3*d^2*e^9 - 7848*A^2*C*a^2*c^4*d^4*e^7 - 6363*A*C^2
*a^4*c^2*d^2*e^9 + 4926*A*C^2*a^3*c^3*d^4*e^7 - 1443*A*C^2*a^2*c^4*d^6*e^5
+ 14283*A^2*B*a^2*c^4*d^3*e^8 - 4617*A*B^2*a^2*c^4*d^4*e^7 - 1944*A*B^2*a^3
*c^3*d^2*e^9 + 791*C^3*a^5*c*d^2*e^9 - 2025*B^3*a^4*c^2*d*e^10 - 1674*A^3*a
*c^5*d^4*e^7 - 90*A^2*C*c^6*d^8*e^3 + 135*A^2*B*c^6*d^7*e^4 - 1728*A^2*C*a^
4*c^2*e^11 + 675*A*B^2*a^4*c^2*e^11 - 225*B^2*C*a^5*c*e^11 + 576*A*C^2*a^5*
c*e^11 - 397*C^3*a^3*c^3*d^6*e^5 - 108*C^3*a^4*c^2*d^4*e^7 - 10*C^3*a^2*c^4
*d^8*e^3 + 3294*B^3*a^3*c^3*d^3*e^8 + 135*B^3*a^2*c^4*d^5*e^6 - 11853*A^3*a
^2*c^4*d^2*e^9 - 189*A^3*c^6*d^6*e^5 + 1728*A^3*a^3*c^3*e^11 - 64*C^3*a^6*e
^11, z, k)*((512*a^13*c^2*d*e^18 + 512*a^5*c^10*d^17*e^2 + 4096*a^6*c^9*d^1
5*e^4 + 14336*a^7*c^8*d^13*e^6 + 28672*a^8*c^7*d^11*e^8 + 35840*a^9*c^6*d^9
*e^10 + 28672*a^10*c^5*d^7*e^12 + 14336*a^11*c^4*d^5*e^14 + 4096*a^12*c^3*d
^3*e^16)/(64*(a^12*e^16 + a^4*c^8*d^16 + 8*a^11*c*d^2*e^14 + 8*a^5*c^7*d^14
*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5*d^10*e^6 + 70*a^8*c^4*d^8*e^8 + 56*
a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12)) + (x*(384*a^13*c^2*e^19 - 128*a^4
*c^11*d^18*e - 640*a^5*c^10*d^16*e^3 - 512*a^6*c^9*d^14*e^5 + 3584*a^7*c^8*
d^12*e^7 + 12544*a^8*c^7*d^10*e^9 + 19712*a^9*c^6*d^8*e^11 + 17920*a^10*c^5

*d^6*e^13 + 9728*a^11*c^4*d^4*e^15 + 2944*a^12*c^3*d^2*e^17))/(64*(a^12*e^1
6 + a^4*c^8*d^16 + 8*a^11*c*d^2*e^14 + 8*a^5*c^7*d^14*e^2 + 28*a^6*c^6*d^12
*e^4 + 56*a^7*c^5*d^10*e^6 + 70*a^8*c^4*d^8*e^8 + 56*a^9*c^3*d^6*e^10 + 28*
a^10*c^2*d^4*e^12))) + (120*B*a^10*c^2*e^16 + 24*A*a^2*c^10*d^15*e + 456*A*
a^9*c^3*d*e^15 + 8*C*a^3*c^9*d^15*e - 232*C*a^10*c^2*d*e^15 + 216*A*a^3*c^9
*d^13*e^3 + 1176*A*a^4*c^8*d^11*e^5 + 3480*A*a^5*c^7*d^9*e^7 + 5640*A*a^6*c
^6*d^7*e^9 + 5064*A*a^7*c^5*d^5*e^11 + 2376*A*a^8*c^4*d^3*e^13 - 24*B*a^3*c
^9*d^14*e^2 - 408*B*a^4*c^8*d^12*e^4 - 1560*B*a^5*c^7*d^10*e^6 - 2520*B*a^6
*c^6*d^8*e^8 - 1800*B*a^7*c^5*d^6*e^10 - 264*B*a^8*c^4*d^4*e^12 + 312*B*a^9
*c^3*d^2*e^14 + 200*C*a^4*c^8*d^13*e^3 + 648*C*a^5*c^7*d^11*e^5 + 520*C*a^6
*c^6*d^9*e^7 - 680*C*a^7*c^5*d^7*e^9 - 1512*C*a^8*c^4*d^5*e^11 - 1000*C*a^9
*c^3*d^3*e^13)/(64*(a^12*e^16 + a^4*c^8*d^16 + 8*a^11*c*d^2*e^14 + 8*a^5*c^
7*d^14*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5*d^10*e^6 + 70*a^8*c^4*d^8*e^8
 + 56*a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12)) + (x*(192*C*a^10*c^2*e^16 -
 576*A*a^9*c^3*e^16 + 1488*B*a^9*c^3*d*e^15 + 48*A*a^2*c^10*d^14*e^2 + 480*
A*a^3*c^9*d^12*e^4 + 4176*A*a^4*c^8*d^10*e^6 + 12288*A*a^5*c^7*d^8*e^8 + 15
312*A*a^6*c^6*d^6*e^10 + 7776*A*a^7*c^5*d^4*e^12 + 432*A*a^8*c^4*d^2*e^14 -
 48*B*a^3*c^9*d^13*e^3 - 1824*B*a^4*c^8*d^11*e^5 - 5328*B*a^5*c^7*d^9*e^7 -
 4032*B*a^6*c^6*d^7*e^9 + 2352*B*a^7*c^5*d^5*e^11 + 4320*B*a^8*c^4*d^3*e^13
 + 16*C*a^3*c^9*d^14*e^2 + 1056*C*a^4*c^8*d^12*e^4 + 2160*C*a^5*c^7*d^10*e^
6 - 1408*C*a^6*c^6*d^8*e^8 - 6672*C*a^7*c^5*d^6*e^10 - 5472*C*a^8*c^4*d^4*e
^12 - 1136*C*a^9*c^3*d^2*e^14))/(64*(a^12*e^16 + a^4*c^8*d^16 + 8*a^11*c*d^
2*e^14 + 8*a^5*c^7*d^14*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5*d^10*e^6 + 7
0*a^8*c^4*d^8*e^8 + 56*a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12))) + (9*A^2*
c^9*d^11*e^2 + 342*A^2*a^2*c^7*d^7*e^6 + 36*A^2*a^3*c^6*d^5*e^8 - 7479*A^2*
a^4*c^5*d^3*e^10 + 9*B^2*a^2*c^7*d^9*e^4 - 108*B^2*a^3*c^6*d^7*e^6 - 3402*B
^2*a^4*c^5*d^5*e^8 + 5076*B^2*a^5*c^4*d^3*e^10 + C^2*a^2*c^7*d^11*e^2 - 36*
C^2*a^3*c^6*d^9*e^4 - 1306*C^2*a^4*c^5*d^7*e^6 + 4708*C^2*a^5*c^4*d^5*e^8 -
 2943*C^2*a^6*c^3*d^3*e^10 + 360*A*B*a^6*c^3*e^13 - 120*B*C*a^7*c^2*e^13 +
108*A^2*a*c^8*d^9*e^4 + 1944*A^2*a^5*c^4*d*e^12 - 855*B^2*a^6*c^3*d*e^12 +
296*C^2*a^7*c^2*d*e^12 - 18*A*B*a*c^8*d^10*e^3 + 6*A*C*a*c^8*d^11*e^2 - 153
6*A*C*a^6*c^3*d*e^12 + 756*A*B*a^3*c^6*d^6*e^7 + 11016*A*B*a^4*c^5*d^4*e^9
 - 7794*A*B*a^5*c^4*d^2*e^11 - 72*A*C*a^2*c^7*d^9*e^4 - 732*A*C*a^3*c^6*d^7*
e^6 - 7368*A*C*a^4*c^5*d^5*e^8 + 10182*A*C*a^5*c^4*d^3*e^10 - 6*B*C*a^2*c^7
*d^10*e^3 + 144*B*C*a^3*c^6*d^8*e^5 + 4284*B*C*a^4*c^5*d^6*e^7 - 10440*B*C*
a^5*c^4*d^4*e^9 + 3738*B*C*a^6*c^3*d^2*e^11)/(64*(a^12*e^16 + a^4*c^8*d^16
 + 8*a^11*c*d^2*e^14 + 8*a^5*c^7*d^14*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5
*d^10*e^6 + 70*a^8*c^4*d^8*e^8 + 56*a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12
)) + (x*(225*B^2*a^6*c^3*e^13 + 9*A^2*c^9*d^10*e^3 - 162*A^2*a^2*c^7*d^6*e^
7 - 2916*A^2*a^3*c^6*d^4*e^9 + 6561*A^2*a^4*c^5*d^2*e^11 + 9*B^2*a^2*c^7*d^
8*e^5 - 468*B^2*a^3*c^6*d^6*e^7 + 6174*B^2*a^4*c^5*d^4*e^9 - 2340*B^2*a^5*c
^4*d^2*e^11 + C^2*a^2*c^7*d^10*e^3 - 116*C^2*a^3*c^6*d^8*e^5 + 3438*C^2*a^4
*c^5*d^6*e^7 - 4292*C^2*a^5*c^4*d^4*e^9 + 1369*C^2*a^6*c^3*d^2*e^11 + 108*A
^2*a*c^8*d^8*e^5 - 18*A*B*a*c^8*d^9*e^4 + 2430*A*B*a^5*c^4*d*e^12 + 6*A*C*a
*c^8*d^10*e^3 - 1110*B*C*a^6*c^3*d*e^12 + 360*A*B*a^2*c^7*d^7*e^6 + 3204*A*
B*a^3*c^6*d^5*e^8 - 13176*A*B*a^4*c^5*d^3*e^10 - 312*A*C*a^2*c^7*d^8*e^5 -
2028*A*C*a^3*c^6*d^6*e^7 + 10728*A*C*a^4*c^5*d^4*e^9 - 5994*A*C*a^5*c^4*d^2
*e^11 - 6*B*C*a^2*c^7*d^9*e^4 + 504*B*C*a^3*c^6*d^7*e^6 - 9300*B*C*a^4*c^5*
d^5*e^8 + 7512*B*C*a^5*c^4*d^3*e^10))/(64*(a^12*e^16 + a^4*c^8*d^16 + 8*a^1
1*c*d^2*e^14 + 8*a^5*c^7*d^14*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5*d^10*e
^6 + 70*a^8*c^4*d^8*e^8 + 56*a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12)))*roo
t(2560*a^14*c*d^2*e^18*z^3 + 64512*a^10*c^5*d^10*e^10*z^3 + 53760*a^11*c^4*
d^8*e^12*z^3 + 53760*a^9*c^6*d^12*e^8*z^3 + 30720*a^12*c^3*d^6*e^14*z^3 + 3
0720*a^8*c^7*d^14*e^6*z^3 + 11520*a^13*c^2*d^4*e^16*z^3 + 11520*a^7*c^8*d^1
6*e^4*z^3 + 2560*a^6*c^9*d^18*e^2*z^3 + 256*a^5*c^10*d^20*z^3 + 256*a^15*e^
20*z^3 - 4806*B*C*a^8*c*d*e^13*z - 18*A*B*a*c^8*d^13*e*z - 147930*B*C*a^6*c
^3*d^5*e^9*z + 74760*B*C*a^5*c^4*d^7*e^7*z + 66588*B*C*a^7*c^2*d^3*e^11*z -
 1050*B*C*a^4*c^5*d^9*e^5*z - 228*B*C*a^3*c^6*d^11*e^3*z + 152052*A*C*a^6*c
^3*d^4*e^10*z - 109830*A*C*a^5*c^4*d^6*e^8*z - 32490*A*C*a^7*c^2*d^2*e^12*z

```
 + 426*A*C*a^3*c^6*d^10*e^4*z - 360*A*C*a^4*c^5*d^8*e^6*z + 180*A*C*a^2*c^7
*d^12*e^2*z + 158130*A*B*a^5*c^4*d^5*e^9*z - 121356*A*B*a^6*c^3*d^3*e^11*z
- 3240*A*B*a^4*c^5*d^7*e^7*z - 1710*A*B*a^3*c^6*d^9*e^5*z - 396*A*B*a^2*c^7
*d^11*e^3*z - 6*B*C*a^2*c^7*d^13*e*z + 13518*A*B*a^7*c^2*d*e^13*z + 67615*C
^2*a^6*c^3*d^6*e^8*z - 47538*C^2*a^7*c^2*d^4*e^10*z - 24860*C^2*a^5*c^4*d^8
*e^6*z + 279*C^2*a^4*c^5*d^10*e^4*z + 46*C^2*a^3*c^6*d^12*e^2*z + 71415*B^2
*a^6*c^3*d^4*e^10*z - 55260*B^2*a^5*c^4*d^6*e^8*z - 19602*B^2*a^7*c^2*d^2*e
^12*z + 1215*B^2*a^4*c^5*d^8*e^6*z + 270*B^2*a^3*c^6*d^10*e^4*z + 9*B^2*a^2
*c^7*d^12*e^2*z - 106722*A^2*a^5*c^4*d^4*e^10*z + 35217*A^2*a^6*c^3*d^2*e^1
2*z + 6615*A^2*a^4*c^5*d^6*e^8*z + 3780*A^2*a^3*c^6*d^8*e^6*z + 1071*A^2*a^
2*c^7*d^10*e^4*z + 1152*A*C*a^8*c*e^14*z + 6*A*C*a*c^8*d^14*z + 7017*C^2*a^
8*c*d^2*e^12*z + 126*A^2*a*c^8*d^12*e^2*z + C^2*a^2*c^7*d^14*z - 1728*A^2*a
^7*c^2*e^14*z + 225*B^2*a^8*c*e^14*z + 9*A^2*c^9*d^14*z - 192*C^2*a^9*e^14*
z + 3168*A*B*C*a^4*c^2*d*e^10 + 270*A*B*C*a*c^5*d^7*e^4 - 6930*A*B*C*a^3*c^
3*d^3*e^8 + 5148*A*B*C*a^2*c^4*d^5*e^6 - 819*A^2*C*a*c^5*d^6*e^5 - 60*A*C^2
*a*c^5*d^8*e^3 - 6102*A^2*B*a^3*c^3*d*e^10 + 1512*A^2*B*a*c^5*d^5*e^6 - 270
*A*B^2*a*c^5*d^6*e^5 - 378*B*C^2*a^5*c*d*e^10 - 5049*B^2*C*a^3*c^3*d^4*e^7
+ 4698*B^2*C*a^4*c^2*d^2*e^9 + 2508*B*C^2*a^3*c^3*d^5*e^6 - 1977*B*C^2*a^4*
c^2*d^3*e^8 - 180*B^2*C*a^2*c^4*d^6*e^5 + 75*B*C^2*a^2*c^4*d^7*e^4 + 15921*
A^2*C*a^3*c^3*d^2*e^9 - 7848*A^2*C*a^2*c^4*d^4*e^7 - 6363*A*C^2*a^4*c^2*d^2
*e^9 + 4926*A*C^2*a^3*c^3*d^4*e^7 - 1443*A*C^2*a^2*c^4*d^6*e^5 + 14283*A^2*
B*a^2*c^4*d^3*e^8 - 4617*A*B^2*a^2*c^4*d^4*e^7 - 1944*A*B^2*a^3*c^3*d^2*e^9
 + 791*C^3*a^5*c*d^2*e^9 - 2025*B^3*a^4*c^2*d*e^10 - 1674*A^3*a*c^5*d^4*e^7
 - 90*A^2*C*c^6*d^8*e^3 + 135*A^2*B*c^6*d^7*e^4 - 1728*A^2*C*a^4*c^2*e^11 +
 675*A*B^2*a^4*c^2*e^11 - 225*B^2*C*a^5*c*e^11 + 576*A*C^2*a^5*c*e^11 - 397
*C^3*a^3*c^3*d^6*e^5 - 108*C^3*a^4*c^2*d^4*e^7 - 10*C^3*a^2*c^4*d^8*e^3 + 3
294*B^3*a^3*c^3*d^3*e^8 + 135*B^3*a^2*c^4*d^5*e^6 - 11853*A^3*a^2*c^4*d^2*e
^9 - 189*A^3*c^6*d^6*e^5 + 1728*A^3*a^3*c^3*e^11 - 64*C^3*a^6*e^11, z, k),
k, 1, 3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.64 $\int \frac{(d+ex)^4(A+Bx+Cx^2)}{(a+cx^2)^4} dx$

Optimal. Leaf size=234

$$\frac{(ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(cd(4aBe+aCd+5Acd)+ae^2(5aC+Ac))}{16a^{7/2}c^{7/2}}-\frac{(d+ex)(ae-cdx)(cd(4aBe+aCd+5Acd)}{16a^3c^3(a+cx^2)}$$

```
[Out] -1/6*(a*B-(A*c-C*a)*x)*(e*x+d)^4/a/c/(c*x^2+a)^3-1/24*(e*x+d)^3*(a*(A*c+5*C
*a)*e-c*(5*A*c*d+4*B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)^2-1/16*(a*(A*c+5*C*a)*
e^2+c*d*(5*A*c*d+4*B*a*e+C*a*d))*(-c*d*x+a*e)*(e*x+d)/a^3/c^3/(c*x^2+a)+1/1
6*(a*e^2+c*d^2)*(a*(A*c+5*C*a)*e^2+c*d*(5*A*c*d+4*B*a*e+C*a*d))*arctan(x*c^
(1/2)/a^(1/2))/a^(7/2)/c^(7/2)
```

Rubi [A] time = 0.29, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1645, 805, 723, 205}

$$\frac{(ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(cd(4aBe+aCd+5Acd)+ae^2(5aC+Ac))}{16a^{7/2}c^{7/2}}-\frac{(d+ex)(ae-cdx)(cd(4aBe+aCd+5Acd)}{16a^3c^3(a+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^4)/(6*a*c*(a + c*x^2)^3) - ((d + e*x)^3*(
a*(A*c + 5*a*C)*e - c*(5*A*c*d + a*C*d + 4*a*B*e)*x))/(24*a^2*c^2*(a + c*x^
2)^2) - ((a*(A*c + 5*a*C)*e^2 + c*d*(5*A*c*d + a*C*d + 4*a*B*e))*(a*e - c*d
*x)*(d + e*x))/(16*a^3*c^3*(a + c*x^2)) + ((c*d^2 + a*e^2)*(a*(A*c + 5*a*C)
*e^2 + c*d*(5*A*c*d + a*C*d + 4*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^
(7/2)*c^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
```

```
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^4\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{\int \frac{(d+ex)^3(-5Acd-aCd-4aBe-(Ac+5aC)ex)}{\left(a+cx^2\right)^3}\,dx}{6ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+aCd+}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+aCd+}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+aCd+}{24a^2c^2\left(a+cx^2\right)^2}$$

Mathematica [A] time = 0.31, size = 437, normalized size = 1.87

$$\frac{\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(ae^2+5cd^2\right)+a\left(5aCe^2+cd(4Be+Cd)\right)\right)}{16a^{7/2}c^{7/2}} + \frac{-a^3e^3(8Be+32Cd+11Cex)+a^2ce^2x}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] (5*A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 4*B*d*e + 6*A*e^2)*x + a^2*c*e^2*(6*C*d
^2 + e*(4*B*d + A*e))*x - a^3*e^3*(32*C*d + 8*B*e + 11*C*e*x))/(16*a^3*c^3*
(a + c*x^2)) + (A*c^3*d^4*x - a^3*e^3*(4*C*d + B*e + C*e*x) - a*c^2*d^2*(4*
A*d*e + C*d^2*x + 6*A*e^2*x + B*d*(d + 4*e*x)) + a^2*c*e*(2*C*d^2*(2*d + 3*
e*x) + e*(A*e*(4*d + e*x) + 2*B*d*(3*d + 2*e*x))))/(6*a*c^3*(a + c*x^2)^3)
+ (5*A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 4*B*d*e + 6*A*e^2)*x + a^3*e^3*(48*C*
d + 12*B*e + 13*C*e*x) - a^2*c*e*(6*C*d^2*(4*d + 7*e*x) + e*(4*B*d*(9*d + 7
*e*x) + A*e*(24*d + 7*e*x))))/(24*a^2*c^3*(a + c*x^2)^2) + ((c*d^2 + a*e^2)
*(A*c*(5*c*d^2 + a*e^2) + a*(5*a*C*e^2 + c*d*(C*d + 4*B*e)))*ArcTan[(Sqrt[c
]*x)/Sqrt[a]])/(16*a^(7/2)*c^(7/2))
```

fricas [B] time = 1.46, size = 1864, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c^3*d^4 + 48*B*a^5*c^2*d^2*e^2 + 16*B*a^6*c*e^4 - 6*(4*B*a
^2*c^5*d^3*e + 4*B*a^3*c^4*d*e^3 + (C*a^2*c^5 + 5*A*a*c^6)*d^4 + 6*(C*a^3*c
^4 + A*a^2*c^5)*d^2*e^2 - (11*C*a^4*c^3 - A*a^3*c^4)*e^4)*x^5 + 32*(C*a^5*c
```

```
^2 + 2*A*a^4*c^3)*d^3*e + 32*(2*C*a^6*c + A*a^5*c^2)*d*e^3 + 48*(4*C*a^4*c^
3*d*e^3 + B*a^4*c^3*e^4)*x^4 - 16*(4*B*a^3*c^4*d^3*e - 4*B*a^4*c^3*d*e^3 +
(C*a^3*c^4 + 5*A*a^2*c^5)*d^4 - 6*(C*a^4*c^3 - A*a^3*c^4)*d^2*e^2 - (5*C*a^
5*c^2 + A*a^4*c^3)*e^4)*x^3 + 48*(2*C*a^4*c^3*d^3*e + 3*B*a^4*c^3*d^2*e^2 +
 B*a^5*c^2*e^4 + 2*(2*C*a^5*c^2 + A*a^4*c^3)*d*e^3)*x^2 + 3*(4*B*a^4*c^2*d^
3*e + 4*B*a^5*c*d*e^3 + (4*B*a*c^5*d^3*e + 4*B*a^2*c^4*d*e^3 + (C*a*c^5 + 5
*A*c^6)*d^4 + 6*(C*a^2*c^4 + A*a*c^5)*d^2*e^2 + (5*C*a^3*c^3 + A*a^2*c^4)*e
^4)*x^6 + (C*a^4*c^2 + 5*A*a^3*c^3)*d^4 + 6*(C*a^5*c + A*a^4*c^2)*d^2*e^2 +
 (5*C*a^6 + A*a^5*c)*e^4 + 3*(4*B*a^2*c^4*d^3*e + 4*B*a^3*c^3*d*e^3 + (C*a^
2*c^4 + 5*A*a*c^5)*d^4 + 6*(C*a^3*c^3 + A*a^2*c^4)*d^2*e^2 + (5*C*a^4*c^2 +
 A*a^3*c^3)*e^4)*x^4 + 3*(4*B*a^3*c^3*d^3*e + 4*B*a^4*c^2*d*e^3 + (C*a^3*c^
3 + 5*A*a^2*c^4)*d^4 + 6*(C*a^4*c^2 + A*a^3*c^3)*d^2*e^2 + (5*C*a^5*c + A*a
^4*c^2)*e^4)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a))
+ 6*(4*B*a^4*c^3*d^3*e + 4*B*a^5*c^2*d*e^3 + (C*a^4*c^3 - 11*A*a^3*c^4)*d^4
 + 6*(C*a^5*c^2 + A*a^4*c^3)*d^2*e^2 + (5*C*a^6*c + A*a^5*c^2)*e^4)*x)/(a^4
*c^7*x^6 + 3*a^5*c^6*x^4 + 3*a^6*c^5*x^2 + a^7*c^4), -1/48*(8*B*a^4*c^3*d^4
 + 24*B*a^5*c^2*d^2*e^2 + 8*B*a^6*c*e^4 - 3*(4*B*a^2*c^5*d^3*e + 4*B*a^3*c^
4*d*e^3 + (C*a^2*c^5 + 5*A*a*c^6)*d^4 + 6*(C*a^3*c^4 + A*a^2*c^5)*d^2*e^2 -
 (11*C*a^4*c^3 - A*a^3*c^4)*e^4)*x^5 + 16*(C*a^5*c^2 + 2*A*a^4*c^3)*d^3*e +
 16*(2*C*a^6*c + A*a^5*c^2)*d*e^3 + 24*(4*C*a^4*c^3*d*e^3 + B*a^4*c^3*e^4)*
x^4 - 8*(4*B*a^3*c^4*d^3*e - 4*B*a^4*c^3*d*e^3 + (C*a^3*c^4 + 5*A*a^2*c^5)*
d^4 - 6*(C*a^4*c^3 - A*a^3*c^4)*d^2*e^2 - (5*C*a^5*c^2 + A*a^4*c^3)*e^4)*x^
3 + 24*(2*C*a^4*c^3*d^3*e + 3*B*a^4*c^3*d^2*e^2 + B*a^5*c^2*e^4 + 2*(2*C*a^
5*c^2 + A*a^4*c^3)*d*e^3)*x^2 - 3*(4*B*a^4*c^2*d^3*e + 4*B*a^5*c*d*e^3 + (4
*B*a*c^5*d^3*e + 4*B*a^2*c^4*d*e^3 + (C*a*c^5 + 5*A*c^6)*d^4 + 6*(C*a^2*c^4
 + A*a*c^5)*d^2*e^2 + (5*C*a^3*c^3 + A*a^2*c^4)*e^4)*x^6 + (C*a^4*c^2 + 5*A
*a^3*c^3)*d^4 + 6*(C*a^5*c + A*a^4*c^2)*d^2*e^2 + (5*C*a^6 + A*a^5*c)*e^4 +
 3*(4*B*a^2*c^4*d^3*e + 4*B*a^3*c^3*d*e^3 + (C*a^2*c^4 + 5*A*a*c^5)*d^4 + 6
*(C*a^3*c^3 + A*a^2*c^4)*d^2*e^2 + (5*C*a^4*c^2 + A*a^3*c^3)*e^4)*x^4 + 3*(
4*B*a^3*c^3*d^3*e + 4*B*a^4*c^2*d*e^3 + (C*a^3*c^3 + 5*A*a^2*c^4)*d^4 + 6*(
C*a^4*c^2 + A*a^3*c^3)*d^2*e^2 + (5*C*a^5*c + A*a^4*c^2)*e^4)*x^2)*sqrt(a*c
)*arctan(sqrt(a*c)*x/a) + 3*(4*B*a^4*c^3*d^3*e + 4*B*a^5*c^2*d*e^3 + (C*a^4
*c^3 - 11*A*a^3*c^4)*d^4 + 6*(C*a^5*c^2 + A*a^4*c^3)*d^2*e^2 + (5*C*a^6*c +
 A*a^5*c^2)*e^4)*x)/(a^4*c^7*x^6 + 3*a^5*c^6*x^4 + 3*a^6*c^5*x^2 + a^7*c^4)
]
```

giac [B] time = 0.18, size = 636, normalized size = 2.72

$$\frac{\left(Cac^2d^4 + 5\,Ac^3d^4 + 4\,Bac^2d^3e + 6\,Ca^2cd^2e^2 + 6\,Aac^2d^2e^2 + 4\,Ba^2cde^3 + 5\,Ca^3e^4 + Aa^2ce^4\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c^3} + \frac{3\,Ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*c^2*d^4 + 5*A*c^3*d^4 + 4*B*a*c^2*d^3*e + 6*C*a^2*c*d^2*e^2 + 6*A
*a*c^2*d^2*e^2 + 4*B*a^2*c*d*e^3 + 5*C*a^3*e^4 + A*a^2*c*e^4)*arctan(c*x/sq
rt(a*c))/(sqrt(a*c)*a^3*c^3) + 1/48*(3*C*a*c^4*d^4*x^5 + 15*A*c^5*d^4*x^5 +
 12*B*a*c^4*d^3*x^5*e + 18*C*a^2*c^3*d^2*x^5*e^2 + 18*A*a*c^4*d^2*x^5*e^2 +
 8*C*a^2*c^3*d^4*x^3 + 40*A*a*c^4*d^4*x^3 + 12*B*a^2*c^3*d*x^5*e^3 + 32*B*a
^2*c^3*d^3*x^3*e - 33*C*a^3*c^2*x^5*e^4 + 3*A*a^2*c^3*x^5*e^4 - 96*C*a^3*c^
2*d*x^4*e^3 - 48*C*a^3*c^2*d^2*x^3*e^2 + 48*A*a^2*c^3*d^2*x^3*e^2 - 48*C*a^
3*c^2*d^3*x^2*e - 3*C*a^3*c^2*d^4*x + 33*A*a^2*c^3*d^4*x - 24*B*a^3*c^2*x^4
*e^4 - 32*B*a^3*c^2*d*x^3*e^3 - 72*B*a^3*c^2*d^2*x^2*e^2 - 12*B*a^3*c^2*d^3
*x*e - 8*B*a^3*c^2*d^4 - 40*C*a^4*c*x^3*e^4 - 8*A*a^3*c^2*x^3*e^4 - 96*C*a^
4*c*d*x^2*e^3 - 48*A*a^3*c^2*d*x^2*e^3 - 18*C*a^4*c*d^2*x*e^2 - 18*A*a^3*c^
2*d^2*x*e^2 - 16*C*a^4*c*d^3*e - 32*A*a^3*c^2*d^3*e - 24*B*a^4*c*x^2*e^4 -
12*B*a^4*c*d*x*e^3 - 24*B*a^4*c*d^2*e^2 - 15*C*a^5*x*e^4 - 3*A*a^4*c*x*e^4
- 32*C*a^5*d*e^3 - 16*A*a^4*c*d*e^3 - 8*B*a^5*e^4)/((c*x^2 + a)^3*a^3*c^3)
```

maple [B] time = 0.01, size = 647, normalized size = 2.76

$$\frac{A\,e^{4}\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a\,c^{2}}+\frac{3A\,d^{2}e^{2}\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;a^{2}c}+\frac{5A\,d^{4}\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^{3}}+\frac{Bd\,e^{3}\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{4\sqrt{ac}\;a\,c^{2}}+\frac{B\,d^{3}e\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{4\sqrt{ac}\;a^{2}c}+\frac{3C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(A*a^2*c*e^4+6*A*a*c^2*d^2*e^2+5*A*c^3*d^4+4*B*a^2*c*d*e^3+4*B*a*c^2*
d^3*e-11*C*a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a^3/c*x^5-1/2*e^3*(B*e+4*
C*d)/c*x^4-1/6*(A*a^2*c*e^4-6*A*a*c^2*d^2*e^2-5*A*c^3*d^4+4*B*a^2*c*d*e^3-4
*B*a*c^2*d^3*e+5*C*a^3*e^4+6*C*a^2*c*d^2*e^2-C*a*c^2*d^4)/a^2/c^2*x^3-1/2*e
*(2*A*c*d*e^2+B*a*e^3+3*B*c*d^2*e+4*C*a*d*e^2+2*C*c*d^3)/c^2*x^2-1/16*(A*a^
2*c*e^4+6*A*a*c^2*d^2*e^2-11*A*c^3*d^4+4*B*a^2*c*d*e^3+4*B*a*c^2*d^3*e+5*C*
a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a/c^3*x-1/6*(2*A*a*c*d*e^3+4*A*c^2*d
^3*e+B*a^2*e^4+3*B*a*c*d^2*e^2+B*c^2*d^4+4*C*a^2*d*e^3+2*C*a*c*d^3*e)/c^3)/
(c*x^2+a)^3+1/16/a/c^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*e^4+3/8/a^2/
c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d^2*e^2+5/16/a^3/(a*c)^(1/2)*arct
an(1/(a*c)^(1/2)*c*x)*A*d^4+1/4/a/c^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)
*B*d*e^3+1/4/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*d^3*e+5/16/c^3/(
a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*e^4+3/8/a/c^2/(a*c)^(1/2)*arctan(1/(
a*c)^(1/2)*c*x)*C*d^2*e^2+1/16/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*
C*d^4
```

maxima [B] time = 1.04, size = 599, normalized size = 2.56

$$-\frac{8\,Ba^{3}c^{2}d^{4}+24\,Ba^{4}cd^{2}e^{2}+8\,Ba^{5}e^{4}-3\left(4\,Bac^{4}d^{3}e+4\,Ba^{2}c^{3}de^{3}+\left(Cac^{4}+5\,Ac^{5}\right)d^{4}+6\left(Ca^{2}c^{3}+Aac^{4}\right)d^{2}e^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(8*B*a^3*c^2*d^4 + 24*B*a^4*c*d^2*e^2 + 8*B*a^5*e^4 - 3*(4*B*a*c^4*d^
3*e + 4*B*a^2*c^3*d*e^3 + (C*a*c^4 + 5*A*c^5)*d^4 + 6*(C*a^2*c^3 + A*a*c^4)
*d^2*e^2 - (11*C*a^3*c^2 - A*a^2*c^3)*e^4)*x^5 + 16*(C*a^4*c + 2*A*a^3*c^2)
*d^3*e + 16*(2*C*a^5 + A*a^4*c)*d*e^3 + 24*(4*C*a^3*c^2*d*e^3 + B*a^3*c^2*e
^4)*x^4 - 8*(4*B*a^2*c^3*d^3*e - 4*B*a^3*c^2*d*e^3 + (C*a^2*c^3 + 5*A*a*c^4
)*d^4 - 6*(C*a^3*c^2 - A*a^2*c^3)*d^2*e^2 - (5*C*a^4*c + A*a^3*c^2)*e^4)*x^
3 + 24*(2*C*a^3*c^2*d^3*e + 3*B*a^3*c^2*d^2*e^2 + B*a^4*c*e^4 + 2*(2*C*a^4*
c + A*a^3*c^2)*d*e^3)*x^2 + 3*(4*B*a^3*c^2*d^3*e + 4*B*a^4*c*d*e^3 + (C*a^3
*c^2 - 11*A*a^2*c^3)*d^4 + 6*(C*a^4*c + A*a^3*c^2)*d^2*e^2 + (5*C*a^5 + A*a
^4*c)*e^4)*x)/(a^3*c^6*x^6 + 3*a^4*c^5*x^4 + 3*a^5*c^4*x^2 + a^6*c^3) + 1/1
6*(4*B*a*c^2*d^3*e + 4*B*a^2*c*d*e^3 + (C*a*c^2 + 5*A*c^3)*d^4 + 6*(C*a^2*c
 + A*a*c^2)*d^2*e^2 + (5*C*a^3 + A*a^2*c)*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(
a*c)*a^3*c^3)
```

mupad [B] time = 4.38, size = 669, normalized size = 2.86

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x\,\left(c\,d^2+a\,e^2\right)\,\left(5\,C\,a^2\,e^2+C\,a\,c\,d^2+4\,B\,a\,c\,d\,e+A\,a\,c\,e^2+5\,A\,c^2\,d^2\right)}{\sqrt{a}\,\left(5\,C\,a^3\,e^4+6\,C\,a^2\,c\,d^2\,e^2+4\,B\,a^2\,c\,d\,e^3+A\,a^2\,c\,e^4+C\,a\,c^2\,d^4+4\,B\,a\,c^2\,d^3\,e+6\,A\,a\,c^2\,d^2\,e^2+5\,A\,c^3\,d^4\right)}\right)\,\left(c\,d^2+a\,e^2\right)\,\left(5\,C\,a^2\,e^2+C\right.}{16\,a^{7/2}\,c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

```
[Out] (atan((c^(1/2)*x*(a*e^2 + c*d^2)*(5*A*c^2*d^2 + 5*C*a^2*e^2 + A*a*c*e^2 + C
*a*c*d^2 + 4*B*a*c*d*e))/(a^(1/2)*(5*A*c^3*d^4 + 5*C*a^3*e^4 + A*a^2*c*e^4
```

```
+ C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 + 6*C*a^2*c*d^2*e^2 + 4*B*a*c^2*d^3*e + 4
*B*a^2*c*d*e^3)))*(a*e^2 + c*d^2)*(5*A*c^2*d^2 + 5*C*a^2*e^2 + A*a*c*e^2 +
C*a*c*d^2 + 4*B*a*c*d*e))/(16*a^(7/2)*c^(7/2)) - ((B*a^2*e^4 + B*c^2*d^4 +
4*A*c^2*d^3*e + 4*C*a^2*d*e^3 + 2*A*a*c*d*e^3 + 2*C*a*c*d^3*e + 3*B*a*c*d^2
*e^2)/(6*c^3) + (x^2*(B*a*e^4 + 2*A*c*d*e^3 + 4*C*a*d*e^3 + 2*C*c*d^3*e + 3
*B*c*d^2*e^2))/(2*c^2) + (x^4*(B*e^4 + 4*C*d*e^3))/(2*c) + (x*(5*C*a^3*e^4
- 11*A*c^3*d^4 + A*a^2*c*e^4 + C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 + 6*C*a^2*c*
d^2*e^2 + 4*B*a*c^2*d^3*e + 4*B*a^2*c*d*e^3))/(16*a*c^3) - (x^3*(5*A*c^3*d^
4 - 5*C*a^3*e^4 - A*a^2*c*e^4 + C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 - 6*C*a^2*c
*d^2*e^2 + 4*B*a*c^2*d^3*e - 4*B*a^2*c*d*e^3))/(6*a^2*c^2) - (x^5*(5*A*c^3*
d^4 - 11*C*a^3*e^4 + A*a^2*c*e^4 + C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 + 6*C*a^
2*c*d^2*e^2 + 4*B*a*c^2*d^3*e + 4*B*a^2*c*d*e^3))/(16*a^3*c))/(a^3 + c^3*x^
6 + 3*a^2*c*x^2 + 3*a*c^2*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

3.65 $\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx$

Optimal. Leaf size=254

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+5cd^2\right)+a\left(ae^2(Be+3Cd)+cd^2(3Be+Cd)\right)\right)}{16a^{7/2}c^{5/2}}-\frac{(d+ex)\left(ae(3aBe+aCd+5Acd)-x\right.}{48\ldots}$$

```
[Out] -1/6*(a*B-(A*c-C*a)*x)*(e*x+d)^3/a/c/(c*x^2+a)^3-1/24*(e*x+d)^2*(2*a*(A*c+2
*C*a)*e-c*(5*A*c*d+3*B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)^2-1/48*(e*x+d)*(a*e*
(5*A*c*d+3*B*a*e+C*a*d)-(4*a*(A*c+2*C*a)*e^2+3*c*d*(5*A*c*d+3*B*a*e+C*a*d))
*x)/a^3/c^2/(c*x^2+a)+1/16*(A*c*d*(3*a*e^2+5*c*d^2)+a*(a*e^2*(B*e+3*C*d)+c*
d^2*(3*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/a^(7/2)/c^(5/2)
```

Rubi [A] time = 0.54, antiderivative size = 288, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1645, 821, 778, 205}

$$-\frac{4ae\left(Ac\left(ae^2+5cd^2\right)+a\left(2aCe^2+cd(3Be+Cd)\right)\right)-cx\left(Acd\left(15cd^2-ae^2\right)+a\left(ae^2(7Cd-3Be)+3cd^2(3Be\ldots\right.\right.}{48a^3c^3\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(6*a*c*(a + c*x^2)^3) - ((d + e*x)^2*(
2*a*(A*c + 2*a*C)*e - c*(5*A*c*d + a*C*d + 3*a*B*e)*x))/(24*a^2*c^2*(a + c*
x^2)^2) - (4*a*e*(A*c*(5*c*d^2 + a*e^2) + a*(2*a*C*e^2 + c*d*(C*d + 3*B*e))
) - c*(A*c*d*(15*c*d^2 - a*e^2) + a*(a*e^2*(7*C*d - 3*B*e) + 3*c*d^2*(C*d +
 3*B*e)))*x)/(48*a^3*c^3*(a + c*x^2)) + ((A*c*d*(5*c*d^2 + 3*a*e^2) + a*(a*
e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*
a^(7/2)*c^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
```

```
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}dx = -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3} - \frac{\int \frac{(d+ex)^2(-5Acd-aCd-3aBe-2(Ac+2aC)ex)}{\left(a+cx^2\right)^3}dx}{6ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd+aCd+3a}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd+aCd+3a}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd+aCd+3a}{24a^2c^2\left(a+cx^2\right)^2}$$

Mathematica [A] time = 0.30, size = 350, normalized size = 1.38

$$-\frac{\frac{3\sqrt{a}\left(8a^3Ce^3-a^2ce^2x(Be+3Cd)-ac^2dx\left(3e(Ae+Bd)+Cd^2\right)-5Ac^3d^3x\right)}{a+cx^2} - \frac{8a^{5/2}\left(a^3Ce^3-a^2ce(e(Ae+3Bd+Bex)+3Cd(d+ex))+ac^2d\left(3Ae(d+ex)+Bd(d+3ex\right.\right.}{\left(a+cx^2\right)^3}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] ((-3*Sqrt[a]*(8*a^3*C*e^3 - 5*A*c^3*d^3*x - a^2*c*e^2*(3*C*d + B*e)*x - a*c
^2*d*(C*d^2 + 3*e*(B*d + A*e))*x))/(a + c*x^2) - (8*a^(5/2)*(a^3*C*e^3 - A*
c^3*d^3*x + a*c^2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d*(d + 3*e*x)) - a^2*c*e
*(3*C*d*(d + e*x) + e*(3*B*d + A*e + B*e*x))))/(a + c*x^2)^3 + (2*a^(3/2)*(
12*a^3*C*e^3 + 5*A*c^3*d^3*x + a*c^2*d*(C*d^2 + 3*e*(B*d + A*e))*x - a^2*c*
e*(3*C*d*(6*d + 7*e*x) + e*(18*B*d + 6*A*e + 7*B*e*x))))/(a + c*x^2)^2 + 3*
Sqrt[c]*(A*c*d*(5*c*d^2 + 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d +
3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(48*a^(7/2)*c^3)
```

fricas [B] time = 0.83, size = 1378, normalized size = 5.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(48*C*a^4*c^2*e^3*x^4 + 16*B*a^4*c^2*d^3 + 24*B*a^5*c*d*e^2 - 6*(3*B
*a^2*c^4*d^2*e + B*a^3*c^3*e^3 + (C*a^2*c^4 + 5*A*a*c^5)*d^3 + 3*(C*a^3*c^3
 + A*a^2*c^4)*d*e^2)*x^5 + 24*(C*a^5*c + 2*A*a^4*c^2)*d^2*e + 8*(2*C*a^6 +
A*a^5*c)*e^3 - 16*(3*B*a^3*c^3*d^2*e - B*a^4*c^2*e^3 + (C*a^3*c^3 + 5*A*a^2
*c^4)*d^3 - 3*(C*a^4*c^2 - A*a^3*c^3)*d*e^2)*x^3 + 24*(3*C*a^4*c^2*d^2*e +
```

```
3*B*a^4*c^2*d*e^2 + (2*C*a^5*c + A*a^4*c^2)*e^3)*x^2 + 3*(3*B*a^4*c*d^2*e +
 B*a^5*e^3 + (3*B*a*c^4*d^2*e + B*a^2*c^3*e^3 + (C*a*c^4 + 5*A*c^5)*d^3 + 3
*(C*a^2*c^3 + A*a*c^4)*d*e^2)*x^6 + 3*(3*B*a^2*c^3*d^2*e + B*a^3*c^2*e^3 +
(C*a^2*c^3 + 5*A*a*c^4)*d^3 + 3*(C*a^3*c^2 + A*a^2*c^3)*d*e^2)*x^4 + (C*a^4
*c + 5*A*a^3*c^2)*d^3 + 3*(C*a^5 + A*a^4*c)*d*e^2 + 3*(3*B*a^3*c^2*d^2*e +
B*a^4*c*e^3 + (C*a^3*c^2 + 5*A*a^2*c^3)*d^3 + 3*(C*a^4*c + A*a^3*c^2)*d*e^2
)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(3*B*a^
4*c^2*d^2*e + B*a^5*c*e^3 + (C*a^4*c^2 - 11*A*a^3*c^3)*d^3 + 3*(C*a^5*c + A
*a^4*c^2)*d*e^2)*x)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3)
, -1/48*(24*C*a^4*c^2*e^3*x^4 + 8*B*a^4*c^2*d^3 + 12*B*a^5*c*d*e^2 - 3*(3*B
*a^2*c^4*d^2*e + B*a^3*c^3*e^3 + (C*a^2*c^4 + 5*A*a*c^5)*d^3 + 3*(C*a^3*c^3
 + A*a^2*c^4)*d*e^2)*x^5 + 12*(C*a^5*c + 2*A*a^4*c^2)*d^2*e + 4*(2*C*a^6 +
A*a^5*c)*e^3 - 8*(3*B*a^3*c^3*d^2*e - B*a^4*c^2*e^3 + (C*a^3*c^3 + 5*A*a^2*
c^4)*d^3 - 3*(C*a^4*c^2 - A*a^3*c^3)*d*e^2)*x^3 + 12*(3*C*a^4*c^2*d^2*e + 3
*B*a^4*c^2*d*e^2 + (2*C*a^5*c + A*a^4*c^2)*e^3)*x^2 - 3*(3*B*a^4*c*d^2*e +
B*a^5*e^3 + (3*B*a*c^4*d^2*e + B*a^2*c^3*e^3 + (C*a*c^4 + 5*A*c^5)*d^3 + 3*
(C*a^2*c^3 + A*a*c^4)*d*e^2)*x^6 + 3*(3*B*a^2*c^3*d^2*e + B*a^3*c^2*e^3 + (
C*a^2*c^3 + 5*A*a*c^4)*d^3 + 3*(C*a^3*c^2 + A*a^2*c^3)*d*e^2)*x^4 + (C*a^4*
c + 5*A*a^3*c^2)*d^3 + 3*(C*a^5 + A*a^4*c)*d*e^2 + 3*(3*B*a^3*c^2*d^2*e + B
*a^4*c*e^3 + (C*a^3*c^2 + 5*A*a^2*c^3)*d^3 + 3*(C*a^4*c + A*a^3*c^2)*d*e^2)
*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(3*B*a^4*c^2*d^2*e + B*a^5*c*e^3
+ (C*a^4*c^2 - 11*A*a^3*c^3)*d^3 + 3*(C*a^5*c + A*a^4*c^2)*d*e^2)*x)/(a^4*c
^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3)]
```

giac [B] time = 0.17, size = 475, normalized size = 1.87

$$\frac{\left(Cacd^3 + 5\,Ac^2d^3 + 3\,Bacd^2e + 3\,Ca^2de^2 + 3\,Aacde^2 + Ba^2e^3\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c^2} + \frac{3\,Cac^4d^3x^5 + 15\,Ac^5d^3x^5 + 9\,Ba\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*c*d^3 + 5*A*c^2*d^3 + 3*B*a*c*d^2*e + 3*C*a^2*d*e^2 + 3*A*a*c*d*e
^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2) + 1/48*(3*C*a*c^4
*d^3*x^5 + 15*A*c^5*d^3*x^5 + 9*B*a*c^4*d^2*x^5*e + 9*C*a^2*c^3*d*x^5*e^2 +
 9*A*a*c^4*d*x^5*e^2 + 8*C*a^2*c^3*d^3*x^3 + 40*A*a*c^4*d^3*x^3 + 3*B*a^2*c
^3*x^5*e^3 + 24*B*a^2*c^3*d^2*x^3*e - 24*C*a^3*c^2*x^4*e^3 - 24*C*a^3*c^2*d
*x^3*e^2 + 24*A*a^2*c^3*d*x^3*e^2 - 36*C*a^3*c^2*d^2*x^2*e - 3*C*a^3*c^2*d^
3*x + 33*A*a^2*c^3*d^3*x - 8*B*a^3*c^2*x^3*e^3 - 36*B*a^3*c^2*d*x^2*e^2 - 9
*B*a^3*c^2*d^2*x*e - 8*B*a^3*c^2*d^3 - 24*C*a^4*c*x^2*e^3 - 12*A*a^3*c^2*x^
2*e^3 - 9*C*a^4*c*d*x*e^2 - 9*A*a^3*c^2*d*x*e^2 - 12*C*a^4*c*d^2*e - 24*A*a
^3*c^2*d^2*e - 3*B*a^4*c*x*e^3 - 12*B*a^4*c*d*e^2 - 8*C*a^5*e^3 - 4*A*a^4*c
*e^3)/((c*x^2 + a)^3*a^3*c^3)
```

maple [A] time = 0.01, size = 464, normalized size = 1.83

$$\frac{3Ad\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\ a^2c} + \frac{5A\,d^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\ a^3} + \frac{B\,e^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\ a\,c^2} + \frac{3B\,d^2e\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\ a^2c} + \frac{3Cd\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\ a\,c^2} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(3*A*a*c*d*e^2+5*A*c^2*d^3+B*a^2*e^3+3*B*a*c*d^2*e+3*C*a^2*d*e^2+C*a*
c*d^3)/a^3*x^5-1/2*C/c*e^3*x^4+1/6*(3*A*a*c*d*e^2+5*A*c^2*d^3-B*a^2*e^3+3*B
*a*c*d^2*e-3*C*a^2*d*e^2+C*a*c*d^3)/a^2/c*x^3-1/4*e*(A*c*e^2+3*B*c*d*e+2*C*
a*e^2+3*C*c*d^2)/c^2*x^2-1/16*(3*A*a*c*d*e^2-11*A*c^2*d^3+B*a^2*e^3+3*B*a*c
```

```
*d^2*e+3*C*a^2*d*e^2+C*a*c*d^3)/a/c^2*x-1/12*(A*a*c*e^3+6*A*c^2*d^2*e+3*B*a
*c*d*e^2+2*B*c^2*d^3+2*C*a^2*e^3+3*C*a*c*d^2*e)/c^3)/(c*x^2+a)^3+3/16/a^2/c
/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d*e^2+5/16/a^3/(a*c)^(1/2)*arctan(
1/(a*c)^(1/2)*c*x)*A*d^3+1/16/a/c^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B
*e^3+3/16/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*d^2*e+3/16/a/c^2/(a
*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d*e^2+1/16/a^2/c/(a*c)^(1/2)*arctan(1
/(a*c)^(1/2)*c*x)*C*d^3
```

maxima [A] time = 1.02, size = 457, normalized size = 1.80

$$-\frac{24\,Ca^3c^2e^3x^4+8\,Ba^3c^2d^3+12\,Ba^4cde^2-3\left(3\,Bac^4d^2e+Ba^2c^3e^3+\left(Cac^4+5\,Ac^5\right)d^3+3\left(Ca^2c^3+Aac^4\right)de^2\right)x^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(24*C*a^3*c^2*e^3*x^4 + 8*B*a^3*c^2*d^3 + 12*B*a^4*c*d*e^2 - 3*(3*B*a
*c^4*d^2*e + B*a^2*c^3*e^3 + (C*a*c^4 + 5*A*c^5)*d^3 + 3*(C*a^2*c^3 + A*a*c
^4)*d*e^2)*x^5 + 12*(C*a^4*c + 2*A*a^3*c^2)*d^2*e + 4*(2*C*a^5 + A*a^4*c)*e
^3 - 8*(3*B*a^2*c^3*d^2*e - B*a^3*c^2*e^3 + (C*a^2*c^3 + 5*A*a*c^4)*d^3 - 3
*(C*a^3*c^2 - A*a^2*c^3)*d*e^2)*x^3 + 12*(3*C*a^3*c^2*d^2*e + 3*B*a^3*c^2*d
*e^2 + (2*C*a^4*c + A*a^3*c^2)*e^3)*x^2 + 3*(3*B*a^3*c^2*d^2*e + B*a^4*c*e^
3 + (C*a^3*c^2 - 11*A*a^2*c^3)*d^3 + 3*(C*a^4*c + A*a^3*c^2)*d*e^2)*x)/(a^3
*c^6*x^6 + 3*a^4*c^5*x^4 + 3*a^5*c^4*x^2 + a^6*c^3) + 1/16*(3*B*a*c*d^2*e +
 B*a^2*e^3 + (C*a*c + 5*A*c^2)*d^3 + 3*(C*a^2 + A*a*c)*d*e^2)*arctan(c*x/sq
rt(a*c))/(sqrt(a*c)*a^3*c^2)
```

mupad [B] time = 4.07, size = 402, normalized size = 1.58

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(3\,C\,a^2\,d\,e^2+B\,a^2\,e^3+C\,a\,c\,d^3+3\,B\,a\,c\,d^2\,e+3\,A\,a\,c\,d\,e^2+5\,A\,c^2\,d^3\right)}{16\,a^{7/2}\,c^{5/2}}-\frac{\frac{2\,C\,a^2\,e^3+3\,C\,a\,c\,d^2\,e+3\,B\,a\,c\,d\,e^2+A\,a\,c}{12\,c^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(5*A*c^2*d^3 + B*a^2*e^3 + C*a*c*d^3 + 3*C*a^2*d
*e^2 + 3*A*a*c*d*e^2 + 3*B*a*c*d^2*e))/(16*a^(7/2)*c^(5/2)) - ((2*B*c^2*d^3
 + 2*C*a^2*e^3 + A*a*c*e^3 + 6*A*c^2*d^2*e + 3*B*a*c*d*e^2 + 3*C*a*c*d^2*e)
/(12*c^3) + (x^2*(A*c*e^3 + 2*C*a*e^3 + 3*B*c*d*e^2 + 3*C*c*d^2*e))/(4*c^2)
 - (x^5*(5*A*c^2*d^3 + B*a^2*e^3 + C*a*c*d^3 + 3*C*a^2*d*e^2 + 3*A*a*c*d*e^
2 + 3*B*a*c*d^2*e))/(16*a^3) + (C*e^3*x^4)/(2*c) - (x^3*(5*A*c^2*d^3 - B*a^
2*e^3 + C*a*c*d^3 - 3*C*a^2*d*e^2 + 3*A*a*c*d*e^2 + 3*B*a*c*d^2*e))/(6*a^2*
c) + (x*(B*a^2*e^3 - 11*A*c^2*d^3 + C*a*c*d^3 + 3*C*a^2*d*e^2 + 3*A*a*c*d*e
^2 + 3*B*a*c*d^2*e))/(16*a*c^2))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

3.66 $\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4} dx$

Optimal. Leaf size=225

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(cd(2aBe+aCd+5Acd)+ae^2(aC+Ac)\right)}{16a^{7/2}c^{5/2}}+\frac{x\left(cd(2aBe+aCd+5Acd)+ae^2(aC+Ac)\right)}{16a^3c^2\left(a+cx^2\right)}-\frac{x\left(3ae^2\right.}{}$$

```
[Out] -1/6*(a*B-(A*c-C*a)*x)*(e*x+d)^2/a/c/(c*x^2+a)^3+1/24*(-2*a*e*(4*A*c*d+B*a*
e+2*C*a*d)-(3*a*(A*c+C*a)*e^2-c*d*(5*A*c*d+2*B*a*e+C*a*d))*x)/a^2/c^2/(c*x^
2+a)^2+1/16*(a*(A*c+C*a)*e^2+c*d*(5*A*c*d+2*B*a*e+C*a*d))*x/a^3/c^2/(c*x^2+
a)+1/16*(a*(A*c+C*a)*e^2+c*d*(5*A*c*d+2*B*a*e+C*a*d))*arctan(x*c^(1/2)/a^(1
/2))/a^(7/2)/c^(5/2)
```

Rubi [A] time = 0.40, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1645, 778, 199, 205}

$$\frac{x\left(cd(2aBe+aCd+5Acd)+ae^2(aC+Ac)\right)}{16a^3c^2\left(a+cx^2\right)}-\frac{x\left(3ae^2(aC+Ac)-cd(2aBe+aCd+5Acd)\right)+2ae(aBe+2aCd}{24a^2c^2\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(6*a*c*(a + c*x^2)^3) - (2*a*e*(4*A*c*
d + 2*a*C*d + a*B*e) + (3*a*(A*c + a*C)*e^2 - c*d*(5*A*c*d + a*C*d + 2*a*B*
e))*x)/(24*a^2*c^2*(a + c*x^2)^2) + ((a*(A*c + a*C)*e^2 + c*d*(5*A*c*d + a*
C*d + 2*a*B*e))*x)/(16*a^3*c^2*(a + c*x^2)) + ((a*(A*c + a*C)*e^2 + c*d*(5*
A*c*d + a*C*d + 2*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(5/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
```

```
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2 \left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3} - \frac{\int \frac{(d+ex)(-5Acd-aCd-2aBe-3(Ac+aC)ex)}{\left(a+cx^2\right)^3}\,dx}{6ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3} - \frac{2ae(4Acd+2aCd+aBe)+\left(3a(Ac+aC)e^2-\right.}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3} - \frac{2ae(4Acd+2aCd+aBe)+\left(3a(Ac+aC)e^2-\right.}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3} - \frac{2ae(4Acd+2aCd+aBe)+\left(3a(Ac+aC)e^2-\right.}{24a^2c^2\left(a+cx^2\right)^2}$$

Mathematica [A] time = 0.16, size = 266, normalized size = 1.18

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(ae^2+5cd^2\right)+a\left(aCe^2+cd(2Be+Cd)\right)\right)}{16a^{7/2}c^{5/2}}+\frac{x\left(Ac\left(ae^2+5cd^2\right)+a\left(aCe^2+cd(2Be+Cd)\right)\right)}{16a^3c^2\left(a+cx^2\right)}+\frac{a^2(\cdot}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] ((A*c*(5*c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x)/(16*a^3*c^2*(
a + c*x^2)) + (5*A*c^2*d^2*x + a*c*(C*d^2 + e*(2*B*d + A*e))*x - a^2*e*(12*
C*d + 6*B*e + 7*C*e*x))/(24*a^2*c^2*(a + c*x^2)^2) + (A*c^2*d^2*x + a^2*e*(
2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x)))/(
6*a*c^2*(a + c*x^2)^3) + ((A*c*(5*c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d +
2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(5/2))
```

fricas [B] time = 0.91, size = 1062, normalized size = 4.72

$$\left[-\frac{16\,Ba^4c^2d^2+8\,Ba^5ce^2-6\left(2\,Ba^2c^4de+\left(Ca^2c^4+5\,Aac^5\right)d^2+\left(Ca^3c^3+Aa^2c^4\right)e^2\right)x^5-16\left(2\,Ba^3c^3de+\left(Ca^3c^3\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c^2*d^2 + 8*B*a^5*c*e^2 - 6*(2*B*a^2*c^4*d*e + (C*a^2*c^4
+ 5*A*a*c^5)*d^2 + (C*a^3*c^3 + A*a^2*c^4)*e^2)*x^5 - 16*(2*B*a^3*c^3*d*e +
 (C*a^3*c^3 + 5*A*a^2*c^4)*d^2 - (C*a^4*c^2 - A*a^3*c^3)*e^2)*x^3 + 16*(C*a
^5*c + 2*A*a^4*c^2)*d*e + 24*(2*C*a^4*c^2*d*e + B*a^4*c^2*e^2)*x^2 + 3*(2*B
*a^4*c*d*e + (2*B*a*c^4*d*e + (C*a*c^4 + 5*A*c^5)*d^2 + (C*a^2*c^3 + A*a*c^
4)*e^2)*x^6 + 3*(2*B*a^2*c^3*d*e + (C*a^2*c^3 + 5*A*a*c^4)*d^2 + (C*a^3*c^2
 + A*a^2*c^3)*e^2)*x^4 + (C*a^4*c + 5*A*a^3*c^2)*d^2 + (C*a^5 + A*a^4*c)*e^
```

```
2 + 3*(2*B*a^3*c^2*d*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d^2 + (C*a^4*c + A*a^3*c
^2)*e^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*
(2*B*a^4*c^2*d*e + (C*a^4*c^2 - 11*A*a^3*c^3)*d^2 + (C*a^5*c + A*a^4*c^2)*e
^2)*x)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3), -1/48*(8*B*
a^4*c^2*d^2 + 4*B*a^5*c*e^2 - 3*(2*B*a^2*c^4*d*e + (C*a^2*c^4 + 5*A*a*c^5)*
d^2 + (C*a^3*c^3 + A*a^2*c^4)*e^2)*x^5 - 8*(2*B*a^3*c^3*d*e + (C*a^3*c^3 +
5*A*a^2*c^4)*d^2 - (C*a^4*c^2 - A*a^3*c^3)*e^2)*x^3 + 8*(C*a^5*c + 2*A*a^4*
c^2)*d*e + 12*(2*C*a^4*c^2*d*e + B*a^4*c^2*e^2)*x^2 - 3*(2*B*a^4*c*d*e + (2
*B*a*c^4*d*e + (C*a*c^4 + 5*A*c^5)*d^2 + (C*a^2*c^3 + A*a*c^4)*e^2)*x^6 + 3
*(2*B*a^2*c^3*d*e + (C*a^2*c^3 + 5*A*a*c^4)*d^2 + (C*a^3*c^2 + A*a^2*c^3)*e
^2)*x^4 + (C*a^4*c + 5*A*a^3*c^2)*d^2 + (C*a^5 + A*a^4*c)*e^2 + 3*(2*B*a^3*
c^2*d*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d^2 + (C*a^4*c + A*a^3*c^2)*e^2)*x^2)*s
qrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(2*B*a^4*c^2*d*e + (C*a^4*c^2 - 11*A*a^3
*c^3)*d^2 + (C*a^5*c + A*a^4*c^2)*e^2)*x)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*
a^6*c^4*x^2 + a^7*c^3)]
```

giac [A] time = 0.17, size = 328, normalized size = 1.46

$$\frac{\left(Cacd^2 + 5\,Ac^2d^2 + 2\,Bacde + Ca^2e^2 + Aace^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c^2} + \frac{3\,Cac^3d^2x^5 + 15\,Ac^4d^2x^5 + 6\,Bac^3dx^5e + 3\,Ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*c*d^2 + 5*A*c^2*d^2 + 2*B*a*c*d*e + C*a^2*e^2 + A*a*c*e^2)*arctan
(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2) + 1/48*(3*C*a*c^3*d^2*x^5 + 15*A*c^4*d^
2*x^5 + 6*B*a*c^3*d*x^5*e + 3*C*a^2*c^2*x^5*e^2 + 3*A*a*c^3*x^5*e^2 + 8*C*a
^2*c^2*d^2*x^3 + 40*A*a*c^3*d^2*x^3 + 16*B*a^2*c^2*d*x^3*e - 8*C*a^3*c*x^3*
e^2 + 8*A*a^2*c^2*x^3*e^2 - 24*C*a^3*c*d*x^2*e - 3*C*a^3*c*d^2*x + 33*A*a^2
*c^2*d^2*x - 12*B*a^3*c*x^2*e^2 - 6*B*a^3*c*d*x*e - 8*B*a^3*c*d^2 - 3*C*a^4
*x*e^2 - 3*A*a^3*c*x*e^2 - 8*C*a^4*d*e - 16*A*a^3*c*d*e - 4*B*a^4*e^2)/((c*
x^2 + a)^3*a^3*c^2)
```

maple [A] time = 0.01, size = 333, normalized size = 1.48

$$\frac{A\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^2c} + \frac{5A\,d^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^3} + \frac{Bde\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;a^2c} + \frac{C\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a\,c^2} + \frac{C\,d^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^2c} + \frac{(Aac\,e^2+5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(A*a*c*e^2+5*A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)/a^3*x^5+1/6*(
A*a*c*e^2+5*A*c^2*d^2+2*B*a*c*d*e-C*a^2*e^2+C*a*c*d^2)/a^2/c*x^3-1/4*(B*e+2
*C*d)/c*e*x^2-1/16*(A*a*c*e^2-11*A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)
/a/c^2*x-1/12*(4*A*c*d*e+B*a*e^2+2*B*c*d^2+2*C*a*d*e)/c^2)/(c*x^2+a)^3+1/16
/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*e^2+5/16/a^3/(a*c)^(1/2)*arc
tan(1/(a*c)^(1/2)*c*x)*A*d^2+1/8/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x
)*B*d*e+1/16/a/c^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*e^2+1/16/a^2/c/(
a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d^2
```

maxima [A] time = 1.01, size = 323, normalized size = 1.44

$$-\frac{8\,Ba^3cd^2 + 4\,Ba^4e^2 - 3\left(2\,Bac^3de + \left(Cac^3 + 5\,Ac^4\right)d^2 + \left(Ca^2c^2 + Aac^3\right)e^2\right)x^5 - 8\left(2\,Ba^2c^2de + \left(Ca^2c^2 + 5\,A\right.\right.}{48\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(8*B*a^3*c*d^2 + 4*B*a^4*e^2 - 3*(2*B*a*c^3*d*e + (C*a*c^3 + 5*A*c^4)
*d^2 + (C*a^2*c^2 + A*a*c^3)*e^2)*x^5 - 8*(2*B*a^2*c^2*d*e + (C*a^2*c^2 + 5
*A*a*c^3)*d^2 - (C*a^3*c - A*a^2*c^2)*e^2)*x^3 + 8*(C*a^4 + 2*A*a^3*c)*d*e
+ 12*(2*C*a^3*c*d*e + B*a^3*c*e^2)*x^2 + 3*(2*B*a^3*c*d*e + (C*a^3*c - 11*A
*a^2*c^2)*d^2 + (C*a^4 + A*a^3*c)*e^2)*x)/(a^3*c^5*x^6 + 3*a^4*c^4*x^4 + 3*
a^5*c^3*x^2 + a^6*c^2) + 1/16*(2*B*a*c*d*e + (C*a*c + 5*A*c^2)*d^2 + (C*a^2
 + A*a*c)*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2)
```

mupad [B] time = 0.23, size = 287, normalized size = 1.28

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2+5\,A\,c^2\,d^2\right)}{16\,a^{7/2}\,c^{5/2}}-\frac{\frac{B\,a\,e^2+2\,B\,c\,d^2+4\,A\,c\,d\,e+2\,C\,a\,d\,e}{12\,c^2}-\frac{x^5\left(C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(5*A*c^2*d^2 + C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^2
 + 2*B*a*c*d*e))/(16*a^(7/2)*c^(5/2)) - ((B*a*e^2 + 2*B*c*d^2 + 4*A*c*d*e +
 2*C*a*d*e)/(12*c^2) - (x^5*(5*A*c^2*d^2 + C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^
2 + 2*B*a*c*d*e))/(16*a^3) + (x^2*(B*e^2 + 2*C*d*e))/(4*c) + (x*(C*a^2*e^2
- 11*A*c^2*d^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(16*a*c^2) - (x^3*(5
*A*c^2*d^2 - C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(6*a^2*c))/(
a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

3.67 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4} dx$

Optimal. Leaf size=165

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+5Acd)}{16a^{7/2}c^{3/2}}+\frac{x(aBe+aCd+5Acd)}{16a^3c\left(a+cx^2\right)}-\frac{2ae(aC+2Ac)-cx(aBe+aCd+5Acd)}{24a^2c^2\left(a+cx^2\right)^2}-\frac{(d+ex)(}{6a}$$

```
[Out] -1/6*(a*B-(A*c-C*a)*x)*(e*x+d)/a/c/(c*x^2+a)^3+1/24*(-2*a*(2*A*c+C*a)*e+c*(
5*A*c*d+B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)^2+1/16*(5*A*c*d+B*a*e+C*a*d)*x/a^
3/c/(c*x^2+a)+1/16*(5*A*c*d+B*a*e+C*a*d)*arctan(x*c^(1/2)/a^(1/2))/a^(7/2)/
c^(3/2)
```

Rubi [A] time = 0.14, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1645, 639, 199, 205}

$$-\frac{2ae(aC+2Ac)-cx(aBe+aCd+5Acd)}{24a^2c^2\left(a+cx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+5Acd)}{16a^{7/2}c^{3/2}}+\frac{x(aBe+aCd+5Acd)}{16a^3c\left(a+cx^2\right)}-\frac{(d+ex}{6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x))/(6*a*c*(a + c*x^2)^3) - (2*a*(2*A*c + a*
C)*e - c*(5*A*c*d + a*C*d + a*B*e)*x)/(24*a^2*c^2*(a + c*x^2)^2) + ((5*A*c*
d + a*C*d + a*B*e)*x)/(16*a^3*c*(a + c*x^2)) + ((5*A*c*d + a*C*d + a*B*e)*A
rcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
```

```
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3} - \frac{\int \frac{-5Acd-a(Cd+Be)-2(2Ac+aC)ex}{\left(a+cx^2\right)^3}\,dx}{6ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3} - \frac{2a(2Ac+aC)e-c(5Acd+aCd+aBe)x}{24a^2c^2\left(a+cx^2\right)^2} + \frac{(5A\cdots}{\cdots}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3} - \frac{2a(2Ac+aC)e-c(5Acd+aCd+aBe)x}{24a^2c^2\left(a+cx^2\right)^2} + \frac{(5A\cdots}{1\cdots}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3} - \frac{2a(2Ac+aC)e-c(5Acd+aCd+aBe)x}{24a^2c^2\left(a+cx^2\right)^2} + \frac{(5A\cdots}{1\cdots}$$

Mathematica [A] time = 0.13, size = 171, normalized size = 1.04

$$\frac{\frac{8a^{5/2}\left(a^2Ce-ac(Ae+B(d+ex)+Cdx)+Ac^2dx\right)}{\left(a+cx^2\right)^3}+\frac{2a^{3/2}\left(-6a^2Ce+acx(Be+Cd)+5Ac^2dx\right)}{\left(a+cx^2\right)^2}+\frac{3\sqrt{a}\,cx(aBe+aCd+5Acd)}{a+cx^2}+3\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+\cdots}{48a^{7/2}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] ((2*a^(3/2)*(-6*a^2*C*e + 5*A*c^2*d*x + a*c*(C*d + B*e)*x))/(a + c*x^2)^2 +
 (3*Sqrt[a]*c*(5*A*c*d + a*C*d + a*B*e)*x)/(a + c*x^2) + (8*a^(5/2)*(a^2*C*
e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x))))/(a + c*x^2)^3 + 3*Sqrt[c]
*(5*A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(48*a^(7/2)*c^2)
```

fricas [B] time = 1.20, size = 636, normalized size = 3.85

$$\left[-\frac{24\,Ca^4cex^2+16\,Ba^4cd-6\left(Ba^2c^3e+\left(Ca^2c^3+5\,Aac^4\right)d\right)x^5-16\left(Ba^3c^2e+\left(Ca^3c^2+5\,Aa^2c^3\right)d\right)x^3+3\left(\left(Bac^3e\cdots\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(24*C*a^4*c*e*x^2 + 16*B*a^4*c*d - 6*(B*a^2*c^3*e + (C*a^2*c^3 + 5*A
*a*c^4)*d)*x^5 - 16*(B*a^3*c^2*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d)*x^3 + 3*((B
*a*c^3*e + (C*a*c^3 + 5*A*c^4)*d)*x^6 + B*a^4*e + 3*(B*a^2*c^2*e + (C*a^2*c
^2 + 5*A*a*c^3)*d)*x^4 + 3*(B*a^3*c*e + (C*a^3*c + 5*A*a^2*c^2)*d)*x^2 + (C
*a^4 + 5*A*a^3*c)*d)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a
)) + 8*(C*a^5 + 2*A*a^4*c)*e + 6*(B*a^4*c*e + (C*a^4*c - 11*A*a^3*c^2)*d)*x
)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2), -1/48*(12*C*a^4*
c*e*x^2 + 8*B*a^4*c*d - 3*(B*a^2*c^3*e + (C*a^2*c^3 + 5*A*a*c^4)*d)*x^5 - 8
*(B*a^3*c^2*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d)*x^3 - 3*((B*a*c^3*e + (C*a*c^3
 + 5*A*c^4)*d)*x^6 + B*a^4*e + 3*(B*a^2*c^2*e + (C*a^2*c^2 + 5*A*a*c^3)*d)*
x^4 + 3*(B*a^3*c*e + (C*a^3*c + 5*A*a^2*c^2)*d)*x^2 + (C*a^4 + 5*A*a^3*c)*d
```

```
)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 4*(C*a^5 + 2*A*a^4*c)*e + 3*(B*a^4*c*e
+ (C*a^4*c - 11*A*a^3*c^2)*d)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x
^2 + a^7*c^2)]
```

giac [A] time = 0.16, size = 194, normalized size = 1.18

$$\frac{(Cad + 5\,Acd + Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c} + \frac{3\,Cac^3dx^5 + 15\,Ac^4dx^5 + 3\,Bac^3x^5e + 8\,Ca^2c^2dx^3 + 40\,Aac^3dx^3 + 8\,Ba^2c}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*d + 5*A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c) + 1/
48*(3*C*a*c^3*d*x^5 + 15*A*c^4*d*x^5 + 3*B*a*c^3*x^5*e + 8*C*a^2*c^2*d*x^3
+ 40*A*a*c^3*d*x^3 + 8*B*a^2*c^2*x^3*e - 12*C*a^3*c*x^2*e - 3*C*a^3*c*d*x +
 33*A*a^2*c^2*d*x - 3*B*a^3*c*x*e - 8*B*a^3*c*d - 4*C*a^4*e - 8*A*a^3*c*e)/
((c*x^2 + a)^3*a^3*c^2)
```

maple [A] time = 0.01, size = 182, normalized size = 1.10

$$\frac{5Ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^3} + \frac{Be\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^2c} + \frac{Cd\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^2c} + \frac{-\frac{Ce\,x^2}{4c} + \frac{(5Acd+Bae+Cad)c\,x^5}{16a^3} + \frac{(5Acd+Bae+Cad)x^3}{6a^2} + \frac{(11Acd}{}}{\left(c\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(5*A*c*d+B*a*e+C*a*d)/a^3*c*x^5+1/6/a^2*(5*A*c*d+B*a*e+C*a*d)*x^3-1/4
*C/c*e*x^2+1/16*(11*A*c*d-B*a*e-C*a*d)/a/c*x-1/12*(2*A*c*e+2*B*c*d+C*a*e)/c
^2)/(c*x^2+a)^3+5/16/a^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d+1/16/a^2
/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*e+1/16/a^2/c/(a*c)^(1/2)*arctan(
1/(a*c)^(1/2)*c*x)*C*d
```

maxima [A] time = 0.98, size = 208, normalized size = 1.26

$$-\frac{12\,Ca^3cex^2 + 8\,Ba^3cd - 3\left(Bac^3e + \left(Cac^3 + 5\,Ac^4\right)d\right)x^5 - 8\left(Ba^2c^2e + \left(Ca^2c^2 + 5\,Aac^3\right)d\right)x^3 + 4\left(Ca^4 + 2\,Aa\right.}{48\left(a^3c^5x^6 + 3\,a^4c^4x^4 + 3\,a^5c^3x^2 + a^6c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(12*C*a^3*c*e*x^2 + 8*B*a^3*c*d - 3*(B*a*c^3*e + (C*a*c^3 + 5*A*c^4)*
d)*x^5 - 8*(B*a^2*c^2*e + (C*a^2*c^2 + 5*A*a*c^3)*d)*x^3 + 4*(C*a^4 + 2*A*a
^3*c)*e + 3*(B*a^3*c*e + (C*a^3*c - 11*A*a^2*c^2)*d)*x)/(a^3*c^5*x^6 + 3*a^
4*c^4*x^4 + 3*a^5*c^3*x^2 + a^6*c^2) + 1/16*(B*a*e + (C*a + 5*A*c)*d)*arcta
n(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c)
```

mupad [B] time = 3.94, size = 164, normalized size = 0.99

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(5\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{16\,a^{7/2}\,c^{3/2}} - \frac{\frac{2\,A\,c\,e+2\,B\,c\,d+C\,a\,e}{12\,c^2} - \frac{x^3\,\left(5\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{6\,a^2} + \frac{C\,e\,x^2}{4\,c} + \frac{x\,\left(B\,a\,e-11\,A\,c\,d+C\,a\,d\right)}{16\,a\,c} - \frac{c\,x^5\,(5}{}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(5*A*c*d + B*a*e + C*a*d))/(16*a^(7/2)*c^(3/2))
- ((2*A*c*e + 2*B*c*d + C*a*e)/(12*c^2) - (x^3*(5*A*c*d + B*a*e + C*a*d))/(
6*a^2) + (C*e*x^2)/(4*c) + (x*(B*a*e - 11*A*c*d + C*a*d))/(16*a*c) - (c*x^5
*(5*A*c*d + B*a*e + C*a*d))/(16*a^3))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^
2*x^4)
```

sympy [A] time = 139.97, size = 298, normalized size = 1.81

$$-\frac{\sqrt{-\frac{1}{a^7c^3}}\,(5Acd+Bae+Cad)\log\left(-a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^7c^3}}\,(5Acd+Bae+Cad)\log\left(a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32}+\frac{-8A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] -sqrt(-1/(a**7*c**3))*(5*A*c*d + B*a*e + C*a*d)*log(-a**4*c*sqrt(-1/(a**7*c
**3)) + x)/32 + sqrt(-1/(a**7*c**3))*(5*A*c*d + B*a*e + C*a*d)*log(a**4*c*s
qrt(-1/(a**7*c**3)) + x)/32 + (-8*A*a**3*c*e - 8*B*a**3*c*d - 4*C*a**4*e -
12*C*a**3*c*e*x**2 + x**5*(15*A*c**4*d + 3*B*a*c**3*e + 3*C*a*c**3*d) + x**
3*(40*A*a*c**3*d + 8*B*a**2*c**2*e + 8*C*a**2*c**2*d) + x*(33*A*a**2*c**2*d
 - 3*B*a**3*c*e - 3*C*a**3*c*d))/(48*a**6*c**2 + 144*a**5*c**3*x**2 + 144*a
**4*c**4*x**4 + 48*a**3*c**5*x**6)
```

3.68 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^4} dx$

Optimal. Leaf size=126

$$\frac{(aC+5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}+\frac{x(aC+5Ac)}{16a^3c\left(a+cx^2\right)}+\frac{x(aC+5Ac)}{24a^2c\left(a+cx^2\right)^2}-\frac{aB-x(Ac-aC)}{6ac\left(a+cx^2\right)^3}$$

```
[Out] 1/6*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^3+1/24*(5*A*c+C*a)*x/a^2/c/(c*x^2+a)^2
+1/16*(5*A*c+C*a)*x/a^3/c/(c*x^2+a)+1/16*(5*A*c+C*a)*arctan(x*c^(1/2)/a^(1/
2))/a^(7/2)/c^(3/2)
```

Rubi [A] time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1814, 12, 199, 205}

$$\frac{(aC+5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}+\frac{x(aC+5Ac)}{16a^3c\left(a+cx^2\right)}+\frac{x(aC+5Ac)}{24a^2c\left(a+cx^2\right)^2}-\frac{aB-x(Ac-aC)}{6ac\left(a+cx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^4,x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(6*a*c*(a + c*x^2)^3) + ((5*A*c + a*C)*x)/(24*a^2*c*
(a + c*x^2)^2) + ((5*A*c + a*C)*x)/(16*a^3*c*(a + c*x^2)) + ((5*A*c + a*C)*
ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^4}\,dx = -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3} - \frac{\int \frac{-5A-\frac{aC}{c}}{\left(a+cx^2\right)^3}\,dx}{6a}$$

$$= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3} + \frac{(5Ac+aC)\int \frac{1}{\left(a+cx^2\right)^3}\,dx}{6ac}$$

$$= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3} + \frac{(5Ac+aC)x}{24a^2c\left(a+cx^2\right)^2} + \frac{(5Ac+aC)\int \frac{1}{\left(a+cx^2\right)^2}\,dx}{8a^2c}$$

$$= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3} + \frac{(5Ac+aC)x}{24a^2c\left(a+cx^2\right)^2} + \frac{(5Ac+aC)x}{16a^3c\left(a+cx^2\right)} + \frac{(5Ac+aC)\int \frac{1}{a+cx^2}\,dx}{16a^3c}$$

$$= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3} + \frac{(5Ac+aC)x}{24a^2c\left(a+cx^2\right)^2} + \frac{(5Ac+aC)x}{16a^3c\left(a+cx^2\right)} + \frac{(5Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}$$

Mathematica [A] time = 0.09, size = 112, normalized size = 0.89

$$\frac{(aC+5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}} + \frac{-a^3(8B+3Cx)+a^2cx\left(33A+8Cx^2\right)+ac^2x^3\left(40A+3Cx^2\right)+15Ac^3x^5}{48a^3c\left(a+cx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^4,x]
```

```
[Out] (15*A*c^3*x^5 - a^3*(8*B + 3*C*x) + a*c^2*x^3*(40*A + 3*C*x^2) + a^2*c*x*(3
3*A + 8*C*x^2))/(48*a^3*c*(a + c*x^2)^3) + ((5*A*c + a*C)*ArcTan[(Sqrt[c]*x
)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

fricas [A] time = 0.67, size = 430, normalized size = 3.41

$$\left[-\frac{16\,Ba^4c-6\left(Ca^2c^3+5\,Aac^4\right)x^5-16\left(Ca^3c^2+5\,Aa^2c^3\right)x^3+3\left(\left(Cac^3+5\,Ac^4\right)x^6+Ca^4+5\,Aa^3c+3\left(Ca^2c^2+\right.\right.}{96\left(a^4c^5x^6+3\,a^5c^4x^4+3\,a^6c^3x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c - 6*(C*a^2*c^3 + 5*A*a*c^4)*x^5 - 16*(C*a^3*c^2 + 5*A*a^
2*c^3)*x^3 + 3*((C*a*c^3 + 5*A*c^4)*x^6 + C*a^4 + 5*A*a^3*c + 3*(C*a^2*c^2
+ 5*A*a*c^3)*x^4 + 3*(C*a^3*c + 5*A*a^2*c^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2
*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(C*a^4*c - 11*A*a^3*c^2)*x)/(a^4*c^5*x^
6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2), -1/48*(8*B*a^4*c - 3*(C*a^2*c
^3 + 5*A*a*c^4)*x^5 - 8*(C*a^3*c^2 + 5*A*a^2*c^3)*x^3 - 3*((C*a*c^3 + 5*A*c
^4)*x^6 + C*a^4 + 5*A*a^3*c + 3*(C*a^2*c^2 + 5*A*a*c^3)*x^4 + 3*(C*a^3*c +
5*A*a^2*c^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(C*a^4*c - 11*A*a^3*c
^2)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2)]
```

giac [A] time = 0.18, size = 109, normalized size = 0.87

$$\frac{(Ca+5\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c} + \frac{3\,Cac^2x^5+15\,Ac^3x^5+8\,Ca^2cx^3+40\,Aac^2x^3-3\,Ca^3x+33\,Aa^2cx-8\,Ba^3}{48\left(cx^2+a\right)^3a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a + 5*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c) + 1/48*(3*C*a*c^
2*x^5 + 15*A*c^3*x^5 + 8*C*a^2*c*x^3 + 40*A*a*c^2*x^3 - 3*C*a^3*x + 33*A*a^
2*c*x - 8*B*a^3)/((c*x^2 + a)^3*a^3*c)
```

maple [A] time = 0.01, size = 113, normalized size = 0.90

$$\frac{5A\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^{3}}+\frac{C\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^{2}c}+\frac{\frac{(5Ac+aC)c\,x^{5}}{16a^{3}}+\frac{(5Ac+aC)x^{3}}{6a^{2}}-\frac{B}{6c}+\frac{(11Ac-aC)x}{16ac}}{\left(c\,x^{2}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(5*A*c+C*a)/a^3*c*x^5+1/6/a^2*(5*A*c+C*a)*x^3+1/16*(11*A*c-C*a)/a/c*x
-1/6*B/c)/(c*x^2+a)^3+5/16/a^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A+1/16
/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C
```

maxima [A] time = 0.98, size = 133, normalized size = 1.06

$$\frac{3\left(Cac^{2}+5\,Ac^{3}\right)x^{5}-8\,Ba^{3}+8\left(Ca^{2}c+5\,Aac^{2}\right)x^{3}-3\left(Ca^{3}-11\,Aa^{2}c\right)x}{48\left(a^{3}c^{4}x^{6}+3\,a^{4}c^{3}x^{4}+3\,a^{5}c^{2}x^{2}+a^{6}c\right)}+\frac{(Ca+5\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^{3}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(C*a*c^2 + 5*A*c^3)*x^5 - 8*B*a^3 + 8*(C*a^2*c + 5*A*a*c^2)*x^3 - 3
*(C*a^3 - 11*A*a^2*c)*x)/(a^3*c^4*x^6 + 3*a^4*c^3*x^4 + 3*a^5*c^2*x^2 + a^6
*c) + 1/16*(C*a + 5*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c)
```

mupad [B] time = 3.89, size = 116, normalized size = 0.92

$$\frac{\frac{x^{3}\,(5\,A\,c+C\,a)}{6\,a^{2}}-\frac{B}{6\,c}+\frac{c\,x^{5}\,(5\,A\,c+C\,a)}{16\,a^{3}}+\frac{x\,(11\,A\,c-C\,a)}{16\,a\,c}}{a^{3}+3\,a^{2}\,c\,x^{2}+3\,a\,c^{2}\,x^{4}+c^{3}\,x^{6}}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(5\,A\,c+C\,a)}{16\,a^{7/2}\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^4,x)
```

```
[Out] ((x^3*(5*A*c + C*a))/(6*a^2) - B/(6*c) + (c*x^5*(5*A*c + C*a))/(16*a^3) + (
x*(11*A*c - C*a))/(16*a*c))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4) + (
atan((c^(1/2)*x)/a^(1/2))*(5*A*c + C*a))/(16*a^(7/2)*c^(3/2))
```

sympy [A] time = 2.08, size = 196, normalized size = 1.56

$$-\frac{\sqrt{-\frac{1}{a^{7}c^{3}}}\,(5Ac+Ca)\log\left(-a^{4}c\sqrt{-\frac{1}{a^{7}c^{3}}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^{7}c^{3}}}\,(5Ac+Ca)\log\left(a^{4}c\sqrt{-\frac{1}{a^{7}c^{3}}}+x\right)}{32}+\frac{-8Ba^{3}+x^{5}\left(15Ac^{3}\right.}{48a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] -sqrt(-1/(a**7*c**3))*(5*A*c + C*a)*log(-a**4*c*sqrt(-1/(a**7*c**3)) + x)/3
2 + sqrt(-1/(a**7*c**3))*(5*A*c + C*a)*log(a**4*c*sqrt(-1/(a**7*c**3)) + x)
/32 + (-8*B*a**3 + x**5*(15*A*c**3 + 3*C*a*c**2) + x**3*(40*A*a*c**2 + 8*C*
a**2*c) + x*(33*A*a**2*c - 3*C*a**3))/(48*a**6*c + 144*a**5*c**2*x**2 + 144
*a**4*c**3*x**4 + 48*a**3*c**4*x**6)
```

3.69 $\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx$

Optimal. Leaf size=43

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-\frac{x^3}{2\left(x^2+1\right)}+\frac{3x}{2}-\frac{3}{2}\tan^{-1}(x)$$

```
[Out] 3/2*x+1/2*x^2-1/2*x^3/(x^2+1)-3/2*arctan(x)-1/2*ln(x^2+1)
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1804, 801, 635, 203, 260}

$$-\frac{x^3}{2\left(x^2+1\right)}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)+\frac{3x}{2}-\frac{3}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] (3*x)/2 + x^2/2 - x^3/(2*(1 + x^2)) - (3*ArcTan[x])/2 - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\left(1+x+x^2\right)}{\left(1+x^2\right)^2}\,dx = -\frac{x^3}{2\left(1+x^2\right)} - \frac{1}{2}\int \frac{(-3-2x)x^2}{1+x^2}\,dx$$
$$= -\frac{x^3}{2\left(1+x^2\right)} - \frac{1}{2}\int \left(-3-2x+\frac{3+2x}{1+x^2}\right)dx$$
$$= \frac{3x}{2} + \frac{x^2}{2} - \frac{x^3}{2\left(1+x^2\right)} - \frac{1}{2}\int \frac{3+2x}{1+x^2}\,dx$$
$$= \frac{3x}{2} + \frac{x^2}{2} - \frac{x^3}{2\left(1+x^2\right)} - \frac{3}{2}\int \frac{1}{1+x^2}\,dx - \int \frac{x}{1+x^2}\,dx$$
$$= \frac{3x}{2} + \frac{x^2}{2} - \frac{x^3}{2\left(1+x^2\right)} - \frac{3}{2}\tan^{-1}(x) - \frac{1}{2}\log\left(1+x^2\right)$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.67

$$\frac{1}{2}\left(x\left(\frac{1}{x^2+1}+x+2\right)-\log\left(x^2+1\right)-3\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + x + x^2))/(1 + x^2)^2,x]
[Out] (x*(2 + x + (1 + x^2)^(-1)) - 3*ArcTan[x] - Log[1 + x^2])/2
```

fricas [A] time = 0.93, size = 46, normalized size = 1.07

$$\frac{x^4+2x^3+x^2-3\left(x^2+1\right)\arctan(x)-\left(x^2+1\right)\log\left(x^2+1\right)+3x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*(x^4 + 2*x^3 + x^2 - 3*(x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + 3
*x)/(x^2 + 1)
```

giac [A] time = 0.15, size = 29, normalized size = 0.67

$$\frac{1}{2}x^2+x+\frac{x}{2\left(x^2+1\right)}-\frac{3}{2}\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
[Out] 1/2*x^2 + x + 1/2*x/(x^2 + 1) - 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

maple [A] time = 0.01, size = 30, normalized size = 0.70

$$\frac{x^2}{2}+x+\frac{x}{2x^2+2}-\frac{3\arctan(x)}{2}-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+x+1)/(x^2+1)^2,x)
[Out] 1/2*x^2+x+1/2*x/(x^2+1)-1/2*ln(x^2+1)-3/2*arctan(x)
```

maxima [A] time = 0.95, size = 29, normalized size = 0.67

$$\frac{1}{2}x^2 + x + \frac{x}{2(x^2+1)} - \frac{3}{2}\arctan(x) - \frac{1}{2}\log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/2*x^2 + x + 1/2*x/(x^2 + 1) - 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

mupad [B] time = 0.04, size = 30, normalized size = 0.70

$$x - \frac{\ln(x^2+1)}{2} - \frac{3\,\mathrm{atan}(x)}{2} + \frac{x}{2(x^2+1)} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x + x^2 + 1))/(x^2 + 1)^2,x)
[Out] x - log(x^2 + 1)/2 - (3*atan(x))/2 + x/(2*(x^2 + 1)) + x^2/2
```

sympy [A] time = 0.13, size = 29, normalized size = 0.67

$$\frac{x^2}{2} + x + \frac{x}{2x^2+2} - \frac{\log(x^2+1)}{2} - \frac{3\,\mathrm{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+x+1)/(x**2+1)**2,x)
[Out] x**2/2 + x + x/(2*x**2 + 2) - log(x**2 + 1)/2 - 3*atan(x)/2
```

3.70 $\int \frac{x^2(1+x+x^2)}{(1+x^2)^2} dx$

Optimal. Leaf size=30

$$-\frac{x^2}{2(x^2+1)}+\frac{1}{2}\log(x^2+1)+x-\tan^{-1}(x)$$

```
[Out] x-1/2*x^2/(x^2+1)-arctan(x)+1/2*ln(x^2+1)
```

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1804, 774, 635, 203, 260}

$$-\frac{x^2}{2(x^2+1)}+\frac{1}{2}\log(x^2+1)+x-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] x - x^2/(2*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(1+x+x^2\right)}{\left(1+x^2\right)^2}\,dx &= -\frac{x^2}{2\left(1+x^2\right)} - \frac{1}{2}\int \frac{(-2-2x)x}{1+x^2}\,dx \\ &= x - \frac{x^2}{2\left(1+x^2\right)} - \frac{1}{2}\int \frac{2-2x}{1+x^2}\,dx \\ &= x - \frac{x^2}{2\left(1+x^2\right)} - \int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx \\ &= x - \frac{x^2}{2\left(1+x^2\right)} - \tan^{-1}(x) + \frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.90

$$\frac{1}{2\left(x^2+1\right)} + \frac{1}{2}\log\left(x^2+1\right) + x - \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] x + 1/(2*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2
```

fricas [A] time = 0.97, size = 40, normalized size = 1.33

$$\frac{2\,x^3 - 2\left(x^2+1\right)\arctan(x) + \left(x^2+1\right)\log\left(x^2+1\right) + 2\,x + 1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^3 - 2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) + 2*x + 1)/(x^2
 + 1)
```

giac [A] time = 0.21, size = 23, normalized size = 0.77

$$x + \frac{1}{2\left(x^2+1\right)} - \arctan(x) + \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] x + 1/2/(x^2 + 1) - arctan(x) + 1/2*log(x^2 + 1)
```

maple [A] time = 0.01, size = 24, normalized size = 0.80

$$x - \arctan(x) + \frac{\ln\left(x^2+1\right)}{2} + \frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2+x+1)/(x^2+1)^2,x)
```

```
[Out] x+1/2/(x^2+1)+1/2*ln(x^2+1)-arctan(x)
```

maxima [A] time = 0.96, size = 23, normalized size = 0.77

$$x + \frac{1}{2\left(x^2+1\right)} - \arctan(x) + \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] x + 1/2/(x^2 + 1) - arctan(x) + 1/2*log(x^2 + 1)
```

mupad [B] time = 0.03, size = 23, normalized size = 0.77

$$x + \frac{\ln\left(x^2+1\right)}{2} - \mathrm{atan}(x) + \frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + x^2 + 1))/(x^2 + 1)^2,x)
[Out] x + log(x^2 + 1)/2 - atan(x) + 1/(2*(x^2 + 1))
```

sympy [A] time = 0.12, size = 20, normalized size = 0.67

$$x + \frac{\log\left(x^2+1\right)}{2} - \mathrm{atan}\left(x\right) + \frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+x+1)/(x**2+1)**2,x)
[Out] x + log(x**2 + 1)/2 - atan(x) + 1/(2*x**2 + 2)
```

3.71 $\int \frac{x\left(1+x+x^2\right)}{\left(1+x^2\right)^2} dx$

Optimal. Leaf size=29

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)+1/2*ln(x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1804, 635, 203, 260}

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2 + Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x\left(1+x+x^2\right)}{\left(1+x^2\right)^2}\,dx = -\frac{x}{2\left(1+x^2\right)} - \frac{1}{2}\int \frac{-1-2x}{1+x^2}\,dx$$
$$= -\frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx$$
$$= -\frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\log\left(1+x^2\right)$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.79

$$\frac{1}{2}\left(-\frac{x}{x^2+1} + \log\left(x^2+1\right) + \tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + x + x^2))/(1 + x^2)^2,x]
[Out] (-(x/(1 + x^2)) + ArcTan[x] + Log[1 + x^2])/2
```

fricas [A] time = 0.96, size = 33, normalized size = 1.14

$$\frac{\left(x^2+1\right)\arctan(x) + \left(x^2+1\right)\log\left(x^2+1\right) - x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - x)/(x^2 + 1)
```

giac [A] time = 0.15, size = 23, normalized size = 0.79

$$-\frac{x}{2\left(x^2+1\right)} + \frac{1}{2}\arctan(x) + \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 24, normalized size = 0.83

$$-\frac{x}{2\left(x^2+1\right)} + \frac{\arctan(x)}{2} + \frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+x+1)/(x^2+1)^2,x)
[Out] -1/2/(x^2+1)*x+1/2*arctan(x)+1/2*ln(x^2+1)
```

maxima [A] time = 0.96, size = 23, normalized size = 0.79

$$-\frac{x}{2\left(x^2+1\right)} + \frac{1}{2}\arctan(x) + \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

mupad [B] time = 0.03, size = 25, normalized size = 0.86

$$\frac{\ln\left(x^2+1\right)}{2}+\frac{\mathrm{atan}(x)}{2}-\frac{x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + x^2 + 1))/(x^2 + 1)^2,x)
```

```
[Out] log(x^2 + 1)/2 + atan(x)/2 - x/(2*(x^2 + 1))
```

sympy [A] time = 0.13, size = 20, normalized size = 0.69

$$-\frac{x}{2x^2+2}+\frac{\log\left(x^2+1\right)}{2}+\frac{\mathrm{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+x+1)/(x**2+1)**2,x)
```

```
[Out] -x/(2*x**2 + 2) + log(x**2 + 1)/2 + atan(x)/2
```

3.72 $\int \frac{1+x+x^2}{\left(1+x^2\right)^2} dx$

Optimal. Leaf size=14

$$\tan^{-1}(x) - \frac{1}{2\left(x^2+1\right)}$$

```
[Out] -1/2/(x^2+1)+arctan(x)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1814, 12, 203}

$$\tan^{-1}(x) - \frac{1}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(1 + x^2)^2,x]
```

```
[Out] -1/(2*(1 + x^2)) + ArcTan[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2}{\left(1+x^2\right)^2} dx &= -\frac{1}{2\left(1+x^2\right)} - \frac{1}{2}\int -\frac{2}{1+x^2} dx \\
&= -\frac{1}{2\left(1+x^2\right)} + \int \frac{1}{1+x^2} dx \\
&= -\frac{1}{2\left(1+x^2\right)} + \tan^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\tan^{-1}(x) - \frac{1}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(1 + x^2)^2,x]
[Out] -1/2*1/(1 + x^2) + ArcTan[x]
```

fricas [A] time = 0.87, size = 20, normalized size = 1.43

$$\frac{2\left(x^2+1\right)\arctan(x)-1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*(2*(x^2 + 1)*arctan(x) - 1)/(x^2 + 1)
```

giac [A] time = 0.15, size = 12, normalized size = 0.86

$$-\frac{1}{2\left(x^2+1\right)}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2/(x^2 + 1) + arctan(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\arctan(x)-\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/(x^2+1)^2,x)
[Out] -1/2/(x^2+1)+arctan(x)
```

maxima [A] time = 0.96, size = 12, normalized size = 0.86

$$-\frac{1}{2\left(x^2+1\right)}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2/(x^2 + 1) + arctan(x)
```

mupad [B] time = 3.80, size = 14, normalized size = 1.00

$$\mathrm{atan}(x)-\frac{1}{2\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^2 + 1)^2,x)
[Out] atan(x) - 1/(2*(x^2 + 1))
```

sympy [A] time = 0.11, size = 10, normalized size = 0.71

$$\operatorname{atan}{\left(x \right)}-\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(x**2+1)**2,x)
```

```
[Out] atan(x) - 1/(2*x**2 + 2)
```

3.73 $\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=31

$$\frac{x}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] 1/2*x/(x^2+1)+1/2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1805, 801, 635, 203, 260}

$$\frac{x}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x*(1 + x^2)^2),x]
```

```
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2 + Log[x] - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx &= \frac{x}{2\left(1+x^2\right)} - \frac{1}{2}\int \frac{-2-x}{x\left(1+x^2\right)}\,dx \\ &= \frac{x}{2\left(1+x^2\right)} - \frac{1}{2}\int \left(-\frac{2}{x} + \frac{-1+2x}{1+x^2}\right)dx \\ &= \frac{x}{2\left(1+x^2\right)} + \log(x) - \frac{1}{2}\int \frac{-1+2x}{1+x^2}\,dx \\ &= \frac{x}{2\left(1+x^2\right)} + \log(x) + \frac{1}{2}\int \frac{1}{1+x^2}\,dx - \int \frac{x}{1+x^2}\,dx \\ &= \frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}(x) + \log(x) - \frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.90

$$\frac{1}{2}\left(\frac{x}{x^2+1} - \log\left(x^2+1\right) + 2\log(x) + \tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x*(1 + x^2)^2),x]
[Out] (x/(1 + x^2) + ArcTan[x] + 2*Log[x] - Log[1 + x^2])/2
```

fricas [A] time = 1.02, size = 41, normalized size = 1.32

$$\frac{\left(x^2+1\right)\arctan(x) - \left(x^2+1\right)\log\left(x^2+1\right) + 2\left(x^2+1\right)\log(x) + x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + 2*(x^2 + 1)*log(x) + x)
/(x^2 + 1)
```

giac [A] time = 0.15, size = 26, normalized size = 0.84

$$\frac{x}{2\left(x^2+1\right)} + \frac{1}{2}\arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="giac")
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

maple [A] time = 0.01, size = 26, normalized size = 0.84

$$\frac{x}{2x^2+2} + \frac{\arctan(x)}{2} + \ln(x) - \frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x/(x^2+1)^2,x)
[Out] 1/2/(x^2+1)*x+1/2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

maxima [A] time = 0.96, size = 25, normalized size = 0.81

$$\frac{x}{2\left(x^2+1\right)} + \frac{1}{2}\arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

mupad [B] time = 0.04, size = 32, normalized size = 1.03

$$\ln(x) + \frac{x}{2\left(x^2+1\right)} + \ln\left(x - \mathrm{i}\right)\left(-\frac{1}{2} - \frac{1}{4}\mathrm{i}\right) + \ln\left(x + 1\mathrm{i}\right)\left(-\frac{1}{2} + \frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x*(x^2 + 1)^2),x)
```

```
[Out] log(x) - log(x + 1i)*(1/2 - 1i/4) - log(x - 1i)*(1/2 + 1i/4) + x/(2*(x^2 +
1))
```

sympy [A] time = 0.16, size = 24, normalized size = 0.77

$$\frac{x}{2x^2+2} + \log\left(x\right) - \frac{\log\left(x^2+1\right)}{2} + \frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x/(x**2+1)**2,x)
```

```
[Out] x/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 + atan(x)/2
```

3.74 $\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=33

$$\frac{1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}+\log(x)-\tan^{-1}(x)$$

```
[Out] -1/x+1/2/(x^2+1)-arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1805, 801, 635, 203, 260}

$$\frac{1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}+\log(x)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x^2*(1 + x^2)^2),x]
```

```
[Out] -x^(-1) + 1/(2*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx &= \frac{1}{2\left(1+x^2\right)} - \frac{1}{2}\int \frac{-2-2x}{x^2\left(1+x^2\right)}\,dx \\
&= \frac{1}{2\left(1+x^2\right)} - \frac{1}{2}\int \left(-\frac{2}{x^2} - \frac{2}{x} + \frac{2(1+x)}{1+x^2}\right) dx \\
&= -\frac{1}{x} + \frac{1}{2\left(1+x^2\right)} + \log(x) - \int \frac{1+x}{1+x^2}\,dx \\
&= -\frac{1}{x} + \frac{1}{2\left(1+x^2\right)} + \log(x) - \int \frac{1}{1+x^2}\,dx - \int \frac{x}{1+x^2}\,dx \\
&= -\frac{1}{x} + \frac{1}{2\left(1+x^2\right)} - \tan^{-1}(x) + \log(x) - \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{1}{2\left(x^2+1\right)} - \frac{1}{2}\log\left(x^2+1\right) - \frac{1}{x} + \log(x) - \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x^2*(1 + x^2)^2),x]
```

```
[Out] -x^(-1) + 1/(2*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

fricas [A] time = 0.71, size = 49, normalized size = 1.48

$$-\frac{2\,x^2 + 2\left(x^3+x\right)\arctan(x) + \left(x^3+x\right)\log\left(x^2+1\right) - 2\left(x^3+x\right)\log(x) - x + 2}{2\left(x^3+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^2 + 2*(x^3 + x)*arctan(x) + (x^3 + x)*log(x^2 + 1) - 2*(x^3 + x)*
log(x) - x + 2)/(x^3 + x)
```

giac [A] time = 0.18, size = 35, normalized size = 1.06

$$-\frac{2\,x^2 - x + 2}{2\left(x^3+x\right)} - \arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*x^2 - x + 2)/(x^3 + x) - arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

maple [A] time = 0.01, size = 30, normalized size = 0.91

$$-\arctan(x) + \ln(x) - \frac{\ln\left(x^2+1\right)}{2} - \frac{1}{x} + \frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x^2/(x^2+1)^2,x)
```

```
[Out] -1/x+1/2/(x^2+1)-arctan(x)+ln(x)-1/2*ln(x^2+1)
```

maxima [A] time = 0.95, size = 34, normalized size = 1.03

$$-\frac{2\,x^2 - x + 2}{2\left(x^3 + x\right)} - \arctan(x) - \frac{1}{2}\,\log\left(x^2 + 1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2*(2*x^2 - x + 2)/(x^3 + x) - arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

mupad [B] time = 3.81, size = 38, normalized size = 1.15

$$\ln(x) - \frac{x^2 - \frac{x}{2} + 1}{x^3 + x} + \ln\left(x - \mathrm{i}\right)\left(-\frac{1}{2} + \frac{1}{2}\mathrm{i}\right) + \ln\left(x + 1\mathrm{i}\right)\left(-\frac{1}{2} - \frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^2*(x^2 + 1)^2),x)
[Out] log(x) - log(x + 1i)*(1/2 + 1i/2) - log(x - 1i)*(1/2 - 1i/2) - (x^2 - x/2 +
 1)/(x + x^3)
```

sympy [A] time = 0.16, size = 31, normalized size = 0.94

$$\log\left(x\right) - \frac{\log\left(x^2 + 1\right)}{2} - \operatorname{atan}\left(x\right) + \frac{-2x^2 + x - 2}{2x^3 + 2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**2/(x**2+1)**2,x)
[Out] log(x) - log(x**2 + 1)/2 - atan(x) + (-2*x**2 + x - 2)/(2*x**3 + 2*x)
```

3.75 $\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=45

$$-\frac{x}{2\left(x^2+1\right)}-\frac{1}{2x^2}+\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}-\log(x)-\frac{3}{2}\tan^{-1}(x)$$

```
[Out] -1/2/x^2-1/x-1/2*x/(x^2+1)-3/2*arctan(x)-ln(x)+1/2*ln(x^2+1)
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1805, 1802, 635, 203, 260}

$$-\frac{x}{2\left(x^2+1\right)}-\frac{1}{2x^2}+\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}-\log(x)-\frac{3}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x^3*(1 + x^2)^2),x]
```

```
[Out] -1/(2*x^2) - x^(-1) - x/(2*(1 + x^2)) - (3*ArcTan[x])/2 - Log[x] + Log[1 +
x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx &= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2-2x+x^3}{x^3\left(1+x^2\right)}\,dx\\ &= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-\frac{2}{x^3}-\frac{2}{x^2}+\frac{2}{x}+\frac{3-2x}{1+x^2}\right)dx\\ &= -\frac{1}{2x^2}-\frac{1}{x}-\frac{x}{2\left(1+x^2\right)}-\log(x)-\frac{1}{2}\int \frac{3-2x}{1+x^2}\,dx\\ &= -\frac{1}{2x^2}-\frac{1}{x}-\frac{x}{2\left(1+x^2\right)}-\log(x)-\frac{3}{2}\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= -\frac{1}{2x^2}-\frac{1}{x}-\frac{x}{2\left(1+x^2\right)}-\frac{3}{2}\tan^{-1}(x)-\log(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.87

$$\frac{1}{2}\left(-\frac{x}{x^2+1}-\frac{1}{x^2}+\log\left(x^2+1\right)-\frac{2}{x}-2\log(x)-3\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x^3*(1 + x^2)^2),x]
[Out] (-x^(-2) - 2/x - x/(1 + x^2) - 3*ArcTan[x] - 2*Log[x] + Log[1 + x^2])/2
```

fricas [A] time = 1.07, size = 61, normalized size = 1.36

$$-\frac{3\,x^3+x^2+3\left(x^4+x^2\right)\arctan(x)-\left(x^4+x^2\right)\log\left(x^2+1\right)+2\left(x^4+x^2\right)\log(x)+2\,x+1}{2\left(x^4+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="fricas")
[Out] -1/2*(3*x^3 + x^2 + 3*(x^4 + x^2)*arctan(x) - (x^4 + x^2)*log(x^2 + 1) + 2*
(x^4 + x^2)*log(x) + 2*x + 1)/(x^4 + x^2)
```

giac [A] time = 0.16, size = 43, normalized size = 0.96

$$-\frac{3\,x^3+x^2+2\,x+1}{2\left(x^2+1\right)x^2}-\frac{3}{2}\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)-\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*(3*x^3 + x^2 + 2*x + 1)/((x^2 + 1)*x^2) - 3/2*arctan(x) + 1/2*log(x^2
+ 1) - log(abs(x))
```

maple [A] time = 0.01, size = 38, normalized size = 0.84

$$-\frac{x}{2\left(x^2+1\right)}-\frac{3\arctan(x)}{2}-\ln(x)+\frac{\ln\left(x^2+1\right)}{2}-\frac{1}{x}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x^3/(x^2+1)^2,x)
```

```
[Out] -1/2/x^2-1/x-1/2/(x^2+1)*x-3/2*arctan(x)-ln(x)+1/2*ln(x^2+1)
```

maxima [A] time = 0.97, size = 41, normalized size = 0.91

$$-\frac{3\,x^3 + x^2 + 2\,x + 1}{2\left(x^4 + x^2\right)} - \frac{3}{2}\,\arctan(x) + \frac{1}{2}\,\log\left(x^2 + 1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*x^3 + x^2 + 2*x + 1)/(x^4 + x^2) - 3/2*arctan(x) + 1/2*log(x^2 + 1)
 - log(x)
```

mupad [B] time = 0.04, size = 47, normalized size = 1.04

$$-\ln(x) - \frac{\frac{3\,x^3}{2} + \frac{x^2}{2} + x + \frac{1}{2}}{x^4 + x^2} + \ln\left(x - \mathrm{i}\right)\left(\frac{1}{2} + \frac{3}{4}\mathrm{i}\right) + \ln\left(x + 1\mathrm{i}\right)\left(\frac{1}{2} - \frac{3}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^3*(x^2 + 1)^2),x)
```

```
[Out] log(x - 1i)*(1/2 + 3i/4) + log(x + 1i)*(1/2 - 3i/4) - log(x) - (x + x^2/2 +
 (3*x^3)/2 + 1/2)/(x^2 + x^4)
```

sympy [A] time = 0.18, size = 42, normalized size = 0.93

$$-\log\left(x\right) + \frac{\log\left(x^2 + 1\right)}{2} - \frac{3\,\mathrm{atan}\left(x\right)}{2} + \frac{-3x^3 - x^2 - 2x - 1}{2x^4 + 2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**3/(x**2+1)**2,x)
```

```
[Out] -log(x) + log(x**2 + 1)/2 - 3*atan(x)/2 + (-3*x**3 - x**2 - 2*x - 1)/(2*x**
4 + 2*x**2)
```

3.76 $\int \frac{1+2x+x^2}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=12

$$\tan^{-1}(x) - \frac{1}{x^2+1}$$

```
[Out] -1/(x^2+1)+arctan(x)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.83, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {27, 723, 203}

$$\tan^{-1}(x) - \frac{(1-x)(x+1)}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x + x^2)/(1 + x^2)^2,x]
[Out] -((1 - x)*(1 + x))/(2*(1 + x^2)) + ArcTan[x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{1+2x+x^2}{\left(1+x^2\right)^2} \, dx &= \int \frac{(1+x)^2}{\left(1+x^2\right)^2} \, dx \\ &= -\frac{(1-x)(1+x)}{2\left(1+x^2\right)} + \int \frac{1}{1+x^2} \, dx \\ &= -\frac{(1-x)(1+x)}{2\left(1+x^2\right)} + \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\tan^{-1}(x) - \frac{1}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + x^2)/(1 + x^2)^2,x]
```

```
[Out] -(1 + x^2)^(-1) + ArcTan[x]
```

fricas [A] time = 1.04, size = 18, normalized size = 1.50

$$\frac{\left(x^{2}+1\right)\arctan(x)-1}{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] ((x^2 + 1)*arctan(x) - 1)/(x^2 + 1)
```

giac [A] time = 0.15, size = 12, normalized size = 1.00

$$-\frac{1}{x^{2}+1}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/(x^2 + 1) + arctan(x)
```

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$\arctan(x)-\frac{1}{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+1)/(x^2+1)^2,x)
```

```
[Out] -1/(x^2+1)+arctan(x)
```

maxima [A] time = 0.95, size = 12, normalized size = 1.00

$$-\frac{1}{x^{2}+1}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/(x^2 + 1) + arctan(x)
```

mupad [B] time = 0.03, size = 12, normalized size = 1.00

$$\mathrm{atan}(x)-\frac{1}{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 + 1)/(x^2 + 1)^2,x)
```

```
[Out] atan(x) - 1/(x^2 + 1)
```

sympy [A] time = 0.12, size = 8, normalized size = 0.67

$$\operatorname{atan}{\left(x \right)}-\frac{1}{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+1)/(x**2+1)**2,x)
```

```
[Out] atan(x) - 1/(x**2 + 1)
```

3.77 $\int \frac{2+12x+3x^2}{\left(4+x^2\right)^2} \, dx$

Optimal. Leaf size=27

$$\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)-\frac{5x+24}{4\left(x^2+4\right)}$$

```
[Out] 1/4*(-24-5*x)/(x^2+4)+7/8*arctan(1/2*x)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1814, 12, 203}

$$\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)-\frac{5x+24}{4\left(x^2+4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 12*x + 3*x^2)/(4 + x^2)^2,x]
```

```
[Out] -(24 + 5*x)/(4*(4 + x^2)) + (7*ArcTan[x/2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+12x+3x^2}{\left(4+x^2\right)^2} \, dx &= -\frac{24+5x}{4\left(4+x^2\right)}-\frac{1}{8}\int -\frac{14}{4+x^2} \, dx \\
&= -\frac{24+5x}{4\left(4+x^2\right)}+\frac{7}{4}\int \frac{1}{4+x^2} \, dx \\
&= -\frac{24+5x}{4\left(4+x^2\right)}+\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{-5x-24}{4\left(x^2+4\right)}+\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 12*x + 3*x^2)/(4 + x^2)^2,x]
```

```
[Out] (-24 - 5*x)/(4*(4 + x^2)) + (7*ArcTan[x/2])/8
```

fricas [A] time = 0.83, size = 25, normalized size = 0.93

$$\frac{7\left(x^2+4\right)\arctan\left(\frac{1}{2}x\right)-10x-48}{8\left(x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(7*(x^2 + 4)*arctan(1/2*x) - 10*x - 48)/(x^2 + 4)
```

giac [A] time = 0.18, size = 21, normalized size = 0.78

$$-\frac{5x+24}{4\left(x^2+4\right)}+\frac{7}{8}\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(5*x + 24)/(x^2 + 4) + 7/8*arctan(1/2*x)
```

maple [A] time = 0.01, size = 21, normalized size = 0.78

$$\frac{7\arctan\left(\frac{x}{2}\right)}{8}+\frac{-\frac{5x}{4}-6}{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+12*x+2)/(x^2+4)^2,x)
```

```
[Out] (-5/4*x-6)/(x^2+4)+7/8*arctan(1/2*x)
```

maxima [A] time = 0.97, size = 21, normalized size = 0.78

$$-\frac{5x+24}{4\left(x^2+4\right)}+\frac{7}{8}\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(5*x + 24)/(x^2 + 4) + 7/8*arctan(1/2*x)
```

mupad [B] time = 3.83, size = 21, normalized size = 0.78

$$\frac{7\,\mathrm{atan}\left(\frac{x}{2}\right)}{8}-\frac{\frac{5x}{4}+6}{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x + 3*x^2 + 2)/(x^2 + 4)^2,x)
```

```
[Out] (7*atan(x/2))/8 - ((5*x)/4 + 6)/(x^2 + 4)
```

sympy [**A**] time = 0.13, size = 20, normalized size = 0.74

$$\frac{-5x-24}{4x^2+16}+\frac{7\,\mathrm{atan}\left(\frac{x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+12*x+2)/(x**2+4)**2,x)
```

```
[Out] (-5*x - 24)/(4*x**2 + 16) + 7*atan(x/2)/8
```

3.78 $\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=390

$$\frac{x\sqrt{a+cx^2}\left(a^2h^2(eh+3fg)-2acg\left(3h(dh+eg)+fg^2\right)+8c^2dg^3\right)}{16c^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(a^2h^2(eh+3fg)-2acg\left(3\right.\right.}{16c^{5/2}}$$

```
[Out] -1/70*(8*a*f*h^2+c*(3*f*g^2-7*h*(2*d*h+e*g)))*(h*x+g)^2*(c*x^2+a)^(3/2)/c^2
/h-1/42*(-7*e*h+3*f*g)*(h*x+g)^3*(c*x^2+a)^(3/2)/c/h+1/7*f*(h*x+g)^4*(c*x^2
+a)^(3/2)/c/h+1/840*(64*a^2*f*h^4-16*a*c*h^2*(15*f*g^2+7*h*(d*h+3*e*g))-8*c
^2*g^2*(3*f*g^2-7*h*(12*d*h+e*g))-3*c*h*(a*h^2*(35*e*h+41*f*g)+2*c*g*(3*f*g
^2-7*h*(7*d*h+e*g)))*x)*(c*x^2+a)^(3/2)/c^3/h+1/16*a*(8*c^2*d*g^3+a^2*h^2*(
e*h+3*f*g)-2*a*c*g*(f*g^2+3*h*(d*h+e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2)
)/c^(5/2)+1/16*(8*c^2*d*g^3+a^2*h^2*(e*h+3*f*g)-2*a*c*g*(f*g^2+3*h*(d*h+e*g
)))*x*(c*x^2+a)^(1/2)/c^2
```

Rubi [A] time = 0.83, antiderivative size = 387, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 833, 780, 195, 217, 206}

$$\frac{\left(a+cx^2\right)^{3/2}\left(8\left(8a^2fh^4-2ach^2\left(7h(dh+3eg)+15fg^2\right)-c^2\left(3fg^4-7g^2h(12dh+eg)\right)\right)-3chx\left(ah^2(35eh+41fg\right.\right.}{840c^3h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((8*c^2*d*g^3 + a^2*h^2*(3*f*g + e*h) - 2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*
x*Sqrt[a + c*x^2])/(16*c^2) - ((3*c*f*g^2 + 8*a*f*h^2 - 7*c*h*(e*g + 2*d*h)
)*(g + h*x)^2*(a + c*x^2)^(3/2))/(70*c^2*h) - ((3*f*g - 7*e*h)*(g + h*x)^3*
(a + c*x^2)^(3/2))/(42*c*h) + (f*(g + h*x)^4*(a + c*x^2)^(3/2))/(7*c*h) + (
(8*(8*a^2*f*h^4 - 2*a*c*h^2*(15*f*g^2 + 7*h*(3*e*g + d*h)) - c^2*(3*f*g^4 -
 7*g^2*h*(e*g + 12*d*h))) - 3*c*h*(6*c*f*g^3 - 14*c*g*h*(e*g + 7*d*h) + a*h
^2*(41*f*g + 35*e*h))*x)*(a + c*x^2)^(3/2))/(840*c^3*h) + (a*(8*c^2*d*g^3 +
 a^2*h^2*(3*f*g + e*h) - 2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c
]*x)/Sqrt[a + c*x^2]])/(16*c^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^4\left(a+cx^2\right)^{3/2}}{7ch} + \frac{\int (g+hx)^3\left((7cd-4af)h^2-ch(3fg-7e\right.}{7ch^2}$$

$$= -\frac{(3fg-7eh)(g+hx)^3\left(a+cx^2\right)^{3/2}}{42ch} + \frac{f(g+hx)^4\left(a+cx^2\right)^{3/2}}{7ch} + \int (g$$

$$= -\frac{\left(3cfg^2+8afh^2-7ch(eg+2dh)\right)(g+hx)^2\left(a+cx^2\right)^{3/2}}{70c^2h} - \frac{(3fg-}{}$$

$$= -\frac{\left(3cfg^2+8afh^2-7ch(eg+2dh)\right)(g+hx)^2\left(a+cx^2\right)^{3/2}}{70c^2h} - \frac{(3fg-}{}$$

$$= \frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2}$$

$$= \frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2}$$

$$= \frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2}$$

Mathematica [A] time = 0.46, size = 362, normalized size = 0.93

$$\frac{105a\sqrt{c}\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)\left(a^2h^2(eh+3fg)-2acg\left(3h(dh+eg)+fg^2\right)+8c^2dg^3\right)+\sqrt{a+cx^2}\left(16cx^2\left(-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(16*a*(8*a^2*f*h^3 + 35*c^2*g^2*(e*g + 3*d*h) - 14*a*c*h*(
3*f*g^2 + h*(3*e*g + d*h))) + 105*c*(8*c^2*d*g^3 - a^2*h^2*(3*f*g + e*h) +
2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*x + 16*c*(-4*a^2*f*h^3 + 35*c^2*g^2*(e*g
 + 3*d*h) + 7*a*c*h*(3*f*g^2 + h*(3*e*g + d*h)))*x^2 + 70*c^2*(a*h^2*(3*f*g
 + e*h) + 6*c*(f*g^3 + 3*g*h*(e*g + d*h)))*x^3 + 48*c^2*h*(a*f*h^2 + 7*c*(3
*f*g^2 + h*(3*e*g + d*h)))*x^4 + 280*c^3*h^2*(3*f*g + e*h)*x^5 + 240*c^3*f*
h^3*x^6) + 105*a*Sqrt[c]*(8*c^2*d*g^3 + a^2*h^2*(3*f*g + e*h) - 2*a*c*g*(f*
g^2 + 3*h*(e*g + d*h)))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(1680*c^3)
```

fricas [A] time = 0.97, size = 855, normalized size = 2.19

$$\left[-\frac{105\left(6\,a^2ceg^2h - a^3eh^3 - 2\left(4\,ac^2d - a^2cf\right)g^3 + 3\left(2\,a^2cd - a^3f\right)gh^2\right)\sqrt{c}\,\log\left(-2\,cx^2 - 2\,\sqrt{cx^2+a}\,\sqrt{c}\,x - a\right) - 2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/3360*(105*(6*a^2*c*e*g^2*h - a^3*e*h^3 - 2*(4*a*c^2*d - a^2*c*f)*g^3 +
3*(2*a^2*c*d - a^3*f)*g*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(
c)*x - a) - 2*(240*c^3*f*h^3*x^6 + 560*a*c^2*e*g^3 - 672*a^2*c*e*g*h^2 + 28
0*(3*c^3*f*g*h^2 + c^3*e*h^3)*x^5 + 48*(21*c^3*f*g^2*h + 21*c^3*e*g*h^2 + (
7*c^3*d + a*c^2*f)*h^3)*x^4 + 336*(5*a*c^2*d - 2*a^2*c*f)*g^2*h - 32*(7*a^2
*c*d - 4*a^3*f)*h^3 + 70*(6*c^3*f*g^3 + 18*c^3*e*g^2*h + a*c^2*e*h^3 + 3*(6
*c^3*d + a*c^2*f)*g*h^2)*x^3 + 16*(35*c^3*e*g^3 + 21*a*c^2*e*g*h^2 + 21*(5*
c^3*d + a*c^2*f)*g^2*h + (7*a*c^2*d - 4*a^2*c*f)*h^3)*x^2 + 105*(6*a*c^2*e*
g^2*h - a^2*c*e*h^3 + 2*(4*c^3*d + a*c^2*f)*g^3 + 3*(2*a*c^2*d - a^2*c*f)*g
*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/1680*(105*(6*a^2*c*e*g^2*h - a^3*e*h^3 - 2
*(4*a*c^2*d - a^2*c*f)*g^3 + 3*(2*a^2*c*d - a^3*f)*g*h^2)*sqrt(-c)*arctan(s
qrt(-c)*x/sqrt(c*x^2 + a)) + (240*c^3*f*h^3*x^6 + 560*a*c^2*e*g^3 - 672*a^2
*c*e*g*h^2 + 280*(3*c^3*f*g*h^2 + c^3*e*h^3)*x^5 + 48*(21*c^3*f*g^2*h + 21*
c^3*e*g*h^2 + (7*c^3*d + a*c^2*f)*h^3)*x^4 + 336*(5*a*c^2*d - 2*a^2*c*f)*g^
2*h - 32*(7*a^2*c*d - 4*a^3*f)*h^3 + 70*(6*c^3*f*g^3 + 18*c^3*e*g^2*h + a*c
^2*e*h^3 + 3*(6*c^3*d + a*c^2*f)*g*h^2)*x^3 + 16*(35*c^3*e*g^3 + 21*a*c^2*e
*g*h^2 + 21*(5*c^3*d + a*c^2*f)*g^2*h + (7*a*c^2*d - 4*a^2*c*f)*h^3)*x^2 +
105*(6*a*c^2*e*g^2*h - a^2*c*e*h^3 + 2*(4*c^3*d + a*c^2*f)*g^3 + 3*(2*a*c^2
*d - a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

giac [A] time = 0.23, size = 475, normalized size = 1.22

$$\frac{1}{1680}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\left(6\,fh^3x + \frac{7\left(3\,c^5fgh^2 + c^5h^3e\right)}{c^5}\right)x + \frac{6\left(21\,c^5fg^2h + 7\,c^5dh^3 + ac^4fh^3 + 21\,c^5gh^2e\right)}{c^5}\right)x + \right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1680*sqrt(c*x^2 + a)*((2*((4*(5*(6*f*h^3*x + 7*(3*c^5*f*g*h^2 + c^5*h^3*e
)/c^5)*x + 6*(21*c^5*f*g^2*h + 7*c^5*d*h^3 + a*c^4*f*h^3 + 21*c^5*g*h^2*e)/
c^5)*x + 35*(6*c^5*f*g^3 + 18*c^5*d*g*h^2 + 3*a*c^4*f*g*h^2 + 18*c^5*g^2*h*
e + a*c^4*h^3*e)/c^5)*x + 8*(105*c^5*d*g^2*h + 21*a*c^4*f*g^2*h + 7*a*c^4*d
*h^3 - 4*a^2*c^3*f*h^3 + 35*c^5*g^3*e + 21*a*c^4*g*h^2*e)/c^5)*x + 105*(8*c
^5*d*g^3 + 2*a*c^4*f*g^3 + 6*a*c^4*d*g*h^2 - 3*a^2*c^3*f*g*h^2 + 6*a*c^4*g^
2*h*e - a^2*c^3*h^3*e)/c^5)*x + 16*(105*a*c^4*d*g^2*h - 42*a^2*c^3*f*g^2*h
- 14*a^2*c^3*d*h^3 + 8*a^3*c^2*f*h^3 + 35*a*c^4*g^3*e - 42*a^2*c^3*g*h^2*e)
/c^5) - 1/16*(8*a*c^2*d*g^3 - 2*a^2*c*f*g^3 - 6*a^2*c*d*g*h^2 + 3*a^3*f*g*h
^2 - 6*a^2*c*g^2*h*e + a^3*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^
(5/2)
```

maple [A] time = 0.02, size = 661, normalized size = 1.69

$$\frac{\left(c\,x^2+a\right)^{\frac{3}{2}} f\,h^3x^4}{7c}+\frac{\left(c\,x^2+a\right)^{\frac{3}{2}} e\,h^3x^3}{6c}+\frac{\left(c\,x^2+a\right)^{\frac{3}{2}} f g\,h^2x^3}{2c}+\frac{a^3e\,h^3\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{16c^{\frac{5}{2}}}+\frac{3a^3f g\,h^2\ln\left(\sqrt{c}\;x\right.}{16c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

```
[Out] 1/3*(c*x^2+a)^(3/2)/c*e*g^3+1/2*d*g^3*x*(c*x^2+a)^(1/2)+3/5*x^2*(c*x^2+a)^(
3/2)/c*e*g*h^2-2/5*a/c^2*(c*x^2+a)^(3/2)*f*g^2*h+3/5*x^2*(c*x^2+a)^(3/2)/c*
f*g^2*h-2/5*a/c^2*(c*x^2+a)^(3/2)*e*g*h^2+3/4*x*(c*x^2+a)^(3/2)/c*d*g*h^2+3
/4*x*(c*x^2+a)^(3/2)/c*e*g^2*h-1/8*a/c*x*(c*x^2+a)^(1/2)*f*g^3-3/8*a^2/c^(3
/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*g^2*h-4/35*f*h^3*a/c^2*x^2*(c*x^2+a)^(3
/2)+1/2*x^3*(c*x^2+a)^(3/2)/c*f*g*h^2-1/8*a/c^2*x*(c*x^2+a)^(3/2)*e*h^3+1/1
6*a^2/c^2*x*(c*x^2+a)^(1/2)*e*h^3+3/16*a^3/c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(
1/2))*f*g*h^2-3/8*a^2/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*d*g*h^2-3/8*a/c
^2*x*(c*x^2+a)^(3/2)*f*g*h^2-3/8*a/c*x*(c*x^2+a)^(1/2)*e*g^2*h+3/16*a^2/c^2
*x*(c*x^2+a)^(1/2)*f*g*h^2-3/8*a/c*x*(c*x^2+a)^(1/2)*d*g*h^2+1/6*x^3*(c*x^2
+a)^(3/2)/c*e*h^3+1/16*a^3/c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*h^3+1/5*
x^2*(c*x^2+a)^(3/2)/c*d*h^3-2/15*a/c^2*(c*x^2+a)^(3/2)*d*h^3+1/4*x*(c*x^2+a
)^(3/2)/c*f*g^3-1/8*a^2/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*f*g^3+1/7*f*h
^3*x^4*(c*x^2+a)^(3/2)/c+8/105*f*h^3*a^2/c^3*(c*x^2+a)^(3/2)+(c*x^2+a)^(3/2
)/c*d*g^2*h+1/2*d*g^3*a/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
```

maxima [A] time = 0.47, size = 436, normalized size = 1.12

$$\frac{\left(cx^2+a\right)^{\frac{3}{2}} fh^3x^4}{7\,c}-\frac{4\left(cx^2+a\right)^{\frac{3}{2}} afh^3x^2}{35\,c^2}+\frac{1}{2}\sqrt{cx^2+a}\,dg^3x+\frac{adg^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}}+\frac{\left(cx^2+a\right)^{\frac{3}{2}} eg^3}{3\,c}+\frac{\left(cx^2+a\right)^{\frac{3}{2}} dg^2h}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(c*x^2 + a)^(3/2)*f*h^3*x^4/c - 4/35*(c*x^2 + a)^(3/2)*a*f*h^3*x^2/c^2
+ 1/2*sqrt(c*x^2 + a)*d*g^3*x + 1/2*a*d*g^3*arcsinh(c*x/sqrt(a*c))/sqrt(c)
+ 1/3*(c*x^2 + a)^(3/2)*e*g^3/c + (c*x^2 + a)^(3/2)*d*g^2*h/c + 8/105*(c*x^
2 + a)^(3/2)*a^2*f*h^3/c^3 + 1/6*(3*f*g*h^2 + e*h^3)*(c*x^2 + a)^(3/2)*x^3/
c + 1/5*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*(c*x^2 + a)^(3/2)*x^2/c - 1/8*(3*f*
g*h^2 + e*h^3)*(c*x^2 + a)^(3/2)*a*x/c^2 + 1/16*(3*f*g*h^2 + e*h^3)*sqrt(c*
x^2 + a)*a^2*x/c^2 + 1/4*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*(c*x^2 + a)^(3/2)*
x/c - 1/8*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*sqrt(c*x^2 + a)*a*x/c + 1/16*(3*f
*g*h^2 + e*h^3)*a^3*arcsinh(c*x/sqrt(a*c))/c^(5/2) - 1/8*(f*g^3 + 3*e*g^2*h
 + 3*d*g*h^2)*a^2*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/15*(3*f*g^2*h + 3*e*g*
h^2 + d*h^3)*(c*x^2 + a)^(3/2)*a/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g+h\,x\right)^3\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

sympy [A] time = 28.37, size = 1088, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] -a**(5/2)*e*h**3*x/(16*c**2*sqrt(1 + c*x**2/a)) - 3*a**(5/2)*f*g*h**2*x/(16
*c**2*sqrt(1 + c*x**2/a)) + 3*a**(3/2)*d*g*h**2*x/(8*c*sqrt(1 + c*x**2/a))
+ 3*a**(3/2)*e*g**2*h*x/(8*c*sqrt(1 + c*x**2/a)) - a**(3/2)*e*h**3*x**3/(48
*c*sqrt(1 + c*x**2/a)) + a**(3/2)*f*g**3*x/(8*c*sqrt(1 + c*x**2/a)) - a**(3
/2)*f*g*h**2*x**3/(16*c*sqrt(1 + c*x**2/a)) + sqrt(a)*d*g**3*x*sqrt(1 + c*x
**2/a)/2 + 9*sqrt(a)*d*g*h**2*x**3/(8*sqrt(1 + c*x**2/a)) + 9*sqrt(a)*e*g**
2*h*x**3/(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*e*h**3*x**5/(24*sqrt(1 + c*x**2
/a)) + 3*sqrt(a)*f*g**3*x**3/(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*f*g*h**2*x*
*5/(8*sqrt(1 + c*x**2/a)) + a**3*e*h**3*asinh(sqrt(c)*x/sqrt(a))/(16*c**(5/
2)) + 3*a**3*f*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(5/2)) - 3*a**2*d*g*h
**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) - 3*a**2*e*g**2*h*asinh(sqrt(c)*x
/sqrt(a))/(8*c**(3/2)) - a**2*f*g**3*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2))
+ a*d*g**3*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + 3*d*g**2*h*Piecewise((sqr
t(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + d*h**3*Piecewi
se((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x
**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + e*g**3*Piecewi
se((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + 3*e*g*h
**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)
/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 3*f
*g**2*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x
**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) +
 f*h**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a
+ c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**
2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 3*c*d*g*h**2*x**5/(4*sqrt(a)*sqr
t(1 + c*x**2/a)) + 3*c*e*g**2*h*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c*e*h
**3*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a)) + c*f*g**3*x**5/(4*sqrt(a)*sqrt(1 +
 c*x**2/a)) + c*f*g*h**2*x**7/(2*sqrt(a)*sqrt(1 + c*x**2/a))
```

3.79 $\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=280

$$\frac{x\sqrt{a+cx^2}\left(a^2fh^2-2ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)}{16c^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(a^2fh^2-2ac\left(h(dh+2eg)+fg^2\right)\right.}{16c^{5/2}}$$

```
[Out] -1/10*(-2*e*h+f*g)*(h*x+g)^2*(c*x^2+a)^(3/2)/c/h+1/6*f*(h*x+g)^3*(c*x^2+a)^
(3/2)/c/h-1/120*(16*a*h^2*(e*h+2*f*g)+8*c*g*(f*g^2-2*h*(5*d*h+e*g))-3*h*(5*
(-a*f+2*c*d)*h^2-2*c*g*(-2*e*h+f*g))*x)*(c*x^2+a)^(3/2)/c^2/h+1/16*a*(8*c^2
*d*g^2+a^2*f*h^2-2*a*c*(f*g^2+h*(d*h+2*e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(
1/2))/c^(5/2)+1/16*(8*c^2*d*g^2+a^2*f*h^2-2*a*c*(f*g^2+h*(d*h+2*e*g)))*x*(c
*x^2+a)^(1/2)/c^2
```

Rubi [A] time = 0.50, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 833, 780, 195, 217, 206}

$$\frac{x\sqrt{a+cx^2}\left(a^2fh^2-2ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)}{16c^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(a^2fh^2-2ac\left(h(dh+2eg)+fg^2\right)\right.}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c*(f*g^2 + h*(2*e*g + d*h)))*x*Sqrt[a + c*x
^2])/(16*c^2) - ((f*g - 2*e*h)*(g + h*x)^2*(a + c*x^2)^(3/2))/(10*c*h) + (f
*(g + h*x)^3*(a + c*x^2)^(3/2))/(6*c*h) - ((8*(c*f*g^3 - 2*c*g*h*(e*g + 5*d
*h) + 2*a*h^2*(2*f*g + e*h)) - 3*h*(5*(2*c*d - a*f)*h^2 - 2*c*g*(f*g - 2*e*
h))*x)*(a + c*x^2)^(3/2))/(120*c^2*h) + (a*(8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c
*(f*g^2 + h*(2*e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(5/
2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^2\sqrt{a+cx^2}\,\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch} + \frac{\int (g+hx)^2\left(3(2cd-af)h^2-3ch(fg-2eh)x\right.}{6ch^2}$$

$$= -\frac{(fg-2eh)(g+hx)^2\left(a+cx^2\right)^{3/2}}{10ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch} + \frac{\int (g+h}{}$$

$$= -\frac{(fg-2eh)(g+hx)^2\left(a+cx^2\right)^{3/2}}{10ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch} - \frac{\left(8\left(cfg\right.\right.}{}$$

$$= \frac{\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{(fg-2eh)(}{}$$

$$= \frac{\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{(fg-2eh)(}{}$$

$$= \frac{\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{(fg-2eh)(}{}$$

Mathematica [A] time = 0.65, size = 256, normalized size = 0.91

$$\frac{\sqrt{a+cx^2}\left(\sqrt{c}\left(a^2(-h)(32eh+64fg+15fhx)+2ac\left(5dh(16g+3hx)+e\left(40g^2+30ghx+8h^2x^2\right)+fx\left(15g^2+1\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(Sqrt[c]*(-(a^2*h*(64*f*g + 32*e*h + 15*f*h*x)) + 2*a*c*(5
*d*h*(16*g + 3*h*x) + f*x*(15*g^2 + 16*g*h*x + 5*h^2*x^2) + e*(40*g^2 + 30*
g*h*x + 8*h^2*x^2)) + 4*c^2*x*(5*d*(6*g^2 + 8*g*h*x + 3*h^2*x^2) + x*(2*e*(
10*g^2 + 15*g*h*x + 6*h^2*x^2) + f*x*(15*g^2 + 24*g*h*x + 10*h^2*x^2)))) +
```

```
(15*Sqrt[a]*(8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c*(f*g^2 + h*(2*e*g + d*h)))*Arc
Sinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a]))/(240*c^(5/2))
```

fricas [A] time = 1.05, size = 595, normalized size = 2.12

$$\left[-\frac{15\left(4\,a^2cegh-2\left(4\,ac^2d-a^2cf\right)g^2+\left(2\,a^2cd-a^3f\right)h^2\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\left(40\,c^3fh^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/480*(15*(4*a^2*c*e*g*h - 2*(4*a*c^2*d - a^2*c*f)*g^2 + (2*a^2*c*d - a^3
*f)*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(40*c^
3*f*h^2*x^5 + 80*a*c^2*e*g^2 - 32*a^2*c*e*h^2 + 48*(2*c^3*f*g*h + c^3*e*h^2
)*x^4 + 10*(6*c^3*f*g^2 + 12*c^3*e*g*h + (6*c^3*d + a*c^2*f)*h^2)*x^3 + 32*
(5*a*c^2*d - 2*a^2*c*f)*g*h + 16*(5*c^3*e*g^2 + a*c^2*e*h^2 + 2*(5*c^3*d +
a*c^2*f)*g*h)*x^2 + 15*(4*a*c^2*e*g*h + 2*(4*c^3*d + a*c^2*f)*g^2 + (2*a*c^
2*d - a^2*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/240*(15*(4*a^2*c*e*g*h - 2*(
4*a*c^2*d - a^2*c*f)*g^2 + (2*a^2*c*d - a^3*f)*h^2)*sqrt(-c)*arctan(sqrt(-c
)*x/sqrt(c*x^2 + a)) + (40*c^3*f*h^2*x^5 + 80*a*c^2*e*g^2 - 32*a^2*c*e*h^2
+ 48*(2*c^3*f*g*h + c^3*e*h^2)*x^4 + 10*(6*c^3*f*g^2 + 12*c^3*e*g*h + (6*c^
3*d + a*c^2*f)*h^2)*x^3 + 32*(5*a*c^2*d - 2*a^2*c*f)*g*h + 16*(5*c^3*e*g^2
+ a*c^2*e*h^2 + 2*(5*c^3*d + a*c^2*f)*g*h)*x^2 + 15*(4*a*c^2*e*g*h + 2*(4*c
^3*d + a*c^2*f)*g^2 + (2*a*c^2*d - a^2*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

giac [A] time = 0.21, size = 321, normalized size = 1.15

$$\frac{1}{240}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\,fh^2x+\frac{6\left(2\,c^4fgh+c^4h^2e\right)}{c^4}\right)x+\frac{5\left(6\,c^4fg^2+6\,c^4dh^2+ac^3fh^2+12\,c^4ghe\right)}{c^4}\right)x+\frac{8\left(10\right.}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/240*sqrt(c*x^2 + a)*((2*((4*(5*f*h^2*x + 6*(2*c^4*f*g*h + c^4*h^2*e)/c^4)
*x + 5*(6*c^4*f*g^2 + 6*c^4*d*h^2 + a*c^3*f*h^2 + 12*c^4*g*h*e)/c^4)*x + 8*
(10*c^4*d*g*h + 2*a*c^3*f*g*h + 5*c^4*g^2*e + a*c^3*h^2*e)/c^4)*x + 15*(8*c
^4*d*g^2 + 2*a*c^3*f*g^2 + 2*a*c^3*d*h^2 - a^2*c^2*f*h^2 + 4*a*c^3*g*h*e)/c
^4)*x + 16*(10*a*c^3*d*g*h - 4*a^2*c^2*f*g*h + 5*a*c^3*g^2*e - 2*a^2*c^2*h^
2*e)/c^4) - 1/16*(8*a*c^2*d*g^2 - 2*a^2*c*f*g^2 - 2*a^2*c*d*h^2 + a^3*f*h^2
 - 4*a^2*c*g*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

maple [A] time = 0.01, size = 446, normalized size = 1.59

$$\frac{\left(c\,x^2+a\right)^{\frac{3}{2}} f\,h^2x^3}{6c}+\frac{a^3f\,h^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{16c^{\frac{5}{2}}}-\frac{a^2d\,h^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{8c^{\frac{3}{2}}}-\frac{a^2egh\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right.}{4c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

```
[Out] 1/6*f*h^2*x^3*(c*x^2+a)^(3/2)/c-1/8*f*h^2*a/c^2*x*(c*x^2+a)^(3/2)+1/16*f*h^
2*a^2/c^2*x*(c*x^2+a)^(1/2)+1/16*f*h^2*a^3/c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(
1/2))+1/5*x^2*(c*x^2+a)^(3/2)/c*e*h^2+2/5*x^2*(c*x^2+a)^(3/2)/c*f*g*h-2/15*
a/c^2*(c*x^2+a)^(3/2)*e*h^2-4/15*a/c^2*(c*x^2+a)^(3/2)*f*g*h+1/4*x*(c*x^2+a
)^(3/2)/c*d*h^2+1/2*x*(c*x^2+a)^(3/2)/c*e*g*h+1/4*x*(c*x^2+a)^(3/2)/c*f*g^2
-1/8*a/c*x*(c*x^2+a)^(1/2)*d*h^2-1/4*a/c*x*(c*x^2+a)^(1/2)*e*g*h-1/8*a/c*x*
```

```
(c*x^2+a)^(1/2)*f*g^2-1/8*a^2/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*d*h^2-1
/4*a^2/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*g*h-1/8*a^2/c^(3/2)*ln(c^(1/
2)*x+(c*x^2+a)^(1/2))*f*g^2+2/3*(c*x^2+a)^(3/2)/c*d*g*h+1/3*(c*x^2+a)^(3/2)
/c*e*g^2+1/2*d*g^2*x*(c*x^2+a)^(1/2)+1/2*d*g^2*a/c^(1/2)*ln(c^(1/2)*x+(c*x^
2+a)^(1/2))
```

maxima [A] time = 0.46, size = 305, normalized size = 1.09

$$\frac{\left(cx^2+a\right)^{\frac{3}{2}}fh^2x^3}{6\,c}+\frac{1}{2}\sqrt{cx^2+a}\,dg^2x-\frac{\left(cx^2+a\right)^{\frac{3}{2}}afh^2x}{8\,c^2}+\frac{\sqrt{cx^2+a}\,a^2fh^2x}{16\,c^2}+\frac{adg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}}+\frac{a^3fh^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{a\dots}}\right.}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2 + a)^(3/2)*f*h^2*x^3/c + 1/2*sqrt(c*x^2 + a)*d*g^2*x - 1/8*(c*x^
2 + a)^(3/2)*a*f*h^2*x/c^2 + 1/16*sqrt(c*x^2 + a)*a^2*f*h^2*x/c^2 + 1/2*a*d
*g^2*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 1/16*a^3*f*h^2*arcsinh(c*x/sqrt(a*c))
/c^(5/2) + 1/3*(c*x^2 + a)^(3/2)*e*g^2/c + 2/3*(c*x^2 + a)^(3/2)*d*g*h/c +
1/5*(2*f*g*h + e*h^2)*(c*x^2 + a)^(3/2)*x^2/c + 1/4*(f*g^2 + 2*e*g*h + d*h^
2)*(c*x^2 + a)^(3/2)*x/c - 1/8*(f*g^2 + 2*e*g*h + d*h^2)*sqrt(c*x^2 + a)*a*
x/c - 1/8*(f*g^2 + 2*e*g*h + d*h^2)*a^2*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/
15*(2*f*g*h + e*h^2)*(c*x^2 + a)^(3/2)*a/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g+h\,x\right)^2\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

sympy [A] time = 21.00, size = 738, normalized size = 2.64

$$-\frac{a^{\frac{5}{2}}fh^2x}{16c^2\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}dh^2x}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}eghx}{4c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}fg^2x}{8c\sqrt{1+\frac{cx^2}{a}}}-\frac{a^{\frac{3}{2}}fh^2x^3}{48c\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}\,dg^2x\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3\sqrt{a}\,dh^2x^3}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}\,e\dots}{4\sqrt{1\dots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] -a**(5/2)*f*h**2*x/(16*c**2*sqrt(1 + c*x**2/a)) + a**(3/2)*d*h**2*x/(8*c*sq
rt(1 + c*x**2/a)) + a**(3/2)*e*g*h*x/(4*c*sqrt(1 + c*x**2/a)) + a**(3/2)*f*
g**2*x/(8*c*sqrt(1 + c*x**2/a)) - a**(3/2)*f*h**2*x**3/(48*c*sqrt(1 + c*x**
2/a)) + sqrt(a)*d*g**2*x*sqrt(1 + c*x**2/a)/2 + 3*sqrt(a)*d*h**2*x**3/(8*sq
rt(1 + c*x**2/a)) + 3*sqrt(a)*e*g*h*x**3/(4*sqrt(1 + c*x**2/a)) + 3*sqrt(a)
*f*g**2*x**3/(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*f*h**2*x**5/(24*sqrt(1 + c*
x**2/a)) + a**3*f*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(5/2)) - a**2*d*h**2
*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) - a**2*e*g*h*asinh(sqrt(c)*x/sqrt(a)
)/(4*c**(3/2)) - a**2*f*g**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) + a*d*g*
*2*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + 2*d*g*h*Piecewise((sqrt(a)*x**2/2
, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + e*g**2*Piecewise((sqrt(a)
*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + e*h**2*Piecewise((
-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*
sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 2*f*g*h*Piecewise(
```

```
(-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4
*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c*d*h**2*x**5/(4*
sqrt(a)*sqrt(1 + c*x**2/a)) + c*e*g*h*x**5/(2*sqrt(a)*sqrt(1 + c*x**2/a)) +
 c*f*g**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c*f*h**2*x**7/(6*sqrt(a)*sq
rt(1 + c*x**2/a))
```

3.80 $\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=175

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-aeh-afg+4cdg)}{8c^{3/2}}-\frac{\left(a+cx^2\right)^{3/2}\left(4\left(2afh^2+c\left(3fg^2-5h(dh+eg)\right)\right)+3chx(3fg-5eh)\right)}{60c^2h}+$$

```
[Out] 1/5*f*(h*x+g)^2*(c*x^2+a)^(3/2)/c/h-1/60*(8*a*f*h^2+4*c*(3*f*g^2-5*h*(d*h+e
*g))+3*c*h*(-5*e*h+3*f*g)*x)*(c*x^2+a)^(3/2)/c^2/h+1/8*a*(-a*e*h-a*f*g+4*c*
d*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+1/8*(4*c*d*g-a*(e*h+f*g))*x
*(c*x^2+a)^(1/2)/c
```

Rubi [A] time = 0.27, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1654, 780, 195, 217, 206}

$$-\frac{\left(a+cx^2\right)^{3/2}\left(4\left(2afh^2+c\left(3fg^2-5h(dh+eg)\right)\right)+3chx(3fg-5eh)\right)}{60c^2h}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-aeh-afg+4cdg)}{8c^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((4*c*d*g - a*(f*g + e*h))*x*Sqrt[a + c*x^2])/(8*c) + (f*(g + h*x)^2*(a + c
*x^2)^(3/2))/(5*c*h) - ((4*(2*a*f*h^2 + c*(3*f*g^2 - 5*h*(e*g + d*h))) + 3*
c*h*(3*f*g - 5*e*h)*x)*(a + c*x^2)^(3/2))/(60*c^2*h) + (a*(4*c*d*g - a*f*g
- a*e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
```

```
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch} + \frac{\int (g+hx)\left((5cd-2af)h^2-ch(3fg-5eh)\right.}{5ch^2}$$

$$= \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch} - \frac{\left(4\left(2afh^2+c\left(3fg^2-5h(eg+dh)\right)\right)+3ch\right.}{60c^2h}$$

$$= \frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c} + \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch} - \frac{\left(4\left(2afh\right.\right.}{}$$

$$= \frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c} + \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch} - \frac{\left(4\left(2afh\right.\right.}{}$$

$$= \frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c} + \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch} - \frac{\left(4\left(2afh\right.\right.}{}$$

Mathematica [A] time = 0.42, size = 153, normalized size = 0.87

$$\frac{\sqrt{a+cx^2}\left(-16a^2fh-\frac{15\sqrt{a}\sqrt{c}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aeh+afg-4cdg)}{\sqrt{\frac{cx^2}{a}+1}}+ac(40dh+5e(8g+3hx)+fx(15g+8hx))+2c^2x(10d(\right.}{120c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-16*a^2*f*h + a*c*(40*d*h + 5*e*(8*g + 3*h*x) + f*x*(15*g
 + 8*h*x)) + 2*c^2*x*(10*d*(3*g + 2*h*x) + x*(5*e*(4*g + 3*h*x) + 3*f*x*(5*
g + 4*h*x))) - (15*Sqrt[a]*Sqrt[c]*(-4*c*d*g + a*f*g + a*e*h)*ArcSinh[(Sqrt
[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a]))/(120*c^2)
```

fricas [A] time = 1.00, size = 329, normalized size = 1.88

$$\left[\frac{15\left(a^2eh-\left(4acd-a^2f\right)g\right)\sqrt{c}\log\left(-2cx^2+2\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(24c^2fhx^4+40aceg+30\left(c^2fg+c^2eh\right.\right.}{240c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*(a^2*e*h - (4*a*c*d - a^2*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*
x^2 + a)*sqrt(c)*x - a) + 2*(24*c^2*f*h*x^4 + 40*a*c*e*g + 30*(c^2*f*g + c^
2*e*h)*x^3 + 8*(5*c^2*e*g + (5*c^2*d + a*c*f)*h)*x^2 + 8*(5*a*c*d - 2*a^2*f
)*h + 15*(a*c*e*h + (4*c^2*d + a*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2, 1/120*(15
*(a^2*e*h - (4*a*c*d - a^2*f)*g)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)
```

```
) + (24*c^2*f*h*x^4 + 40*a*c*e*g + 30*(c^2*f*g + c^2*e*h)*x^3 + 8*(5*c^2*e*
g + (5*c^2*d + a*c*f)*h)*x^2 + 8*(5*a*c*d - 2*a^2*f)*h + 15*(a*c*e*h + (4*c
^2*d + a*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2]
```

giac [A] time = 0.21, size = 180, normalized size = 1.03

$$\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(4\,fhx+\frac{5\left(c^3fg+c^3he\right)}{c^3}\right)x+\frac{4\left(5\,c^3dh+ac^2fh+5\,c^3ge\right)}{c^3}\right)x+\frac{15\left(4\,c^3dg+ac^2fg+ac^2he\right)}{c^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(c*x^2 + a)*((2*(3*(4*f*h*x + 5*(c^3*f*g + c^3*h*e)/c^3)*x + 4*(5
*c^3*d*h + a*c^2*f*h + 5*c^3*g*e)/c^3)*x + 15*(4*c^3*d*g + a*c^2*f*g + a*c^
2*h*e)/c^3)*x + 8*(5*a*c^2*d*h - 2*a^2*c*f*h + 5*a*c^2*g*e)/c^3) - 1/8*(4*a
*c*d*g - a^2*f*g - a^2*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

maple [A] time = 0.01, size = 230, normalized size = 1.31

$$-\frac{a^2eh\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{8c^{\frac{3}{2}}}-\frac{a^2fg\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{8c^{\frac{3}{2}}}+\frac{adg\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}-\frac{\sqrt{c\,x^2+a}\,aehx}{8c}-\frac{\sqrt{c\,x^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

```
[Out] 1/5*h*f*x^2*(c*x^2+a)^(3/2)/c-2/15*h*f*a/c^2*(c*x^2+a)^(3/2)+1/4*x*(c*x^2+a
)^(3/2)/c*e*h+1/4*x*(c*x^2+a)^(3/2)/c*f*g-1/8*a/c*x*(c*x^2+a)^(1/2)*e*h-1/8
*a/c*x*(c*x^2+a)^(1/2)*f*g-1/8*a^2/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*
h-1/8*a^2/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*f*g+1/3*(c*x^2+a)^(3/2)/c*d
*h+1/3*(c*x^2+a)^(3/2)/c*e*g+1/2*d*g*x*(c*x^2+a)^(1/2)+1/2*d*g*a/c^(1/2)*ln
(c^(1/2)*x+(c*x^2+a)^(1/2))
```

maxima [A] time = 0.45, size = 169, normalized size = 0.97

$$\frac{\left(cx^2+a\right)^{\frac{3}{2}}fhx^2}{5\,c}+\frac{1}{2}\sqrt{cx^2+a}\,dgx+\frac{adg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}eg}{3\,c}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}dh}{3\,c}-\frac{2\left(cx^2+a\right)^{\frac{3}{2}}afh}{15\,c^2}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(c*x^2 + a)^(3/2)*f*h*x^2/c + 1/2*sqrt(c*x^2 + a)*d*g*x + 1/2*a*d*g*arc
sinh(c*x/sqrt(a*c))/sqrt(c) + 1/3*(c*x^2 + a)^(3/2)*e*g/c + 1/3*(c*x^2 + a)
^(3/2)*d*h/c - 2/15*(c*x^2 + a)^(3/2)*a*f*h/c^2 + 1/4*(c*x^2 + a)^(3/2)*(f*
g + e*h)*x/c - 1/8*sqrt(c*x^2 + a)*(f*g + e*h)*a*x/c - 1/8*(f*g + e*h)*a^2*
arcsinh(c*x/sqrt(a*c))/c^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(g+h\,x\right)\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

sympy [A] time = 11.88, size = 384, normalized size = 2.19

$$\frac{a^{\frac{3}{2}}ehx}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}fgx}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}\,dgx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3\sqrt{a}\,ehx^3}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}\,fgx^3}{8\sqrt{1+\frac{cx^2}{a}}}-\frac{a^2eh\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}}-\frac{a^2fg\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] a**(3/2)*e*h*x/(8*c*sqrt(1 + c*x**2/a)) + a**(3/2)*f*g*x/(8*c*sqrt(1 + c*x*
*2/a)) + sqrt(a)*d*g*x*sqrt(1 + c*x**2/a)/2 + 3*sqrt(a)*e*h*x**3/(8*sqrt(1
+ c*x**2/a)) + 3*sqrt(a)*f*g*x**3/(8*sqrt(1 + c*x**2/a)) - a**2*e*h*asinh(s
qrt(c)*x/sqrt(a))/(8*c**(3/2)) - a**2*f*g*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3
/2)) + a*d*g*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + d*h*Piecewise((sqrt(a)*
x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + e*g*Piecewise((sqrt
(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + f*h*Piecewise((
-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*
sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c*e*h*x**5/(4*sqrt
(a)*sqrt(1 + c*x**2/a)) + c*f*g*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a))
```

3.81 $\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=106

$$\frac{a(4cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}+\frac{x\sqrt{a+cx^2}(4cd-af)}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}$$

```
[Out] 1/3*e*(c*x^2+a)^(3/2)/c+1/4*f*x*(c*x^2+a)^(3/2)/c+1/8*a*(-a*f+4*c*d)*arctan
h(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+1/8*(-a*f+4*c*d)*x*(c*x^2+a)^(1/2)/c
```

Rubi [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1815, 641, 195, 217, 206}

$$\frac{a(4cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}+\frac{x\sqrt{a+cx^2}(4cd-af)}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((4*c*d - a*f)*x*Sqrt[a + c*x^2])/(8*c) + (e*(a + c*x^2)^(3/2))/(3*c) + (f*
x*(a + c*x^2)^(3/2))/(4*c) + (a*(4*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/Sqrt[a +
c*x^2]])/(8*c^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)dx = \frac{fx\left(a+cx^2\right)^{3/2}}{4c}+\frac{\int (4cd-af+4cex)\sqrt{a+cx^2}\,dx}{4c}$$
$$=\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}+\frac{(4cd-af)\int \sqrt{a+cx^2}\,dx}{4c}$$
$$=\frac{(4cd-af)x\sqrt{a+cx^2}}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}+\frac{(a(4cd-af))\int \frac{}{}}{8c}$$
$$=\frac{(4cd-af)x\sqrt{a+cx^2}}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}+\frac{(a(4cd-af))\mathrm{Su}}{}$$
$$=\frac{(4cd-af)x\sqrt{a+cx^2}}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}+\frac{a(4cd-af)\tanh}{8c^{3/2}}$$

Mathematica [A] time = 0.20, size = 98, normalized size = 0.92

$$\frac{\sqrt{a+cx^2}\left(\sqrt{c}\,(a(8e+3fx)+2cx(6d+x(4e+3fx)))-\frac{3\sqrt{a}\,(af-4cd)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{cx^2}{a}+1}}\right)}{24c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(Sqrt[c]*(a*(8*e + 3*f*x) + 2*c*x*(6*d + x*(4*e + 3*f*x)))
 - (3*Sqrt[a]*(-4*c*d + a*f)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)
/a]))/(24*c^(3/2))
```

fricas [A] time = 0.95, size = 190, normalized size = 1.79

$$\left[-\frac{3\left(4\,acd-a^2f\right)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\left(6\,c^2fx^3+8\,c^2ex^2+8\,ace+3\left(4\,c^2d+acf\right)x\right)\sqrt{c}}{48\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(4*a*c*d - a^2*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c
)*x - a) - 2*(6*c^2*f*x^3 + 8*c^2*e*x^2 + 8*a*c*e + 3*(4*c^2*d + a*c*f)*x)*
sqrt(c*x^2 + a))/c^2, -1/24*(3*(4*a*c*d - a^2*f)*sqrt(-c)*arctan(sqrt(-c)*x
/sqrt(c*x^2 + a)) - (6*c^2*f*x^3 + 8*c^2*e*x^2 + 8*a*c*e + 3*(4*c^2*d + a*c
*f)*x)*sqrt(c*x^2 + a))/c^2]
```

giac [A] time = 0.18, size = 87, normalized size = 0.82

$$\frac{1}{24}\,\sqrt{cx^2+a}\left(\left(2\left(3\,fx+4\,e\right)x+\frac{3\left(4\,c^2d+acf\right)}{c^2}\right)x+\frac{8\,ae}{c}\right)-\frac{\left(4\,acd-a^2f\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{8\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + a)*((2*(3*f*x + 4*e)*x + 3*(4*c^2*d + a*c*f)/c^2)*x + 8*a
*e/c) - 1/8*(4*a*c*d - a^2*f)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2
)
```

maple [A] time = 0.00, size = 111, normalized size = 1.05

$$-\frac{a^2 f \ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{8c^{\frac{3}{2}}}+\frac{ad \ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}-\frac{\sqrt{c\,x^2+a}\,afx}{8c}+\frac{\sqrt{c\,x^2+a}\,dx}{2}+\frac{\left(c\,x^2+a\right)^{\frac{3}{2}} fx}{4c}+\frac{\left(c\,x^2+\right.}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

```
[Out] 1/4*f*x*(c*x^2+a)^(3/2)/c-1/8*f*a/c*x*(c*x^2+a)^(1/2)-1/8*f*a^2/c^(3/2)*ln(
c^(1/2)*x+(c*x^2+a)^(1/2))+1/3*e*(c*x^2+a)^(3/2)/c+1/2*d*x*(c*x^2+a)^(1/2)+
1/2*d*a/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
```

maxima [A] time = 0.45, size = 96, normalized size = 0.91

$$\frac{1}{2}\sqrt{cx^2+a}\,dx+\frac{\left(cx^2+a\right)^{\frac{3}{2}} fx}{4\,c}-\frac{\sqrt{cx^2+a}\,afx}{8\,c}+\frac{ad\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}}-\frac{a^2 f\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{3}{2}}}+\frac{\left(cx^2+a\right)^{\frac{3}{2}} e}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*d*x + 1/4*(c*x^2 + a)^(3/2)*f*x/c - 1/8*sqrt(c*x^2 + a)
*a*f*x/c + 1/2*a*d*arcsinh(c*x/sqrt(a*c))/sqrt(c) - 1/8*a^2*f*arcsinh(c*x/s
qrt(a*c))/c^(3/2) + 1/3*(c*x^2 + a)^(3/2)*e/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

sympy [A] time = 6.90, size = 170, normalized size = 1.60

$$\frac{a^{\frac{3}{2}} fx}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}\,dx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3\sqrt{a}\,fx^3}{8\sqrt{1+\frac{cx^2}{a}}}-\frac{a^2 f\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}}+\frac{ad\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\sqrt{c}}+e\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for } c=0\\ \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)+\frac{}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] a**(3/2)*f*x/(8*c*sqrt(1 + c*x**2/a)) + sqrt(a)*d*x*sqrt(1 + c*x**2/a)/2 +
3*sqrt(a)*f*x**3/(8*sqrt(1 + c*x**2/a)) - a**2*f*asinh(sqrt(c)*x/sqrt(a))/(
8*c**(3/2)) + a*d*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + e*Piecewise((sqrt(
a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + c*f*x**5/(4*sqrt
(a)*sqrt(1 + c*x**2/a))
```

3.82 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{g+hx} dx$

Optimal. Leaf size=206

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(\left(ah^2+2cg^2\right)(fg-eh)+2cdgh^2\right)}{2\sqrt{c}h^4}-\frac{\sqrt{ah^2+cg^2}\left(dh^2-egh+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h^4}$$

```
[Out] 1/3*f*(c*x^2+a)^(3/2)/c/h-1/2*(2*c*d*g*h^2+(-e*h+f*g)*(a*h^2+2*c*g^2))*arct
anh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^4/c^(1/2)-(d*h^2-e*g*h+f*g^2)*arctanh((-c*
g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))*(a*h^2+c*g^2)^(1/2)/h^4+1/2*(
2*d*h^2-2*e*g*h+2*f*g^2-h*(-e*h+f*g)*x)*(c*x^2+a)^(1/2)/h^3
```

Rubi [A] time = 0.39, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 815, 844, 217, 206, 725}

$$\frac{\sqrt{a+cx^2}\left(2\left(dh^2-egh+fg^2\right)-hx(fg-eh)\right)}{2h^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(\left(ah^2+2cg^2\right)(fg-eh)+2cdgh^2\right)}{2\sqrt{c}h^4}-\frac{\sqrt{ah^2+c}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((2*(f*g^2 - e*g*h + d*h^2) - h*(f*g - e*h)*x)*Sqrt[a + c*x^2])/(2*h^3) + (
f*(a + c*x^2)^(3/2))/(3*c*h) - ((2*c*d*g*h^2 + (f*g - e*h)*(2*c*g^2 + a*h^2
))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*h^4) - (Sqrt[c*g^2 + a*
h^2]*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2]])])/h^4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
```

```
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{g+hx}\,dx = \frac{f\left(a+cx^2\right)^{3/2}}{3ch} + \frac{\int \frac{\left(3cdh^2-3ch(fg-eh)x\right)\sqrt{a+cx^2}}{g+hx}\,dx}{3ch^2}$$

$$= \frac{\left(2\left(fg^2-egh+dh^2\right)-h(fg-eh)x\right)\sqrt{a+cx^2}}{2h^3} + \frac{f\left(a+cx^2\right)^{3/2}}{3ch} + \frac{\int \frac{3ac^2h^2\left(fg^2-\right.}{}}{}$$

$$= \frac{\left(2\left(fg^2-egh+dh^2\right)-h(fg-eh)x\right)\sqrt{a+cx^2}}{2h^3} + \frac{f\left(a+cx^2\right)^{3/2}}{3ch} + \frac{\left(\left(cg^2+ah^2\right)\right.}{}$$

$$= \frac{\left(2\left(fg^2-egh+dh^2\right)-h(fg-eh)x\right)\sqrt{a+cx^2}}{2h^3} + \frac{f\left(a+cx^2\right)^{3/2}}{3ch} - \frac{\left(\left(cg^2+ah^2\right)\right.}{}$$

$$= \frac{\left(2\left(fg^2-egh+dh^2\right)-h(fg-eh)x\right)\sqrt{a+cx^2}}{2h^3} + \frac{f\left(a+cx^2\right)^{3/2}}{3ch} - \frac{\left(2cdgh^2+(f\right.}{}$$

Mathematica [A] time = 0.44, size = 224, normalized size = 1.09

$$\frac{\left(h(dh-eg)+fg^2\right)\left(-\sqrt{ah^2+cg^2}\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)-\sqrt{c}\,g\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)+h\sqrt{a+cx^2}\right)}{h^4}+\frac{\sqrt{a+cx^2}\left(\sqrt{\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] (f*(a + c*x^2)^(3/2))/(3*c*h) + ((-(f*g) + e*h)*Sqrt[a + c*x^2]*(Sqrt[c]*x*
Sqrt[1 + (c*x^2)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/(2*Sqrt[c]*h^2
*Sqrt[1 + (c*x^2)/a]) + ((f*g^2 + h*(-(e*g) + d*h))*(h*Sqrt[a + c*x^2] - Sq
rt[c]*g*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[c*g^2 + a*h^2]*ArcTanh[
(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])]))/h^4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.22, size = 278, normalized size = 1.35

$$\frac{1}{6}\sqrt{cx^2+a}\left(\left(\frac{2\,fx}{h}-\frac{3\left(cfgh^8-ch^9e\right)}{ch^{10}}\right)x+\frac{2\left(3\,cfg^2h^7+3\,cdh^9+afh^9-3\,cgh^8e\right)}{ch^{10}}\right)+\frac{2\left(cfg^4+cdg^2h^2+afg^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2 + a)*((2*f*x/h - 3*(c*f*g*h^8 - c*h^9*e)/(c*h^10))*x + 2*(3*
c*f*g^2*h^7 + 3*c*d*h^9 + a*f*h^9 - 3*c*g*h^8*e)/(c*h^10)) + 2*(c*f*g^4 + c
*d*g^2*h^2 + a*f*g^2*h^2 + a*d*h^4 - c*g^3*h*e - a*g*h^3*e)*arctan(-((sqrt(
c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 -
 a*h^2)*h^4) + 1/2*(2*c^(3/2)*f*g^3 + 2*c^(3/2)*d*g*h^2 + a*sqrt(c)*f*g*h^2
 - 2*c^(3/2)*g^2*h*e - a*sqrt(c)*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a
)))/(c*h^4)
```

maple [B] time = 0.02, size = 1265, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x)
```

```
[Out] 1/3*f*(c*x^2+a)^(3/2)/c/h+1/2/h*e*x*(c*x^2+a)^(1/2)+1/2/h*e*a/c^(1/2)*ln(c^
(1/2)*x+(c*x^2+a)^(1/2))-1/2/h^2*f*g*x*(c*x^2+a)^(1/2)-1/2/h^2*f*g*a/c^(1/2
)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+1/h*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g
^2)/h^2)^(1/2)*d-1/h^2*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2
)*e*g+1/h^3*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*f*g^2-1/h
^2*c^(1/2)*g*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*
h^2+c*g^2)/h^2)^(1/2))*d+1/h^3*c^(1/2)*g^2*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((
x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*e-1/h^4*c^(1/2)*g^3*ln
((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)
^(1/2))*f-1/h/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+
g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)
/h^2)^(1/2))/(x+g/h))*a*d+1/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^
2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+
g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*e*g-1/h^3/((a*h^2+c*g^2)/h^2)^(1/
2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+
g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*f*g^2-1/h^3/(
(a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+
c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x
+g/h))*c*g^2*d+1/h^4/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*
g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2
+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^3*e-1/h^5/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*
(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-
2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^4*f
```

maxima [A] time = 0.61, size = 362, normalized size = 1.76

$$-\frac{\sqrt{cx^2+a}\,fgx}{2\,h^2}+\frac{\sqrt{cx^2+a}\,ex}{2\,h}-\frac{\sqrt{c}\,fg^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4}+\frac{\sqrt{c}\,eg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^3}-\frac{\sqrt{c}\,dg\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^2}-\frac{afg\operatorname{arsinh}\left(\frac{c}{\sqrt{}}\right.}{2\sqrt{c}\,h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^2 + a)*f*g*x/h^2 + 1/2*sqrt(c*x^2 + a)*e*x/h - sqrt(c)*f*g^3*
arcsinh(c*x/sqrt(a*c))/h^4 + sqrt(c)*e*g^2*arcsinh(c*x/sqrt(a*c))/h^3 - sqr
t(c)*d*g*arcsinh(c*x/sqrt(a*c))/h^2 - 1/2*a*f*g*arcsinh(c*x/sqrt(a*c))/(sqr
t(c)*h^2) + 1/2*a*e*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h) + sqrt(a + c*g^2/h^2
)*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g
)))/h^3 - sqrt(a + c*g^2/h^2)*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
a*h/(sqrt(a*c)*abs(h*x + g)))/h^2 + sqrt(a + c*g^2/h^2)*d*arcsinh(c*g*x/(sq
rt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h + sqrt(c*x^2 + a)*f
*g^2/h^3 - sqrt(c*x^2 + a)*e*g/h^2 + sqrt(c*x^2 + a)*d/h + 1/3*(c*x^2 + a)^
(3/2)*f/(c*h)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

```
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\,\left(d+ex+fx^2\right)}{g+hx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x), x)
```

3.83 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx$

Optimal. Leaf size=308

$$-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(afh^2+2c\left(3fg^2-h(2eg-dh)\right)\right)}{2\sqrt{c}h^4}+\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right.}{}$$

```
[Out] -(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)+1/2*(a*f*h^2+2
*c*(3*f*g^2-h*(-d*h+2*e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^4/c^(1/2)
+(a*h^2*(-e*h+2*f*g)+c*g*(3*f*g^2-h*(-d*h+2*e*g)))*arctanh((-c*g*x+a*h)/(a*
h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^4/(a*h^2+c*g^2)^(1/2)-1/2*(2*a*h^2*(-e*
h+2*f*g)+2*c*g*(3*f*g^2-h*(-d*h+2*e*g))-h*(a*f*h^2+c*(3*f*g^2-2*h*(-d*h+e*g
)))*x)*(c*x^2+a)^(1/2)/h^3/(a*h^2+c*g^2)
```

Rubi [A] time = 0.51, antiderivative size = 303, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 815, 844, 217, 206, 725}

$$-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}-\frac{\sqrt{a+cx^2}\left(2\left(ah^2(2fg-eh)-cgh(2eg-dh)+3cfg^3\right)-hx\left(afh^2-2ch(eg-\right.\right.}{2h^3\left(ah^2+cg^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -((2*(3*c*f*g^3 - c*g*h*(2*e*g - d*h) + a*h^2*(2*f*g - e*h)) - h*(3*c*f*g^2
 + a*f*h^2 - 2*c*h*(e*g - d*h))*x)*Sqrt[a + c*x^2])/(2*h^3*(c*g^2 + a*h^2))
 - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(h*(c*g^2 + a*h^2)*(g + h*x)
) + ((6*c*f*g^2 + a*f*h^2 - 2*c*h*(2*e*g - d*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a
 + c*x^2]])/(2*Sqrt[c]*h^4) + ((3*c*f*g^3 - c*g*h*(2*e*g - d*h) + a*h^2*(2*
f*g - e*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(
h^4*Sqrt[c*g^2 + a*h^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{h\left(cg^2+ah^2\right)(g+hx)} - \frac{\int \frac{\left(-cdg+afg-aeh-\left(afh-c\left(2eg-\frac{3fg^2}{h}-2dh\right)\right)x\right)\sqrt{a+cx^2}}{g+hx}\ldots}{cg^2+ah^2}$$

$$= -\frac{\left(2\left(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh)\right)-h\left(3cfg^2+afh^2-2ch(eg-dh\ldots\right.\right.}{2h^3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(2\left(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh)\right)-h\left(3cfg^2+afh^2-2ch(eg-dh\ldots\right.\right.}{2h^3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(2\left(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh)\right)-h\left(3cfg^2+afh^2-2ch(eg-dh\ldots\right.\right.}{2h^3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(2\left(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh)\right)-h\left(3cfg^2+afh^2-2ch(eg-dh\ldots\right.\right.}{2h^3\left(cg^2+ah^2\right)}$$

Mathematica [**A**] time = 0.26, size = 264, normalized size = 0.86

$$\frac{\frac{h\sqrt{a+cx^2}\left(2h(-dh+2eg+ehx)+f\left(-6g^2-3ghx+h^2x^2\right)\right)}{g+hx}+\frac{\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)\left(afh^2+2ch(dh-2eg)+6cfg^2\right)}{\sqrt{c}}+\frac{2\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(ah^2\ldots\right.}{\sqrt{ah^2+cg\ldots}}}{2h^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(2*h*(2*e*g - d*h + e*h*x) + f*(-6*g^2 - 3*g*h*x + h^2*
x^2)))/(g + h*x) - (2*(3*c*f*g^3 + c*g*h*(-2*e*g + d*h) + a*h^2*(2*f*g - e*
```

```
h))*Log[g + h*x])/Sqrt[c*g^2 + a*h^2] + ((6*c*f*g^2 + a*f*h^2 + 2*c*h*(-2*e
*g + d*h))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/Sqrt[c] + (2*(3*c*f*g^3 + c*
g*h*(-2*e*g + d*h) + a*h^2*(2*f*g - e*h))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*
h^2]*Sqrt[a + c*x^2]])/Sqrt[c*g^2 + a*h^2])/(2*h^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.02, size = 2818, normalized size = 9.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x)
[Out] 1/h/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^
(3/2)*e*g-1/h^2/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+
c*g^2)/h^2)^(3/2)*f*g^2-1/h*c*g/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c
+(a*h^2+c*g^2)/h^2)^(1/2)*d+1/h^2*c*g^2/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+
g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e-1/h^3*c*g^3/(a*h^2+c*g^2)*(-2*(x+g/h)*c
*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f+1/h^2*c^(3/2)*g^2/(a*h^2+c*g^2)
*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/
h^2)^(1/2))*d-1/h^3*c^(3/2)*g^3/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)
+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*e+1/h^4*c^(3/2)*g^
4/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c
+(a*h^2+c*g^2)/h^2)^(1/2))*f+2/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)
*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x
+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*f*g-1/h^4/((a*h^2+c*g^2)/h^2
)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2
)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^2*e+
2/h^5/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*
((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^
(1/2))/(x+g/h))*c*g^3*f-1/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2
*c+(a*h^2+c*g^2)/h^2)^(3/2)*d-2/h^3*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*
g^2)/h^2)^(1/2)*f*g+1/2*f/h^2*x*(c*x^2+a)^(1/2)+1/h^2*c^(1/2)/(a*h^2+c*g^2)
*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/
h^2)^(1/2))*a*f*g^2+1/h^3*c^2*g^3/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*l
n((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+
g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d-1/h^4*c^2*g^4/(
a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)
/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2
)/h^2)^(1/2))/(x+g/h))*e+1/h^5*c^2*g^5/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1
/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-
2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f-1/h*c/(a*h
```

```
^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*e*g+1/h^
2*c/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*
f*g^2-1/h*c^(1/2)/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c
*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*a*e*g+1/h^3*c*g^3/(a*h^2+c*g^2)/
((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^
2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))
/(x+g/h))*a*f+1/h*c*g/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h
)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(
x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*d-1/h^2*c*g^2/(a*h^2+c*g^2)
/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h
^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)
)/(x+g/h))*a*e+1/2*f/h^2*a/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+c/(a*h^2+c
*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*d+1/h^2*(-2*
(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e+c^(1/2)/(a*h^2+c*g^2)*
ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h
^2)^(1/2))*a*d-1/h^3*c^(1/2)*g*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*
g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*e+2/h^4*c^(1/2)*g^2*ln((-c*g/h+(x
+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*f-
1/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*
((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^
(1/2))/(x+g/h))*a*e
```

maxima [A] time = 0.65, size = 478, normalized size = 1.55

$$-\frac{\sqrt{cx^2+a}\,fg^2}{h^4x+gh^3}+\frac{\sqrt{cx^2+a}\,eg}{h^3x+gh^2}-\frac{\sqrt{cx^2+a}\,d}{h^2x+gh}+\frac{\sqrt{cx^2+a}\,fx}{2\,h^2}+\frac{3\,\sqrt{c}\,fg^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4}-\frac{2\,\sqrt{c}\,eg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^3}+\frac{\sqrt{c}\,d\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2 + a)*f*g^2/(h^4*x + g*h^3) + sqrt(c*x^2 + a)*e*g/(h^3*x + g*h^2
) - sqrt(c*x^2 + a)*d/(h^2*x + g*h) + 1/2*sqrt(c*x^2 + a)*f*x/h^2 + 3*sqrt(
c)*f*g^2*arcsinh(c*x/sqrt(a*c))/h^4 - 2*sqrt(c)*e*g*arcsinh(c*x/sqrt(a*c))/
h^3 + sqrt(c)*d*arcsinh(c*x/sqrt(a*c))/h^2 + 1/2*a*f*arcsinh(c*x/sqrt(a*c))
/(sqrt(c)*h^2) - c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt
(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^5) + c*e*g^2*arcsinh(c*g*x/(sqr
t(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h
^4) - c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
 + g)))/(sqrt(a + c*g^2/h^2)*h^3) - 2*sqrt(a + c*g^2/h^2)*f*g*arcsinh(c*g*x
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 + sqrt(a + c*
g^2/h^2)*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
+ g)))/h^2 - 2*sqrt(c*x^2 + a)*f*g/h^3 + sqrt(c*x^2 + a)*e/h^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

```
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\,\left(d+ex+fx^2\right)}{\left(g+hx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**2,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

3.84 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

Optimal. Leaf size=296

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4+ach^2\left(9fg^2-h(3eg-dh)\right)+2c^2g^3(3fg-eh)\right)}{2h^4\left(ah^2+cg^2\right)^{3/2}}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+f\cdots\right.}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

```
[Out] -1/2*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^2-1/2*(2*a
^2*f*h^4+2*c^2*g^3*(-e*h+3*f*g)+a*c*h^2*(9*f*g^2-h*(-d*h+3*e*g)))*arctanh((
-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^4/(a*h^2+c*g^2)^(3/2)-(-
e*h+3*f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/h^4+1/2*(2*(-e*h+3*f*
g)*(a*h^2+c*g^2)+h*(2*a*f*h^2+c*(3*f*g^2-h*(-d*h+e*g)))*x)*(c*x^2+a)^(1/2)/
h^3/(a*h^2+c*g^2)/(h*x+g)
```

Rubi [A] time = 0.55, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 813, 844, 217, 206, 725}

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4+ach^2\left(9fg^2-h(3eg-dh)\right)+2c^2g^3(3fg-eh)\right)}{2h^4\left(ah^2+cg^2\right)^{3/2}}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+f\cdots\right.}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((2*(3*f*g - e*h)*(c*g^2 + a*h^2) + h*(3*c*f*g^2 + 2*a*f*h^2 - c*h*(e*g - d
*h))*x)*Sqrt[a + c*x^2])/(2*h^3*(c*g^2 + a*h^2)*(g + h*x)) - ((f*g^2 - e*g*
h + d*h^2)*(a + c*x^2)^(3/2))/(2*h*(c*g^2 + a*h^2)*(g + h*x)^2) - (Sqrt[c]*
(3*f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/h^4 - ((2*a^2*f*h^4 + 2
*c^2*g^3*(3*f*g - e*h) + a*c*h^2*(9*f*g^2 - h*(3*e*g - d*h)))*ArcTanh[(a*h
- c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^4*(c*g^2 + a*h^2)^(3/
2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
```

```
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{2h\left(cg^2+ah^2\right)(g+hx)^2} - \frac{\int \frac{\left(-2(cdg-afg+aeh)-\left(2afh-c\left(eg-\frac{3fg^2}{h}-dh\right)\right)x\right)\sqrt{a+\cdots}}{(g+hx)^2}}{2\left(cg^2+ah^2\right)}$$

$$= \frac{\left(2(3fg-eh)\left(cg^2+ah^2\right)+h\left(3cfg^2+2afh^2-ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} - \cdots$$

$$= \frac{\left(2(3fg-eh)\left(cg^2+ah^2\right)+h\left(3cfg^2+2afh^2-ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} - \cdots$$

$$= \frac{\left(2(3fg-eh)\left(cg^2+ah^2\right)+h\left(3cfg^2+2afh^2-ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} - \cdots$$

$$= \frac{\left(2(3fg-eh)\left(cg^2+ah^2\right)+h\left(3cfg^2+2afh^2-ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} - \cdots$$

Mathematica [A] time = 0.56, size = 318, normalized size = 1.07

$$\frac{-\frac{\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(2a^2fh^4+ach^2\left(h(dh-3eg)+9fg^2\right)+2c^2g^3(3fg-eh)\right)}{\left(ah^2+cg^2\right)^{3/2}}+\frac{\log(g+hx)\left(2a^2fh^4+ach^2\left(h(dh-3eg)+9fg^2\right)+2c^2g^3(3fg-\cdots\right)}{\left(ah^2+cg^2\right)^{3/2}}}{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] (h*Sqrt[a + c*x^2]*(2*f + (-(f*g^2) + h*(e*g - d*h))/(g + h*x)^2 + (5*c*f*g
^3 + c*g*h*(-3*e*g + d*h) - 2*a*h^2*(-2*f*g + e*h))/((c*g^2 + a*h^2)*(g + h
```

```
*x))) + ((2*a^2*f*h^4 + 2*c^2*g^3*(3*f*g - e*h) + a*c*h^2*(9*f*g^2 + h*(-3*
e*g + d*h)))*Log[g + h*x])/(c*g^2 + a*h^2)^(3/2) + 2*Sqrt[c]*(-3*f*g + e*h)
*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] - ((2*a^2*f*h^4 + 2*c^2*g^3*(3*f*g - e*
h) + a*c*h^2*(9*f*g^2 + h*(-3*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a
*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(3/2))/(2*h^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.35, size = 923, normalized size = 3.12

$$-\frac{\left(6\,c^2fg^4+9\,acfg^2h^2+acdh^4+2\,a^2fh^4-2\,c^2g^3he-3\,acgh^3e\right)\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)h+\sqrt{c}g}{\sqrt{-cg^2-ah^2}}\right)}{\left(cg^2h^4+ah^6\right)\sqrt{-cg^2-ah^2}}+\frac{\sqrt{cx^2+a}\,f}{h^3}+\frac{6\left(\sqrt{\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] -(6*c^2*f*g^4 + 9*a*c*f*g^2*h^2 + a*c*d*h^4 + 2*a^2*f*h^4 - 2*c^2*g^3*h*e -
 3*a*c*g*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-
c*g^2 - a*h^2))/((c*g^2*h^4 + a*h^6)*sqrt(-c*g^2 - a*h^2)) + sqrt(c*x^2 + a
)*f/h^3 + (6*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h + 2*(sqrt(c)*x - s
qrt(c*x^2 + a))^3*c^2*d*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g
^2*h^3 + (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*d*h^5 - 4*(sqrt(c)*x - sqrt(c*
x^2 + a))^3*c^2*g^3*h^2*e - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*g*h^4*e +
 10*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 + 2*(sqrt(c)*x - sqrt(c*x
^2 + a))^2*c^(5/2)*d*g^3*h^2 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*
f*g^3*h^2 - (sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*
x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 - 6*(sqrt(c)*x - sqrt(c*x^2 + a)
)^2*c^(5/2)*g^4*h*e - (sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*g^2*h^3*e +
 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*h^5*e - 14*(sqrt(c)*x - sqrt
(c*x^2 + a))*a*c^2*f*g^4*h - 2*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2*h^
3 - 11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^3 + (sqrt(c)*x - sqrt(c*
x^2 + a))*a^2*c*d*h^5 + 8*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*g^3*h^2*e + 5
*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*g*h^4*e + 5*a^2*c^(3/2)*f*g^3*h^2 + a^
2*c^(3/2)*d*g*h^4 + 4*a^3*sqrt(c)*f*g*h^4 - 3*a^2*c^(3/2)*g^2*h^3*e - 2*a^3
*sqrt(c)*h^5*e)/((c*g^2*h^4 + a*h^6)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2
*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2) + (3*sqrt(c)*f*g - sqrt(
c)*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^4
```

maple [B] time = 0.02, size = 4432, normalized size = 14.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x)
```

```
[Out] 5/2/h^3*c*g^2/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2
)^(1/2)*f+3/2/h^3*c^(3/2)*g^2/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(
-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*e-5/2/h^4*c^(3/2)*g^
3/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c
```

```
+(a*h^2+c*g^2)/h^2)^(1/2))*f+1/h*c/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^
2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*e+1/h*c^(1/2)/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/
h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*a*e+2
/h^2/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)
^(3/2)*f*g-1/2*c*g/(a*h^2+c*g^2)^2/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a
*h^2+c*g^2)/h^2)^(3/2)*d+1/2*c^(3/2)*g/(a*h^2+c*g^2)^2*ln((-c*g/h+(x+g/h)*c
)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*a*d+1/2*c
^2*g/(a*h^2+c*g^2)^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)
*x*d+1/2/h^2*c^(5/2)*g^3/(a*h^2+c*g^2)^2*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*
(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*d-1/2/h^3*c^(5/2)*g^4/(
a*h^2+c*g^2)^2*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+
(a*h^2+c*g^2)/h^2)^(1/2))*e+1/2/h^4*c^(5/2)*g^5/(a*h^2+c*g^2)^2*ln((-c*g/h+
(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*
f-1/2/h^2*c^(3/2)/(a*h^2+c*g^2)*g*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)
*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*d+1/2/h^2/(a*h^2+c*g^2)/(x+g/h
)^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(3/2)*e*g-1/2/h^3/(a*h
^2+c*g^2)/(x+g/h)^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(3/2)*
f*g^2-1/2/h*c^2*g^2/(a*h^2+c*g^2)^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*
g^2)/h^2)^(1/2)*d+1/2/h^2*c^2*g^3/(a*h^2+c*g^2)^2*(-2*(x+g/h)*c*g/h+(x+g/h)
^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e-1/2/h^3*c^2*g^4/(a*h^2+c*g^2)^2*(-2*(x+g/h)
*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f-f/h^5/((a*h^2+c*g^2)/h^2)^(1/
2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2
*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^2-3/2/h^2
*c*g/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e
-1/2/h^2*c^2*g^3/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c
*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g
/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*e+1/2/h*c^2*g^2/(a*h^2+c*g^2)^
2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*
h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2
))/(x+g/h))*a*d-5/2/h^3*c*g^2/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-
2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)
*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*f+3/2/h^2*c*g/(a*h^
2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2
+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^
2)^(1/2))/(x+g/h))*a*e+1/2/h^3*c^2*g^4/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^
(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*
(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*f+f/h^3*
(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)+3/2/h^4*c^2*g^3/(a*h
^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^
2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h
^2)^(1/2))/(x+g/h))*e-5/2/h^5*c^2*g^4/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/
2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2
*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f-2/h^2*c/(a*
h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*f*g-2/h
^2*c^(1/2)/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x
+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*a*f*g-1/2/h*c/(a*h^2+c*g^2)/((a*h^2+c*g
^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^
2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a
*d-1/2/h^3*c^2/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h
+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^
2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*g^2*d+1/2/h^2*c^2*g^3/(a*h^2+c*g^2)^
2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*f-1/2/h*c^(3/2)*
g^2/(a*h^2+c*g^2)^2*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)
^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*a*e+1/2/h^5*c^3*g^6/(a*h^2+c*g^2)^2/((a*h^2+
c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)
/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h)
)*f-1/2/h^2*c*g^3/(a*h^2+c*g^2)^2/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*
h^2+c*g^2)/h^2)^(3/2)*f+1/2/h^3*c^3*g^4/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)
^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)
```

```
*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d-1/2/h^4
*c^3*g^5/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(
a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+
(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e-1/2/h*c^2*g^2/(a*h^2+c*g^2)^2*(-2*(x+g
/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*e+1/2/h*c*g^2/(a*h^2+c*g^2
)^2/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(3/2)*e+1/2/h^
2*c^(3/2)*g^3/(a*h^2+c*g^2)^2*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g
/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*a*f-1/h/(a*h^2+c*g^2)/(x+g/h)*(-2*
(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(3/2)*e-f/h^4*c^(1/2)*g*ln((-c
*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1
/2))-f/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h
^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/
h^2)^(1/2))/(x+g/h))*a+1/2/h*c/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+
(a*h^2+c*g^2)/h^2)^(1/2)*d-1/2/h/(a*h^2+c*g^2)/(x+g/h)^2*(-2*(x+g/h)*c*g/h+
(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(3/2)*d
```

maxima [B] time = 0.70, size = 927, normalized size = 3.13

$$-\frac{\sqrt{cx^2+a}\,cfg^3}{2\left(cg^2h^4x+ah^6x+cg^3h^3+agh^5\right)}+\frac{\sqrt{cx^2+a}\,ceg^2}{2\left(cg^2h^3x+ah^5x+cg^3h^2+agh^4\right)}-\frac{\left(cx^2+a\right)^{\frac{3}{2}}fg^2}{2\left(cg^2h^3x^2+ah^5x^2+2\,cg^3h^2x+2\,agh^4\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^2 + a)*c*f*g^3/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 + a*g*h^5)
+ 1/2*sqrt(c*x^2 + a)*c*e*g^2/(c*g^2*h^3*x + a*h^5*x + c*g^3*h^2 + a*g*h^4)
 - 1/2*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2*h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x +
 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*sqrt(c*x^2 + a)*c*f*g^2/(c*g^2*h^
3 + a*h^5) - 1/2*sqrt(c*x^2 + a)*c*d*g/(c*g^2*h^2*x + a*h^4*x + c*g^3*h + a
*g*h^3) + 1/2*(c*x^2 + a)^(3/2)*e*g/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h*
x + 2*a*g*h^3*x + c*g^4 + a*g^2*h^2) - 1/2*sqrt(c*x^2 + a)*c*e*g/(c*g^2*h^2
 + a*h^4) - 1/2*(c*x^2 + a)^(3/2)*d/(c*g^2*h*x^2 + a*h^3*x^2 + 2*c*g^3*x +
2*a*g*h^2*x + c*g^4/h + a*g^2*h) + 1/2*sqrt(c*x^2 + a)*c*d/(c*g^2*h + a*h^3
) + 2*sqrt(c*x^2 + a)*f*g/(h^4*x + g*h^3) - sqrt(c*x^2 + a)*e/(h^3*x + g*h^
2) - 3*sqrt(c)*f*g*arcsinh(c*x/sqrt(a*c))/h^4 + sqrt(c)*e*arcsinh(c*x/sqrt(
a*c))/h^3 - 1/2*c^2*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqr
t(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) + 1/2*c^2*e*g^3*arcsinh(c
*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h
^2)^(3/2)*h^6) - 1/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h
/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^5) + 5/2*c*f*g^2*arcsin
h(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a +
c*g^2/h^2)*h^5) - 3/2*c*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(s
qrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^4) + 1/2*c*d*arcsinh(c*g*x/(
sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2
)*h^3) + sqrt(a + c*g^2/h^2)*f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h
/(sqrt(a*c)*abs(h*x + g)))/h^3 + sqrt(c*x^2 + a)*f/h^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
```

```
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{\left(g+hx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**3,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

3.85 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$

Optimal. Leaf size=314

$$\frac{c\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(a^2h^4(4fg-eh)+acgh^2\left(5fg^2-dh^2\right)+2c^2fg^5\right)}{2h^4\left(ah^2+cg^2\right)^{5/2}}-\frac{\sqrt{a+cx^2}\left(hx\left(2a^2fh^4+acgh^2(6fg\right.\right.}{}$$

```
[Out] -1/3*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^3+1/2*c*(2
*c^2*f*g^5+a^2*h^4*(-e*h+4*f*g)+a*c*g*h^2*(-d*h^2+5*f*g^2))*arctanh((-c*g*x
+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^4/(a*h^2+c*g^2)^(5/2)+f*arctan
h(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/h^4-1/2*(2*c^2*f*g^5+a^2*e*h^5+a*c*g*h
^2*(d*h^2+3*f*g^2)+h*(2*a^2*f*h^4+a*c*g*h^2*(-e*h+6*f*g)+c^2*(-d*g^2*h^2+3*
f*g^4))*x)*(c*x^2+a)^(1/2)/h^3/(a*h^2+c*g^2)^2/(h*x+g)^2
```

Rubi [A] time = 0.51, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1651, 811, 844, 217, 206, 725}

$$-\frac{\sqrt{a+cx^2}\left(hx\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\left(3fg^4-dg^2h^2\right)\right)+a^2eh^5+acgh^2\left(dh^2+3fg^2\right)+2c^2fg^5\right)}{2h^3(g+hx)^2\left(ah^2+cg^2\right)^2}+\frac{c\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -((2*c^2*f*g^5 + a^2*e*h^5 + a*c*g*h^2*(3*f*g^2 + d*h^2) + h*(2*a^2*f*h^4 +
 a*c*g*h^2*(6*f*g - e*h) + c^2*(3*f*g^4 - d*g^2*h^2))*x)*Sqrt[a + c*x^2])/(
2*h^3*(c*g^2 + a*h^2)^2*(g + h*x)^2) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)
^(3/2))/(3*h*(c*g^2 + a*h^2)*(g + h*x)^3) + (Sqrt[c]*f*ArcTanh[(Sqrt[c]*x)/
Sqrt[a + c*x^2]])/h^4 + (c*(2*c^2*f*g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h^2
*(5*f*g^2 - d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x
^2])])/(2*h^4*(c*g^2 + a*h^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
```

```
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{3h\left(cg^2+ah^2\right)(g+hx)^3} - \frac{\int \frac{\left(-3(cdg-afg+aeh)-3f\left(\frac{cg^2}{h}+ah\right)x\right)\sqrt{a+cx^2}}{(g+hx)^3}\,dx}{3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(2c^2fg^5+a^2eh^5+acgh^2\left(3fg^2+dh^2\right)+h\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\right.\right.}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}$$

$$= -\frac{\left(2c^2fg^5+a^2eh^5+acgh^2\left(3fg^2+dh^2\right)+h\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\right.\right.}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}$$

$$= -\frac{\left(2c^2fg^5+a^2eh^5+acgh^2\left(3fg^2+dh^2\right)+h\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\right.\right.}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}$$

$$= -\frac{\left(2c^2fg^5+a^2eh^5+acgh^2\left(3fg^2+dh^2\right)+h\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\right.\right.}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}$$

Mathematica [A] time = 0.82, size = 382, normalized size = 1.22

$$\frac{\frac{3c\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(a^2h^4(4fg-eh)+acgh^2\left(5fg^2-dh^2\right)+2c^2fg^5\right)}{\left(ah^2+cg^2\right)^{5/2}} - \frac{3c\log(g+hx)\left(a^2h^4(4fg-eh)+acgh^2\left(5fg^2-dh^2\right)+2c^2fg^5\right)}{\left(ah^2+cg^2\right)^{5/2}} + \cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(-2*(f*g^2 + h*(-(e*g) + d*h)) + ((7*c*f*g^3 + c*g*h*(-
4*e*g + d*h) - 3*a*h^2*(-2*f*g + e*h))*(g + h*x))/(c*g^2 + a*h^2) - ((6*a^2
*f*h^4 + c^2*(11*f*g^4 - g^2*h*(2*e*g + d*h)) + a*c*h^2*(20*f*g^2 + h*(-5*e
*g + 2*d*h)))*(g + h*x)^2)/(c*g^2 + a*h^2)^2))/(g + h*x)^3 - (3*c*(2*c^2*f*
g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h^2*(5*f*g^2 - d*h^2))*Log[g + h*x])/(c
*g^2 + a*h^2)^(5/2) + 6*Sqrt[c]*f*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c
*(2*c^2*f*g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h^2*(5*f*g^2 - d*h^2))*Log[a*
h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(5/2))/(6
*h^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.50, size = 1719, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] -(2*c^3*f*g^5 + 5*a*c^2*f*g^3*h^2 - a*c^2*d*g*h^4 + 4*a^2*c*f*g*h^4 - a^2*c
*h^5*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 -
 a*h^2))/((c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8)*sqrt(-c*g^2 - a*h^2)) - s
qrt(c)*f*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^4 - 1/3*(18*(sqrt(c)*x -
sqrt(c*x^2 + a))^5*c^3*f*g^5*h^2 + 33*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2
*f*g^3*h^4 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*d*g*h^6 + 12*(sqrt(c)*
x - sqrt(c*x^2 + a))^5*a^2*c*f*g*h^6 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^
3*g^4*h^3*e - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*g^2*h^5*e - 3*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a^2*c*h^7*e + 54*(sqrt(c)*x - sqrt(c*x^2 + a))^4*
c^(7/2)*f*g^6*h - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*d*g^4*h^3 + 87*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*f*g^4*h^3 + 3*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a*c^(5/2)*d*g^2*h^5 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c
^(3/2)*f*g^2*h^5 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*d*h^7 - 6*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*sqrt(c)*f*h^7 - 12*(sqrt(c)*x - sqrt(c*
x^2 + a))^4*c^(7/2)*g^5*h^2*e - 24*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2
)*g^3*h^4*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*g*h^6*e + 44*(s
qrt(c)*x - sqrt(c*x^2 + a))^3*c^4*f*g^7 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^3
*c^4*d*g^5*h^2 + 14*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*f*g^5*h^2 + 14*(s
qrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*d*g^3*h^4 - 96*(sqrt(c)*x - sqrt(c*x^2
+ a))^3*a^2*c^2*f*g^3*h^4 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*d*g*
h^6 - 36*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c*f*g*h^6 - 8*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*c^4*g^6*h*e - 8*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*g^4*h
^3*e + 30*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*g^2*h^5*e - 78*(sqrt(c)*x
 - sqrt(c*x^2 + a))^2*a*c^(7/2)*f*g^6*h + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*a*c^(7/2)*d*g^4*h^3 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)*f*g^
4*h^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)*d*g^2*h^5 + 12*(sqrt
(c)*x - sqrt(c*x^2 + a))^2*a^4*sqrt(c)*f*h^7 + 12*(sqrt(c)*x - sqrt(c*x^2 +
 a))^2*a*c^(7/2)*g^5*h^2*e + 30*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)
*g^3*h^4*e - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*g*h^6*e + 48*(s
qrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*f*g^5*h^2 - 6*(sqrt(c)*x - sqrt(c*x^2 +
 a))*a^2*c^3*d*g^3*h^4 + 87*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*f*g^3*h^4
 + 9*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d*g*h^6 + 24*(sqrt(c)*x - sqrt(c
*x^2 + a))*a^4*c*f*g*h^6 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*g^4*h^3*
e - 18*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*g^2*h^5*e + 3*(sqrt(c)*x - sqr
```

```
t(c*x^2 + a))*a^4*c*h^7*e - 11*a^3*c^(5/2)*f*g^4*h^3 + a^3*c^(5/2)*d*g^2*h^
5 - 20*a^4*c^(3/2)*f*g^2*h^5 - 2*a^4*c^(3/2)*d*h^7 - 6*a^5*sqrt(c)*f*h^7 +
2*a^3*c^(5/2)*g^3*h^4*e + 5*a^4*c^(3/2)*g*h^6*e)/((c^2*g^4*h^4 + 2*a*c*g^2*
h^6 + a^2*h^8)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x
^2 + a))*sqrt(c)*g - a*h)^3)
```

maple [B] time = 0.02, size = 5565, normalized size = 17.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.84, size = 1772, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^2 + a)*c^2*f*g^4/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x
 + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) + 1/2*sqrt(c*x^2 + a)*c^2*e*g^3
/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*h^4
 + a^2*g*h^6) - 1/2*(c*x^2 + a)^(3/2)*c*f*g^3/(c^2*g^4*h^3*x^2 + 2*a*c*g^2*
h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6*x +
 c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 1/2*sqrt(c*x^2 + a)*c^2*f*g^3/(
c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/2*sqrt(c*x^2 + a)*c^2*d*g^2/(c^2
*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*
g*h^5) + 1/2*(c*x^2 + a)^(3/2)*c*e*g^2/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2
 + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6
+ 2*a*c*g^4*h^2 + a^2*g^2*h^4) - 1/2*sqrt(c*x^2 + a)*c^2*e*g^2/(c^2*g^4*h^2
 + 2*a*c*g^2*h^4 + a^2*h^6) - 1/2*(c*x^2 + a)^(3/2)*c*d*g/(c^2*g^4*h*x^2 +
2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 + 2*c^2*g^5*x + 4*a*c*g^3*h^2*x + 2*a^2*g*h
^4*x + c^2*g^6/h + 2*a*c*g^4*h + a^2*g^2*h^3) + 1/2*sqrt(c*x^2 + a)*c^2*d*g
/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*h^5) - 1/3*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2
*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*
a*g^2*h^4*x + c*g^5*h + a*g^3*h^3) + sqrt(c*x^2 + a)*c*f*g^2/(c*g^2*h^4*x +
 a*h^6*x + c*g^3*h^3 + a*g*h^5) + 1/3*(c*x^2 + a)^(3/2)*e*g/(c*g^2*h^3*x^3
+ a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h^4*x^2 + 3*c*g^4*h*x + 3*a*g^2*h^3*x
 + c*g^5 + a*g^3*h^2) - 1/2*sqrt(c*x^2 + a)*c*e*g/(c*g^2*h^3*x + a*h^5*x +
c*g^3*h^2 + a*g*h^4) + (c*x^2 + a)^(3/2)*f*g/(c*g^2*h^3*x^2 + a*h^5*x^2 + 2
*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) - sqrt(c*x^2 + a)*c*f*g/(
c*g^2*h^3 + a*h^5) - 1/3*(c*x^2 + a)^(3/2)*d/(c*g^2*h^2*x^3 + a*h^4*x^3 + 3
*c*g^3*h*x^2 + 3*a*g*h^3*x^2 + 3*c*g^4*x + 3*a*g^2*h^2*x + c*g^5/h + a*g^3*
h) - 1/2*(c*x^2 + a)^(3/2)*e/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h*x + 2*a
*g*h^3*x + c*g^4 + a*g^2*h^2) + 1/2*sqrt(c*x^2 + a)*c*e/(c*g^2*h^2 + a*h^4)
 - sqrt(c*x^2 + a)*f/(h^4*x + g*h^3) + sqrt(c)*f*arcsinh(c*x/sqrt(a*c))/h^4
 - 1/2*c^3*f*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*ab
s(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^9) + 1/2*c^3*e*g^4*arcsinh(c*g*x/(sqr
t(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)
*h^8) - 1/2*c^3*d*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*
c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) + 3/2*c^2*f*g^3*arcsinh(c*g*x
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^
(3/2)*h^7) - c^2*e*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a
*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^6) + 1/2*c^2*d*g*arcsinh(c*g*x/
(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(
3/2)*h^5) - 2*c*f*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)
```

```
*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^5) + 1/2*c*e*arcsinh(c*g*x/(sqrt(a*c
)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
```

```
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\,\left(d+ex+fx^2\right)}{\left(g+hx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**4,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

3.86 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx$

Optimal. Leaf size=313

$$-\frac{\sqrt{a+cx^2}\,(ah-cgx)\left(4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)+4c^2dg^2\right)}{8(g+hx)^2\left(ah^2+cg^2\right)^3}-\frac{ac\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\,\sqrt{ah^2+cg^2}}\right)\left(4a^2fh^2-ac\left(\ldots\right.\right.}{8\left(ah^2+cg^2\right)^{7/\ldots}}$$

```
[Out] -1/4*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^4+1/12*(4*
a*h^2*(-e*h+2*f*g)+c*g*(3*f*g^2+h*(-5*d*h+e*g)))*(c*x^2+a)^(3/2)/h/(a*h^2+c
*g^2)^2/(h*x+g)^3-1/8*a*c*(4*c^2*d*g^2+4*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+5*e*g
)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)
^(7/2)-1/8*(4*c^2*d*g^2+4*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+5*e*g)))*(-c*g*x+a*h
)*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^3/(h*x+g)^2
```

Rubi [A] time = 0.43, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1651, 807, 721, 725, 206}

$$-\frac{\sqrt{a+cx^2}\,(ah-cgx)\left(4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)+4c^2dg^2\right)}{8(g+hx)^2\left(ah^2+cg^2\right)^3}-\frac{ac\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\,\sqrt{ah^2+cg^2}}\right)\left(4a^2fh^2-ac\left(\ldots\right.\right.}{8\left(ah^2+cg^2\right)^{7/\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] -((4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 - h*(5*e*g - d*h)))*(a*h - c*g*x)
*Sqrt[a + c*x^2])/(8*(c*g^2 + a*h^2)^3*(g + h*x)^2) - ((f*g^2 - e*g*h + d*h
^2)*(a + c*x^2)^(3/2))/(4*h*(c*g^2 + a*h^2)*(g + h*x)^4) + ((3*c*f*g^3 + c*
g*h*(e*g - 5*d*h) + 4*a*h^2*(2*f*g - e*h))*(a + c*x^2)^(3/2))/(12*h*(c*g^2
+ a*h^2)^2*(g + h*x)^3) - (a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 - h*
(5*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])
])/(8*(c*g^2 + a*h^2)^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{4h\left(cg^2+ah^2\right)(g+hx)^4} - \frac{\int \frac{\left(-4(cdg-afg+aeh)-\left(4afh+c\left(eg+\frac{3fg^2}{h}-dh\right)\right)x\right)\sqrt{a+cx^2}}{(g+hx)^4}}{4\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{4h\left(cg^2+ah^2\right)(g+hx)^4} + \frac{\left(3cfg^3+cgh(eg-5dh)+4ah^2(2fg-eh)\right)(}{12h\left(cg^2+ah^2\right)^2(g+hx)^3}$$

$$= -\frac{\left(4c^2dg^2+4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^3(g+hx)^2} - \frac{\left(fg^2-eg\right.}{4h\left(cg^2\right.}$$

$$= -\frac{\left(4c^2dg^2+4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^3(g+hx)^2} - \frac{\left(fg^2-eg\right.}{4h\left(cg^2\right.}$$

$$= -\frac{\left(4c^2dg^2+4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^3(g+hx)^2} - \frac{\left(fg^2-eg\right.}{4h\left(cg^2\right.}$$

Mathematica [A] time = 1.31, size = 439, normalized size = 1.40

$$\frac{1}{24}\left(-\frac{3ac\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(4a^2fh^2-ac\left(h(dh-5eg)+fg^2\right)+4c^2dg^2\right)}{\left(ah^2+cg^2\right)^{7/2}} + \frac{3ac\log(g+hx)(}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (-((Sqrt[a + c*x^2]*(6*(c*g^2 + a*h^2)^3*(f*g^2 + h*(-(e*g) + d*h)) - 2*(c*
g^2 + a*h^2)^2*(9*c*f*g^3 + c*g*h*(-5*e*g + d*h) - 4*a*h^2*(-2*f*g + e*h))*
(g + h*x) + (c*g^2 + a*h^2)*(12*a^2*f*h^4 + 2*c^2*(9*f*g^4 - g^2*h*(e*g + d
*h)) + a*c*h^2*(35*f*g^2 + h*(-7*e*g + 3*d*h)))*(g + h*x)^2 - c*(4*a^2*h^4*
(7*f*g - 2*e*h) + a*c*g*h^2*(19*f*g^2 + h*(9*e*g - 13*d*h)) + 2*c^2*(3*f*g^
5 + g^3*h*(e*g + d*h)))*(g + h*x)^3))/((c*g^2*h + a*h^3)^3*(g + h*x)^4)) +
(3*a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 + h*(-5*e*g + d*h)))*Log[g +
 h*x])/(c*g^2 + a*h^2)^(7/2) - (3*a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g
^2 + h*(-5*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^
2]])/(c*g^2 + a*h^2)^(7/2))/24
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.02, size = 7237, normalized size = 23.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x)
[Out] result too large to display
```

maxima [B] time = 1.14, size = 3404, normalized size = 10.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="maxima")
[Out] -5/8*sqrt(c*x^2 + a)*c^3*f*g^5/(c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c
*g^2*h^8*x + a^3*h^10*x + c^3*g^7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 +
 a^3*g*h^9) + 5/8*sqrt(c*x^2 + a)*c^3*e*g^4/(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^
5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2
*c*g^3*h^6 + a^3*g*h^8) - 5/8*(c*x^2 + a)^(3/2)*c^2*f*g^4/(c^3*g^6*h^3*x^2
+ 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x
 + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*
c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2*h^7) + 5/8*sqrt(c*x^2 + a)*c^3*f*g^
4/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 5/8*sqrt(c*
x^2 + a)*c^3*d*g^3/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g^2*h^6*x +
 a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7) + 5
/8*(c*x^2 + a)^(3/2)*c^2*e*g^3/(c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h^4*x^2 + 3*a
^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 + 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^3*x + 6*a^2*
c*g^3*h^5*x + 2*a^3*g*h^7*x + c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4 +
 a^3*g^2*h^6) - 5/8*sqrt(c*x^2 + a)*c^3*e*g^3/(c^3*g^6*h^2 + 3*a*c^2*g^4*h^
4 + 3*a^2*c*g^2*h^6 + a^3*h^8) - 5/8*(c*x^2 + a)^(3/2)*c^2*d*g^2/(c^3*g^6*h
*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*
x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a
*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^3*g^2*h^5) + 5/8*sqrt(c*x^2 + a)*c^3*d*g^2
/(c^3*g^6*h + 3*a*c^2*g^4*h^3 + 3*a^2*c*g^2*h^5 + a^3*h^7) - 5/12*(c*x^2 +
a)^(3/2)*c*f*g^3/(c^2*g^4*h^4*x^3 + 2*a*c*g^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2
*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*
c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^5) +
9/8*sqrt(c*x^2 + a)*c^2*f*g^3/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x
+ c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) + 5/12*(c*x^2 + a)^(3/2)*c*e*g^2
```

```
/(c^2*g^4*h^3*x^3 + 2*a*c*g^2*h^5*x^3 + a^2*h^7*x^3 + 3*c^2*g^5*h^2*x^2 + 6
*a*c*g^3*h^4*x^2 + 3*a^2*g*h^6*x^2 + 3*c^2*g^6*h*x + 6*a*c*g^4*h^3*x + 3*a^
2*g^2*h^5*x + c^2*g^7 + 2*a*c*g^5*h^2 + a^2*g^3*h^4) - 5/8*sqrt(c*x^2 + a)*
c^2*e*g^2/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*
c*g^3*h^4 + a^2*g*h^6) + 9/8*(c*x^2 + a)^(3/2)*c*f*g^2/(c^2*g^4*h^3*x^2 + 2
*a*c*g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*
g*h^6*x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) - 9/8*sqrt(c*x^2 + a)*c^
2*f*g^2/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 5/12*(c*x^2 + a)^(3/2)*c*
d*g/(c^2*g^4*h^2*x^3 + 2*a*c*g^2*h^4*x^3 + a^2*h^6*x^3 + 3*c^2*g^5*h*x^2 +
6*a*c*g^3*h^3*x^2 + 3*a^2*g*h^5*x^2 + 3*c^2*g^6*x + 6*a*c*g^4*h^2*x + 3*a^2
*g^2*h^4*x + c^2*g^7/h + 2*a*c*g^5*h + a^2*g^3*h^3) + 1/8*sqrt(c*x^2 + a)*c
^2*d*g/(c^2*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3
*h^3 + a^2*g*h^5) - 5/8*(c*x^2 + a)^(3/2)*c*e*g/(c^2*g^4*h^2*x^2 + 2*a*c*g^
2*h^4*x^2 + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x +
 c^2*g^6 + 2*a*c*g^4*h^2 + a^2*g^2*h^4) + 5/8*sqrt(c*x^2 + a)*c^2*e*g/(c^2*
g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) - 1/4*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2*h^5
*x^4 + a*h^7*x^4 + 4*c*g^3*h^4*x^3 + 4*a*g*h^6*x^3 + 6*c*g^4*h^3*x^2 + 6*a*
g^2*h^5*x^2 + 4*c*g^5*h^2*x + 4*a*g^3*h^4*x + c*g^6*h + a*g^4*h^3) + 1/8*(c
*x^2 + a)^(3/2)*c*d/(c^2*g^4*h*x^2 + 2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 + 2*c^
2*g^5*x + 4*a*c*g^3*h^2*x + 2*a^2*g*h^4*x + c^2*g^6/h + 2*a*c*g^4*h + a^2*g
^2*h^3) - 1/8*sqrt(c*x^2 + a)*c^2*d/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*h^5) +
 1/4*(c*x^2 + a)^(3/2)*e*g/(c*g^2*h^4*x^4 + a*h^6*x^4 + 4*c*g^3*h^3*x^3 + 4
*a*g*h^5*x^3 + 6*c*g^4*h^2*x^2 + 6*a*g^2*h^4*x^2 + 4*c*g^5*h*x + 4*a*g^3*h^
3*x + c*g^6 + a*g^4*h^2) + 2/3*(c*x^2 + a)^(3/2)*f*g/(c*g^2*h^4*x^3 + a*h^6
*x^3 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c*
g^5*h + a*g^3*h^3) - 1/2*sqrt(c*x^2 + a)*c*f*g/(c*g^2*h^4*x + a*h^6*x + c*g
^3*h^3 + a*g*h^5) - 1/4*(c*x^2 + a)^(3/2)*d/(c*g^2*h^3*x^4 + a*h^5*x^4 + 4*
c*g^3*h^2*x^3 + 4*a*g*h^4*x^3 + 6*c*g^4*h*x^2 + 6*a*g^2*h^3*x^2 + 4*c*g^5*x
 + 4*a*g^3*h^2*x + c*g^6/h + a*g^4*h) - 1/3*(c*x^2 + a)^(3/2)*e/(c*g^2*h^3*
x^3 + a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h^4*x^2 + 3*c*g^4*h*x + 3*a*g^2*h
^3*x + c*g^5 + a*g^3*h^2) - 1/2*(c*x^2 + a)^(3/2)*f/(c*g^2*h^3*x^2 + a*h^5*
x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*sqrt(c*x^2 +
 a)*c*f/(c*g^2*h^3 + a*h^5) - 5/8*c^4*f*g^6*arcsinh(c*g*x/(sqrt(a*c)*abs(h*
x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^11) + 5/8*
c^4*e*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x +
 g)))/((a + c*g^2/h^2)^(7/2)*h^10) - 5/8*c^4*d*g^4*arcsinh(c*g*x/(sqrt(a*c)
*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^9)
+ 7/4*c^3*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs
(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^9) - 5/4*c^3*e*g^3*arcsinh(c*g*x/(sqrt
(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*
h^8) + 3/4*c^3*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c
)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) - 13/8*c^2*f*g^2*arcsinh(c*g*x
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^
(3/2)*h^7) + 5/8*c^2*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt
(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^6) - 1/8*c^2*d*arcsinh(c*g*x/
(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(
3/2)*h^5) + 1/2*c*f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)
*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{\left(g+hx\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**5,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

3.87 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

Optimal. Leaf size=433

$$-\frac{\left(a+cx^2\right)^{3/2}\left(20a^2fh^4-ach^2\left(18fg^2-h(33eg-8dh)\right)-c^2g^2\left(h(2eg-27dh)+3fg^2\right)\right)}{60h(g+hx)^3\left(ah^2+cg^2\right)^3}-\frac{c\sqrt{a+cx^2}\,(ah-cgx)\left(a\right.}{}$$

```
[Out] -1/5*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^5+1/20*(5*
a*h^2*(-e*h+2*f*g)+c*g*(3*f*g^2+h*(-7*d*h+2*e*g)))*(c*x^2+a)^(3/2)/h/(a*h^2
+c*g^2)^2/(h*x+g)^4-1/60*(20*a^2*f*h^4-c^2*g^2*(3*f*g^2+h*(-27*d*h+2*e*g))-
a*c*h^2*(18*f*g^2-h*(-8*d*h+33*e*g)))*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)^3/(h*
x+g)^3-1/8*a*c^2*(4*c^2*d*g^3+a^2*h^2*(-e*h+6*f*g)-a*c*g*(f*g^2-3*h*(-d*h+2
*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*
g^2)^(9/2)-1/8*c*(4*c^2*d*g^3+a^2*h^2*(-e*h+6*f*g)-a*c*g*(f*g^2-3*h*(-d*h+2
*e*g)))*(-c*g*x+a*h)*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^4/(h*x+g)^2
```

Rubi [A] time = 0.74, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1651, 835, 807, 721, 725, 206}

$$-\frac{c\sqrt{a+cx^2}\,(ah-cgx)\left(a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)+4c^2dg^3\right)}{8(g+hx)^2\left(ah^2+cg^2\right)^4}-\frac{\left(a+cx^2\right)^{3/2}\left(20a^2fh^4-ach^2\left(18f\right.\right.}{60h}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -(c*(4*c^2*d*g^3 + a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(2*e*g - d*h)
))*(a*h - c*g*x)*Sqrt[a + c*x^2])/(8*(c*g^2 + a*h^2)^4*(g + h*x)^2) - ((f*g
^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(5*h*(c*g^2 + a*h^2)*(g + h*x)^5) +
((3*c*f*g^3 + c*g*h*(2*e*g - 7*d*h) + 5*a*h^2*(2*f*g - e*h))*(a + c*x^2)^(3
/2))/(20*h*(c*g^2 + a*h^2)^2*(g + h*x)^4) - ((20*a^2*f*h^4 - c^2*(3*f*g^4 +
 g^2*h*(2*e*g - 27*d*h)) - a*c*h^2*(18*f*g^2 - h*(33*e*g - 8*d*h)))*(a + c*
x^2)^(3/2))/(60*h*(c*g^2 + a*h^2)^3*(g + h*x)^3) - (a*c^2*(4*c^2*d*g^3 + a^
2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(2*e*g - d*h)))*ArcTanh[(a*h - c*g
*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(8*(c*g^2 + a*h^2)^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} - \frac{\int \frac{\left(-5(cdg-afg+aeh)-\left(5afh+c\left(2eg+\frac{3fg^2}{h}-2dh\right)\right)x\right)\sqrt{\cdots}}{(g+hx)^5}}{5\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} + \frac{\left(3cfg^3+cgh(2eg-7dh)+5ah^2(2fg-e\cdots\right.}{20h\left(cg^2+ah^2\right)^2(g+hx)^4\cdots}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} + \frac{\left(3cfg^3+cgh(2eg-7dh)+5ah^2(2fg-e\cdots\right.}{20h\left(cg^2+ah^2\right)^2(g+hx)^4\cdots}$$

$$= -\frac{c\left(4c^2dg^3+a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx\cdots}}{8\left(cg^2+ah^2\right)^4(g+hx)^2}$$

$$= -\frac{c\left(4c^2dg^3+a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx\cdots}}{8\left(cg^2+ah^2\right)^4(g+hx)^2}$$

$$= -\frac{c\left(4c^2dg^3+a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx\cdots}}{8\left(cg^2+ah^2\right)^4(g+hx)^2}$$

Mathematica [A] time = 1.62, size = 583, normalized size = 1.35

$$-\frac{ac^2\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(a^2h^2(6fg-eh)-acg\left(3h(dh-2eg)+fg^2\right)+4c^2dg^3\right)}{8\left(ah^2+cg^2\right)^{9/2}}+\frac{ac^2\log(g+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -1/120*(Sqrt[a + c*x^2]*(24*(c*g^2 + a*h^2)^4*(f*g^2 + h*(-(e*g) + d*h)) -
6*(c*g^2 + a*h^2)^3*(11*c*f*g^3 + c*g*h*(-6*e*g + d*h) - 5*a*h^2*(-2*f*g +
e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^2*(20*a^2*f*h^4 + c^2*(27*f*g^4 - g^2*h
*(2*e*g + 3*d*h)) + a*c*h^2*(54*f*g^2 + h*(-9*e*g + 4*d*h)))*(g + h*x)^2 -
c*(c*g^2 + a*h^2)*(5*a^2*h^4*(10*f*g - 3*e*h) + a*c*g*h^2*(21*f*g^2 + h*(24
*e*g - 29*d*h)) + c^2*(6*f*g^5 + 2*g^3*h*(2*e*g + 3*d*h)))*(g + h*x)^3 - c*
(-40*a^3*f*h^6 + a*c^2*g^2*h^2*(27*f*g^2 + h*(28*e*g - 83*d*h)) + c^3*(6*f*
g^6 + 2*g^4*h*(2*e*g + 3*d*h)) + a^2*c*h^4*(86*f*g^2 + h*(-81*e*g + 16*d*h)
))*(g + h*x)^4))/(h^3*(c*g^2 + a*h^2)^4*(g + h*x)^5) + (a*c^2*(4*c^2*d*g^3
+ a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 + 3*h*(-2*e*g + d*h)))*Log[g + h*x])
/(8*(c*g^2 + a*h^2)^(9/2)) - (a*c^2*(4*c^2*d*g^3 + a^2*h^2*(6*f*g - e*h) -
a*c*g*(f*g^2 + 3*h*(-2*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*S
qrt[a + c*x^2]])/(8*(c*g^2 + a*h^2)^(9/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.66, size = 4212, normalized size = 9.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] -1/4*(4*a*c^4*d*g^3 - a^2*c^3*f*g^3 - 3*a^2*c^3*d*g*h^2 + 6*a^3*c^2*f*g*h^2
 + 6*a^2*c^3*g^2*h*e - a^3*c^2*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))
*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^4*g^8 + 4*a*c^3*g^6*h^2 + 6*a^2*c
^2*g^4*h^4 + 4*a^3*c*g^2*h^6 + a^4*h^8)*sqrt(-c*g^2 - a*h^2)) - 1/60*(60*(s
qrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*d*g^3*h^8 - 15*(sqrt(c)*x - sqrt(c*x^2
+ a))^9*a^2*c^3*f*g^3*h^8 - 45*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*d*g*
h^10 + 90*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^2*f*g*h^10 + 90*(sqrt(c)*x
- sqrt(c*x^2 + a))^9*a^2*c^3*g^2*h^9*e - 15*(sqrt(c)*x - sqrt(c*x^2 + a))^9
*a^3*c^2*h^11*e - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*f*g^8*h^3 -
480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/2)*f*g^6*h^5 + 540*(sqrt(c)*x -
sqrt(c*x^2 + a))^8*a*c^(9/2)*d*g^4*h^7 - 855*(sqrt(c)*x - sqrt(c*x^2 + a))^
8*a^2*c^(7/2)*f*g^4*h^7 - 405*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*d
*g^2*h^9 + 330*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(5/2)*f*g^2*h^9 - 120*
(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(3/2)*f*h^11 + 810*(sqrt(c)*x - sqrt(
c*x^2 + a))^8*a^2*c^(7/2)*g^3*h^8*e - 135*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a
^3*c^(5/2)*g*h^10*e - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*f*g^9*h^2 - 9
60*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*f*g^7*h^4 + 1880*(sqrt(c)*x - sqrt
(c*x^2 + a))^7*a*c^5*d*g^5*h^6 - 1910*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c
^4*f*g^5*h^6 - 1690*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*d*g^3*h^8 + 193
0*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*f*g^3*h^8 + 210*(sqrt(c)*x - sqrt
```

(c*x^2 + a))^7*a^3*c^3*d*g*h^10 - 660*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^2*f*g*h^10 - 160*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*g^8*h^3*e - 640*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*g^6*h^5*e + 1860*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*g^4*h^7*e - 1530*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*g^2*h^9*e - 90*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^2*h^11*e - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(13/2)*f*g^10*h - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(13/2)*d*g^8*h^3 - 720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*f*g^8*h^3 + 2120*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*d*g^6*h^5 - 1250*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*f*g^6*h^5 - 5710*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*d*g^4*h^7 + 5590*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*f*g^4*h^7 + 510*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*d*g^2*h^9 - 2220*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*f*g^2*h^9 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*d*h^11 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(3/2)*f*h^11 - 160*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(13/2)*g^9*h^2*e - 640*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*g^7*h^4*e + 3660*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*g^5*h^6*e - 4350*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*g^3*h^8*e + 330*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*g*h^10*e - 96*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*f*g^11 - 96*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*d*g^9*h^2 + 48*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^6*f*g^9*h^2 + 1808*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^6*d*g^7*h^4 + 604*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*f*g^7*h^4 - 7076*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*d*g^5*h^6 + 6710*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^4*f*g^5*h^6 + 3770*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^4*d*g^3*h^8 - 5780*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*f*g^3*h^8 - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*d*g*h^10 + 1200*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^2*f*g*h^10 - 64*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*g^10*h*e - 128*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^6*g^8*h^3*e + 3416*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*g^6*h^5*e - 7320*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^4*g^4*h^7*e + 2430*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*g^2*h^9*e + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*f*g^10*h + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*d*g^8*h^3 + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*f*g^8*h^3 - 5240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*d*g^6*h^5 + 2450*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*f*g^6*h^5 + 5590*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*d*g^4*h^7 - 7660*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*f*g^4*h^7 - 2240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*d*g^2*h^9 + 3440*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2)*f*g^2*h^9 - 80*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2)*d*h^11 - 160*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(3/2)*f*h^11 + 160*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*g^9*h^2*e + 1120*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*g^7*h^4*e - 6140*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*g^5*h^6*e + 5650*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*g^3*h^8*e - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2)*g*h^10*e - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*f*g^9*h^2 - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*d*g^7*h^4 - 960*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5*f*g^7*h^4 + 5000*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5*d*g^5*h^6 - 3890*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*f*g^5*h^6 - 2910*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*d*g^3*h^8 + 4710*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^3*f*g^3*h^8 + 430*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^3*d*g*h^10 - 940*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^2*f*g*h^10 - 160*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*g^8*h^3*e - 1440*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5*g^6*h^5*e + 5740*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*g^4*h^7*e - 1710*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^3*g^2*h^9*e + 90*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^2*h^11*e + 120*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(11/2)*f*g^8*h^3 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(11/2)*d*g^6*h^5 + 570*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*f*g^6*h^5 - 2810*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*d*g^4*h^7 + 2450*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(7/2)*f*g^4*h^7 + 650*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(7/2)*d*g^2*h^9 - 1700*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(5/2)*f*g^2*h^9 - 80*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(5/2)*d*h^11 + 80*(sqrt(c)*x

```
- sqrt(c*x^2 + a))^2*a^7*c^(3/2)*f*h^11 + 160*(sqrt(c)*x - sqrt(c*x^2 + a))
^2*a^3*c^(11/2)*g^7*h^4*e + 1100*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2
)*g^5*h^6*e - 2570*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(7/2)*g^3*h^8*e +
270*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(5/2)*g*h^10*e - 60*(sqrt(c)*x -
sqrt(c*x^2 + a))*a^4*c^5*f*g^7*h^4 - 60*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c
^5*d*g^5*h^6 - 270*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^4*f*g^5*h^6 + 770*(s
qrt(c)*x - sqrt(c*x^2 + a))*a^5*c^4*d*g^3*h^8 - 845*(sqrt(c)*x - sqrt(c*x^2
 + a))*a^6*c^3*f*g^3*h^8 - 115*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*d*g*h^
10 + 310*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^2*f*g*h^10 - 40*(sqrt(c)*x - s
qrt(c*x^2 + a))*a^4*c^5*g^6*h^5*e - 280*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c
^4*g^4*h^7*e + 720*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*g^2*h^9*e + 15*(sq
rt(c)*x - sqrt(c*x^2 + a))*a^7*c^2*h^11*e + 6*a^5*c^(9/2)*f*g^6*h^5 + 6*a^5
*c^(9/2)*d*g^4*h^7 + 27*a^6*c^(7/2)*f*g^4*h^7 - 83*a^6*c^(7/2)*d*g^2*h^9 +
86*a^7*c^(5/2)*f*g^2*h^9 + 16*a^7*c^(5/2)*d*h^11 - 40*a^8*c^(3/2)*f*h^11 +
4*a^5*c^(9/2)*g^5*h^6*e + 28*a^6*c^(7/2)*g^3*h^8*e - 81*a^7*c^(5/2)*g*h^10*
e)/((c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 +
 a^4*h^12)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 +
 a))*sqrt(c)*g - a*h)^5)
```

maple [B] time = 0.02, size = 8546, normalized size = 19.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x)
```

```
[Out] result too large to display
```

maxima [B] time = 1.53, size = 5793, normalized size = 13.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="maxima")
```

```
[Out] -7/8*sqrt(c*x^2 + a)*c^4*f*g^6/(c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x + 6*a^2*c
^2*g^4*h^8*x + 4*a^3*c*g^2*h^10*x + a^4*h^12*x + c^4*g^9*h^3 + 4*a*c^3*g^7*
h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^3*c*g^3*h^9 + a^4*g*h^11) + 7/8*sqrt(c*x^2 +
a)*c^4*e*g^5/(c^4*g^8*h^3*x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^4*h^7*x + 4*a
^3*c*g^2*h^9*x + a^4*h^11*x + c^4*g^9*h^2 + 4*a*c^3*g^7*h^4 + 6*a^2*c^2*g^5
*h^6 + 4*a^3*c*g^3*h^8 + a^4*g*h^10) - 7/8*(c*x^2 + a)^(3/2)*c^3*f*g^5/(c^4
*g^8*h^3*x^2 + 4*a*c^3*g^6*h^5*x^2 + 6*a^2*c^2*g^4*h^7*x^2 + 4*a^3*c*g^2*h^
9*x^2 + a^4*h^11*x^2 + 2*c^4*g^9*h^2*x + 8*a*c^3*g^7*h^4*x + 12*a^2*c^2*g^5
*h^6*x + 8*a^3*c*g^3*h^8*x + 2*a^4*g*h^10*x + c^4*g^10*h + 4*a*c^3*g^8*h^3
+ 6*a^2*c^2*g^6*h^5 + 4*a^3*c*g^4*h^7 + a^4*g^2*h^9) + 7/8*sqrt(c*x^2 + a)*
c^4*f*g^5/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*
h^9 + a^4*h^11) - 7/8*sqrt(c*x^2 + a)*c^4*d*g^4/(c^4*g^8*h^2*x + 4*a*c^3*g^
6*h^4*x + 6*a^2*c^2*g^4*h^6*x + 4*a^3*c*g^2*h^8*x + a^4*h^10*x + c^4*g^9*h
+ 4*a*c^3*g^7*h^3 + 6*a^2*c^2*g^5*h^5 + 4*a^3*c*g^3*h^7 + a^4*g*h^9) + 7/8*
(c*x^2 + a)^(3/2)*c^3*e*g^4/(c^4*g^8*h^2*x^2 + 4*a*c^3*g^6*h^4*x^2 + 6*a^2*
c^2*g^4*h^6*x^2 + 4*a^3*c*g^2*h^8*x^2 + a^4*h^10*x^2 + 2*c^4*g^9*h*x + 8*a*
c^3*g^7*h^3*x + 12*a^2*c^2*g^5*h^5*x + 8*a^3*c*g^3*h^7*x + 2*a^4*g*h^9*x +
c^4*g^10 + 4*a*c^3*g^8*h^2 + 6*a^2*c^2*g^6*h^4 + 4*a^3*c*g^4*h^6 + a^4*g^2*
h^8) - 7/8*sqrt(c*x^2 + a)*c^4*e*g^4/(c^4*g^8*h^2 + 4*a*c^3*g^6*h^4 + 6*a^2
*c^2*g^4*h^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) - 7/8*(c*x^2 + a)^(3/2)*c^3*d*g^
3/(c^4*g^8*h*x^2 + 4*a*c^3*g^6*h^3*x^2 + 6*a^2*c^2*g^4*h^5*x^2 + 4*a^3*c*g^
2*h^7*x^2 + a^4*h^9*x^2 + 2*c^4*g^9*x + 8*a*c^3*g^7*h^2*x + 12*a^2*c^2*g^5*
h^4*x + 8*a^3*c*g^3*h^6*x + 2*a^4*g*h^8*x + c^4*g^10/h + 4*a*c^3*g^8*h + 6*
a^2*c^2*g^6*h^3 + 4*a^3*c*g^4*h^5 + a^4*g^2*h^7) + 7/8*sqrt(c*x^2 + a)*c^4*
d*g^3/(c^4*g^8*h + 4*a*c^3*g^6*h^3 + 6*a^2*c^2*g^4*h^5 + 4*a^3*c*g^2*h^7 +
```

a^4*h^9) - 7/12*(c*x^2 + a)^(3/2)*c^2*f*g^4/(c^3*g^6*h^4*x^3 + 3*a*c^2*g^4*h^6*x^3 + 3*a^2*c*g^2*h^8*x^3 + a^3*h^10*x^3 + 3*c^3*g^7*h^3*x^2 + 9*a*c^2*g^5*h^5*x^2 + 9*a^2*c*g^3*h^7*x^2 + 3*a^3*g*h^9*x^2 + 3*c^3*g^8*h^2*x + 9*a*c^2*g^6*h^4*x + 9*a^2*c*g^4*h^6*x + 3*a^3*g^2*h^8*x + c^3*g^9*h + 3*a*c^2*g^7*h^3 + 3*a^2*c*g^5*h^5 + a^3*g^3*h^7) + 13/8*sqrt(c*x^2 + a)*c^3*f*g^4/(c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h^10*x + c^3*g^7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 + a^3*g*h^9) + 7/12*(c*x^2 + a)^(3/2)*c^2*e*g^3/(c^3*g^6*h^3*x^3 + 3*a*c^2*g^4*h^5*x^3 + 3*a^2*c*g^2*h^7*x^3 + a^3*h^9*x^3 + 3*c^3*g^7*h^2*x^2 + 9*a*c^2*g^5*h^4*x^2 + 9*a^2*c*g^3*h^6*x^2 + 3*a^3*g*h^8*x^2 + 3*c^3*g^8*h*x + 9*a*c^2*g^6*h^3*x + 9*a^2*c*g^4*h^5*x + 3*a^3*g^2*h^7*x + c^3*g^9 + 3*a*c^2*g^7*h^2 + 3*a^2*c*g^5*h^4 + a^3*g^3*h^6) - sqrt(c*x^2 + a)*c^3*e*g^3/(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2*c*g^3*h^6 + a^3*g*h^8) + 13/8*(c*x^2 + a)^(3/2)*c^2*f*g^3/(c^3*g^6*h^3*x^2 + 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2*h^7) - 13/8*sqrt(c*x^2 + a)*c^3*f*g^3/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 7/12*(c*x^2 + a)^(3/2)*c^2*d*g^2/(c^3*g^6*h^2*x^3 + 3*a*c^2*g^4*h^4*x^3 + 3*a^2*c*g^2*h^6*x^3 + a^3*h^8*x^3 + 3*c^3*g^7*h*x^2 + 9*a*c^2*g^5*h^3*x^2 + 9*a^2*c*g^3*h^5*x^2 + 3*a^3*g*h^7*x^2 + 3*c^3*g^8*x + 9*a*c^2*g^6*h^2*x + 9*a^2*c*g^4*h^4*x + 3*a^3*g^2*h^6*x + c^3*g^9/h + 3*a*c^2*g^7*h + 3*a^2*c*g^5*h^3 + a^3*g^3*h^5) + 3/8*sqrt(c*x^2 + a)*c^3*d*g^2/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g^2*h^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7) - (c*x^2 + a)^(3/2)*c^2*e*g^2/(c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h^4*x^2 + 3*a^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 + 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^3*x + 6*a^2*c*g^3*h^5*x + 2*a^3*g*h^7*x + c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4 + a^3*g^2*h^6) + sqrt(c*x^2 + a)*c^3*e*g^2/(c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) - 7/20*(c*x^2 + a)^(3/2)*c*f*g^3/(c^2*g^4*h^5*x^4 + 2*a*c*g^2*h^7*x^4 + a^2*h^9*x^4 + 4*c^2*g^5*h^4*x^3 + 8*a*c*g^3*h^6*x^3 + 4*a^2*g*h^8*x^3 + 6*c^2*g^6*h^3*x^2 + 12*a*c*g^4*h^5*x^2 + 6*a^2*g^2*h^7*x^2 + 4*c^2*g^7*h^2*x + 8*a*c*g^5*h^4*x + 4*a^2*g^3*h^6*x + c^2*g^8*h + 2*a*c*g^6*h^3 + a^2*g^4*h^5) + 3/8*(c*x^2 + a)^(3/2)*c^2*d*g/(c^3*g^6*h*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^3*g^2*h^5) - 3/8*sqrt(c*x^2 + a)*c^3*d*g/(c^3*g^6*h + 3*a*c^2*g^4*h^3 + 3*a^2*c*g^2*h^5 + a^3*h^7) + 7/20*(c*x^2 + a)^(3/2)*c*e*g^2/(c^2*g^4*h^4*x^4 + 2*a*c*g^2*h^6*x^4 + a^2*h^8*x^4 + 4*c^2*g^5*h^3*x^3 + 8*a*c*g^3*h^5*x^3 + 4*a^2*g*h^7*x^3 + 6*c^2*g^6*h^2*x^2 + 12*a*c*g^4*h^4*x^2 + 6*a^2*g^2*h^6*x^2 + 4*c^2*g^7*h*x + 8*a*c*g^5*h^3*x + 4*a^2*g^3*h^5*x + c^2*g^8 + 2*a*c*g^6*h^2 + a^2*g^4*h^4) + 29/30*(c*x^2 + a)^(3/2)*c*f*g^2/(c^2*g^4*h^4*x^3 + 2*a*c*g^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^5) - 3/4*sqrt(c*x^2 + a)*c^2*f*g^2/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) - 7/20*(c*x^2 + a)^(3/2)*c*d*g/(c^2*g^4*h^3*x^4 + 2*a*c*g^2*h^5*x^4 + a^2*h^7*x^4 + 4*c^2*g^5*h^2*x^3 + 8*a*c*g^3*h^4*x^3 + 4*a^2*g*h^6*x^3 + 6*c^2*g^6*h*x^2 + 12*a*c*g^4*h^3*x^2 + 6*a^2*g^2*h^5*x^2 + 4*c^2*g^7*x + 8*a*c*g^5*h^2*x + 4*a^2*g^3*h^4*x + c^2*g^8/h + 2*a*c*g^6*h + a^2*g^4*h^3) - 11/20*(c*x^2 + a)^(3/2)*c*e*g/(c^2*g^4*h^3*x^3 + 2*a*c*g^2*h^5*x^3 + a^2*h^7*x^3 + 3*c^2*g^5*h^2*x^2 + 6*a*c*g^3*h^4*x^2 + 3*a^2*g*h^6*x^2 + 3*c^2*g^6*h*x + 6*a*c*g^4*h^3*x + 3*a^2*g^2*h^5*x + c^2*g^7 + 2*a*c*g^5*h^2 + a^2*g^3*h^4) + 1/8*sqrt(c*x^2 + a)*c^2*e*g/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*h^4 + a^2*g*h^6) - 3/4*(c*x^2 + a)^(3/2)*c*f*g/(c^2*g^4*h^3*x^2 + 2*a*c*g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6*x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 3/4*sqrt(c*x^2 + a)*c^2*f*g/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/5*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2*h^6*x^5 + a*h^8*x^5 + 5*c*g^3*h^5*x^4 + 5*a*g*h^7*x^4 + 10*c*g^4*h^4*x^3 + 10*a*g

```
^2*h^6*x^3 + 10*c*g^5*h^3*x^2 + 10*a*g^3*h^5*x^2 + 5*c*g^6*h^2*x + 5*a*g^4*
h^4*x + c*g^7*h + a*g^5*h^3) + 2/15*(c*x^2 + a)^(3/2)*c*d/(c^2*g^4*h^2*x^3
+ 2*a*c*g^2*h^4*x^3 + a^2*h^6*x^3 + 3*c^2*g^5*h*x^2 + 6*a*c*g^3*h^3*x^2 + 3
*a^2*g*h^5*x^2 + 3*c^2*g^6*x + 6*a*c*g^4*h^2*x + 3*a^2*g^2*h^4*x + c^2*g^7/
h + 2*a*c*g^5*h + a^2*g^3*h^3) + 1/8*(c*x^2 + a)^(3/2)*c*e/(c^2*g^4*h^2*x^2
 + 2*a*c*g^2*h^4*x^2 + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^
2*g*h^5*x + c^2*g^6 + 2*a*c*g^4*h^2 + a^2*g^2*h^4) - 1/8*sqrt(c*x^2 + a)*c^
2*e/(c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) + 1/5*(c*x^2 + a)^(3/2)*e*g/(c*
g^2*h^5*x^5 + a*h^7*x^5 + 5*c*g^3*h^4*x^4 + 5*a*g*h^6*x^4 + 10*c*g^4*h^3*x^
3 + 10*a*g^2*h^5*x^3 + 10*c*g^5*h^2*x^2 + 10*a*g^3*h^4*x^2 + 5*c*g^6*h*x +
5*a*g^4*h^3*x + c*g^7 + a*g^5*h^2) + 1/2*(c*x^2 + a)^(3/2)*f*g/(c*g^2*h^5*x
^4 + a*h^7*x^4 + 4*c*g^3*h^4*x^3 + 4*a*g*h^6*x^3 + 6*c*g^4*h^3*x^2 + 6*a*g^
2*h^5*x^2 + 4*c*g^5*h^2*x + 4*a*g^3*h^4*x + c*g^6*h + a*g^4*h^3) - 1/5*(c*x
^2 + a)^(3/2)*d/(c*g^2*h^4*x^5 + a*h^6*x^5 + 5*c*g^3*h^3*x^4 + 5*a*g*h^5*x^
4 + 10*c*g^4*h^2*x^3 + 10*a*g^2*h^4*x^3 + 10*c*g^5*h*x^2 + 10*a*g^3*h^3*x^2
 + 5*c*g^6*x + 5*a*g^4*h^2*x + c*g^7/h + a*g^5*h) - 1/4*(c*x^2 + a)^(3/2)*e
/(c*g^2*h^4*x^4 + a*h^6*x^4 + 4*c*g^3*h^3*x^3 + 4*a*g*h^5*x^3 + 6*c*g^4*h^2
*x^2 + 6*a*g^2*h^4*x^2 + 4*c*g^5*h*x + 4*a*g^3*h^3*x + c*g^6 + a*g^4*h^2) -
 1/3*(c*x^2 + a)^(3/2)*f/(c*g^2*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3*x^2 + 3*a
*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c*g^5*h + a*g^3*h^3) - 7/8*c^5
*f*g^7*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)
))/((a + c*g^2/h^2)^(9/2)*h^13) + 7/8*c^5*e*g^6*arcsinh(c*g*x/(sqrt(a*c)*ab
s(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(9/2)*h^12) -
7/8*c^5*d*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h
*x + g)))/((a + c*g^2/h^2)^(9/2)*h^11) + 5/2*c^4*f*g^5*arcsinh(c*g*x/(sqrt(
a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h
^11) - 15/8*c^4*e*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*
c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^10) + 5/4*c^4*d*g^3*arcsinh(c*g*
x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)
^(7/2)*h^9) - 19/8*c^3*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(
sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^9) + 9/8*c^3*e*g^2*arcsin
h(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^
2/h^2)^(5/2)*h^8) - 3/8*c^3*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*
h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) + 3/4*c^2*f*g*arcsi
nh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g
^2/h^2)^(3/2)*h^7) - 1/8*c^2*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h
/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
```

```
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**6,x)
```

```
[Out] Timed out
```

3.88 $\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=462

$$\frac{x\left(a+cx^2\right)^{3/2}\left(3a^2h^2(eh+3fg)-8acg\left(3h(dh+eg)+fg^2\right)+48c^2dg^3\right)}{192c^2}+\frac{ax\sqrt{a+cx^2}\left(3a^2h^2(eh+3fg)-8acg\right.}{128c}$$

```
[Out] 1/192*(48*c^2*d*g^3+3*a^2*h^2*(e*h+3*f*g)-8*a*c*g*(f*g^2+3*h*(d*h+e*g)))*x*
(c*x^2+a)^(3/2)/c^2+1/504*(8*(-4*a*f+9*c*d)*h^2-3*c*g*(-9*e*h+5*f*g))*(h*x+
g)^2*(c*x^2+a)^(5/2)/c^2/h-1/72*(-9*e*h+5*f*g)*(h*x+g)^3*(c*x^2+a)^(5/2)/c/
h+1/9*f*(h*x+g)^4*(c*x^2+a)^(5/2)/c/h+1/5040*(128*a^2*f*h^4-32*a*c*h^2*(17*
f*g^2+9*h*(d*h+3*e*g))-12*c^2*g^2*(5*f*g^2-3*h*(64*d*h+3*e*g))-5*c*h*(a*h^2
*(63*e*h+61*f*g)+2*c*g*(5*f*g^2-9*h*(12*d*h+e*g)))*x)*(c*x^2+a)^(5/2)/c^3/h
+1/128*a^2*(48*c^2*d*g^3+3*a^2*h^2*(e*h+3*f*g)-8*a*c*g*(f*g^2+3*h*(d*h+e*g)
))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(5/2)+1/128*a*(48*c^2*d*g^3+3*a^2*h
^2*(e*h+3*f*g)-8*a*c*g*(f*g^2+3*h*(d*h+e*g)))*x*(c*x^2+a)^(1/2)/c^2
```

Rubi [A] time = 1.13, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 833, 780, 195, 217, 206}

$$\frac{\left(a+cx^2\right)^{5/2}\left(4\left(32a^2fh^4-8ach^2\left(9h(dh+3eg)+17fg^2\right)-c^2\left(15fg^4-9g^2h(64dh+3eg)\right)\right)-5chx\left(ah^2(63eh\right.\right.}{5040c^3h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*
h)))*x*Sqrt[a + c*x^2])/(128*c^2) + ((48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h
) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*x*(a + c*x^2)^(3/2))/(192*c^2) + ((8
*(9*c*d - 4*a*f)*h^2 - 3*c*g*(5*f*g - 9*e*h))*(g + h*x)^2*(a + c*x^2)^(5/2)
)/(504*c^2*h) - ((5*f*g - 9*e*h)*(g + h*x)^3*(a + c*x^2)^(5/2))/(72*c*h) +
(f*(g + h*x)^4*(a + c*x^2)^(5/2))/(9*c*h) + ((4*(32*a^2*f*h^4 - 8*a*c*h^2*(
17*f*g^2 + 9*h*(3*e*g + d*h)) - c^2*(15*f*g^4 - 9*g^2*h*(3*e*g + 64*d*h)))
- 5*c*h*(a*h^2*(61*f*g + 63*e*h) + 2*c*(5*f*g^3 - 9*g*h*(e*g + 12*d*h)))*x)
*(a + c*x^2)^(5/2))/(5040*c^3*h) + (a^2*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g +
e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^
2]])/(128*c^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^3\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx=\frac{f(g+hx)^4\left(a+cx^2\right)^{5/2}}{9ch}+\frac{\int (g+hx)^3\left((9cd-4af)h^2-ch(5fg-9eh\right.}{9ch^2}$$

$$=-\frac{(5fg-9eh)(g+hx)^3\left(a+cx^2\right)^{5/2}}{72ch}+\frac{f(g+hx)^4\left(a+cx^2\right)^{5/2}}{9ch}+\frac{\int (g}{}$$

$$=\frac{\left(8(9cd-4af)h^2-3cg(5fg-9eh)\right)(g+hx)^2\left(a+cx^2\right)^{5/2}}{504c^2h}-\frac{(5fg-}{}$$

$$=\frac{\left(8(9cd-4af)h^2-3cg(5fg-9eh)\right)(g+hx)^2\left(a+cx^2\right)^{5/2}}{504c^2h}-\frac{(5fg-}{}$$

$$=\frac{\left(48c^2dg^3+3a^2h^2(3fg+eh)-8acg\left(fg^2+3h(eg+dh)\right)\right)x\left(a+cx^2\right.}{192c^2}$$

$$=\frac{a\left(48c^2dg^3+3a^2h^2(3fg+eh)-8acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+c}}{128c^2}$$

$$=\frac{a\left(48c^2dg^3+3a^2h^2(3fg+eh)-8acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+c}}{128c^2}$$

$$=\frac{a\left(48c^2dg^3+3a^2h^2(3fg+eh)-8acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+c}}{128c^2}$$

Mathematica [A] time = 0.54, size = 481, normalized size = 1.04

$$\frac{315a^2\sqrt{c}\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)\left(3a^2h^2(eh+3fg)-8acg\left(3h(dh+eg)+fg^2\right)+48c^2dg^3\right)+\sqrt{a+cx^2}\left(384c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(128*a^2*(8*a^2*f*h^3 + 63*c^2*g^2*(e*g + 3*d*h) - 18*a*c*
h*(3*f*g^2 + h*(3*e*g + d*h))) + 315*a*c*(80*c^2*d*g^3 - 3*a^2*h^2*(3*f*g +
 e*h) + 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*x + 128*a*c*(-4*a^2*f*h^3 + 126*
c^2*g^2*(e*g + 3*d*h) + 9*a*c*h*(3*f*g^2 + h*(3*e*g + d*h)))*x^2 + 210*c^2*
(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) + 56*a*c*g*(f*g^2 + 3*h*(e*g + d*h)
))*x^3 + 384*c^2*(a^2*f*h^3 + 21*c^2*g^2*(e*g + 3*d*h) + 24*a*c*h*(3*f*g^2
+ h*(3*e*g + d*h)))*x^4 + 840*c^3*(9*a*h^2*(3*f*g + e*h) + 8*c*(f*g^3 + 3*g
*h*(e*g + d*h)))*x^5 + 640*c^3*h*(10*a*f*h^2 + 9*c*(3*f*g^2 + h*(3*e*g + d*
h)))*x^6 + 5040*c^4*h^2*(3*f*g + e*h)*x^7 + 4480*c^4*f*h^3*x^8) + 315*a^2*S
qrt[c]*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g
+ d*h)))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(40320*c^3)
```

fricas [A] time = 1.48, size = 1177, normalized size = 2.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/80640*(315*(24*a^3*c*e*g^2*h - 3*a^4*e*h^3 - 8*(6*a^2*c^2*d - a^3*c*f)*
g^3 + 3*(8*a^3*c*d - 3*a^4*f)*g*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 +
a)*sqrt(c)*x - a) - 2*(4480*c^4*f*h^3*x^8 + 8064*a^2*c^2*e*g^3 - 6912*a^3*c
*e*g*h^2 + 5040*(3*c^4*f*g*h^2 + c^4*e*h^3)*x^7 + 640*(27*c^4*f*g^2*h + 27*
c^4*e*g*h^2 + (9*c^4*d + 10*a*c^3*f)*h^3)*x^6 + 840*(8*c^4*f*g^3 + 24*c^4*e
*g^2*h + 9*a*c^3*e*h^3 + 3*(8*c^4*d + 9*a*c^3*f)*g*h^2)*x^5 + 384*(21*c^4*e
*g^3 + 72*a*c^3*e*g*h^2 + 9*(7*c^4*d + 8*a*c^3*f)*g^2*h + (24*a*c^3*d + a^2
*c^2*f)*h^3)*x^4 + 3456*(7*a^2*c^2*d - 2*a^3*c*f)*g^2*h - 256*(9*a^3*c*d -
4*a^4*f)*h^3 + 210*(168*a*c^3*e*g^2*h + 3*a^2*c^2*e*h^3 + 8*(6*c^4*d + 7*a*
c^3*f)*g^3 + 3*(56*a*c^3*d + 3*a^2*c^2*f)*g*h^2)*x^3 + 128*(126*a*c^3*e*g^3
 + 27*a^2*c^2*e*g*h^2 + 27*(14*a*c^3*d + a^2*c^2*f)*g^2*h + (9*a^2*c^2*d -
4*a^3*c*f)*h^3)*x^2 + 315*(24*a^2*c^2*e*g^2*h - 3*a^3*c*e*h^3 + 8*(10*a*c^3
*d + a^2*c^2*f)*g^3 + 3*(8*a^2*c^2*d - 3*a^3*c*f)*g*h^2)*x)*sqrt(c*x^2 + a)
)/c^3, 1/40320*(315*(24*a^3*c*e*g^2*h - 3*a^4*e*h^3 - 8*(6*a^2*c^2*d - a^3*
c*f)*g^3 + 3*(8*a^3*c*d - 3*a^4*f)*g*h^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) + (4480*c^4*f*h^3*x^8 + 8064*a^2*c^2*e*g^3 - 6912*a^3*c*e*g*h^2
+ 5040*(3*c^4*f*g*h^2 + c^4*e*h^3)*x^7 + 640*(27*c^4*f*g^2*h + 27*c^4*e*g*h
^2 + (9*c^4*d + 10*a*c^3*f)*h^3)*x^6 + 840*(8*c^4*f*g^3 + 24*c^4*e*g^2*h +
9*a*c^3*e*h^3 + 3*(8*c^4*d + 9*a*c^3*f)*g*h^2)*x^5 + 384*(21*c^4*e*g^3 + 72
*a*c^3*e*g*h^2 + 9*(7*c^4*d + 8*a*c^3*f)*g^2*h + (24*a*c^3*d + a^2*c^2*f)*h
^3)*x^4 + 3456*(7*a^2*c^2*d - 2*a^3*c*f)*g^2*h - 256*(9*a^3*c*d - 4*a^4*f)*
h^3 + 210*(168*a*c^3*e*g^2*h + 3*a^2*c^2*e*h^3 + 8*(6*c^4*d + 7*a*c^3*f)*g^
3 + 3*(56*a*c^3*d + 3*a^2*c^2*f)*g*h^2)*x^3 + 128*(126*a*c^3*e*g^3 + 27*a^2
*c^2*e*g*h^2 + 27*(14*a*c^3*d + a^2*c^2*f)*g^2*h + (9*a^2*c^2*d - 4*a^3*c*f
)*h^3)*x^2 + 315*(24*a^2*c^2*e*g^2*h - 3*a^3*c*e*h^3 + 8*(10*a*c^3*d + a^2*
c^2*f)*g^3 + 3*(8*a^2*c^2*d - 3*a^3*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

giac [A] time = 0.27, size = 652, normalized size = 1.41

$$\frac{1}{40320}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\left(2\left(7\left(8\,cfh^3x+\frac{9\left(3\,c^8fgh^2+c^8h^3e\right)}{c^7}\right)x+\frac{8\left(27\,c^8fg^2h+9\,c^8dh^3+10\,ac^7fh^3+27\right.}{c^7}\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")

[Out] 1/40320*sqrt(c*x^2 + a)*((2*((4*(5*(2*(7*(8*c*f*h^3*x + 9*(3*c^8*f*g*h^2 + c^8*h^3*e)/c^7)*x + 8*(27*c^8*f*g^2*h + 9*c^8*d*h^3 + 10*a*c^7*f*h^3 + 27*c^8*g*h^2*e)/c^7)*x + 21*(8*c^8*f*g^3 + 24*c^8*d*g*h^2 + 27*a*c^7*f*g*h^2 + 24*c^8*g^2*h*e + 9*a*c^7*h^3*e)/c^7)*x + 48*(63*c^8*d*g^2*h + 72*a*c^7*f*g^2*h + 24*a*c^7*d*h^3 + a^2*c^6*f*h^3 + 21*c^8*g^3*e + 72*a*c^7*g*h^2*e)/c^7)*x + 105*(48*c^8*d*g^3 + 56*a*c^7*f*g^3 + 168*a*c^7*d*g*h^2 + 9*a^2*c^6*f*g*h^2 + 168*a*c^7*g^2*h*e + 3*a^2*c^6*h^3*e)/c^7)*x + 64*(378*a*c^7*d*g^2*h + 27*a^2*c^6*f*g^2*h + 9*a^2*c^6*d*h^3 - 4*a^3*c^5*f*h^3 + 126*a*c^7*g^3*e + 27*a^2*c^6*g*h^2*e)/c^7)*x + 315*(80*a*c^7*d*g^3 + 8*a^2*c^6*f*g^3 + 24*a^2*c^6*d*g*h^2 - 9*a^3*c^5*f*g*h^2 + 24*a^2*c^6*g^2*h*e - 3*a^3*c^5*h^3*e)/c^7)*x + 128*(189*a^2*c^6*d*g^2*h - 54*a^3*c^5*f*g^2*h - 18*a^3*c^5*d*h^3 + 8*a^4*c^4*f*h^3 + 63*a^2*c^6*g^3*e - 54*a^3*c^5*g*h^2*e)/c^7) - 1/128*(48*a^2*c^2*d*g^3 - 8*a^3*c*f*g^3 - 24*a^3*c*d*g*h^2 + 9*a^4*f*g*h^2 - 24*a^3*c*g^2*h*e + 3*a^4*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)

maple [A] time = 0.02, size = 794, normalized size = 1.72

$$\frac{\left(c\,x^{2}+a\right)^{\frac{5}{2}} f\,h^{3} x^{4}}{9 c}+\frac{3 a^{4} e\,h^{3} \ln \left(\sqrt{c}\, x+\sqrt{c\,x^{2}+a}\right)}{128 c^{\frac{5}{2}}}+\frac{9 a^{4} f g\,h^{2} \ln \left(\sqrt{c}\, x+\sqrt{c\,x^{2}+a}\right)}{128 c^{\frac{5}{2}}}-\frac{3 a^{3} d g\,h^{2} \ln \left(\sqrt{c}\, x+\sqrt{c\,x^{2}\right.}}{16 c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x)

[Out] -3/16*a^3/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*d*g*h^2-3/16*a^3/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*g^2*h+3/7*x^2*(c*x^2+a)^(5/2)/c*e*g*h^2+3/7*x^2*(c*x^2+a)^(5/2)/c*f*g^2*h-6/35*a/c^2*(c*x^2+a)^(5/2)*e*g*h^2-6/35*a/c^2*(c*x^2+a)^(5/2)*f*g^2*h-4/63*f*h^3*a/c^2*x^2*(c*x^2+a)^(5/2)+3/8*x^3*(c*x^2+a)^(5/2)/c*f*g*h^2-1/16*a/c^2*x*(c*x^2+a)^(5/2)*e*h^3+1/64*a^2/c^2*x*(c*x^2+a)^(3/2)*e*h^3+3/128*a^3/c^2*x*(c*x^2+a)^(1/2)*e*h^3+9/128*a^4/c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*f*g*h^2+1/2*x*(c*x^2+a)^(5/2)/c*d*g*h^2+1/2*x*(c*x^2+a)^(5/2)/c*e*g^2*h-1/24*a/c*x*(c*x^2+a)^(3/2)*f*g^3-1/16*a^2/c*x*(c*x^2+a)^(1/2)*f*g^3+1/5*(c*x^2+a)^(5/2)/c*e*g^3+1/4*d*g^3*x*(c*x^2+a)^(3/2)-3/16*a^2/c*x*(c*x^2+a)^(1/2)*d*g*h^2-3/16*a^2/c*x*(c*x^2+a)^(1/2)*e*g^2*h-1/8*a/c*x*(c*x^2+a)^(3/2)*e*g^2*h-3/16*a/c^2*x*(c*x^2+a)^(5/2)*f*g*h^2+3/64*a^2/c^2*x*(c*x^2+a)^(3/2)*f*g*h^2+9/128*a^3/c^2*x*(c*x^2+a)^(1/2)*f*g*h^2-1/8*a/c*x*(c*x^2+a)^(3/2)*d*g*h^2+1/9*f*h^3*x^4*(c*x^2+a)^(5/2)/c+8/315*f*h^3*a^2/c^3*(c*x^2+a)^(5/2)+1/8*x^3*(c*x^2+a)^(5/2)/c*e*h^3+3/128*a^4/c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*h^3+1/7*x^2*(c*x^2+a)^(5/2)/c*d*h^3-2/35*a/c^2*(c*x^2+a)^(5/2)*d*h^3+1/6*x*(c*x^2+a)^(5/2)/c*f*g^3-1/16*a^3/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*f*g^3+3/5*(c*x^2+a)^(5/2)/c*d*g^2*h+3/8*d*g^3*a*x*(c*x^2+a)^(1/2)+3/8*d*g^3*a^2/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))

maxima [A] time = 0.46, size = 525, normalized size = 1.14

$$\frac{\left(cx^{2}+a\right)^{\frac{5}{2}} f h^{3} x^{4}}{9\,c}-\frac{4\left(cx^{2}+a\right)^{\frac{5}{2}} a f h^{3} x^{2}}{63\,c^{2}}+\frac{1}{4}\left(cx^{2}+a\right)^{\frac{3}{2}} d g^{3} x+\frac{3}{8}\sqrt{cx^{2}+a}\,a d g^{3} x+\frac{3\,a^{2} d g^{3} \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}}+\frac{\left(cx^{2}+a\right)^{\frac{5}{2}}}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")

[Out] 1/9*(c*x^2 + a)^(5/2)*f*h^3*x^4/c - 4/63*(c*x^2 + a)^(5/2)*a*f*h^3*x^2/c^2 + 1/4*(c*x^2 + a)^(3/2)*d*g^3*x + 3/8*sqrt(c*x^2 + a)*a*d*g^3*x + 3/8*a^2*d*g^3*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 1/5*(c*x^2 + a)^(5/2)*e*g^3/c + 3/5*(c*x^2 + a)^(5/2)*d*g^2*h/c + 8/315*(c*x^2 + a)^(5/2)*a^2*f*h^3/c^3 + 1/8*(3

```
*f*g*h^2 + e*h^3)*(c*x^2 + a)^(5/2)*x^3/c + 1/7*(3*f*g^2*h + 3*e*g*h^2 + d*
h^3)*(c*x^2 + a)^(5/2)*x^2/c - 1/16*(3*f*g*h^2 + e*h^3)*(c*x^2 + a)^(5/2)*a
*x/c^2 + 1/64*(3*f*g*h^2 + e*h^3)*(c*x^2 + a)^(3/2)*a^2*x/c^2 + 3/128*(3*f*
g*h^2 + e*h^3)*sqrt(c*x^2 + a)*a^3*x/c^2 + 1/6*(f*g^3 + 3*e*g^2*h + 3*d*g*h
^2)*(c*x^2 + a)^(5/2)*x/c - 1/24*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*(c*x^2 + a
)^(3/2)*a*x/c - 1/16*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*sqrt(c*x^2 + a)*a^2*x/
c + 3/128*(3*f*g*h^2 + e*h^3)*a^4*arcsinh(c*x/sqrt(a*c))/c^(5/2) - 1/16*(f*
g^3 + 3*e*g^2*h + 3*d*g*h^2)*a^3*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/35*(3*f
*g^2*h + 3*e*g*h^2 + d*h^3)*(c*x^2 + a)^(5/2)*a/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g+h\,x\right)^3\,{\left(c\,x^2+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

sympy [A] time = 72.41, size = 1916, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
[Out] -3*a**(7/2)*e*h**3*x/(128*c**2*sqrt(1 + c*x**2/a)) - 9*a**(7/2)*f*g*h**2*x/
(128*c**2*sqrt(1 + c*x**2/a)) + 3*a**(5/2)*d*g*h**2*x/(16*c*sqrt(1 + c*x**2
/a)) + 3*a**(5/2)*e*g**2*h*x/(16*c*sqrt(1 + c*x**2/a)) - a**(5/2)*e*h**3*x*
*3/(128*c*sqrt(1 + c*x**2/a)) + a**(5/2)*f*g**3*x/(16*c*sqrt(1 + c*x**2/a))
 - 3*a**(5/2)*f*g*h**2*x**3/(128*c*sqrt(1 + c*x**2/a)) + a**(3/2)*d*g**3*x*
sqrt(1 + c*x**2/a)/2 + a**(3/2)*d*g**3*x/(8*sqrt(1 + c*x**2/a)) + 17*a**(3/
2)*d*g*h**2*x**3/(16*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*e*g**2*h*x**3/(16*sq
rt(1 + c*x**2/a)) + 13*a**(3/2)*e*h**3*x**5/(64*sqrt(1 + c*x**2/a)) + 17*a*
*(3/2)*f*g**3*x**3/(48*sqrt(1 + c*x**2/a)) + 39*a**(3/2)*f*g*h**2*x**5/(64*
sqrt(1 + c*x**2/a)) + 3*sqrt(a)*c*d*g**3*x**3/(8*sqrt(1 + c*x**2/a)) + 11*s
qrt(a)*c*d*g*h**2*x**5/(8*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*e*g**2*h*x**5/
(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*c*e*h**3*x**7/(16*sqrt(1 + c*x**2/a)) +
11*sqrt(a)*c*f*g**3*x**5/(24*sqrt(1 + c*x**2/a)) + 15*sqrt(a)*c*f*g*h**2*x*
*7/(16*sqrt(1 + c*x**2/a)) + 3*a**4*e*h**3*asinh(sqrt(c)*x/sqrt(a))/(128*c*
*(5/2)) + 9*a**4*f*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(128*c**(5/2)) - 3*a**3*
d*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) - 3*a**3*e*g**2*h*asinh(sqr
t(c)*x/sqrt(a))/(16*c**(3/2)) - a**3*f*g**3*asinh(sqrt(c)*x/sqrt(a))/(16*c*
*(3/2)) + 3*a**2*d*g**3*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + 3*a*d*g**2*h
*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) +
 a*d*h**3*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c
*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True))
 + a*e*g**3*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c
), True)) + 3*a*e*g*h**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*
x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)
*x**4/4, True)) + 3*a*f*g**2*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2
) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (s
qrt(a)*x**4/4, True)) + a*f*h**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c*
*3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35
*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 3*c*d*g
**2*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**
2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c
*d*h**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a
+ c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**
```

```
2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + c*e*g**3*Piecewise((-2*a**2*sqrt
(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x
**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 3*c*e*g*h**2*Piecewise((8*a**3
*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*
x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)
*x**6/6, True)) + 3*c*f*g**2*h*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3
) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c
) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + c*f*h**3*
Piecewise((-16*a**4*sqrt(a + c*x**2)/(315*c**4) + 8*a**3*x**2*sqrt(a + c*x*
*2)/(315*c**3) - 2*a**2*x**4*sqrt(a + c*x**2)/(105*c**2) + a*x**6*sqrt(a +
c*x**2)/(63*c) + x**8*sqrt(a + c*x**2)/9, Ne(c, 0)), (sqrt(a)*x**8/8, True)
) + c**2*d*g**3*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*d*g*h**2*x**7/(2
*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*e*g**2*h*x**7/(2*sqrt(a)*sqrt(1 + c*x**
2/a)) + c**2*e*h**3*x**9/(8*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*f*g**3*x**7/
(6*sqrt(a)*sqrt(1 + c*x**2/a)) + 3*c**2*f*g*h**2*x**9/(8*sqrt(a)*sqrt(1 + c
*x**2/a))
```

3.89 $\int (g+hx)^2 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=346

$$\frac{x\left(a+cx^2\right)^{3/2}\left(3a^2fh^2-8ac\left(h(dh+2eg)+fg^2\right)+48c^2dg^2\right)}{192c^2}+\frac{ax\sqrt{a+cx^2}\left(3a^2fh^2-8ac\left(h(dh+2eg)+fg^2\right)\right.}{128c^2}$$

```
[Out] 1/192*(48*c^2*d*g^2+3*a^2*f*h^2-8*a*c*(f*g^2+h*(d*h+2*e*g)))*x*(c*x^2+a)^(3
/2)/c^2-1/56*(-8*e*h+5*f*g)*(h*x+g)^2*(c*x^2+a)^(5/2)/c/h+1/8*f*(h*x+g)^3*(
c*x^2+a)^(5/2)/c/h-1/1680*(96*a*h^2*(e*h+2*f*g)+12*c*g*(5*f*g^2-8*h*(7*d*h+
e*g))-5*h*(7*(-3*a*f+8*c*d)*h^2-2*c*g*(-8*e*h+5*f*g))*x)*(c*x^2+a)^(5/2)/c^
2/h+1/128*a^2*(48*c^2*d*g^2+3*a^2*f*h^2-8*a*c*(f*g^2+h*(d*h+2*e*g)))*arctan
h(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(5/2)+1/128*a*(48*c^2*d*g^2+3*a^2*f*h^2-8*a*
c*(f*g^2+h*(d*h+2*e*g)))*x*(c*x^2+a)^(1/2)/c^2
```

Rubi [A] time = 0.52, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 833, 780, 195, 217, 206}

$$\frac{x\left(a+cx^2\right)^{3/2}\left(3a^2fh^2-8ac\left(h(dh+2eg)+fg^2\right)+48c^2dg^2\right)}{192c^2}+\frac{ax\sqrt{a+cx^2}\left(3a^2fh^2-8ac\left(h(dh+2eg)+fg^2\right)\right.}{128c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*g + d*h)))*x*Sqrt[a
 + c*x^2])/(128*c^2) + ((48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*
g + d*h)))*x*(a + c*x^2)^(3/2))/(192*c^2) - ((5*f*g - 8*e*h)*(g + h*x)^2*(a
 + c*x^2)^(5/2))/(56*c*h) + (f*(g + h*x)^3*(a + c*x^2)^(5/2))/(8*c*h) - ((1
2*(5*c*f*g^3 - 8*c*g*h*(e*g + 7*d*h) + 8*a*h^2*(2*f*g + e*h)) - 5*h*(7*(8*c
*d - 3*a*f)*h^2 - 2*c*g*(5*f*g - 8*e*h))*x)*(a + c*x^2)^(5/2))/(1680*c^2*h)
 + (a^2*(48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*g + d*h)))*ArcT
anh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
```

```
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch} + \frac{\int (g+hx)^2\left((8cd-3af)h^2-ch(5fg-8eh\cdots}{8ch^2}$$

$$= -\frac{(5fg-8eh)(g+hx)^2\left(a+cx^2\right)^{5/2}}{56ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch} + \frac{\int(g\cdots}{\cdots}$$

$$= -\frac{(5fg-8eh)(g+hx)^2\left(a+cx^2\right)^{5/2}}{56ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch} - \frac{(12\cdots}{\cdots}$$

$$= \frac{\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\left(a+cx^2\right)^{3/2}}{192c^2} - \frac{(5f\cdots}{\cdots}$$

$$= \frac{a\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{128c^2} + \frac{\left(48c\cdots\right.}{\cdots}$$

$$= \frac{a\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{128c^2} + \frac{\left(48c\cdots\right.}{\cdots}$$

$$= \frac{a\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{128c^2} + \frac{\left(48c\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.09, size = 346, normalized size = 1.00

$$\sqrt{a+cx^2}\left(-\frac{280a\left(3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)+\sqrt{c}x\left(5a+2cx^2\right)\sqrt{\frac{cx^2}{a}+1}\right)\left(h(dh+2eg)+fg^2\right)}{c^{3/2}\sqrt{\frac{cx^2}{a}+1}} + 1680dg^2\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{c}\sqrt{\frac{cx^2}{a}+1}} + 5ax + 2cx^3\right) + \frac{105\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*((2688*g*(e*g + 2*d*h)*(a + c*x^2)^2)/c + (2240*(f*g^2 + h
*(2*e*g + d*h))*x*(a + c*x^2)^2)/c + (1680*f*h^2*x^3*(a + c*x^2)^2)/c + (38
4*h*(2*f*g + e*h)*(a + c*x^2)^2*(-2*a + 5*c*x^2))/c^2 - (280*a*(f*g^2 + h*(
2*e*g + d*h))*(Sqrt[c]*x*(5*a + 2*c*x^2)*Sqrt[1 + (c*x^2)/a] + 3*a^(3/2)*Ar
cSinh[(Sqrt[c]*x)/Sqrt[a]]))/(c^(3/2)*Sqrt[1 + (c*x^2)/a]) + (105*a*f*h^2*(
-(Sqrt[c]*x*(3*a^2 + 14*a*c*x^2 + 8*c^2*x^4)) + (3*a^(5/2)*ArcSinh[(Sqrt[c]
*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a]))/c^(5/2) + 1680*d*g^2*(5*a*x + 2*c*x^3 +
 (3*a^(3/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[c]*Sqrt[1 + (c*x^2)/a]))))/
13440
```

fricas [A] time = 1.34, size = 831, normalized size = 2.40

$$\left[-\frac{105\left(16\,a^{3}cegh-8\left(6\,a^{2}c^{2}d-a^{3}cf\right)g^{2}+\left(8\,a^{3}cd-3\,a^{4}f\right)h^{2}\right)\sqrt{c}\,\log\left(-2\,cx^{2}-2\,\sqrt{cx^{2}+a}\,\sqrt{c}\,x-a\right)-2\,(168\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/26880*(105*(16*a^3*c*e*g*h - 8*(6*a^2*c^2*d - a^3*c*f)*g^2 + (8*a^3*c*d
 - 3*a^4*f)*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) -
2*(1680*c^4*f*h^2*x^7 + 2688*a^2*c^2*e*g^2 - 768*a^3*c*e*h^2 + 1920*(2*c^4*
f*g*h + c^4*e*h^2)*x^6 + 280*(8*c^4*f*g^2 + 16*c^4*e*g*h + (8*c^4*d + 9*a*c
^3*f)*h^2)*x^5 + 384*(7*c^4*e*g^2 + 8*a*c^3*e*h^2 + 2*(7*c^4*d + 8*a*c^3*f)
*g*h)*x^4 + 70*(112*a*c^3*e*g*h + 8*(6*c^4*d + 7*a*c^3*f)*g^2 + (56*a*c^3*d
 + 3*a^2*c^2*f)*h^2)*x^3 + 768*(7*a^2*c^2*d - 2*a^3*c*f)*g*h + 384*(14*a*c^
3*e*g^2 + a^2*c^2*e*h^2 + 2*(14*a*c^3*d + a^2*c^2*f)*g*h)*x^2 + 105*(16*a^2
*c^2*e*g*h + 8*(10*a*c^3*d + a^2*c^2*f)*g^2 + (8*a^2*c^2*d - 3*a^3*c*f)*h^2
)*x)*sqrt(c*x^2 + a))/c^3, 1/13440*(105*(16*a^3*c*e*g*h - 8*(6*a^2*c^2*d -
a^3*c*f)*g^2 + (8*a^3*c*d - 3*a^4*f)*h^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) + (1680*c^4*f*h^2*x^7 + 2688*a^2*c^2*e*g^2 - 768*a^3*c*e*h^2 + 1
920*(2*c^4*f*g*h + c^4*e*h^2)*x^6 + 280*(8*c^4*f*g^2 + 16*c^4*e*g*h + (8*c^
4*d + 9*a*c^3*f)*h^2)*x^5 + 384*(7*c^4*e*g^2 + 8*a*c^3*e*h^2 + 2*(7*c^4*d +
 8*a*c^3*f)*g*h)*x^4 + 70*(112*a*c^3*e*g*h + 8*(6*c^4*d + 7*a*c^3*f)*g^2 +
(56*a*c^3*d + 3*a^2*c^2*f)*h^2)*x^3 + 768*(7*a^2*c^2*d - 2*a^3*c*f)*g*h + 3
84*(14*a*c^3*e*g^2 + a^2*c^2*e*h^2 + 2*(14*a*c^3*d + a^2*c^2*f)*g*h)*x^2 +
105*(16*a^2*c^2*e*g*h + 8*(10*a*c^3*d + a^2*c^2*f)*g^2 + (8*a^2*c^2*d - 3*a
^3*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

giac [A] time = 0.26, size = 452, normalized size = 1.31

$$\frac{1}{13440}\sqrt{cx^{2}+a}\left(\left(2\left(\left(4\left(5\left(6\left(7\,cfh^{2}x+\frac{8\left(2\,c^{7}fgh+c^{7}h^{2}e\right)}{c^{6}}\right)x+\frac{7\left(8\,c^{7}fg^{2}+8\,c^{7}dh^{2}+9\,ac^{6}fh^{2}+16\,c^{7}ghe\right)}{c^{6}}\ldots\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/13440*sqrt(c*x^2 + a)*((2*((4*(5*(6*(7*c*f*h^2*x + 8*(2*c^7*f*g*h + c^7*h
^2*e)/c^6)*x + 7*(8*c^7*f*g^2 + 8*c^7*d*h^2 + 9*a*c^6*f*h^2 + 16*c^7*g*h*e)
/c^6)*x + 48*(14*c^7*d*g*h + 16*a*c^6*f*g*h + 7*c^7*g^2*e + 8*a*c^6*h^2*e)/
c^6)*x + 35*(48*c^7*d*g^2 + 56*a*c^6*f*g^2 + 56*a*c^6*d*h^2 + 3*a^2*c^5*f*h
^2 + 112*a*c^6*g*h*e)/c^6)*x + 192*(28*a*c^6*d*g*h + 2*a^2*c^5*f*g*h + 14*a
*c^6*g^2*e + a^2*c^5*h^2*e)/c^6)*x + 105*(80*a*c^6*d*g^2 + 8*a^2*c^5*f*g^2
+ 8*a^2*c^5*d*h^2 - 3*a^3*c^4*f*h^2 + 16*a^2*c^5*g*h*e)/c^6)*x + 384*(14*a^
```

```
2*c^5*d*g*h - 4*a^3*c^4*f*g*h + 7*a^2*c^5*g^2*e - 2*a^3*c^4*h^2*e)/c^6) - 1
/128*(48*a^2*c^2*d*g^2 - 8*a^3*c*f*g^2 - 8*a^3*c*d*h^2 + 3*a^4*f*h^2 - 16*a
^3*c*g*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

maple [A] time = 0.01, size = 552, normalized size = 1.60

$$\frac{3a^4 f h^2 \ln\left(\sqrt{c}\, x+\sqrt{c\,x^2+a}\right)}{128c^{\frac{5}{2}}}-\frac{a^3 d h^2 \ln\left(\sqrt{c}\, x+\sqrt{c\,x^2+a}\right)}{16c^{\frac{3}{2}}}-\frac{a^3 egh \ln\left(\sqrt{c}\, x+\sqrt{c\,x^2+a}\right)}{8c^{\frac{3}{2}}}-\frac{a^3 f g^2 \ln\left(\sqrt{c}\, x+\right.}{16c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/3*x*(c*x^2+a)^(5/2)/c*e*g*h-1/24*a/c*x*(c*x^2+a)^(3/2)*d*h^2-1/24*a/c*x*(
c*x^2+a)^(3/2)*f*g^2-1/16*a^2/c*x*(c*x^2+a)^(1/2)*d*h^2-1/16*f*h^2*a/c^2*x*
(c*x^2+a)^(5/2)+1/64*f*h^2*a^2/c^2*x*(c*x^2+a)^(3/2)+3/128*f*h^2*a^3/c^2*x*
(c*x^2+a)^(1/2)+2/7*x^2*(c*x^2+a)^(5/2)/c*f*g*h-4/35*a/c^2*(c*x^2+a)^(5/2)*
f*g*h-1/8*a^3/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*g*h-1/16*a^2/c*x*(c*x
^2+a)^(1/2)*f*g^2+1/4*d*g^2*x*(c*x^2+a)^(3/2)+1/5*(c*x^2+a)^(5/2)/c*e*g^2+1
/8*f*h^2*x^3*(c*x^2+a)^(5/2)/c+3/128*f*h^2*a^4/c^(5/2)*ln(c^(1/2)*x+(c*x^2+
a)^(1/2))-1/12*a/c*x*(c*x^2+a)^(3/2)*e*g*h-1/8*a^2/c*x*(c*x^2+a)^(1/2)*e*g*
h+1/6*x*(c*x^2+a)^(5/2)/c*d*h^2+1/6*x*(c*x^2+a)^(5/2)/c*f*g^2-1/16*a^3/c^(3
/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*d*h^2-1/16*a^3/c^(3/2)*ln(c^(1/2)*x+(c*x^
2+a)^(1/2))*f*g^2+2/5*(c*x^2+a)^(5/2)/c*d*g*h+3/8*d*g^2*a*x*(c*x^2+a)^(1/2)
+3/8*d*g^2*a^2/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+1/7*x^2*(c*x^2+a)^(5/2
)/c*e*h^2-2/35*a/c^2*(c*x^2+a)^(5/2)*e*h^2
```

maxima [A] time = 0.45, size = 380, normalized size = 1.10

$$\frac{\left(cx^2+a\right)^{\frac{5}{2}} fh^2x^3}{8\,c}+\frac{1}{4}\left(cx^2+a\right)^{\frac{3}{2}} dg^2x+\frac{3}{8}\sqrt{cx^2+a}\,adg^2x-\frac{\left(cx^2+a\right)^{\frac{5}{2}} afh^2x}{16\,c^2}+\frac{\left(cx^2+a\right)^{\frac{3}{2}} a^2fh^2x}{64\,c^2}+\frac{3\sqrt{cx^2+a}\,a^3fh^2}{128\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^2 + a)^(5/2)*f*h^2*x^3/c + 1/4*(c*x^2 + a)^(3/2)*d*g^2*x + 3/8*sqr
t(c*x^2 + a)*a*d*g^2*x - 1/16*(c*x^2 + a)^(5/2)*a*f*h^2*x/c^2 + 1/64*(c*x^2
 + a)^(3/2)*a^2*f*h^2*x/c^2 + 3/128*sqrt(c*x^2 + a)*a^3*f*h^2*x/c^2 + 3/8*a
^2*d*g^2*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 3/128*a^4*f*h^2*arcsinh(c*x/sqrt(
a*c))/c^(5/2) + 1/5*(c*x^2 + a)^(5/2)*e*g^2/c + 2/5*(c*x^2 + a)^(5/2)*d*g*h
/c + 1/7*(2*f*g*h + e*h^2)*(c*x^2 + a)^(5/2)*x^2/c + 1/6*(f*g^2 + 2*e*g*h +
 d*h^2)*(c*x^2 + a)^(5/2)*x/c - 1/24*(f*g^2 + 2*e*g*h + d*h^2)*(c*x^2 + a)^
(3/2)*a*x/c - 1/16*(f*g^2 + 2*e*g*h + d*h^2)*sqrt(c*x^2 + a)*a^2*x/c - 1/16
*(f*g^2 + 2*e*g*h + d*h^2)*a^3*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/35*(2*f*g
*h + e*h^2)*(c*x^2 + a)^(5/2)*a/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g+h\,x\right)}^2\,{\left(c\,x^2+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

sympy [A] time = 54.50, size = 1304, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] -3*a**(7/2)*f*h**2*x/(128*c**2*sqrt(1 + c*x**2/a)) + a**(5/2)*d*h**2*x/(16*
c*sqrt(1 + c*x**2/a)) + a**(5/2)*e*g*h*x/(8*c*sqrt(1 + c*x**2/a)) + a**(5/2
)*f*g**2*x/(16*c*sqrt(1 + c*x**2/a)) - a**(5/2)*f*h**2*x**3/(128*c*sqrt(1 +
 c*x**2/a)) + a**(3/2)*d*g**2*x*sqrt(1 + c*x**2/a)/2 + a**(3/2)*d*g**2*x/(8
*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*d*h**2*x**3/(48*sqrt(1 + c*x**2/a)) + 17
*a**(3/2)*e*g*h*x**3/(24*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*f*g**2*x**3/(48*
sqrt(1 + c*x**2/a)) + 13*a**(3/2)*f*h**2*x**5/(64*sqrt(1 + c*x**2/a)) + 3*s
qrt(a)*c*d*g**2*x**3/(8*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*d*h**2*x**5/(24*
sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*e*g*h*x**5/(12*sqrt(1 + c*x**2/a)) + 11*
sqrt(a)*c*f*g**2*x**5/(24*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*c*f*h**2*x**7/(16
*sqrt(1 + c*x**2/a)) + 3*a**4*f*h**2*asinh(sqrt(c)*x/sqrt(a))/(128*c**(5/2)
) - a**3*d*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) - a**3*e*g*h*asinh(s
qrt(c)*x/sqrt(a))/(8*c**(3/2)) - a**3*f*g**2*asinh(sqrt(c)*x/sqrt(a))/(16*c
**(3/2)) + 3*a**2*d*g**2*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + 2*a*d*g*h*P
iecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + a
*e*g**2*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), T
rue)) + a*e*h**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqr
t(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4,
 True)) + 2*a*f*g*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*
sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4
/4, True)) + 2*c*d*g*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x*
*2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x
**4/4, True)) + c*e*g**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*
x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)
*x**4/4, True)) + c*e*h**2*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) -
4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) +
x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 2*c*f*g*h*Pie
cewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(
105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c,
 0)), (sqrt(a)*x**6/6, True)) + c**2*d*g**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**2
/a)) + c**2*d*h**2*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*e*g*h*x**7/(3
*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*f*g**2*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/
a)) + c**2*f*h**2*x**9/(8*sqrt(a)*sqrt(1 + c*x**2/a))
```

3.90 $\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=213

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(-aeh-afg+6cdg)}{16c^{3/2}}-\frac{\left(a+cx^2\right)^{5/2}\left(6\left(2afh^2+c\left(5fg^2-7h(dh+eg)\right)\right)+5chx(5fg-7eh)\right)}{210c^2h}+$$

```
[Out] 1/24*(6*c*d*g-a*(e*h+f*g))*x*(c*x^2+a)^(3/2)/c+1/7*f*(h*x+g)^2*(c*x^2+a)^(5
/2)/c/h-1/210*(12*a*f*h^2+6*c*(5*f*g^2-7*h*(d*h+e*g))+5*c*h*(-7*e*h+5*f*g)*
x)*(c*x^2+a)^(5/2)/c^2/h+1/16*a^2*(-a*e*h-a*f*g+6*c*d*g)*arctanh(x*c^(1/2)/
(c*x^2+a)^(1/2))/c^(3/2)+1/16*a*(-a*e*h-a*f*g+6*c*d*g)*x*(c*x^2+a)^(1/2)/c
```

Rubi [A] time = 0.27, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1654, 780, 195, 217, 206}

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(-aeh-afg+6cdg)}{16c^{3/2}}-\frac{\left(a+cx^2\right)^{5/2}\left(6\left(2afh^2-7ch(dh+eg)+5cfg^2\right)+5chx(5fg-7eh)\right)}{210c^2h}+\frac{x}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(6*c*d*g - a*f*g - a*e*h)*x*Sqrt[a + c*x^2])/(16*c) + ((6*c*d*g - a*(f*g
 + e*h))*x*(a + c*x^2)^(3/2))/(24*c) + (f*(g + h*x)^2*(a + c*x^2)^(5/2))/(7
*c*h) - ((6*(5*c*f*g^2 + 2*a*f*h^2 - 7*c*h*(e*g + d*h)) + 5*c*h*(5*f*g - 7*
e*h)*x)*(a + c*x^2)^(5/2))/(210*c^2*h) + (a^2*(6*c*d*g - a*f*g - a*e*h)*Arc
Tanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} + \frac{\int (g+hx)\left((7cd-2af)h^2-ch(5fg-7e\right.}{7ch^2}$$

$$= \frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} - \frac{\left(6\left(5cfg^2+2afh^2-7ch(eg+dh)\right)+5c\right.}{210c^2h}$$

$$= \frac{(6cdg-a(fg+eh))x\left(a+cx^2\right)^{3/2}}{24c} + \frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} - \frac{(6\left(\right.}{}$$

$$= \frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c} + \frac{(6cdg-a(fg+eh))x\left(a+cx^2\right)^{3}}{24c}$$

$$= \frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c} + \frac{(6cdg-a(fg+eh))x\left(a+cx^2\right)^{3}}{24c}$$

$$= \frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c} + \frac{(6cdg-a(fg+eh))x\left(a+cx^2\right)^{3}}{24c}$$

Mathematica [A] time = 0.63, size = 209, normalized size = 0.98

$$\frac{\sqrt{a+cx^2}\left(-\frac{105a^{5/2}\sqrt{\frac{cx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aeh+afg-6cdg)}{c^{3/2}\left(a+cx^2\right)} - \frac{96a^3fh}{c^2} + \frac{3a^2(112dh+7e(16g+5hx)+fx(35g+16hx))}{c} + 2ax(21d(25g+\right.}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*((-96*a^3*f*h)/c^2 + (3*a^2*(112*d*h + 7*e*(16*g + 5*h*x)
+ f*x*(35*g + 16*h*x)))/c + 4*c*x^3*(21*d*(5*g + 4*h*x) + 2*x*(7*e*(6*g + 5
*h*x) + 5*f*x*(7*g + 6*h*x))) + 2*a*x*(21*d*(25*g + 16*h*x) + x*(7*e*(48*g
+ 35*h*x) + f*x*(245*g + 192*h*x))) - (105*a^(5/2)*(-6*c*d*g + a*f*g + a*e*
h)*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(c^(3/2)*(a + c*x^2)))
)/1680
```

fricas [A] time = 1.02, size = 477, normalized size = 2.24

$$\left[\frac{105\left(a^3eh-\left(6a^2cd-a^3f\right)g\right)\sqrt{c}\,\log\left(-2cx^2+2\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(240c^3fhx^6+280\left(c^3fg+c^3eh\right)x^5+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/3360*(105*(a^3*e*h - (6*a^2*c*d - a^3*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqr
t(c*x^2 + a)*sqrt(c)*x - a) + 2*(240*c^3*f*h*x^6 + 280*(c^3*f*g + c^3*e*h)*
x^5 + 336*a^2*c*e*g + 48*(7*c^3*e*g + (7*c^3*d + 8*a*c^2*f)*h)*x^4 + 70*(7*
a*c^2*e*h + (6*c^3*d + 7*a*c^2*f)*g)*x^3 + 48*(14*a*c^2*e*g + (14*a*c^2*d +
 a^2*c*f)*h)*x^2 + 48*(7*a^2*c*d - 2*a^3*f)*h + 105*(a^2*c*e*h + (10*a*c^2*
d + a^2*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2, 1/1680*(105*(a^3*e*h - (6*a^2*c*d
- a^3*f)*g)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (240*c^3*f*h*x^6
+ 280*(c^3*f*g + c^3*e*h)*x^5 + 336*a^2*c*e*g + 48*(7*c^3*e*g + (7*c^3*d +
8*a*c^2*f)*h)*x^4 + 70*(7*a*c^2*e*h + (6*c^3*d + 7*a*c^2*f)*g)*x^3 + 48*(14
*a*c^2*e*g + (14*a*c^2*d + a^2*c*f)*h)*x^2 + 48*(7*a^2*c*d - 2*a^3*f)*h + 1
05*(a^2*c*e*h + (10*a*c^2*d + a^2*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2]
```

giac [A] time = 0.25, size = 264, normalized size = 1.24

$$\frac{1}{1680}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\left(6\,cfhx+\frac{7\left(c^6fg+c^6he\right)}{c^5}\right)x+\frac{6\left(7\,c^6dh+8\,ac^5fh+7\,c^6ge\right)}{c^5}\right)x+\frac{35\left(6\,c^6dg+7\,ac^5fg\right.}{c^5}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/1680*sqrt(c*x^2 + a)*((2*((4*(5*(6*c*f*h*x + 7*(c^6*f*g + c^6*h*e)/c^5)*x
 + 6*(7*c^6*d*h + 8*a*c^5*f*h + 7*c^6*g*e)/c^5)*x + 35*(6*c^6*d*g + 7*a*c^5
*f*g + 7*a*c^5*h*e)/c^5)*x + 24*(14*a*c^5*d*h + a^2*c^4*f*h + 14*a*c^5*g*e)
/c^5)*x + 105*(10*a*c^5*d*g + a^2*c^4*f*g + a^2*c^4*h*e)/c^5)*x + 48*(7*a^2
*c^4*d*h - 2*a^3*c^3*f*h + 7*a^2*c^4*g*e)/c^5) - 1/16*(6*a^2*c*d*g - a^3*f*
g - a^3*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

maple [A] time = 0.00, size = 287, normalized size = 1.35

$$-\frac{a^3eh\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{16c^{\frac{3}{2}}}-\frac{a^3fg\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{16c^{\frac{3}{2}}}+\frac{3a^2dg\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{8\sqrt{c}}-\frac{\sqrt{c\,x^2+a}\,a^2ehx}{16c}-\frac{\sqrt{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/7*h*f*x^2*(c*x^2+a)^(5/2)/c-2/35*h*f*a/c^2*(c*x^2+a)^(5/2)+1/6*x*(c*x^2+a
)^(5/2)/c*e*h+1/6*x*(c*x^2+a)^(5/2)/c*f*g-1/24*a/c*x*(c*x^2+a)^(3/2)*e*h-1/
24*a/c*x*(c*x^2+a)^(3/2)*f*g-1/16*a^2/c*x*(c*x^2+a)^(1/2)*e*h-1/16*a^2/c*x*
(c*x^2+a)^(1/2)*f*g-1/16*a^3/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*h-1/16
*a^3/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*f*g+1/5*(c*x^2+a)^(5/2)/c*d*h+1/
5*(c*x^2+a)^(5/2)/c*e*g+1/4*d*g*x*(c*x^2+a)^(3/2)+3/8*d*g*a*x*(c*x^2+a)^(1/
2)+3/8*d*g*a^2/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
```

maxima [A] time = 0.45, size = 211, normalized size = 0.99

$$\frac{\left(cx^2+a\right)^{\frac{5}{2}}fhx^2}{7\,c}+\frac{1}{4}\left(cx^2+a\right)^{\frac{3}{2}}dgx+\frac{3}{8}\sqrt{cx^2+a}\,adgx+\frac{3\,a^2dg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}}+\frac{\left(cx^2+a\right)^{\frac{5}{2}}eg}{5\,c}+\frac{\left(cx^2+a\right)^{\frac{5}{2}}dh}{5\,c}-\frac{2\left(cx^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/7*(c*x^2 + a)^(5/2)*f*h*x^2/c + 1/4*(c*x^2 + a)^(3/2)*d*g*x + 3/8*sqrt(c*
x^2 + a)*a*d*g*x + 3/8*a^2*d*g*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 1/5*(c*x^2
+ a)^(5/2)*e*g/c + 1/5*(c*x^2 + a)^(5/2)*d*h/c - 2/35*(c*x^2 + a)^(5/2)*a*f
*h/c^2 + 1/6*(c*x^2 + a)^(5/2)*(f*g + e*h)*x/c - 1/24*(c*x^2 + a)^(3/2)*(f*
```

```
g + e*h)*a*x/c - 1/16*sqrt(c*x^2 + a)*(f*g + e*h)*a^2*x/c - 1/16*(f*g + e*h
)*a^3*arcsinh(c*x/sqrt(a*c))/c^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g+h\,x\right)\,{\left(c\,x^2+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

sympy [A] time = 27.89, size = 768, normalized size = 3.61

$$\frac{a^{\frac{5}{2}}ehx}{16c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{5}{2}}fgx}{16c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}dgx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{a^{\frac{3}{2}}dgx}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{17a^{\frac{3}{2}}ehx^3}{48\sqrt{1+\frac{cx^2}{a}}}+\frac{17a^{\frac{3}{2}}fgx^3}{48\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}\,cdgx^3}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{11\sqrt{a}\,ce}{24\sqrt{1+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] a**(5/2)*e*h*x/(16*c*sqrt(1 + c*x**2/a)) + a**(5/2)*f*g*x/(16*c*sqrt(1 + c*
x**2/a)) + a**(3/2)*d*g*x*sqrt(1 + c*x**2/a)/2 + a**(3/2)*d*g*x/(8*sqrt(1 +
 c*x**2/a)) + 17*a**(3/2)*e*h*x**3/(48*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*f*
g*x**3/(48*sqrt(1 + c*x**2/a)) + 3*sqrt(a)*c*d*g*x**3/(8*sqrt(1 + c*x**2/a)
) + 11*sqrt(a)*c*e*h*x**5/(24*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*f*g*x**5/(
24*sqrt(1 + c*x**2/a)) - a**3*e*h*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) -
a**3*f*g*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) + 3*a**2*d*g*asinh(sqrt(c)*
x/sqrt(a))/(8*sqrt(c)) + a*d*h*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a +
c*x**2)**(3/2)/(3*c), True)) + a*e*g*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)),
((a + c*x**2)**(3/2)/(3*c), True)) + a*f*h*Piecewise((-2*a**2*sqrt(a + c*x*
*2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, N
e(c, 0)), (sqrt(a)*x**4/4, True)) + c*d*h*Piecewise((-2*a**2*sqrt(a + c*x**
2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne
(c, 0)), (sqrt(a)*x**4/4, True)) + c*e*g*Piecewise((-2*a**2*sqrt(a + c*x**2
)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(
c, 0)), (sqrt(a)*x**4/4, True)) + c*f*h*Piecewise((8*a**3*sqrt(a + c*x**2)/
(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x*
*2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) +
c**2*d*g*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*e*h*x**7/(6*sqrt(a)*sqr
t(1 + c*x**2/a)) + c**2*f*g*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a))
```

3.91 $\int \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=137

$$\frac{a^2(6cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}+\frac{x\left(a+cx^2\right)^{3/2}(6cd-af)}{24c}+\frac{ax\sqrt{a+cx^2}\,(6cd-af)}{16c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}$$

```
[Out] 1/24*(-a*f+6*c*d)*x*(c*x^2+a)^(3/2)/c+1/5*e*(c*x^2+a)^(5/2)/c+1/6*f*x*(c*x^
2+a)^(5/2)/c+1/16*a^2*(-a*f+6*c*d)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/
2)+1/16*a*(-a*f+6*c*d)*x*(c*x^2+a)^(1/2)/c
```

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1815, 641, 195, 217, 206}

$$\frac{a^2(6cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}+\frac{x\left(a+cx^2\right)^{3/2}(6cd-af)}{24c}+\frac{ax\sqrt{a+cx^2}\,(6cd-af)}{16c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(6*c*d - a*f)*x*Sqrt[a + c*x^2])/(16*c) + ((6*c*d - a*f)*x*(a + c*x^2)^(
3/2))/(24*c) + (e*(a + c*x^2)^(5/2))/(5*c) + (f*x*(a + c*x^2)^(5/2))/(6*c)
+ (a^2*(6*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{fx\left(a+cx^2\right)^{5/2}}{6c}+\frac{\int(6cd-af+6cex)\left(a+cx^2\right)^{3/2}dx}{6c}$$
$$=\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}+\frac{(6cd-af)\int\left(a+cx^2\right)^{3/2}dx}{6c}$$
$$=\frac{(6cd-af)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}+\frac{(a(6cd-af)}{}$$
$$=\frac{a(6cd-af)x\sqrt{a+cx^2}}{16c}+\frac{(6cd-af)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx(}{}$$
$$=\frac{a(6cd-af)x\sqrt{a+cx^2}}{16c}+\frac{(6cd-af)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx(}{}$$
$$=\frac{a(6cd-af)x\sqrt{a+cx^2}}{16c}+\frac{(6cd-af)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx(}{}$$

Mathematica [A] time = 0.26, size = 125, normalized size = 0.91

$$\frac{\sqrt{a+cx^2}\left(\sqrt{c}\left(3a^2(16e+5fx)+2acx(75d+x(48e+35fx))+4c^2x^3(15d+2x(6e+5fx))\right)-\frac{15a^{3/2}(af-6cd)\sinh}{\sqrt{\frac{cx^2}{a}+1}}\right.}{240c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(Sqrt[c]*(3*a^2*(16*e + 5*f*x) + 4*c^2*x^3*(15*d + 2*x*(6*
e + 5*f*x)) + 2*a*c*x*(75*d + x*(48*e + 35*f*x))) - (15*a^(3/2)*(-6*c*d + a
*f)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a]))/(240*c^(3/2))
```

fricas [A] time = 1.19, size = 262, normalized size = 1.91

$$\left[-\frac{15\left(6a^2cd-a^3f\right)\sqrt{c}\log\left(-2cx^2+2\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(40c^3fx^5+48c^3ex^4+96ac^2ex^2+48a^2ce+10\right.}{480c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/480*(15*(6*a^2*c*d - a^3*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sq
rt(c)*x - a) - 2*(40*c^3*f*x^5 + 48*c^3*e*x^4 + 96*a*c^2*e*x^2 + 48*a^2*c*e
 + 10*(6*c^3*d + 7*a*c^2*f)*x^3 + 15*(10*a*c^2*d + a^2*c*f)*x)*sqrt(c*x^2 +
 a))/c^2, -1/240*(15*(6*a^2*c*d - a^3*f)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*
x^2 + a)) - (40*c^3*f*x^5 + 48*c^3*e*x^4 + 96*a*c^2*e*x^2 + 48*a^2*c*e + 10
*(6*c^3*d + 7*a*c^2*f)*x^3 + 15*(10*a*c^2*d + a^2*c*f)*x)*sqrt(c*x^2 + a))/
c^2]
```

giac [A] time = 0.23, size = 129, normalized size = 0.94

$$\frac{1}{240}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5cfx+6ce\right)x+\frac{5\left(6c^5d+7ac^4f\right)}{c^4}\right)x+48ae\right)x+\frac{15\left(10ac^4d+a^2c^3f\right)}{c^4}\right)x+\frac{48a^2e}{c}\right)-\frac{(6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/240*sqrt(c*x^2 + a)*((2*((4*(5*c*f*x + 6*c*e)*x + 5*(6*c^5*d + 7*a*c^4*f)
/c^4)*x + 48*a*e)*x + 15*(10*a*c^4*d + a^2*c^3*f)/c^4)*x + 48*a^2*e/c) - 1/
16*(6*a^2*c*d - a^3*f)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

maple [A] time = 0.01, size = 146, normalized size = 1.07

$$-\frac{a^3 f \ln\left(\sqrt{c}\, x+\sqrt{c\,x^2+a}\right)}{16 c^{\frac{3}{2}}}+\frac{3a^2 d \ln\left(\sqrt{c}\, x+\sqrt{c\,x^2+a}\right)}{8\sqrt{c}}-\frac{\sqrt{c\,x^2+a}\; a^2 f x}{16c}+\frac{3\sqrt{c\,x^2+a}\; a d x}{8}-\frac{\left(c\,x^2+a\right)^{\frac{3}{2}} a f x}{24c}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/6*f*x*(c*x^2+a)^(5/2)/c-1/24*f*a/c*x*(c*x^2+a)^(3/2)-1/16*f*a^2/c*x*(c*x^
2+a)^(1/2)-1/16*f*a^3/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+1/5*e*(c*x^2+a)
^(5/2)/c+1/4*d*x*(c*x^2+a)^(3/2)+3/8*d*a*x*(c*x^2+a)^(1/2)+3/8*d*a^2/c^(1/2
)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
```

maxima [A] time = 0.45, size = 131, normalized size = 0.96

$$\frac{1}{4}\left(cx^2+a\right)^{\frac{3}{2}} dx+\frac{3}{8}\sqrt{cx^2+a}\, adx+\frac{\left(cx^2+a\right)^{\frac{5}{2}} fx}{6c}-\frac{\left(cx^2+a\right)^{\frac{3}{2}} afx}{24c}-\frac{\sqrt{cx^2+a}\, a^2 fx}{16c}+\frac{3a^2 d \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{c}}-\frac{a^3 f \operatorname{arsin}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2 + a)^(3/2)*d*x + 3/8*sqrt(c*x^2 + a)*a*d*x + 1/6*(c*x^2 + a)^(5/
2)*f*x/c - 1/24*(c*x^2 + a)^(3/2)*a*f*x/c - 1/16*sqrt(c*x^2 + a)*a^2*f*x/c
+ 3/8*a^2*d*arcsinh(c*x/sqrt(a*c))/sqrt(c) - 1/16*a^3*f*arcsinh(c*x/sqrt(a*
c))/c^(3/2) + 1/5*(c*x^2 + a)^(5/2)*e/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

sympy [A] time = 17.01, size = 348, normalized size = 2.54

$$\frac{a^{\frac{5}{2}} fx}{16c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}} dx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{a^{\frac{3}{2}} dx}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{17a^{\frac{3}{2}} fx^3}{48\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}\, cdx^3}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{11\sqrt{a}\, cfx^5}{24\sqrt{1+\frac{cx^2}{a}}}-\frac{a^3 f \operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16c^{\frac{3}{2}}}+\frac{3a^2 d \operatorname{asi}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] a**(5/2)*f*x/(16*c*sqrt(1 + c*x**2/a)) + a**(3/2)*d*x*sqrt(1 + c*x**2/a)/2
+ a**(3/2)*d*x/(8*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*f*x**3/(48*sqrt(1 + c*x
```

```
**2/a)) + 3*sqrt(a)*c*d*x**3/(8*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*f*x**5/(
24*sqrt(1 + c*x**2/a)) - a**3*f*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) + 3*
a**2*d*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + a*e*Piecewise((sqrt(a)*x**2/2
, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + c*e*Piecewise((-2*a**2*sq
rt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c
*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c**2*d*x**5/(4*sqrt(a)*sqrt(
1 + c*x**2/a)) + c**2*f*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a))
```

3.92 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{g+hx}\,dx$

Optimal. Leaf size=326

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3a^2h^4(fg-eh)+12acgh^2\left(fg^2-h(eg-dh)\right)+8c^2g^3\left(fg^2-h(eg-dh)\right)\right)}{8\sqrt{c}\,h^6}-\frac{\left(ah^2+cg^2\right)^{3/2}\left(dh^2\right.}{}$$

```
[Out] 1/12*(4*d*h^2-4*e*g*h+4*f*g^2-3*h*(-e*h+f*g)*x)*(c*x^2+a)^(3/2)/h^3+1/5*f*(
c*x^2+a)^(5/2)/c/h-(a*h^2+c*g^2)^(3/2)*(d*h^2-e*g*h+f*g^2)*arctanh((-c*g*x+
a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^6-1/8*(3*a^2*h^4*(-e*h+f*g)+8*c
^2*g^3*(f*g^2-h*(-d*h+e*g))+12*a*c*g*h^2*(f*g^2-h*(-d*h+e*g)))*arctanh(x*c^
(1/2)/(c*x^2+a)^(1/2))/h^6/c^(1/2)+1/8*(8*(a*h^2+c*g^2)*(d*h^2-e*g*h+f*g^2)
-h*(4*c*d*g*h^2+(-e*h+f*g)*(3*a*h^2+4*c*g^2))*x)*(c*x^2+a)^(1/2)/h^5
```

Rubi [A] time = 0.77, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1654, 815, 844, 217, 206, 725}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3a^2h^4(fg-eh)+12acgh^2\left(fg^2-h(eg-dh)\right)+8c^2\left(fg^5-g^3h(eg-dh)\right)\right)}{8\sqrt{c}\,h^6}+\frac{\left(a+cx^2\right)^{3/2}\left(4\left(dh\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((8*(c*g^2 + a*h^2)*(f*g^2 - e*g*h + d*h^2) - h*(4*c*d*g*h^2 + (f*g - e*h)*
(4*c*g^2 + 3*a*h^2))*x)*Sqrt[a + c*x^2])/(8*h^5) + ((4*(f*g^2 - e*g*h + d*h
^2) - 3*h*(f*g - e*h)*x)*(a + c*x^2)^(3/2))/(12*h^3) + (f*(a + c*x^2)^(5/2)
)/(5*c*h) - ((3*a^2*h^4*(f*g - e*h) + 12*a*c*g*h^2*(f*g^2 - h*(e*g - d*h))
+ 8*c^2*(f*g^5 - g^3*h*(e*g - d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/
(8*Sqrt[c]*h^6) - ((c*g^2 + a*h^2)^(3/2)*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(a
*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/h^6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
```

```
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{g+hx}\,dx = \frac{f\left(a+cx^2\right)^{5/2}}{5ch} + \frac{\int \frac{\left(5cdh^2-5ch(fg-eh)x\right)\left(a+cx^2\right)^{3/2}}{g+hx}\,dx}{5ch^2}$$

$$= \frac{\left(4\left(fg^2-egh+dh^2\right)-3h(fg-eh)x\right)\left(a+cx^2\right)^{3/2}}{12h^3} + \frac{f\left(a+cx^2\right)^{5/2}}{5ch} + \frac{\int \frac{(5}{}}{}$$

$$= \frac{\left(8\left(cg^2+ah^2\right)\left(fg^2-egh+dh^2\right)-h\left(4cdgh^2+(fg-eh)\left(4cg^2+3ah^2\right)\right)x}{8h^5}$$

$$= \frac{\left(8\left(cg^2+ah^2\right)\left(fg^2-egh+dh^2\right)-h\left(4cdgh^2+(fg-eh)\left(4cg^2+3ah^2\right)\right)x}{8h^5}$$

$$= \frac{\left(8\left(cg^2+ah^2\right)\left(fg^2-egh+dh^2\right)-h\left(4cdgh^2+(fg-eh)\left(4cg^2+3ah^2\right)\right)x}{8h^5}$$

$$= \frac{\left(8\left(cg^2+ah^2\right)\left(fg^2-egh+dh^2\right)-h\left(4cdgh^2+(fg-eh)\left(4cg^2+3ah^2\right)\right)x}{8h^5}$$

Mathematica [A] time = 1.21, size = 348, normalized size = 1.07

$$\frac{\sqrt{a+cx^2}\left(3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)+\sqrt{c}\,x\left(5a+2cx^2\right)\sqrt{\frac{cx^2}{a}+1}\right)(eh-fg)}{8\sqrt{c}\,h^2\sqrt{\frac{cx^2}{a}+1}} - \frac{\left(h(dh-eg)+fg^2\right)\left(\sqrt{\frac{cx^2}{a}+1}\left(-h\sqrt{a}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] (f*(a + c*x^2)^(5/2))/(5*c*h) + ((-(f*g) + e*h)*Sqrt[a + c*x^2]*(Sqrt[c]*x*
(5*a + 2*c*x^2)*Sqrt[1 + (c*x^2)/a] + 3*a^(3/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]
]))/(8*Sqrt[c]*h^2*Sqrt[1 + (c*x^2)/a]) - ((f*g^2 + h*(-(e*g) + d*h))*(3*Sq
rt[a]*Sqrt[c]*g*h^2*Sqrt[a + c*x^2]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]] + Sqrt[1 +
 (c*x^2)/a]*(-(h*Sqrt[a + c*x^2]*(6*c*g^2 + 8*a*h^2 - 3*c*g*h*x + 2*c*h^2*x
^2)) + 6*Sqrt[c]*g*(c*g^2 + a*h^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + 6
*(c*g^2 + a*h^2)^(3/2)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a +
c*x^2])])))/(6*h^6*Sqrt[1 + (c*x^2)/a])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.29, size = 551, normalized size = 1.69

$$\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(\frac{4\,cfx}{h}-\frac{5\left(c^4fgh^{19}-c^4h^{20}e\right)}{c^3h^{21}}\right)x+\frac{4\left(5\,c^4fg^2h^{18}+5\,c^4dh^{20}+6\,ac^3fh^{20}-5\,c^4gh^{19}e\right)}{c^3h^{21}}\right)x-\frac{15}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(c*x^2 + a)*((2*(3*(4*c*f*x/h - 5*(c^4*f*g*h^19 - c^4*h^20*e)/(c^
3*h^21))*x + 4*(5*c^4*f*g^2*h^18 + 5*c^4*d*h^20 + 6*a*c^3*f*h^20 - 5*c^4*g*
h^19*e)/(c^3*h^21))*x - 15*(4*c^4*f*g^3*h^17 + 4*c^4*d*g*h^19 + 5*a*c^3*f*g
*h^19 - 4*c^4*g^2*h^18*e - 5*a*c^3*h^20*e)/(c^3*h^21))*x + 8*(15*c^4*f*g^4*
h^16 + 15*c^4*d*g^2*h^18 + 20*a*c^3*f*g^2*h^18 + 20*a*c^3*d*h^20 + 3*a^2*c^
2*f*h^20 - 15*c^4*g^3*h^17*e - 20*a*c^3*g*h^19*e)/(c^3*h^21)) + 2*(c^2*f*g^
6 + c^2*d*g^4*h^2 + 2*a*c*f*g^4*h^2 + 2*a*c*d*g^2*h^4 + a^2*f*g^2*h^4 + a^2
*d*h^6 - c^2*g^5*h*e - 2*a*c*g^3*h^3*e - a^2*g*h^5*e)*arctan(-((sqrt(c)*x -
 sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 - a*h^2
)*h^6) + 1/8*(8*c^(5/2)*f*g^5 + 8*c^(5/2)*d*g^3*h^2 + 12*a*c^(3/2)*f*g^3*h^
2 + 12*a*c^(3/2)*d*g*h^4 + 3*a^2*sqrt(c)*f*g*h^4 - 8*c^(5/2)*g^4*h*e - 12*a
*c^(3/2)*g^2*h^3*e - 3*a^2*sqrt(c)*h^5*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
 a)))/(c*h^6)
```

maple [B] time = 0.01, size = 2420, normalized size = 7.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x)
```

```
[Out] 1/5*f*(c*x^2+a)^(5/2)/c/h+1/2/h^3*c*g^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^
2+c*g^2)/h^2)^(1/2)*x*e-1/2/h^4*c*g^3*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+
c*g^2)/h^2)^(1/2)*x*f-3/2/h^2*c^(1/2)*g*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(
x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*a*d-3/8/h^2*f*g*a*x*(c*x
^2+a)^(1/2)-3/8/h^2*f*g*a^2/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+3/2/h^3*c
^(1/2)*g^2*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h
^2+c*g^2)/h^2)^(1/2))*a*e-3/2/h^4*c^(1/2)*g^3*ln((-c*g/h+(x+g/h)*c)/c^(1/2)
+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*a*f+1/h^2/((a*h^2+
c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)
/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h)
```

```
)*a^2*e*g-1/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g
^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*
g^2)/h^2)^(1/2))/(x+g/h))*a^2*f*g^2+1/h^6/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*
(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c
*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*c^2*g^5*e-1/h^7/((a*h^2
+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2
)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h
))*c^2*g^6*f-1/h^5/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+
c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2
+c*g^2)/h^2)^(1/2))/(x+g/h))*c^2*g^4*d-1/2/h^2*c*g*(-2*(x+g/h)*c*g/h+(x+g/h
)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*d-1/3/h^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a
*h^2+c*g^2)/h^2)^(3/2)*e*g+1/h*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/
h^2)^(1/2)*a*d+1/3/h^3*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(3/
2)*f*g^2+1/4/h*e*x*(c*x^2+a)^(3/2)-2/h^5/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(
x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*
g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*c*g^4*f-2/h^3/((a*h^2+
c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)
/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h)
)*a*c*g^2*d+2/h^4/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c
*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+
c*g^2)/h^2)^(1/2))/(x+g/h))*a*c*g^3*e+1/3/h*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(
a*h^2+c*g^2)/h^2)^(3/2)*d+3/8/h*e*a*x*(c*x^2+a)^(1/2)+3/8/h*e*a^2/c^(1/2)*l
n(c^(1/2)*x+(c*x^2+a)^(1/2))-1/4/h^2*f*g*x*(c*x^2+a)^(3/2)+1/h^3*(-2*(x+g/h
)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*a*f*g^2-1/h^2*(-2*(x+g/h)*c*g/
h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*a*e*g+1/h^3*(-2*(x+g/h)*c*g/h+(x+g/h
)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*c*g^2*d-1/h^4*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+
(a*h^2+c*g^2)/h^2)^(1/2)*c*g^3*e+1/h^5*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2
+c*g^2)/h^2)^(1/2)*c*g^4*f-1/h^4*c^(3/2)*g^3*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+
(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*d+1/h^5*c^(3/2)*g^4
*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/
h^2)^(1/2))*e-1/h^6*c^(3/2)*g^5*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c
*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*f-1/h/((a*h^2+c*g^2)/h^2)^(1/2)*
ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x
+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a^2*d
```

maxima [B] time = 0.79, size = 632, normalized size = 1.94

$$-\frac{\sqrt{cx^2+a}\,cfg^3x}{2\,h^4}+\frac{\sqrt{cx^2+a}\,ceg^2x}{2\,h^3}-\frac{\sqrt{cx^2+a}\,cdgx}{2\,h^2}-\frac{\left(cx^2+a\right)^{\frac{3}{2}}fgx}{4\,h^2}-\frac{3\,\sqrt{cx^2+a}\,afgx}{8\,h^2}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}ex}{4\,h}+\frac{3\,\sqrt{cx^2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^2 + a)*c*f*g^3*x/h^4 + 1/2*sqrt(c*x^2 + a)*c*e*g^2*x/h^3 - 1/
2*sqrt(c*x^2 + a)*c*d*g*x/h^2 - 1/4*(c*x^2 + a)^(3/2)*f*g*x/h^2 - 3/8*sqrt(
c*x^2 + a)*a*f*g*x/h^2 + 1/4*(c*x^2 + a)^(3/2)*e*x/h + 3/8*sqrt(c*x^2 + a)*
a*e*x/h - c^(3/2)*f*g^5*arcsinh(c*x/sqrt(a*c))/h^6 + c^(3/2)*e*g^4*arcsinh(
c*x/sqrt(a*c))/h^5 - c^(3/2)*d*g^3*arcsinh(c*x/sqrt(a*c))/h^4 - 3/2*a*sqrt(
c)*f*g^3*arcsinh(c*x/sqrt(a*c))/h^4 + 3/2*a*sqrt(c)*e*g^2*arcsinh(c*x/sqrt(
a*c))/h^3 - 3/2*a*sqrt(c)*d*g*arcsinh(c*x/sqrt(a*c))/h^2 - 3/8*a^2*f*g*arcs
inh(c*x/sqrt(a*c))/(sqrt(c)*h^2) + 3/8*a^2*e*arcsinh(c*x/sqrt(a*c))/(sqrt(c
)*h) + (a + c*g^2/h^2)^(3/2)*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
 a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 - (a + c*g^2/h^2)^(3/2)*e*g*arcsinh(c*g*
x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^2 + (a + c*g^2
/h^2)^(3/2)*d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h
*x + g)))/h + sqrt(c*x^2 + a)*c*f*g^4/h^5 - sqrt(c*x^2 + a)*c*e*g^3/h^4 + s
qrt(c*x^2 + a)*c*d*g^2/h^3 + 1/3*(c*x^2 + a)^(3/2)*f*g^2/h^3 + sqrt(c*x^2 +
 a)*a*f*g^2/h^3 - 1/3*(c*x^2 + a)^(3/2)*e*g/h^2 - sqrt(c*x^2 + a)*a*e*g/h^2
```

```
 + 1/3*(c*x^2 + a)^(3/2)*d/h + sqrt(c*x^2 + a)*a*d/h + 1/5*(c*x^2 + a)^(5/2
)*f/(c*h)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

```
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{g+hx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x), x)
```

3.93 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

Optimal. Leaf size=432

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^4+12ach^2\left(3fg^2-h(2eg-dh)\right)+8c^2g^2\left(5fg^2-h(4eg-3dh)\right)\right)}{8\sqrt{c}\,h^6}-\frac{(a+cx^2)^{5/2}\left(dh^2-\right.}{h(g+hx)\left(ah^2\right.}$$

```
[Out] -1/12*(4*a*h^2*(-e*h+2*f*g)+4*c*g*(5*f*g^2-h*(-3*d*h+4*e*g))-3*h*(a*f*h^2+c
*(5*f*g^2-4*h*(-d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^3/(a*h^2+c*g^2)-(d*h^2-e*g*
h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)+1/8*(3*a^2*f*h^4+8*c^2*g^2
*(5*f*g^2-h*(-3*d*h+4*e*g))+12*a*c*h^2*(3*f*g^2-h*(-d*h+2*e*g)))*arctanh(x*
c^(1/2)/(c*x^2+a)^(1/2))/h^6/c^(1/2)+(a*h^2*(-e*h+2*f*g)+c*g*(5*f*g^2-h*(-3
*d*h+4*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))*(a*
h^2+c*g^2)^(1/2)/h^6-1/8*(8*a*h^2*(-e*h+2*f*g)+8*c*g*(5*f*g^2-h*(-3*d*h+4*e
*g))-h*(3*a*f*h^2+12*c*d*h^2-16*c*e*g*h+20*c*f*g^2)*x)*(c*x^2+a)^(1/2)/h^5
```

Rubi [A] time = 0.90, antiderivative size = 428, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 815, 844, 217, 206, 725}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^4+12ach^2\left(3fg^2-h(2eg-dh)\right)+8c^2\left(5fg^4-g^2h(4eg-3dh)\right)\right)}{8\sqrt{c}\,h^6}-\frac{(a+cx^2)^{5/2}\left(dh^2-\right.}{h(g+hx)\left(ah^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -((8*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2*(2*f*g - e*h)) - h*(20*c*f*
g^2 - 16*c*e*g*h + 12*c*d*h^2 + 3*a*f*h^2)*x)*Sqrt[a + c*x^2])/(8*h^5) - ((
4*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2*(2*f*g - e*h)) - 3*h*(5*c*f*g^
2 + a*f*h^2 - 4*c*h*(e*g - d*h))*x)*(a + c*x^2)^(3/2))/(12*h^3*(c*g^2 + a*h
^2)) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(h*(c*g^2 + a*h^2)*(g +
h*x)) + ((3*a^2*f*h^4 + 8*c^2*(5*f*g^4 - g^2*h*(4*e*g - 3*d*h)) + 12*a*c*h^
2*(3*f*g^2 - h*(2*e*g - d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqr
t[c]*h^6) + (Sqrt[c*g^2 + a*h^2]*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2
*(2*f*g - e*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])
])/h^6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{h\left(cg^2+ah^2\right)(g+hx)} - \frac{\int \frac{\left(-cdg+afg-aeh-\left(afh-c\left(4eg-\frac{5fg^2}{h}-4dh\right)\right)x\right)\left(a+cx^2\right.}{g+hx}}{cg^2+ah^2}$$

$$= -\frac{\left(4\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-3h\left(5cfg^2+afh^2-4ch(eg\right.\right.}{12h^3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(8\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-h\left(20cfg^2-16cegh+12cdh\right.\right.}{8h^5}$$

$$= -\frac{\left(8\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-h\left(20cfg^2-16cegh+12cdh\right.\right.}{8h^5}$$

$$= -\frac{\left(8\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-h\left(20cfg^2-16cegh+12cdh\right.\right.}{8h^5}$$

$$= -\frac{\left(8\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-h\left(20cfg^2-16cegh+12cdh\right.\right.}{8h^5}$$

Mathematica [A] time = 0.53, size = 392, normalized size = 0.91

$$\frac{\frac{3\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)\left(3a^2fh^4+12ach^2\left(h(dh-2eg)+3fg^2\right)+8c^2\left(g^2h(3dh-4eg)+5fg^4\right)\right)}{\sqrt{c}}+24\sqrt{ah^2+cg^2}\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] (h*Sqrt[a + c*x^2]*(8*(4*a*h^2*(-2*f*g + e*h) - 3*c*(4*f*g^3 + g*h*(-3*e*g
+ 2*d*h))) + 3*h*(5*a*f*h^2 + 4*c*(3*f*g^2 + h*(-2*e*g + d*h)))*x + 8*c*h^2
*(-2*f*g + e*h)*x^2 + 6*c*f*h^3*x^3 - (24*(c*g^2 + a*h^2)*(f*g^2 + h*(-(e*g
) + d*h)))/(g + h*x)) - 24*Sqrt[c*g^2 + a*h^2]*(5*c*f*g^3 + c*g*h*(-4*e*g +
 3*d*h) + a*h^2*(2*f*g - e*h))*Log[g + h*x] + (3*(3*a^2*f*h^4 + 12*a*c*h^2*
(3*f*g^2 + h*(-2*e*g + d*h)) + 8*c^2*(5*f*g^4 + g^2*h*(-4*e*g + 3*d*h)))*Lo
g[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/Sqrt[c] + 24*Sqrt[c*g^2 + a*h^2]*(5*c*f*g
^3 + c*g*h*(-4*e*g + 3*d*h) + a*h^2*(2*f*g - e*h))*Log[a*h - c*g*x + Sqrt[c
*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(24*h^6)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 5121, normalized size = 11.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.78, size = 708, normalized size = 1.64

$$-\frac{\left(cx^2+a\right)^{\frac{3}{2}}fg^2}{h^4x+gh^3}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}eg}{h^3x+gh^2}-\frac{\left(cx^2+a\right)^{\frac{3}{2}}d}{h^2x+gh}+\frac{5\sqrt{cx^2+a}\,cfg^2x}{2\,h^4}-\frac{2\sqrt{cx^2+a}\,cegx}{h^3}+\frac{3\sqrt{cx^2+a}\,cdx}{2\,h^2}+\frac{\left(cx^2+a\right)}{4\,h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="maxima")
```

```
[Out] -(c*x^2 + a)^(3/2)*f*g^2/(h^4*x + g*h^3) + (c*x^2 + a)^(3/2)*e*g/(h^3*x + g
*h^2) - (c*x^2 + a)^(3/2)*d/(h^2*x + g*h) + 5/2*sqrt(c*x^2 + a)*c*f*g^2*x/h
^4 - 2*sqrt(c*x^2 + a)*c*e*g*x/h^3 + 3/2*sqrt(c*x^2 + a)*c*d*x/h^2 + 1/4*(c
*x^2 + a)^(3/2)*f*x/h^2 + 3/8*sqrt(c*x^2 + a)*a*f*x/h^2 + 5*c^(3/2)*f*g^4*a
rcsinh(c*x/sqrt(a*c))/h^6 - 4*c^(3/2)*e*g^3*arcsinh(c*x/sqrt(a*c))/h^5 + 3*
c^(3/2)*d*g^2*arcsinh(c*x/sqrt(a*c))/h^4 + 9/2*a*sqrt(c)*f*g^2*arcsinh(c*x/
sqrt(a*c))/h^4 - 3*a*sqrt(c)*e*g*arcsinh(c*x/sqrt(a*c))/h^3 + 3/2*a*sqrt(c)
*d*arcsinh(c*x/sqrt(a*c))/h^2 + 3/8*a^2*f*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h
^2) - 3*sqrt(a + c*g^2/h^2)*c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g))
```

```
- a*h/(sqrt(a*c)*abs(h*x + g)))/h^5 + 3*sqrt(a + c*g^2/h^2)*c*e*g^2*arcsinh
(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^4 - 3*sqr
t(a + c*g^2/h^2)*c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a
*c)*abs(h*x + g)))/h^3 - 2*(a + c*g^2/h^2)^(3/2)*f*g*arcsinh(c*g*x/(sqrt(a*
c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 + (a + c*g^2/h^2)^(3/2
)*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/
h^2 - 5*sqrt(c*x^2 + a)*c*f*g^3/h^5 + 4*sqrt(c*x^2 + a)*c*e*g^2/h^4 - 3*sqr
t(c*x^2 + a)*c*d*g/h^3 - 2/3*(c*x^2 + a)^(3/2)*f*g/h^3 - 2*sqrt(c*x^2 + a)*
a*f*g/h^3 + 1/3*(c*x^2 + a)^(3/2)*e/h^2 + sqrt(c*x^2 + a)*a*e/h^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**2,x)
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

3.94 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

Optimal. Leaf size=488

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4+ach^2\left(19fg^2-3h(3eg-dh)\right)+2c^2g^2\left(10fg^2-3h(2eg-dh)\right)\right)}{2h^6\sqrt{ah^2+cg^2}}+\sqrt{a+cx^2}$$

```
[Out] -1/6*(2*c*g*(6*e*g-10*f*g^2/h-3*d*h)-2*a*h*(-3*e*h+7*f*g)-(2*a*f*h^2+c*(5*f
*g^2-3*h*(-d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^2/(a*h^2+c*g^2)/(h*x+g)-1/2*(d*h
^2-e*g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^2-1/2*(3*a*h^2*(-e*
h+3*f*g)+2*c*g*(10*f*g^2-3*h*(-d*h+2*e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/
2))*c^(1/2)/h^6-1/2*(2*a^2*f*h^4+2*c^2*g^2*(10*f*g^2-3*h*(-d*h+2*e*g))+a*c*
h^2*(19*f*g^2-3*h*(-d*h+3*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(
c*x^2+a)^(1/2))/h^6/(a*h^2+c*g^2)^(1/2)+1/2*(2*a^2*f*h^4+2*c^2*g^2*(10*f*g^
2-3*h*(-d*h+2*e*g))+a*c*h^2*(19*f*g^2-3*h*(-d*h+3*e*g))-c*h*(a*h^2*(-3*e*h+
7*f*g)+c*g*(10*f*g^2-3*h*(-d*h+2*e*g)))*x)*(c*x^2+a)^(1/2)/h^5/(a*h^2+c*g^2
)
```

Rubi [A] time = 0.92, antiderivative size = 480, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1651, 813, 815, 844, 217, 206, 725}

$$\frac{\sqrt{a+cx^2}\left(2a^2fh^3-cx\left(ah^2(7fg-3eh)-3cgh(2eg-dh)+10cfg^3\right)+ach\left(19fg^2-3h(3eg-dh)\right)-2c^2g^2\left(-\right.\right.}{2h^4\left(ah^2+cg^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((2*a^2*f*h^3 - 2*c^2*g^2*(6*e*g - (10*f*g^2)/h - 3*d*h) + a*c*h*(19*f*g^2
- 3*h*(3*e*g - d*h)) - c*(10*c*f*g^3 - 3*c*g*h*(2*e*g - d*h) + a*h^2*(7*f*g
 - 3*e*h))*x)*Sqrt[a + c*x^2])/(2*h^4*(c*g^2 + a*h^2)) - ((2*(c*g*(6*e*g -
(10*f*g^2)/h - 3*d*h) - a*h*(7*f*g - 3*e*h)) - (5*c*f*g^2 + 2*a*f*h^2 - 3*c
*h*(e*g - d*h))*x)*(a + c*x^2)^(3/2))/(6*h^2*(c*g^2 + a*h^2)*(g + h*x)) - (
(f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(2*h*(c*g^2 + a*h^2)*(g + h*x)^2
) - (Sqrt[c]*(20*c*f*g^3 - 6*c*g*h*(2*e*g - d*h) + 3*a*h^2*(3*f*g - e*h))*A
rcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*h^6) - ((2*a^2*f*h^4 + 2*c^2*(10*f*
g^4 - 3*g^2*h*(2*e*g - d*h)) + a*c*h^2*(19*f*g^2 - 3*h*(3*e*g - d*h)))*ArcT
anh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^6*Sqrt[c*g^2
 + a*h^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{2h\left(cg^2+ah^2\right)(g+hx)^2} - \frac{\int \frac{\left(-2(cdg-afg+aeh)-\left(2afh-c\left(3eg-\frac{5fg^2}{h}-3dh\right)\right)x}{(g+hx)^2}}{2\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(2\left(cg\left(6eg-\frac{10fg^2}{h}-3dh\right)-ah(7fg-3eh)\right)-\left(5cfg^2+2afh^2-3ch(eg\right.\right.}{6h^2\left(cg^2+ah^2\right)(g+hx)}$$

$$= \frac{\left(2a^2fh^3-2c^2g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)+ach\left(19fg^2-3h(3eg-dh)\right)-c\left(10\right.\right.}{2h^4\left(cg^2+ah^2\right)}$$

$$= \frac{\left(2a^2fh^3-2c^2g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)+ach\left(19fg^2-3h(3eg-dh)\right)-c\left(10\right.\right.}{2h^4\left(cg^2+ah^2\right)}$$

$$= \frac{\left(2a^2fh^3-2c^2g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)+ach\left(19fg^2-3h(3eg-dh)\right)-c\left(10\right.\right.}{2h^4\left(cg^2+ah^2\right)}$$

$$= \frac{\left(2a^2fh^3-2c^2g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)+ach\left(19fg^2-3h(3eg-dh)\right)-c\left(10\right.\right.}{2h^4\left(cg^2+ah^2\right)}$$

Mathematica [A] time = 0.65, size = 435, normalized size = 0.89

$$-\frac{\frac{3\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(2a^2fh^4+ach^2\left(3h(dh-3eg)+19fg^2\right)+2c^2\left(3g^2h(dh-2eg)+10fg^4\right)\right)}{\sqrt{ah^2+cg^2}}+\frac{3\log(g+hx)\left(2a^2fh^4+ach^2\left(3h(dh-3eg)+\right.\right.}{\sqrt{ah^2+}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(a*h^2*(-3*h*(e*g + d*h + 2*e*h*x) + f*(17*g^2 + 28*g*h
*x + 8*h^2*x^2)) + c*(f*(60*g^4 + 90*g^3*h*x + 20*g^2*h^2*x^2 - 5*g*h^3*x^3
 + 2*h^4*x^4) + 3*h*(d*h*(6*g^2 + 9*g*h*x + 2*h^2*x^2) + e*(-12*g^3 - 18*g^
2*h*x - 4*g*h^2*x^2 + h^3*x^3)))))/(g + h*x)^2 + (3*(2*a^2*f*h^4 + a*c*h^2*
(19*f*g^2 + 3*h*(-3*e*g + d*h)) + 2*c^2*(10*f*g^4 + 3*g^2*h*(-2*e*g + d*h))
)*Log[g + h*x])/Sqrt[c*g^2 + a*h^2] - 3*Sqrt[c]*(20*c*f*g^3 + 6*c*g*h*(-2*e
*g + d*h) - 3*a*h^2*(-3*f*g + e*h))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] - (3
*(2*a^2*f*h^4 + a*c*h^2*(19*f*g^2 + 3*h*(-3*e*g + d*h)) + 2*c^2*(10*f*g^4 +
 3*g^2*h*(-2*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*
x^2]])/Sqrt[c*g^2 + a*h^2])/(6*h^6)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.43, size = 1036, normalized size = 2.12

$$\frac{1}{6}\sqrt{cx^2+a}\left(x\left(\frac{2\,cfx}{h^3}-\frac{3\left(3\,c^2fgh^{14}-c^2h^{15}e\right)}{ch^{18}}\right)+\frac{2\left(18\,c^2fg^2h^{13}+3\,c^2dh^{15}+4\,acfh^{15}-9\,c^2gh^{14}e\right)}{ch^{18}}\right)+\frac{\left(20\,c^{\frac{3}{2}}fg^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2 + a)*(x*(2*c*f*x/h^3 - 3*(3*c^2*f*g*h^14 - c^2*h^15*e)/(c*h^
18)) + 2*(18*c^2*f*g^2*h^13 + 3*c^2*d*h^15 + 4*a*c*f*h^15 - 9*c^2*g*h^14*e)
/(c*h^18)) + 1/2*(20*c^(3/2)*f*g^3 + 6*c^(3/2)*d*g*h^2 + 9*a*sqrt(c)*f*g*h^
2 - 12*c^(3/2)*g^2*h*e - 3*a*sqrt(c)*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2
 + a)))/h^6 + (20*c^2*f*g^4 + 6*c^2*d*g^2*h^2 + 19*a*c*f*g^2*h^2 + 3*a*c*d*
h^4 + 2*a^2*f*h^4 - 12*c^2*g^3*h*e - 9*a*c*g*h^3*e)*arctan(-((sqrt(c)*x - s
qrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 - a*h^2)*
h^6) + (10*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h + 6*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*c^2*d*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g^2
*h^3 + (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*d*h^5 - 8*(sqrt(c)*x - sqrt(c*x^
2 + a))^3*c^2*g^3*h^2*e - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*g*h^4*e + 1
8*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 + 10*(sqrt(c)*x - sqrt(c*x^
2 + a))^2*c^(5/2)*d*g^3*h^2 - (sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*f*g
^3*h^2 - 5*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*x
 - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 - 14*(sqrt(c)*x - sqrt(c*x^2 + a)
)^2*c^(5/2)*g^4*h*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*g^2*h^3*e
 + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*h^5*e - 26*(sqrt(c)*x - sq
rt(c*x^2 + a))*a*c^2*f*g^4*h - 14*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2
*h^3 - 11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^3 + (sqrt(c)*x - sqrt
(c*x^2 + a))*a^2*c*d*h^5 + 20*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*g^3*h^2*e
 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*g*h^4*e + 9*a^2*c^(3/2)*f*g^3*h^2
+ 5*a^2*c^(3/2)*d*g*h^4 + 4*a^3*sqrt(c)*f*g*h^4 - 7*a^2*c^(3/2)*g^2*h^3*e -
 2*a^3*sqrt(c)*h^5*e)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x -
sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2*h^6)
```

maple [B] time = 0.02, size = 7817, normalized size = 16.02

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.88, size = 1299, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(c*x^2 + a)*c^2*f*g^4/(c*g^2*h^5 + a*h^7) - 3/2*sqrt(c*x^2 + a)*c^2
*f*g^3*x/(c*g^2*h^4 + a*h^6) - 3/2*sqrt(c*x^2 + a)*c^2*e*g^3/(c*g^2*h^4 + a
*h^6) + 1/2*(c*x^2 + a)^(3/2)*c*f*g^3/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 +
a*g*h^5) + 3/2*sqrt(c*x^2 + a)*c^2*e*g^2*x/(c*g^2*h^3 + a*h^5) + 3/2*sqrt(c
*x^2 + a)*c^2*d*g^2/(c*g^2*h^3 + a*h^5) - 1/2*(c*x^2 + a)^(3/2)*c*e*g^2/(c*
g^2*h^3*x + a*h^5*x + c*g^3*h^2 + a*g*h^4) - 1/2*(c*x^2 + a)^(5/2)*f*g^2/(c
*g^2*h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^
```

```
3) + 1/2*(c*x^2 + a)^(3/2)*c*f*g^2/(c*g^2*h^3 + a*h^5) - 3/2*sqrt(c*x^2 + a
)*c^2*d*g*x/(c*g^2*h^2 + a*h^4) + 1/2*(c*x^2 + a)^(3/2)*c*d*g/(c*g^2*h^2*x
+ a*h^4*x + c*g^3*h + a*g*h^3) + 1/2*(c*x^2 + a)^(5/2)*e*g/(c*g^2*h^2*x^2 +
 a*h^4*x^2 + 2*c*g^3*h*x + 2*a*g*h^3*x + c*g^4 + a*g^2*h^2) - 1/2*(c*x^2 +
a)^(3/2)*c*e*g/(c*g^2*h^2 + a*h^4) - 1/2*(c*x^2 + a)^(5/2)*d/(c*g^2*h*x^2 +
 a*h^3*x^2 + 2*c*g^3*x + 2*a*g*h^2*x + c*g^4/h + a*g^2*h) + 1/2*(c*x^2 + a)
^(3/2)*c*d/(c*g^2*h + a*h^3) + 2*(c*x^2 + a)^(3/2)*f*g/(h^4*x + g*h^3) - (c
*x^2 + a)^(3/2)*e/(h^3*x + g*h^2) - 7/2*sqrt(c*x^2 + a)*c*f*g*x/h^4 + 3/2*s
qrt(c*x^2 + a)*c*e*x/h^3 - 10*c^(3/2)*f*g^3*arcsinh(c*x/sqrt(a*c))/h^6 + 6*
c^(3/2)*e*g^2*arcsinh(c*x/sqrt(a*c))/h^5 - 3*c^(3/2)*d*g*arcsinh(c*x/sqrt(a
*c))/h^4 - 9/2*a*sqrt(c)*f*g*arcsinh(c*x/sqrt(a*c))/h^4 + 3/2*a*sqrt(c)*e*a
rcsinh(c*x/sqrt(a*c))/h^3 + 3/2*c^2*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x
+ g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^7) - 3/2*c^2*e
*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))
/(sqrt(a + c*g^2/h^2)*h^6) + 3/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x
 + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^5) + 15/2*sqr
t(a + c*g^2/h^2)*c*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt
(a*c)*abs(h*x + g)))/h^5 - 9/2*sqrt(a + c*g^2/h^2)*c*e*g*arcsinh(c*g*x/(sqr
t(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^4 + 3/2*sqrt(a + c*g
^2/h^2)*c*d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
 + g)))/h^3 + (a + c*g^2/h^2)^(3/2)*f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)
) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 + 17/2*sqrt(c*x^2 + a)*c*f*g^2/h^5 -
9/2*sqrt(c*x^2 + a)*c*e*g/h^4 + 3/2*sqrt(c*x^2 + a)*c*d/h^3 + 1/3*(c*x^2 +
a)^(3/2)*f/h^3 + sqrt(c*x^2 + a)*a*f/h^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{\left(g+hx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**3,x)
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

3.95 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$

Optimal. Leaf size=475

$$\frac{c\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(3a^2h^4(4fg-eh)+3acgh^2\left(11fg^2-h(4eg-dh)\right)+2c^2g^3\left(10fg^2-h(4eg-dh)\right)\right)}{2h^6\left(ah^2+cg^2\right)^{3/2}}-\frac{(a}{}$$

```
[Out] -1/6*(c*g*(4*e*g-10*f*g^2/h-d*h)-3*a*h*(-e*h+3*f*g)-(3*a*f*h^2+c*(5*f*g^2-2
*h*(-d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^2/(a*h^2+c*g^2)/(h*x+g)^2-1/3*(d*h^2-e
*g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^3+1/2*c*(3*a^2*h^4*(-e*
h+4*f*g)+2*c^2*g^3*(10*f*g^2-h*(-d*h+4*e*g))+3*a*c*g*h^2*(11*f*g^2-h*(-d*h+
4*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^6/(a*h
^2+c*g^2)^(3/2)+1/2*(3*a*f*h^2+2*c*(10*f*g^2-h*(-d*h+4*e*g)))*arctanh(x*c^(
1/2)/(c*x^2+a)^(1/2))*c^(1/2)/h^6-1/2*((a*h^2+c*g^2)*(3*a*f*h^2+2*c*(10*f*g
^2-h*(-d*h+4*e*g)))+c*h*(3*a*h^2*(-e*h+3*f*g)+c*g*(10*f*g^2-h*(-d*h+4*e*g))
)*x)*(c*x^2+a)^(1/2)/h^5/(a*h^2+c*g^2)/(h*x+g)
```

Rubi [A] time = 0.84, antiderivative size = 469, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 813, 844, 217, 206, 725}

$$\frac{c\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(3a^2h^4(4fg-eh)+3acgh^2\left(11fg^2-h(4eg-dh)\right)+2c^2\left(10fg^5-g^3h(4eg-dh)\right)\right)}{2h^6\left(ah^2+cg^2\right)^{3/2}}-\frac{(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -(((c*g^2 + a*h^2)*(20*c*f*g^2 + 3*a*f*h^2 - 2*c*h*(4*e*g - d*h)) + c*h*(10
*c*f*g^3 - c*g*h*(4*e*g - d*h) + 3*a*h^2*(3*f*g - e*h))*x)*Sqrt[a + c*x^2])
/(2*h^5*(c*g^2 + a*h^2)*(g + h*x)) - ((c*g*(4*e*g - (10*f*g^2)/h - d*h) - 3
*a*h*(3*f*g - e*h) - (5*c*f*g^2 + 3*a*f*h^2 - 2*c*h*(e*g - d*h))*x)*(a + c*
x^2)^(3/2))/(6*h^2*(c*g^2 + a*h^2)*(g + h*x)^2) - ((f*g^2 - e*g*h + d*h^2)*
(a + c*x^2)^(5/2))/(3*h*(c*g^2 + a*h^2)*(g + h*x)^3) + (Sqrt[c]*(20*c*f*g^2
 + 3*a*f*h^2 - 2*c*h*(4*e*g - d*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(
2*h^6) + (c*(3*a^2*h^4*(4*f*g - e*h) + 3*a*c*g*h^2*(11*f*g^2 - h*(4*e*g - d
*h)) + 2*c^2*(10*f*g^5 - g^3*h*(4*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[
c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^6*(c*g^2 + a*h^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{3h\left(cg^2+ah^2\right)(g+hx)^3} - \frac{\int \frac{\left(-3(cdg-afg+aeh)-\left(3afh-c\left(2eg-\frac{5fg^2}{h}-2dh\right)\right)x\right.}{(g+hx)^3}}{3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(cg\left(4eg-\frac{10fg^2}{h}-dh\right)-3ah(3fg-eh)-\left(5cfg^2+3afh^2-2ch(eg-dh\right)\right.}{6h^2\left(cg^2+ah^2\right)(g+hx)^2}$$

$$= -\frac{\left(\left(cg^2+ah^2\right)\left(20cfg^2+3afh^2-2ch(4eg-dh)\right)+ch\left(10cfg^3-cgh(4eg\right.\right.}{2h^5\left(cg^2+ah^2\right)(g+hx)}$$

$$= -\frac{\left(\left(cg^2+ah^2\right)\left(20cfg^2+3afh^2-2ch(4eg-dh)\right)+ch\left(10cfg^3-cgh(4eg\right.\right.}{2h^5\left(cg^2+ah^2\right)(g+hx)}$$

$$= -\frac{\left(\left(cg^2+ah^2\right)\left(20cfg^2+3afh^2-2ch(4eg-dh)\right)+ch\left(10cfg^3-cgh(4eg\right.\right.}{2h^5\left(cg^2+ah^2\right)(g+hx)}$$

$$= -\frac{\left(\left(cg^2+ah^2\right)\left(20cfg^2+3afh^2-2ch(4eg-dh)\right)+ch\left(10cfg^3-cgh(4eg\right.\right.}{2h^5\left(cg^2+ah^2\right)(g+hx)}$$

Mathematica [A] time = 1.23, size = 517, normalized size = 1.09

$$\frac{\frac{3c\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(-3a^2h^4(eh-4fg)+3acgh^2\left(h(dh-4eg)+11fg^2\right)+2c^2\left(g^3h(dh-4eg)+10fg^5\right)\right)}{\left(ah^2+cg^2\right)^{3/2}}-\frac{3c\log(g+hx)\left(-3a^2h^4(eh-4fg)+3acg\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] (-((h*Sqrt[a + c*x^2]*(2*(c*g^2 + a*h^2)^2*(f*g^2 + h*(-(e*g) + d*h)) - (c*
g^2 + a*h^2)*(13*c*f*g^3 + c*g*h*(-10*e*g + 7*d*h) - 3*a*h^2*(-2*f*g + e*h)
)*(g + h*x) + (6*a^2*f*h^4 + a*c*h^2*(50*f*g^2 + h*(-23*e*g + 8*d*h)) + c^2
*(47*f*g^4 + g^2*h*(-26*e*g + 11*d*h)))*(g + h*x)^2 + 6*c*(4*f*g - e*h)*(c*
g^2 + a*h^2)*(g + h*x)^3 - 3*c*f*h*(c*g^2 + a*h^2)*x*(g + h*x)^3))/((c*g^2
+ a*h^2)*(g + h*x)^3)) - (3*c*(-3*a^2*h^4*(-4*f*g + e*h) + 3*a*c*g*h^2*(11*
f*g^2 + h*(-4*e*g + d*h)) + 2*c^2*(10*f*g^5 + g^3*h*(-4*e*g + d*h)))*Log[g
+ h*x])/(c*g^2 + a*h^2)^(3/2) + 3*Sqrt[c]*(20*c*f*g^2 + 3*a*f*h^2 + 2*c*h*(
-4*e*g + d*h))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c*(-3*a^2*h^4*(-4*f*
g + e*h) + 3*a*c*g*h^2*(11*f*g^2 + h*(-4*e*g + d*h)) + 2*c^2*(10*f*g^5 + g^
3*h*(-4*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]]
)/(c*g^2 + a*h^2)^(3/2))/(6*h^6)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.61, size = 1900, normalized size = 4.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*(c*f*x/h^4 - 2*(4*c*f*g*h^10 - c*h^11*e)/h^15) - (20*c^
3*f*g^5 + 2*c^3*d*g^3*h^2 + 33*a*c^2*f*g^3*h^2 + 3*a*c^2*d*g*h^4 + 12*a^2*c
*f*g*h^4 - 8*c^3*g^4*h*e - 12*a*c^2*g^2*h^3*e - 3*a^2*c*h^5*e)*arctan(-((sq
rt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c*g^2*h^6
 + a*h^8)*sqrt(-c*g^2 - a*h^2)) - 1/3*(60*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c
^3*f*g^5*h^2 + 18*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^3*d*g^3*h^4 + 69*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a*c^2*f*g^3*h^4 + 15*(sqrt(c)*x - sqrt(c*x^2 + a)
)^5*a*c^2*d*g*h^6 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*f*g*h^6 - 36*(
sqrt(c)*x - sqrt(c*x^2 + a))^5*c^3*g^4*h^3*e - 36*(sqrt(c)*x - sqrt(c*x^2 +
 a))^5*a*c^2*g^2*h^5*e - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*h^7*e + 21
0*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*f*g^6*h + 54*(sqrt(c)*x - sqrt(c*
x^2 + a))^4*c^(7/2)*d*g^4*h^3 + 183*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/
2)*f*g^4*h^3 + 27*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*d*g^2*h^5 - 18*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*f*g^2*h^5 - 12*(sqrt(c)*x - sqr
t(c*x^2 + a))^4*a^2*c^(3/2)*d*h^7 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*s
qrt(c)*f*h^7 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*g^5*h^2*e - 84*(
sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*g^3*h^4*e + 21*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a^2*c^(3/2)*g*h^6*e + 188*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*
f*g^7 + 44*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*d*g^5*h^2 - 82*(sqrt(c)*x -
sqrt(c*x^2 + a))^3*a*c^3*f*g^5*h^2 - 34*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c
```

```
^3*d*g^3*h^4 - 276*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*f*g^3*h^4 - 48*(
sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*d*g*h^6 - 36*(sqrt(c)*x - sqrt(c*x^2
 + a))^3*a^3*c*f*g*h^6 - 104*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*g^6*h*e +
64*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*g^4*h^3*e + 138*(sqrt(c)*x - sqrt(
c*x^2 + a))^3*a^2*c^2*g^2*h^5*e - 354*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(
7/2)*f*g^6*h - 78*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(7/2)*d*g^4*h^3 - 276
*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)*f*g^4*h^3 - 36*(sqrt(c)*x - sq
rt(c*x^2 + a))^2*a^2*c^(5/2)*d*g^2*h^5 + 60*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*a^3*c^(3/2)*f*g^2*h^5 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*d*h
^7 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*sqrt(c)*f*h^7 + 192*(sqrt(c)*x
- sqrt(c*x^2 + a))^2*a*c^(7/2)*g^5*h^2*e + 114*(sqrt(c)*x - sqrt(c*x^2 + a)
)^2*a^2*c^(5/2)*g^3*h^4*e - 48*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*
g*h^6*e + 222*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*f*g^5*h^2 + 48*(sqrt(c)
*x - sqrt(c*x^2 + a))*a^2*c^3*d*g^3*h^4 + 231*(sqrt(c)*x - sqrt(c*x^2 + a))
*a^3*c^2*f*g^3*h^4 + 33*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d*g*h^6 + 24*
(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c*f*g*h^6 - 120*(sqrt(c)*x - sqrt(c*x^2 +
 a))*a^2*c^3*g^4*h^3*e - 102*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*g^2*h^5*
e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c*h^7*e - 47*a^3*c^(5/2)*f*g^4*h^3
- 11*a^3*c^(5/2)*d*g^2*h^5 - 50*a^4*c^(3/2)*f*g^2*h^5 - 8*a^4*c^(3/2)*d*h^7
 - 6*a^5*sqrt(c)*f*h^7 + 26*a^3*c^(5/2)*g^3*h^4*e + 23*a^4*c^(3/2)*g*h^6*e)
/((c*g^2*h^6 + a*h^8)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - s
qrt(c*x^2 + a))*sqrt(c)*g - a*h)^3) - 1/2*(20*c^(3/2)*f*g^2 + 2*c^(3/2)*d*h
^2 + 3*a*sqrt(c)*f*h^2 - 8*c^(3/2)*g*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
 a)))/h^6
```

maple [B] time = 0.02, size = 9835, normalized size = 20.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x)
```

```
[Out] result too large to display
```

maxima [B] time = 1.10, size = 2415, normalized size = 5.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*c^3*f*g^5/(c^2*g^4*h^5 + 2*a*c*g^2*h^7 + a^2*h^9) - 1/2
*sqrt(c*x^2 + a)*c^3*f*g^4*x/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) - 1/2*
sqrt(c*x^2 + a)*c^3*e*g^4/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) + 1/6*(c*
x^2 + a)^(3/2)*c^2*f*g^4/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2
*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) + 1/2*sqrt(c*x^2 + a)*c^3*e*g^3*x/(c^
2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) + 1/2*sqrt(c*x^2 + a)*c^3*d*g^3/(c^2*g
^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/6*(c*x^2 + a)^(3/2)*c^2*e*g^3/(c^2*g^
4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*h^4 + a^2*g
*h^6) - 1/6*(c*x^2 + a)^(5/2)*c*f*g^3/(c^2*g^4*h^3*x^2 + 2*a*c*g^2*h^5*x^2
+ a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6*x + c^2*g^6
*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 1/6*(c*x^2 + a)^(3/2)*c^2*f*g^3/(c^2*g^
4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/2*sqrt(c*x^2 + a)*c^3*d*g^2*x/(c^2*g^4
*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) + 1/6*(c*x^2 + a)^(3/2)*c^2*d*g^2/(c^2*g^4*
h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*g*h^5
) + 1/6*(c*x^2 + a)^(5/2)*c*e*g^2/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2 + a^
2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6 + 2*a
*c*g^4*h^2 + a^2*g^2*h^4) - 1/6*(c*x^2 + a)^(3/2)*c^2*e*g^2/(c^2*g^4*h^2 +
2*a*c*g^2*h^4 + a^2*h^6) - 9/2*sqrt(c*x^2 + a)*c^2*f*g^3/(c*g^2*h^5 + a*h^7
) + 4*sqrt(c*x^2 + a)*c^2*f*g^2*x/(c*g^2*h^4 + a*h^6) - 1/6*(c*x^2 + a)^(5/
```

```
2)*c*d*g/(c^2*g^4*h*x^2 + 2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 + 2*c^2*g^5*x + 4
*a*c*g^3*h^2*x + 2*a^2*g*h^4*x + c^2*g^6/h + 2*a*c*g^4*h + a^2*g^2*h^3) + 1
/6*(c*x^2 + a)^(3/2)*c^2*d*g/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*h^5) + 3*sqrt
(c*x^2 + a)*c^2*e*g^2/(c*g^2*h^4 + a*h^6) - 1/3*(c*x^2 + a)^(5/2)*f*g^2/(c*
g^2*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x +
 3*a*g^2*h^4*x + c*g^5*h + a*g^3*h^3) - 5/3*(c*x^2 + a)^(3/2)*c*f*g^2/(c*g^
2*h^4*x + a*h^6*x + c*g^3*h^3 + a*g*h^5) - 5/2*sqrt(c*x^2 + a)*c^2*e*g*x/(c
*g^2*h^3 + a*h^5) - 3/2*sqrt(c*x^2 + a)*c^2*d*g/(c*g^2*h^3 + a*h^5) + 1/3*(
c*x^2 + a)^(5/2)*e*g/(c*g^2*h^3*x^3 + a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h
^4*x^2 + 3*c*g^4*h*x + 3*a*g^2*h^3*x + c*g^5 + a*g^3*h^2) + 7/6*(c*x^2 + a)
^(3/2)*c*e*g/(c*g^2*h^3*x + a*h^5*x + c*g^3*h^2 + a*g*h^4) + (c*x^2 + a)^(5
/2)*f*g/(c*g^2*h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h
+ a*g^2*h^3) - (c*x^2 + a)^(3/2)*c*f*g/(c*g^2*h^3 + a*h^5) + sqrt(c*x^2 + a
)*c^2*d*x/(c*g^2*h^2 + a*h^4) - 1/3*(c*x^2 + a)^(5/2)*d/(c*g^2*h^2*x^3 + a*
h^4*x^3 + 3*c*g^3*h*x^2 + 3*a*g*h^3*x^2 + 3*c*g^4*x + 3*a*g^2*h^2*x + c*g^5
/h + a*g^3*h) - 2/3*(c*x^2 + a)^(3/2)*c*d/(c*g^2*h^2*x + a*h^4*x + c*g^3*h
+ a*g*h^3) - 1/2*(c*x^2 + a)^(5/2)*e/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h
*x + 2*a*g*h^3*x + c*g^4 + a*g^2*h^2) + 1/2*(c*x^2 + a)^(3/2)*c*e/(c*g^2*h^
2 + a*h^4) - (c*x^2 + a)^(3/2)*f/(h^4*x + g*h^3) + 3/2*sqrt(c*x^2 + a)*c*f*
x/h^4 + 10*c^(3/2)*f*g^2*arcsinh(c*x/sqrt(a*c))/h^6 - 4*c^(3/2)*e*g*arcsinh
(c*x/sqrt(a*c))/h^5 + c^(3/2)*d*arcsinh(c*x/sqrt(a*c))/h^4 + 3/2*a*sqrt(c)*
f*arcsinh(c*x/sqrt(a*c))/h^4 + 1/2*c^3*f*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h
*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^9) - 1/2*
c^3*e*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x +
 g)))/((a + c*g^2/h^2)^(3/2)*h^8) + 1/2*c^3*d*g^3*arcsinh(c*g*x/(sqrt(a*c)*
abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) -
 9/2*c^2*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(
h*x + g)))/(sqrt(a + c*g^2/h^2)*h^7) + 3*c^2*e*g^2*arcsinh(c*g*x/(sqrt(a*c)
*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^6) -
3/2*c^2*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
 + g)))/(sqrt(a + c*g^2/h^2)*h^5) - 6*sqrt(a + c*g^2/h^2)*c*f*g*arcsinh(c*g
*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^5 + 3/2*sqrt(
a + c*g^2/h^2)*c*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*
abs(h*x + g)))/h^4 - 6*sqrt(c*x^2 + a)*c*f*g/h^5 + 3/2*sqrt(c*x^2 + a)*c*e/
h^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**4,x)
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

3.96 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx$

Optimal. Leaf size=511

$$\frac{(a+cx^2)^{3/2}\left(-3hx\left(4a^2fh^4+ach^2\left(17fg^2-h(5eg-dh)\right)+2c^2g^2\left(5fg^2-h(dh+eg)\right)\right)+4a^2h^4(fg-2eh)-a\cdots}{24h^3(g+hx)^3\left(ah^2+cg^2\right)^2}$$

```
[Out] 1/24*(4*a^2*h^4*(-2*e*h+f*g)-4*c^2*g^4*(-e*h+5*f*g)-a*c*g*h^2*(25*f*g^2-h*(
-9*d*h+5*e*g))-3*h*(4*a^2*f*h^4+a*c*h^2*(17*f*g^2-h*(-d*h+5*e*g))+2*c^2*g^2
*(5*f*g^2-h*(d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^3/(a*h^2+c*g^2)^2/(h*x+g)^3-1/
4*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^4-c^(3/2)*(-e
*h+5*f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^6-1/8*c*(12*a^3*f*h^6+8*c^3*
g^5*(-e*h+5*f*g)+20*a*c^2*g^3*h^2*(-e*h+5*f*g)+3*a^2*c*h^4*(25*f*g^2-h*(-d*
h+5*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^6/(a
*h^2+c*g^2)^(5/2)+1/8*c*(8*(-e*h+5*f*g)*(a*h^2+c*g^2)^2+h*(12*a^2*f*h^4+4*c
^2*g^3*(-e*h+5*f*g)+a*c*h^2*(35*f*g^2-h*(-3*d*h+7*e*g)))*x)*(c*x^2+a)^(1/2)
/h^5/(a*h^2+c*g^2)^2/(h*x+g)
```

Rubi [A] time = 1.09, antiderivative size = 511, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1651, 811, 813, 844, 217, 206, 725}

$$\frac{(a+cx^2)^{3/2}\left(-3x\left(4a^2fh^4+ach^2\left(17fg^2-h(5eg-dh)\right)+2c^2\left(5fg^4-g^2h(dh+eg)\right)\right)+4a^2h^3(fg-2eh)-acg\cdots}{24h^2(g+hx)^3\left(ah^2+cg^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (c*(8*(5*f*g - e*h)*(c*g^2 + a*h^2)^2 + h*(12*a^2*f*h^4 + 4*c^2*g^3*(5*f*g
- e*h) + a*c*h^2*(35*f*g^2 - h*(7*e*g - 3*d*h)))*x)*Sqrt[a + c*x^2])/(8*h^5
*(c*g^2 + a*h^2)^2*(g + h*x)) + ((4*a^2*h^3*(f*g - 2*e*h) - (4*c^2*g^4*(5*f
*g - e*h))/h - a*c*g*h*(25*f*g^2 - h*(5*e*g - 9*d*h)) - 3*(4*a^2*f*h^4 + a*
c*h^2*(17*f*g^2 - h*(5*e*g - d*h)) + 2*c^2*(5*f*g^4 - g^2*h*(e*g + d*h)))*x
)*(a + c*x^2)^(3/2))/(24*h^2*(c*g^2 + a*h^2)^2*(g + h*x)^3) - ((f*g^2 - e*g
*h + d*h^2)*(a + c*x^2)^(5/2))/(4*h*(c*g^2 + a*h^2)*(g + h*x)^4) - (c^(3/2)
*(5*f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/h^6 - (c*(12*a^3*f*h^6
 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) + 3*a^2*c*h^4*(
25*f*g^2 - h*(5*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2])])/(8*h^6*(c*g^2 + a*h^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{4h\left(cg^2+ah^2\right)(g+hx)^4} - \frac{\int \frac{\left(-4(cdg-afg+aeh)-\left(4afh-c\left(eg-\frac{5fg^2}{h}-dh\right)\right)x\right)(\cdots}{(g+hx)^4}}{4\left(cg^2+ah^2\right)}$$

$$= \frac{\left(4a^2h^3(fg-2eh)-\frac{4c^2g^4(5fg-eh)}{h}-acgh\left(25fg^2-h(5eg-9dh)\right)-3\left(4a^2f\cdots\right.\right.}{24h^2\left(cg^2+\cdots\right.}$$

$$= \frac{c\left(8(5fg-eh)\left(cg^2+ah^2\right)^2+h\left(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2\left(35fg\cdots\right.\right.\right.}{8h^5\left(cg^2+ah^2\right)^2(g+hx)}$$

$$= \frac{c\left(8(5fg-eh)\left(cg^2+ah^2\right)^2+h\left(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2\left(35fg\cdots\right.\right.\right.}{8h^5\left(cg^2+ah^2\right)^2(g+hx)}$$

$$= \frac{c\left(8(5fg-eh)\left(cg^2+ah^2\right)^2+h\left(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2\left(35fg\cdots\right.\right.\right.}{8h^5\left(cg^2+ah^2\right)^2(g+hx)}$$

$$= \frac{c\left(8(5fg-eh)\left(cg^2+ah^2\right)^2+h\left(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2\left(35fg\cdots\right.\right.\right.}{8h^5\left(cg^2+ah^2\right)^2(g+hx)}$$

Mathematica [A] time = 2.15, size = 575, normalized size = 1.13

$$-\frac{\frac{h\sqrt{a+cx^2}\left((g+hx)^2\left(ah^2+cg^2\right)\left(12a^2fh^4+ach^2\left(h(15dh-43eg)+95fg^2\right)+2c^2\left(g^2h(9dh-23eg)+43fg^4\right)\right)-c(g+hx)^3\left(4a^2h^4(31fg-8eh)+acgh^2\left(h(15dh-\cdots\right.\right.\right.\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] -1/24*((h*Sqrt[a + c*x^2]*(6*(c*g^2 + a*h^2)^3*(f*g^2 + h*(-(e*g) + d*h)) -
 2*(c*g^2 + a*h^2)^2*(17*c*f*g^3 + c*g*h*(-13*e*g + 9*d*h) - 4*a*h^2*(-2*f*
g + e*h))*(g + h*x) + (c*g^2 + a*h^2)*(12*a^2*f*h^4 + 2*c^2*(43*f*g^4 + g^2
*h*(-23*e*g + 9*d*h)) + a*c*h^2*(95*f*g^2 + h*(-43*e*g + 15*d*h)))*(g + h*x
)^2 - c*(4*a^2*h^4*(31*f*g - 8*e*h) + 2*c^2*(77*f*g^5 + g^3*h*(-25*e*g + 3*
d*h)) + a*c*g*h^2*(287*f*g^2 + h*(-91*e*g + 15*d*h)))*(g + h*x)^3 - 24*c*f*
(c*g^2 + a*h^2)^2*(g + h*x)^4))/((c*g^2 + a*h^2)^2*(g + h*x)^4) - (3*c*(12*
a^3*f*h^6 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) + 3*a^
2*c*h^4*(25*f*g^2 + h*(-5*e*g + d*h)))*Log[g + h*x])/(c*g^2 + a*h^2)^(5/2)
+ 24*c^(3/2)*(5*f*g - e*h)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c*(12*a^
3*f*h^6 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) + 3*a^2*
c*h^4*(25*f*g^2 + h*(-5*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*
Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(5/2))/h^6
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.02, size = 12481, normalized size = 24.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x)
[Out] result too large to display
```

maxima [B] time = 1.50, size = 4326, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="maxima")
[Out] 3/8*sqrt(c*x^2 + a)*c^4*f*g^6/(c^3*g^6*h^5 + 3*a*c^2*g^4*h^7 + 3*a^2*c*g^2*
h^9 + a^3*h^11) - 3/8*sqrt(c*x^2 + a)*c^4*f*g^5*x/(c^3*g^6*h^4 + 3*a*c^2*g^
4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) - 3/8*sqrt(c*x^2 + a)*c^4*e*g^5/(c^3*g^
6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) + 1/8*(c*x^2 + a)^(3/
2)*c^3*f*g^5/(c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h
^10*x + c^3*g^7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 + a^3*g*h^9) + 3/8*
sqrt(c*x^2 + a)*c^4*e*g^4*x/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^
7 + a^3*h^9) + 3/8*sqrt(c*x^2 + a)*c^4*d*g^4/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5
 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 1/8*(c*x^2 + a)^(3/2)*c^3*e*g^4/(c^3*g^6*h^
3*x + 3*a*c^2*g^4*h^5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^7*h^2 + 3*a
*c^2*g^5*h^4 + 3*a^2*c*g^3*h^6 + a^3*g*h^8) - 1/8*(c*x^2 + a)^(5/2)*c^2*f*g
^4/(c^3*g^6*h^3*x^2 + 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x
^2 + 2*c^3*g^7*h^2*x + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*
x + c^3*g^8*h + 3*a*c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2*h^7) + 1/8*(c*x
^2 + a)^(3/2)*c^3*f*g^4/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 +
a^3*h^9) - 3/8*sqrt(c*x^2 + a)*c^4*d*g^3*x/(c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 +
 3*a^2*c*g^2*h^6 + a^3*h^8) + 1/8*(c*x^2 + a)^(3/2)*c^3*d*g^3/(c^3*g^6*h^2*
x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g^2*h^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2
*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7) + 1/8*(c*x^2 + a)^(5/2)*c^2*e*g^3/(
c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h^4*x^2 + 3*a^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 +
 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^3*x + 6*a^2*c*g^3*h^5*x + 2*a^3*g*h^7*x + c^
3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4 + a^3*g^2*h^6) - 1/8*(c*x^2 + a)^
(3/2)*c^3*e*g^3/(c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8)
 - 7/4*sqrt(c*x^2 + a)*c^3*f*g^4/(c^2*g^4*h^5 + 2*a*c*g^2*h^7 + a^2*h^9) +
11/8*sqrt(c*x^2 + a)*c^3*f*g^3*x/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) -
1/8*(c*x^2 + a)^(5/2)*c^2*d*g^2/(c^3*g^6*h*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^
2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g
^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^
3*g^2*h^5) + 1/8*(c*x^2 + a)^(3/2)*c^3*d*g^2/(c^3*g^6*h + 3*a*c^2*g^4*h^3 +
 3*a^2*c*g^2*h^5 + a^3*h^7) + 5/4*sqrt(c*x^2 + a)*c^3*e*g^3/(c^2*g^4*h^4 +
2*a*c*g^2*h^6 + a^2*h^8) - 1/4*(c*x^2 + a)^(5/2)*c*f*g^3/(c^2*g^4*h^4*x^3 +
 2*a*c*g^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 +
```

3*a^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2
*g^7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^5) - 17/24*(c*x^2 + a)^(3/2)*c^2*f*g^3/(
c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2*g^5*h^3 + 2*a*c*g^3*h^5 +
 a^2*g*h^7) - 7/8*sqrt(c*x^2 + a)*c^3*e*g^2*x/(c^2*g^4*h^3 + 2*a*c*g^2*h^5
+ a^2*h^7) - 3/4*sqrt(c*x^2 + a)*c^3*d*g^2/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a
^2*h^7) + 1/4*(c*x^2 + a)^(5/2)*c*e*g^2/(c^2*g^4*h^3*x^3 + 2*a*c*g^2*h^5*x^
3 + a^2*h^7*x^3 + 3*c^2*g^5*h^2*x^2 + 6*a*c*g^3*h^4*x^2 + 3*a^2*g*h^6*x^2 +
 3*c^2*g^6*h*x + 6*a*c*g^4*h^3*x + 3*a^2*g^2*h^5*x + c^2*g^7 + 2*a*c*g^5*h^
2 + a^2*g^3*h^4) + 13/24*(c*x^2 + a)^(3/2)*c^2*e*g^2/(c^2*g^4*h^3*x + 2*a*c
*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*h^4 + a^2*g*h^6) + 5/24*(c
*x^2 + a)^(5/2)*c*f*g^2/(c^2*g^4*h^3*x^2 + 2*a*c*g^2*h^5*x^2 + a^2*h^7*x^2
+ 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6*x + c^2*g^6*h + 2*a*c*g^4
*h^3 + a^2*g^2*h^5) - 5/24*(c*x^2 + a)^(3/2)*c^2*f*g^2/(c^2*g^4*h^3 + 2*a*c
*g^2*h^5 + a^2*h^7) + 3/8*sqrt(c*x^2 + a)*c^3*d*g*x/(c^2*g^4*h^2 + 2*a*c*g^
2*h^4 + a^2*h^6) - 1/4*(c*x^2 + a)^(5/2)*c*d*g/(c^2*g^4*h^2*x^3 + 2*a*c*g^2
*h^4*x^3 + a^2*h^6*x^3 + 3*c^2*g^5*h*x^2 + 6*a*c*g^3*h^3*x^2 + 3*a^2*g*h^5*
x^2 + 3*c^2*g^6*x + 6*a*c*g^4*h^2*x + 3*a^2*g^2*h^4*x + c^2*g^7/h + 2*a*c*g
^5*h + a^2*g^3*h^3) - 3/8*(c*x^2 + a)^(3/2)*c^2*d*g/(c^2*g^4*h^2*x + 2*a*c*
g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*g*h^5) - 1/24*(c*x^
2 + a)^(5/2)*c*e*g/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2 + a^2*h^6*x^2 + 2*c
^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6 + 2*a*c*g^4*h^2 + a^
2*g^2*h^4) + 1/24*(c*x^2 + a)^(3/2)*c^2*e*g/(c^2*g^4*h^2 + 2*a*c*g^2*h^4 +
a^2*h^6) - 1/4*(c*x^2 + a)^(5/2)*f*g^2/(c*g^2*h^5*x^4 + a*h^7*x^4 + 4*c*g^3
*h^4*x^3 + 4*a*g*h^6*x^3 + 6*c*g^4*h^3*x^2 + 6*a*g^2*h^5*x^2 + 4*c*g^5*h^2*
x + 4*a*g^3*h^4*x + c*g^6*h + a*g^4*h^3) + 39/8*sqrt(c*x^2 + a)*c^2*f*g^2/(
c*g^2*h^5 + a*h^7) - 7/2*sqrt(c*x^2 + a)*c^2*f*g*x/(c*g^2*h^4 + a*h^6) - 1/
8*(c*x^2 + a)^(5/2)*c*d/(c^2*g^4*h*x^2 + 2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 +
2*c^2*g^5*x + 4*a*c*g^3*h^2*x + 2*a^2*g*h^4*x + c^2*g^6/h + 2*a*c*g^4*h + a
^2*g^2*h^3) + 1/8*(c*x^2 + a)^(3/2)*c^2*d/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*
h^5) + 1/4*(c*x^2 + a)^(5/2)*e*g/(c*g^2*h^4*x^4 + a*h^6*x^4 + 4*c*g^3*h^3*x
^3 + 4*a*g*h^5*x^3 + 6*c*g^4*h^2*x^2 + 6*a*g^2*h^4*x^2 + 4*c*g^5*h*x + 4*a*
g^3*h^3*x + c*g^6 + a*g^4*h^2) - 15/8*sqrt(c*x^2 + a)*c^2*e*g/(c*g^2*h^4 +
a*h^6) + 2/3*(c*x^2 + a)^(5/2)*f*g/(c*g^2*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3
*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c*g^5*h + a*g^3*h^3)
 + 11/6*(c*x^2 + a)^(3/2)*c*f*g/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 + a*g*h^
5) + sqrt(c*x^2 + a)*c^2*e*x/(c*g^2*h^3 + a*h^5) - 1/4*(c*x^2 + a)^(5/2)*d/
(c*g^2*h^3*x^4 + a*h^5*x^4 + 4*c*g^3*h^2*x^3 + 4*a*g*h^4*x^3 + 6*c*g^4*h*x^
2 + 6*a*g^2*h^3*x^2 + 4*c*g^5*x + 4*a*g^3*h^2*x + c*g^6/h + a*g^4*h) + 3/8*
sqrt(c*x^2 + a)*c^2*d/(c*g^2*h^3 + a*h^5) - 1/3*(c*x^2 + a)^(5/2)*e/(c*g^2*
h^3*x^3 + a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h^4*x^2 + 3*c*g^4*h*x + 3*a*g
^2*h^3*x + c*g^5 + a*g^3*h^2) - 2/3*(c*x^2 + a)^(3/2)*c*e/(c*g^2*h^3*x + a*
h^5*x + c*g^3*h^2 + a*g*h^4) - 1/2*(c*x^2 + a)^(5/2)*f/(c*g^2*h^3*x^2 + a*h
^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*(c*x^2 +
a)^(3/2)*c*f/(c*g^2*h^3 + a*h^5) - 5*c^(3/2)*f*g*arcsinh(c*x/sqrt(a*c))/h^6
 + c^(3/2)*e*arcsinh(c*x/sqrt(a*c))/h^5 + 3/8*c^4*f*g^6*arcsinh(c*g*x/(sqrt
(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*
h^11) - 3/8*c^4*e*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*
c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^10) + 3/8*c^4*d*g^4*arcsinh(c*g*
x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)
^(5/2)*h^9) - 7/4*c^3*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(s
qrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^9) + 5/4*c^3*e*g^3*arcsinh
(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2
/h^2)^(3/2)*h^8) - 3/4*c^3*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a
*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) + 39/8*c^2*f*g^2*a
rcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt
(a + c*g^2/h^2)*h^7) - 15/8*c^2*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g))
- a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^6) + 3/8*c^2*d*arcsi
nh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a +
 c*g^2/h^2)*h^5) + 3/2*sqrt(a + c*g^2/h^2)*c*f*arcsinh(c*g*x/(sqrt(a*c)*abs

```
(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^5 + 3/2*sqrt(c*x^2 + a)*c*f/h^
5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{\left(g+hx\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**5,x)
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

3.97 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

Optimal. Leaf size=507

$$-\frac{(a+cx^2)^{3/2}\left(hx\left(4a^2fh^4+acgh^2(14fg-3eh)+c^2\left(7fg^4-3dg^2h^2\right)\right)-a^2h^4(2fg-3eh)+acgh^2\left(3dh^2+5fg\right.\right.}{12h^3(g+hx)^4\left(ah^2+cg^2\right)^2}$$

```
[Out] -1/12*(4*c^2*f*g^5-a^2*h^4*(-3*e*h+2*f*g)+a*c*g*h^2*(3*d*h^2+5*f*g^2)+h*(4*
a^2*f*h^4+a*c*g*h^2*(-3*e*h+14*f*g)+c^2*(-3*d*g^2*h^2+7*f*g^4))*x)*(c*x^2+a
)^(3/2)/h^3/(a*h^2+c*g^2)^2/(h*x+g)^4-1/5*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(5/
2)/h/(a*h^2+c*g^2)/(h*x+g)^5+c^(3/2)*f*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h
^6+1/8*c^2*(8*c^3*f*g^7+28*a*c^2*f*g^5*h^2+3*a^3*h^6*(-e*h+6*f*g)+a^2*c*g*h
^4*(-3*d*h^2+35*f*g^2))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^
(1/2))/h^6/(a*h^2+c*g^2)^(7/2)-1/8*c*(8*c^3*f*g^7+20*a*c^2*f*g^5*h^2-a^3*h^
6*(-3*e*h+2*f*g)+a^2*c*g*h^4*(3*d*h^2+13*f*g^2)+h*(12*c^3*f*g^6+8*a^3*f*h^6
+a^2*c*g*h^4*(-3*e*h+34*f*g)+a*c^2*g^2*h^2*(-3*d*h^2+35*f*g^2))*x)*(c*x^2+a
)^(1/2)/h^5/(a*h^2+c*g^2)^3/(h*x+g)^2
```

Rubi [A] time = 0.86, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = $\{1651, 811, 844, 217, 206, 725\}$

$$-\frac{(a+cx^2)^{3/2}\left(hx\left(4a^2fh^4+acgh^2(14fg-3eh)+c^2\left(7fg^4-3dg^2h^2\right)\right)-a^2h^4(2fg-3eh)+acgh^2\left(3dh^2+5fg\right.\right.}{12h^3(g+hx)^4\left(ah^2+cg^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -(c*(8*c^3*f*g^7 + 20*a*c^2*f*g^5*h^2 - a^3*h^6*(2*f*g - 3*e*h) + a^2*c*g*h
^4*(13*f*g^2 + 3*d*h^2) + h*(12*c^3*f*g^6 + 8*a^3*f*h^6 + a^2*c*g*h^4*(34*f
*g - 3*e*h) + a*c^2*g^2*h^2*(35*f*g^2 - 3*d*h^2))*x)*Sqrt[a + c*x^2])/(8*h^
5*(c*g^2 + a*h^2)^3*(g + h*x)^2) - ((4*c^2*f*g^5 - a^2*h^4*(2*f*g - 3*e*h)
+ a*c*g*h^2*(5*f*g^2 + 3*d*h^2) + h*(4*a^2*f*h^4 + a*c*g*h^2*(14*f*g - 3*e*
h) + c^2*(7*f*g^4 - 3*d*g^2*h^2))*x)*(a + c*x^2)^(3/2))/(12*h^3*(c*g^2 + a*
h^2)^2*(g + h*x)^4) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(5*h*(c*g
^2 + a*h^2)*(g + h*x)^5) + (c^(3/2)*f*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])
/h^6 + (c^2*(8*c^3*f*g^7 + 28*a*c^2*f*g^5*h^2 + 3*a^3*h^6*(6*f*g - e*h) + a
^2*c*g*h^4*(35*f*g^2 - 3*d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]
*Sqrt[a + c*x^2])])/(8*h^6*(c*g^2 + a*h^2)^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} - \frac{\int \frac{\left(-5(cdg-afg+aeh)-5f\left(\frac{cg^2}{h}+ah\right)x\right)\left(a+cx^2\right)^{3/2}}{(g+hx)^5}\,d}{5\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(4c^2fg^5-a^2h^4(2fg-3eh)+acgh^2\left(5fg^2+3dh^2\right)+h\left(4a^2fh^4+acgh^2(\right.\right.}{12h^3\left(cg^2+ah^2\right)^2(g+hx}$$

$$= -\frac{c\left(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4\left(13fg^2+3dh^2\right)+h\right.}{8h^5\left(cg^2\right.}$$

$$= -\frac{c\left(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4\left(13fg^2+3dh^2\right)+h\right.}{8h^5\left(cg^2\right.}$$

$$= -\frac{c\left(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4\left(13fg^2+3dh^2\right)+h\right.}{8h^5\left(cg^2\right.}$$

$$= -\frac{c\left(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4\left(13fg^2+3dh^2\right)+h\right.}{8h^5\left(cg^2\right.}$$

Mathematica [A] time = 2.27, size = 639, normalized size = 1.26

$$\frac{\frac{15c^2\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(-3a^3h^6(eh-6fg)+a^2cgh^4\left(35fg^2-3dh^2\right)+28ac^2fg^5h^2+8c^3fg^7\right)}{\left(ah^2+cg^2\right)^{7/2}} - \frac{15c^2\log(g+hx)\left(-3a^3h^6(eh-6fg)+a^2cgh\right.}{\left(ah^2+cg\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] (-((h*Sqrt[a + c*x^2]*(24*(c*g^2 + a*h^2)^4*(f*g^2 + h*(-(e*g) + d*h)) - 6*
(c*g^2 + a*h^2)^3*(21*c*f*g^3 + c*g*h*(-16*e*g + 11*d*h) - 5*a*h^2*(-2*f*g
+ e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^2*(20*a^2*f*h^4 + c^2*(137*f*g^4 + 9*
g^2*h*(-8*e*g + 3*d*h)) + a*c*h^2*(154*f*g^2 + 3*h*(-23*e*g + 8*d*h)))*(g +
 h*x)^2 - c*(c*g^2 + a*h^2)*(5*a^2*h^4*(58*f*g - 15*e*h) + c^2*(326*f*g^5 +
 6*g^3*h*(-16*e*g + d*h)) + a*c*g*h^2*(631*f*g^2 + 3*h*(-62*e*g + 7*d*h)))*
(g + h*x)^3 + c*(160*a^3*f*h^6 + c^3*(274*f*g^6 - 6*g^4*h*(4*e*g + d*h)) +
3*a^2*c*h^4*(238*f*g^2 + h*(-33*e*g + 8*d*h)) + 3*a*c^2*g^2*h^2*(261*f*g^2
- h*(26*e*g + 9*d*h)))*(g + h*x)^4))/((c*g^2 + a*h^2)^3*(g + h*x)^5)) - (15
*c^2*(8*c^3*f*g^7 + 28*a*c^2*f*g^5*h^2 - 3*a^3*h^6*(-6*f*g + e*h) + a^2*c*g
*h^4*(35*f*g^2 - 3*d*h^2))*Log[g + h*x])/(c*g^2 + a*h^2)^(7/2) + 120*c^(3/2
)*f*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (15*c^2*(8*c^3*f*g^7 + 28*a*c^2*f*
g^5*h^2 - 3*a^3*h^6*(-6*f*g + e*h) + a^2*c*g*h^4*(35*f*g^2 - 3*d*h^2))*Log[
a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(7/2))/
(120*h^6)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.33, size = 4408, normalized size = 8.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] -1/4*(8*c^5*f*g^7 + 28*a*c^4*f*g^5*h^2 + 35*a^2*c^3*f*g^3*h^4 - 3*a^2*c^3*d
*g*h^6 + 18*a^3*c^2*f*g*h^6 - 3*a^3*c^2*h^7*e)*arctan(-((sqrt(c)*x - sqrt(c
*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^3*g^6*h^6 + 3*a*c^2*g^4
*h^8 + 3*a^2*c*g^2*h^10 + a^3*h^12)*sqrt(-c*g^2 - a*h^2)) - c^(3/2)*f*log(a
bs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^6 - 1/60*(600*(sqrt(c)*x - sqrt(c*x^2 +
 a))^9*c^5*f*g^7*h^4 + 1740*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*f*g^5*h^6
 + 1635*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*f*g^3*h^8 + 45*(sqrt(c)*x -
 sqrt(c*x^2 + a))^9*a^2*c^3*d*g*h^10 + 450*(sqrt(c)*x - sqrt(c*x^2 + a))^9*
a^3*c^2*f*g*h^10 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^5*g^6*h^5*e - 360*
(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*g^4*h^7*e - 360*(sqrt(c)*x - sqrt(c*x
^2 + a))^9*a^2*c^3*g^2*h^9*e - 75*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^2*h
^11*e + 3600*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*f*g^8*h^3 - 120*(sqrt
(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*d*g^6*h^5 + 10020*(sqrt(c)*x - sqrt(c*x
^2 + a))^8*a*c^(9/2)*f*g^6*h^5 - 360*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9
/2)*d*g^4*h^7 + 8595*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*f*g^4*h^7
+ 45*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*d*g^2*h^9 + 1530*(sqrt(c)*
x - sqrt(c*x^2 + a))^8*a^3*c^(5/2)*f*g^2*h^9 - 120*(sqrt(c)*x - sqrt(c*x^2
+ a))^8*a^3*c^(5/2)*d*h^11 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(3/2
)*f*h^11 - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*g^7*h^4*e - 1440*(s
qrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/2)*g^5*h^6*e - 1440*(sqrt(c)*x - sqrt(
c*x^2 + a))^8*a^2*c^(7/2)*g^3*h^8*e - 75*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^
3*c^(5/2)*g*h^10*e + 8800*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*f*g^9*h^2 - 2
40*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*d*g^7*h^4 + 21240*(sqrt(c)*x - sqrt(
c*x^2 + a))^7*a*c^5*f*g^7*h^4 - 720*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*d
*g^5*h^6 + 11670*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*f*g^5*h^6 + 690*(s
qrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*d*g^3*h^8 - 4970*(sqrt(c)*x - sqrt(c*
x^2 + a))^7*a^3*c^3*f*g^3*h^8 - 450*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3
*d*g*h^10 - 2580*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^2*f*g*h^10 - 960*(sq
rt(c)*x - sqrt(c*x^2 + a))^7*c^6*g^8*h^3*e - 2640*(sqrt(c)*x - sqrt(c*x^2 +
 a))^7*a*c^5*g^6*h^5*e - 2160*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*g^4*h
^7*e + 1170*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*g^2*h^9*e + 30*(sqrt(c)
*x - sqrt(c*x^2 + a))^7*a^4*c^2*h^11*e + 10000*(sqrt(c)*x - sqrt(c*x^2 + a)
)^6*c^(13/2)*f*g^10*h - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(13/2)*d*g^8*
h^3 + 14040*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*f*g^8*h^3 - 720*(sqr
t(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*d*g^6*h^5 - 14430*(sqrt(c)*x - sqrt(
c*x^2 + a))^6*a^2*c^(9/2)*f*g^6*h^5 + 1590*(sqrt(c)*x - sqrt(c*x^2 + a))^6*
a^2*c^(9/2)*d*g^4*h^7 - 28790*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*f
*g^4*h^7 - 1710*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*d*g^2*h^9 - 582
0*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*f*g^2*h^9 + 720*(sqrt(c)*x -
sqrt(c*x^2 + a))^6*a^5*c^(3/2)*f*h^11 - 960*(sqrt(c)*x - sqrt(c*x^2 + a))^6
*c^(13/2)*g^9*h^2*e - 1680*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*g^7*h
^4*e + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*g^5*h^6*e + 4950*(sq
rt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*g^3*h^8*e - 270*(sqrt(c)*x - sqrt(
c*x^2 + a))^6*a^4*c^(5/2)*g*h^10*e + 4384*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c
^7*f*g^11 - 96*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*d*g^9*h^2 - 9392*(sqrt(c
)*x - sqrt(c*x^2 + a))^5*a*c^6*f*g^9*h^2 + 48*(sqrt(c)*x - sqrt(c*x^2 + a))
^5*a*c^6*d*g^7*h^4 - 42996*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*f*g^7*h^
4 + 2364*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*d*g^5*h^6 - 31070*(sqrt(c)
*x - sqrt(c*x^2 + a))^5*a^3*c^4*f*g^5*h^6 - 2730*(sqrt(c)*x - sqrt(c*x^2 +
```

```
a))^5*a^3*c^4*d*g^3*h^8 + 8620*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*f*g^
3*h^8 + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*d*g*h^10 + 4800*(sqrt(c
)*x - sqrt(c*x^2 + a))^5*a^5*c^2*f*g*h^10 - 384*(sqrt(c)*x - sqrt(c*x^2 + a
))^5*c^7*g^10*h*e + 672*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^6*g^8*h^3*e + 3
936*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*g^6*h^5*e + 5580*(sqrt(c)*x - s
qrt(c*x^2 + a))^5*a^3*c^4*g^4*h^7*e - 2970*(sqrt(c)*x - sqrt(c*x^2 + a))^5*
a^4*c^3*g^2*h^9*e - 11920*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*f*g^10
*h + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*d*g^8*h^3 - 15720*(sqrt
(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*f*g^8*h^3 + 720*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a^2*c^(11/2)*d*g^6*h^5 + 19670*(sqrt(c)*x - sqrt(c*x^2 + a))^4
*a^3*c^(9/2)*f*g^6*h^5 - 3510*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*d
*g^4*h^7 + 36260*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*f*g^4*h^7 + 14
40*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*d*g^2*h^9 + 6240*(sqrt(c)*x
- sqrt(c*x^2 + a))^4*a^5*c^(5/2)*f*g^2*h^9 - 240*(sqrt(c)*x - sqrt(c*x^2 +
a))^4*a^5*c^(5/2)*d*h^11 - 880*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(3/2)*
f*h^11 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*g^9*h^2*e + 1680*(s
qrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*g^7*h^4*e - 480*(sqrt(c)*x - sqr
t(c*x^2 + a))^4*a^3*c^(9/2)*g^5*h^6*e - 6150*(sqrt(c)*x - sqrt(c*x^2 + a))^
4*a^4*c^(7/2)*g^3*h^8*e + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2)*g
*h^10*e + 13120*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*f*g^9*h^2 - 240*(sq
rt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*d*g^7*h^4 + 30440*(sqrt(c)*x - sqrt(c*
x^2 + a))^3*a^3*c^5*f*g^7*h^4 - 1200*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^
5*d*g^5*h^6 + 14130*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*f*g^5*h^6 + 231
0*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*d*g^3*h^8 - 10790*(sqrt(c)*x - sq
rt(c*x^2 + a))^3*a^5*c^3*f*g^3*h^8 - 510*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^
5*c^3*d*g*h^10 - 3820*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^2*f*g*h^10 - 96
0*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*g^8*h^3*e - 2640*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*a^3*c^5*g^6*h^5*e - 2640*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^
4*c^4*g^4*h^7*e + 2790*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^3*g^2*h^9*e -
30*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^2*h^11*e - 7360*(sqrt(c)*x - sqrt(
c*x^2 + a))^2*a^3*c^(11/2)*f*g^8*h^3 + 120*(sqrt(c)*x - sqrt(c*x^2 + a))^2*
a^3*c^(11/2)*d*g^6*h^5 - 19930*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*
f*g^6*h^5 + 690*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*d*g^4*h^7 - 160
50*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(7/2)*f*g^4*h^7 - 1050*(sqrt(c)*x
- sqrt(c*x^2 + a))^2*a^5*c^(7/2)*d*g^2*h^9 - 1300*(sqrt(c)*x - sqrt(c*x^2 +
 a))^2*a^6*c^(5/2)*f*g^2*h^9 + 560*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(3
/2)*f*h^11 + 480*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(11/2)*g^7*h^4*e + 1
560*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*g^5*h^6*e + 2130*(sqrt(c)*x
 - sqrt(c*x^2 + a))^2*a^5*c^(7/2)*g^3*h^8*e - 570*(sqrt(c)*x - sqrt(c*x^2 +
 a))^2*a^6*c^(5/2)*g*h^10*e + 2140*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c^5*f*
g^7*h^4 - 60*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c^5*d*g^5*h^6 + 6090*(sqrt(c
)*x - sqrt(c*x^2 + a))*a^5*c^4*f*g^5*h^6 - 270*(sqrt(c)*x - sqrt(c*x^2 + a)
)*a^5*c^4*d*g^3*h^8 + 5505*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*f*g^3*h^8
+ 195*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*d*g*h^10 + 1150*(sqrt(c)*x - sq
rt(c*x^2 + a))*a^7*c^2*f*g*h^10 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c^5
*g^6*h^5*e - 420*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^4*g^4*h^7*e - 630*(sqr
t(c)*x - sqrt(c*x^2 + a))*a^6*c^3*g^2*h^9*e + 75*(sqrt(c)*x - sqrt(c*x^2 +
a))*a^7*c^2*h^11*e - 274*a^5*c^(9/2)*f*g^6*h^5 + 6*a^5*c^(9/2)*d*g^4*h^7 -
783*a^6*c^(7/2)*f*g^4*h^7 + 27*a^6*c^(7/2)*d*g^2*h^9 - 714*a^7*c^(5/2)*f*g^
2*h^9 - 24*a^7*c^(5/2)*d*h^11 - 160*a^8*c^(3/2)*f*h^11 + 24*a^5*c^(9/2)*g^5
*h^6*e + 78*a^6*c^(7/2)*g^3*h^8*e + 99*a^7*c^(5/2)*g*h^10*e)/((c^3*g^6*h^6
+ 3*a*c^2*g^4*h^8 + 3*a^2*c*g^2*h^10 + a^3*h^12)*((sqrt(c)*x - sqrt(c*x^2 +
 a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^5)
```

maple [B] time = 0.03, size = 14169, normalized size = 27.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x)
```

[Out] result too large to display

maxima [B] time = 1.89, size = 6650, normalized size = 13.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(c*x^2 + a)*c^5*f*g^7/(c^4*g^8*h^5 + 4*a*c^3*g^6*h^7 + 6*a^2*c^2*g^
4*h^9 + 4*a^3*c*g^2*h^11 + a^4*h^13) - 3/8*sqrt(c*x^2 + a)*c^5*f*g^6*x/(c^4
*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^1
2) - 3/8*sqrt(c*x^2 + a)*c^5*e*g^6/(c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c
^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^12) + 1/8*(c*x^2 + a)^(3/2)*c^4*f*g^6
/(c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x + 6*a^2*c^2*g^4*h^8*x + 4*a^3*c*g^2*h^1
0*x + a^4*h^12*x + c^4*g^9*h^3 + 4*a*c^3*g^7*h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^
3*c*g^3*h^9 + a^4*g*h^11) + 3/8*sqrt(c*x^2 + a)*c^5*e*g^5*x/(c^4*g^8*h^3 +
4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) + 3/8*sqr
t(c*x^2 + a)*c^5*d*g^5/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 +
 4*a^3*c*g^2*h^9 + a^4*h^11) - 1/8*(c*x^2 + a)^(3/2)*c^4*e*g^5/(c^4*g^8*h^3
*x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^4*h^7*x + 4*a^3*c*g^2*h^9*x + a^4*h^11
*x + c^4*g^9*h^2 + 4*a*c^3*g^7*h^4 + 6*a^2*c^2*g^5*h^6 + 4*a^3*c*g^3*h^8 +
a^4*g*h^10) - 1/8*(c*x^2 + a)^(5/2)*c^3*f*g^5/(c^4*g^8*h^3*x^2 + 4*a*c^3*g^
6*h^5*x^2 + 6*a^2*c^2*g^4*h^7*x^2 + 4*a^3*c*g^2*h^9*x^2 + a^4*h^11*x^2 + 2*
c^4*g^9*h^2*x + 8*a*c^3*g^7*h^4*x + 12*a^2*c^2*g^5*h^6*x + 8*a^3*c*g^3*h^8*
x + 2*a^4*g*h^10*x + c^4*g^10*h + 4*a*c^3*g^8*h^3 + 6*a^2*c^2*g^6*h^5 + 4*a
^3*c*g^4*h^7 + a^4*g^2*h^9) + 1/8*(c*x^2 + a)^(3/2)*c^4*f*g^5/(c^4*g^8*h^3
+ 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) - 3/8*s
qrt(c*x^2 + a)*c^5*d*g^4*x/(c^4*g^8*h^2 + 4*a*c^3*g^6*h^4 + 6*a^2*c^2*g^4*h
^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) + 1/8*(c*x^2 + a)^(3/2)*c^4*d*g^4/(c^4*g^8
*h^2*x + 4*a*c^3*g^6*h^4*x + 6*a^2*c^2*g^4*h^6*x + 4*a^3*c*g^2*h^8*x + a^4*
h^10*x + c^4*g^9*h + 4*a*c^3*g^7*h^3 + 6*a^2*c^2*g^5*h^5 + 4*a^3*c*g^3*h^7
+ a^4*g*h^9) + 1/8*(c*x^2 + a)^(5/2)*c^3*e*g^4/(c^4*g^8*h^2*x^2 + 4*a*c^3*g
^6*h^4*x^2 + 6*a^2*c^2*g^4*h^6*x^2 + 4*a^3*c*g^2*h^8*x^2 + a^4*h^10*x^2 + 2
*c^4*g^9*h*x + 8*a*c^3*g^7*h^3*x + 12*a^2*c^2*g^5*h^5*x + 8*a^3*c*g^3*h^7*x
 + 2*a^4*g*h^9*x + c^4*g^10 + 4*a*c^3*g^8*h^2 + 6*a^2*c^2*g^6*h^4 + 4*a^3*c
*g^4*h^6 + a^4*g^2*h^8) - 1/8*(c*x^2 + a)^(3/2)*c^4*e*g^4/(c^4*g^8*h^2 + 4*
a*c^3*g^6*h^4 + 6*a^2*c^2*g^4*h^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) - 3/2*sqrt(
c*x^2 + a)*c^4*f*g^5/(c^3*g^6*h^5 + 3*a*c^2*g^4*h^7 + 3*a^2*c*g^2*h^9 + a^3
*h^11) + 9/8*sqrt(c*x^2 + a)*c^4*f*g^4*x/(c^3*g^6*h^4 + 3*a*c^2*g^4*h^6 + 3
*a^2*c*g^2*h^8 + a^3*h^10) - 1/8*(c*x^2 + a)^(5/2)*c^3*d*g^3/(c^4*g^8*h*x^2
 + 4*a*c^3*g^6*h^3*x^2 + 6*a^2*c^2*g^4*h^5*x^2 + 4*a^3*c*g^2*h^7*x^2 + a^4*
h^9*x^2 + 2*c^4*g^9*x + 8*a*c^3*g^7*h^2*x + 12*a^2*c^2*g^5*h^4*x + 8*a^3*c*
g^3*h^6*x + 2*a^4*g*h^8*x + c^4*g^10/h + 4*a*c^3*g^8*h + 6*a^2*c^2*g^6*h^3
+ 4*a^3*c*g^4*h^5 + a^4*g^2*h^7) + 1/8*(c*x^2 + a)^(3/2)*c^4*d*g^3/(c^4*g^8
*h + 4*a*c^3*g^6*h^3 + 6*a^2*c^2*g^4*h^5 + 4*a^3*c*g^2*h^7 + a^4*h^9) + 9/8
*sqrt(c*x^2 + a)*c^4*e*g^4/(c^3*g^6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2*c*g^2*h^8
 + a^3*h^10) - 1/4*(c*x^2 + a)^(5/2)*c^2*f*g^4/(c^3*g^6*h^4*x^3 + 3*a*c^2*g
^4*h^6*x^3 + 3*a^2*c*g^2*h^8*x^3 + a^3*h^10*x^3 + 3*c^3*g^7*h^3*x^2 + 9*a*c
^2*g^5*h^5*x^2 + 9*a^2*c*g^3*h^7*x^2 + 3*a^3*g*h^9*x^2 + 3*c^3*g^8*h^2*x +
9*a*c^2*g^6*h^4*x + 9*a^2*c*g^4*h^6*x + 3*a^3*g^2*h^8*x + c^3*g^9*h + 3*a*c
^2*g^7*h^3 + 3*a^2*c*g^5*h^5 + a^3*g^3*h^7) - 5/8*(c*x^2 + a)^(3/2)*c^3*f*g
^4/(c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h^10*x + c^
3*g^7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 + a^3*g*h^9) - 3/4*sqrt(c*x^2
 + a)*c^4*e*g^3*x/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^
9) - 3/4*sqrt(c*x^2 + a)*c^4*d*g^3/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c
*g^2*h^7 + a^3*h^9) + 1/4*(c*x^2 + a)^(5/2)*c^2*e*g^3/(c^3*g^6*h^3*x^3 + 3*
a*c^2*g^4*h^5*x^3 + 3*a^2*c*g^2*h^7*x^3 + a^3*h^9*x^3 + 3*c^3*g^7*h^2*x^2 +
 9*a*c^2*g^5*h^4*x^2 + 9*a^2*c*g^3*h^6*x^2 + 3*a^3*g*h^8*x^2 + 3*c^3*g^8*h*
x + 9*a*c^2*g^6*h^3*x + 9*a^2*c*g^4*h^5*x + 3*a^3*g^2*h^7*x + c^3*g^9 + 3*a
```

```
*c^2*g^7*h^2 + 3*a^2*c*g^5*h^4 + a^3*g^3*h^6) + 1/2*(c*x^2 + a)^(3/2)*c^3*e
*g^3/(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c
^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2*c*g^3*h^6 + a^3*g*h^8) + 1/8*(c*x^2 +
a)^(5/2)*c^2*f*g^3/(c^3*g^6*h^3*x^2 + 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7
*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*
x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2
*h^7) - 1/8*(c*x^2 + a)^(3/2)*c^3*f*g^3/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*
a^2*c*g^2*h^7 + a^3*h^9) + 3/8*sqrt(c*x^2 + a)*c^4*d*g^2*x/(c^3*g^6*h^2 + 3
*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) - 1/4*(c*x^2 + a)^(5/2)*c^2*d*g
^2/(c^3*g^6*h^2*x^3 + 3*a*c^2*g^4*h^4*x^3 + 3*a^2*c*g^2*h^6*x^3 + a^3*h^8*x
^3 + 3*c^3*g^7*h*x^2 + 9*a*c^2*g^5*h^3*x^2 + 9*a^2*c*g^3*h^5*x^2 + 3*a^3*g*
h^7*x^2 + 3*c^3*g^8*x + 9*a*c^2*g^6*h^2*x + 9*a^2*c*g^4*h^4*x + 3*a^3*g^2*h
^6*x + c^3*g^9/h + 3*a*c^2*g^7*h + 3*a^2*c*g^5*h^3 + a^3*g^3*h^5) - 3/8*(c*
x^2 + a)^(3/2)*c^3*d*g^2/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g^2*h
^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^
7) - 1/4*(c*x^2 + a)^(5/2)*c*f*g^3/(c^2*g^4*h^5*x^4 + 2*a*c*g^2*h^7*x^4 + a
^2*h^9*x^4 + 4*c^2*g^5*h^4*x^3 + 8*a*c*g^3*h^6*x^3 + 4*a^2*g*h^8*x^3 + 6*c^
2*g^6*h^3*x^2 + 12*a*c*g^4*h^5*x^2 + 6*a^2*g^2*h^7*x^2 + 4*c^2*g^7*h^2*x +
8*a*c*g^5*h^4*x + 4*a^2*g^3*h^6*x + c^2*g^8*h + 2*a*c*g^6*h^3 + a^2*g^4*h^5
) + 19/8*sqrt(c*x^2 + a)*c^3*f*g^3/(c^2*g^4*h^5 + 2*a*c*g^2*h^7 + a^2*h^9)
- 5/4*sqrt(c*x^2 + a)*c^3*f*g^2*x/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) -
 1/8*(c*x^2 + a)^(5/2)*c^2*d*g/(c^3*g^6*h*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^2
*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g^
3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^3
*g^2*h^5) + 1/8*(c*x^2 + a)^(3/2)*c^3*d*g/(c^3*g^6*h + 3*a*c^2*g^4*h^3 + 3*
a^2*c*g^2*h^5 + a^3*h^7) + 1/4*(c*x^2 + a)^(5/2)*c*e*g^2/(c^2*g^4*h^4*x^4 +
 2*a*c*g^2*h^6*x^4 + a^2*h^8*x^4 + 4*c^2*g^5*h^3*x^3 + 8*a*c*g^3*h^5*x^3 +
4*a^2*g*h^7*x^3 + 6*c^2*g^6*h^2*x^2 + 12*a*c*g^4*h^4*x^2 + 6*a^2*g^2*h^6*x^
2 + 4*c^2*g^7*h*x + 8*a*c*g^5*h^3*x + 4*a^2*g^3*h^5*x + c^2*g^8 + 2*a*c*g^6
*h^2 + a^2*g^4*h^4) - 9/8*sqrt(c*x^2 + a)*c^3*e*g^2/(c^2*g^4*h^4 + 2*a*c*g^
2*h^6 + a^2*h^8) + 1/2*(c*x^2 + a)^(5/2)*c*f*g^2/(c^2*g^4*h^4*x^3 + 2*a*c*g
^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a^2*g*
h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^7*h +
 2*a*c*g^5*h^3 + a^2*g^3*h^5) + 11/12*(c*x^2 + a)^(3/2)*c^2*f*g^2/(c^2*g^4*
h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h
^7) + 3/8*sqrt(c*x^2 + a)*c^3*e*g*x/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7)
 - 1/4*(c*x^2 + a)^(5/2)*c*d*g/(c^2*g^4*h^3*x^4 + 2*a*c*g^2*h^5*x^4 + a^2*h
^7*x^4 + 4*c^2*g^5*h^2*x^3 + 8*a*c*g^3*h^4*x^3 + 4*a^2*g*h^6*x^3 + 6*c^2*g^
6*h*x^2 + 12*a*c*g^4*h^3*x^2 + 6*a^2*g^2*h^5*x^2 + 4*c^2*g^7*x + 8*a*c*g^5*
h^2*x + 4*a^2*g^3*h^4*x + c^2*g^8/h + 2*a*c*g^6*h + a^2*g^4*h^3) + 3/8*sqrt
(c*x^2 + a)*c^3*d*g/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/4*(c*x^2 +
a)^(5/2)*c*e*g/(c^2*g^4*h^3*x^3 + 2*a*c*g^2*h^5*x^3 + a^2*h^7*x^3 + 3*c^2*g
^5*h^2*x^2 + 6*a*c*g^3*h^4*x^2 + 3*a^2*g*h^6*x^2 + 3*c^2*g^6*h*x + 6*a*c*g^
4*h^3*x + 3*a^2*g^2*h^5*x + c^2*g^7 + 2*a*c*g^5*h^2 + a^2*g^3*h^4) - 3/8*(c
*x^2 + a)^(3/2)*c^2*e*g/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*
g^5*h^2 + 2*a*c*g^3*h^4 + a^2*g*h^6) + 1/12*(c*x^2 + a)^(5/2)*c*f*g/(c^2*g^
4*h^3*x^2 + 2*a*c*g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h
^4*x + 2*a^2*g*h^6*x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) - 1/12*(c*x
^2 + a)^(3/2)*c^2*f*g/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/5*(c*x^2
+ a)^(5/2)*f*g^2/(c*g^2*h^6*x^5 + a*h^8*x^5 + 5*c*g^3*h^5*x^4 + 5*a*g*h^7*x
^4 + 10*c*g^4*h^4*x^3 + 10*a*g^2*h^6*x^3 + 10*c*g^5*h^3*x^2 + 10*a*g^3*h^5*
x^2 + 5*c*g^6*h^2*x + 5*a*g^4*h^4*x + c*g^7*h + a*g^5*h^3) - 1/8*(c*x^2 + a
)^(5/2)*c*e/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2 + a^2*h^6*x^2 + 2*c^2*g^5*
h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6 + 2*a*c*g^4*h^2 + a^2*g^2*h
^4) + 1/8*(c*x^2 + a)^(3/2)*c^2*e/(c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) +
 1/5*(c*x^2 + a)^(5/2)*e*g/(c*g^2*h^5*x^5 + a*h^7*x^5 + 5*c*g^3*h^4*x^4 + 5
*a*g*h^6*x^4 + 10*c*g^4*h^3*x^3 + 10*a*g^2*h^5*x^3 + 10*c*g^5*h^2*x^2 + 10*
a*g^3*h^4*x^2 + 5*c*g^6*h*x + 5*a*g^4*h^3*x + c*g^7 + a*g^5*h^2) + 1/2*(c*x
^2 + a)^(5/2)*f*g/(c*g^2*h^5*x^4 + a*h^7*x^4 + 4*c*g^3*h^4*x^3 + 4*a*g*h^6*
```

```
x^3 + 6*c*g^4*h^3*x^2 + 6*a*g^2*h^5*x^2 + 4*c*g^5*h^2*x + 4*a*g^3*h^4*x + c
*g^6*h + a*g^4*h^3) - 9/4*sqrt(c*x^2 + a)*c^2*f*g/(c*g^2*h^5 + a*h^7) + sqr
t(c*x^2 + a)*c^2*f*x/(c*g^2*h^4 + a*h^6) - 1/5*(c*x^2 + a)^(5/2)*d/(c*g^2*h
^4*x^5 + a*h^6*x^5 + 5*c*g^3*h^3*x^4 + 5*a*g*h^5*x^4 + 10*c*g^4*h^2*x^3 + 1
0*a*g^2*h^4*x^3 + 10*c*g^5*h*x^2 + 10*a*g^3*h^3*x^2 + 5*c*g^6*x + 5*a*g^4*h
^2*x + c*g^7/h + a*g^5*h) - 1/4*(c*x^2 + a)^(5/2)*e/(c*g^2*h^4*x^4 + a*h^6*
x^4 + 4*c*g^3*h^3*x^3 + 4*a*g*h^5*x^3 + 6*c*g^4*h^2*x^2 + 6*a*g^2*h^4*x^2 +
 4*c*g^5*h*x + 4*a*g^3*h^3*x + c*g^6 + a*g^4*h^2) + 3/8*sqrt(c*x^2 + a)*c^2
*e/(c*g^2*h^4 + a*h^6) - 1/3*(c*x^2 + a)^(5/2)*f/(c*g^2*h^4*x^3 + a*h^6*x^3
 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c*g^5*
h + a*g^3*h^3) - 2/3*(c*x^2 + a)^(3/2)*c*f/(c*g^2*h^4*x + a*h^6*x + c*g^3*h
^3 + a*g*h^5) + c^(3/2)*f*arcsinh(c*x/sqrt(a*c))/h^6 + 3/8*c^5*f*g^7*arcsin
h(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^
2/h^2)^(7/2)*h^13) - 3/8*c^5*e*g^6*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
 a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^12) + 3/8*c^5*d*g^5
*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a
 + c*g^2/h^2)^(7/2)*h^11) - 3/2*c^4*f*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x
+ g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^11) + 9/8*c^
4*e*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g
)))/((a + c*g^2/h^2)^(5/2)*h^10) - 3/4*c^4*d*g^3*arcsinh(c*g*x/(sqrt(a*c)*a
bs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^9) +
19/8*c^3*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(
h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^9) - 9/8*c^3*e*g^2*arcsinh(c*g*x/(sqrt(
a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h
^8) + 3/8*c^3*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*a
bs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) - 9/4*c^2*f*g*arcsinh(c*g*x/(sqrt
(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^
7) + 3/8*c^2*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(
h*x + g)))/(sqrt(a + c*g^2/h^2)*h^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**6,x)
[Out] Timed out
```

3.98 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx$

Optimal. Leaf size=404

$$-\frac{(a+cx^2)^{3/2}(ah-cgx)\left(6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)+6c^2dg^2\right)}{24(g+hx)^4\left(ah^2+cg^2\right)^3}-\frac{ac\sqrt{a+cx^2}\,(ah-cgx)\left(6a^2fh^2-ac\left(fg\right.\right.}{16(g+hx)^2\left(ah^2+\right.}$$

[Out] -1/24*(6*c^2*d*g^2+6*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+7*e*g)))*(-c*g*x+a*h)*(c*x^2+a)^(3/2)/(a*h^2+c*g^2)^3/(h*x+g)^4-1/6*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^6+1/30*(6*a*h^2*(-e*h+2*f*g)+c*g*(5*f*g^2+h*(-7*d*h+e*g)))*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)^2/(h*x+g)^5-1/16*a^2*c^2*(6*c^2*d*g^2+6*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+7*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(9/2)-1/16*a*c*(6*c^2*d*g^2+6*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+7*e*g)))*(-c*g*x+a*h)*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^4/(h*x+g)^2

Rubi [A] time = 0.55, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1651, 807, 721, 725, 206}

$$-\frac{(a+cx^2)^{3/2}(ah-cgx)\left(6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)+6c^2dg^2\right)}{24(g+hx)^4\left(ah^2+cg^2\right)^3}-\frac{ac\sqrt{a+cx^2}\,(ah-cgx)\left(6a^2fh^2-ac\left(fg\right.\right.}{16(g+hx)^2\left(ah^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] -(a*c*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*(a*h - c*g*x)*Sqrt[a + c*x^2])/(16*(c*g^2 + a*h^2)^4*(g + h*x)^2) - ((6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*(a*h - c*g*x)*(a + c*x^2)^(3/2))/(24*(c*g^2 + a*h^2)^3*(g + h*x)^4) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(6*h*(c*g^2 + a*h^2)*(g + h*x)^6) + ((5*c*f*g^3 + c*g*h*(e*g - 7*d*h) + 6*a*h^2*(2*f*g - e*h))*(a + c*x^2)^(5/2))/(30*h*(c*g^2 + a*h^2)^2*(g + h*x)^5) - (a^2*c^2*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(16*(c*g^2 + a*h^2)^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 + a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^7}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{6h\left(cg^2+ah^2\right)(g+hx)^6} - \frac{\int \frac{\left(-6(cdg-afg+aeh)-\left(6afh+c\left(eg+\frac{5fg^2}{h}-dh\right)\right)x\right)\left(a+c\right.}{(g+hx)^6}}{6\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{6h\left(cg^2+ah^2\right)(g+hx)^6} + \frac{\left(5cfg^3+cgh(eg-7dh)+6ah^2(2fg-eh)\right.}{30h\left(cg^2+ah^2\right)^2(g+hx)^5}$$

$$= -\frac{\left(6c^2dg^2+6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)\right)(ah-cgx)\left(a+cx^2\right)^{3/2}}{24\left(cg^2+ah^2\right)^3(g+hx)^4} - \frac{\left(fg^2\right.}{6h}$$

$$= -\frac{ac\left(6c^2dg^2+6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{16\left(cg^2+ah^2\right)^4(g+hx)^2} - \frac{\left(6c^2d\right.}{}$$

$$= -\frac{ac\left(6c^2dg^2+6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{16\left(cg^2+ah^2\right)^4(g+hx)^2} - \frac{\left(6c^2d\right.}{}$$

$$= -\frac{ac\left(6c^2dg^2+6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{16\left(cg^2+ah^2\right)^4(g+hx)^2} - \frac{\left(6c^2d\right.}{}$$

Mathematica [A] time = 2.47, size = 696, normalized size = 1.72

$$\frac{1}{240}\left(-\frac{15a^2c^2\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(6a^2fh^2-ac\left(h(dh-7eg)+fg^2\right)+6c^2dg^2\right)}{\left(ah^2+cg^2\right)^{9/2}} + \frac{15a^2c^2\log(g}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] (-((Sqrt[a + c*x^2]*(40*(c*g^2 + a*h^2)^5*(f*g^2 + h*(-(e*g) + d*h)) - 8*(c
*g^2 + a*h^2)^4*(25*c*f*g^3 + c*g*h*(-19*e*g + 13*d*h) - 6*a*h^2*(-2*f*g +
e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^3*(30*a^2*f*h^4 + 2*c^2*(100*f*g^4 + g^
2*h*(-52*e*g + 19*d*h)) + a*c*h^2*(227*f*g^2 + h*(-101*e*g + 35*d*h)))*(g +
 h*x)^2 - 2*c*(c*g^2 + a*h^2)^2*(6*a^2*h^4*(31*f*g - 8*e*h) + 2*c^2*(100*f*
g^5 + g^3*h*(-28*e*g + d*h)) + 3*a*c*g*h^2*(131*f*g^2 + h*(-37*e*g + 3*d*h)
))*(g + h*x)^3 + c*(c*g^2 + a*h^2)*(150*a^3*f*h^6 + 4*c^3*(50*f*g^6 - g^4*h
*(2*e*g + d*h)) + 6*a*c^2*g^2*h^2*(99*f*g^2 - h*(5*e*g + 4*d*h)) + 3*a^2*c*
h^4*(193*f*g^2 + h*(-19*e*g + 5*d*h)))*(g + h*x)^4 - c^2*(6*a^3*h^6*(41*f*g
 - 8*e*h) + 3*a^2*c*g*h^4*(89*f*g^2 + h*(29*e*g - 27*d*h)) + 4*c^3*(10*f*g^
7 + g^5*h*(2*e*g + d*h)) + 2*a*c^2*g^3*h^2*(83*f*g^2 + h*(19*e*g + 14*d*h))
)*(g + h*x)^5))/(h^5*(c*g^2 + a*h^2)^4*(g + h*x)^6)) + (15*a^2*c^2*(6*c^2*d
*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 + h*(-7*e*g + d*h)))*Log[g + h*x])/(c*g^2 +
 a*h^2)^(9/2) - (15*a^2*c^2*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 + h*(-7
*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^
2 + a*h^2)^(9/2))/240
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.12, size = 6122, normalized size = 15.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="giac")
```

```
[Out] 1/8*(6*a^2*c^4*d*g^2 - a^3*c^3*f*g^2 - a^3*c^3*d*h^2 + 6*a^4*c^2*f*h^2 + 7*
a^3*c^3*g*h*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-
c*g^2 - a*h^2))/((c^4*g^8 + 4*a*c^3*g^6*h^2 + 6*a^2*c^2*g^4*h^4 + 4*a^3*c*g
^2*h^6 + a^4*h^8)*sqrt(-c*g^2 - a*h^2)) + 1/120*(240*(sqrt(c)*x - sqrt(c*x^
2 + a))^11*c^6*f*g^8*h^5 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a*c^5*f*g^6
*h^7 + 1440*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^2*c^4*f*g^4*h^9 - 90*(sqrt(c
)*x - sqrt(c*x^2 + a))^11*a^2*c^4*d*g^2*h^11 + 975*(sqrt(c)*x - sqrt(c*x^2
+ a))^11*a^3*c^3*f*g^2*h^11 + 15*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^3*d
*h^13 + 150*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^4*c^2*f*h^13 - 105*(sqrt(c)*
x - sqrt(c*x^2 + a))^11*a^3*c^3*g*h^12*e + 1200*(sqrt(c)*x - sqrt(c*x^2 + a
))^10*c^(13/2)*f*g^9*h^4 + 4800*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(11/2)
*f*g^7*h^6 + 7200*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(9/2)*f*g^5*h^8 -
990*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(9/2)*d*g^3*h^10 + 4965*(sqrt(c)
*x - sqrt(c*x^2 + a))^10*a^3*c^(7/2)*f*g^3*h^10 + 165*(sqrt(c)*x - sqrt(c*x
^2 + a))^10*a^3*c^(7/2)*d*g*h^12 + 210*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^4
*c^(5/2)*f*g*h^12 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^10*c^(13/2)*g^8*h^5*e
 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(11/2)*g^6*h^7*e + 1440*(sqrt(c
)*x - sqrt(c*x^2 + a))^10*a^2*c^(9/2)*g^4*h^9*e - 195*(sqrt(c)*x - sqrt(c*x
^2 + a))^10*a^3*c^(7/2)*g^2*h^11*e + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a
^4*c^(5/2)*h^13*e + 3200*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^7*f*g^10*h^3 + 3
20*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^7*d*g^8*h^5 + 12080*(sqrt(c)*x - sqrt(
c*x^2 + a))^9*a*c^6*f*g^8*h^5 + 1280*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^6*
d*g^6*h^7 + 16320*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^5*f*g^6*h^7 - 2520*
(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^5*d*g^4*h^9 + 9220*(sqrt(c)*x - sqrt(
c*x^2 + a))^9*a^3*c^4*f*g^4*h^9 + 2530*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*
c^4*d*g^2*h^11 - 4205*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^3*f*g^2*h^11 +
235*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^3*d*h^13 - 210*(sqrt(c)*x - sqrt(
```

c*x^2 + a))^9*a^5*c^2*f*h^13 + 640*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^7*g^9*
h^4*e + 2560*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^6*g^7*h^6*e + 3840*(sqrt(c
)*x - sqrt(c*x^2 + a))^9*a^2*c^5*g^5*h^8*e - 2620*(sqrt(c)*x - sqrt(c*x^2 +
 a))^9*a^3*c^4*g^3*h^10*e + 1235*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^3*g*
h^12*e + 4800*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(15/2)*f*g^11*h^2 + 480*(sq
rt(c)*x - sqrt(c*x^2 + a))^8*c^(15/2)*d*g^9*h^4 + 15120*(sqrt(c)*x - sqrt(c
*x^2 + a))^8*a*c^(13/2)*f*g^9*h^4 + 1920*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*
c^(13/2)*d*g^7*h^6 + 12480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(11/2)*f*g
^7*h^6 - 7380*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(11/2)*d*g^5*h^8 - 3570
*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(9/2)*f*g^5*h^8 + 8220*(sqrt(c)*x -
sqrt(c*x^2 + a))^8*a^3*c^(9/2)*d*g^3*h^10 - 22545*(sqrt(c)*x - sqrt(c*x^2 +
 a))^8*a^4*c^(7/2)*f*g^3*h^10 - 285*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(
7/2)*d*g*h^12 + 510*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(5/2)*f*g*h^12 +
960*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(15/2)*g^10*h^3*e + 3600*(sqrt(c)*x -
 sqrt(c*x^2 + a))^8*a*c^(13/2)*g^8*h^5*e + 4800*(sqrt(c)*x - sqrt(c*x^2 + a
))^8*a^2*c^(11/2)*g^6*h^7*e - 9570*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(9
/2)*g^4*h^9*e + 5355*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(7/2)*g^2*h^11*e
 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(5/2)*h^13*e + 3840*(sqrt(c)*x
 - sqrt(c*x^2 + a))^7*c^8*f*g^12*h + 384*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^
8*d*g^10*h^3 + 6336*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^7*f*g^10*h^3 + 1728
*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^7*d*g^8*h^5 - 11808*(sqrt(c)*x - sqrt(
c*x^2 + a))^7*a^2*c^6*f*g^8*h^5 - 9456*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*
c^6*d*g^6*h^7 - 31704*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^5*f*g^6*h^7 + 2
0760*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^5*d*g^4*h^9 - 39960*(sqrt(c)*x -
 sqrt(c*x^2 + a))^7*a^4*c^4*f*g^4*h^9 - 2700*(sqrt(c)*x - sqrt(c*x^2 + a))^
7*a^4*c^4*d*g^2*h^11 + 12150*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^3*f*g^2*
h^11 + 390*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^3*d*h^13 + 60*(sqrt(c)*x -
 sqrt(c*x^2 + a))^7*a^6*c^2*f*h^13 + 768*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^
8*g^11*h^2*e + 1728*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^7*g^9*h^4*e - 768*(
sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^6*g^7*h^6*e - 19608*(sqrt(c)*x - sqrt(
c*x^2 + a))^7*a^3*c^5*g^5*h^8*e + 14040*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4
*c^4*g^3*h^10*e - 2730*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^3*g*h^12*e + 1
280*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(17/2)*f*g^13 + 128*(sqrt(c)*x - sqrt
(c*x^2 + a))^6*c^(17/2)*d*g^11*h^2 - 4288*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a
*c^(15/2)*f*g^11*h^2 - 64*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(15/2)*d*g^9*
h^4 - 24096*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(13/2)*f*g^9*h^4 - 8592*(
sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(13/2)*d*g^7*h^6 - 26728*(sqrt(c)*x -
sqrt(c*x^2 + a))^6*a^3*c^(11/2)*f*g^7*h^6 + 24440*(sqrt(c)*x - sqrt(c*x^2 +
 a))^6*a^3*c^(11/2)*d*g^5*h^8 - 12640*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c
^(9/2)*f*g^5*h^8 - 14860*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(9/2)*d*g^3*
h^10 + 41610*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(7/2)*f*g^3*h^10 + 810*(
sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(7/2)*d*g*h^12 - 2460*(sqrt(c)*x - sqr
t(c*x^2 + a))^6*a^6*c^(5/2)*f*g*h^12 + 256*(sqrt(c)*x - sqrt(c*x^2 + a))^6*
c^(17/2)*g^12*h*e - 704*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(15/2)*g^10*h^3
e - 4896(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(13/2)*g^8*h^5*e - 15656*(s
qrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(11/2)*g^6*h^7*e + 26800*(sqrt(c)*x - s
qrt(c*x^2 + a))^6*a^4*c^(9/2)*g^4*h^9*e - 9510*(sqrt(c)*x - sqrt(c*x^2 + a)
)^6*a^5*c^(7/2)*g^2*h^11*e + 480*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^6*c^(5/2
)*h^13*e - 3840*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^8*f*g^12*h - 384*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a*c^8*d*g^10*h^3 - 6336*(sqrt(c)*x - sqrt(c*x^2 +
 a))^5*a^2*c^7*f*g^10*h^3 - 1728*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^7*d*
g^8*h^5 + 11808*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^6*f*g^8*h^5 + 19056*(
sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^6*d*g^6*h^7 + 29304*(sqrt(c)*x - sqrt(
c*x^2 + a))^5*a^4*c^5*f*g^6*h^7 - 21480*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4
*c^5*d*g^4*h^9 + 46080*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^4*f*g^4*h^9 +
7020*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^4*d*g^2*h^11 - 17370*(sqrt(c)*x
 - sqrt(c*x^2 + a))^5*a^6*c^3*f*g^2*h^11 + 390*(sqrt(c)*x - sqrt(c*x^2 + a))
^5*a^6*c^3*d*h^13 + 60*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^2*f*h^13 - 768
*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^8*g^11*h^2*e - 1728*(sqrt(c)*x - sqrt(

c*x^2 + a))^5*a^2*c^7*g^9*h^4*e - 192*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c
^6*g^7*h^6*e + 26808*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^5*g^5*h^8*e - 19
440*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^4*g^3*h^10*e + 3030*(sqrt(c)*x -
sqrt(c*x^2 + a))^5*a^6*c^3*g*h^12*e + 4800*(sqrt(c)*x - sqrt(c*x^2 + a))^4*
a^2*c^(15/2)*f*g^11*h^2 + 480*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(15/2)*
d*g^9*h^4 + 15120*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(13/2)*f*g^9*h^4 +
3840*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(13/2)*d*g^7*h^6 + 12360*(sqrt(c
)*x - sqrt(c*x^2 + a))^4*a^4*c^(11/2)*f*g^7*h^6 - 18720*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a^4*c^(11/2)*d*g^5*h^8 + 1020*(sqrt(c)*x - sqrt(c*x^2 + a))^4*
a^5*c^(9/2)*f*g^5*h^8 + 11640*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(9/2)*d
*g^3*h^10 - 32490*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(7/2)*f*g^3*h^10 -
930*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(7/2)*d*g*h^12 + 3180*(sqrt(c)*x
- sqrt(c*x^2 + a))^4*a^7*c^(5/2)*f*g*h^12 + 960*(sqrt(c)*x - sqrt(c*x^2 + a
))^4*a^2*c^(15/2)*g^10*h^3*e + 3600*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(
13/2)*g^8*h^5*e + 7080*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(11/2)*g^6*h^7
e - 22260(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(9/2)*g^4*h^9*e + 7470*(sq
rt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(7/2)*g^2*h^11*e - 480*(sqrt(c)*x - sqrt
(c*x^2 + a))^4*a^7*c^(5/2)*h^13*e - 3200*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^
3*c^7*f*g^10*h^3 - 320*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^7*d*g^8*h^5 -
12080*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^6*f*g^8*h^5 - 2960*(sqrt(c)*x -
sqrt(c*x^2 + a))^3*a^4*c^6*d*g^6*h^7 - 16440*(sqrt(c)*x - sqrt(c*x^2 + a))
^3*a^5*c^5*f*g^6*h^7 + 12120*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^5*d*g^4*
h^9 - 14120*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^4*f*g^4*h^9 - 2330*(sqrt(
c)*x - sqrt(c*x^2 + a))^3*a^6*c^4*d*g^2*h^11 + 10555*(sqrt(c)*x - sqrt(c*x^
2 + a))^3*a^7*c^3*f*g^2*h^11 + 235*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^3*
d*h^13 - 210*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^8*c^2*f*h^13 - 640*(sqrt(c)*
x - sqrt(c*x^2 + a))^3*a^3*c^7*g^9*h^4*e - 3040*(sqrt(c)*x - sqrt(c*x^2 + a
))^3*a^4*c^6*g^7*h^6*e - 7800*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^5*g^5*h
^8*e + 10280*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^4*g^3*h^10*e - 1645*(sqr
t(c)*x - sqrt(c*x^2 + a))^3*a^7*c^3*g*h^12*e + 1200*(sqrt(c)*x - sqrt(c*x^2
+ a))^2*a^4*c^(13/2)*f*g^9*h^4 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c
^(13/2)*d*g^7*h^6 + 4920*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(11/2)*f*g^7
h^6 + 1656(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(11/2)*d*g^5*h^8 + 7824*(
sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(9/2)*f*g^5*h^8 - 4038*(sqrt(c)*x - sq
rt(c*x^2 + a))^2*a^6*c^(9/2)*d*g^3*h^10 + 8193*(sqrt(c)*x - sqrt(c*x^2 + a)
)^2*a^7*c^(7/2)*f*g^3*h^10 + 321*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(7/2
)*d*g*h^12 - 1686*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(5/2)*f*g*h^12 + 24
0*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(13/2)*g^8*h^5*e + 1272*(sqrt(c)*x
- sqrt(c*x^2 + a))^2*a^5*c^(11/2)*g^6*h^7*e + 3552*(sqrt(c)*x - sqrt(c*x^2
+ a))^2*a^6*c^(9/2)*g^4*h^9*e - 3207*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^
(7/2)*g^2*h^11*e + 48*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(5/2)*h^13*e -
240*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^6*f*g^8*h^5 - 48*(sqrt(c)*x - sqrt(
c*x^2 + a))*a^5*c^6*d*g^6*h^7 - 1032*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^5*
f*g^6*h^7 - 336*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^5*d*g^4*h^9 - 1764*(sqr
t(c)*x - sqrt(c*x^2 + a))*a^7*c^4*f*g^4*h^9 + 882*(sqrt(c)*x - sqrt(c*x^2 +
a))*a^7*c^4*d*g^2*h^11 - 1977*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^3*f*g^2*
h^11 + 15*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^3*d*h^13 + 150*(sqrt(c)*x - s
qrt(c*x^2 + a))*a^9*c^2*f*h^13 - 96*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^6*g
^7*h^6*e - 456*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^5*g^5*h^8*e - 1044*(sqrt
(c)*x - sqrt(c*x^2 + a))*a^7*c^4*g^3*h^10*e + 471*(sqrt(c)*x - sqrt(c*x^2 +
a))*a^8*c^3*g*h^12*e + 40*a^6*c^(11/2)*f*g^7*h^6 + 4*a^6*c^(11/2)*d*g^5*h^
8 + 166*a^7*c^(9/2)*f*g^5*h^8 + 28*a^7*c^(9/2)*d*g^3*h^10 + 267*a^8*c^(7/2)
*f*g^3*h^10 - 81*a^8*c^(7/2)*d*g*h^12 + 246*a^9*c^(5/2)*f*g*h^12 + 8*a^6*c^
(11/2)*g^6*h^7*e + 38*a^7*c^(9/2)*g^4*h^9*e + 87*a^8*c^(7/2)*g^2*h^11*e - 4
8*a^9*c^(5/2)*h^13*e)/((c^4*g^8*h^6 + 4*a*c^3*g^6*h^8 + 6*a^2*c^2*g^4*h^10
+ 4*a^3*c*g^2*h^12 + a^4*h^14)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt
(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^6)

maple [**B**] time = 0.04, size = 17026, normalized size = 42.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x)
```

```
[Out] result too large to display
```

maxima [**B**] time = 2.73, size = 10724, normalized size = 26.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="maxima")
```

```
[Out] 7/16*sqrt(c*x^2 + a)*c^6*f*g^8/(c^5*g^10*h^5 + 5*a*c^4*g^8*h^7 + 10*a^2*c^3
*g^6*h^9 + 10*a^3*c^2*g^4*h^11 + 5*a^4*c*g^2*h^13 + a^5*h^15) - 7/16*sqrt(c
*x^2 + a)*c^6*f*g^7*x/(c^5*g^10*h^4 + 5*a*c^4*g^8*h^6 + 10*a^2*c^3*g^6*h^8
+ 10*a^3*c^2*g^4*h^10 + 5*a^4*c*g^2*h^12 + a^5*h^14) - 7/16*sqrt(c*x^2 + a)
*c^6*e*g^7/(c^5*g^10*h^4 + 5*a*c^4*g^8*h^6 + 10*a^2*c^3*g^6*h^8 + 10*a^3*c^
2*g^4*h^10 + 5*a^4*c*g^2*h^12 + a^5*h^14) + 7/48*(c*x^2 + a)^(3/2)*c^5*f*g^
7/(c^5*g^10*h^4*x + 5*a*c^4*g^8*h^6*x + 10*a^2*c^3*g^6*h^8*x + 10*a^3*c^2*g
^4*h^10*x + 5*a^4*c*g^2*h^12*x + a^5*h^14*x + c^5*g^11*h^3 + 5*a*c^4*g^9*h^
5 + 10*a^2*c^3*g^7*h^7 + 10*a^3*c^2*g^5*h^9 + 5*a^4*c*g^3*h^11 + a^5*g*h^13
) + 7/16*sqrt(c*x^2 + a)*c^6*e*g^6*x/(c^5*g^10*h^3 + 5*a*c^4*g^8*h^5 + 10*a
^2*c^3*g^6*h^7 + 10*a^3*c^2*g^4*h^9 + 5*a^4*c*g^2*h^11 + a^5*h^13) + 7/16*s
qrt(c*x^2 + a)*c^6*d*g^6/(c^5*g^10*h^3 + 5*a*c^4*g^8*h^5 + 10*a^2*c^3*g^6*h
^7 + 10*a^3*c^2*g^4*h^9 + 5*a^4*c*g^2*h^11 + a^5*h^13) - 7/48*(c*x^2 + a)^(
3/2)*c^5*e*g^6/(c^5*g^10*h^3*x + 5*a*c^4*g^8*h^5*x + 10*a^2*c^3*g^6*h^7*x +
 10*a^3*c^2*g^4*h^9*x + 5*a^4*c*g^2*h^11*x + a^5*h^13*x + c^5*g^11*h^2 + 5*
a*c^4*g^9*h^4 + 10*a^2*c^3*g^7*h^6 + 10*a^3*c^2*g^5*h^8 + 5*a^4*c*g^3*h^10
+ a^5*g*h^12) - 7/48*(c*x^2 + a)^(5/2)*c^4*f*g^6/(c^5*g^10*h^3*x^2 + 5*a*c^
4*g^8*h^5*x^2 + 10*a^2*c^3*g^6*h^7*x^2 + 10*a^3*c^2*g^4*h^9*x^2 + 5*a^4*c*g
^2*h^11*x^2 + a^5*h^13*x^2 + 2*c^5*g^11*h^2*x + 10*a*c^4*g^9*h^4*x + 20*a^2
*c^3*g^7*h^6*x + 20*a^3*c^2*g^5*h^8*x + 10*a^4*c*g^3*h^10*x + 2*a^5*g*h^12*
x + c^5*g^12*h + 5*a*c^4*g^10*h^3 + 10*a^2*c^3*g^8*h^5 + 10*a^3*c^2*g^6*h^7
 + 5*a^4*c*g^4*h^9 + a^5*g^2*h^11) + 7/48*(c*x^2 + a)^(3/2)*c^5*f*g^6/(c^5*
g^10*h^3 + 5*a*c^4*g^8*h^5 + 10*a^2*c^3*g^6*h^7 + 10*a^3*c^2*g^4*h^9 + 5*a^
4*c*g^2*h^11 + a^5*h^13) - 7/16*sqrt(c*x^2 + a)*c^6*d*g^5*x/(c^5*g^10*h^2 +
 5*a*c^4*g^8*h^4 + 10*a^2*c^3*g^6*h^6 + 10*a^3*c^2*g^4*h^8 + 5*a^4*c*g^2*h^
10 + a^5*h^12) + 7/48*(c*x^2 + a)^(3/2)*c^5*d*g^5/(c^5*g^10*h^2*x + 5*a*c^4
*g^8*h^4*x + 10*a^2*c^3*g^6*h^6*x + 10*a^3*c^2*g^4*h^8*x + 5*a^4*c*g^2*h^10
*x + a^5*h^12*x + c^5*g^11*h + 5*a*c^4*g^9*h^3 + 10*a^2*c^3*g^7*h^5 + 10*a^
3*c^2*g^5*h^7 + 5*a^4*c*g^3*h^9 + a^5*g*h^11) + 7/48*(c*x^2 + a)^(5/2)*c^4*
e*g^5/(c^5*g^10*h^2*x^2 + 5*a*c^4*g^8*h^4*x^2 + 10*a^2*c^3*g^6*h^6*x^2 + 10
*a^3*c^2*g^4*h^8*x^2 + 5*a^4*c*g^2*h^10*x^2 + a^5*h^12*x^2 + 2*c^5*g^11*h*x
 + 10*a*c^4*g^9*h^3*x + 20*a^2*c^3*g^7*h^5*x + 20*a^3*c^2*g^5*h^7*x + 10*a^
4*c*g^3*h^9*x + 2*a^5*g*h^11*x + c^5*g^12 + 5*a*c^4*g^10*h^2 + 10*a^2*c^3*g
^8*h^4 + 10*a^3*c^2*g^6*h^6 + 5*a^4*c*g^4*h^8 + a^5*g^2*h^10) - 7/48*(c*x^2
 + a)^(3/2)*c^5*e*g^5/(c^5*g^10*h^2 + 5*a*c^4*g^8*h^4 + 10*a^2*c^3*g^6*h^6
+ 10*a^3*c^2*g^4*h^8 + 5*a^4*c*g^2*h^10 + a^5*h^12) - 27/16*sqrt(c*x^2 + a)
*c^5*f*g^6/(c^4*g^8*h^5 + 4*a*c^3*g^6*h^7 + 6*a^2*c^2*g^4*h^9 + 4*a^3*c*g^2
*h^11 + a^4*h^13) + 5/4*sqrt(c*x^2 + a)*c^5*f*g^5*x/(c^4*g^8*h^4 + 4*a*c^3*
g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^12) - 7/48*(c*x^2 +
a)^(5/2)*c^4*d*g^4/(c^5*g^10*h*x^2 + 5*a*c^4*g^8*h^3*x^2 + 10*a^2*c^3*g^6*h
^5*x^2 + 10*a^3*c^2*g^4*h^7*x^2 + 5*a^4*c*g^2*h^9*x^2 + a^5*h^11*x^2 + 2*c^
5*g^11*x + 10*a*c^4*g^9*h^2*x + 20*a^2*c^3*g^7*h^4*x + 20*a^3*c^2*g^5*h^6*x
 + 10*a^4*c*g^3*h^8*x + 2*a^5*g*h^10*x + c^5*g^12/h + 5*a*c^4*g^10*h + 10*a
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^2*c^3*g^8*h^3 + 10*a^3*c^2*g^6*h^5 + 5*a^4*c*g^4*h^7 + a^5*g^2*h^9) + 7/48 *(c*x^2 + a)^(3/2)*c^5*d*g^4/(c^5*g^10*h + 5*a*c^4*g^8*h^3 + 10*a^2*c^3*g^6 *h^5 + 10*a^3*c^2*g^4*h^7 + 5*a^4*c*g^2*h^9 + a^5*h^11) + 21/16*sqrt(c*x^2 + a)*c^5*e*g^5/(c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c *g^2*h^10 + a^4*h^12) - 7/24*(c*x^2 + a)^(5/2)*c^3*f*g^5/(c^4*g^8*h^4*x^3 + 4*a*c^3*g^6*h^6*x^3 + 6*a^2*c^2*g^4*h^8*x^3 + 4*a^3*c*g^2*h^10*x^3 + a^4*h ^12*x^3 + 3*c^4*g^9*h^3*x^2 + 12*a*c^3*g^7*h^5*x^2 + 18*a^2*c^2*g^5*h^7*x^2 + 12*a^3*c*g^3*h^9*x^2 + 3*a^4*g*h^11*x^2 + 3*c^4*g^10*h^2*x + 12*a*c^3*g^ 8*h^4*x + 18*a^2*c^2*g^6*h^6*x + 12*a^3*c*g^4*h^8*x + 3*a^4*g^2*h^10*x + c^ 4*g^11*h + 4*a*c^3*g^9*h^3 + 6*a^2*c^2*g^7*h^5 + 4*a^3*c*g^5*h^7 + a^4*g^3* h^9) - 17/24*(c*x^2 + a)^(3/2)*c^4*f*g^5/(c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x + 6*a^2*c^2*g^4*h^8*x + 4*a^3*c*g^2*h^10*x + a^4*h^12*x + c^4*g^9*h^3 + 4* a*c^3*g^7*h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^3*c*g^3*h^9 + a^4*g*h^11) - 7/8*sqr t(c*x^2 + a)*c^5*e*g^4*x/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) - 15/16*sqrt(c*x^2 + a)*c^5*d*g^4/(c^4*g^8*h ^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) + 7/ 24*(c*x^2 + a)^(5/2)*c^3*e*g^4/(c^4*g^8*h^3*x^3 + 4*a*c^3*g^6*h^5*x^3 + 6*a ^2*c^2*g^4*h^7*x^3 + 4*a^3*c*g^2*h^9*x^3 + a^4*h^11*x^3 + 3*c^4*g^9*h^2*x^2 + 12*a*c^3*g^7*h^4*x^2 + 18*a^2*c^2*g^5*h^6*x^2 + 12*a^3*c*g^3*h^8*x^2 + 3 *a^4*g*h^10*x^2 + 3*c^4*g^10*h*x + 12*a*c^3*g^8*h^3*x + 18*a^2*c^2*g^6*h^5* x + 12*a^3*c*g^4*h^7*x + 3*a^4*g^2*h^9*x + c^4*g^11 + 4*a*c^3*g^9*h^2 + 6*a ^2*c^2*g^7*h^4 + 4*a^3*c*g^5*h^6 + a^4*g^3*h^8) + 7/12*(c*x^2 + a)^(3/2)*c^ 4*e*g^4/(c^4*g^8*h^3*x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^4*h^7*x + 4*a^3*c* g^2*h^9*x + a^4*h^11*x + c^4*g^9*h^2 + 4*a*c^3*g^7*h^4 + 6*a^2*c^2*g^5*h^6 + 4*a^3*c*g^3*h^8 + a^4*g*h^10) + 1/8*(c*x^2 + a)^(5/2)*c^3*f*g^4/(c^4*g^8* h^3*x^2 + 4*a*c^3*g^6*h^5*x^2 + 6*a^2*c^2*g^4*h^7*x^2 + 4*a^3*c*g^2*h^9*x^2 + a^4*h^11*x^2 + 2*c^4*g^9*h^2*x + 8*a*c^3*g^7*h^4*x + 12*a^2*c^2*g^5*h^6* x + 8*a^3*c*g^3*h^8*x + 2*a^4*g*h^10*x + c^4*g^10*h + 4*a*c^3*g^8*h^3 + 6*a ^2*c^2*g^6*h^5 + 4*a^3*c*g^4*h^7 + a^4*g^2*h^9) - 1/8*(c*x^2 + a)^(3/2)*c^4 *f*g^4/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) + 1/2*sqrt(c*x^2 + a)*c^5*d*g^3*x/(c^4*g^8*h^2 + 4*a*c^3*g^6*h ^4 + 6*a^2*c^2*g^4*h^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) - 7/24*(c*x^2 + a)^(5/ 2)*c^3*d*g^3/(c^4*g^8*h^2*x^3 + 4*a*c^3*g^6*h^4*x^3 + 6*a^2*c^2*g^4*h^6*x^3 + 4*a^3*c*g^2*h^8*x^3 + a^4*h^10*x^3 + 3*c^4*g^9*h*x^2 + 12*a*c^3*g^7*h^3* x^2 + 18*a^2*c^2*g^5*h^5*x^2 + 12*a^3*c*g^3*h^7*x^2 + 3*a^4*g*h^9*x^2 + 3*c ^4*g^10*x + 12*a*c^3*g^8*h^2*x + 18*a^2*c^2*g^6*h^4*x + 12*a^3*c*g^4*h^6*x + 3*a^4*g^2*h^8*x + c^4*g^11/h + 4*a*c^3*g^9*h + 6*a^2*c^2*g^7*h^3 + 4*a^3* c*g^5*h^5 + a^4*g^3*h^7) - 11/24*(c*x^2 + a)^(3/2)*c^4*d*g^3/(c^4*g^8*h^2*x + 4*a*c^3*g^6*h^4*x + 6*a^2*c^2*g^4*h^6*x + 4*a^3*c*g^2*h^8*x + a^4*h^10*x + c^4*g^9*h + 4*a*c^3*g^7*h^3 + 6*a^2*c^2*g^5*h^5 + 4*a^3*c*g^3*h^7 + a^4* g*h^9) - 7/24*(c*x^2 + a)^(5/2)*c^2*f*g^4/(c^3*g^6*h^5*x^4 + 3*a*c^2*g^4*h^ 7*x^4 + 3*a^2*c*g^2*h^9*x^4 + a^3*h^11*x^4 + 4*c^3*g^7*h^4*x^3 + 12*a*c^2*g ^5*h^6*x^3 + 12*a^2*c*g^3*h^8*x^3 + 4*a^3*g*h^10*x^3 + 6*c^3*g^8*h^3*x^2 + 18*a*c^2*g^6*h^5*x^2 + 18*a^2*c*g^4*h^7*x^2 + 6*a^3*g^2*h^9*x^2 + 4*c^3*g^9 *h^2*x + 12*a*c^2*g^7*h^4*x + 12*a^2*c*g^5*h^6*x + 4*a^3*g^3*h^8*x + c^3*g^ 10*h + 3*a*c^2*g^8*h^3 + 3*a^2*c*g^6*h^5 + a^3*g^4*h^7) + 39/16*sqrt(c*x^2 + a)*c^4*f*g^4/(c^3*g^6*h^5 + 3*a*c^2*g^4*h^7 + 3*a^2*c*g^2*h^9 + a^3*h^11) - 19/16*sqrt(c*x^2 + a)*c^4*f*g^3*x/(c^3*g^6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2 *c*g^2*h^8 + a^3*h^10) - 1/8*(c*x^2 + a)^(5/2)*c^3*d*g^2/(c^4*g^8*h*x^2 + 4 *a*c^3*g^6*h^3*x^2 + 6*a^2*c^2*g^4*h^5*x^2 + 4*a^3*c*g^2*h^7*x^2 + a^4*h^9* x^2 + 2*c^4*g^9*x + 8*a*c^3*g^7*h^2*x + 12*a^2*c^2*g^5*h^4*x + 8*a^3*c*g^3* h^6*x + 2*a^4*g*h^8*x + c^4*g^10/h + 4*a*c^3*g^8*h + 6*a^2*c^2*g^6*h^3 + 4* a^3*c*g^4*h^5 + a^4*g^2*h^7) + 1/8*(c*x^2 + a)^(3/2)*c^4*d*g^2/(c^4*g^8*h + 4*a*c^3*g^6*h^3 + 6*a^2*c^2*g^4*h^5 + 4*a^3*c*g^2*h^7 + a^4*h^9) + 7/24*(c *x^2 + a)^(5/2)*c^2*e*g^3/(c^3*g^6*h^4*x^4 + 3*a*c^2*g^4*h^6*x^4 + 3*a^2*c* g^2*h^8*x^4 + a^3*h^10*x^4 + 4*c^3*g^7*h^3*x^3 + 12*a*c^2*g^5*h^5*x^3 + 12* a^2*c*g^3*h^7*x^3 + 4*a^3*g*h^9*x^3 + 6*c^3*g^8*h^2*x^2 + 18*a*c^2*g^6*h^4* x^2 + 18*a^2*c*g^4*h^6*x^2 + 6*a^3*g^2*h^8*x^2 + 4*c^3*g^9*h*x + 12*a*c^2*g ^7*h^3*x + 12*a^2*c*g^5*h^5*x + 4*a^3*g^3*h^7*x + c^3*g^10 + 3*a*c^2*g^8*h^

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2 + 3*a^2*c*g^6*h^4 + a^3*g^4*h^6) - 21/16*sqrt(c*x^2 + a)*c^4*e*g^3/(c^3*g
^6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) + 13/24*(c*x^2 + a)^
(5/2)*c^2*f*g^3/(c^3*g^6*h^4*x^3 + 3*a*c^2*g^4*h^6*x^3 + 3*a^2*c*g^2*h^8*x^
3 + a^3*h^10*x^3 + 3*c^3*g^7*h^3*x^2 + 9*a*c^2*g^5*h^5*x^2 + 9*a^2*c*g^3*h^
7*x^2 + 3*a^3*g*h^9*x^2 + 3*c^3*g^8*h^2*x + 9*a*c^2*g^6*h^4*x + 9*a^2*c*g^4
*h^6*x + 3*a^3*g^2*h^8*x + c^3*g^9*h + 3*a*c^2*g^7*h^3 + 3*a^2*c*g^5*h^5 +
a^3*g^3*h^7) + 15/16*(c*x^2 + a)^(3/2)*c^3*f*g^3/(c^3*g^6*h^4*x + 3*a*c^2*g
^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h^10*x + c^3*g^7*h^3 + 3*a*c^2*g^5*h^5 +
 3*a^2*c*g^3*h^7 + a^3*g*h^9) + 7/16*sqrt(c*x^2 + a)*c^4*e*g^2*x/(c^3*g^6*h
^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 7/24*(c*x^2 + a)^(5/2)*
c^2*d*g^2/(c^3*g^6*h^3*x^4 + 3*a*c^2*g^4*h^5*x^4 + 3*a^2*c*g^2*h^7*x^4 + a^
3*h^9*x^4 + 4*c^3*g^7*h^2*x^3 + 12*a*c^2*g^5*h^4*x^3 + 12*a^2*c*g^3*h^6*x^3
 + 4*a^3*g*h^8*x^3 + 6*c^3*g^8*h*x^2 + 18*a*c^2*g^6*h^3*x^2 + 18*a^2*c*g^4*
h^5*x^2 + 6*a^3*g^2*h^7*x^2 + 4*c^3*g^9*x + 12*a*c^2*g^7*h^2*x + 12*a^2*c*g
^5*h^4*x + 4*a^3*g^3*h^6*x + c^3*g^10/h + 3*a*c^2*g^8*h + 3*a^2*c*g^6*h^3 +
 a^3*g^4*h^5) + 9/16*sqrt(c*x^2 + a)*c^4*d*g^2/(c^3*g^6*h^3 + 3*a*c^2*g^4*h
^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 7/24*(c*x^2 + a)^(5/2)*c^2*e*g^2/(c^3*g^6
*h^3*x^3 + 3*a*c^2*g^4*h^5*x^3 + 3*a^2*c*g^2*h^7*x^3 + a^3*h^9*x^3 + 3*c^3*
g^7*h^2*x^2 + 9*a*c^2*g^5*h^4*x^2 + 9*a^2*c*g^3*h^6*x^2 + 3*a^3*g*h^8*x^2 +
 3*c^3*g^8*h*x + 9*a*c^2*g^6*h^3*x + 9*a^2*c*g^4*h^5*x + 3*a^3*g^2*h^7*x +
c^3*g^9 + 3*a*c^2*g^7*h^2 + 3*a^2*c*g^5*h^4 + a^3*g^3*h^6) - 7/16*(c*x^2 +
a)^(3/2)*c^3*e*g^2/(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^5*x + 3*a^2*c*g^2*h^7*x +
 a^3*h^9*x + c^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2*c*g^3*h^6 + a^3*g*h^8) +
 7/48*(c*x^2 + a)^(5/2)*c^2*f*g^2/(c^3*g^6*h^3*x^2 + 3*a*c^2*g^4*h^5*x^2 +
3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x + 6*a*c^2*g^5*h^4*x + 6
*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*c^2*g^6*h^3 + 3*a^2*c*g^
4*h^5 + a^3*g^2*h^7) - 7/48*(c*x^2 + a)^(3/2)*c^3*f*g^2/(c^3*g^6*h^3 + 3*a*
c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 7/30*(c*x^2 + a)^(5/2)*c*f*g^3/(
c^2*g^4*h^6*x^5 + 2*a*c*g^2*h^8*x^5 + a^2*h^10*x^5 + 5*c^2*g^5*h^5*x^4 + 10
*a*c*g^3*h^7*x^4 + 5*a^2*g*h^9*x^4 + 10*c^2*g^6*h^4*x^3 + 20*a*c*g^4*h^6*x^
3 + 10*a^2*g^2*h^8*x^3 + 10*c^2*g^7*h^3*x^2 + 20*a*c*g^5*h^5*x^2 + 10*a^2*g
^3*h^7*x^2 + 5*c^2*g^8*h^2*x + 10*a*c*g^6*h^4*x + 5*a^2*g^4*h^6*x + c^2*g^9
*h + 2*a*c*g^7*h^3 + a^2*g^5*h^5) - 1/16*sqrt(c*x^2 + a)*c^4*d*g*x/(c^3*g^6
*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) + 1/24*(c*x^2 + a)^(5/2
)*c^2*d*g/(c^3*g^6*h^2*x^3 + 3*a*c^2*g^4*h^4*x^3 + 3*a^2*c*g^2*h^6*x^3 + a^
3*h^8*x^3 + 3*c^3*g^7*h*x^2 + 9*a*c^2*g^5*h^3*x^2 + 9*a^2*c*g^3*h^5*x^2 + 3
*a^3*g*h^7*x^2 + 3*c^3*g^8*x + 9*a*c^2*g^6*h^2*x + 9*a^2*c*g^4*h^4*x + 3*a^
3*g^2*h^6*x + c^3*g^9/h + 3*a*c^2*g^7*h + 3*a^2*c*g^5*h^3 + a^3*g^3*h^5) +
1/16*(c*x^2 + a)^(3/2)*c^3*d*g/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c
*g^2*h^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^
3*g*h^7) - 7/48*(c*x^2 + a)^(5/2)*c^2*e*g/(c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h^
4*x^2 + 3*a^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 + 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^3
*x + 6*a^2*c*g^3*h^5*x + 2*a^3*g*h^7*x + c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*
c*g^4*h^4 + a^3*g^2*h^6) + 7/48*(c*x^2 + a)^(3/2)*c^3*e*g/(c^3*g^6*h^2 + 3*
a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) + 7/30*(c*x^2 + a)^(5/2)*c*e*g^2
/(c^2*g^4*h^5*x^5 + 2*a*c*g^2*h^7*x^5 + a^2*h^9*x^5 + 5*c^2*g^5*h^4*x^4 + 1
0*a*c*g^3*h^6*x^4 + 5*a^2*g*h^8*x^4 + 10*c^2*g^6*h^3*x^3 + 20*a*c*g^4*h^5*x
^3 + 10*a^2*g^2*h^7*x^3 + 10*c^2*g^7*h^2*x^2 + 20*a*c*g^5*h^4*x^2 + 10*a^2*
g^3*h^6*x^2 + 5*c^2*g^8*h*x + 10*a*c*g^6*h^3*x + 5*a^2*g^4*h^5*x + c^2*g^9
+ 2*a*c*g^7*h^2 + a^2*g^5*h^4) + 13/24*(c*x^2 + a)^(5/2)*c*f*g^2/(c^2*g^4*h
^5*x^4 + 2*a*c*g^2*h^7*x^4 + a^2*h^9*x^4 + 4*c^2*g^5*h^4*x^3 + 8*a*c*g^3*h^
6*x^3 + 4*a^2*g*h^8*x^3 + 6*c^2*g^6*h^3*x^2 + 12*a*c*g^4*h^5*x^2 + 6*a^2*g^
2*h^7*x^2 + 4*c^2*g^7*h^2*x + 8*a*c*g^5*h^4*x + 4*a^2*g^3*h^6*x + c^2*g^8*h
 + 2*a*c*g^6*h^3 + a^2*g^4*h^5) - 25/16*sqrt(c*x^2 + a)*c^3*f*g^2/(c^2*g^4*
h^5 + 2*a*c*g^2*h^7 + a^2*h^9) + 3/8*sqrt(c*x^2 + a)*c^3*f*g*x/(c^2*g^4*h^4
 + 2*a*c*g^2*h^6 + a^2*h^8) + 1/48*(c*x^2 + a)^(5/2)*c^2*d/(c^3*g^6*h*x^2 +
 3*a*c^2*g^4*h^3*x^2 + 3*a^2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*x + 6*
a*c^2*g^5*h^2*x + 6*a^2*c*g^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a*c^2*g
^6*h + 3*a^2*c*g^4*h^3 + a^3*g^2*h^5) - 1/48*(c*x^2 + a)^(3/2)*c^3*d/(c^3*g
```

```
^6*h + 3*a*c^2*g^4*h^3 + 3*a^2*c*g^2*h^5 + a^3*h^7) - 7/30*(c*x^2 + a)^(5/2
)*c*d*g/(c^2*g^4*h^4*x^5 + 2*a*c*g^2*h^6*x^5 + a^2*h^8*x^5 + 5*c^2*g^5*h^3*
x^4 + 10*a*c*g^3*h^5*x^4 + 5*a^2*g*h^7*x^4 + 10*c^2*g^6*h^2*x^3 + 20*a*c*g^
4*h^4*x^3 + 10*a^2*g^2*h^6*x^3 + 10*c^2*g^7*h*x^2 + 20*a*c*g^5*h^3*x^2 + 10
*a^2*g^3*h^5*x^2 + 5*c^2*g^8*x + 10*a*c*g^6*h^2*x + 5*a^2*g^4*h^4*x + c^2*g
^9/h + 2*a*c*g^7*h + a^2*g^5*h^3) - 7/24*(c*x^2 + a)^(5/2)*c*e*g/(c^2*g^4*h
^4*x^4 + 2*a*c*g^2*h^6*x^4 + a^2*h^8*x^4 + 4*c^2*g^5*h^3*x^3 + 8*a*c*g^3*h^
5*x^3 + 4*a^2*g*h^7*x^3 + 6*c^2*g^6*h^2*x^2 + 12*a*c*g^4*h^4*x^2 + 6*a^2*g^
2*h^6*x^2 + 4*c^2*g^7*h*x + 8*a*c*g^5*h^3*x + 4*a^2*g^3*h^5*x + c^2*g^8 + 2
*a*c*g^6*h^2 + a^2*g^4*h^4) + 7/16*sqrt(c*x^2 + a)*c^3*e*g/(c^2*g^4*h^4 + 2
*a*c*g^2*h^6 + a^2*h^8) - 1/4*(c*x^2 + a)^(5/2)*c*f*g/(c^2*g^4*h^4*x^3 + 2*
a*c*g^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a
^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^
7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^5) - 3/8*(c*x^2 + a)^(3/2)*c^2*f*g/(c^2*g^4
*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*
h^7) - 1/6*(c*x^2 + a)^(5/2)*f*g^2/(c*g^2*h^7*x^6 + a*h^9*x^6 + 6*c*g^3*h^6
*x^5 + 6*a*g*h^8*x^5 + 15*c*g^4*h^5*x^4 + 15*a*g^2*h^7*x^4 + 20*c*g^5*h^4*x
^3 + 20*a*g^3*h^6*x^3 + 15*c*g^6*h^3*x^2 + 15*a*g^4*h^5*x^2 + 6*c*g^7*h^2*x
 + 6*a*g^5*h^4*x + c*g^8*h + a*g^6*h^3) + 1/24*(c*x^2 + a)^(5/2)*c*d/(c^2*g
^4*h^3*x^4 + 2*a*c*g^2*h^5*x^4 + a^2*h^7*x^4 + 4*c^2*g^5*h^2*x^3 + 8*a*c*g^
3*h^4*x^3 + 4*a^2*g*h^6*x^3 + 6*c^2*g^6*h*x^2 + 12*a*c*g^4*h^3*x^2 + 6*a^2*
g^2*h^5*x^2 + 4*c^2*g^7*x + 8*a*c*g^5*h^2*x + 4*a^2*g^3*h^4*x + c^2*g^8/h +
 2*a*c*g^6*h + a^2*g^4*h^3) - 1/16*sqrt(c*x^2 + a)*c^3*d/(c^2*g^4*h^3 + 2*a
*c*g^2*h^5 + a^2*h^7) - 1/8*(c*x^2 + a)^(5/2)*c*f/(c^2*g^4*h^3*x^2 + 2*a*c*
g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6
*x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 1/8*(c*x^2 + a)^(3/2)*c^2*f
/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) + 1/6*(c*x^2 + a)^(5/2)*e*g/(c*g^2
*h^6*x^6 + a*h^8*x^6 + 6*c*g^3*h^5*x^5 + 6*a*g*h^7*x^5 + 15*c*g^4*h^4*x^4 +
 15*a*g^2*h^6*x^4 + 20*c*g^5*h^3*x^3 + 20*a*g^3*h^5*x^3 + 15*c*g^6*h^2*x^2
+ 15*a*g^4*h^4*x^2 + 6*c*g^7*h*x + 6*a*g^5*h^3*x + c*g^8 + a*g^6*h^2) + 2/5
*(c*x^2 + a)^(5/2)*f*g/(c*g^2*h^6*x^5 + a*h^8*x^5 + 5*c*g^3*h^5*x^4 + 5*a*g
*h^7*x^4 + 10*c*g^4*h^4*x^3 + 10*a*g^2*h^6*x^3 + 10*c*g^5*h^3*x^2 + 10*a*g^
3*h^5*x^2 + 5*c*g^6*h^2*x + 5*a*g^4*h^4*x + c*g^7*h + a*g^5*h^3) - 1/6*(c*x
^2 + a)^(5/2)*d/(c*g^2*h^5*x^6 + a*h^7*x^6 + 6*c*g^3*h^4*x^5 + 6*a*g*h^6*x^
5 + 15*c*g^4*h^3*x^4 + 15*a*g^2*h^5*x^4 + 20*c*g^5*h^2*x^3 + 20*a*g^3*h^4*x
^3 + 15*c*g^6*h*x^2 + 15*a*g^4*h^3*x^2 + 6*c*g^7*x + 6*a*g^5*h^2*x + c*g^8/
h + a*g^6*h) - 1/5*(c*x^2 + a)^(5/2)*e/(c*g^2*h^5*x^5 + a*h^7*x^5 + 5*c*g^3
*h^4*x^4 + 5*a*g*h^6*x^4 + 10*c*g^4*h^3*x^3 + 10*a*g^2*h^5*x^3 + 10*c*g^5*h
^2*x^2 + 10*a*g^3*h^4*x^2 + 5*c*g^6*h*x + 5*a*g^4*h^3*x + c*g^7 + a*g^5*h^2
) - 1/4*(c*x^2 + a)^(5/2)*f/(c*g^2*h^5*x^4 + a*h^7*x^4 + 4*c*g^3*h^4*x^3 +
4*a*g*h^6*x^3 + 6*c*g^4*h^3*x^2 + 6*a*g^2*h^5*x^2 + 4*c*g^5*h^2*x + 4*a*g^3
*h^4*x + c*g^6*h + a*g^4*h^3) + 3/8*sqrt(c*x^2 + a)*c^2*f/(c*g^2*h^5 + a*h^
7) + 7/16*c^6*f*g^8*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)
*abs(h*x + g)))/((a + c*g^2/h^2)^(9/2)*h^15) - 7/16*c^6*e*g^7*arcsinh(c*g*x
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^
(9/2)*h^14) + 7/16*c^6*d*g^6*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(
sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(9/2)*h^13) - 27/16*c^5*f*g^6*arc
sinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c
*g^2/h^2)^(7/2)*h^13) + 21/16*c^5*e*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x +
g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^12) - 15/16*c^
5*d*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g
)))/((a + c*g^2/h^2)^(7/2)*h^11) + 39/16*c^4*f*g^4*arcsinh(c*g*x/(sqrt(a*c)
*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^11)
 - 21/16*c^4*e*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*
abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^10) + 9/16*c^4*d*g^2*arcsinh(c*g*x/
(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(
5/2)*h^9) - 25/16*c^3*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(s
qrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^9) + 7/16*c^3*e*g*arcsinh(
c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/
```

```
h^2)^(3/2)*h^8) - 1/16*c^3*d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(
sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) + 3/8*c^2*f*arcsinh(c*
g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^
2/h^2)*h^7)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**7,x)
[Out] Timed out
```

3.99 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx$

Optimal. Leaf size=532

$$-\frac{(a+cx^2)^{5/2}\left(42a^2fh^4-ach^2\left(26fg^2-h(61eg-12dh)\right)-c^2g^2\left(h(2eg-51dh)+5fg^2\right)\right)}{210h(g+hx)^5\left(ah^2+cg^2\right)^3}-\frac{ac^2\sqrt{a+cx^2}\,(ah-c$$

```
[Out] -1/24*c*(6*c^2*d*g^3+a^2*h^2*(-e*h+8*f*g)-a*c*g*(f*g^2-h*(-3*d*h+8*e*g)))*(
-c*g*x+a*h)*(c*x^2+a)^(3/2)/(a*h^2+c*g^2)^4/(h*x+g)^4-1/7*(d*h^2-e*g*h+f*g^
2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^7+1/42*(7*a*h^2*(-e*h+2*f*g)+c*g
*(5*f*g^2+h*(-9*d*h+2*e*g)))*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)^2/(h*x+g)^6-1/
210*(42*a^2*f*h^4-c^2*g^2*(5*f*g^2+h*(-51*d*h+2*e*g))-a*c*h^2*(26*f*g^2-h*(
-12*d*h+61*e*g)))*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)^3/(h*x+g)^5-1/16*a^2*c^3*
(6*c^2*d*g^3+a^2*h^2*(-e*h+8*f*g)-a*c*g*(f*g^2-h*(-3*d*h+8*e*g)))*arctanh((
-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(11/2)-1/16*
a*c^2*(6*c^2*d*g^3+a^2*h^2*(-e*h+8*f*g)-a*c*g*(f*g^2-h*(-3*d*h+8*e*g)))*(-c
*g*x+a*h)*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^5/(h*x+g)^2
```

Rubi [A] time = 0.89, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1651, 835, 807, 721, 725, 206}

$$-\frac{ac^2\sqrt{a+cx^2}\,(ah-cgx)\left(a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)+6c^2dg^3\right)}{16(g+hx)^2\left(ah^2+cg^2\right)^5}-\frac{c\left(a+cx^2\right)^{3/2}(ah-cgx)\left(a^2h\right.}{24}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -(a*c^2*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*
d*h)))*(a*h - c*g*x)*Sqrt[a + c*x^2])/(16*(c*g^2 + a*h^2)^5*(g + h*x)^2) -
(c*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*d*h))
)*(a*h - c*g*x)*(a + c*x^2)^(3/2))/(24*(c*g^2 + a*h^2)^4*(g + h*x)^4) - ((f
*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(7*h*(c*g^2 + a*h^2)*(g + h*x)^7)
+ ((5*c*f*g^3 + c*g*h*(2*e*g - 9*d*h) + 7*a*h^2*(2*f*g - e*h))*(a + c*x^2)^
(5/2))/(42*h*(c*g^2 + a*h^2)^2*(g + h*x)^6) - ((42*a^2*f*h^4 - c^2*(5*f*g^4
 + g^2*h*(2*e*g - 51*d*h)) - a*c*h^2*(26*f*g^2 - h*(61*e*g - 12*d*h)))*(a +
 c*x^2)^(5/2))/(210*h*(c*g^2 + a*h^2)^3*(g + h*x)^5) - (a^2*c^3*(6*c^2*d*g^
3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*d*h)))*ArcTanh[(a*h
 - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(16*(c*g^2 + a*h^2)^(11/2
))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{7h\left(cg^2+ah^2\right)(g+hx)^7} - \frac{\int \frac{\left(-7(cdg-afg+aeh)-\left(7afh+c\left(2eg+\frac{5fg^2}{h}-2dh\right)\right)x}{(g+hx)^7}}{7\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{7h\left(cg^2+ah^2\right)(g+hx)^7} + \frac{\left(5cfg^3+cgh(2eg-9dh)+7ah^2(2fg-\right.}{42h\left(cg^2+ah^2\right)^2(g+h}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{7h\left(cg^2+ah^2\right)(g+hx)^7} + \frac{\left(5cfg^3+cgh(2eg-9dh)+7ah^2(2fg-\right.}{42h\left(cg^2+ah^2\right)^2(g+h}$$

$$= -\frac{c\left(6c^2dg^3+a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)\right)(ah-cgx)\left(a+\right.}{24\left(cg^2+ah^2\right)^4(g+hx)^4}$$

$$= -\frac{ac^2\left(6c^2dg^3+a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)\right)(ah-cgx)\sqrt{a}}{16\left(cg^2+ah^2\right)^5(g+hx)^2}$$

$$= -\frac{ac^2\left(6c^2dg^3+a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)\right)(ah-cgx)\sqrt{a}}{16\left(cg^2+ah^2\right)^5(g+hx)^2}$$

$$= -\frac{ac^2\left(6c^2dg^3+a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)\right)(ah-cgx)\sqrt{a}}{16\left(cg^2+ah^2\right)^5(g+hx)^2}$$

Mathematica [A] time = 2.50, size = 863, normalized size = 1.62

$$\frac{a^2\left(6c^2dg^3-ac\left(fg^2+h(3dh-8eg)\right)g+a^2h^2(8fg-eh)\right)\log(g+hx)c^3}{16\left(cg^2+ah^2\right)^{11/2}} - \frac{a^2\left(6c^2dg^3-ac\left(fg^2+h(3dh-8eg)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -1/1680*(Sqrt[a + c*x^2]*(240*(c*g^2 + a*h^2)^6*(f*g^2 + h*(-(e*g) + d*h))
- 40*(c*g^2 + a*h^2)^5*(29*c*f*g^3 + c*g*h*(-22*e*g + 15*d*h) - 7*a*h^2*(-2
*f*g + e*h))*(g + h*x) + 8*(c*g^2 + a*h^2)^4*(42*a^2*f*h^4 + a*c*h^2*(314*f
*g^2 + h*(-139*e*g + 48*d*h)) + c^2*(275*f*g^4 + g^2*h*(-142*e*g + 51*d*h))
)*(g + h*x)^2 - 2*c*(c*g^2 + a*h^2)^3*(7*a^2*h^4*(136*f*g - 35*e*h) + 2*c^2
*(500*f*g^5 + g^3*h*(-136*e*g + 3*d*h)) + a*c*g*h^2*(1979*f*g^2 + h*(-544*e
*g + 33*d*h)))*(g + h*x)^3 + 2*c*(c*g^2 + a*h^2)^2*(336*a^3*f*h^6 + c^3*(40
0*f*g^6 - 2*g^4*h*(4*e*g + 3*d*h)) + 3*a^2*c*h^4*(400*f*g^2 + h*(-29*e*g +
8*d*h)) + a*c^2*g^2*h^2*(1201*f*g^2 - h*(32*e*g + 45*d*h)))*(g + h*x)^4 - c
^2*(c*g^2 + a*h^2)*(21*a^3*h^6*(24*f*g - 5*e*h) + 2*a*c^2*g^3*h^2*(89*f*g^2
 + 44*e*g*h + 54*d*h^2) + 3*a^2*c*g*h^4*(109*f*g^2 + h*(94*e*g - 73*d*h)) +
 4*c^3*(10*f*g^7 + g^5*h*(4*e*g + 3*d*h)))*(g + h*x)^5 - c^2*(-336*a^4*f*h^
8 + 2*a*c^3*g^4*h^2*(109*f*g^2 + 52*e*g*h + 60*d*h^2) + a^2*c^2*g^2*h^4*(50
5*f*g^2 + h*(370*e*g - 741*d*h)) + 4*c^4*(10*f*g^8 + g^6*h*(4*e*g + 3*d*h))
 + 3*a^3*c*h^6*(312*f*g^2 + h*(-221*e*g + 32*d*h)))*(g + h*x)^6))/((c*g^2*h
 + a*h^3)^5*(g + h*x)^7) + (a^2*c^3*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) -
a*c*g*(f*g^2 + h*(-8*e*g + 3*d*h)))*Log[g + h*x])/(16*(c*g^2 + a*h^2)^(11/2
)) - (a^2*c^3*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 + h*(-8*e
```

```
*g + 3*d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(16*(
c*g^2 + a*h^2)^(11/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.12, size = 7936, normalized size = 14.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="giac")
```

```
[Out] -1/8*(6*a^2*c^5*d*g^3 - a^3*c^4*f*g^3 - 3*a^3*c^4*d*g*h^2 + 8*a^4*c^3*f*g*h
^2 + 8*a^3*c^4*g^2*h*e - a^4*c^3*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a
))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^5*g^10 + 5*a*c^4*g^8*h^2 + 10*a
^2*c^3*g^6*h^4 + 10*a^3*c^2*g^4*h^6 + 5*a^4*c*g^2*h^8 + a^5*h^10)*sqrt(-c*g
^2 - a*h^2)) - 1/840*(630*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^2*c^5*d*g^3*h^
12 - 105*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^3*c^4*f*g^3*h^12 - 315*(sqrt(c)
*x - sqrt(c*x^2 + a))^13*a^3*c^4*d*g*h^14 + 840*(sqrt(c)*x - sqrt(c*x^2 + a
))^13*a^4*c^3*f*g*h^14 + 840*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^3*c^4*g^2*h
^13*e - 105*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^4*c^3*h^15*e - 1680*(sqrt(c)
*x - sqrt(c*x^2 + a))^12*c^(15/2)*f*g^10*h^5 - 8400*(sqrt(c)*x - sqrt(c*x^2
 + a))^12*a*c^(13/2)*f*g^8*h^7 - 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^2
*c^(11/2)*f*g^6*h^9 + 8190*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^2*c^(11/2)*d*
g^4*h^11 - 18165*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^3*c^(9/2)*f*g^4*h^11 -
4095*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^3*c^(9/2)*d*g^2*h^13 + 2520*(sqrt(c
)*x - sqrt(c*x^2 + a))^12*a^4*c^(7/2)*f*g^2*h^13 - 1680*(sqrt(c)*x - sqrt(c
*x^2 + a))^12*a^5*c^(5/2)*f*h^15 + 10920*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a
^3*c^(9/2)*g^3*h^12*e - 1365*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^4*c^(7/2)*g
*h^14*e - 5600*(sqrt(c)*x - sqrt(c*x^2 + a))^11*c^8*f*g^11*h^4 - 28000*(sqr
t(c)*x - sqrt(c*x^2 + a))^11*a*c^7*f*g^9*h^6 - 56000*(sqrt(c)*x - sqrt(c*x^
2 + a))^11*a^2*c^6*f*g^7*h^8 + 44940*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^2*c
^6*d*g^5*h^10 - 63490*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^5*f*g^5*h^10 -
 26670*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^5*d*g^3*h^12 + 32620*(sqrt(c)
*x - sqrt(c*x^2 + a))^11*a^4*c^4*f*g^3*h^12 + 2100*(sqrt(c)*x - sqrt(c*x^2
+ a))^11*a^4*c^4*d*g*h^14 - 11200*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^5*c^3*
f*g*h^14 - 2240*(sqrt(c)*x - sqrt(c*x^2 + a))^11*c^8*g^10*h^5*e - 11200*(sq
rt(c)*x - sqrt(c*x^2 + a))^11*a*c^7*g^8*h^7*e - 22400*(sqrt(c)*x - sqrt(c*x
^2 + a))^11*a^2*c^6*g^6*h^9*e + 37520*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*
c^5*g^4*h^11*e - 24290*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^4*c^4*g^2*h^13*e
- 1540*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^5*c^3*h^15*e - 11200*(sqrt(c)*x -
 sqrt(c*x^2 + a))^10*c^(17/2)*f*g^12*h^3 - 3360*(sqrt(c)*x - sqrt(c*x^2 + a
))^10*c^(17/2)*d*g^10*h^5 - 52640*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(15/
2)*f*g^10*h^5 - 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(15/2)*d*g^8*h^7
 - 95200*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(13/2)*f*g^8*h^7 + 100380*(
sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(13/2)*d*g^6*h^9 - 100730*(sqrt(c)*x
- sqrt(c*x^2 + a))^10*a^3*c^(11/2)*f*g^6*h^9 - 146790*(sqrt(c)*x - sqrt(c*x
^2 + a))^10*a^3*c^(11/2)*d*g^4*h^11 + 163940*(sqrt(c)*x - sqrt(c*x^2 + a))^
10*a^4*c^(9/2)*f*g^4*h^11 + 6300*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^4*c^(9/
2)*d*g^2*h^13 - 56000*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^5*c^(7/2)*f*g^2*h^
13 - 3360*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^5*c^(7/2)*d*h^15 + 3360*(sqrt(
c)*x - sqrt(c*x^2 + a))^10*a^6*c^(5/2)*f*h^15 - 4480*(sqrt(c)*x - sqrt(c*x^
2 + a))^10*c^(17/2)*g^11*h^4*e - 22400*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c
```

^(15/2)*g^9*h^6*e - 44800*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(13/2)*g^7*h^8*e + 133840*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^3*c^(11/2)*g^5*h^10*e - 106330*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^4*c^(9/2)*g^3*h^12*e + 3220*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^5*c^(7/2)*g*h^14*e - 13440*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^9*f*g^13*h^2 - 4032*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^9*d*g^11*h^4 - 50848*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^8*f*g^11*h^4 - 20160*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^8*d*g^9*h^6 - 52640*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^7*f*g^9*h^6 + 191016*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^7*d*g^7*h^8 - 9436*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^6*f*g^7*h^8 - 363216*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^6*d*g^5*h^10 + 439306*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^5*f*g^5*h^10 + 95340*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^5*d*g^3*h^12 - 209965*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^5*c^4*f*g^3*h^12 - 9975*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^5*c^4*d*g*h^14 + 32200*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^6*c^3*f*g*h^14 - 5376*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^9*g^12*h^3*e - 25984*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^8*g^10*h^5*e - 49280*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^7*g^8*h^7*e + 263648*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^6*g^6*h^9*e - 332780*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^5*g^4*h^11*e + 49490*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^5*c^4*g^2*h^13*e - 1085*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^6*c^3*h^15*e - 8960*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(19/2)*f*g^14*h - 2688*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(19/2)*d*g^12*h^3 - 15232*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(17/2)*f*g^12*h^3 - 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(17/2)*d*g^10*h^5 + 53200*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(15/2)*f*g^10*h^5 + 181104*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(15/2)*d*g^8*h^7 + 143416*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(13/2)*f*g^8*h^7 - 651924*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(13/2)*d*g^6*h^9 + 580034*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(11/2)*f*g^6*h^9 + 299460*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(11/2)*d*g^4*h^11 - 568085*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(9/2)*f*g^4*h^11 - 72975*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(9/2)*d*g^2*h^13 + 147000*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^6*c^(7/2)*f*g^2*h^13 - 3360*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^6*c^(7/2)*d*h^15 - 5040*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^7*c^(5/2)*f*h^15 - 3584*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(19/2)*g^13*h^2*e - 9856*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(17/2)*g^11*h^4*e + 4480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(15/2)*g^9*h^6*e + 344512*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(13/2)*g^7*h^8*e - 613480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(11/2)*g^5*h^10*e + 259210*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(9/2)*g^3*h^12*e - 9765*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^6*c^(7/2)*g*h^14*e - 2560*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^10*f*g^15 - 768*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^10*d*g^13*h^2 + 12928*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^9*f*g^13*h^2 + 384*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^9*d*g^11*h^4 + 80576*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^8*f*g^11*h^4 + 117984*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^8*d*g^9*h^6 + 101936*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^7*f*g^9*h^6 - 603216*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^7*d*g^7*h^8 + 256816*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^6*f*g^7*h^8 + 703752*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^6*d*g^5*h^10 - 941332*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^5*f*g^5*h^10 - 184380*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^5*d*g^3*h^12 + 413280*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^6*c^4*f*g^3*h^12 + 13440*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^6*c^4*d*g*h^14 - 47040*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^7*c^3*f*g*h^14 - 1024*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^10*g^14*h*e + 4096*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^9*g^12*h^3*e + 32768*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^8*g^10*h^5*e + 205952*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^7*g^8*h^7*e - 741776*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^6*g^6*h^9*e + 608720*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^5*g^4*h^11*e - 92820*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^6*c^4*g^2*h^13*e + 8960*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(19/2)*f*g^14*h + 2688*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(19/2)*d*g^12*h^3 + 15232*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(17/2)*f*g^12*h^3 + 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(17/2)*d*g^10*h^5 - 53200*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(15/2)*f*g^10*h^5 - 342384*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(15/2

)*d*g^8*h^7 - 103936*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(13/2)*f*g^8*h^7 + 736344*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(13/2)*d*g^6*h^9 - 726404*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(11/2)*f*g^6*h^9 - 488460*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(11/2)*d*g^4*h^11 + 764960*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^6*c^(9/2)*f*g^4*h^11 + 33600*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^6*c^(9/2)*d*g^2*h^13 - 168000*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^7*c^(7/2)*f*g^2*h^13 - 6720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^7*c^(7/2)*d*h^15 + 6720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^8*c^(5/2)*f*h^15 + 3584*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(19/2)*g^13*h^2*e + 9856*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(17/2)*g^11*h^4*e + 8960*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(15/2)*g^9*h^6*e - 487312*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(13/2)*g^7*h^8*e + 807520*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(11/2)*g^5*h^10*e - 310660*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^6*c^(9/2)*g^3*h^12*e + 13440*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^7*c^(7/2)*g*h^14*e - 13440*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^9*f*g^13*h^2 - 4032*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^9*d*g^11*h^4 - 50848*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^8*f*g^11*h^4 - 47040*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^8*d*g^9*h^6 - 50960*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^7*f*g^9*h^6 + 438816*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^7*d*g^7*h^8 - 99736*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^6*f*g^7*h^8 - 556416*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^6*d*g^5*h^10 + 728756*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^5*f*g^5*h^10 + 167790*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^5*d*g^3*h^12 - 362915*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^4*f*g^3*h^12 - 10185*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^4*d*g*h^14 + 38360*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^8*c^3*f*g*h^14 - 5376*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^9*g^12*h^3*e - 25984*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^8*g^10*h^5*e - 86240*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^7*g^8*h^7*e + 574448*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^6*g^6*h^9*e - 487480*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^5*g^4*h^11*e + 89740*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^4*g^2*h^13*e + 1085*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^8*c^3*h^15*e + 11200*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(17/2)*f*g^12*h^3 + 3360*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(17/2)*d*g^10*h^5 + 52640*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(15/2)*f*g^10*h^5 + 45360*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(15/2)*d*g^8*h^7 + 96880*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(13/2)*f*g^8*h^7 - 364728*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(13/2)*d*g^6*h^9 + 215908*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(11/2)*f*g^6*h^9 + 220710*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(11/2)*d*g^4*h^11 - 406735*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^7*c^(9/2)*f*g^4*h^11 - 49581*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^7*c^(9/2)*d*g^2*h^13 + 104776*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^8*c^(7/2)*f*g^2*h^13 - 1344*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^8*c^(7/2)*d*h^15 - 3696*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^9*c^(5/2)*f*h^15 + 4480*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(17/2)*g^11*h^4*e + 29120*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(15/2)*g^9*h^6*e + 119056*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(13/2)*g^7*h^8*e - 390656*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(11/2)*g^5*h^10*e + 179900*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^7*c^(9/2)*g^3*h^12*e - 10703*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^8*c^(7/2)*g*h^14*e - 5600*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^8*f*g^11*h^4 - 3360*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^8*d*g^9*h^6 - 29680*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^7*f*g^9*h^6 - 32592*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^7*d*g^7*h^8 - 67088*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^6*f*g^7*h^8 + 172620*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^6*d*g^5*h^10 - 156170*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^5*f*g^5*h^10 - 62454*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^5*d*g^3*h^12 + 140084*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^8*c^4*f*g^3*h^12 + 5964*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^8*c^4*d*g*h^14 - 17024*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^9*c^3*f*g*h^14 - 2240*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^8*g^10*h^5*e - 16576*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^7*g^8*h^7*e - 72464*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^6*g^6*h^9*e + 179200*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^5*g^4*h^11*e - 31402*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^8*c^4*g^2*h^13*e + 1540*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^9*c^3*h^15*e + 1680*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(15/2)*f*

```
g^10*h^5 + 1008*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(15/2)*d*g^8*h^7 + 96
32*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(13/2)*f*g^8*h^7 + 9996*(sqrt(c)*x
 - sqrt(c*x^2 + a))^2*a^6*c^(13/2)*d*g^6*h^9 + 24094*(sqrt(c)*x - sqrt(c*x^
2 + a))^2*a^7*c^(11/2)*f*g^6*h^9 - 54894*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^
7*c^(11/2)*d*g^4*h^11 + 56924*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(9/2)*f
*g^4*h^11 + 9156*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(9/2)*d*g^2*h^13 - 3
2256*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^9*c^(7/2)*f*g^2*h^13 - 672*(sqrt(c)*
x - sqrt(c*x^2 + a))^2*a^9*c^(7/2)*d*h^15 + 672*(sqrt(c)*x - sqrt(c*x^2 + a
))^2*a^10*c^(5/2)*f*h^15 + 1344*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(15/2
)*g^9*h^6*e + 8624*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(13/2)*g^7*h^8*e +
 30352*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(11/2)*g^5*h^10*e - 47362*(sqr
t(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(9/2)*g^3*h^12*e + 3276*(sqrt(c)*x - sqrt
(c*x^2 + a))^2*a^9*c^(7/2)*g*h^14*e - 560*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6
*c^7*f*g^9*h^6 - 168*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^7*d*g^7*h^8 - 3052
*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^6*f*g^7*h^8 - 1680*(sqrt(c)*x - sqrt(c
*x^2 + a))*a^7*c^6*d*g^5*h^10 - 7070*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^5*
f*g^5*h^10 + 9744*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^5*d*g^3*h^12 - 12999*
(sqrt(c)*x - sqrt(c*x^2 + a))*a^9*c^4*f*g^3*h^12 - 1029*(sqrt(c)*x - sqrt(c
*x^2 + a))*a^9*c^4*d*g*h^14 + 3864*(sqrt(c)*x - sqrt(c*x^2 + a))*a^10*c^3*f
*g*h^14 - 224*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^7*g^8*h^7*e - 1456*(sqrt(
c)*x - sqrt(c*x^2 + a))*a^7*c^6*g^6*h^9*e - 5180*(sqrt(c)*x - sqrt(c*x^2 +
a))*a^8*c^5*g^4*h^11*e + 8442*(sqrt(c)*x - sqrt(c*x^2 + a))*a^9*c^4*g^2*h^1
3*e + 105*(sqrt(c)*x - sqrt(c*x^2 + a))*a^10*c^3*h^15*e + 40*a^7*c^(13/2)*f
*g^8*h^7 + 12*a^7*c^(13/2)*d*g^6*h^9 + 218*a^8*c^(11/2)*f*g^6*h^9 + 120*a^8
*c^(11/2)*d*g^4*h^11 + 505*a^9*c^(9/2)*f*g^4*h^11 - 741*a^9*c^(9/2)*d*g^2*h
^13 + 936*a^10*c^(7/2)*f*g^2*h^13 + 96*a^10*c^(7/2)*d*h^15 - 336*a^11*c^(5/
2)*f*h^15 + 16*a^7*c^(13/2)*g^7*h^8*e + 104*a^8*c^(11/2)*g^5*h^10*e + 370*a
^9*c^(9/2)*g^3*h^12*e - 663*a^10*c^(7/2)*g*h^14*e)/((c^5*g^10*h^6 + 5*a*c^4
*g^8*h^8 + 10*a^2*c^3*g^6*h^10 + 10*a^3*c^2*g^4*h^12 + 5*a^4*c*g^2*h^14 + a
^5*h^16)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a
))*sqrt(c)*g - a*h)^7)
```

maple [B] time = 0.05, size = 19093, normalized size = 35.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^2+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^8} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x)
```

```
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**8,x)
```

```
[Out] Timed out
```

3.100 $\int (a+cx^2)^{5/2} (A+Bx+Cx^2)\,dx$

Optimal. Leaf size=168

$$\frac{5a^3(8Ac-aC)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}}+\frac{5a^2x\sqrt{a+cx^2}\,(8Ac-aC)}{128c}+\frac{x\left(a+cx^2\right)^{5/2}(8Ac-aC)}{48c}+\frac{5ax\left(a+cx^2\right)^{3/2}(8Ac}{192c}$$

```
[Out] 5/192*a*(8*A*c-C*a)*x*(c*x^2+a)^(3/2)/c+1/48*(8*A*c-C*a)*x*(c*x^2+a)^(5/2)/
c+1/7*B*(c*x^2+a)^(7/2)/c+1/8*C*x*(c*x^2+a)^(7/2)/c+5/128*a^3*(8*A*c-C*a)*a
rctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+5/128*a^2*(8*A*c-C*a)*x*(c*x^2+a)
^(1/2)/c
```

Rubi [A] time = 0.10, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1815, 641, 195, 217, 206}

$$\frac{5a^3(8Ac-aC)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}}+\frac{5a^2x\sqrt{a+cx^2}\,(8Ac-aC)}{128c}+\frac{x\left(a+cx^2\right)^{5/2}(8Ac-aC)}{48c}+\frac{5ax\left(a+cx^2\right)^{3/2}(8Ac}{192c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)*(A + B*x + C*x^2),x]
```

```
[Out] (5*a^2*(8*A*c - a*C)*x*Sqrt[a + c*x^2])/(128*c) + (5*a*(8*A*c - a*C)*x*(a +
 c*x^2)^(3/2))/(192*c) + ((8*A*c - a*C)*x*(a + c*x^2)^(5/2))/(48*c) + (B*(a
 + c*x^2)^(7/2))/(7*c) + (C*x*(a + c*x^2)^(7/2))/(8*c) + (5*a^3*(8*A*c - a*
C)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
```

```
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+cx^2\right)^{5/2}\left(A+Bx+Cx^2\right)dx &= \frac{Cx\left(a+cx^2\right)^{7/2}}{8c}+\frac{\int (8Ac-aC+8Bcx)\left(a+cx^2\right)^{5/2}dx}{8c}\\
&= \frac{B\left(a+cx^2\right)^{7/2}}{7c}+\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}+\frac{(8Ac-aC)\int \left(a+cx^2\right)^{5/2}dx}{8c}\\
&= \frac{(8Ac-aC)x\left(a+cx^2\right)^{5/2}}{48c}+\frac{B\left(a+cx^2\right)^{7/2}}{7c}+\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}+\frac{(5a(8Ac-a}{}\\
&= \frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-aC)x\left(a+cx^2\right)^{5/2}}{48c}+\frac{B\left(a+cx^2\right)^{7/2}}{7c}+\\
&= \frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c}+\frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-aC)x\left(a\right.}{48c}\\
&= \frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c}+\frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-aC)x\left(a\right.}{48c}\\
&= \frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c}+\frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-aC)x\left(a\right.}{48c}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 150, normalized size = 0.89

$$\frac{\sqrt{a+cx^2}\left(\sqrt{c}\left(3a^3(128B+35Cx)+2a^2cx(924A+x(576B+413Cx))+8ac^2x^3(182A+x(144B+119Cx))+16c\right.\right.}{2688c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(Sqrt[c]*(3*a^3*(128*B + 35*C*x) + 16*c^3*x^5*(28*A + 3*x*
(8*B + 7*C*x)) + 8*a*c^2*x^3*(182*A + x*(144*B + 119*C*x)) + 2*a^2*c*x*(924
*A + x*(576*B + 413*C*x))) - (105*a^(5/2)*(-8*A*c + a*C)*ArcSinh[(Sqrt[c]*x
)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a]))/(2688*c^(3/2))
```

fricas [A] time = 1.32, size = 333, normalized size = 1.98

$$\left[-\frac{105\left(Ca^4-8\,Aa^3c\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\left(336\,Cc^4x^7+384\,Bc^4x^6+1152\,Bac^3x^4+1152\,B\right.}{53}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] [-1/5376*(105*(C*a^4 - 8*A*a^3*c)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*
sqrt(c)*x - a) - 2*(336*C*c^4*x^7 + 384*B*c^4*x^6 + 1152*B*a*c^3*x^4 + 1152
*B*a^2*c^2*x^2 + 56*(17*C*a*c^3 + 8*A*c^4)*x^5 + 384*B*a^3*c + 14*(59*C*a^2
*c^2 + 104*A*a*c^3)*x^3 + 21*(5*C*a^3*c + 88*A*a^2*c^2)*x)*sqrt(c*x^2 + a))
/c^2, 1/2688*(105*(C*a^4 - 8*A*a^3*c)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2
 + a)) + (336*C*c^4*x^7 + 384*B*c^4*x^6 + 1152*B*a*c^3*x^4 + 1152*B*a^2*c^2
*x^2 + 56*(17*C*a*c^3 + 8*A*c^4)*x^5 + 384*B*a^3*c + 14*(59*C*a^2*c^2 + 104
*A*a*c^3)*x^3 + 21*(5*C*a^3*c + 88*A*a^2*c^2)*x)*sqrt(c*x^2 + a))/c^2]
```

giac [A] time = 0.21, size = 168, normalized size = 1.00

$$\frac{1}{2688}\left(\frac{384\,Ba^3}{c} + \left(2\left(576\,Ba^2 + \left(4\left(144\,Bac + \left(6\left(7\,Cc^2x + 8\,Bc^2\right)x + \frac{7\left(17\,Cac^7 + 8\,Ac^8\right)}{c^6}\right)x\right)x + \frac{7\left(59\,Ca^2c^6\right.}{c}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/2688*(384*B*a^3/c + (2*(576*B*a^2 + (4*(144*B*a*c + (6*(7*C*c^2*x + 8*B*c
^2)*x + 7*(17*C*a*c^7 + 8*A*c^8)/c^6)*x)*x + 7*(59*C*a^2*c^6 + 104*A*a*c^7)
/c^6)*x)*x + 21*(5*C*a^3*c^5 + 88*A*a^2*c^6)/c^6)*x)*sqrt(c*x^2 + a) + 5/12
8*(C*a^4 - 8*A*a^3*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

maple [A] time = 0.01, size = 181, normalized size = 1.08

$$\frac{5A\,a^3\ln\left(\sqrt{c}\,x + \sqrt{c\,x^2 + a}\right)}{16\sqrt{c}} - \frac{5C\,a^4\ln\left(\sqrt{c}\,x + \sqrt{c\,x^2 + a}\right)}{128c^{\frac{3}{2}}} + \frac{5\sqrt{c\,x^2 + a}\;A\,a^2x}{16} - \frac{5\sqrt{c\,x^2 + a}\;C\,a^3x}{128c} + \frac{5\left(c\,x^2 + \right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x)
```

```
[Out] 1/8*C*x*(c*x^2+a)^(7/2)/c-1/48*C*a/c*x*(c*x^2+a)^(5/2)-5/192*C*a^2/c*x*(c*x
^2+a)^(3/2)-5/128*C*a^3/c*x*(c*x^2+a)^(1/2)-5/128*C*a^4/c^(3/2)*ln(c^(1/2)*
x+(c*x^2+a)^(1/2))+1/7*B*(c*x^2+a)^(7/2)/c+1/6*A*x*(c*x^2+a)^(5/2)+5/24*A*a
*x*(c*x^2+a)^(3/2)+5/16*A*a^2*x*(c*x^2+a)^(1/2)+5/16*A*a^3/c^(1/2)*ln(c^(1/
2)*x+(c*x^2+a)^(1/2))
```

maxima [A] time = 0.45, size = 166, normalized size = 0.99

$$\frac{1}{6}\left(cx^2 + a\right)^{\frac{5}{2}}Ax + \frac{5}{24}\left(cx^2 + a\right)^{\frac{3}{2}}Aax + \frac{5}{16}\sqrt{cx^2 + a}\,Aa^2x + \frac{\left(cx^2 + a\right)^{\frac{7}{2}}Cx}{8\,c} - \frac{\left(cx^2 + a\right)^{\frac{5}{2}}Cax}{48\,c} - \frac{5\left(cx^2 + a\right)^{\frac{3}{2}}Ca^2x}{192\,c} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2 + a)^(5/2)*A*x + 5/24*(c*x^2 + a)^(3/2)*A*a*x + 5/16*sqrt(c*x^2
+ a)*A*a^2*x + 1/8*(c*x^2 + a)^(7/2)*C*x/c - 1/48*(c*x^2 + a)^(5/2)*C*a*x/c
 - 5/192*(c*x^2 + a)^(3/2)*C*a^2*x/c - 5/128*sqrt(c*x^2 + a)*C*a^3*x/c - 5/
128*C*a^4*arcsinh(c*x/sqrt(a*c))/c^(3/2) + 5/16*A*a^3*arcsinh(c*x/sqrt(a*c)
)/sqrt(c) + 1/7*(c*x^2 + a)^(7/2)*B/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,x^2 + a\right)^{5/2}\,\left(C\,x^2 + B\,x + A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(5/2)*(A + B*x + C*x^2),x)
```

```
[Out] int((a + c*x^2)^(5/2)*(A + B*x + C*x^2), x)
```

sympy [A] time = 32.92, size = 510, normalized size = 3.04

$$\frac{Aa^{\frac{5}{2}}x\sqrt{1 + \frac{cx^2}{a}}}{2} + \frac{3Aa^{\frac{5}{2}}x}{16\sqrt{1 + \frac{cx^2}{a}}} + \frac{35Aa^{\frac{3}{2}}cx^3}{48\sqrt{1 + \frac{cx^2}{a}}} + \frac{17A\sqrt{a}\,c^2x^5}{24\sqrt{1 + \frac{cx^2}{a}}} + \frac{5Aa^3\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16\sqrt{c}} + \frac{Ac^3x^7}{6\sqrt{a}\sqrt{1 + \frac{cx^2}{a}}} + Ba^2\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} \\ \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{3c}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)*(C*x**2+B*x+A),x)
```

```
[Out] A*a**(5/2)*x*sqrt(1 + c*x**2/a)/2 + 3*A*a**(5/2)*x/(16*sqrt(1 + c*x**2/a))
+ 35*A*a**(3/2)*c*x**3/(48*sqrt(1 + c*x**2/a)) + 17*A*sqrt(a)*c**2*x**5/(24
*sqrt(1 + c*x**2/a)) + 5*A*a**3*asinh(sqrt(c)*x/sqrt(a))/(16*sqrt(c)) + A*c
**3*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a)) + B*a**2*Piecewise((sqrt(a)*x**2/2,
 Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + 2*B*a*c*Piecewise((-2*a**2
*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a
+ c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + B*c**2*Piecewise((8*a**3*
sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x
**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*
x**6/6, True)) + 5*C*a**(7/2)*x/(128*c*sqrt(1 + c*x**2/a)) + 133*C*a**(5/2)
*x**3/(384*sqrt(1 + c*x**2/a)) + 127*C*a**(3/2)*c*x**5/(192*sqrt(1 + c*x**2
/a)) + 23*C*sqrt(a)*c**2*x**7/(48*sqrt(1 + c*x**2/a)) - 5*C*a**4*asinh(sqrt
(c)*x/sqrt(a))/(128*c**(3/2)) + C*c**3*x**9/(8*sqrt(a)*sqrt(1 + c*x**2/a))
```

3.101 $\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=325

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3a^2h^2(eh+3fg)-4acg\left(3h(dh+eg)+fg^2\right)+8c^2dg^3\right)}{8c^{5/2}}+\frac{\sqrt{a+cx^2}\left(4\left(16a^2fh^4-4ach^2\left(5h(\right.\right.\right.}{}$$

```
[Out] 1/8*(8*c^2*d*g^3+3*a^2*h^2*(e*h+3*f*g)-4*a*c*g*(f*g^2+3*h*(d*h+e*g)))*arcta
nh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(5/2)+1/60*(4*(-4*a*f+5*c*d)*h^2-3*c*g*(-5*
e*h+f*g))*(h*x+g)^2*(c*x^2+a)^(1/2)/c^2/h-1/20*(-5*e*h+f*g)*(h*x+g)^3*(c*x^
2+a)^(1/2)/c/h+1/5*f*(h*x+g)^4*(c*x^2+a)^(1/2)/c/h+1/120*(64*a^2*f*h^4-16*a
*c*h^2*(13*f*g^2+5*h*(d*h+3*e*g))-4*c^2*g^2*(3*f*g^2-5*h*(16*d*h+3*e*g))-c*
h*(a*h^2*(45*e*h+71*f*g)+2*c*g*(3*f*g^2-5*h*(10*d*h+3*e*g)))*x)*(c*x^2+a)^(
1/2)/c^3/h
```

Rubi [A] time = 0.66, antiderivative size = 323, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1654, 833, 780, 217, 206}

$$\frac{\sqrt{a+cx^2}\left(4\left(16a^2fh^4-4ach^2\left(5h(dh+3eg)+13fg^2\right)-c^2g^2\left(3fg^2-5h(16dh+3eg)\right)\right)-chx\left(ah^2(45eh+71\right.\right.}{120c^3h}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] ((4*(5*c*d - 4*a*f)*h^2 - 3*c*g*(f*g - 5*e*h))*(g + h*x)^2*Sqrt[a + c*x^2])
/(60*c^2*h) - ((f*g - 5*e*h)*(g + h*x)^3*Sqrt[a + c*x^2])/(20*c*h) + (f*(g
+ h*x)^4*Sqrt[a + c*x^2])/(5*c*h) + ((4*(16*a^2*f*h^4 - 4*a*c*h^2*(13*f*g^2
 + 5*h*(3*e*g + d*h)) - c^2*g^2*(3*f*g^2 - 5*h*(3*e*g + 16*d*h))) - c*h*(6*
c*f*g^3 - 10*c*g*h*(3*e*g + 10*d*h) + a*h^2*(71*f*g + 45*e*h))*x)*Sqrt[a +
c*x^2])/(120*c^3*h) + ((8*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 4*a*c*g*(f*
g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
```

```
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}dx = \frac{f(g+hx)^4\sqrt{a+cx^2}}{5ch} + \frac{\int \frac{(g+hx)^3\left((5cd-4af)h^2-ch(fg-5eh)x\right)}{\sqrt{a+cx^2}}dx}{5ch^2}$$

$$= -\frac{(fg-5eh)(g+hx)^3\sqrt{a+cx^2}}{20ch} + \frac{f(g+hx)^4\sqrt{a+cx^2}}{5ch} + \frac{\int \frac{(g+hx)^2\left(ch^2(20cdg-13af\right.}{}}{}$$

$$= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+hx)^3\sqrt{a}}{20ch}$$

$$= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+hx)^3\sqrt{a}}{20ch}$$

$$= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+hx)^3\sqrt{a}}{20ch}$$

$$= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+hx)^3\sqrt{a}}{20ch}$$

Mathematica [A] time = 0.35, size = 252, normalized size = 0.78

$$15\sqrt{c}\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)\left(3a^2h^2(eh+3fg)-4acg\left(3h(dh+eg)+fg^2\right)+8c^2dg^3\right)+\sqrt{a+cx^2}\left(8\left(8a^2fh^3-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[a + c*x^2]*(8*(8*a^2*f*h^3 + 15*c^2*g^2*(e*g + 3*d*h) - 10*a*c*h*(3*f
*g^2 + h*(3*e*g + d*h))) + 15*c*(-3*a*h^2*(3*f*g + e*h) + 4*c*(f*g^3 + 3*g*
h*(e*g + d*h)))*x + 8*c*h*(-4*a*f*h^2 + 5*c*(3*f*g^2 + h*(3*e*g + d*h)))*x^
2 + 30*c^2*h^2*(3*f*g + e*h)*x^3 + 24*c^2*f*h^3*x^4) + 15*Sqrt[c]*(8*c^2*d*
g^3 + 3*a^2*h^2*(3*f*g + e*h) - 4*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*Log[c*x
+ Sqrt[c]*Sqrt[a + c*x^2]])/(120*c^3)
```

fricas [A] time = 1.22, size = 559, normalized size = 1.72

$$\left[-\frac{15\left(12\,aceg^2h-3\,a^2eh^3-4\left(2\,c^2d-acf\right)g^3+3\left(4\,acd-3\,a^2f\right)gh^2\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/240*(15*(12*a*c*e*g^2*h - 3*a^2*e*h^3 - 4*(2*c^2*d - a*c*f)*g^3 + 3*(4*
a*c*d - 3*a^2*f)*g*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x
- a) - 2*(24*c^2*f*h^3*x^4 + 120*c^2*e*g^3 - 240*a*c*e*g*h^2 + 120*(3*c^2*d
 - 2*a*c*f)*g^2*h - 16*(5*a*c*d - 4*a^2*f)*h^3 + 30*(3*c^2*f*g*h^2 + c^2*e*
h^3)*x^3 + 8*(15*c^2*f*g^2*h + 15*c^2*e*g*h^2 + (5*c^2*d - 4*a*c*f)*h^3)*x^
2 + 15*(4*c^2*f*g^3 + 12*c^2*e*g^2*h - 3*a*c*e*h^3 + 3*(4*c^2*d - 3*a*c*f)*
g*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/120*(15*(12*a*c*e*g^2*h - 3*a^2*e*h^3 - 4
*(2*c^2*d - a*c*f)*g^3 + 3*(4*a*c*d - 3*a^2*f)*g*h^2)*sqrt(-c)*arctan(sqrt(
-c)*x/sqrt(c*x^2 + a)) + (24*c^2*f*h^3*x^4 + 120*c^2*e*g^3 - 240*a*c*e*g*h^
2 + 120*(3*c^2*d - 2*a*c*f)*g^2*h - 16*(5*a*c*d - 4*a^2*f)*h^3 + 30*(3*c^2*
f*g*h^2 + c^2*e*h^3)*x^3 + 8*(15*c^2*f*g^2*h + 15*c^2*e*g*h^2 + (5*c^2*d -
4*a*c*f)*h^3)*x^2 + 15*(4*c^2*f*g^3 + 12*c^2*e*g^2*h - 3*a*c*e*h^3 + 3*(4*c
^2*d - 3*a*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

giac [A] time = 0.26, size = 314, normalized size = 0.97

$$\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(\frac{4\,fh^3x}{c}+\frac{5\left(3\,c^4fgh^2+c^4h^3e\right)}{c^5}\right)x+\frac{4\left(15\,c^4fg^2h+5\,c^4dh^3-4\,ac^3fh^3+15\,c^4gh^2e\right)}{c^5}\right)x+\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(c*x^2 + a)*((2*(3*(4*f*h^3*x/c + 5*(3*c^4*f*g*h^2 + c^4*h^3*e)/c
^5)*x + 4*(15*c^4*f*g^2*h + 5*c^4*d*h^3 - 4*a*c^3*f*h^3 + 15*c^4*g*h^2*e)/c
^5)*x + 15*(4*c^4*f*g^3 + 12*c^4*d*g*h^2 - 9*a*c^3*f*g*h^2 + 12*c^4*g^2*h*e
 - 3*a*c^3*h^3*e)/c^5)*x + 8*(45*c^4*d*g^2*h - 30*a*c^3*f*g^2*h - 10*a*c^3*
d*h^3 + 8*a^2*c^2*f*h^3 + 15*c^4*g^3*e - 30*a*c^3*g*h^2*e)/c^5) - 1/8*(8*c^
2*d*g^3 - 4*a*c*f*g^3 - 12*a*c*d*g*h^2 + 9*a^2*f*g*h^2 - 12*a*c*g^2*h*e + 3
*a^2*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

maple [A] time = 0.02, size = 528, normalized size = 1.62

$$\frac{\sqrt{c\,x^2+a}\;f\,h^3x^4}{5c}+\frac{\sqrt{c\,x^2+a}\;e\,h^3x^3}{4c}+\frac{3\sqrt{c\,x^2+a}\;fg\,h^2x^3}{4c}-\frac{4\sqrt{c\,x^2+a}\;af\,h^3x^2}{15c^2}+\frac{\sqrt{c\,x^2+a}\;d\,h^3x^2}{3c}+\frac{\sqrt{c\,x^2+a}\;\cdots}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/5*h^3*f*x^4/c*(c*x^2+a)^(1/2)-4/15*h^3*f*a/c^2*x^2*(c*x^2+a)^(1/2)+8/15*h
^3*f*a^2/c^3*(c*x^2+a)^(1/2)+1/4*x^3/c*(c*x^2+a)^(1/2)*h^3*e+3/4*x^3/c*(c*x
^2+a)^(1/2)*g*h^2*f-3/8*a/c^2*x*(c*x^2+a)^(1/2)*h^3*e-9/8*a/c^2*x*(c*x^2+a)
^(1/2)*g*h^2*f+3/8*a^2/c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*h^3*e+9/8*a^2/
c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*g*h^2*f+1/3*x^2/c*(c*x^2+a)^(1/2)*h^3
*d+x^2/c*(c*x^2+a)^(1/2)*g*h^2*e+x^2/c*(c*x^2+a)^(1/2)*g^2*h*f-2/3*a/c^2*(c
*x^2+a)^(1/2)*h^3*d-2*a/c^2*(c*x^2+a)^(1/2)*g*h^2*e-2*a/c^2*(c*x^2+a)^(1/2)
*g^2*h*f+3/2*x/c*(c*x^2+a)^(1/2)*g*h^2*d+3/2*x/c*(c*x^2+a)^(1/2)*g^2*h*e+1/
2*x/c*(c*x^2+a)^(1/2)*g^3*f-3/2*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*g*h
```

```
^2*d-3/2*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*g^2*h*e-1/2*a/c^(3/2)*ln(c
^(1/2)*x+(c*x^2+a)^(1/2))*g^3*f+3/c*(c*x^2+a)^(1/2)*g^2*h*d+1/c*(c*x^2+a)^(
1/2)*g^3*e+g^3*d*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)
```

maxima [A] time = 0.45, size = 349, normalized size = 1.07

$$\frac{\sqrt{cx^2+a}\,fh^3x^4}{5\,c}-\frac{4\,\sqrt{cx^2+a}\,afh^3x^2}{15\,c^2}+\frac{dg^3\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}+\frac{\sqrt{cx^2+a}\,eg^3}{c}+\frac{3\,\sqrt{cx^2+a}\,dg^2h}{c}+\frac{8\,\sqrt{cx^2+a}\,a^2fh^3}{15\,c^3}+\frac{(3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(c*x^2 + a)*f*h^3*x^4/c - 4/15*sqrt(c*x^2 + a)*a*f*h^3*x^2/c^2 + d*
g^3*arcsinh(c*x/sqrt(a*c))/sqrt(c) + sqrt(c*x^2 + a)*e*g^3/c + 3*sqrt(c*x^2
 + a)*d*g^2*h/c + 8/15*sqrt(c*x^2 + a)*a^2*f*h^3/c^3 + 1/4*(3*f*g*h^2 + e*h
^3)*sqrt(c*x^2 + a)*x^3/c + 1/3*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*sqrt(c*x^2
+ a)*x^2/c - 3/8*(3*f*g*h^2 + e*h^3)*sqrt(c*x^2 + a)*a*x/c^2 + 1/2*(f*g^3 +
 3*e*g^2*h + 3*d*g*h^2)*sqrt(c*x^2 + a)*x/c + 3/8*(3*f*g*h^2 + e*h^3)*a^2*a
rcsinh(c*x/sqrt(a*c))/c^(5/2) - 1/2*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*a*arcsi
nh(c*x/sqrt(a*c))/c^(3/2) - 2/3*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*sqrt(c*x^2
+ a)*a/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g+h\,x\right)^3\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(1/2),x)
```

```
[Out] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(1/2), x)
```

sympy [A] time = 22.20, size = 796, normalized size = 2.45

$$-\frac{3a^{\frac{3}{2}}eh^3x}{8c^2\sqrt{1+\frac{cx^2}{a}}}-\frac{9a^{\frac{3}{2}}fgh^2x}{8c^2\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}\,dgh^2x\sqrt{1+\frac{cx^2}{a}}}{2c}+\frac{3\sqrt{a}\,eg^2hx\sqrt{1+\frac{cx^2}{a}}}{2c}-\frac{\sqrt{a}\,eh^3x^3}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}\,fg^3x\sqrt{1+\frac{cx^2}{a}}}{2c}-\frac{3\sqrt{}}{8c\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*e*h**3*x/(8*c**2*sqrt(1 + c*x**2/a)) - 9*a**(3/2)*f*g*h**2*x/(8
*c**2*sqrt(1 + c*x**2/a)) + 3*sqrt(a)*d*g*h**2*x*sqrt(1 + c*x**2/a)/(2*c) +
 3*sqrt(a)*e*g**2*h*x*sqrt(1 + c*x**2/a)/(2*c) - sqrt(a)*e*h**3*x**3/(8*c*s
qrt(1 + c*x**2/a)) + sqrt(a)*f*g**3*x*sqrt(1 + c*x**2/a)/(2*c) - 3*sqrt(a)*
f*g*h**2*x**3/(8*c*sqrt(1 + c*x**2/a)) + 3*a**2*e*h**3*asinh(sqrt(c)*x/sqrt
(a))/(8*c**(5/2)) + 9*a**2*f*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(5/2)) -
 3*a*d*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(2*c**(3/2)) - 3*a*e*g**2*h*asinh(sq
rt(c)*x/sqrt(a))/(2*c**(3/2)) - a*f*g**3*asinh(sqrt(c)*x/sqrt(a))/(2*c**(3/
2)) + d*g**3*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a), (a > 0) & (c
 < 0)), (sqrt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c > 0)), (sqrt(-a
/c)*acosh(x*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + 3*d*g**2*h*Piecewis
```

```
e((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True)) + d*h**3*Piecew
ise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c), Ne(c, 0)
), (x**4/(4*sqrt(a)), True)) + e*g**3*Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)
), (sqrt(a + c*x**2)/c, True)) + 3*e*g*h**2*Piecewise((-2*a*sqrt(a + c*x**2
)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c), Ne(c, 0)), (x**4/(4*sqrt(a)), Tru
e)) + 3*f*g**2*h*Piecewise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a +
c*x**2)/(3*c), Ne(c, 0)), (x**4/(4*sqrt(a)), True)) + f*h**3*Piecewise((8*a
**2*sqrt(a + c*x**2)/(15*c**3) - 4*a*x**2*sqrt(a + c*x**2)/(15*c**2) + x**4
*sqrt(a + c*x**2)/(5*c), Ne(c, 0)), (x**6/(6*sqrt(a)), True)) + e*h**3*x**5
/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + 3*f*g*h**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**
2/a))
```

3.102 $\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=223

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^2-4ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)}{8c^{5/2}}-\frac{\sqrt{a+cx^2}\left(4\left(4ah^2(eh+2fg)+cg\left(fg^2-4h(3dh\right.\right.\right.}{24}$$

```
[Out] 1/8*(8*c^2*d*g^2+3*a^2*f*h^2-4*a*c*(f*g^2+h*(d*h+2*e*g)))*arctanh(x*c^(1/2)
/(c*x^2+a)^(1/2))/c^(5/2)-1/12*(-4*e*h+f*g)*(h*x+g)^2*(c*x^2+a)^(1/2)/c/h+1
/4*f*(h*x+g)^3*(c*x^2+a)^(1/2)/c/h-1/24*(16*a*h^2*(e*h+2*f*g)+4*c*g*(f*g^2-
4*h*(3*d*h+e*g))-h*(3*(-3*a*f+4*c*d)*h^2-2*c*g*(-4*e*h+f*g))*x)*(c*x^2+a)^(
1/2)/c^2/h
```

Rubi [A] time = 0.37, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1654, 833, 780, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^2-4ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)}{8c^{5/2}}-\frac{\sqrt{a+cx^2}\left(4\left(4ah^2(eh+2fg)-4cgh(3dh+eg)+\right.\right.}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] -((f*g - 4*e*h)*(g + h*x)^2*Sqrt[a + c*x^2])/(12*c*h) + (f*(g + h*x)^3*Sqrt
[a + c*x^2])/(4*c*h) - ((4*(c*f*g^3 - 4*c*g*h*(e*g + 3*d*h) + 4*a*h^2*(2*f*
g + e*h)) - h*(3*(4*c*d - 3*a*f)*h^2 - 2*c*g*(f*g - 4*e*h))*x)*Sqrt[a + c*x
^2])/(24*c^2*h) + ((8*c^2*d*g^2 + 3*a^2*f*h^2 - 4*a*c*(f*g^2 + h*(2*e*g + d
*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx = \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} + \frac{\int \frac{(g+hx)^2\left((4cd-3af)h^2-ch(fg-4eh)x\right)}{\sqrt{a+cx^2}}\,dx}{4ch^2}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} + \frac{\int \frac{(g+hx)\left(ch^2(12cdg-7a\right.}{}}{}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} - \frac{\left(4\left(cfg^3-4cgh(eg\right.\right.}{}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} - \frac{\left(4\left(cfg^3-4cgh(eg\right.\right.}{}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} - \frac{\left(4\left(cfg^3-4cgh(eg\right.\right.}{}$$

Mathematica [A] time = 0.23, size = 164, normalized size = 0.74

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^2-4ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)+\sqrt{c}\sqrt{a+cx^2}\left(2c\left(6dh(4g+hx)+4e\left(3g^2+\right.\right.\right.}{24c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a + c*x^2]*(-(a*h*(32*f*g + 16*e*h + 9*f*h*x)) + 2*c*(6*d*h*(
4*g + h*x) + 4*e*(3*g^2 + 3*g*h*x + h^2*x^2) + f*x*(6*g^2 + 8*g*h*x + 3*h^2
*x^2))) + 3*(8*c^2*d*g^2 + 3*a^2*f*h^2 - 4*a*c*(f*g^2 + h*(2*e*g + d*h)))*A
rcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(24*c^(5/2))
```

fricas [A] time = 0.68, size = 381, normalized size = 1.71

$$\left[-\frac{3\left(8acegh-4\left(2c^2d-acf\right)g^2+\left(4acd-3a^2f\right)h^2\right)\sqrt{c}\log\left(-2cx^2-2\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(6c^2fh^2x^3+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(8*a*c*e*g*h - 4*(2*c^2*d - a*c*f)*g^2 + (4*a*c*d - 3*a^2*f)*h^2)
*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(6*c^2*f*h^2*x
```

```
^3 + 24*c^2*e*g^2 - 16*a*c*e*h^2 + 16*(3*c^2*d - 2*a*c*f)*g*h + 8*(2*c^2*f*
g*h + c^2*e*h^2)*x^2 + 3*(4*c^2*f*g^2 + 8*c^2*e*g*h + (4*c^2*d - 3*a*c*f)*h
^2)*x)*sqrt(c*x^2 + a))/c^3, 1/24*(3*(8*a*c*e*g*h - 4*(2*c^2*d - a*c*f)*g^2
 + (4*a*c*d - 3*a^2*f)*h^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (
6*c^2*f*h^2*x^3 + 24*c^2*e*g^2 - 16*a*c*e*h^2 + 16*(3*c^2*d - 2*a*c*f)*g*h
+ 8*(2*c^2*f*g*h + c^2*e*h^2)*x^2 + 3*(4*c^2*f*g^2 + 8*c^2*e*g*h + (4*c^2*d
 - 3*a*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

giac [A] time = 0.23, size = 206, normalized size = 0.92

$$\frac{1}{24}\sqrt{cx^2+a}\left(\left(2\left(\frac{3fh^2x}{c}+\frac{4\left(2c^3fgh+c^3h^2e\right)}{c^4}\right)x+\frac{3\left(4c^3fg^2+4c^3dh^2-3ac^2fh^2+8c^3ghe\right)}{c^4}\right)x+\frac{8\left(6c^3dgh-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + a)*((2*(3*f*h^2*x/c + 4*(2*c^3*f*g*h + c^3*h^2*e)/c^4)*x
+ 3*(4*c^3*f*g^2 + 4*c^3*d*h^2 - 3*a*c^2*f*h^2 + 8*c^3*g*h*e)/c^4)*x + 8*(6
*c^3*d*g*h - 4*a*c^2*f*g*h + 3*c^3*g^2*e - 2*a*c^2*h^2*e)/c^4) - 1/8*(8*c^2
*d*g^2 - 4*a*c*f*g^2 - 4*a*c*d*h^2 + 3*a^2*f*h^2 - 8*a*c*g*h*e)*log(abs(-sq
rt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

maple [A] time = 0.01, size = 339, normalized size = 1.52

$$\frac{\sqrt{c\,x^2+a}\;f\,h^2x^3}{4c}+\frac{\sqrt{c\,x^2+a}\;e\,h^2x^2}{3c}+\frac{2\sqrt{c\,x^2+a}\;fgh\,x^2}{3c}+\frac{3a^2f\,h^2\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{8c^{\frac{5}{2}}}-\frac{ad\,h^2\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2\right.}}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/4*h^2*f*x^3/c*(c*x^2+a)^(1/2)-3/8*h^2*f*a/c^2*x*(c*x^2+a)^(1/2)+3/8*h^2*f
*a^2/c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+1/3*x^2/c*(c*x^2+a)^(1/2)*h^2*e+
2/3*x^2/c*(c*x^2+a)^(1/2)*g*h*f-2/3*a/c^2*(c*x^2+a)^(1/2)*h^2*e-4/3*a/c^2*(
c*x^2+a)^(1/2)*g*h*f+1/2*x/c*(c*x^2+a)^(1/2)*d*h^2+x/c*(c*x^2+a)^(1/2)*e*g*
h+1/2*x/c*(c*x^2+a)^(1/2)*f*g^2-1/2*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
*d*h^2-a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*g*h-1/2*a/c^(3/2)*ln(c^(1/
2)*x+(c*x^2+a)^(1/2))*f*g^2+2/c*(c*x^2+a)^(1/2)*g*h*d+1/c*(c*x^2+a)^(1/2)*g
^2*e+g^2*d*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)
```

maxima [A] time = 0.45, size = 230, normalized size = 1.03

$$\frac{\sqrt{cx^2+a}\,fh^2x^3}{4\,c}-\frac{3\,\sqrt{cx^2+a}\,afh^2x}{8\,c^2}+\frac{dg^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}+\frac{3\,a^2fh^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{5}{2}}}+\frac{\sqrt{cx^2+a}\,eg^2}{c}+\frac{2\,\sqrt{cx^2+a}\,dgh}{c}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^2 + a)*f*h^2*x^3/c - 3/8*sqrt(c*x^2 + a)*a*f*h^2*x/c^2 + d*g^2
*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 3/8*a^2*f*h^2*arcsinh(c*x/sqrt(a*c))/c^(5
/2) + sqrt(c*x^2 + a)*e*g^2/c + 2*sqrt(c*x^2 + a)*d*g*h/c + 1/3*(2*f*g*h +
e*h^2)*sqrt(c*x^2 + a)*x^2/c + 1/2*(f*g^2 + 2*e*g*h + d*h^2)*sqrt(c*x^2 + a
)*x/c - 1/2*(f*g^2 + 2*e*g*h + d*h^2)*a*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/
3*(2*f*g*h + e*h^2)*sqrt(c*x^2 + a)*a/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(g+h\,x\right)^2\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(1/2),x)
```

```
[Out] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(1/2), x)
```

sympy [A] time = 15.78, size = 518, normalized size = 2.32

$$-\frac{3a^{\frac{3}{2}}fh^2x}{8c^2\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}\,dh^2x\sqrt{1+\frac{cx^2}{a}}}{2c}+\frac{\sqrt{a}\,eghx\sqrt{1+\frac{cx^2}{a}}}{c}+\frac{\sqrt{a}\,fg^2x\sqrt{1+\frac{cx^2}{a}}}{2c}-\frac{\sqrt{a}\,fh^2x^3}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{3a^2fh^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*f*h**2*x/(8*c**2*sqrt(1 + c*x**2/a)) + sqrt(a)*d*h**2*x*sqrt(1
+ c*x**2/a)/(2*c) + sqrt(a)*e*g*h*x*sqrt(1 + c*x**2/a)/c + sqrt(a)*f*g**2*x
*sqrt(1 + c*x**2/a)/(2*c) - sqrt(a)*f*h**2*x**3/(8*c*sqrt(1 + c*x**2/a)) +
3*a**2*f*h**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(5/2)) - a*d*h**2*asinh(sqrt(c
)*x/sqrt(a))/(2*c**(3/2)) - a*e*g*h*asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - a*f
*g**2*asinh(sqrt(c)*x/sqrt(a))/(2*c**(3/2)) + d*g**2*Piecewise((sqrt(-a/c)*
asin(x*sqrt(-c/a))/sqrt(a), (a > 0) & (c < 0)), (sqrt(a/c)*asinh(x*sqrt(c/a
))/sqrt(a), (a > 0) & (c > 0)), (sqrt(-a/c)*acosh(x*sqrt(-c/a))/sqrt(-a), (
c > 0) & (a < 0))) + 2*d*g*h*Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(
a + c*x**2)/c, True)) + e*g**2*Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqr
t(a + c*x**2)/c, True)) + e*h**2*Piecewise((-2*a*sqrt(a + c*x**2)/(3*c**2)
+ x**2*sqrt(a + c*x**2)/(3*c), Ne(c, 0)), (x**4/(4*sqrt(a)), True)) + 2*f*g
*h*Piecewise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c),
 Ne(c, 0)), (x**4/(4*sqrt(a)), True)) + f*h**2*x**5/(4*sqrt(a)*sqrt(1 + c*x
**2/a))
```

3.103 $\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=136

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(2cdg-a(eh+fg))}{2c^{3/2}}-\frac{\sqrt{a+cx^2}\,\left(2\left(2afh^2+c\left(fg^2-3h(dh+eg)\right)\right)+chx(fg-3eh)\right)}{6c^2h}+\frac{f\sqrt{a+cx}}{3}$$

```
[Out] 1/2*(2*c*d*g-a*(e*h+f*g))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+1/3*f*
(h*x+g)^2*(c*x^2+a)^(1/2)/c/h-1/6*(4*a*f*h^2+2*c*(f*g^2-3*h*(d*h+e*g))+c*h*
(-3*e*h+f*g)*x)*(c*x^2+a)^(1/2)/c^2/h
```

Rubi [A] time = 0.18, antiderivative size = 135, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1654, 780, 217, 206}

$$-\frac{\sqrt{a+cx^2}\,\left(2\left(2afh^2-3ch(dh+eg)+cfg^2\right)+chx(fg-3eh)\right)}{6c^2h}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(2cdg-a(eh+fg))}{2c^{3/2}}+\frac{f\sqrt{a+cx}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (f*(g + h*x)^2*Sqrt[a + c*x^2])/(3*c*h) - ((2*(c*f*g^2 + 2*a*f*h^2 - 3*c*h*
(e*g + d*h)) + c*h*(f*g - 3*e*h)*x)*Sqrt[a + c*x^2])/(6*c^2*h) + ((2*c*d*g
- a*(f*g + e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx = \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} + \frac{\int \frac{(g+hx)\left((3cd-2af)h^2-ch(fg-3eh)x\right)}{\sqrt{a+cx^2}}\,dx}{3ch^2}$$

$$= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} - \frac{\left(2\left(cfg^2+2afh^2-3ch(eg+dh)\right)+ch(fg-3eh)x\right)\sqrt{}}{6c^2h}$$

$$= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} - \frac{\left(2\left(cfg^2+2afh^2-3ch(eg+dh)\right)+ch(fg-3eh)x\right)\sqrt{}}{6c^2h}$$

$$= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} - \frac{\left(2\left(cfg^2+2afh^2-3ch(eg+dh)\right)+ch(fg-3eh)x\right)\sqrt{}}{6c^2h}$$

Mathematica [A] time = 0.10, size = 96, normalized size = 0.71

$$\frac{\sqrt{a+cx^2}\left(c\left(6dh+6eg+3ehx+3fgx+2fhx^2\right)-4afh\right)+3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(2cdg-a(eh+fg))}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[a + c*x^2]*(-4*a*f*h + c*(6*e*g + 6*d*h + 3*f*g*x + 3*e*h*x + 2*f*h*x
^2)) + 3*Sqrt[c]*(2*c*d*g - a*(f*g + e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x
^2]])/(6*c^2)
```

fricas [A] time = 0.87, size = 199, normalized size = 1.46

$$\left[\frac{3\left(aeh-\left(2cd-af\right)g\right)\sqrt{c}\log\left(-2cx^2+2\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(2cfhx^2+6ceg+2\left(3cd-2af\right)h+3\left(cfg\right.\right.}{12c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*e*h - (2*c*d - a*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)
*sqrt(c)*x - a) + 2*(2*c*f*h*x^2 + 6*c*e*g + 2*(3*c*d - 2*a*f)*h + 3*(c*f*g
 + c*e*h)*x)*sqrt(c*x^2 + a))/c^2, 1/6*(3*(a*e*h - (2*c*d - a*f)*g)*sqrt(-c
)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2*c*f*h*x^2 + 6*c*e*g + 2*(3*c*d -
2*a*f)*h + 3*(c*f*g + c*e*h)*x)*sqrt(c*x^2 + a))/c^2]
```

giac [A] time = 0.21, size = 110, normalized size = 0.81

$$\frac{1}{6}\sqrt{cx^2+a}\left(\left(\frac{2fhx}{c}+\frac{3\left(c^2fg+c^2he\right)}{c^3}\right)x+\frac{2\left(3c^2dh-2acfh+3c^2ge\right)}{c^3}\right)-\frac{\left(2cdg-afg-ahe\right)\log\left(\left|-\sqrt{c}x+\right.\right.}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2 + a)*((2*f*h*x/c + 3*(c^2*f*g + c^2*h*e)/c^3)*x + 2*(3*c^2*d
*h - 2*a*c*f*h + 3*c^2*g*e)/c^3) - 1/2*(2*c*d*g - a*f*g - a*h*e)*log(abs(-s
qrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

maple [A] time = 0.01, size = 172, normalized size = 1.26

$$\frac{\sqrt{c\,x^2+a}\;fh\,x^2}{3c}-\frac{aeh\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}-\frac{afg\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}+\frac{dg\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}+\frac{\sqrt{c\,x^2+a}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/3*h*f*x^2/c*(c*x^2+a)^(1/2)-2/3*h*f*a/c^2*(c*x^2+a)^(1/2)+1/2*x/c*(c*x^2+
a)^(1/2)*e*h+1/2*x/c*(c*x^2+a)^(1/2)*f*g-1/2*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+
a)^(1/2))*e*h-1/2*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*f*g+1/c*(c*x^2+a)
^(1/2)*d*h+1/c*(c*x^2+a)^(1/2)*e*g+d*g*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2
)
```

maxima [A] time = 0.44, size = 126, normalized size = 0.93

$$\frac{\sqrt{cx^2+a}\,fhx^2}{3\,c}+\frac{dg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}+\frac{\sqrt{cx^2+a}\,eg}{c}+\frac{\sqrt{cx^2+a}\,dh}{c}-\frac{2\,\sqrt{cx^2+a}\,afh}{3\,c^2}+\frac{\sqrt{cx^2+a}\,(fg+eh)x}{2\,c}-\frac{(fg+eh)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c*x^2 + a)*f*h*x^2/c + d*g*arcsinh(c*x/sqrt(a*c))/sqrt(c) + sqrt(c
*x^2 + a)*e*g/c + sqrt(c*x^2 + a)*d*h/c - 2/3*sqrt(c*x^2 + a)*a*f*h/c^2 + 1
/2*sqrt(c*x^2 + a)*(f*g + e*h)*x/c - 1/2*(f*g + e*h)*a*arcsinh(c*x/sqrt(a*c
))/c^(3/2)
```

mupad [B] time = 5.17, size = 227, normalized size = 1.67

$$\begin{cases} \frac{2\,f\,g\,x^3+3\,e\,g\,x^2+6\,d\,g\,x}{6\,\sqrt{a}}+\frac{3\,f\,h\,x^4+4\,e\,h\,x^3+6\,d\,h\,x^2}{12\,\sqrt{a}} \\ \frac{d\,g\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}+\frac{d\,h\,\sqrt{c\,x^2+a}}{c}+\frac{e\,g\,\sqrt{c\,x^2+a}}{c}+\frac{e\,h\,x\,\sqrt{c\,x^2+a}}{2\,c}+\frac{f\,g\,x\,\sqrt{c\,x^2+a}}{2\,c}-\frac{f\,h\,\sqrt{c\,x^2+a}\,\left(2\,a-c\,x^2\right)}{3\,c^2}-\frac{a\,e\,h\,\ln\left(2\,\sqrt{c}\,x+2\,\sqrt{c\,x^2}\right.}{2\,c^{3/2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(1/2),x)
```

```
[Out] piecewise(c == 0, (3*e*g*x^2 + 2*f*g*x^3 + 6*d*g*x)/(6*a^(1/2)) + (6*d*h*x^
2 + 4*e*h*x^3 + 3*f*h*x^4)/(12*a^(1/2)), c ~= 0, (d*g*log(c^(1/2)*x + (a +
c*x^2)^(1/2)))/c^(1/2) + (d*h*(a + c*x^2)^(1/2))/c + (e*g*(a + c*x^2)^(1/2)
)/c + (e*h*x*(a + c*x^2)^(1/2))/(2*c) + (f*g*x*(a + c*x^2)^(1/2))/(2*c) - (
f*h*(a + c*x^2)^(1/2)*(2*a - c*x^2))/(3*c^2) - (a*e*h*log(2*c^(1/2)*x + 2*(
a + c*x^2)^(1/2)))/(2*c^(3/2)) - (a*f*g*log(2*c^(1/2)*x + 2*(a + c*x^2)^(1/
2)))/(2*c^(3/2)))
```

sympy [A] time = 9.02, size = 282, normalized size = 2.07

$$\frac{\sqrt{a}\,ehx\sqrt{1+\frac{cx^2}{a}}}{2c}+\frac{\sqrt{a}\,fgx\sqrt{1+\frac{cx^2}{a}}}{2c}-\frac{aeh\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}}-\frac{afg\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}}+dg\begin{cases}\frac{\sqrt{-\frac{a}{c}}\,\mathrm{asin}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge c \\ \frac{\sqrt{\frac{a}{c}}\,\mathrm{asinh}\left(x\sqrt{\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge c \\ \frac{\sqrt{-\frac{a}{c}}\,\mathrm{acosh}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{-a}} & \text{for } c>0\wedge a\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*e*h*x*sqrt(1 + c*x**2/a)/(2*c) + sqrt(a)*f*g*x*sqrt(1 + c*x**2/a)/(
2*c) - a*e*h*asinh(sqrt(c)*x/sqrt(a))/(2*c**(3/2)) - a*f*g*asinh(sqrt(c)*x/
sqrt(a))/(2*c**(3/2)) + d*g*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a
), (a > 0) & (c < 0)), (sqrt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c
> 0)), (sqrt(-a/c)*acosh(x*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + d*h*
Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True)) + e*g*P
iecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True)) + f*h*Pi
ecewise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c), Ne(c
, 0)), (x**4/(4*sqrt(a)), True))
```

3.104 $\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=74

$$\frac{(2cd-af)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}+\frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}$$

```
[Out] 1/2*(-a*f+2*c*d)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+e*(c*x^2+a)^(1/
2)/c+1/2*f*x*(c*x^2+a)^(1/2)/c
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1815, 641, 217, 206}

$$\frac{(2cd-af)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}+\frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/Sqrt[a + c*x^2],x]
```

```
[Out] (e*Sqrt[a + c*x^2])/c + (f*x*Sqrt[a + c*x^2])/(2*c) + ((2*c*d - a*f)*ArcTan
h[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx = \frac{fx\sqrt{a+cx^2}}{2c} + \frac{\int \frac{2cd-af+2cex}{\sqrt{a+cx^2}}\,dx}{2c}$$

$$= \frac{e\sqrt{a+cx^2}}{c} + \frac{fx\sqrt{a+cx^2}}{2c} + \frac{(2cd-af)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2c}$$

$$= \frac{e\sqrt{a+cx^2}}{c} + \frac{fx\sqrt{a+cx^2}}{2c} + \frac{(2cd-af)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2c}$$

$$= \frac{e\sqrt{a+cx^2}}{c} + \frac{fx\sqrt{a+cx^2}}{2c} + \frac{(2cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.85

$$\frac{(2cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right) + \sqrt{c}\sqrt{a+cx^2}\,(2e+fx)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[c]*(2*e + f*x)*Sqrt[a + c*x^2] + (2*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/Sq
rt[a + c*x^2]])/(2*c^(3/2))
```

fricas [A] time = 0.60, size = 124, normalized size = 1.68

$$\left[-\frac{\left(2\,cd-af\right)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\,\left(cfx+2\,ce\right)\sqrt{cx^2+a}}{4\,c^2}, -\frac{\left(2\,cd-af\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-}}{\sqrt{cx^2}}\right.}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*c*d - a*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a
) - 2*(c*f*x + 2*c*e)*sqrt(c*x^2 + a))/c^2, -1/2*((2*c*d - a*f)*sqrt(-c)*ar
ctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*f*x + 2*c*e)*sqrt(c*x^2 + a))/c^2]
```

giac [A] time = 0.20, size = 58, normalized size = 0.78

$$\frac{1}{2}\sqrt{cx^2+a}\left(\frac{fx}{c}+\frac{2\,e}{c}\right)-\frac{\left(2\,cd-af\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*(f*x/c + 2*e/c) - 1/2*(2*c*d - a*f)*log(abs(-sqrt(c)*x
+ sqrt(c*x^2 + a)))/c^(3/2)
```

maple [A] time = 0.01, size = 76, normalized size = 1.03

$$-\frac{af\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}+\frac{d\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}+\frac{\sqrt{c\,x^2+a}\,fx}{2c}+\frac{\sqrt{c\,x^2+a}\,e}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/2*f*x*(c*x^2+a)^(1/2)/c-1/2*f*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+e*(
c*x^2+a)^(1/2)/c+d*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)
```

maxima [A] time = 0.43, size = 61, normalized size = 0.82

$$\frac{\sqrt{cx^2+a}\,fx}{2\,c}+\frac{d\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}-\frac{af\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^2+a}\,e}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*f*x/c + d*arcsinh(c*x/sqrt(a*c))/sqrt(c) - 1/2*a*f*arcs
inh(c*x/sqrt(a*c))/c^(3/2) + sqrt(c*x^2 + a)*e/c
```

mupad [B] time = 4.56, size = 107, normalized size = 1.45

$$\begin{cases} \frac{2\,f\,x^3+3\,e\,x^2+6\,d\,x}{6\,\sqrt{a}} & \text{if } c=0\\ \frac{e\,\sqrt{c\,x^2+a}}{c}+\frac{d\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}-\frac{a\,f\,\ln\left(2\,\sqrt{c}\,x+2\,\sqrt{c\,x^2+a}\right)}{2\,c^{3/2}}+\frac{f\,x\,\sqrt{c\,x^2+a}}{2\,c} & \text{if } c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + c*x^2)^(1/2),x)
```

```
[Out] piecewise(c == 0, (6*d*x + 3*e*x^2 + 2*f*x^3)/(6*a^(1/2)), c ~= 0, (e*(a +
c*x^2)^(1/2))/c + (d*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/c^(1/2) - (a*f*log
(2*c^(1/2)*x + 2*(a + c*x^2)^(1/2)))/(2*c^(3/2)) + (f*x*(a + c*x^2)^(1/2))/
(2*c))
```

sympy [A] time = 3.50, size = 150, normalized size = 2.03

$$\frac{\sqrt{a}\,fx\sqrt{1+\frac{cx^2}{a}}}{2c}-\frac{af\,\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}}+d\left(\begin{cases}\frac{\sqrt{-\frac{a}{c}}\,\operatorname{asin}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge c<0\\ \frac{\sqrt{\frac{a}{c}}\,\operatorname{asinh}\left(x\sqrt{\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge c>0\\ \frac{\sqrt{-\frac{a}{c}}\,\operatorname{acosh}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{-a}} & \text{for } c>0\wedge a<0\end{cases}\right)+e\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for } c=0\\ \frac{\sqrt{a+cx^2}}{c} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*f*x*sqrt(1 + c*x**2/a)/(2*c) - a*f*asinh(sqrt(c)*x/sqrt(a))/(2*c**(
3/2)) + d*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a), (a > 0) & (c <
0)), (sqrt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c > 0)), (sqrt(-a/c)
*acosh(x*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + e*Piecewise((x**2/(2*s
qrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True))
```

3.105 $\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=130

$$-\frac{\left(dh^2-egh+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h^2\sqrt{ah^2+cg^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(fg-eh)}{\sqrt{c}\,h^2}+\frac{f\sqrt{a+cx^2}}{ch}$$

```
[Out] -(-e*h+f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^2/c^(1/2)-(d*h^2-e*g*h+f*g
^2)*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^2/(a*h^2+c*
g^2)^(1/2)+f*(c*x^2+a)^(1/2)/c/h
```

Rubi [A] time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1654, 844, 217, 206, 725}

$$-\frac{\left(dh^2-egh+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h^2\sqrt{ah^2+cg^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(fg-eh)}{\sqrt{c}\,h^2}+\frac{f\sqrt{a+cx^2}}{ch}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (f*Sqrt[a + c*x^2])/(c*h) - ((f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2
]])/(Sqrt[c]*h^2) - ((f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*
g^2 + a*h^2]*Sqrt[a + c*x^2])])/(h^2*Sqrt[c*g^2 + a*h^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
```

```
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx = \frac{f\sqrt{a+cx^2}}{ch} + \frac{\int \frac{cdh^2-ch(fg-eh)x}{(g+hx)\sqrt{a+cx^2}}\,dx}{ch^2}$$
$$= \frac{f\sqrt{a+cx^2}}{ch} - \frac{(fg-eh)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{h^2} + \frac{\left(fg^2-egh+dh^2\right)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{h^2}$$
$$= \frac{f\sqrt{a+cx^2}}{ch} - \frac{(fg-eh)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{h^2} - \frac{\left(fg^2-egh+dh^2\right)\text{Subst}\left(\int \frac{}{cg}\right.}{h^2}$$
$$= \frac{f\sqrt{a+cx^2}}{ch} - \frac{(fg-eh)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,h^2} - \frac{\left(fg^2-egh+dh^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\,\sqrt{a+cx^2}}\right.}{h^2\sqrt{cg^2+ah^2}}$$

Mathematica [A] time = 0.22, size = 125, normalized size = 0.96

$$\frac{-\frac{\left(h(dh-eg)+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\,\sqrt{ah^2+cg^2}}\right)}{\sqrt{ah^2+cg^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(eh-fg)}{\sqrt{c}} + \frac{fh\sqrt{a+cx^2}}{c}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((f*h*Sqrt[a + c*x^2])/c + ((-(f*g) + e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x
^2]])/Sqrt[c] - ((f*g^2 + h*(-(e*g) + d*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g
^2 + a*h^2]*Sqrt[a + c*x^2])])/Sqrt[c*g^2 + a*h^2])/h^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.22, size = 138, normalized size = 1.06

$$\frac{\sqrt{cx^2+a}\,f}{ch} + \frac{2\left(fg^2+dh^2-ghe\right)\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)h+\sqrt{c}\,g}{\sqrt{-cg^2-ah^2}}\right)}{\sqrt{-cg^2-ah^2}\,h^2} + \frac{\left(\sqrt{c}\,fg-\sqrt{c}\,he\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{ch^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x^2 + a)*f/(c*h) + 2*(f*g^2 + d*h^2 - g*h*e)*arctan(-((sqrt(c)*x - s
qrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 - a*h^2)*
```

```
h^2) + (sqrt(c)*f*g - sqrt(c)*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/(
c*h^2)
```

maple [B] time = 0.01, size = 453, normalized size = 3.48

$$-\frac{d\ln\left(\frac{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+\frac{2a\,h^2+2c\,g^2}{h^2}+2\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\,\sqrt{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+\left(x+\frac{g}{h}\right)^2c+\frac{a\,h^2+c\,g^2}{h^2}}}{x+\frac{g}{h}}\right)}{\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\;h}+\frac{eg\ln\left(\frac{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+\frac{2a\,h^2+2c\,g^2}{h^2}+2\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\,\sqrt{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+}}{x+\frac{g}{h}}\right.}{\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\;h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x)
```

```
[Out] f*(c*x^2+a)^(1/2)/c/h+1/h*e*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)-1/h^2*f*g
*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)-1/h/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2
*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*
c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d+1/h^2/((a*h^2+c*g^2)
/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^
(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e*g-
1/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*
((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^
(1/2))/(x+g/h))*f*g^2
```

maxima [A] time = 0.56, size = 218, normalized size = 1.68

$$-\frac{fg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,h^2}+\frac{e\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,h}+\frac{fg^2\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}\,h^3}-\frac{eg\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}\,h^2}+\frac{d\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -f*g*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h^2) + e*arcsinh(c*x/sqrt(a*c))/(sqrt(
c)*h) + f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h
*x + g)))/(sqrt(a + c*g^2/h^2)*h^3) - e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x
+ g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^2) + d*arcsinh
(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c
*g^2/h^2)*h) + sqrt(c*x^2 + a)*f/(c*h)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)\,\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}\,\left(g+hx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)), x)
```

3.106 $\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=168

$$-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}+\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(ah^2(2fg-eh)+c\left(fg^3-dgh^2\right)\right)}{h^2\left(ah^2+cg^2\right)^{3/2}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,h^2}$$

```
[Out] (a*h^2*(-e*h+2*f*g)+c*(-d*g*h^2+f*g^3))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^
(1/2)/(c*x^2+a)^(1/2))/h^2/(a*h^2+c*g^2)^(3/2)+f*arctanh(x*c^(1/2)/(c*x^2+a
)^(1/2))/h^2/c^(1/2)-(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1/2)/h/(a*h^2+c*g^2)/(h
*x+g)
```

Rubi [A] time = 0.23, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1651, 844, 217, 206, 725}

$$-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}+\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(ah^2(2fg-eh)+c\left(fg^3-dgh^2\right)\right)}{h^2\left(ah^2+cg^2\right)^{3/2}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,h^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -(((f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/(h*(c*g^2 + a*h^2)*(g + h*x)))
+ (f*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*h^2) + ((a*h^2*(2*f*g -
 e*h) + c*(f*g^3 - d*g*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2])])/(h^2*(c*g^2 + a*h^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
```

```
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{h\left(cg^2+ah^2\right)(g+hx)} - \frac{\int \frac{-cdg+afg-aeh-f\left(\frac{cg^2}{h}+ah\right)x}{(g+hx)\sqrt{a+cx^2}}\,dx}{cg^2+ah^2}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{h\left(cg^2+ah^2\right)(g+hx)} + \frac{f\int \frac{1}{\sqrt{a+cx^2}}\,dx}{h^2} + \frac{\left(cdg-2afg-\frac{cfg^3}{h^2}+aeh\right)\int \frac{\ldots}{(g+\ldots}}{cg^2+ah^2}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{h\left(cg^2+ah^2\right)(g+hx)} + \frac{f\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{h^2} - \frac{\left(cdg-2afg-\ldots\right.}{\ldots}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{h\left(cg^2+ah^2\right)(g+hx)} + \frac{f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,h^2} - \frac{\left(cdg-2afg-\frac{cfg^3}{h^2}+aeh\right)\mathrm{t}\ldots}{\left(cg^2+a\ldots\right.}$$

Mathematica [A] time = 0.36, size = 218, normalized size = 1.30

$$\frac{-\frac{h\sqrt{a+cx^2}\left(h(dh-eg)+fg^2\right)}{(g+hx)\left(ah^2+cg^2\right)} + \frac{\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(ah^2(2fg-eh)+c\left(fg^3-dgh^2\right)\right)}{\left(ah^2+cg^2\right)^{3/2}} + \frac{\log(g+hx)\left(ah^2(eh-2fg)+c\left(dgh^2-fg^3\right)\right)}{\left(ah^2+cg^2\right)^{3/2}} + \frac{f\,\mathrm{l}\ldots}{\ldots}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (-((h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)*(g + h*x
))) + ((a*h^2*(-2*f*g + e*h) + c*(-(f*g^3) + d*g*h^2))*Log[g + h*x])/(c*g^2
 + a*h^2)^(3/2) + (f*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/Sqrt[c] + ((a*h^2*
(2*f*g - e*h) + c*(f*g^3 - d*g*h^2))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*
Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(3/2))/h^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Error: Bad Argument Type
```

maple [B] time = 0.02, size = 923, normalized size = 5.49

$$-\frac{cdg\ln\left(\frac{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+\frac{2ah^2+2cg^2}{h^2}+2\sqrt{\frac{ah^2+cg^2}{h^2}}\sqrt{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+\left(x+\frac{g}{h}\right)^2c+\frac{ah^2+cg^2}{h^2}}}{x+\frac{g}{h}}\right)}{\left(ah^2+cg^2\right)\sqrt{\frac{ah^2+cg^2}{h^2}}\,h}+\frac{ceg^2\ln\left(\frac{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+\frac{2ah^2+2cg^2}{h^2}+2\sqrt{\frac{ah^2+cg^2}{h^2}}\sqrt{-\frac{2\left(x+\frac{g}{h}\right)c}{h}}}{x+\frac{g}{h}}\right)}{\left(ah^2+cg^2\right)\sqrt{\frac{ah^2+cg^2}{h^2}}\,h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] f/h^2*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)-1/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+
g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*d+1/h/(a*h^2+c*g^2)/(x+g/h)
*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e*g-1/h^2/(a*h^2+c*
g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f*g^2-1
/h*c*g/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^
2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h
^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d+1/h^2*c*g^2/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h
^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1
/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e-1/h^
3*c*g^3/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h
^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*
h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f-1/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x
+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g
/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e+2/h^3/((a*h^2+c*g^2)/h^
2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/
2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f*g
```

maxima [B] time = 0.58, size = 419, normalized size = 2.49

$$-\frac{\sqrt{cx^2+a}\,fg^2}{cg^2h^2x+ah^4x+cg^3h+agh^3}+\frac{\sqrt{cx^2+a}\,eg}{cg^2hx+ah^3x+cg^3+agh^2}-\frac{\sqrt{cx^2+a}\,d}{cg^2x+ah^2x+\frac{cg^3}{h}+agh}+\frac{f\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,h^2}+\frac{cfg^3\operatorname{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2 + a)*f*g^2/(c*g^2*h^2*x + a*h^4*x + c*g^3*h + a*g*h^3) + sqrt(c
*x^2 + a)*e*g/(c*g^2*h*x + a*h^3*x + c*g^3 + a*g*h^2) - sqrt(c*x^2 + a)*d/(
c*g^2*x + a*h^2*x + c*g^3/h + a*g*h) + f*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h^
2) + c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*
x + g)))/((a + c*g^2/h^2)^(3/2)*h^5) - c*e*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs
(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^4) + c*
d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/
((a + c*g^2/h^2)^(3/2)*h^3) - 2*f*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g))
- a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^3) + e*arcsinh(c*g*x
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h
^2)*h^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^2\,\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2}{\sqrt{a + cx^2}\left(g + hx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+a)**(1/2),x)
[Out] Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)**2), x)
```

3.107 $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=225

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^2-ac\left(fg^2-h(3eg-dh)\right)+2c^2dg^2\right)}{2\left(ah^2+cg^2\right)^{5/2}}-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}+\frac{\sqrt{a+cx^2}}{}$$

```
[Out] -1/2*(2*c^2*d*g^2+2*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+3*e*g)))*arctanh((-c*g*x+a
*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(5/2)-1/2*(d*h^2-e*g
*h+f*g^2)*(c*x^2+a)^(1/2)/h/(a*h^2+c*g^2)/(h*x+g)^2+1/2*(2*a*h^2*(-e*h+2*f*
g)+c*g*(f*g^2+h*(-3*d*h+e*g)))*(c*x^2+a)^(1/2)/h/(a*h^2+c*g^2)^2/(h*x+g)
```

Rubi [A] time = 0.29, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1651, 807, 725, 206}

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^2-ac\left(fg^2-h(3eg-dh)\right)+2c^2dg^2\right)}{2\left(ah^2+cg^2\right)^{5/2}}-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}+\frac{\sqrt{a+cx^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] -((f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/(2*h*(c*g^2 + a*h^2)*(g + h*x)^2
) + ((c*f*g^3 + c*g*h*(e*g - 3*d*h) + 2*a*h^2*(2*f*g - e*h))*Sqrt[a + c*x^2
])/(2*h*(c*g^2 + a*h^2)^2*(g + h*x)) - ((2*c^2*d*g^2 + 2*a^2*f*h^2 - a*c*(f
*g^2 - h*(3*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a
+ c*x^2]])/(2*(c*g^2 + a*h^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
```

```
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)(g+hx)^2} - \frac{\int \frac{-2(cdg-afg+aeh)-\left(2afh+c\left(eg+\frac{fg^2}{h}-dh\right)\right)x}{(g+hx)^2\sqrt{a+cx^2}}\,dx}{2\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)(g+hx)^2} + \frac{\left(cfg^3+cgh(eg-3dh)+2ah^2(2fg-eh)\right)\sqrt{a+c}}{2h\left(cg^2+ah^2\right)^2(g+hx)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)(g+hx)^2} + \frac{\left(cfg^3+cgh(eg-3dh)+2ah^2(2fg-eh)\right)\sqrt{a+c}}{2h\left(cg^2+ah^2\right)^2(g+hx)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)(g+hx)^2} + \frac{\left(cfg^3+cgh(eg-3dh)+2ah^2(2fg-eh)\right)\sqrt{a+c}}{2h\left(cg^2+ah^2\right)^2(g+hx)}$$

Mathematica [A] time = 0.45, size = 254, normalized size = 1.13

$$\frac{(g+hx)^2\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(-2a^2fh^2+ac\left(h(dh-3eg)+fg^2\right)-2c^2dg^2\right)+(g+hx)^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]*(c*g*(f*g^2*x + e*g*(2*g + h*x) - d*h*
(4*g + 3*h*x)) - a*h*(-(f*g*(3*g + 4*h*x)) + h*(d*h + e*(g + 2*h*x)))) + (2
*c^2*d*g^2 + 2*a^2*f*h^2 - a*c*(f*g^2 + h*(-3*e*g + d*h)))*(g + h*x)^2*Log[
g + h*x] + (-2*c^2*d*g^2 - 2*a^2*f*h^2 + a*c*(f*g^2 + h*(-3*e*g + d*h)))*(g
 + h*x)^2*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(2*(c*g^2
 + a*h^2)^(5/2)*(g + h*x)^2)
```

fricas [B] time = 22.09, size = 1088, normalized size = 4.84

$$\left[\frac{\left(3\,aceg^3h+\left(2\,c^2d-acf\right)g^4-\left(acd-2\,a^2f\right)g^2h^2+\left(3\,acegh^3+\left(2\,c^2d-acf\right)g^2h^2-\left(acd-2\,a^2f\right)h^4\right)x^2+2\left(3\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((3*a*c*e*g^3*h + (2*c^2*d - a*c*f)*g^4 - (a*c*d - 2*a^2*f)*g^2*h^2 +
(3*a*c*e*g*h^3 + (2*c^2*d - a*c*f)*g^2*h^2 - (a*c*d - 2*a^2*f)*h^4)*x^2 + 2
*(3*a*c*e*g^2*h^2 + (2*c^2*d - a*c*f)*g^3*h - (a*c*d - 2*a^2*f)*g*h^3)*x)*s
qrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*
c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2
+ 2*g*h*x + g^2)) + 2*(2*c^2*e*g^5 + a*c*e*g^3*h^2 - a^2*e*g*h^4 - a^2*d*h^
5 - (4*c^2*d - 3*a*c*f)*g^4*h - (5*a*c*d - 3*a^2*f)*g^2*h^3 + (c^2*f*g^5 +
c^2*e*g^4*h - a*c*e*g^2*h^3 - 2*a^2*e*h^5 - (3*c^2*d - 5*a*c*f)*g^3*h^2 - (
```

```
3*a*c*d - 4*a^2*f)*g*h^4)*x)*sqrt(c*x^2 + a))/(c^3*g^8 + 3*a*c^2*g^6*h^2 +
3*a^2*c*g^4*h^4 + a^3*g^2*h^6 + (c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^
2*h^6 + a^3*h^8)*x^2 + 2*(c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a
^3*g*h^7)*x), -1/2*((3*a*c*e*g^3*h + (2*c^2*d - a*c*f)*g^4 - (a*c*d - 2*a^2
*f)*g^2*h^2 + (3*a*c*e*g*h^3 + (2*c^2*d - a*c*f)*g^2*h^2 - (a*c*d - 2*a^2*f
)*h^4)*x^2 + 2*(3*a*c*e*g^2*h^2 + (2*c^2*d - a*c*f)*g^3*h - (a*c*d - 2*a^2*
f)*g*h^3)*x)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2 - a*h^2)*(c*g*x - a*h)
*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2)*x^2)) - (2*c^2*e*
g^5 + a*c*e*g^3*h^2 - a^2*e*g*h^4 - a^2*d*h^5 - (4*c^2*d - 3*a*c*f)*g^4*h -
 (5*a*c*d - 3*a^2*f)*g^2*h^3 + (c^2*f*g^5 + c^2*e*g^4*h - a*c*e*g^2*h^3 - 2
*a^2*e*h^5 - (3*c^2*d - 5*a*c*f)*g^3*h^2 - (3*a*c*d - 4*a^2*f)*g*h^4)*x)*sq
rt(c*x^2 + a))/(c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4 + a^3*g^2*h^6 +
 (c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8)*x^2 + 2*(c^3*g
^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7)*x)]
```

giac [B] time = 0.26, size = 848, normalized size = 3.77

$$-\frac{\left(2\,c^2dg^2-acfg^2-acdh^2+2\,a^2fh^2+3\,acghe\right)\arctan\left(\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)h+\sqrt{c}\,g}{\sqrt{-cg^2-ah^2}}\right)}{\left(c^2g^4+2\,acg^2h^2+a^2h^4\right)\sqrt{-cg^2-ah^2}}+\frac{2\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^3c^2fg^4h-2\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*c^2*d*g^2 - a*c*f*g^2 - a*c*d*h^2 + 2*a^2*f*h^2 + 3*a*c*g*h*e)*arctan((
(sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^2*g^
4 + 2*a*c*g^2*h^2 + a^2*h^4)*sqrt(-c*g^2 - a*h^2)) + (2*(sqrt(c)*x - sqrt(c
*x^2 + a))^3*c^2*f*g^4*h - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*d*g^2*h^3
+ 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g^2*h^3 + (sqrt(c)*x - sqrt(c*x^2
 + a))^3*a*c*d*h^5 - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*g*h^4*e + 2*(sqr
t(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))
^2*c^(5/2)*d*g^3*h^2 + 7*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*f*g^3*h^
2 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*x - sq
rt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^
(5/2)*g^4*h*e - 5*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*g^2*h^3*e + 2*(
sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*h^5*e - 2*(sqrt(c)*x - sqrt(c*x^
2 + a))*a*c^2*f*g^4*h + 10*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2*h^3 -
11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^3 + (sqrt(c)*x - sqrt(c*x^2
+ a))*a^2*c*d*h^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*g^3*h^2*e + 5*(sq
rt(c)*x - sqrt(c*x^2 + a))*a^2*c*g*h^4*e + a^2*c^(3/2)*f*g^3*h^2 - 3*a^2*c^
(3/2)*d*g*h^4 + 4*a^3*sqrt(c)*f*g*h^4 + a^2*c^(3/2)*g^2*h^3*e - 2*a^3*sqrt(
c)*h^5*e)/((c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6)*((sqrt(c)*x - sqrt(c*x^2
 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2)
```

maple [B] time = 0.02, size = 1574, normalized size = 7.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/h/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)
^(1/2)*e+2/h^2/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c
*g^2)/h^2)^(1/2)*f*g-3/2/h^2*c*g/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln
((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g
/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e+5/2/h^3*c*g^2/(a
*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/
```

```
h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)
/h^2)^(1/2))/(x+g/h))*f-f/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/
h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)
^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))-1/2/h/(a*h^2+c*g^2)/(x+g/h)^2*(-2*(
x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*d+1/2/h^2/(a*h^2+c*g^2)/(
x+g/h)^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e*g-1/2/h^3
/(a*h^2+c*g^2)/(x+g/h)^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(
1/2)*f*g^2-3/2*c*g/(a*h^2+c*g^2)^2/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a
*h^2+c*g^2)/h^2)^(1/2)*d+3/2/h*c*g^2/(a*h^2+c*g^2)^2/(x+g/h)*(-2*(x+g/h)*c*
g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e-3/2/h^2*c*g^3/(a*h^2+c*g^2)^2/(x
+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f-3/2/h*c^2*g^
2/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c
*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+
c*g^2)/h^2)^(1/2))/(x+g/h))*d+3/2/h^2*c^2*g^3/(a*h^2+c*g^2)^2/((a*h^2+c*g^2
)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)
^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e-3
/2/h^3*c^2*g^4/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g
/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h
)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f+1/2/h*c/(a*h^2+c*g^2)/((a*h^2+c*
g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h
^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*
d
```

maxima [B] time = 0.67, size = 896, normalized size = 3.98

$$-\frac{3\sqrt{cx^2+a}\,cfg^3}{2\left(c^2g^4h^2x+2\,acg^2h^4x+a^2h^6x+c^2g^5h+2\,acg^3h^3+a^2gh^5\right)}+\frac{3\sqrt{cx^2+a}\,ceg^2}{2\left(c^2g^4hx+2\,acg^2h^3x+a^2h^5x+c^2g^5+2\,acg\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(c*x^2 + a)*c*f*g^3/(c^2*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x +
 c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*g*h^5) + 3/2*sqrt(c*x^2 + a)*c*e*g^2/(c^2*
g^4*h*x + 2*a*c*g^2*h^3*x + a^2*h^5*x + c^2*g^5 + 2*a*c*g^3*h^2 + a^2*g*h^4
) - 3/2*sqrt(c*x^2 + a)*c*d*g/(c^2*g^4*x + 2*a*c*g^2*h^2*x + a^2*h^4*x + c^
2*g^5/h + 2*a*c*g^3*h + a^2*g*h^3) - 1/2*sqrt(c*x^2 + a)*f*g^2/(c*g^2*h^3*x
^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*s
qrt(c*x^2 + a)*e*g/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h*x + 2*a*g*h^3*x +
 c*g^4 + a*g^2*h^2) + 2*sqrt(c*x^2 + a)*f*g/(c*g^2*h^2*x + a*h^4*x + c*g^3*
h + a*g*h^3) - 1/2*sqrt(c*x^2 + a)*d/(c*g^2*h*x^2 + a*h^3*x^2 + 2*c*g^3*x +
 2*a*g*h^2*x + c*g^4/h + a*g^2*h) - sqrt(c*x^2 + a)*e/(c*g^2*h*x + a*h^3*x
+ c*g^3 + a*g*h^2) + 3/2*c^2*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
 a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) - 3/2*c^2*e*g^3*
arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a
+ c*g^2/h^2)^(5/2)*h^6) + 3/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x +
g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^5) - 5/2*c*f*g
^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(
(a + c*g^2/h^2)^(3/2)*h^5) + 3/2*c*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g
)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^4) - 1/2*c*d*ar
csinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a +
c*g^2/h^2)^(3/2)*h^3) + f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqr
t(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2}{\sqrt{a + cx^2}\,\left(g + hx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)**3), x)
```

3.108 $\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{h\sqrt{a+cx^2}\left(4\left(4a^2fh^2-ac\left(3h(dh+3eg)+7fg^2\right)+3c^2dg^2\right)+chx(-9aeh-11afg+6cdg)\right)}{6ac^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{}$$

```
[Out] -1/2*(3*a*h^2*(e*h+3*f*g)-2*c*g*(f*g^2+3*h*(d*h+e*g)))*arctanh(x*c^(1/2)/(c
*x^2+a)^(1/2))/c^(5/2)-(a*e-(-a*f+c*d)*x)*(h*x+g)^3/a/c/(c*x^2+a)^(1/2)-1/3
*(-4*a*f+3*c*d)*h*(h*x+g)^2*(c*x^2+a)^(1/2)/a/c^2-1/6*h*(12*c^2*d*g^2+16*a^
2*f*h^2-4*a*c*(7*f*g^2+3*h*(d*h+3*e*g))+c*h*(-9*a*e*h-11*a*f*g+6*c*d*g)*x)*
(c*x^2+a)^(1/2)/a/c^3
```

Rubi [A] time = 0.32, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1645, 833, 780, 217, 206}

$$-\frac{h\sqrt{a+cx^2}\left(4\left(4a^2fh^2-ac\left(3h(dh+3eg)+7fg^2\right)+3c^2dg^2\right)+chx(-9aeh-11afg+6cdg)\right)}{6ac^3}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x)^3)/(a*c*Sqrt[a + c*x^2])) - ((3*c*d - 4*
a*f)*h*(g + h*x)^2*Sqrt[a + c*x^2])/(3*a*c^2) - (h*(4*(3*c^2*d*g^2 + 4*a^2*
f*h^2 - a*c*(7*f*g^2 + 3*h*(3*e*g + d*h))) + c*h*(6*c*d*g - 11*a*f*g - 9*a*
e*h)*x)*Sqrt[a + c*x^2])/(6*a*c^3) + ((2*c*f*g^3 + 6*c*g*h*(e*g + d*h) - 3*
a*h^2*(3*f*g + e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}dx = -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{\int \frac{(g+hx)^2(-a(fg+3eh)+(3cd-4af)hx)}{\sqrt{a+cx^2}}dx}{ac}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{\int \frac{(g+hx)(-a(2(3}{}}{}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{h\left(4\left(3c^2dg^2+\right.\right.}{}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{h\left(4\left(3c^2dg^2+\right.\right.}{}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{h\left(4\left(3c^2dg^2+\right.\right.}{}$$

Mathematica [A] time = 0.45, size = 246, normalized size = 1.07

$$\frac{\frac{-16a^3fh^3+a^2ch\left(3h(4dh+3e(4g+hx))+f\left(36g^2+27ghx-8h^2x^2\right)\right)+ac^2\left(6dh\left(-3g^2-3ghx+h^2x^2\right)-3e\left(2g^3+6g^2hx-6gh^2x^2-h^3x^3\right)+fx\left(-6g^3+18g^2hx+9gh^2x^2\right.\right.}{a\sqrt{a+cx^2}}}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] ((-16*a^3*f*h^3 + 6*c^3*d*g^3*x + a*c^2*(6*d*h*(-3*g^2 - 3*g*h*x + h^2*x^2)
 - 3*e*(2*g^3 + 6*g^2*h*x - 6*g*h^2*x^2 - h^3*x^3) + f*x*(-6*g^3 + 18*g^2*h
*x + 9*g*h^2*x^2 + 2*h^3*x^3)) + a^2*c*h*(f*(36*g^2 + 27*g*h*x - 8*h^2*x^2)
 + 3*h*(4*d*h + 3*e*(4*g + h*x))))/(a*Sqrt[a + c*x^2]) + 3*Sqrt[c]*(2*c*f*g
^3 + 6*c*g*h*(e*g + d*h) - 3*a*h^2*(3*f*g + e*h))*Log[c*x + Sqrt[c]*Sqrt[a
+ c*x^2]])/(6*c^3)
```

fricas [A] time = 1.22, size = 758, normalized size = 3.31

$$\left[-\frac{3\left(2a^2cfg^3+6a^2ceg^2h-3a^3eh^3+3\left(2a^2cd-3a^3f\right)gh^2+\left(2ac^2fg^3+6ac^2eg^2h-3a^2ceh^3+3\left(2ac^2d-3a^2\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*a^2*c*f*g^3 + 6*a^2*c*e*g^2*h - 3*a^3*e*h^3 + 3*(2*a^2*c*d - 3
*a^3*f)*g*h^2 + (2*a*c^2*f*g^3 + 6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 + 3*(2*a*c
^2*d - 3*a^2*c*f)*g*h^2)*x^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt
(c)*x - a) - 2*(2*a*c^2*f*h^3*x^4 - 6*a*c^2*e*g^3 + 36*a^2*c*e*g*h^2 - 18*(
a*c^2*d - 2*a^2*c*f)*g^2*h + 4*(3*a^2*c*d - 4*a^3*f)*h^3 + 3*(3*a*c^2*f*g*h
^2 + a*c^2*e*h^3)*x^3 + 2*(9*a*c^2*f*g^2*h + 9*a*c^2*e*g*h^2 + (3*a*c^2*d -
 4*a^2*c*f)*h^3)*x^2 - 3*(6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 - 2*(c^3*d - a*c^
2*f)*g^3 + 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/(a*c^4*x^2
+ a^2*c^3), -1/6*(3*(2*a^2*c*f*g^3 + 6*a^2*c*e*g^2*h - 3*a^3*e*h^3 + 3*(2*a
^2*c*d - 3*a^3*f)*g*h^2 + (2*a*c^2*f*g^3 + 6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3
+ 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x
^2 + a)) - (2*a*c^2*f*h^3*x^4 - 6*a*c^2*e*g^3 + 36*a^2*c*e*g*h^2 - 18*(a*c^
2*d - 2*a^2*c*f)*g^2*h + 4*(3*a^2*c*d - 4*a^3*f)*h^3 + 3*(3*a*c^2*f*g*h^2 +
 a*c^2*e*h^3)*x^3 + 2*(9*a*c^2*f*g^2*h + 9*a*c^2*e*g*h^2 + (3*a*c^2*d - 4*a
^2*c*f)*h^3)*x^2 - 3*(6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 - 2*(c^3*d - a*c^2*f)
*g^3 + 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/(a*c^4*x^2 + a^
2*c^3)]
```

giac [**A**] time = 0.25, size = 339, normalized size = 1.48

$$\frac{\left(\left(\left(\frac{2\, fh^3x}{c} + \frac{3\left(3\, ac^4fgh^2+ac^4h^3e\right)}{ac^5}\right)x + \frac{2\left(9\, ac^4fg^2h+3\, ac^4dh^3-4\, a^2c^3fh^3+9\, ac^4gh^2e\right)}{ac^5}\right)x + \frac{3\left(2\, c^5dg^3-2\, ac^4fg^3-6\, ac^4dgh^2+9\, a^2c^3fgh^2-6\, ac\right.}{ac^5}}{6\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*((((2*f*h^3*x/c + 3*(3*a*c^4*f*g*h^2 + a*c^4*h^3*e)/(a*c^5))*x + 2*(9*a
*c^4*f*g^2*h + 3*a*c^4*d*h^3 - 4*a^2*c^3*f*h^3 + 9*a*c^4*g*h^2*e)/(a*c^5))*
x + 3*(2*c^5*d*g^3 - 2*a*c^4*f*g^3 - 6*a*c^4*d*g*h^2 + 9*a^2*c^3*f*g*h^2 -
6*a*c^4*g^2*h*e + 3*a^2*c^3*h^3*e)/(a*c^5))*x - 2*(9*a*c^4*d*g^2*h - 18*a^2
*c^3*f*g^2*h - 6*a^2*c^3*d*h^3 + 8*a^3*c^2*f*h^3 + 3*a*c^4*g^3*e - 18*a^2*c
^3*g*h^2*e)/(a*c^5))/sqrt(c*x^2 + a) - 1/2*(2*c*f*g^3 + 6*c*d*g*h^2 - 9*a*f
*g*h^2 + 6*c*g^2*h*e - 3*a*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^
(5/2)
```

maple [**B**] time = 0.02, size = 516, normalized size = 2.25

$$\frac{f\,h^3x^4}{3\sqrt{c\,x^2+a}\;c}+\frac{e\,h^3x^3}{2\sqrt{c\,x^2+a}\;c}+\frac{3fg\,h^2x^3}{2\sqrt{c\,x^2+a}\;c}-\frac{4af\,h^3x^2}{3\sqrt{c\,x^2+a}\;c^2}+\frac{d\,h^3x^2}{\sqrt{c\,x^2+a}\;c}+\frac{3eg\,h^2x^2}{\sqrt{c\,x^2+a}\;c}+\frac{3f\,g^2h\,x^2}{\sqrt{c\,x^2+a}\;c}+\frac{3ae\,h^3}{2\sqrt{c\,x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/3*h^3*f*x^4/c/(c*x^2+a)^(1/2)-4/3*h^3*f*a/c^2*x^2/(c*x^2+a)^(1/2)-8/3*h^3
*f*a^2/c^3/(c*x^2+a)^(1/2)+1/2*x^3/c/(c*x^2+a)^(1/2)*h^3*e+3/2*x^3/c/(c*x^2
+a)^(1/2)*g*h^2*f+3/2*a/c^2*x/(c*x^2+a)^(1/2)*h^3*e+9/2*a/c^2*x/(c*x^2+a)^(
1/2)*g*h^2*f-3/2*a/c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*h^3*e-9/2*a/c^(5/2
)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*g*h^2*f+x^2/c/(c*x^2+a)^(1/2)*h^3*d+3*x^2/c
/(c*x^2+a)^(1/2)*g*h^2*e+3*x^2/c/(c*x^2+a)^(1/2)*g^2*h*f+2*a/c^2/(c*x^2+a)^
(1/2)*h^3*d+6*a/c^2/(c*x^2+a)^(1/2)*g*h^2*e+6*a/c^2/(c*x^2+a)^(1/2)*g^2*h*f
-3*x/c/(c*x^2+a)^(1/2)*g*h^2*d-3*x/c/(c*x^2+a)^(1/2)*g^2*h*e-x/c/(c*x^2+a)^
(1/2)*g^3*f+3/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*g*h^2*d+3/c^(3/2)*ln(c^
(1/2)*x+(c*x^2+a)^(1/2))*g^2*h*e+1/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*g^
3*f-3/c/(c*x^2+a)^(1/2)*g^2*h*d-1/c/(c*x^2+a)^(1/2)*g^3*e+g^3*d*x/a/(c*x^2+
a)^(1/2)
```

maxima [A] time = 0.46, size = 346, normalized size = 1.51

$$\frac{fh^3x^4}{3\sqrt{cx^2+a}\,c}-\frac{4\,afh^3x^2}{3\sqrt{cx^2+a}\,c^2}+\frac{dg^3x}{\sqrt{cx^2+a}\,a}-\frac{eg^3}{\sqrt{cx^2+a}\,c}-\frac{3\,dg^2h}{\sqrt{cx^2+a}\,c}-\frac{8\,a^2fh^3}{3\sqrt{cx^2+a}\,c^3}+\frac{\left(3\,fgh^2+eh^3\right)x^3}{2\sqrt{cx^2+a}\,c}+\frac{\left(3\,fg^2h+\right.}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*f*h^3*x^4/(sqrt(c*x^2 + a)*c) - 4/3*a*f*h^3*x^2/(sqrt(c*x^2 + a)*c^2) +
 d*g^3*x/(sqrt(c*x^2 + a)*a) - e*g^3/(sqrt(c*x^2 + a)*c) - 3*d*g^2*h/(sqrt(
c*x^2 + a)*c) - 8/3*a^2*f*h^3/(sqrt(c*x^2 + a)*c^3) + 1/2*(3*f*g*h^2 + e*h^
3)*x^3/(sqrt(c*x^2 + a)*c) + (3*f*g^2*h + 3*e*g*h^2 + d*h^3)*x^2/(sqrt(c*x^
2 + a)*c) + 3/2*(3*f*g*h^2 + e*h^3)*a*x/(sqrt(c*x^2 + a)*c^2) - (f*g^3 + 3*
e*g^2*h + 3*d*g*h^2)*x/(sqrt(c*x^2 + a)*c) - 3/2*(3*f*g*h^2 + e*h^3)*a*arcs
inh(c*x/sqrt(a*c))/c^(5/2) + (f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*arcsinh(c*x/sq
rt(a*c))/c^(3/2) + 2*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*a/(sqrt(c*x^2 + a)*c^2
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g+h\,x\right)^3\,\left(f\,x^2+e\,x+d\right)}{\left(c\,x^2+a\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g+hx\right)^{3}\left(d+ex+fx^{2}\right)}{\left(a+cx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**3*(d + e*x + f*x**2)/(a + c*x**2)**(3/2), x)
```

3.109 $\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=149

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(h^2(2cd-3af)+2cg(2eh+fg)\right)}{2c^{5/2}}-\frac{h\sqrt{a+cx^2}\,(4(cdg-a(eh+2fg))+hx(2cd-3af))}{2ac^2}-\frac{(g+h}{}$$

```
[Out] 1/2*((-3*a*f+2*c*d)*h^2+2*c*g*(2*e*h+f*g))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2
))/c^(5/2)-(a*e-(-a*f+c*d)*x)*(h*x+g)^2/a/c/(c*x^2+a)^(1/2)-1/2*h*(4*c*d*g-
4*a*(e*h+2*f*g)+(-3*a*f+2*c*d)*h*x)*(c*x^2+a)^(1/2)/a/c^2
```

Rubi [A] time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = $\{1645, 780, 217, 206\}$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(h^2(2cd-3af)+2cg(2eh+fg)\right)}{2c^{5/2}}-\frac{h\sqrt{a+cx^2}\,(4(cdg-a(eh+2fg))+hx(2cd-3af))}{2ac^2}-\frac{(g+h}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x)^2)/(a*c*Sqrt[a + c*x^2])) - (h*(4*(c*d*g
 - a*(2*f*g + e*h)) + (2*c*d - 3*a*f)*h*x)*Sqrt[a + c*x^2])/(2*a*c^2) + (((
2*c*d - 3*a*f)*h^2 + 2*c*g*(f*g + 2*e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^
2]])/(2*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^2 \left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} - \frac{\int \frac{(g+hx)(-a(fg+2eh)+(2cd-3af)hx)}{\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} - \frac{h(4(cdg-a(2fg+eh))+(2cd-3af)hx)\sqrt{a+cx}}{2ac^2}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} - \frac{h(4(cdg-a(2fg+eh))+(2cd-3af)hx)\sqrt{a+cx}}{2ac^2}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} - \frac{h(4(cdg-a(2fg+eh))+(2cd-3af)hx)\sqrt{a+cx}}{2ac^2}$$

Mathematica [A] time = 0.28, size = 177, normalized size = 1.19

$$\frac{\sqrt{c}\left(a^2h(4eh+8fg+3fhx)+ac\left(-2dh(2g+hx)-2e\left(g^2+2ghx-h^2x^2\right)+fx\left(-2g^2+4ghx+h^2x^2\right)\right)+2c^2dg^2x\right.}{2ac^{5/2}\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*(2*c^2*d*g^2*x + a^2*h*(8*f*g + 4*e*h + 3*f*h*x) + a*c*(-2*d*h*(2*
g + h*x) - 2*e*(g^2 + 2*g*h*x - h^2*x^2) + f*x*(-2*g^2 + 4*g*h*x + h^2*x^2)
)) - a^(3/2)*(3*a*f*h^2 - 2*c*(f*g^2 + h*(2*e*g + d*h)))*Sqrt[1 + (c*x^2)/a
]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(2*a*c^(5/2)*Sqrt[a + c*x^2])
```

fricas [A] time = 0.89, size = 530, normalized size = 3.56

$$\left[-\frac{\left(2\,a^2cfg^2+4\,a^2cegh+\left(2\,a^2cd-3\,a^3f\right)h^2+\left(2\,ac^2fg^2+4\,ac^2egh+\left(2\,ac^2d-3\,a^2cf\right)h^2\right)x^2\right)\sqrt{c}\,\log\left(-2\,cx^2+2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*a^2*c*f*g^2 + 4*a^2*c*e*g*h + (2*a^2*c*d - 3*a^3*f)*h^2 + (2*a*c^
2*f*g^2 + 4*a*c^2*e*g*h + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x^2)*sqrt(c)*log(-2*
c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(a*c^2*f*h^2*x^3 - 2*a*c^2*e*g
^2 + 4*a^2*c*e*h^2 - 4*(a*c^2*d - 2*a^2*c*f)*g*h + 2*(2*a*c^2*f*g*h + a*c^2
*e*h^2)*x^2 - (4*a*c^2*e*g*h - 2*(c^3*d - a*c^2*f)*g^2 + (2*a*c^2*d - 3*a^2
*c*f)*h^2)*x)*sqrt(c*x^2 + a))/(a*c^4*x^2 + a^2*c^3), -1/2*((2*a^2*c*f*g^2
+ 4*a^2*c*e*g*h + (2*a^2*c*d - 3*a^3*f)*h^2 + (2*a*c^2*f*g^2 + 4*a*c^2*e*g*
h + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2
 + a)) - (a*c^2*f*h^2*x^3 - 2*a*c^2*e*g^2 + 4*a^2*c*e*h^2 - 4*(a*c^2*d - 2*
a^2*c*f)*g*h + 2*(2*a*c^2*f*g*h + a*c^2*e*h^2)*x^2 - (4*a*c^2*e*g*h - 2*(c^
3*d - a*c^2*f)*g^2 + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x)*sqrt(c*x^2 + a))/(a*c^
4*x^2 + a^2*c^3)]
```

giac [A] time = 0.25, size = 219, normalized size = 1.47

$$\frac{\left(\left(\frac{fh^2x}{c}+\frac{2\left(2\,ac^3fgh+ac^3h^2e\right)}{ac^4}\right)x+\frac{2\,c^4dg^2-2\,ac^3fg^2-2\,ac^3dh^2+3\,a^2c^2fh^2-4\,ac^3ghe}{ac^4}\right)x-\frac{2\left(2\,ac^3dgh-4\,a^2c^2fgh+ac^3g^2e-2\,a^2c^2h^2e\right)}{ac^4}}{2\sqrt{cx^2+a}}-\frac{\left(2\,cfg^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(((f*h^2*x/c + 2*(2*a*c^3*f*g*h + a*c^3*h^2*e)/(a*c^4))*x + (2*c^4*d*g^
2 - 2*a*c^3*f*g^2 - 2*a*c^3*d*h^2 + 3*a^2*c^2*f*h^2 - 4*a*c^3*g*h*e)/(a*c^4
))*x - 2*(2*a*c^3*d*g*h - 4*a^2*c^2*f*g*h + a*c^3*g^2*e - 2*a^2*c^2*h^2*e)/
(a*c^4))/sqrt(c*x^2 + a) - 1/2*(2*c*f*g^2 + 2*c*d*h^2 - 3*a*f*h^2 + 4*c*g*h
*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

maple [B] time = 0.01, size = 327, normalized size = 2.19

$$\frac{f\,h^{2}x^{3}}{2\sqrt{c\,x^{2}+a}\;c}+\frac{e\,h^{2}x^{2}}{\sqrt{c\,x^{2}+a}\;c}+\frac{2 f g h\,x^{2}}{\sqrt{c\,x^{2}+a}\;c}+\frac{3 a f\,h^{2}x}{2\sqrt{c\,x^{2}+a}\;c^{2}}+\frac{d\,g^{2}x}{\sqrt{c\,x^{2}+a}\;a}-\frac{d\,h^{2}x}{\sqrt{c\,x^{2}+a}\;c}-\frac{2 e g h x}{\sqrt{c\,x^{2}+a}\;c}-\frac{f\,g^{2}x}{\sqrt{c\,x^{2}+a}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/2*h^2*f*x^3/c/(c*x^2+a)^(1/2)+3/2*h^2*f*a/c^2*x/(c*x^2+a)^(1/2)-3/2*h^2*f
*a/c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+x^2/c/(c*x^2+a)^(1/2)*h^2*e+2*x^2/
c/(c*x^2+a)^(1/2)*g*h*f+2*a/c^2/(c*x^2+a)^(1/2)*h^2*e+4*a/c^2/(c*x^2+a)^(1/
2)*g*h*f-x/c/(c*x^2+a)^(1/2)*d*h^2-2*x/c/(c*x^2+a)^(1/2)*e*g*h-x/c/(c*x^2+a
)^(1/2)*f*g^2+1/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*d*h^2+2/c^(3/2)*ln(c^
(1/2)*x+(c*x^2+a)^(1/2))*e*g*h+1/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*f*g^
2-2/c/(c*x^2+a)^(1/2)*g*h*d-1/c/(c*x^2+a)^(1/2)*g^2*e+g^2*d*x/a/(c*x^2+a)^(
1/2)
```

maxima [A] time = 0.45, size = 227, normalized size = 1.52

$$\frac{fh^{2}x^{3}}{2\sqrt{cx^{2}+a}\,c}+\frac{dg^{2}x}{\sqrt{cx^{2}+a}\,a}+\frac{3\,afh^{2}x}{2\sqrt{cx^{2}+a}\,c^{2}}-\frac{3\,afh^{2}\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{5}{2}}}-\frac{eg^{2}}{\sqrt{cx^{2}+a}\,c}-\frac{2\,dgh}{\sqrt{cx^{2}+a}\,c}+\frac{\left(2\,fgh+eh^{2}\right)x^{2}}{\sqrt{cx^{2}+a}\,c}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*f*h^2*x^3/(sqrt(c*x^2 + a)*c) + d*g^2*x/(sqrt(c*x^2 + a)*a) + 3/2*a*f*h
^2*x/(sqrt(c*x^2 + a)*c^2) - 3/2*a*f*h^2*arcsinh(c*x/sqrt(a*c))/c^(5/2) - e
*g^2/(sqrt(c*x^2 + a)*c) - 2*d*g*h/(sqrt(c*x^2 + a)*c) + (2*f*g*h + e*h^2)*
x^2/(sqrt(c*x^2 + a)*c) - (f*g^2 + 2*e*g*h + d*h^2)*x/(sqrt(c*x^2 + a)*c) +
 (f*g^2 + 2*e*g*h + d*h^2)*arcsinh(c*x/sqrt(a*c))/c^(3/2) + 2*(2*f*g*h + e*
h^2)*a/(sqrt(c*x^2 + a)*c^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g+h\,x\right)}^2\,\left(f\,x^2+e\,x+d\right)}{{\left(c\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g + hx\right)^{2} \left(d + ex + fx^{2}\right)}{\left(a + cx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**2*(d + e*x + f*x**2)/(a + c*x**2)**(3/2), x)
```

3.110 $\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(eh+fg)}{c^{3/2}}-\frac{h\sqrt{a+cx^2}\,(cd-2af)}{ac^2}-\frac{(g+hx)(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

```
[Out] (e*h+f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)-(a*e-(-a*f+c*d)*x)*(h*
x+g)/a/c/(c*x^2+a)^(1/2)-(-2*a*f+c*d)*h*(c*x^2+a)^(1/2)/a/c^2
```

Rubi [A] time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1645, 641, 217, 206}

$$-\frac{h\sqrt{a+cx^2}\,(cd-2af)}{ac^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(eh+fg)}{c^{3/2}}-\frac{(g+hx)(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x))/(a*c*Sqrt[a + c*x^2])) - ((c*d - 2*a*f)
*h*Sqrt[a + c*x^2])/(a*c^2) + ((f*g + e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x
^2]])/c^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{\int \frac{-a(fg+eh)+(cd-2af)hx}{\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2} + \frac{(fg+eh)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{c}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2} + \frac{(fg+eh)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,d\right.}{c}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2} + \frac{(fg+eh)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.14, size = 102, normalized size = 1.02

$$\frac{a^{3/2}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(eh+fg)+2a^2fh-ac(dh+e(g+hx)+fx(g-hx))+c^2dgx}{ac^2\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] (2*a^2*f*h + c^2*d*g*x - a*c*(d*h + f*x*(g - h*x) + e*(g + h*x)) + a^(3/2)*
Sqrt[c]*(f*g + e*h)*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(a*c^
2*Sqrt[a + c*x^2])
```

fricas [A] time = 1.14, size = 278, normalized size = 2.78

$$\left[\frac{\left(a^2fg+a^2eh+\left(acfg+aceh\right)x^2\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\left(acfhx^2-aceg-\left(acd-2\,a^2f\right)h-\left(\right.\right.}{2\left(ac^3x^2+a^2c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*f*g + a^2*e*h + (a*c*f*g + a*c*e*h)*x^2)*sqrt(c)*log(-2*c*x^2 -
2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(a*c*f*h*x^2 - a*c*e*g - (a*c*d - 2*a^
2*f)*h - (a*c*e*h - (c^2*d - a*c*f)*g)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2
*c^2), -((a^2*f*g + a^2*e*h + (a*c*f*g + a*c*e*h)*x^2)*sqrt(-c)*arctan(sqrt
(-c)*x/sqrt(c*x^2 + a)) - (a*c*f*h*x^2 - a*c*e*g - (a*c*d - 2*a^2*f)*h - (a
*c*e*h - (c^2*d - a*c*f)*g)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*c^2)]
```

giac [A] time = 0.25, size = 116, normalized size = 1.16

$$\frac{\left(\frac{fhx}{c}+\frac{c^3dg-ac^2fg-ac^2he}{ac^3}\right)x-\frac{ac^2dh-2\,a^2cfh+ac^2ge}{ac^3}}{\sqrt{cx^2+a}}-\frac{\left(fg+he\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((f*h*x/c + (c^3*d*g - a*c^2*f*g - a*c^2*h*e)/(a*c^3))*x - (a*c^2*d*h - 2*a
^2*c*f*h + a*c^2*g*e)/(a*c^3))/sqrt(c*x^2 + a) - (f*g + h*e)*log(abs(-sqrt(
c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

maple [A] time = 0.00, size = 163, normalized size = 1.63

$$\frac{fh\,x^2}{\sqrt{c\,x^2+a}\;c}+\frac{dgx}{\sqrt{c\,x^2+a}\;a}-\frac{ehx}{\sqrt{c\,x^2+a}\;c}-\frac{fgx}{\sqrt{c\,x^2+a}\;c}+\frac{eh\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{c^{\frac{3}{2}}}+\frac{fg\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{c^{\frac{3}{2}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] h*f*x^2/c/(c*x^2+a)^(1/2)+2*h*f*a/c^2/(c*x^2+a)^(1/2)-x/c/(c*x^2+a)^(1/2)*e
*h-x/c/(c*x^2+a)^(1/2)*f*g+1/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*h+1/c^
(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*f*g-1/c/(c*x^2+a)^(1/2)*d*h-1/c/(c*x^2+
a)^(1/2)*e*g+d*g*x/a/(c*x^2+a)^(1/2)
```

maxima [A] time = 0.44, size = 126, normalized size = 1.26

$$\frac{fhx^2}{\sqrt{cx^2+a}\,c}+\frac{dgx}{\sqrt{cx^2+a}\,a}-\frac{eg}{\sqrt{cx^2+a}\,c}-\frac{dh}{\sqrt{cx^2+a}\,c}+\frac{2\,afh}{\sqrt{cx^2+a}\,c^2}-\frac{\left(fg+eh\right)x}{\sqrt{cx^2+a}\,c}+\frac{\left(fg+eh\right)\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] f*h*x^2/(sqrt(c*x^2 + a)*c) + d*g*x/(sqrt(c*x^2 + a)*a) - e*g/(sqrt(c*x^2 +
 a)*c) - d*h/(sqrt(c*x^2 + a)*c) + 2*a*f*h/(sqrt(c*x^2 + a)*c^2) - (f*g + e
*h)*x/(sqrt(c*x^2 + a)*c) + (f*g + e*h)*arcsinh(c*x/sqrt(a*c))/c^(3/2)
```

mupad [B] time = 5.28, size = 151, normalized size = 1.51

$$\frac{e\,h\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{c^{3/2}}+\frac{f\,g\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{c^{3/2}}-\frac{d\,h}{c\,\sqrt{c\,x^2+a}}-\frac{e\,g}{c\,\sqrt{c\,x^2+a}}+\frac{d\,g\,x}{a\,\sqrt{c\,x^2+a}}-\frac{e\,h\,x}{c\,\sqrt{c\,x^2+a}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x)
```

```
[Out] (e*h*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/c^(3/2) + (f*g*log(c^(1/2)*x + (a
+ c*x^2)^(1/2)))/c^(3/2) - (d*h)/(c*(a + c*x^2)^(1/2)) - (e*g)/(c*(a + c*x^
2)^(1/2)) + (d*g*x)/(a*(a + c*x^2)^(1/2)) - (e*h*x)/(c*(a + c*x^2)^(1/2)) -
 (f*g*x)/(c*(a + c*x^2)^(1/2)) + (f*h*(2*a + c*x^2))/(c^2*(a + c*x^2)^(1/2)
)
```

sympy [A] time = 18.84, size = 209, normalized size = 2.09

$$dh\left(\begin{cases}-\frac{1}{c\sqrt{a+cx^2}} & \text{for}\ c\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+eg\left(\begin{cases}-\frac{1}{c\sqrt{a+cx^2}} & \text{for}\ c\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+eh\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,c\sqrt{1+\frac{cx^2}{a}}}\right)+fg\left(\frac{\operatorname{asinh}\left(\right.}{c^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] d*h*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)), True
)) + e*g*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)),
 True)) + e*h*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 + c*
x**2/a))) + f*g*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 +
c*x**2/a))) + f*h*Piecewise((2*a/(c**2*sqrt(a + c*x**2)) + x**2/(c*sqrt(a +
 c*x**2)), Ne(c, 0)), (x**4/(4*a**(3/2)), True)) + d*g*x/(a**(3/2)*sqrt(1 +
 c*x**2/a))
```

3.111 $\int \frac{d+ex+fx^2}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=61

$$\frac{f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}-\frac{ae-x(cd-af)}{ac\sqrt{a+cx^2}}$$

[Out] f*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+(-a*e+(-a*f+c*d)*x)/a/c/(c*x^2+a)^(1/2)

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1814, 12, 217, 206}

$$\frac{f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}-\frac{ae-x(cd-af)}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*x^2)/(a + c*x^2)^(3/2),x]

[Out] -((a*e - (c*d - a*f)*x)/(a*c*Sqrt[a + c*x^2])) + (f*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/c^(3/2)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}} + \frac{\int \frac{af}{c\sqrt{a+cx^2}}\,dx}{a}$$
$$= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}} + \frac{f\int \frac{1}{\sqrt{a+cx^2}}\,dx}{c}$$
$$= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}} + \frac{f\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{c}$$
$$= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}} + \frac{f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.06, size = 74, normalized size = 1.21

$$\frac{a^{3/2}f\sqrt{\frac{cx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)+\sqrt{c}\,(cdx-a(e+fx))}{ac^{3/2}\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*(c*d*x - a*(e + f*x)) + a^(3/2)*f*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqr
t[c]*x)/Sqrt[a]])/(a*c^(3/2)*Sqrt[a + c*x^2])
```

fricas [A] time = 0.63, size = 181, normalized size = 2.97

$$\left[\frac{\left(acfx^2+a^2f\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\left(ace-\left(c^2d-acf\right)x\right)\sqrt{cx^2+a}}{2\left(ac^3x^2+a^2c^2\right)}, -\frac{\left(acfx^2+a^2f\right)\sqrt{-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c*f*x^2 + a^2*f)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*
x - a) - 2*(a*c*e - (c^2*d - a*c*f)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*c^
2), -((a*c*f*x^2 + a^2*f)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (a*
c*e - (c^2*d - a*c*f)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*c^2)]
```

giac [A] time = 0.20, size = 63, normalized size = 1.03

$$-\frac{\frac{e}{c}-\frac{\left(c^2d-acf\right)x}{ac^2}}{\sqrt{cx^2+a}}-\frac{f\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(e/c - (c^2*d - a*c*f)*x/(a*c^2))/sqrt(c*x^2 + a) - f*log(abs(-sqrt(c)*x +
 sqrt(c*x^2 + a)))/c^(3/2)
```

maple [A] time = 0.01, size = 69, normalized size = 1.13

$$\frac{dx}{\sqrt{c\,x^2+a}\;a}-\frac{fx}{\sqrt{c\,x^2+a}\;c}+\frac{f\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{c^{\frac{3}{2}}}-\frac{e}{\sqrt{c\,x^2+a}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] -f*x/c/(c*x^2+a)^(1/2)+f/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))-e/c/(c*x^2+a
)^(1/2)+d*x/a/(c*x^2+a)^(1/2)
```

maxima [A] time = 0.43, size = 61, normalized size = 1.00

$$\frac{dx}{\sqrt{cx^2+a}\,a} - \frac{fx}{\sqrt{cx^2+a}\,c} + \frac{f\,\text{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}} - \frac{e}{\sqrt{cx^2+a}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] d*x/(sqrt(c*x^2 + a)*a) - f*x/(sqrt(c*x^2 + a)*c) + f*arcsinh(c*x/sqrt(a*c)
)/c^(3/2) - e/(sqrt(c*x^2 + a)*c)
```

mupad [B] time = 4.33, size = 68, normalized size = 1.11

$$\frac{f\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{c^{3/2}} - \frac{e}{c\,\sqrt{c\,x^2+a}} + \frac{d\,x}{a\,\sqrt{c\,x^2+a}} - \frac{f\,x}{c\,\sqrt{c\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + c*x^2)^(3/2),x)
```

```
[Out] (f*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/c^(3/2) - e/(c*(a + c*x^2)^(1/2)) +
(d*x)/(a*(a + c*x^2)^(1/2)) - (f*x)/(c*(a + c*x^2)^(1/2))
```

sympy [A] time = 8.86, size = 87, normalized size = 1.43

$$e\left(\begin{cases} -\frac{1}{c\sqrt{a+cx^2}} & \text{for } c \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + f\left(\frac{\text{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}} - \frac{x}{\sqrt{a}\,c\sqrt{1+\frac{cx^2}{a}}}\right) + \frac{dx}{a^{\frac{3}{2}}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] e*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)), True))
 + f*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 + c*x**2/a)))
 + d*x/(a**(3/2)*sqrt(1 + c*x**2/a))
```

3.112 $\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=138

$$-\frac{a(afh-cdh+ceg)-cx(aeh-afg+cdg)}{ac\sqrt{a+cx^2}\left(ah^2+cg^2\right)}-\frac{\left(dh^2-egh+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{\left(ah^2+cg^2\right)^{3/2}}$$

```
[Out] -(d*h^2-e*g*h+f*g^2)*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/
2))/(a*h^2+c*g^2)^(3/2)+(-a*(a*f*h-c*d*h+c*e*g)+c*(a*e*h-a*f*g+c*d*g)*x)/a/
c/(a*h^2+c*g^2)/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1647, 12, 725, 206}

$$-\frac{a(afh-cdh+ceg)-cx(aeh-afg+cdg)}{ac\sqrt{a+cx^2}\left(ah^2+cg^2\right)}-\frac{\left(dh^2-egh+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{\left(ah^2+cg^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*(c*e*g - c*d*h + a*f*h) - c*(c*d*g - a*f*g + a*e*h)*x)/(a*c*(c*g^2 + a
*h^2)*Sqrt[a + c*x^2])) - ((f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(S
qrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{ac\left(fg^2-egh+dh^2\right)}{\left(cg^2+ah^2\right)(g+hx)\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}} + \frac{\left(fg^2-egh+dh^2\right)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{cg^2+ah^2}$$

$$= -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}} - \frac{\left(fg^2-egh+dh^2\right)\text{Subst}\left(\int \frac{1}{cg^2+ah^2-x}\right.}{cg^2+ah^2}$$

$$= -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}} - \frac{\left(fg^2-egh+dh^2\right)\tanh^{-1}\left(\frac{ah-cg}{\sqrt{cg^2+ah^2}\,\sqrt{}}\right.}{\left(cg^2+ah^2\right)^{3/2}}$$

Mathematica [A] time = 0.18, size = 137, normalized size = 0.99

$$\frac{a^2(-f)h+ac(dh-eg+ehx-fgx)+c^2dgx}{ac\sqrt{a+cx^2}\left(ah^2+cg^2\right)} - \frac{\left(h(dh-eg)+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\,\sqrt{ah^2+cg^2}}\right)}{\left(ah^2+cg^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-(a^2*f*h) + c^2*d*g*x + a*c*(-(e*g) + d*h - f*g*x + e*h*x))/(a*c*(c*g^2 +
 a*h^2)*Sqrt[a + c*x^2]) - ((f*g^2 + h*(-(e*g) + d*h))*ArcTanh[(a*h - c*g*x
)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(3/2)
```

fricas [B] time = 4.00, size = 721, normalized size = 5.22

$$\left[\frac{\left(a^2cfg^2-a^2cegh+a^2cdh^2+\left(ac^2fg^2-ac^2egh+ac^2dh^2\right)x^2\right)\sqrt{cg^2+ah^2}\,\log\left(\frac{2\,acghx-acg^2-2\,a^2h^2-\left(2\,c^2g^2+ach^2\right)x^2-2\,\sqrt{c}}{h^2x^2+2\,ghx+g^2}\right.}{2\left(a^2c^3g^4+2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*c*f*g^2 - a^2*c*e*g*h + a^2*c*d*h^2 + (a*c^2*f*g^2 - a*c^2*e*g*h
 + a*c^2*d*h^2)*x^2)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2
*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt
(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) - 2*(a*c^2*e*g^3 + a^2*c*e*g*h^2 -
(a*c^2*d - a^2*c*f)*g^2*h - (a^2*c*d - a^3*f)*h^3 - (a*c^2*e*g^2*h + a^2*c*
e*h^3 + (c^3*d - a*c^2*f)*g^3 + (a*c^2*d - a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 +
a))/(a^2*c^3*g^4 + 2*a^3*c^2*g^2*h^2 + a^4*c*h^4 + (a*c^4*g^4 + 2*a^2*c^3*g
^2*h^2 + a^3*c^2*h^4)*x^2), -((a^2*c*f*g^2 - a^2*c*e*g*h + a^2*c*d*h^2 + (a
*c^2*f*g^2 - a*c^2*e*g*h + a*c^2*d*h^2)*x^2)*sqrt(-c*g^2 - a*h^2)*arctan(sq
rt(-c*g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*
g^2 + a*c*h^2)*x^2)) + (a*c^2*e*g^3 + a^2*c*e*g*h^2 - (a*c^2*d - a^2*c*f)*g
^2*h - (a^2*c*d - a^3*f)*h^3 - (a*c^2*e*g^2*h + a^2*c*e*h^3 + (c^3*d - a*c^
2*f)*g^3 + (a*c^2*d - a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/(a^2*c^3*g^4 + 2*
a^3*c^2*g^2*h^2 + a^4*c*h^4 + (a*c^4*g^4 + 2*a^2*c^3*g^2*h^2 + a^3*c^2*h^4)
*x^2)]
```

giac [B] time = 0.29, size = 294, normalized size = 2.13

(c^3dg^3−ac^2fg^3+ac^2dgh^2−a^2cfgh^2+ac^2g^2he+a^2ch^3e)x / (ac^3g^4+2a^2c^2g^2h^2+a^3ch^4) + (ac^2dg^2h−a^2cfg^2h+a^2cdh^3−a^3fh^3−ac^2g^3e−a^2cgh^2e) / (ac^3g^4+2a^2c^2g^2h^2+a^3ch^4), all over √(cx^2 + a), − 2(fg^2 + dh^2 − ghe) arcta… / ((cg^2 + ah^2)√…

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((c^3*d*g^3 - a*c^2*f*g^3 + a*c^2*d*g*h^2 - a^2*c*f*g*h^2 + a*c^2*g^2*h*e +
 a^2*c*h^3*e)*x/(a*c^3*g^4 + 2*a^2*c^2*g^2*h^2 + a^3*c*h^4) + (a*c^2*d*g^2*
h - a^2*c*f*g^2*h + a^2*c*d*h^3 - a^3*f*h^3 - a*c^2*g^3*e - a^2*c*g*h^2*e)/
(a*c^3*g^4 + 2*a^2*c^2*g^2*h^2 + a^3*c*h^4))/sqrt(c*x^2 + a) - 2*(f*g^2 + d
*h^2 - g*h*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*
g^2 - a*h^2))/((c*g^2 + a*h^2)*sqrt(-c*g^2 - a*h^2))
```

maple [B] time = 0.02, size = 862, normalized size = 6.25

$$\frac{cdgx}{\left(ah^2+cg^2\right)\sqrt{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+\left(x+\frac{g}{h}\right)^2c+\frac{ah^2+cg^2}{h^2}}\,a}-\frac{ceg^2x}{\left(ah^2+cg^2\right)\sqrt{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+\left(x+\frac{g}{h}\right)^2c+\frac{ah^2+cg^2}{h^2}}\,ah}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x)
```

```
[Out] -1/h*f/c/(c*x^2+a)^(1/2)+1/h*e*x/a/(c*x^2+a)^(1/2)-1/h^2*f*g*x/a/(c*x^2+a)^
(1/2)+h/(a*h^2+c*g^2)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2
)*d-1/(a*h^2+c*g^2)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*
e*g+1/h/(a*h^2+c*g^2)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2
)*f*g^2+g/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^
(1/2)*x*c*d-1/h*g^2/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*
g^2)/h^2)^(1/2)*x*c*e+1/h^2*g^3/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2
*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*c*f-h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)
*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(
x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d+1/(a*h^2+c*g^
2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a
*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/
2))/(x+g/h))*e*g-1/h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)
*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x
+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f*g^2
```

maxima [B] time = 0.62, size = 453, normalized size = 3.28

$$\frac{cfg^3x}{\sqrt{cx^2+a}\,acg^2h^2+\sqrt{cx^2+a}\,a^2h^4}-\frac{ceg^2x}{\sqrt{cx^2+a}\,acg^2h+\sqrt{cx^2+a}\,a^2h^3}+\frac{cdgx}{\sqrt{cx^2+a}\,acg^2+\sqrt{cx^2+a}\,a^2h^2}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] c*f*g^3*x/(sqrt(c*x^2 + a)*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) - c*e*g^2
*x/(sqrt(c*x^2 + a)*a*c*g^2*h + sqrt(c*x^2 + a)*a^2*h^3) + c*d*g*x/(sqrt(c*
x^2 + a)*a*c*g^2 + sqrt(c*x^2 + a)*a^2*h^2) + f*g^2/(sqrt(c*x^2 + a)*c*g^2*
```

```
h + sqrt(c*x^2 + a)*a*h^3) - e*g/(sqrt(c*x^2 + a)*c*g^2 + sqrt(c*x^2 + a)*a
*h^2) + d/(sqrt(c*x^2 + a)*c*g^2/h + sqrt(c*x^2 + a)*a*h) - f*g*x/(sqrt(c*x
^2 + a)*a*h^2) + e*x/(sqrt(c*x^2 + a)*a*h) + f*g^2*arcsinh(c*g*x/(sqrt(a*c)
*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^3)
- e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g))
)/((a + c*g^2/h^2)^(3/2)*h^2) + d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h) - f/(sqrt(c*x^2 + a
)*c*h)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)\,{\left(c\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\left(a+cx^2\right)^{\frac{3}{2}}\left(g+hx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((a + c*x**2)**(3/2)*(g + h*x)), x)
```

3.113 $\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{a(ah(2fg-eh)+cg(eg-2dh))-x\left(a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)+c^2dg^2\right)}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^2}-\frac{h\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{(g+hx)\left(ah^2+cg^2\right)^2}$$

```
[Out] (a*h^2*(-e*h+2*f*g)-c*g*(f*g^2-h*(-3*d*h+2*e*g)))*arctanh((-c*g*x+a*h)/(a*h
^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(5/2)+(-a*(c*g*(-2*d*h+e*g)+
a*h*(-e*h+2*f*g))+(c^2*d*g^2+a^2*f*h^2-a*c*(f*g^2-h*(-d*h+2*e*g)))*x)/a/(a*
h^2+c*g^2)^2/(c*x^2+a)^(1/2)-h*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1/2)/(a*h^2+c
*g^2)^2/(h*x+g)
```

Rubi [A] time = 0.42, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1647, 807, 725, 206}

$$-\frac{a(ah(2fg-eh)+cg(eg-2dh))-x\left(a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)+c^2dg^2\right)}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^2}-\frac{h\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{(g+hx)\left(ah^2+cg^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*(c*g*(e*g - 2*d*h) + a*h*(2*f*g - e*h)) - (c^2*d*g^2 + a^2*f*h^2 - a*c
*(f*g^2 - h*(2*e*g - d*h)))*x)/(a*(c*g^2 + a*h^2)^2*Sqrt[a + c*x^2])) - (h*
(f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)^2*(g + h*x)) - ((
c*f*g^3 - c*g*h*(2*e*g - 3*d*h) - a*h^2*(2*f*g - e*h))*ArcTanh[(a*h - c*g*x
)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
```

```
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx = -\frac{a(cg(eg-2dh)+ah(2fg-eh))-\left(c^2dg^2+a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)\right)x}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}} - \ldots$$

$$= -\frac{a(cg(eg-2dh)+ah(2fg-eh))-\left(c^2dg^2+a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)\right)x}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}} - \ldots$$

$$= -\frac{a(cg(eg-2dh)+ah(2fg-eh))-\left(c^2dg^2+a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)\right)x}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}} - \ldots$$

$$= -\frac{a(cg(eg-2dh)+ah(2fg-eh))-\left(c^2dg^2+a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)\right)x}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}} - \ldots$$

Mathematica [A] time = 0.61, size = 285, normalized size = 1.19

$$\frac{\left(ah^2+cg^2\right)^{3/2}\left(-a^2fh^2+ac(2h(dh-eg+ehx)+fg(g-2hx))+2c^2dghx\right)+2h\left(h\left(a+cx^2\right)\sqrt{ah^2+cg^2}\left(a^2fh^2+\ldots\right.\right.}{2a\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-(a*f*(c*g^2 + a*h^2)^(5/2)) + (c*g^2 + a*h^2)^(3/2)*(-(a^2*f*h^2) + 2*c^2
*d*g*h*x + a*c*(f*g*(g - 2*h*x) + 2*h*(-(e*g) + d*h + e*h*x))) + 2*h*(h*Sqr
t[c*g^2 + a*h^2]*(c^2*d*g^2 + a^2*f*h^2 + a*c*(-2*f*g^2 + h*(3*e*g - 2*d*h)
))*(a + c*x^2) - a*c*(c*f*g^3 + c*g*h*(-2*e*g + 3*d*h) + a*h^2*(-2*f*g + e*
h))*(g + h*x)*Sqrt[a + c*x^2]*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sq
rt[a + c*x^2])]))/(2*a*c*h*(c*g^2 + a*h^2)^(5/2)*(g + h*x)*Sqrt[a + c*x^2])
```

fricas [B] time = 7.18, size = 1573, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*c*f*g^4 - 2*a^2*c*e*g^3*h + a^3*e*g*h^3 + (3*a^2*c*d - 2*a^3*f)
*g^2*h^2 + (a*c^2*f*g^3*h - 2*a*c^2*e*g^2*h^2 + a^2*c*e*h^4 + (3*a*c^2*d -
2*a^2*c*f)*g*h^3)*x^3 + (a*c^2*f*g^4 - 2*a*c^2*e*g^3*h + a^2*c*e*g*h^3 + (3
*a*c^2*d - 2*a^2*c*f)*g^2*h^2)*x^2 + (a^2*c*f*g^3*h - 2*a^2*c*e*g^2*h^2 + a
^3*e*h^4 + (3*a^2*c*d - 2*a^3*f)*g*h^3)*x)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g
*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 + 2*sqrt(c*g^2 + a*h
^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) + 2*(a*c^2*e*
```

```
g^5 - a^2*c*e*g^3*h^2 - 2*a^3*e*g*h^4 + a^3*d*h^5 - (2*a*c^2*d - 3*a^2*c*f)
*g^4*h - (a^2*c*d - 3*a^3*f)*g^2*h^3 - (3*a*c^2*e*g^3*h^2 + 3*a^2*c*e*g*h^4
 + (c^3*d - 2*a*c^2*f)*g^4*h - (a*c^2*d + a^2*c*f)*g^2*h^3 - (2*a^2*c*d - a
^3*f)*h^5)*x^2 - (a*c^2*e*g^4*h + 2*a^2*c*e*g^2*h^3 + a^3*e*h^5 + (c^3*d -
a*c^2*f)*g^5 + 2*(a*c^2*d - a^2*c*f)*g^3*h^2 + (a^2*c*d - a^3*f)*g*h^4)*x)*
sqrt(c*x^2 + a))/(a^2*c^3*g^7 + 3*a^3*c^2*g^5*h^2 + 3*a^4*c*g^3*h^4 + a^5*g
*h^6 + (a*c^4*g^6*h + 3*a^2*c^3*g^4*h^3 + 3*a^3*c^2*g^2*h^5 + a^4*c*h^7)*x^
3 + (a*c^4*g^7 + 3*a^2*c^3*g^5*h^2 + 3*a^3*c^2*g^3*h^4 + a^4*c*g*h^6)*x^2 +
 (a^2*c^3*g^6*h + 3*a^3*c^2*g^4*h^3 + 3*a^4*c*g^2*h^5 + a^5*h^7)*x), -((a^2
*c*f*g^4 - 2*a^2*c*e*g^3*h + a^3*e*g*h^3 + (3*a^2*c*d - 2*a^3*f)*g^2*h^2 +
(a*c^2*f*g^3*h - 2*a*c^2*e*g^2*h^2 + a^2*c*e*h^4 + (3*a*c^2*d - 2*a^2*c*f)*
g*h^3)*x^3 + (a*c^2*f*g^4 - 2*a*c^2*e*g^3*h + a^2*c*e*g*h^3 + (3*a*c^2*d -
2*a^2*c*f)*g^2*h^2)*x^2 + (a^2*c*f*g^3*h - 2*a^2*c*e*g^2*h^2 + a^3*e*h^4 +
(3*a^2*c*d - 2*a^3*f)*g*h^3)*x)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2 - a
*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2
)*x^2)) + (a*c^2*e*g^5 - a^2*c*e*g^3*h^2 - 2*a^3*e*g*h^4 + a^3*d*h^5 - (2*a
*c^2*d - 3*a^2*c*f)*g^4*h - (a^2*c*d - 3*a^3*f)*g^2*h^3 - (3*a*c^2*e*g^3*h^
2 + 3*a^2*c*e*g*h^4 + (c^3*d - 2*a*c^2*f)*g^4*h - (a*c^2*d + a^2*c*f)*g^2*h
^3 - (2*a^2*c*d - a^3*f)*h^5)*x^2 - (a*c^2*e*g^4*h + 2*a^2*c*e*g^2*h^3 + a^
3*e*h^5 + (c^3*d - a*c^2*f)*g^5 + 2*(a*c^2*d - a^2*c*f)*g^3*h^2 + (a^2*c*d
- a^3*f)*g*h^4)*x)*sqrt(c*x^2 + a))/(a^2*c^3*g^7 + 3*a^3*c^2*g^5*h^2 + 3*a^
4*c*g^3*h^4 + a^5*g*h^6 + (a*c^4*g^6*h + 3*a^2*c^3*g^4*h^3 + 3*a^3*c^2*g^2*
h^5 + a^4*c*h^7)*x^3 + (a*c^4*g^7 + 3*a^2*c^3*g^5*h^2 + 3*a^3*c^2*g^3*h^4 +
 a^4*c*g*h^6)*x^2 + (a^2*c^3*g^6*h + 3*a^3*c^2*g^4*h^3 + 3*a^4*c*g^2*h^5 +
a^5*h^7)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 1663, normalized size = 6.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x)
```

```
[Out] f/h^2*x/a/(c*x^2+a)^(1/2)-1/(a*h^2+c*g^2)/(x+g/h)/(-2*(x+g/h)*c*g/h+(x+g/h)
^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*d+1/h/(a*h^2+c*g^2)/(x+g/h)/(-2*(x+g/h)*c*g/h
+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e*g-1/h^2/(a*h^2+c*g^2)/(x+g/h)/(-2*(
x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f*g^2+3*h*c*g/(a*h^2+c*g^
2)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*d-3*c*g^2/(a*h^
2+c*g^2)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e+3/h*c*g
^3/(a*h^2+c*g^2)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f
+3*c^2*g^2/(a*h^2+c*g^2)^2/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^
2)^(1/2)*x*d-3/h*c^2*g^3/(a*h^2+c*g^2)^2/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a
*h^2+c*g^2)/h^2)^(1/2)*x*e+3/h^2*c^2*g^4/(a*h^2+c*g^2)^2/a/(-2*(x+g/h)*c*g/
h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*f-3*h*c*g/(a*h^2+c*g^2)^2/((a*h^2+
c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)
/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h)
)*d+3*c*g^2/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+
2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2
*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e-3/h*c*g^3/(a*h^2+c*g^2)^2/((a*h^2+c
*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/
```

```
h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))
*f-2/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)
*x*c*d+3/h/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)
^(1/2)*x*c*e*g-4/h^2/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c
*g^2)/h^2)^(1/2)*x*c*f*g^2+1/(a*h^2+c*g^2)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a
*h^2+c*g^2)/h^2)^(1/2)*e-2/h/(a*h^2+c*g^2)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a
*h^2+c*g^2)/h^2)^(1/2)*f*g-1/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2
*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*
c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e+2/h/(a*h^2+c*g^2)/((
a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+
c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(
x+g/h))*f*g
```

maxima [B] time = 0.77, size = 1085, normalized size = 4.54

$$\frac{3\,c^2fg^4x}{\sqrt{cx^2+a}\,ac^2g^4h^2+2\,\sqrt{cx^2+a}\,a^2cg^2h^4+\sqrt{cx^2+a}\,a^3h^6}-\frac{3\,c^2eg^3x}{\sqrt{cx^2+a}\,ac^2g^4h+2\,\sqrt{cx^2+a}\,a^2cg^2h^3+\sqrt{cx^2+a}\,a^3h^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 3*c^2*f*g^4*x/(sqrt(c*x^2 + a)*a*c^2*g^4*h^2 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*
h^4 + sqrt(c*x^2 + a)*a^3*h^6) - 3*c^2*e*g^3*x/(sqrt(c*x^2 + a)*a*c^2*g^4*h
 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^3 + sqrt(c*x^2 + a)*a^3*h^5) + 3*c^2*d*g^2
*x/(sqrt(c*x^2 + a)*a*c^2*g^4 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^2 + sqrt(c*x^
2 + a)*a^3*h^4) + 3*c*f*g^3/(sqrt(c*x^2 + a)*c^2*g^4*h + 2*sqrt(c*x^2 + a)*
a*c*g^2*h^3 + sqrt(c*x^2 + a)*a^2*h^5) - 4*c*f*g^2*x/(sqrt(c*x^2 + a)*a*c*g
^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) - 3*c*e*g^2/(sqrt(c*x^2 + a)*c^2*g^4 + 2*
sqrt(c*x^2 + a)*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) + 3*c*e*g*x/(sqrt(c*
x^2 + a)*a*c*g^2*h + sqrt(c*x^2 + a)*a^2*h^3) + 3*c*d*g/(sqrt(c*x^2 + a)*c^
2*g^4/h + 2*sqrt(c*x^2 + a)*a*c*g^2*h + sqrt(c*x^2 + a)*a^2*h^3) - f*g^2/(s
qrt(c*x^2 + a)*c*g^2*h^2*x + sqrt(c*x^2 + a)*a*h^4*x + sqrt(c*x^2 + a)*c*g^
3*h + sqrt(c*x^2 + a)*a*g*h^3) - 2*c*d*x/(sqrt(c*x^2 + a)*a*c*g^2 + sqrt(c*
x^2 + a)*a^2*h^2) + e*g/(sqrt(c*x^2 + a)*c*g^2*h*x + sqrt(c*x^2 + a)*a*h^3*
x + sqrt(c*x^2 + a)*c*g^3 + sqrt(c*x^2 + a)*a*g*h^2) - 2*f*g/(sqrt(c*x^2 +
a)*c*g^2*h + sqrt(c*x^2 + a)*a*h^3) - d/(sqrt(c*x^2 + a)*c*g^2*x + sqrt(c*x
^2 + a)*a*h^2*x + sqrt(c*x^2 + a)*c*g^3/h + sqrt(c*x^2 + a)*a*g*h) + e/(sqr
t(c*x^2 + a)*c*g^2 + sqrt(c*x^2 + a)*a*h^2) + f*x/(sqrt(c*x^2 + a)*a*h^2) +
 3*c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
+ g)))/((a + c*g^2/h^2)^(5/2)*h^5) - 3*c*e*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs
(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^4) + 3*
c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g))
)/((a + c*g^2/h^2)^(5/2)*h^3) - 2*f*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)
) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^3) + e*arcsinh(c
*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h
^2)^(3/2)*h^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^2\,{\left(c\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)), x)
```

<u>**sympy**</u> [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

3.114 $\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=374

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4-ach^2\left(3dh^2-9egh+11fg^2\right)+2c^2g^2\left(6dh^2-3egh+fg^2\right)\right)}{2\left(ah^2+cg^2\right)^{7/2}}+\frac{a\left(a^2fh^3-ach\right.}{}$$

```
[Out] -1/2*(2*a^2*f*h^4-a*c*h^2*(3*d*h^2-9*e*g*h+11*f*g^2)+2*c^2*g^2*(6*d*h^2-3*e
*g*h+f*g^2))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h
^2+c*g^2)^(7/2)+(a*(a^2*f*h^3-c^2*g^2*(-3*d*h+e*g)-a*c*h*(3*f*g^2-h*(-d*h+3
*e*g)))+c*(c^2*d*g^3+a^2*h^2*(-e*h+3*f*g)-a*c*g*(f*g^2-3*h*(-d*h+e*g)))*x)/
a/(a*h^2+c*g^2)^3/(c*x^2+a)^(1/2)-1/2*h*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1/2)
/(a*h^2+c*g^2)^2/(h*x+g)^2+1/2*h*(2*a*h^2*(-e*h+2*f*g)-c*g*(3*f*g^2-h*(-7*d
*h+5*e*g)))*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^3/(h*x+g)
```

Rubi [A] time = 1.03, antiderivative size = 372, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1647, 1651, 807, 725, 206}

$$\frac{cx\left(a^2h^2(3fg-eh)-acg\left(fg^2-3h(eg-dh)\right)+c^2dg^3\right)+a\left(a^2fh^3-ach\left(3fg^2-h(3eg-dh)\right)-c^2g^2(eg-3dh)\right)}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*(a^2*f*h^3 - c^2*g^2*(e*g - 3*d*h) - a*c*h*(3*f*g^2 - h*(3*e*g - d*h)))
+ c*(c^2*d*g^3 + a^2*h^2*(3*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(e*g - d*h)))*x
)/(a*(c*g^2 + a*h^2)^3*Sqrt[a + c*x^2]) - (h*(f*g^2 - e*g*h + d*h^2)*Sqrt[a
 + c*x^2])/(2*(c*g^2 + a*h^2)^2*(g + h*x)^2) - (h*(3*c*f*g^3 - c*g*h*(5*e*g
 - 7*d*h) - 2*a*h^2*(2*f*g - e*h))*Sqrt[a + c*x^2])/(2*(c*g^2 + a*h^2)^3*(g
 + h*x)) - ((2*a^2*f*h^4 - a*c*h^2*(11*f*g^2 - 9*e*g*h + 3*d*h^2) + 2*c^2*g
^2*(f*g^2 - 3*e*g*h + 6*d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*
Sqrt[a + c*x^2]])])/(2*(c*g^2 + a*h^2)^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx = \frac{a\left(a^2fh^3-c^2g^2(eg-3dh)-ach\left(3fg^2-h(3eg-dh)\right)\right)+c\left(c^2dg^3+a^2h^2(3fg-\right.}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

$$= \frac{a\left(a^2fh^3-c^2g^2(eg-3dh)-ach\left(3fg^2-h(3eg-dh)\right)\right)+c\left(c^2dg^3+a^2h^2(3fg-\right.}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

$$= \frac{a\left(a^2fh^3-c^2g^2(eg-3dh)-ach\left(3fg^2-h(3eg-dh)\right)\right)+c\left(c^2dg^3+a^2h^2(3fg-\right.}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

$$= \frac{a\left(a^2fh^3-c^2g^2(eg-3dh)-ach\left(3fg^2-h(3eg-dh)\right)\right)+c\left(c^2dg^3+a^2h^2(3fg-\right.}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

$$= \frac{a\left(a^2fh^3-c^2g^2(eg-3dh)-ach\left(3fg^2-h(3eg-dh)\right)\right)+c\left(c^2dg^3+a^2h^2(3fg-\right.}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

Mathematica [A] time = 1.18, size = 404, normalized size = 1.08

$$\frac{1}{2}\left(-\frac{\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(2a^2fh^4+ach^2\left(-3dh^2+9egh-11fg^2\right)+2c^2g^2\left(6dh^2-3egh+f\right.\right.}{\left(ah^2+cg^2\right)^{7/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-((Sqrt[a + c*x^2]*((h*(c*g^2 + a*h^2)*(f*g^2 + h*(-(e*g) + d*h)))/(g + h*
x)^2 + (h*(3*c*f*g^3 + c*g*h*(-5*e*g + 7*d*h) + 2*a*h^2*(-2*f*g + e*h)))/(g
```

```
 + h*x) + (2*(-(a^3*f*h^3) - c^3*d*g^3*x + a*c^2*g*(f*g^2*x + e*g*(g - 3*h*
x) + 3*d*h*(-g + h*x)) + a^2*c*h*(3*f*g*(g - h*x) + h*(-3*e*g + d*h + e*h*x
))))/(a*(a + c*x^2))))/(c*g^2 + a*h^2)^3) + ((2*a^2*f*h^4 + a*c*h^2*(-11*f*
g^2 + 9*e*g*h - 3*d*h^2) + 2*c^2*g^2*(f*g^2 - 3*e*g*h + 6*d*h^2))*Log[g + h
*x])/(c*g^2 + a*h^2)^(7/2) - ((2*a^2*f*h^4 + a*c*h^2*(-11*f*g^2 + 9*e*g*h -
 3*d*h^2) + 2*c^2*g^2*(f*g^2 - 3*e*g*h + 6*d*h^2))*Log[a*h - c*g*x + Sqrt[c
*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(7/2))/2
```

fricas [B] time = 33.90, size = 2853, normalized size = 7.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*a^2*c^2*f*g^6 - 6*a^2*c^2*e*g^5*h + 9*a^3*c*e*g^3*h^3 + (12*a^2*c^
2*d - 11*a^3*c*f)*g^4*h^2 - (3*a^3*c*d - 2*a^4*f)*g^2*h^4 + (2*a*c^3*f*g^4*
h^2 - 6*a*c^3*e*g^3*h^3 + 9*a^2*c^2*e*g*h^5 + (12*a*c^3*d - 11*a^2*c^2*f)*g
^2*h^4 - (3*a^2*c^2*d - 2*a^3*c*f)*h^6)*x^4 + 2*(2*a*c^3*f*g^5*h - 6*a*c^3*
e*g^4*h^2 + 9*a^2*c^2*e*g^2*h^4 + (12*a*c^3*d - 11*a^2*c^2*f)*g^3*h^3 - (3*
a^2*c^2*d - 2*a^3*c*f)*g*h^5)*x^3 + (2*a*c^3*f*g^6 - 6*a*c^3*e*g^5*h + 3*a^
2*c^2*e*g^3*h^3 + 9*a^3*c*e*g*h^5 + 3*(4*a*c^3*d - 3*a^2*c^2*f)*g^4*h^2 + 9
*(a^2*c^2*d - a^3*c*f)*g^2*h^4 - (3*a^3*c*d - 2*a^4*f)*h^6)*x^2 + 2*(2*a^2*
c^2*f*g^5*h - 6*a^2*c^2*e*g^4*h^2 + 9*a^3*c*e*g^2*h^4 + (12*a^2*c^2*d - 11*
a^3*c*f)*g^3*h^3 - (3*a^3*c*d - 2*a^4*f)*g*h^5)*x)*sqrt(c*g^2 + a*h^2)*log(
(2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 - 2*sqrt(c*g
^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) - 2*(
2*a*c^3*e*g^7 - 10*a^2*c^2*e*g^5*h^2 - 11*a^3*c*e*g^3*h^4 + a^4*e*g*h^6 + a
^4*d*h^7 - 2*(3*a*c^3*d - 5*a^2*c^2*f)*g^6*h + (4*a^2*c^2*d + 5*a^3*c*f)*g^
4*h^3 + (11*a^3*c*d - 5*a^4*f)*g^2*h^5 - (11*a*c^3*e*g^4*h^3 + 7*a^2*c^2*e*
g^2*h^5 - 4*a^3*c*e*h^7 + (2*c^4*d - 5*a*c^3*f)*g^5*h^2 - (11*a*c^3*d - 5*a
^2*c^2*f)*g^3*h^4 - (13*a^2*c^2*d - 10*a^3*c*f)*g*h^6)*x^3 - (16*a*c^3*e*g^
5*h^2 + 17*a^2*c^2*e*g^3*h^4 + a^3*c*e*g*h^6 + 4*(c^4*d - 2*a*c^3*f)*g^6*h
 - (10*a*c^3*d - a^2*c^2*f)*g^4*h^3 - (17*a^2*c^2*d - 11*a^3*c*f)*g^2*h^5 -
(3*a^3*c*d - 2*a^4*f)*h^7)*x^2 - (2*a*c^3*e*g^6*h + 17*a^2*c^2*e*g^4*h^3 +
13*a^3*c*e*g^2*h^5 - 2*a^4*e*h^7 + 2*(c^4*d - a*c^3*f)*g^7 + (8*a*c^3*d - 1
1*a^2*c^2*f)*g^5*h^2 - (5*a^2*c^2*d + a^3*c*f)*g^3*h^4 - (11*a^3*c*d - 8*a^
4*f)*g*h^6)*x)*sqrt(c*x^2 + a))/(a^2*c^4*g^10 + 4*a^3*c^3*g^8*h^2 + 6*a^4*c
^2*g^6*h^4 + 4*a^5*c*g^4*h^6 + a^6*g^2*h^8 + (a*c^5*g^8*h^2 + 4*a^2*c^4*g^6
*h^4 + 6*a^3*c^3*g^4*h^6 + 4*a^4*c^2*g^2*h^8 + a^5*c*h^10)*x^4 + 2*(a*c^5*g
^9*h + 4*a^2*c^4*g^7*h^3 + 6*a^3*c^3*g^5*h^5 + 4*a^4*c^2*g^3*h^7 + a^5*c*g*
h^9)*x^3 + (a*c^5*g^10 + 5*a^2*c^4*g^8*h^2 + 10*a^3*c^3*g^6*h^4 + 10*a^4*c^
2*g^4*h^6 + 5*a^5*c*g^2*h^8 + a^6*h^10)*x^2 + 2*(a^2*c^4*g^9*h + 4*a^3*c^3*
g^7*h^3 + 6*a^4*c^2*g^5*h^5 + 4*a^5*c*g^3*h^7 + a^6*g*h^9)*x), -1/2*((2*a^2
*c^2*f*g^6 - 6*a^2*c^2*e*g^5*h + 9*a^3*c*e*g^3*h^3 + (12*a^2*c^2*d - 11*a^3
*c*f)*g^4*h^2 - (3*a^3*c*d - 2*a^4*f)*g^2*h^4 + (2*a*c^3*f*g^4*h^2 - 6*a*c^
3*e*g^3*h^3 + 9*a^2*c^2*e*g*h^5 + (12*a*c^3*d - 11*a^2*c^2*f)*g^2*h^4 - (3*
a^2*c^2*d - 2*a^3*c*f)*h^6)*x^4 + 2*(2*a*c^3*f*g^5*h - 6*a*c^3*e*g^4*h^2 +
9*a^2*c^2*e*g^2*h^4 + (12*a*c^3*d - 11*a^2*c^2*f)*g^3*h^3 - (3*a^2*c^2*d -
2*a^3*c*f)*g*h^5)*x^3 + (2*a*c^3*f*g^6 - 6*a*c^3*e*g^5*h + 3*a^2*c^2*e*g^3*
h^3 + 9*a^3*c*e*g*h^5 + 3*(4*a*c^3*d - 3*a^2*c^2*f)*g^4*h^2 + 9*(a^2*c^2*d
- a^3*c*f)*g^2*h^4 - (3*a^3*c*d - 2*a^4*f)*h^6)*x^2 + 2*(2*a^2*c^2*f*g^5*h
- 6*a^2*c^2*e*g^4*h^2 + 9*a^3*c*e*g^2*h^4 + (12*a^2*c^2*d - 11*a^3*c*f)*g^3
*h^3 - (3*a^3*c*d - 2*a^4*f)*g*h^5)*x)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*
g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 +
a*c*h^2)*x^2)) + (2*a*c^3*e*g^7 - 10*a^2*c^2*e*g^5*h^2 - 11*a^3*c*e*g^3*h^4
 + a^4*e*g*h^6 + a^4*d*h^7 - 2*(3*a*c^3*d - 5*a^2*c^2*f)*g^6*h + (4*a^2*c^2
*d + 5*a^3*c*f)*g^4*h^3 + (11*a^3*c*d - 5*a^4*f)*g^2*h^5 - (11*a*c^3*e*g^4*
h^3 + 7*a^2*c^2*e*g^2*h^5 - 4*a^3*c*e*h^7 + (2*c^4*d - 5*a*c^3*f)*g^5*h^2 -
 (11*a*c^3*d - 5*a^2*c^2*f)*g^3*h^4 - (13*a^2*c^2*d - 10*a^3*c*f)*g*h^6)*x^
```

```
3 - (16*a*c^3*e*g^5*h^2 + 17*a^2*c^2*e*g^3*h^4 + a^3*c*e*g*h^6 + 4*(c^4*d -
 2*a*c^3*f)*g^6*h - (10*a*c^3*d - a^2*c^2*f)*g^4*h^3 - (17*a^2*c^2*d - 11*a
^3*c*f)*g^2*h^5 - (3*a^3*c*d - 2*a^4*f)*h^7)*x^2 - (2*a*c^3*e*g^6*h + 17*a^
2*c^2*e*g^4*h^3 + 13*a^3*c*e*g^2*h^5 - 2*a^4*e*h^7 + 2*(c^4*d - a*c^3*f)*g^
7 + (8*a*c^3*d - 11*a^2*c^2*f)*g^5*h^2 - (5*a^2*c^2*d + a^3*c*f)*g^3*h^4 -
(11*a^3*c*d - 8*a^4*f)*g*h^6)*x)*sqrt(c*x^2 + a))/(a^2*c^4*g^10 + 4*a^3*c^3
*g^8*h^2 + 6*a^4*c^2*g^6*h^4 + 4*a^5*c*g^4*h^6 + a^6*g^2*h^8 + (a*c^5*g^8*h
^2 + 4*a^2*c^4*g^6*h^4 + 6*a^3*c^3*g^4*h^6 + 4*a^4*c^2*g^2*h^8 + a^5*c*h^10
)*x^4 + 2*(a*c^5*g^9*h + 4*a^2*c^4*g^7*h^3 + 6*a^3*c^3*g^5*h^5 + 4*a^4*c^2*
g^3*h^7 + a^5*c*g*h^9)*x^3 + (a*c^5*g^10 + 5*a^2*c^4*g^8*h^2 + 10*a^3*c^3*g
^6*h^4 + 10*a^4*c^2*g^4*h^6 + 5*a^5*c*g^2*h^8 + a^6*h^10)*x^2 + 2*(a^2*c^4*
g^9*h + 4*a^3*c^3*g^7*h^3 + 6*a^4*c^2*g^5*h^5 + 4*a^5*c*g^3*h^7 + a^6*g*h^9
)*x)]
```

giac [B] time = 0.39, size = 1440, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((c^6*d*g^9 - a*c^5*f*g^9 - 6*a^2*c^4*d*g^5*h^4 + 6*a^3*c^3*f*g^5*h^4 - 8*a
^3*c^3*d*g^3*h^6 + 8*a^4*c^2*f*g^3*h^6 - 3*a^4*c^2*d*g*h^8 + 3*a^5*c*f*g*h^
8 + 3*a*c^5*g^8*h*e + 8*a^2*c^4*g^6*h^3*e + 6*a^3*c^3*g^4*h^5*e - a^5*c*h^9
*e)*x/(a*c^6*g^12 + 6*a^2*c^5*g^10*h^2 + 15*a^3*c^4*g^8*h^4 + 20*a^4*c^3*g^
6*h^6 + 15*a^5*c^2*g^4*h^8 + 6*a^6*c*g^2*h^10 + a^7*h^12) + (3*a*c^5*d*g^8*
h - 3*a^2*c^4*f*g^8*h + 8*a^2*c^4*d*g^6*h^3 - 8*a^3*c^3*f*g^6*h^3 + 6*a^3*c
^3*d*g^4*h^5 - 6*a^4*c^2*f*g^4*h^5 - a^5*c*d*h^9 + a^6*f*h^9 - a*c^5*g^9*e
+ 6*a^3*c^3*g^5*h^4*e + 8*a^4*c^2*g^3*h^6*e + 3*a^5*c*g*h^8*e)/(a*c^6*g^12
+ 6*a^2*c^5*g^10*h^2 + 15*a^3*c^4*g^8*h^4 + 20*a^4*c^3*g^6*h^6 + 15*a^5*c^2
*g^4*h^8 + 6*a^6*c*g^2*h^10 + a^7*h^12))/sqrt(c*x^2 + a) - (2*c^2*f*g^4 + 1
2*c^2*d*g^2*h^2 - 11*a*c*f*g^2*h^2 - 3*a*c*d*h^4 + 2*a^2*f*h^4 - 6*c^2*g^3*
h*e + 9*a*c*g*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/s
qrt(-c*g^2 - a*h^2))/((c^3*g^6 + 3*a*c^2*g^4*h^2 + 3*a^2*c*g^2*h^4 + a^3*h^
6)*sqrt(-c*g^2 - a*h^2)) - (2*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h +
 6*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*d*g^2*h^3 - 5*(sqrt(c)*x - sqrt(c*x^
2 + a))^3*a*c*f*g^2*h^3 - (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*d*h^5 - 4*(sq
rt(c)*x - sqrt(c*x^2 + a))^3*c^2*g^3*h^2*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a)
)^3*a*c*g*h^4*e + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 + 14*(sqr
t(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*d*g^3*h^2 - 11*(sqrt(c)*x - sqrt(c*x^2
+ a))^2*a*c^(3/2)*f*g^3*h^2 - 7*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d
*g*h^4 + 4*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 - 10*(sqrt(c
)*x - sqrt(c*x^2 + a))^2*c^(5/2)*g^4*h*e + 9*(sqrt(c)*x - sqrt(c*x^2 + a))^
2*a*c^(3/2)*g^2*h^3*e - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*h^5*e
 - 10*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*f*g^4*h - 22*(sqrt(c)*x - sqrt(c*
x^2 + a))*a*c^2*d*g^2*h^3 + 11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^
3 - (sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*d*h^5 + 16*(sqrt(c)*x - sqrt(c*x^2
+ a))*a*c^2*g^3*h^2*e - 5*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*g*h^4*e + 3*a
^2*c^(3/2)*f*g^3*h^2 + 7*a^2*c^(3/2)*d*g*h^4 - 4*a^3*sqrt(c)*f*g*h^4 - 5*a^
2*c^(3/2)*g^2*h^3*e + 2*a^3*sqrt(c)*h^5*e)/((c^3*g^6 + 3*a*c^2*g^4*h^2 + 3*
a^2*c*g^2*h^4 + a^3*h^6)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x
- sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2)
```

maple [B] time = 0.02, size = 2584, normalized size = 6.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x)
```

```
[Out] f/h/(a*h^2+c*g^2)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)+5/
2/h*c*g^2/(a*h^2+c*g^2)^2/(x+g/h)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^
2)/h^2)^(1/2)*e-5/2/h^2*c*g^3/(a*h^2+c*g^2)^2/(x+g/h)/(-2*(x+g/h)*c*g/h+(x+
g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f+15/2*c^3*g^3/(a*h^2+c*g^2)^3/a/(-2*(x+g
/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*d-15/2*h*c^2*g^2/(a*h^2+c*
g^2)^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2
*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)
^(1/2))/(x+g/h))*d-15/2/h*c^2*g^4/(a*h^2+c*g^2)^3/((a*h^2+c*g^2)/h^2)^(1/2)
*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(
x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f-13/2*c^2*g/(a
*h^2+c*g^2)^2/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*d-
2/h/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*
x*c*e+15/2/h*c*g^2/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)
*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x
+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f+5/h^2/(a*h^2+c*g^2)/a/(-2*(x
+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*c*f*g-15/2/h*c^3*g^4/(a*
h^2+c*g^2)^3/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*e+1
5/2/h^2*c^3*g^5/(a*h^2+c*g^2)^3/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^
2)/h^2)^(1/2)*x*f+19/2/h*c^2*g^2/(a*h^2+c*g^2)^2/a/(-2*(x+g/h)*c*g/h+(x+g/h
)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*e-25/2/h^2*c^2*g^3/(a*h^2+c*g^2)^2/a/(-2*(
x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*f-1/h/(a*h^2+c*g^2)/(x+
g/h)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e+9/2*c*g/(a*h^
2+c*g^2)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e+2/h^2/(
a*h^2+c*g^2)/(x+g/h)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)
*f*g-15/2/h*c*g^2/(a*h^2+c*g^2)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^
2)/h^2)^(1/2)*f-9/2*c*g/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x
+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g
/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e+1/2/h^2/(a*h^2+c*g^2)/(
x+g/h)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e*g-1/2/h^3
/(a*h^2+c*g^2)/(x+g/h)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(
1/2)*f*g^2-5/2*c*g/(a*h^2+c*g^2)^2/(x+g/h)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a
*h^2+c*g^2)/h^2)^(1/2)*d+15/2*h*c^2*g^2/(a*h^2+c*g^2)^3/(-2*(x+g/h)*c*g/h+(
x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*d+15/2/h*c^2*g^4/(a*h^2+c*g^2)^3/(-2*(x
+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f+15/2*c^2*g^3/(a*h^2+c*g^
2)^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*(
(a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(
1/2))/(x+g/h))*e+3/2*h*c/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(
x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*
g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d-f/h/(a*h^2+c*g^2)/((a*
h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*
g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+
g/h))-1/2/h/(a*h^2+c*g^2)/(x+g/h)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*
g^2)/h^2)^(1/2)*d-15/2*c^2*g^3/(a*h^2+c*g^2)^3/(-2*(x+g/h)*c*g/h+(x+g/h)^2*
c+(a*h^2+c*g^2)/h^2)^(1/2)*e-3/2*h*c/(a*h^2+c*g^2)^2/(-2*(x+g/h)*c*g/h+(x+g
/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*d
```

maxima [B] time = 1.02, size = 2254, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 15/2*c^3*f*g^5*x/(sqrt(c*x^2 + a)*a*c^3*g^6*h^2 + 3*sqrt(c*x^2 + a)*a^2*c^2
*g^4*h^4 + 3*sqrt(c*x^2 + a)*a^3*c*g^2*h^6 + sqrt(c*x^2 + a)*a^4*h^8) - 15/
2*c^3*e*g^4*x/(sqrt(c*x^2 + a)*a*c^3*g^6*h + 3*sqrt(c*x^2 + a)*a^2*c^2*g^4*
h^3 + 3*sqrt(c*x^2 + a)*a^3*c*g^2*h^5 + sqrt(c*x^2 + a)*a^4*h^7) + 15/2*c^3
*d*g^3*x/(sqrt(c*x^2 + a)*a*c^3*g^6 + 3*sqrt(c*x^2 + a)*a^2*c^2*g^4*h^2 + 3
*sqrt(c*x^2 + a)*a^3*c*g^2*h^4 + sqrt(c*x^2 + a)*a^4*h^6) + 15/2*c^2*f*g^4/
(sqrt(c*x^2 + a)*c^3*g^6*h + 3*sqrt(c*x^2 + a)*a*c^2*g^4*h^3 + 3*sqrt(c*x^2
```

```
 + a)*a^2*c*g^2*h^5 + sqrt(c*x^2 + a)*a^3*h^7) - 25/2*c^2*f*g^3*x/(sqrt(c*x
^2 + a)*a*c^2*g^4*h^2 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^4 + sqrt(c*x^2 + a)*a
^3*h^6) - 15/2*c^2*e*g^3/(sqrt(c*x^2 + a)*c^3*g^6 + 3*sqrt(c*x^2 + a)*a*c^2
*g^4*h^2 + 3*sqrt(c*x^2 + a)*a^2*c*g^2*h^4 + sqrt(c*x^2 + a)*a^3*h^6) + 19/
2*c^2*e*g^2*x/(sqrt(c*x^2 + a)*a*c^2*g^4*h + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^
3 + sqrt(c*x^2 + a)*a^3*h^5) + 15/2*c^2*d*g^2/(sqrt(c*x^2 + a)*c^3*g^6/h +
3*sqrt(c*x^2 + a)*a*c^2*g^4*h + 3*sqrt(c*x^2 + a)*a^2*c*g^2*h^3 + sqrt(c*x^
2 + a)*a^3*h^5) - 5/2*c*f*g^3/(sqrt(c*x^2 + a)*c^2*g^4*h^2*x + 2*sqrt(c*x^2
 + a)*a*c*g^2*h^4*x + sqrt(c*x^2 + a)*a^2*h^6*x + sqrt(c*x^2 + a)*c^2*g^5*h
 + 2*sqrt(c*x^2 + a)*a*c*g^3*h^3 + sqrt(c*x^2 + a)*a^2*g*h^5) - 13/2*c^2*d*
g*x/(sqrt(c*x^2 + a)*a*c^2*g^4 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^2 + sqrt(c*x
^2 + a)*a^3*h^4) + 5/2*c*e*g^2/(sqrt(c*x^2 + a)*c^2*g^4*h*x + 2*sqrt(c*x^2
+ a)*a*c*g^2*h^3*x + sqrt(c*x^2 + a)*a^2*h^5*x + sqrt(c*x^2 + a)*c^2*g^5 +
2*sqrt(c*x^2 + a)*a*c*g^3*h^2 + sqrt(c*x^2 + a)*a^2*g*h^4) - 15/2*c*f*g^2/(
sqrt(c*x^2 + a)*c^2*g^4*h + 2*sqrt(c*x^2 + a)*a*c*g^2*h^3 + sqrt(c*x^2 + a)
*a^2*h^5) + 5*c*f*g*x/(sqrt(c*x^2 + a)*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^
4) - 5/2*c*d*g/(sqrt(c*x^2 + a)*c^2*g^4*x + 2*sqrt(c*x^2 + a)*a*c*g^2*h^2*x
 + sqrt(c*x^2 + a)*a^2*h^4*x + sqrt(c*x^2 + a)*c^2*g^5/h + 2*sqrt(c*x^2 + a
)*a*c*g^3*h + sqrt(c*x^2 + a)*a^2*g*h^3) + 9/2*c*e*g/(sqrt(c*x^2 + a)*c^2*g
^4 + 2*sqrt(c*x^2 + a)*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) - 1/2*f*g^2/(
sqrt(c*x^2 + a)*c*g^2*h^3*x^2 + sqrt(c*x^2 + a)*a*h^5*x^2 + 2*sqrt(c*x^2 +
a)*c*g^3*h^2*x + 2*sqrt(c*x^2 + a)*a*g*h^4*x + sqrt(c*x^2 + a)*c*g^4*h + sq
rt(c*x^2 + a)*a*g^2*h^3) - 2*c*e*x/(sqrt(c*x^2 + a)*a*c*g^2*h + sqrt(c*x^2
+ a)*a^2*h^3) - 3/2*c*d/(sqrt(c*x^2 + a)*c^2*g^4/h + 2*sqrt(c*x^2 + a)*a*c*
g^2*h + sqrt(c*x^2 + a)*a^2*h^3) + 1/2*e*g/(sqrt(c*x^2 + a)*c*g^2*h^2*x^2 +
 sqrt(c*x^2 + a)*a*h^4*x^2 + 2*sqrt(c*x^2 + a)*c*g^3*h*x + 2*sqrt(c*x^2 + a
)*a*g*h^3*x + sqrt(c*x^2 + a)*c*g^4 + sqrt(c*x^2 + a)*a*g^2*h^2) + 2*f*g/(s
qrt(c*x^2 + a)*c*g^2*h^2*x + sqrt(c*x^2 + a)*a*h^4*x + sqrt(c*x^2 + a)*c*g^
3*h + sqrt(c*x^2 + a)*a*g*h^3) - 1/2*d/(sqrt(c*x^2 + a)*c*g^2*h*x^2 + sqrt(
c*x^2 + a)*a*h^3*x^2 + 2*sqrt(c*x^2 + a)*c*g^3*x + 2*sqrt(c*x^2 + a)*a*g*h^
2*x + sqrt(c*x^2 + a)*c*g^4/h + sqrt(c*x^2 + a)*a*g^2*h) - e/(sqrt(c*x^2 +
a)*c*g^2*h*x + sqrt(c*x^2 + a)*a*h^3*x + sqrt(c*x^2 + a)*c*g^3 + sqrt(c*x^2
 + a)*a*g*h^2) + f/(sqrt(c*x^2 + a)*c*g^2*h + sqrt(c*x^2 + a)*a*h^3) + 15/2
*c^2*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
+ g)))/((a + c*g^2/h^2)^(7/2)*h^7) - 15/2*c^2*e*g^3*arcsinh(c*g*x/(sqrt(a*c
)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^6)
 + 15/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*a
bs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^5) - 15/2*c*f*g^2*arcsinh(c*g*x/(sqr
t(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)
*h^5) + 9/2*c*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*a
bs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^4) - 3/2*c*d*arcsinh(c*g*x/(sqrt(a*c
)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^3)
 + f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))
/((a + c*g^2/h^2)^(3/2)*h^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,{\left(c\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+a)**(3/2),x)

[Out] Timed out
```

3.115 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=67

$$\frac{x(aC+2Ac)}{3a^2c\sqrt{a+cx^2}}-\frac{aB-x(Ac-aC)}{3ac\left(a+cx^2\right)^{3/2}}$$

```
[Out] 1/3*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^(3/2)+1/3*(2*A*c+C*a)*x/a^2/c/(c*x^2+a
)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1814, 12, 191}

$$\frac{x(aC+2Ac)}{3a^2c\sqrt{a+cx^2}}-\frac{aB-x(Ac-aC)}{3ac\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^(5/2),x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(3*a*c*(a + c*x^2)^(3/2)) + ((2*A*c + a*C)*x)/(3*a^2
*c*Sqrt[a + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{5/2}}\,dx &= -\frac{aB-(Ac-aC)x}{3ac\left(a+cx^2\right)^{3/2}}-\frac{\int \frac{-2A-\frac{aC}{c}}{\left(a+cx^2\right)^{3/2}}\,dx}{3a}\\ &= -\frac{aB-(Ac-aC)x}{3ac\left(a+cx^2\right)^{3/2}}+\frac{(2Ac+aC)\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{3ac}\\ &= -\frac{aB-(Ac-aC)x}{3ac\left(a+cx^2\right)^{3/2}}+\frac{(2Ac+aC)x}{3a^2c\sqrt{a+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.75

$$\frac{-a^2B+acx\left(3A+Cx^2\right)+2Ac^2x^3}{3a^2c\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(5/2),x]
[Out] (-(a^2*B) + 2*A*c^2*x^3 + a*c*x*(3*A + C*x^2))/(3*a^2*c*(a + c*x^2)^(3/2))
```

fricas [A] time = 0.79, size = 68, normalized size = 1.01

$$\frac{\left(3\,Aacx+\left(Cac+2\,Ac^2\right)x^3-Ba^2\right)\sqrt{cx^2+a}}{3\left(a^2c^3x^4+2\,a^3c^2x^2+a^4c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="fricas")
[Out] 1/3*(3*A*a*c*x + (C*a*c + 2*A*c^2)*x^3 - B*a^2)*sqrt(c*x^2 + a)/(a^2*c^3*x^
4 + 2*a^3*c^2*x^2 + a^4*c)
```

giac [A] time = 0.21, size = 48, normalized size = 0.72

$$\frac{x\left(\frac{3\,A}{a}+\frac{\left(Cac+2\,Ac^2\right)x^2}{a^2c}\right)-\frac{B}{c}}{3\left(cx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="giac")
[Out] 1/3*(x*(3*A/a + (C*a*c + 2*A*c^2)*x^2/(a^2*c)) - B/c)/(c*x^2 + a)^(3/2)
```

maple [A] time = 0.00, size = 47, normalized size = 0.70

$$\frac{2A\,c^2x^3+Cac\,x^3+3Axac-B\,a^2}{3\left(c\,x^2+a\right)^{\frac{3}{2}}a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x)
[Out] 1/3*(2*A*c^2*x^3+C*a*c*x^3+3*A*a*c*x-B*a^2)/(c*x^2+a)^(3/2)/a^2/c
```

maxima [A] time = 0.43, size = 83, normalized size = 1.24

$$\frac{2\,Ax}{3\,\sqrt{cx^2+a}\,a^2}+\frac{Ax}{3\left(cx^2+a\right)^{\frac{3}{2}}a}-\frac{Cx}{3\left(cx^2+a\right)^{\frac{3}{2}}c}+\frac{Cx}{3\,\sqrt{cx^2+a}\,ac}-\frac{B}{3\left(cx^2+a\right)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="maxima")
[Out] 2/3*A*x/(sqrt(c*x^2 + a)*a^2) + 1/3*A*x/((c*x^2 + a)^(3/2)*a) - 1/3*C*x/((c
*x^2 + a)^(3/2)*c) + 1/3*C*x/(sqrt(c*x^2 + a)*a*c) - 1/3*B/((c*x^2 + a)^(3/
2)*c)
```

mupad [B] time = 4.22, size = 59, normalized size = 0.88

$$\frac{2\,A\,c\,x\,\left(c\,x^2+a\right)-C\,a^2\,x-B\,a^2+C\,a\,x\,\left(c\,x^2+a\right)+A\,a\,c\,x}{3\,a^2\,c\,{\left(c\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^(5/2),x)
```

```
[Out] (2*A*c*x*(a + c*x^2) - C*a^2*x - B*a^2 + C*a*x*(a + c*x^2) + A*a*c*x)/(3*a^
2*c*(a + c*x^2)^(3/2))
```

sympy [A] time = 17.13, size = 194, normalized size = 2.90

$$A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^2}{a}}+3a^{\frac{5}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}+\frac{2cx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^2}{a}}+3a^{\frac{5}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}\right)+B\left(\begin{cases}-\frac{1}{3ac\sqrt{a+cx^2}+3c^2x^2\sqrt{a+cx^2}} & \text{for } c \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{other}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a*x/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 + c*x**2
/a)) + 2*c*x**3/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 +
 c*x**2/a))) + B*Piecewise((-1/(3*a*c*sqrt(a + c*x**2) + 3*c**2*x**2*sqrt(a
 + c*x**2)), Ne(c, 0)), (x**2/(2*a**(5/2)), True)) + C*x**3/(3*a**(5/2)*sqr
t(1 + c*x**2/a) + 3*a**(3/2)*c*x**2*sqrt(1 + c*x**2/a))
```

3.116 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=97

$$\frac{2x(aC+4Ac)}{15a^3c\sqrt{a+cx^2}}+\frac{x(aC+4Ac)}{15a^2c\left(a+cx^2\right)^{3/2}}-\frac{aB-x(Ac-aC)}{5ac\left(a+cx^2\right)^{5/2}}$$

```
[Out] 1/5*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^(5/2)+1/15*(4*A*c+C*a)*x/a^2/c/(c*x^2+
a)^(3/2)+2/15*(4*A*c+C*a)*x/a^3/c/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1814, 12, 192, 191}

$$\frac{2x(aC+4Ac)}{15a^3c\sqrt{a+cx^2}}+\frac{x(aC+4Ac)}{15a^2c\left(a+cx^2\right)^{3/2}}-\frac{aB-x(Ac-aC)}{5ac\left(a+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^(7/2),x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(5*a*c*(a + c*x^2)^(5/2)) + ((4*A*c + a*C)*x)/(15*a^
2*c*(a + c*x^2)^(3/2)) + (2*(4*A*c + a*C)*x)/(15*a^3*c*Sqrt[a + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{7/2}}\,dx = -\frac{aB-(Ac-aC)x}{5ac\left(a+cx^2\right)^{5/2}} - \frac{\int \frac{-4A-\frac{aC}{c}}{\left(a+cx^2\right)^{5/2}}\,dx}{5a}$$

$$= -\frac{aB-(Ac-aC)x}{5ac\left(a+cx^2\right)^{5/2}} + \frac{(4Ac+aC)\int \frac{1}{\left(a+cx^2\right)^{5/2}}\,dx}{5ac}$$

$$= -\frac{aB-(Ac-aC)x}{5ac\left(a+cx^2\right)^{5/2}} + \frac{(4Ac+aC)x}{15a^2c\left(a+cx^2\right)^{3/2}} + \frac{(2(4Ac+aC))\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{15a^2c}$$

$$= -\frac{aB-(Ac-aC)x}{5ac\left(a+cx^2\right)^{5/2}} + \frac{(4Ac+aC)x}{15a^2c\left(a+cx^2\right)^{3/2}} + \frac{2(4Ac+aC)x}{15a^3c\sqrt{a+cx^2}}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.73

$$\frac{-3a^3B+5a^2cx\left(3A+Cx^2\right)+2ac^2x^3\left(10A+Cx^2\right)+8Ac^3x^5}{15a^3c\left(a+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(7/2),x]
```

```
[Out] (-3*a^3*B + 8*A*c^3*x^5 + 5*a^2*c*x*(3*A + C*x^2) + 2*a*c^2*x^3*(10*A + C*x
^2))/(15*a^3*c*(a + c*x^2)^(5/2))
```

fricas [A] time = 0.89, size = 103, normalized size = 1.06

$$\frac{\left(2\left(Cac^2+4\,Ac^3\right)x^5+15\,Aa^2cx-3\,Ba^3+5\left(Ca^2c+4\,Aac^2\right)x^3\right)\sqrt{cx^2+a}}{15\left(a^3c^4x^6+3\,a^4c^3x^4+3\,a^5c^2x^2+a^6c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(C*a*c^2 + 4*A*c^3)*x^5 + 15*A*a^2*c*x - 3*B*a^3 + 5*(C*a^2*c + 4*A
*a*c^2)*x^3)*sqrt(c*x^2 + a)/(a^3*c^4*x^6 + 3*a^4*c^3*x^4 + 3*a^5*c^2*x^2 +
 a^6*c)
```

giac [A] time = 0.26, size = 80, normalized size = 0.82

$$\frac{\left(x^2\left(\frac{2\left(Cac^3+4\,Ac^4\right)x^2}{a^3c^2}+\frac{5\left(Ca^2c^2+4\,Aac^3\right)}{a^3c^2}\right)+\frac{15\,A}{a}\right)x-\frac{3\,B}{c}}{15\left(cx^2+a\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((x^2*(2*(C*a*c^3 + 4*A*c^4)*x^2/(a^3*c^2) + 5*(C*a^2*c^2 + 4*A*a*c^3)
/(a^3*c^2)) + 15*A/a)*x - 3*B/c)/(c*x^2 + a)^(5/2)
```

maple [A] time = 0.00, size = 72, normalized size = 0.74

$$\frac{8A\,c^3x^5+2Ca\,c^2x^5+20Aa\,c^2x^3+5C\,a^2c\,x^3+15Ax\,a^2c-3B\,a^3}{15\left(c\,x^2+a\right)^{\frac{5}{2}}a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x)
```

```
[Out] 1/15*(8*A*c^3*x^5+2*C*a*c^2*x^5+20*A*a*c^2*x^3+5*C*a^2*c*x^3+15*A*a^2*c*x-3
*B*a^3)/(c*x^2+a)^(5/2)/a^3/c
```

maxima [A] time = 0.44, size = 118, normalized size = 1.22

$$\frac{8\,Ax}{15\,\sqrt{cx^2+a}\,a^3}+\frac{4\,Ax}{15\,\left(cx^2+a\right)^{\frac{3}{2}}a^2}+\frac{Ax}{5\,\left(cx^2+a\right)^{\frac{5}{2}}a}-\frac{Cx}{5\,\left(cx^2+a\right)^{\frac{5}{2}}c}+\frac{2\,Cx}{15\,\sqrt{cx^2+a}\,a^2c}+\frac{Cx}{15\,\left(cx^2+a\right)^{\frac{3}{2}}ac}-\frac{B}{5\,\left(cx^2+a\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*A*x/(sqrt(c*x^2 + a)*a^3) + 4/15*A*x/((c*x^2 + a)^(3/2)*a^2) + 1/5*A*x
/((c*x^2 + a)^(5/2)*a) - 1/5*C*x/((c*x^2 + a)^(5/2)*c) + 2/15*C*x/(sqrt(c*x
^2 + a)*a^2*c) + 1/15*C*x/((c*x^2 + a)^(3/2)*a*c) - 1/5*B/((c*x^2 + a)^(5/2
)*c)
```

mupad [B] time = 4.28, size = 93, normalized size = 0.96

$$\frac{8\,A\,c\,x\,{\left(c\,x^2+a\right)}^2-3\,C\,a^3\,x-3\,B\,a^3+2\,C\,a\,x\,{\left(c\,x^2+a\right)}^2+C\,a^2\,x\,\left(c\,x^2+a\right)+3\,A\,a^2\,c\,x+4\,A\,a\,c\,x\,\left(c\,x^2+a\right)}{15\,a^3\,c\,{\left(c\,x^2+a\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^(7/2),x)
```

```
[Out] (8*A*c*x*(a + c*x^2)^2 - 3*C*a^3*x - 3*B*a^3 + 2*C*a*x*(a + c*x^2)^2 + C*a^
2*x*(a + c*x^2) + 3*A*a^2*c*x + 4*A*a*c*x*(a + c*x^2))/(15*a^3*c*(a + c*x^2
)^(5/2))
```

sympy [B] time = 37.22, size = 638, normalized size = 6.58

$$A\left(\frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}+\frac{}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**(7/2),x)
```

```
[Out] A*(15*a**5*x/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1
+ c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3
*x**6*sqrt(1 + c*x**2/a)) + 35*a**4*c*x**3/(15*a**(17/2)*sqrt(1 + c*x**2/a)
 + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 +
 c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a)) + 28*a**3*c**2*x**5
/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a)
+ 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1
 + c*x**2/a)) + 8*a**2*c**3*x**7/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(
15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a)
 + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a))) + B*Piecewise((-1/(5*a**2*c*
sqrt(a + c*x**2) + 10*a*c**2*x**2*sqrt(a + c*x**2) + 5*c**3*x**4*sqrt(a + c
*x**2)), Ne(c, 0)), (x**2/(2*a**(7/2)), True)) + C*(5*a*x**3/(15*a**(9/2)*s
qrt(1 + c*x**2/a) + 30*a**(7/2)*c*x**2*sqrt(1 + c*x**2/a) + 15*a**(5/2)*c**
2*x**4*sqrt(1 + c*x**2/a)) + 2*c*x**5/(15*a**(9/2)*sqrt(1 + c*x**2/a) + 30*
a**(7/2)*c*x**2*sqrt(1 + c*x**2/a) + 15*a**(5/2)*c**2*x**4*sqrt(1 + c*x**2/
a)))
```

3.117 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=127

$$\frac{8x(aC+6Ac)}{105a^4c\sqrt{a+cx^2}}+\frac{4x(aC+6Ac)}{105a^3c\left(a+cx^2\right)^{3/2}}+\frac{x(aC+6Ac)}{35a^2c\left(a+cx^2\right)^{5/2}}-\frac{aB-x(Ac-aC)}{7ac\left(a+cx^2\right)^{7/2}}$$

```
[Out] 1/7*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^(7/2)+1/35*(6*A*c+C*a)*x/a^2/c/(c*x^2+
a)^(5/2)+4/105*(6*A*c+C*a)*x/a^3/c/(c*x^2+a)^(3/2)+8/105*(6*A*c+C*a)*x/a^4/
c/(c*x^2+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1814, 12, 192, 191}

$$\frac{8x(aC+6Ac)}{105a^4c\sqrt{a+cx^2}}+\frac{4x(aC+6Ac)}{105a^3c\left(a+cx^2\right)^{3/2}}+\frac{x(aC+6Ac)}{35a^2c\left(a+cx^2\right)^{5/2}}-\frac{aB-x(Ac-aC)}{7ac\left(a+cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^(9/2),x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(7*a*c*(a + c*x^2)^(7/2)) + ((6*A*c + a*C)*x)/(35*a^
2*c*(a + c*x^2)^(5/2)) + (4*(6*A*c + a*C)*x)/(105*a^3*c*(a + c*x^2)^(3/2))
+ (8*(6*A*c + a*C)*x)/(105*a^4*c*Sqrt[a + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}}\,dx = -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}} - \frac{\int \frac{-6A-\frac{aC}{c}}{\left(a+cx^2\right)^{7/2}}\,dx}{7a}$$

$$= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}} + \frac{(6Ac+aC)\int \frac{1}{\left(a+cx^2\right)^{7/2}}\,dx}{7ac}$$

$$= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}} + \frac{(6Ac+aC)x}{35a^2c\left(a+cx^2\right)^{5/2}} + \frac{(4(6Ac+aC))\int \frac{1}{\left(a+cx^2\right)^{5/2}}\,dx}{35a^2c}$$

$$= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}} + \frac{(6Ac+aC)x}{35a^2c\left(a+cx^2\right)^{5/2}} + \frac{4(6Ac+aC)x}{105a^3c\left(a+cx^2\right)^{3/2}} + \frac{(8(6Ac+aC))\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{105a^3c}$$

$$= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}} + \frac{(6Ac+aC)x}{35a^2c\left(a+cx^2\right)^{5/2}} + \frac{4(6Ac+aC)x}{105a^3c\left(a+cx^2\right)^{3/2}} + \frac{8(6Ac+aC)x}{105a^4c\sqrt{a+cx^2}}$$

Mathematica [A] time = 0.06, size = 92, normalized size = 0.72

$$\frac{-15a^4B+35a^3cx\left(3A+Cx^2\right)+14a^2c^2x^3\left(15A+2Cx^2\right)+8ac^3x^5\left(21A+Cx^2\right)+48Ac^4x^7}{105a^4c\left(a+cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(9/2),x]
```

```
[Out] (-15*a^4*B + 48*A*c^4*x^7 + 35*a^3*c*x*(3*A + C*x^2) + 8*a*c^3*x^5*(21*A +
C*x^2) + 14*a^2*c^2*x^3*(15*A + 2*C*x^2))/(105*a^4*c*(a + c*x^2)^(7/2))
```

fricas [A] time = 0.85, size = 137, normalized size = 1.08

$$\frac{\left(8\left(Cac^3+6\,Ac^4\right)x^7+105\,Aa^3cx+28\left(Ca^2c^2+6\,Aac^3\right)x^5-15\,Ba^4+35\left(Ca^3c+6\,Aa^2c^2\right)x^3\right)\sqrt{cx^2+a}}{105\left(a^4c^5x^8+4\,a^5c^4x^6+6\,a^6c^3x^4+4\,a^7c^2x^2+a^8c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(C*a*c^3 + 6*A*c^4)*x^7 + 105*A*a^3*c*x + 28*(C*a^2*c^2 + 6*A*a*c^
3)*x^5 - 15*B*a^4 + 35*(C*a^3*c + 6*A*a^2*c^2)*x^3)*sqrt(c*x^2 + a)/(a^4*c^
5*x^8 + 4*a^5*c^4*x^6 + 6*a^6*c^3*x^4 + 4*a^7*c^2*x^2 + a^8*c)
```

giac [A] time = 0.27, size = 112, normalized size = 0.88

$$\frac{\left(\left(4\,x^2\left(\frac{2\left(Cac^5+6\,Ac^6\right)x^2}{a^4c^3}+\frac{7\left(Ca^2c^4+6\,Aac^5\right)}{a^4c^3}\right)+\frac{35\left(Ca^3c^3+6\,Aa^2c^4\right)}{a^4c^3}\right)x^2+\frac{105\,A}{a}\right)x-\frac{15\,B}{c}}{105\left(cx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*x^2*(2*(C*a*c^5 + 6*A*c^6)*x^2/(a^4*c^3) + 7*(C*a^2*c^4 + 6*A*a*
c^5)/(a^4*c^3)) + 35*(C*a^3*c^3 + 6*A*a^2*c^4)/(a^4*c^3))*x^2 + 105*A/a)*x
- 15*B/c)/(c*x^2 + a)^(7/2)
```

maple [A] time = 0.01, size = 96, normalized size = 0.76

$$\frac{48A\,c^4x^7+8Ca\,c^3x^7+168Aa\,c^3x^5+28C\,a^2c^2x^5+210A\,a^2c^2x^3+35C\,a^3c\,x^3+105Ax\,a^3c-15B\,a^4}{105\left(c\,x^2+a\right)^{\frac{7}{2}}a^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x)
```

```
[Out] 1/105*(48*A*c^4*x^7+8*C*a*c^3*x^7+168*A*a*c^3*x^5+28*C*a^2*c^2*x^5+210*A*a^
2*c^2*x^3+35*C*a^3*c*x^3+105*A*a^3*c*x-15*B*a^4)/(c*x^2+a)^(7/2)/a^4/c
```

maxima [A] time = 0.45, size = 153, normalized size = 1.20

$$\frac{16\,Ax}{35\,\sqrt{cx^2+a}\,a^4}+\frac{8\,Ax}{35\left(cx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,Ax}{35\left(cx^2+a\right)^{\frac{5}{2}}a^2}+\frac{Ax}{7\left(cx^2+a\right)^{\frac{7}{2}}a}-\frac{Cx}{7\left(cx^2+a\right)^{\frac{7}{2}}c}+\frac{8\,Cx}{105\,\sqrt{cx^2+a}\,a^3c}+\frac{4}{105\,(cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*A*x/(sqrt(c*x^2 + a)*a^4) + 8/35*A*x/((c*x^2 + a)^(3/2)*a^3) + 6/35*A
*x/((c*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((c*x^2 + a)^(7/2)*a) - 1/7*C*x/((c*x^
2 + a)^(7/2)*c) + 8/105*C*x/(sqrt(c*x^2 + a)*a^3*c) + 4/105*C*x/((c*x^2 + a
)^(3/2)*a^2*c) + 1/35*C*x/((c*x^2 + a)^(5/2)*a*c) - 1/7*B/((c*x^2 + a)^(7/2
)*c)
```

mupad [B] time = 4.37, size = 115, normalized size = 0.91

$$\frac{x\,\left(6\,A\,c+C\,a\right)}{35\,a^2\,c\,{\left(c\,x^2+a\right)}^{5/2}}-\frac{\frac{B}{7\,c}-x\,\left(\frac{A}{7\,a}-\frac{C}{7\,c}\right)}{{\left(c\,x^2+a\right)}^{7/2}}+\frac{x\,\left(24\,A\,c+4\,C\,a\right)}{105\,a^3\,c\,{\left(c\,x^2+a\right)}^{3/2}}+\frac{x\,\left(48\,A\,c+8\,C\,a\right)}{105\,a^4\,c\,\sqrt{c\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^(9/2),x)
```

```
[Out] (x*(6*A*c + C*a))/(35*a^2*c*(a + c*x^2)^(5/2)) - (B/(7*c) - x*(A/(7*a) - C/
(7*c)))/(a + c*x^2)^(7/2) + (x*(24*A*c + 4*C*a))/(105*a^3*c*(a + c*x^2)^(3/
2)) + (x*(48*A*c + 8*C*a))/(105*a^4*c*(a + c*x^2)^(1/2))
```

sympy [B] time = 78.30, size = 1880, normalized size = 14.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(
1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*
c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) +
 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt
(1 + c*x**2/a)) + 175*a**13*c*x**3/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a
**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**
2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8
*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(
25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 371*a**12*c**2*x**5/(35*a**(37/2)*sq
rt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*
c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) +
```

```
 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt
(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 429*a**11*c*
*3*x**7/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*
x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x
**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a
**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c
*x**2/a)) + 286*a**10*c**4*x**9/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(
35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a
) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sq
rt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/
2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 104*a**9*c**5*x**11/(35*a**(37/2)*sqrt(
1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**
2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 52
5*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1
+ c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 16*a**8*c**6*x*
*13/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2
/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*
sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(2
7/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**
2/a))) + B*Piecewise((-1/(7*a**3*c*sqrt(a + c*x**2) + 21*a**2*c**2*x**2*sqr
t(a + c*x**2) + 21*a*c**3*x**4*sqrt(a + c*x**2) + 7*c**4*x**6*sqrt(a + c*x*
*2)), Ne(c, 0)), (x**2/(2*a**(9/2)), True)) + C*(35*a**5*x**3/(105*a**(19/2
)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**2/a) + 630*a**(15
/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6*sqrt(1 + c*x**2/
a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)) + 63*a**4*c*x**5/(105*a**(
19/2)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**2/a) + 630*a*
*(15/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6*sqrt(1 + c*x
**2/a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)) + 36*a**3*c**2*x**7/(1
05*a**(19/2)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**2/a) +
 630*a**(15/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6*sqrt(
1 + c*x**2/a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)) + 8*a**2*c**3*x
**9/(105*a**(19/2)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**
2/a) + 630*a**(15/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6
*sqrt(1 + c*x**2/a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)))
```

3.118 $\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=106

$$\frac{2}{15}\sqrt{3x^2+2}\,(2x+1)^4+\frac{13}{60}\sqrt{3x^2+2}\,(2x+1)^3-\frac{19}{540}\sqrt{3x^2+2}\,(2x+1)^2-\frac{1}{810}(2073x+3937)\sqrt{3x^2+2}+\frac{5\sinh^{-1}\left(\sqrt{}\right.}{3\sqrt{3}}$$

```
[Out] 5/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)-19/540*(1+2*x)^2*(3*x^2+2)^(1/2)+13/60*(
1+2*x)^3*(3*x^2+2)^(1/2)+2/15*(1+2*x)^4*(3*x^2+2)^(1/2)-1/810*(3937+2073*x)
*(3*x^2+2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1654, 833, 780, 215}

$$\frac{2}{15}\sqrt{3x^2+2}\,(2x+1)^4+\frac{13}{60}\sqrt{3x^2+2}\,(2x+1)^3-\frac{19}{540}\sqrt{3x^2+2}\,(2x+1)^2-\frac{1}{810}(2073x+3937)\sqrt{3x^2+2}+\frac{5\sinh^{-1}\left(\sqrt{}\right.}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (-19*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/540 + (13*(1 + 2*x)^3*Sqrt[2 + 3*x^2])/60
 + (2*(1 + 2*x)^4*Sqrt[2 + 3*x^2])/15 - ((3937 + 2073*x)*Sqrt[2 + 3*x^2])/8
10 + (5*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
```

```
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} + \frac{1}{60}\int \frac{(1+2x)^3(-68+156x)}{\sqrt{2+3x^2}}\,dx$$

$$= \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} + \frac{1}{720}\int \frac{(-2688-228x)(1+2x}{\sqrt{2+3x^2}}$$

$$= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2} + \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} + \frac{\int}{}$$

$$= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2} + \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} - \frac{1}{81}$$

$$= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2} + \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} - \frac{1}{81}$$

Mathematica [A] time = 0.07, size = 54, normalized size = 0.51

$$\frac{1}{405}\left(\sqrt{3x^2+2}\left(864x^4+2430x^3+2292x^2-135x-1841\right)+225\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (Sqrt[2 + 3*x^2]*(-1841 - 135*x + 2292*x^2 + 2430*x^3 + 864*x^4) + 225*Sqrt
[3]*ArcSinh[Sqrt[3/2]*x])/405
```

fricas [A] time = 1.01, size = 60, normalized size = 0.57

$$\frac{1}{405}\left(864\,x^4+2430\,x^3+2292\,x^2-135\,x-1841\right)\sqrt{3\,x^2+2}+\frac{5}{18}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}\,x-3\,x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/405*(864*x^4 + 2430*x^3 + 2292*x^2 - 135*x - 1841)*sqrt(3*x^2 + 2) + 5/18
*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

giac [A] time = 0.23, size = 54, normalized size = 0.51

$$\frac{1}{405}\left(3\left(2\left(9\left(16\,x+45\right)x+382\right)x-45\right)x-1841\right)\sqrt{3\,x^2+2}-\frac{5}{9}\sqrt{3}\log\left(-\sqrt{3}\,x+\sqrt{3\,x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/405*(3*(2*(9*(16*x + 45)*x + 382)*x - 45)*x - 1841)*sqrt(3*x^2 + 2) - 5/9
*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

maple [A] time = 0.01, size = 79, normalized size = 0.75

$$\frac{32\sqrt{3x^2+2}\,x^4}{15}+6\sqrt{3x^2+2}\,x^3+\frac{764\sqrt{3x^2+2}\,x^2}{135}-\frac{\sqrt{3x^2+2}\,x}{3}+\frac{5\sqrt{3}\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}-\frac{1841\sqrt{3x^2+2}}{405}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x)
```

```
[Out] 32/15*x^4*(3*x^2+2)^(1/2)+764/135*x^2*(3*x^2+2)^(1/2)-1841/405*(3*x^2+2)^(1
/2)+6*x^3*(3*x^2+2)^(1/2)-1/3*x*(3*x^2+2)^(1/2)+5/9*arcsinh(1/2*x*6^(1/2))*
3^(1/2)
```

maxima [A] time = 0.95, size = 78, normalized size = 0.74

$$\frac{32}{15}\sqrt{3x^2+2}\,x^4+6\sqrt{3x^2+2}\,x^3+\frac{764}{135}\sqrt{3x^2+2}\,x^2-\frac{1}{3}\sqrt{3x^2+2}\,x+\frac{5}{9}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}\,x\right)-\frac{1841}{405}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/15*sqrt(3*x^2 + 2)*x^4 + 6*sqrt(3*x^2 + 2)*x^3 + 764/135*sqrt(3*x^2 + 2)
*x^2 - 1/3*sqrt(3*x^2 + 2)*x + 5/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 1841/40
5*sqrt(3*x^2 + 2)
```

mupad [B] time = 0.05, size = 45, normalized size = 0.42

$$\frac{5\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}+\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{32\,x^4}{5}+18\,x^3+\frac{764\,x^2}{45}-x-\frac{1841}{135}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

```
[Out] (5*3^(1/2)*asinh((6^(1/2)*x)/2))/9 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((764*x^2)/
45 - x + 18*x^3 + (32*x^4)/5 - 1841/135))/3
```

sympy [A] time = 2.20, size = 94, normalized size = 0.89

$$\frac{32x^4\sqrt{3x^2+2}}{15}+6x^3\sqrt{3x^2+2}+\frac{764x^2\sqrt{3x^2+2}}{135}-\frac{x\sqrt{3x^2+2}}{3}-\frac{1841\sqrt{3x^2+2}}{405}+\frac{5\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

```
[Out] 32*x**4*sqrt(3*x**2 + 2)/15 + 6*x**3*sqrt(3*x**2 + 2) + 764*x**2*sqrt(3*x**
2 + 2)/135 - x*sqrt(3*x**2 + 2)/3 - 1841*sqrt(3*x**2 + 2)/405 + 5*sqrt(3)*a
sinh(sqrt(6)*x/2)/9
```

3.119 $\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=82

$$\frac{1}{6}\sqrt{3x^2+2}\,(2x+1)^3+\frac{5}{18}\sqrt{3x^2+2}\,(2x+1)^2-\frac{1}{27}(3x+61)\sqrt{3x^2+2}-\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)$$

```
[Out] -arcsinh(1/2*x*6^(1/2))*3^(1/2)+5/18*(1+2*x)^2*(3*x^2+2)^(1/2)+1/6*(1+2*x)^
3*(3*x^2+2)^(1/2)-1/27*(61+3*x)*(3*x^2+2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1654, 833, 780, 215}

$$\frac{1}{6}\sqrt{3x^2+2}\,(2x+1)^3+\frac{5}{18}\sqrt{3x^2+2}\,(2x+1)^2-\frac{1}{27}(3x+61)\sqrt{3x^2+2}-\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (5*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/18 + ((1 + 2*x)^3*Sqrt[2 + 3*x^2])/6 - ((61
 + 3*x)*Sqrt[2 + 3*x^2])/27 - Sqrt[3]*ArcSinh[Sqrt[3/2]*x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
```

```
1/2, 0]))
```

Rubi steps

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{1}{6}(1+2x)^3\sqrt{2+3x^2} + \frac{1}{48}\int \frac{(1+2x)^2(-48+120x)}{\sqrt{2+3x^2}}\,dx$$
$$= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2+3x^2} + \frac{1}{432}\int \frac{(-1392-144x)(1+2}{\sqrt{2+3x^2}}$$
$$= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2+3x^2} - \frac{1}{27}(61+3x)\sqrt{2+3x^2} - 3\int$$
$$= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2+3x^2} - \frac{1}{27}(61+3x)\sqrt{2+3x^2} - \sqrt{3}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.59

$$\frac{1}{27}\sqrt{3x^2+2}\left(36x^3+84x^2+54x-49\right)-\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (Sqrt[2 + 3*x^2]*(-49 + 54*x + 84*x^2 + 36*x^3))/27 - Sqrt[3]*ArcSinh[Sqrt[
3/2]*x]
```

fricas [A] time = 0.90, size = 54, normalized size = 0.66

$$\frac{1}{27}\left(36\,x^3+84\,x^2+54\,x-49\right)\sqrt{3\,x^2+2}+\frac{1}{2}\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^2+2}\,x-3\,x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(36*x^3 + 84*x^2 + 54*x - 49)*sqrt(3*x^2 + 2) + 1/2*sqrt(3)*log(sqrt(3
)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

giac [A] time = 0.20, size = 48, normalized size = 0.59

$$\frac{1}{27}\left(6\left(2\left(3\,x+7\right)x+9\right)x-49\right)\sqrt{3\,x^2+2}+\sqrt{3}\log\left(-\sqrt{3}\,x+\sqrt{3\,x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/27*(6*(2*(3*x + 7)*x + 9)*x - 49)*sqrt(3*x^2 + 2) + sqrt(3)*log(-sqrt(3)*
x + sqrt(3*x^2 + 2))
```

maple [A] time = 0.00, size = 65, normalized size = 0.79

$$\frac{4\sqrt{3x^2+2}\,x^3}{3}+\frac{28\sqrt{3x^2+2}\,x^2}{9}+2\sqrt{3x^2+2}\,x-\sqrt{3}\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)-\frac{49\sqrt{3x^2+2}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x)
```

```
[Out] 4/3*(3*x^2+2)^(1/2)*x^3+2*(3*x^2+2)^(1/2)*x-arcsinh(1/2*6^(1/2)*x)*3^(1/2)+
28/9*(3*x^2+2)^(1/2)*x^2-49/27*(3*x^2+2)^(1/2)
```

maxima [A] time = 0.96, size = 64, normalized size = 0.78

$$\frac{4}{3}\sqrt{3x^2+2}\,x^3+\frac{28}{9}\sqrt{3x^2+2}\,x^2+2\sqrt{3x^2+2}\,x-\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}\,x\right)-\frac{49}{27}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3*x^2 + 2)*x^3 + 28/9*sqrt(3*x^2 + 2)*x^2 + 2*sqrt(3*x^2 + 2)*x -
sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 49/27*sqrt(3*x^2 + 2)
```

mupad [B] time = 4.10, size = 40, normalized size = 0.49

$$\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(4x^3+\frac{28x^2}{3}+6x-\frac{49}{9}\right)}{3}-\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

```
[Out] (3^(1/2)*(x^2 + 2/3)^(1/2)*(6*x + (28*x^2)/3 + 4*x^3 - 49/9))/3 - 3^(1/2)*a
sinh((6^(1/2)*x)/2)
```

sympy [A] time = 1.19, size = 75, normalized size = 0.91

$$\frac{4x^3\sqrt{3x^2+2}}{3}+\frac{28x^2\sqrt{3x^2+2}}{9}+2x\sqrt{3x^2+2}-\frac{49\sqrt{3x^2+2}}{27}-\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

```
[Out] 4*x**3*sqrt(3*x**2 + 2)/3 + 28*x**2*sqrt(3*x**2 + 2)/9 + 2*x*sqrt(3*x**2 +
2) - 49*sqrt(3*x**2 + 2)/27 - sqrt(3)*asinh(sqrt(6)*x/2)
```

3.120 $\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=62

$$\frac{2}{9}\sqrt{3x^2+2}\,(2x+1)^2+\frac{7}{27}(3x+1)\sqrt{3x^2+2}-\frac{7\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

```
[Out] -7/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+2/9*(1+2*x)^2*(3*x^2+2)^(1/2)+7/27*(1+3
*x)*(3*x^2+2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1654, 780, 215}

$$\frac{2}{9}\sqrt{3x^2+2}\,(2x+1)^2+\frac{7}{27}(3x+1)\sqrt{3x^2+2}-\frac{7\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (2*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/9 + (7*(1 + 3*x)*Sqrt[2 + 3*x^2])/27 - (7*A
rcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{2}{9}(1+2x)^2\sqrt{2+3x^2} + \frac{1}{36}\int \frac{(1+2x)(-28+84x)}{\sqrt{2+3x^2}}\,dx$$
$$= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2} + \frac{7}{27}(1+3x)\sqrt{2+3x^2} - \frac{7}{3}\int \frac{1}{\sqrt{2+3x^2}}\,dx$$
$$= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2} + \frac{7}{27}(1+3x)\sqrt{2+3x^2} - \frac{7\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.71

$$\frac{1}{27}\left(\sqrt{3x^2+2}\left(24x^2+45x+13\right)-21\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (Sqrt[2 + 3*x^2]*(13 + 45*x + 24*x^2) - 21*Sqrt[3]*ArcSinh[Sqrt[3/2]*x])/27
```

fricas [A] time = 0.80, size = 49, normalized size = 0.79

$$\frac{1}{27}\left(24x^2+45x+13\right)\sqrt{3x^2+2}+\frac{7}{18}\sqrt{3}\log\left(\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(24*x^2 + 45*x + 13)*sqrt(3*x^2 + 2) + 7/18*sqrt(3)*log(sqrt(3)*sqrt(3
*x^2 + 2)*x - 3*x^2 - 1)
```

giac [A] time = 0.19, size = 44, normalized size = 0.71

$$\frac{1}{27}(3(8x+15)x+13)\sqrt{3x^2+2}+\frac{7}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/27*(3*(8*x + 15)*x + 13)*sqrt(3*x^2 + 2) + 7/9*sqrt(3)*log(-sqrt(3)*x + s
qrt(3*x^2 + 2))
```

maple [A] time = 0.01, size = 51, normalized size = 0.82

$$\frac{8\sqrt{3x^2+2}\,x^2}{9}+\frac{5\sqrt{3x^2+2}\,x}{3}-\frac{7\sqrt{3}\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}+\frac{13\sqrt{3x^2+2}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x)
```

```
[Out] 8/9*(3*x^2+2)^(1/2)*x^2+13/27*(3*x^2+2)^(1/2)+5/3*(3*x^2+2)^(1/2)*x-7/9*arc
sinh(1/2*6^(1/2)*x)*3^(1/2)
```

maxima [A] time = 0.96, size = 50, normalized size = 0.81

$$\frac{8}{9}\sqrt{3x^2+2}x^2+\frac{5}{3}\sqrt{3x^2+2}x-\frac{7}{9}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)+\frac{13}{27}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*sqrt(3*x^2 + 2)*x^2 + 5/3*sqrt(3*x^2 + 2)*x - 7/9*sqrt(3)*arcsinh(1/2*s
qrt(6)*x) + 13/27*sqrt(3*x^2 + 2)
```

mupad [B] time = 0.03, size = 35, normalized size = 0.56

$$\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{8\,x^2}{3}+5\,x+\frac{13}{9}\right)}{3}-\frac{7\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

```
[Out] (3^(1/2)*(x^2 + 2/3)^(1/2)*(5*x + (8*x^2)/3 + 13/9))/3 - (7*3^(1/2)*asinh((
6^(1/2)*x)/2))/9
```

sympy [A] time = 0.55, size = 63, normalized size = 1.02

$$\frac{8x^2\sqrt{3x^2+2}}{9}+\frac{5x\sqrt{3x^2+2}}{3}+\frac{13\sqrt{3x^2+2}}{27}-\frac{7\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

```
[Out] 8*x**2*sqrt(3*x**2 + 2)/9 + 5*x*sqrt(3*x**2 + 2)/3 + 13*sqrt(3*x**2 + 2)/27
 - 7*sqrt(3)*asinh(sqrt(6)*x/2)/9
```

3.121 $\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}} \, dx$

Optimal. Leaf size=67

$$\frac{2}{3}\sqrt{3x^2+2} - \frac{\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{2\sqrt{11}} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}$$

```
[Out] 1/6*arcsinh(1/2*x*6^(1/2))*3^(1/2)-1/22*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^
2+2)^(1/2))*11^(1/2)+2/3*(3*x^2+2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1654, 844, 215, 725, 206}

$$\frac{2}{3}\sqrt{3x^2+2} - \frac{\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{2\sqrt{11}} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (2*Sqrt[2 + 3*x^2])/3 + ArcSinh[Sqrt[3/2]*x]/(2*Sqrt[3]) - ArcTanh[(4 - 3*x
)/(Sqrt[11]*Sqrt[2 + 3*x^2])]/(2*Sqrt[11])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
```

```
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx &= \frac{2}{3}\sqrt{2+3x^2}+\frac{1}{12}\int \frac{12+12x}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= \frac{2}{3}\sqrt{2+3x^2}+\frac{1}{2}\int \frac{1}{\sqrt{2+3x^2}}\,dx+\frac{1}{2}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= \frac{2}{3}\sqrt{2+3x^2}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\
&= \frac{2}{3}\sqrt{2+3x^2}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{2\sqrt{11}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.90

$$\frac{1}{66}\left(44\sqrt{3x^2+2}-3\sqrt{11}\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)+11\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (44*Sqrt[2 + 3*x^2] + 11*Sqrt[3]*ArcSinh[Sqrt[3/2]*x] - 3*Sqrt[11]*ArcTanh[
(4 - 3*x)/Sqrt[22 + 33*x^2]])/66
```

fricas [A] time = 0.91, size = 88, normalized size = 1.31

$$\frac{1}{12}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3x^2+2}\,x-3x^2-1\right)+\frac{1}{44}\sqrt{11}\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}\,(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+\frac{2}{3}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + 1/44*sqrt(11)*lo
g(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x +
 1)) + 2/3*sqrt(3*x^2 + 2)
```

giac [B] time = 0.23, size = 99, normalized size = 1.48

$$-\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)+\frac{1}{22}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{2}{3}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/22*sqrt(11)*log(-abs(-2*
sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(11)
 + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 2/3*sqrt(3*x^2 + 2)
```

maple [A] time = 0.01, size = 55, normalized size = 0.82

$$\frac{\sqrt{3}\ \operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}-\frac{\sqrt{11}\ \operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^{2}+5}}\right)}{22}+\frac{2\sqrt{3x^{2}+2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x)
```

```
[Out] 2/3*(3*x^2+2)^(1/2)+1/6*arcsinh(1/2*6^(1/2)*x)*3^(1/2)-1/22*11^(1/2)*arctan
h(2/11*(4-3*x)*11^(1/2)/(12*(1/2+x)^2-12*x+5)^(1/2))
```

maxima [A] time = 0.96, size = 58, normalized size = 0.87

$$\frac{1}{6}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}\,x\right)+\frac{1}{22}\sqrt{11}\operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{2}{3}\sqrt{3\,x^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 1/22*sqrt(11)*arcsinh(1/2*sqrt(6)*x/ab
s(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1)) + 2/3*sqrt(3*x^2 + 2)
```

mupad [B] time = 0.19, size = 61, normalized size = 0.91

$$\frac{\sqrt{11}\left(2\ln\left(x+\frac{1}{2}\right)-2\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^{2}+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{44}+\frac{2\sqrt{3}\sqrt{x^{2}+\frac{2}{3}}}{3}+\frac{\sqrt{3}\ \operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(1/2)),x)
```

```
[Out] (11^(1/2)*(2*log(x + 1/2) - 2*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/
3 - 4/3)))/44 + (2*3^(1/2)*(x^2 + 2/3)^(1/2))/3 + (3^(1/2)*asinh((2^(1/2)*3
^(1/2)*x)/2))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{4x^{2}+3x+1}{(2x+1)\sqrt{3x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*sqrt(3*x**2 + 2)), x)
```

3.122 $\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{3x^2+2}}{11(2x+1)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

```
[Out] 1/3*arcsinh(1/2*x*6^(1/2))*3^(1/2)+4/121*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x
^2+2)^(1/2))*11^(1/2)-1/11*(3*x^2+2)^(1/2)/(1+2*x)
```

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1651, 844, 215, 725, 206}

$$-\frac{\sqrt{3x^2+2}}{11(2x+1)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 + 3*x^2]),x]
```

```
[Out] -Sqrt[2 + 3*x^2]/(11*(1 + 2*x)) + ArcSinh[Sqrt[3/2]*x]/Sqrt[3] + (4*ArcTanh
[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(11*Sqrt[11])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx &= -\frac{\sqrt{2+3x^2}}{11(1+2x)} - \frac{1}{11}\int \frac{-7-22x}{(1+2x)\sqrt{2+3x^2}}\,dx \\ &= -\frac{\sqrt{2+3x^2}}{11(1+2x)} - \frac{4}{11}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx + \int \frac{1}{\sqrt{2+3x^2}}\,dx \\ &= -\frac{\sqrt{2+3x^2}}{11(1+2x)} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}} + \frac{4}{11}\text{Subst}\left(\int \frac{1}{11-x^2}\,dx, x, \frac{4-3x}{\sqrt{2+3x^2}}\right) \\ &= -\frac{\sqrt{2+3x^2}}{11(1+2x)} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}} + \frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{11\sqrt{11}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 64, normalized size = 0.90

$$-\frac{\sqrt{3x^2+2}}{22x+11} + \frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{11\sqrt{11}} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 + 3*x^2]),x]
```

```
[Out] -(Sqrt[2 + 3*x^2]/(11 + 22*x)) + ArcSinh[Sqrt[3/2]*x]/Sqrt[3] + (4*ArcTanh[
(4 - 3*x)/Sqrt[22 + 33*x^2]])/(11*Sqrt[11])
```

fricas [A] time = 1.00, size = 106, normalized size = 1.49

$$\frac{121\sqrt{3}(2x+1)\log\left(-\sqrt{3}\sqrt{3x^2+2}\,x-3x^2-1\right)+12\sqrt{11}(2x+1)\log\left(\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)-21x^2+12x-19}{4x^2+4x+1}\right)-66\sqrt{3x}}{726(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/726*(121*sqrt(3)*(2*x + 1)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
12*sqrt(11)*(2*x + 1)*log((sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) - 21*x^2 + 12
*x - 19)/(4*x^2 + 4*x + 1)) - 66*sqrt(3*x^2 + 2))/(2*x + 1)
```

giac [A] time = 0.32, size = 48, normalized size = 0.68

$$\frac{1}{22}\sqrt{3}\,\text{sgn}\left(\frac{1}{2x+1}\right) - \frac{\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}}{22\,\text{sgn}\left(\frac{1}{2x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/22*sqrt(3)*sgn(1/(2*x + 1)) - 1/22*sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3
)/sgn(1/(2*x + 1))
```

maple [A] time = 0.01, size = 65, normalized size = 0.92

$$\frac{\sqrt{3}\ \text{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{3} + \frac{4\sqrt{11}\ \text{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{121} - \frac{\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{22\left(x+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*arcsinh(1/2*6^(1/2)*x)*3^(1/2)+4/121*11^(1/2)*arctanh(2/11*(-3*x+4)*11^
(1/2)/(-12*x+12*(x+1/2)^2+5)^(1/2))-1/22/(x+1/2)*(3*(x+1/2)^2-3*x+5/4)^(1/2
)
```

maxima [A] time = 0.97, size = 65, normalized size = 0.92

$$\frac{1}{3}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}\,x\right)-\frac{4}{121}\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)-\frac{\sqrt{3\,x^2+2}}{11\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 4/121*sqrt(11)*arcsinh(1/2*sqrt(6)*x/a
bs(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1)) - 1/11*sqrt(3*x^2 + 2)/(2*x + 1)
```

mupad [B] time = 0.11, size = 68, normalized size = 0.96

$$\frac{\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{3}-\frac{4\,\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{121}+\frac{4\,\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{121}-\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{22\,\left(x+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(1/2)),x)
```

```
[Out] (3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/3 - (4*11^(1/2)*log(x + 1/2))/121 +
(4*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/121 - (3
^(1/2)*(x^2 + 2/3)^(1/2))/(22*(x + 1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^2\,\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*sqrt(3*x**2 + 2)), x)
```

3.123 $\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=77

$$\frac{13\sqrt{3x^2+2}}{242(2x+1)}-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}-\frac{103\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

```
[Out] -103/1331*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)-1/22*(3*x
^2+2)^(1/2)/(1+2*x)^2+13/242*(3*x^2+2)^(1/2)/(1+2*x)
```

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1651, 807, 725, 206}

$$\frac{13\sqrt{3x^2+2}}{242(2x+1)}-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}-\frac{103\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 + 3*x^2]),x]
```

```
[Out] -Sqrt[2 + 3*x^2]/(22*(1 + 2*x)^2) + (13*Sqrt[2 + 3*x^2])/(242*(1 + 2*x)) -
(103*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = -\frac{\sqrt{2+3x^2}}{22(1+2x)^2} - \frac{1}{22}\int \frac{-14-41x}{(1+2x)^2\sqrt{2+3x^2}}\,dx$$
$$= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2} + \frac{13\sqrt{2+3x^2}}{242(1+2x)} + \frac{103}{121}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx$$
$$= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2} + \frac{13\sqrt{2+3x^2}}{242(1+2x)} - \frac{103}{121}\,\text{Subst}\left(\int \frac{1}{11-x^2}\,dx, x, \frac{4-3x}{\sqrt{2+3x^2}}\right)$$
$$= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2} + \frac{13\sqrt{2+3x^2}}{242(1+2x)} - \frac{103\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 0.71

$$\frac{\frac{11(13x+1)\sqrt{3x^2+2}}{(2x+1)^2} - 103\sqrt{11}\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{1331}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 + 3*x^2]),x]
```

```
[Out] ((11*(1 + 13*x)*Sqrt[2 + 3*x^2])/(1 + 2*x)^2 - 103*Sqrt[11]*ArcTanh[(4 - 3*
x)/Sqrt[22 + 33*x^2]])/1331
```

fricas [A] time = 0.78, size = 89, normalized size = 1.16

$$\frac{103\sqrt{11}\left(4x^2+4x+1\right)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right) + 22\sqrt{3x^2+2}(13x+1)}{2662\left(4x^2+4x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2662*(103*sqrt(11)*(4*x^2 + 4*x + 1)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x
- 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 22*sqrt(3*x^2 + 2)*(13*x +
1))/(4*x^2 + 4*x + 1)
```

giac [B] time = 0.26, size = 180, normalized size = 2.34

$$\frac{103}{1331}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right) + \frac{72\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^3 - 13\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)}{484\left(\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2+\sqrt{3}\left(\sqrt{\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 103/1331*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2
 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/484*(72*
(sqrt(3)*x - sqrt(3*x^2 + 2))^3 - 13*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^
2 - 168*sqrt(3)*x + 104*sqrt(3) + 168*sqrt(3*x^2 + 2))/((sqrt(3)*x - sqrt(3
*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^2
```

maple [A] time = 0.01, size = 74, normalized size = 0.96

$$-\frac{103\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{1331}+\frac{13\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{484\left(x+\frac{1}{2}\right)}-\frac{\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{88\left(x+\frac{1}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x)
```

```
[Out] -103/1331*11^(1/2)*arctanh(2/11*(-3*x+4)*11^(1/2)/(-12*x+12*(x+1/2)^2+5)^(1
/2))+13/484/(x+1/2)*(-3*x+3*(x+1/2)^2+5/4)^(1/2)-1/88/(x+1/2)^2*(-3*x+3*(x+
1/2)^2+5/4)^(1/2)
```

maxima [A] time = 0.97, size = 76, normalized size = 0.99

$$\frac{103}{1331}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)-\frac{\sqrt{3\,x^2+2}}{22\left(4\,x^2+4\,x+1\right)}+\frac{13\,\sqrt{3\,x^2+2}}{242\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 103/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
+ 1)) - 1/22*sqrt(3*x^2 + 2)/(4*x^2 + 4*x + 1) + 13/242*sqrt(3*x^2 + 2)/(2*
x + 1)
```

mupad [B] time = 0.11, size = 77, normalized size = 1.00

$$\frac{103\,\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{1331}-\frac{103\,\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331}-\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{88\left(x^2+x+\frac{1}{4}\right)}+\frac{13\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{484\left(x+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(1/2)),x)
```

```
[Out] (103*11^(1/2)*log(x + 1/2))/1331 - (103*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*
(x^2 + 2/3)^(1/2))/3 - 4/3))/1331 - (3^(1/2)*(x^2 + 2/3)^(1/2))/(88*(x + x^
2 + 1/4)) + (13*3^(1/2)*(x^2 + 2/3)^(1/2))/(484*(x + 1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(1/2),x)
```

```
[Out] Timed out
```

3.124 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{32}{27}\sqrt{3x^2+2}\,x^2+4\sqrt{3x^2+2}\,x+\frac{292}{81}\sqrt{3x^2+2}+\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{38\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

```
[Out] -38/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/54*(398+279*x)/(3*x^2+2)^(1/2)+292/8
1*(3*x^2+2)^(1/2)+4*x*(3*x^2+2)^(1/2)+32/27*x^2*(3*x^2+2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1814, 1815, 641, 215}

$$\frac{32}{27}\sqrt{3x^2+2}\,x^2+4\sqrt{3x^2+2}\,x+\frac{292}{81}\sqrt{3x^2+2}+\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{38\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (398 + 279*x)/(54*Sqrt[2 + 3*x^2]) + (292*Sqrt[2 + 3*x^2])/81 + 4*x*Sqrt[2
+ 3*x^2] + (32*x^2*Sqrt[2 + 3*x^2])/27 - (38*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[
3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{398+279x}{54\sqrt{2+3x^2}} - \frac{1}{2}\int \frac{\frac{28}{3}-\frac{280x}{9}-48x^2-\frac{64x^3}{3}}{\sqrt{2+3x^2}}\,dx$$

$$= \frac{398+279x}{54\sqrt{2+3x^2}} + \frac{32}{27}x^2\sqrt{2+3x^2} - \frac{1}{18}\int \frac{84-\frac{584x}{3}-432x^2}{\sqrt{2+3x^2}}\,dx$$

$$= \frac{398+279x}{54\sqrt{2+3x^2}} + 4x\sqrt{2+3x^2} + \frac{32}{27}x^2\sqrt{2+3x^2} - \frac{1}{108}\int \frac{1368-1168x}{\sqrt{2+3x^2}}\,dx$$

$$= \frac{398+279x}{54\sqrt{2+3x^2}} + \frac{292}{81}\sqrt{2+3x^2} + 4x\sqrt{2+3x^2} + \frac{32}{27}x^2\sqrt{2+3x^2} - \frac{38}{3}\int \frac{1}{\sqrt{2+3\ldots}}$$

$$= \frac{398+279x}{54\sqrt{2+3x^2}} + \frac{292}{81}\sqrt{2+3x^2} + 4x\sqrt{2+3x^2} + \frac{32}{27}x^2\sqrt{2+3x^2} - \frac{38\sinh^{-1}\left(\sqrt{\ldots}\right.}{3\sqrt{3}}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 0.67

$$\frac{576x^4+1944x^3+2136x^2-684\sqrt{9x^2+6}\,\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)+2133x+2362}{162\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (2362 + 2133*x + 2136*x^2 + 1944*x^3 + 576*x^4 - 684*Sqrt[6 + 9*x^2]*ArcSin
h[Sqrt[3/2]*x])/(162*Sqrt[2 + 3*x^2])
```

fricas [A] time = 0.85, size = 76, normalized size = 0.87

$$\frac{342\sqrt{3}\left(3x^2+2\right)\log\left(\sqrt{3}\sqrt{3x^2+2}\,x-3x^2-1\right)+\left(576x^4+1944x^3+2136x^2+2133x+2362\right)\sqrt{3x^2+2}}{162\left(3x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/162*(342*sqrt(3)*(3*x^2 + 2)*log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
 (576*x^4 + 1944*x^3 + 2136*x^2 + 2133*x + 2362)*sqrt(3*x^2 + 2))/(3*x^2 +
2)
```

giac [A] time = 0.21, size = 54, normalized size = 0.62

$$\frac{38}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)+\frac{3\left(8\left(3\left(8x+27\right)x+89\right)x+711\right)x+2362}{162\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] 38/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/162*(3*(8*(3*(8*x + 27)*
x + 89)*x + 711)*x + 2362)/sqrt(3*x^2 + 2)
```

maple [A] time = 0.01, size = 79, normalized size = 0.91

$$\frac{32x^4}{9\sqrt{3x^2+2}}+\frac{12x^3}{\sqrt{3x^2+2}}+\frac{356x^2}{27\sqrt{3x^2+2}}+\frac{79x}{6\sqrt{3x^2+2}}-\frac{38\sqrt{3}\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}+\frac{1181}{81\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x)
```

```
[Out] 32/9*x^4/(3*x^2+2)^(1/2)+356/27*x^2/(3*x^2+2)^(1/2)+1181/81/(3*x^2+2)^(1/2)
+12*x^3/(3*x^2+2)^(1/2)+79/6*x/(3*x^2+2)^(1/2)-38/9*arcsinh(1/2*6^(1/2)*x)*
3^(1/2)
```

maxima [A] time = 0.96, size = 78, normalized size = 0.90

$$\frac{32\,x^4}{9\,\sqrt{3\,x^2+2}}+\frac{12\,x^3}{\sqrt{3\,x^2+2}}+\frac{356\,x^2}{27\,\sqrt{3\,x^2+2}}-\frac{38}{9}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{6}\,x\right)+\frac{79\,x}{6\,\sqrt{3\,x^2+2}}+\frac{1181}{81\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 32/9*x^4/sqrt(3*x^2 + 2) + 12*x^3/sqrt(3*x^2 + 2) + 356/27*x^2/sqrt(3*x^2 +
 2) - 38/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 79/6*x/sqrt(3*x^2 + 2) + 1181/8
1/sqrt(3*x^2 + 2)
```

mupad [B] time = 0.06, size = 110, normalized size = 1.26

$$\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{32\,x^2}{9}+12\,x+\frac{292}{27}\right)}{3}-\frac{38\,\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{9}-\frac{\sqrt{3}\,\sqrt{6}\,\left(-1194+\sqrt{6}\,279\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\,\sqrt{6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

```
[Out] (3^(1/2)*(x^2 + 2/3)^(1/2)*(12*x + (32*x^2)/9 + 292/27))/3 - (38*3^(1/2)*as
inh((2^(1/2)*3^(1/2)*x)/2))/9 - (3^(1/2)*6^(1/2)*(6^(1/2)*279i - 1194)*(x^2
 + 2/3)^(1/2)*1i)/(1944*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*2
79i + 1194)*(x^2 + 2/3)^(1/2)*1i)/(1944*(x - (6^(1/2)*1i)/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(3/2), x)
```

3.125 $\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}} dx$

Optimal. Leaf size=71

$$\frac{70-47x}{18\sqrt{3x^2+2}}+\frac{8}{9}x\sqrt{3x^2+2}+\frac{28}{9}\sqrt{3x^2+2}+\frac{4\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

```
[Out] 4/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/18*(70-47*x)/(3*x^2+2)^(1/2)+28/9*(3*x
^2+2)^(1/2)+8/9*x*(3*x^2+2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1814, 1815, 641, 215}

$$\frac{70-47x}{18\sqrt{3x^2+2}}+\frac{8}{9}x\sqrt{3x^2+2}+\frac{28}{9}\sqrt{3x^2+2}+\frac{4\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (70 - 47*x)/(18*Sqrt[2 + 3*x^2]) + (28*Sqrt[2 + 3*x^2])/9 + (8*x*Sqrt[2 + 3
*x^2])/9 + (4*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{70-47x}{18\sqrt{2+3x^2}} - \frac{1}{2}\int \frac{-\frac{56}{9}-\frac{56x}{3}-\frac{32x^2}{3}}{\sqrt{2+3x^2}}\,dx$$
$$= \frac{70-47x}{18\sqrt{2+3x^2}} + \frac{8}{9}x\sqrt{2+3x^2} - \frac{1}{12}\int \frac{-16-112x}{\sqrt{2+3x^2}}\,dx$$
$$= \frac{70-47x}{18\sqrt{2+3x^2}} + \frac{28}{9}\sqrt{2+3x^2} + \frac{8}{9}x\sqrt{2+3x^2} + \frac{4}{3}\int \frac{1}{\sqrt{2+3x^2}}\,dx$$
$$= \frac{70-47x}{18\sqrt{2+3x^2}} + \frac{28}{9}\sqrt{2+3x^2} + \frac{8}{9}x\sqrt{2+3x^2} + \frac{4\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 0.75

$$\frac{48x^3+168x^2+8\sqrt{9x^2+6}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)-15x+182}{18\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
[Out] (182 - 15*x + 168*x^2 + 48*x^3 + 8*Sqrt[6 + 9*x^2]*ArcSinh[Sqrt[3/2]*x])/(1
8*Sqrt[2 + 3*x^2])
```

fricas [A] time = 0.86, size = 72, normalized size = 1.01

$$\frac{4\sqrt{3}\left(3x^2+2\right)\log\left(-\sqrt{3}\sqrt{3x^2+2}\,x-3x^2-1\right)+\left(48x^3+168x^2-15x+182\right)\sqrt{3x^2+2}}{18\left(3x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
[Out] 1/18*(4*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + (
48*x^3 + 168*x^2 - 15*x + 182)*sqrt(3*x^2 + 2))/(3*x^2 + 2)
```

giac [A] time = 0.20, size = 49, normalized size = 0.69

$$-\frac{4}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)+\frac{3(8(2x+7)x-5)x+182}{18\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
[Out] -4/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/18*(3*(8*(2*x + 7)*x - 5
)*x + 182)/sqrt(3*x^2 + 2)
```

maple [A] time = 0.01, size = 65, normalized size = 0.92

$$\frac{8x^3}{3\sqrt{3x^2+2}}+\frac{28x^2}{3\sqrt{3x^2+2}}-\frac{5x}{6\sqrt{3x^2+2}}+\frac{4\sqrt{3}\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}+\frac{91}{9\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x)
```

```
[Out] 8/3/(3*x^2+2)^(1/2)*x^3-5/6/(3*x^2+2)^(1/2)*x+4/9*arcsinh(1/2*6^(1/2)*x)*3^
(1/2)+28/3/(3*x^2+2)^(1/2)*x^2+91/9/(3*x^2+2)^(1/2)
```

maxima [A] time = 0.96, size = 64, normalized size = 0.90

$$\frac{8\,x^3}{3\,\sqrt{3\,x^2+2}}+\frac{28\,x^2}{3\,\sqrt{3\,x^2+2}}+\frac{4}{9}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{6}\,x\right)-\frac{5\,x}{6\,\sqrt{3\,x^2+2}}+\frac{91}{9\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^3/sqrt(3*x^2 + 2) + 28/3*x^2/sqrt(3*x^2 + 2) + 4/9*sqrt(3)*arcsinh(1/
2*sqrt(6)*x) - 5/6*x/sqrt(3*x^2 + 2) + 91/9/sqrt(3*x^2 + 2)
```

mupad [B] time = 4.07, size = 105, normalized size = 1.48

$$\frac{4\,\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{9}+\frac{\sqrt{3}\,\left(\frac{8\,x}{3}+\frac{28}{3}\right)\,\sqrt{x^2+\frac{2}{3}}}{3}+\frac{\sqrt{3}\,\sqrt{6}\,\left(-630+\sqrt{6}\,141\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{\sqrt{3}\,\sqrt{6}\,\left(630+\sqrt{6}\,14\right.}{1944\,\left(x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

```
[Out] (4*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/9 + (3^(1/2)*((8*x)/3 + 28/3)*(x^2
 + 2/3)^(1/2))/3 + (3^(1/2)*6^(1/2)*(6^(1/2)*141i - 630)*(x^2 + 2/3)^(1/2)*
1i)/(1944*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*141i + 630)*(x^
2 + 2/3)^(1/2)*1i)/(1944*(x + (6^(1/2)*1i)/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^2\left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(3/2), x)
```

3.126 $\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{2-51x}{18\sqrt{3x^2+2}}+\frac{8}{9}\sqrt{3x^2+2}+\frac{10\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

```
[Out] 10/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/18*(2-51*x)/(3*x^2+2)^(1/2)+8/9*(3*x^
2+2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1814, 641, 215}

$$\frac{2-51x}{18\sqrt{3x^2+2}}+\frac{8}{9}\sqrt{3x^2+2}+\frac{10\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (2 - 51*x)/(18*Sqrt[2 + 3*x^2]) + (8*Sqrt[2 + 3*x^2])/9 + (10*ArcSinh[Sqrt[
3/2]*x])/(3*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx &= \frac{2-51x}{18\sqrt{2+3x^2}}-\frac{1}{2}\int \frac{-\frac{20}{3}-\frac{16x}{3}}{\sqrt{2+3x^2}}\,dx\\
&= \frac{2-51x}{18\sqrt{2+3x^2}}+\frac{8}{9}\sqrt{2+3x^2}+\frac{10}{3}\int \frac{1}{\sqrt{2+3x^2}}\,dx\\
&= \frac{2-51x}{18\sqrt{2+3x^2}}+\frac{8}{9}\sqrt{2+3x^2}+\frac{10\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.87

$$\frac{48x^2+20\sqrt{9x^2+6}\,\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)-51x+34}{18\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (34 - 51*x + 48*x^2 + 20*Sqrt[6 + 9*x^2]*ArcSinh[Sqrt[3/2]*x])/(18*Sqrt[2 +
 3*x^2])
```

fricas [A] time = 0.76, size = 67, normalized size = 1.22

$$\frac{10\sqrt{3}\left(3x^2+2\right)\log\left(-\sqrt{3}\sqrt{3x^2+2}\,x-3x^2-1\right)+\left(48x^2-51x+34\right)\sqrt{3x^2+2}}{18\left(3x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/18*(10*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
(48*x^2 - 51*x + 34)*sqrt(3*x^2 + 2))/(3*x^2 + 2)
```

giac [A] time = 0.23, size = 44, normalized size = 0.80

$$-\frac{10}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)+\frac{3(16x-17)x+34}{18\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] -10/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/18*(3*(16*x - 17)*x + 3
4)/sqrt(3*x^2 + 2)
```

maple [A] time = 0.00, size = 51, normalized size = 0.93

$$\frac{8x^2}{3\sqrt{3x^2+2}}-\frac{17x}{6\sqrt{3x^2+2}}+\frac{10\sqrt{3}\ \operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}+\frac{17}{9\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x)
```

```
[Out] 8/3/(3*x^2+2)^(1/2)*x^2+17/9/(3*x^2+2)^(1/2)-17/6/(3*x^2+2)^(1/2)*x+10/9*ar
csinh(1/2*6^(1/2)*x)*3^(1/2)
```

maxima [A] time = 0.96, size = 50, normalized size = 0.91

$$\frac{8x^2}{3\sqrt{3x^2+2}}+\frac{10}{9}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)-\frac{17x}{6\sqrt{3x^2+2}}+\frac{17}{9\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^2/sqrt(3*x^2 + 2) + 10/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 17/6*x/sqrt
(3*x^2 + 2) + 17/9/sqrt(3*x^2 + 2)
```

mupad [B] time = 0.04, size = 100, normalized size = 1.82

$$\frac{8\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{9}+\frac{10\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{9}+\frac{\sqrt{3}\,\sqrt{6}\,\left(-6+\sqrt{6}\,51\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{648\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{\sqrt{3}\,\sqrt{6}\,\left(6+\sqrt{6}\,51\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}}{648\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

```
[Out] (8*3^(1/2)*(x^2 + 2/3)^(1/2))/9 + (10*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))
/9 + (3^(1/2)*6^(1/2)*(6^(1/2)*51i - 6)*(x^2 + 2/3)^(1/2)*1i)/(648*(x - (6^
(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*51i + 6)*(x^2 + 2/3)^(1/2)*1i)/(6
48*(x + (6^(1/2)*1i)/3))
```

sympy [B] time = 15.96, size = 114, normalized size = 2.07

$$\frac{30\sqrt{3}\,x^2\,\mathrm{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{27x^2+18}+\frac{8x^2}{3\sqrt{3x^2+2}}-\frac{30x\sqrt{3x^2+2}}{27x^2+18}+\frac{x}{2\sqrt{3x^2+2}}+\frac{20\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{27x^2+18}+\frac{17}{9\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

```
[Out] 30*sqrt(3)*x**2*asinh(sqrt(6)*x/2)/(27*x**2 + 18) + 8*x**2/(3*sqrt(3*x**2 +
 2)) - 30*x*sqrt(3*x**2 + 2)/(27*x**2 + 18) + x/(2*sqrt(3*x**2 + 2)) + 20*s
qrt(3)*asinh(sqrt(6)*x/2)/(27*x**2 + 18) + 17/(9*sqrt(3*x**2 + 2))
```

3.127 $\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=53

$$\frac{21x-38}{66\sqrt{3x^2+2}}-\frac{2\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}$$

```
[Out] -2/121*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)+1/66*(-38+21
*x)/(3*x^2+2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1647, 12, 725, 206}

$$-\frac{38-21x}{66\sqrt{3x^2+2}}-\frac{2\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] -(38 - 21*x)/(66*Sqrt[2 + 3*x^2]) - (2*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 +
 3*x^2])])/(11*Sqrt[11])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx = -\frac{38-21x}{66\sqrt{2+3x^2}}-\frac{1}{6}\int -\frac{12}{11(1+2x)\sqrt{2+3x^2}}\,dx$$
$$= -\frac{38-21x}{66\sqrt{2+3x^2}}+\frac{2}{11}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx$$
$$= -\frac{38-21x}{66\sqrt{2+3x^2}}-\frac{2}{11}\text{Subst}\left(\int \frac{1}{11-x^2}\,dx, x, \frac{4-3x}{\sqrt{2+3x^2}}\right)$$
$$= -\frac{38-21x}{66\sqrt{2+3x^2}}-\frac{2\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{11\sqrt{11}}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.96

$$\frac{-12\sqrt{33x^2+22}\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)+231x-418}{726\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] (-418 + 231*x - 12*Sqrt[22 + 33*x^2]*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])/
(726*Sqrt[2 + 3*x^2])
```

fricas [A] time = 0.73, size = 83, normalized size = 1.57

$$\frac{6\sqrt{11}\left(3x^2+2\right)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+11\sqrt{3x^2+2}(21x-38)}{726\left(3x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/726*(6*sqrt(11)*(3*x^2 + 2)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21
*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*sqrt(3*x^2 + 2)*(21*x - 38))/(3*x
^2 + 2)
```

giac [A] time = 0.21, size = 82, normalized size = 1.55

$$\frac{2}{121}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{21x-38}{66\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/121*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 +
2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/66*(21*x -
38)/sqrt(3*x^2 + 2)
```

maple [B] time = 0.01, size = 88, normalized size = 1.66

$$\frac{x}{4\sqrt{3x^2+2}}+\frac{3x}{44\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}-\frac{2\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{121}-\frac{2}{3\sqrt{3x^2+2}}+\frac{1}{11\sqrt{-3x+3\left(x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x)
```

```
[Out] -2/3/(3*x^2+2)^(1/2)+1/4/(3*x^2+2)^(1/2)*x+1/11/(-3*x+3*(x+1/2)^2+5/4)^(1/2
)+3/44*x/(-3*x+3*(x+1/2)^2+5/4)^(1/2)-2/121*11^(1/2)*arctanh(2/11*(-3*x+4)*
11^(1/2)/(-12*x+12*(x+1/2)^2+5)^(1/2))
```

maxima [A] time = 0.96, size = 58, normalized size = 1.09

$$\frac{2}{121}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{7\,x}{22\,\sqrt{3\,x^2+2}}-\frac{19}{33\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/121*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1
)) + 7/22*x/sqrt(3*x^2 + 2) - 19/33/sqrt(3*x^2 + 2)
```

mupad [B] time = 0.14, size = 106, normalized size = 2.00

$$\frac{\sqrt{11}\,\left(2\,\ln\left(x+\frac{1}{2}\right)-2\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{121}-\frac{\sqrt{3}\,\sqrt{6}\,\left(-114+\sqrt{6}\,21\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2376\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\,\sqrt{6}\,(114+\sqrt{6}\,2}{2376\,(x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(3/2)),x)
```

```
[Out] (11^(1/2)*(2*log(x + 1/2) - 2*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/
3 - 4/3)))/121 - (3^(1/2)*6^(1/2)*(6^(1/2)*21i - 114)*(x^2 + 2/3)^(1/2)*1i)
/(2376*(x - (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*21i + 114)*(x^2 +
2/3)^(1/2)*1i)/(2376*(x + (6^(1/2)*1i)/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 + 2)**(3/2)), x)
```

3.128 $\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=75

$$\frac{97x-10}{242\sqrt{3x^2+2}}-\frac{4\sqrt{3x^2+2}}{121(2x+1)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

```
[Out] 4/1331*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)+1/242*(-10+9
7*x)/(3*x^2+2)^(1/2)-4/121*(3*x^2+2)^(1/2)/(1+2*x)
```

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1647, 807, 725, 206}

$$-\frac{10-97x}{242\sqrt{3x^2+2}}-\frac{4\sqrt{3x^2+2}}{121(2x+1)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] -(10 - 97*x)/(242*Sqrt[2 + 3*x^2]) - (4*Sqrt[2 + 3*x^2])/(121*(1 + 2*x)) +
(4*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx = -\frac{10-97x}{242\sqrt{2+3x^2}} - \frac{1}{6}\int \frac{-\frac{72}{121}+\frac{120x}{121}}{(1+2x)^2\sqrt{2+3x^2}}\,dx$$
$$= -\frac{10-97x}{242\sqrt{2+3x^2}} - \frac{4\sqrt{2+3x^2}}{121(1+2x)} - \frac{4}{121}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx$$
$$= -\frac{10-97x}{242\sqrt{2+3x^2}} - \frac{4\sqrt{2+3x^2}}{121(1+2x)} + \frac{4}{121}\,\mathrm{Subst}\left(\int \frac{1}{11-x^2}\,dx, x, \frac{4-3x}{\sqrt{2+3x^2}}\right)$$
$$= -\frac{10-97x}{242\sqrt{2+3x^2}} - \frac{4\sqrt{2+3x^2}}{121(1+2x)} + \frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 0.95

$$\frac{11\left(170x^2+77x-26\right)+8(2x+1)\sqrt{33x^2+22}\,\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{2662(2x+1)\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] (11*(-26 + 77*x + 170*x^2) + 8*(1 + 2*x)*Sqrt[22 + 33*x^2]*ArcTanh[(4 - 3*x
)/Sqrt[22 + 33*x^2]])/(2662*(1 + 2*x)*Sqrt[2 + 3*x^2])
```

fricas [A] time = 1.12, size = 103, normalized size = 1.37

$$\frac{4\sqrt{11}\left(6x^3+3x^2+4x+2\right)\log\left(\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)-21x^2+12x-19}{4x^2+4x+1}\right)+11\left(170x^2+77x-26\right)\sqrt{3x^2+2}}{2662\left(6x^3+3x^2+4x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2662*(4*sqrt(11)*(6*x^3 + 3*x^2 + 4*x + 2)*log((sqrt(11)*sqrt(3*x^2 + 2)*
(3*x - 4) - 21*x^2 + 12*x - 19)/(4*x^2 + 4*x + 1)) + 11*(170*x^2 + 77*x - 2
6)*sqrt(3*x^2 + 2))/(6*x^3 + 3*x^2 + 4*x + 2)
```

giac [B] time = 0.27, size = 168, normalized size = 2.24

$$-\frac{1}{7986}\sqrt{11}\left(85\sqrt{11}\sqrt{3}+24\log\left(\sqrt{11}\sqrt{3}-3\right)\right)\mathrm{sgn}\left(\frac{1}{2x+1}\right)-\frac{\frac{\frac{93}{\mathrm{sgn}\left(\frac{1}{2x+1}\right)}+\frac{44}{(2x+1)\mathrm{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}-\frac{85}{\mathrm{sgn}\left(\frac{1}{2x+1}\right)}}{242\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}}+\frac{4\sqrt{11}\log\Big(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/7986*sqrt(11)*(85*sqrt(11)*sqrt(3) + 24*log(sqrt(11)*sqrt(3) - 3))*sgn(1
/(2*x + 1)) - 1/242*((93/sgn(1/(2*x + 1)) + 44/((2*x + 1)*sgn(1/(2*x + 1)))
)/(2*x + 1) - 85/sgn(1/(2*x + 1)))/sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3)
+ 4/1331*sqrt(11)*log(sqrt(11)*(sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + s
qrt(11)/(2*x + 1)) - 3)/sgn(1/(2*x + 1))
```

maple [A] time = 0.01, size = 98, normalized size = 1.31

$$\frac{x}{2\sqrt{3x^2+2}}-\frac{18x}{121\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}+\frac{4\sqrt{11}\ \operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{1331}-\frac{2}{121\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x)
```

```
[Out] 1/2/(3*x^2+2)^(1/2)*x-2/121/(-3*x+3*(x+1/2)^2+5/4)^(1/2)-18/121/(-3*x+3*(x+
1/2)^2+5/4)^(1/2)*x+4/1331*11^(1/2)*arctanh(2/11*(-3*x+4)*11^(1/2)/(-12*x+1
2*(x+1/2)^2+5)^(1/2))-1/22/(x+1/2)/(-3*x+3*(x+1/2)^2+5/4)^(1/2)
```

maxima [A] time = 0.97, size = 84, normalized size = 1.12

$$-\frac{4}{1331}\sqrt{11}\ \operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{85\,x}{242\,\sqrt{3\,x^2+2}}-\frac{2}{121\,\sqrt{3\,x^2+2}}-\frac{1}{11\left(2\,\sqrt{3\,x^2+2}\,x+\sqrt{3\,x^2+2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x +
 1)) + 85/242*x/sqrt(3*x^2 + 2) - 2/121/sqrt(3*x^2 + 2) - 1/11/(2*sqrt(3*x^
2 + 2)*x + sqrt(3*x^2 + 2))
```

mupad [B] time = 4.14, size = 157, normalized size = 2.09

$$\frac{4\,\sqrt{11}\ \ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331}-\frac{4\,\sqrt{11}\ \ln\left(x+\frac{1}{2}\right)}{1331}+\frac{97\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{1452\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{97\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{1452\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{2\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{121\left(x+\frac{1}{2}\right)}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(3/2)),x)
```

```
[Out] (4*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/1331 - (
4*11^(1/2)*log(x + 1/2))/1331 + (97*3^(1/2)*(x^2 + 2/3)^(1/2))/(1452*(x - (
6^(1/2)*1i)/3)) + (97*3^(1/2)*(x^2 + 2/3)^(1/2))/(1452*(x + (6^(1/2)*1i)/3)
) - (2*3^(1/2)*(x^2 + 2/3)^(1/2))/(121*(x + 1/2)) + (3^(1/2)*6^(1/2)*(x^2 +
 2/3)^(1/2)*5i)/(1452*(x - (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(x^2 + 2/3)^
(1/2)*5i)/(1452*(x + (6^(1/2)*1i)/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(3/2),x)
```

```
[Out] Timed out
```

3.129 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{351x+358}{2662\sqrt{3x^2+2}}+\frac{2\sqrt{3x^2+2}}{1331(2x+1)}-\frac{2\sqrt{3x^2+2}}{121(2x+1)^2}-\frac{322\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

```
[Out] -322/14641*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)+1/2662*(
358+351*x)/(3*x^2+2)^(1/2)-2/121*(3*x^2+2)^(1/2)/(1+2*x)^2+2/1331*(3*x^2+2)
^(1/2)/(1+2*x)
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1647, 1651, 807, 725, 206}

$$\frac{351x+358}{2662\sqrt{3x^2+2}}+\frac{2\sqrt{3x^2+2}}{1331(2x+1)}-\frac{2\sqrt{3x^2+2}}{121(2x+1)^2}-\frac{322\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] (358 + 351*x)/(2662*Sqrt[2 + 3*x^2]) - (2*Sqrt[2 + 3*x^2])/(121*(1 + 2*x)^2
) + (2*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)) - (322*ArcTanh[(4 - 3*x)/(Sqrt[11]
*Sqrt[2 + 3*x^2])])/(1331*Sqrt[11])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx &= \frac{358+351x}{2662\sqrt{2+3x^2}}-\frac{1}{6}\int \frac{-\frac{2940}{1331}-\frac{7272x}{1331}-\frac{8592x^2}{1331}}{(1+2x)^3\sqrt{2+3x^2}}\,dx\\
&= \frac{358+351x}{2662\sqrt{2+3x^2}}-\frac{2\sqrt{2+3x^2}}{121(1+2x)^2}+\frac{1}{132}\int \frac{\frac{3768}{121}+\frac{7800x}{121}}{(1+2x)^2\sqrt{2+3x^2}}\,dx\\
&= \frac{358+351x}{2662\sqrt{2+3x^2}}-\frac{2\sqrt{2+3x^2}}{121(1+2x)^2}+\frac{2\sqrt{2+3x^2}}{1331(1+2x)}+\frac{322\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx}{1331}\\
&= \frac{358+351x}{2662\sqrt{2+3x^2}}-\frac{2\sqrt{2+3x^2}}{121(1+2x)^2}+\frac{2\sqrt{2+3x^2}}{1331(1+2x)}-\frac{322\,\mathrm{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3}{\sqrt{2+}}\right.}{1331}\\
&= \frac{358+351x}{2662\sqrt{2+3x^2}}-\frac{2\sqrt{2+3x^2}}{121(1+2x)^2}+\frac{2\sqrt{2+3x^2}}{1331(1+2x)}-\frac{322\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 78, normalized size = 0.80

$$
\frac{11\left(1428x^3+2716x^2+1799x+278\right)-644(2x+1)^2\sqrt{33x^2+22}\,\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{29282(2x+1)^2\sqrt{3x^2+2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] (11*(278 + 1799*x + 2716*x^2 + 1428*x^3) - 644*(1 + 2*x)^2*Sqrt[22 + 33*x^2
]*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])/(29282*(1 + 2*x)^2*Sqrt[2 + 3*x^2])
```

fricas [A] time = 0.82, size = 119, normalized size = 1.23

$$
\frac{322\sqrt{11}\left(12x^4+12x^3+11x^2+8x+2\right)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+11\left(1428x^3+2716x^2+1799\right.}{29282\left(12x^4+12x^3+11x^2+8x+2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/29282*(322*sqrt(11)*(12*x^4 + 12*x^3 + 11*x^2 + 8*x + 2)*log(-(sqrt(11)*s
qrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*(142
8*x^3 + 2716*x^2 + 1799*x + 278)*sqrt(3*x^2 + 2))/(12*x^4 + 12*x^3 + 11*x^2
 + 8*x + 2)
```

giac [B] time = 0.24, size = 196, normalized size = 2.02

$$\frac{322}{14641}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{351x+358}{2662\sqrt{3x^2+2}}+\frac{36\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^3-\sqrt{3}\left(\sqrt{3}x\right.}{1331\left(\left(\sqrt{3}x-\sqrt{3x^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] 322/14641*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^
2 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/2662*(3
51*x + 358)/sqrt(3*x^2 + 2) + 1/1331*(36*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 -
sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 48*sqrt(3)*x + 8*sqrt(3) - 48*sqr
t(3*x^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(
3*x^2 + 2)) - 2)^2
```

maple [A] time = 0.01, size = 107, normalized size = 1.10

$$\frac{357x}{2662\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}-\frac{322\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{14641}+\frac{161}{1331\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}+\frac{}{484\left(x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x)
```

```
[Out] 161/1331/(-3*x+3*(x+1/2)^2+5/4)^(1/2)+357/2662/(-3*x+3*(x+1/2)^2+5/4)^(1/2)
*x-322/14641*11^(1/2)*arctanh(2/11*(-3*x+4)*11^(1/2)/(-12*x+12*(x+1/2)^2+5)
^(1/2))+7/484/(x+1/2)/(-3*x+3*(x+1/2)^2+5/4)^(1/2)-1/88/(x+1/2)^2/(-3*x+3*(
x+1/2)^2+5/4)^(1/2)
```

maxima [A] time = 0.98, size = 124, normalized size = 1.28

$$\frac{322}{14641}\sqrt{11}\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2|2x+1|}-\frac{2\sqrt{6}}{3|2x+1|}\right)+\frac{357x}{2662\sqrt{3x^2+2}}+\frac{161}{1331\sqrt{3x^2+2}}-\frac{1}{22\left(4\sqrt{3x^2+2}x^2+4\sqrt{3x^2+2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 322/14641*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
 + 1)) + 357/2662*x/sqrt(3*x^2 + 2) + 161/1331/sqrt(3*x^2 + 2) - 1/22/(4*sq
rt(3*x^2 + 2)*x^2 + 4*sqrt(3*x^2 + 2)*x + sqrt(3*x^2 + 2)) + 7/242/(2*sqrt(
3*x^2 + 2)*x + sqrt(3*x^2 + 2))
```

mupad [B] time = 4.17, size = 180, normalized size = 1.86

$$\frac{322\sqrt{11}\ln\left(x+\frac{1}{2}\right)}{14641}-\frac{322\sqrt{11}\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{14641}+\frac{117\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{5324\left(x-\frac{\sqrt{6}\,1i}{3}\right)}+\frac{117\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{5324\left(x+\frac{\sqrt{6}\,1i}{3}\right)}-\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{242\left(x^2+x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(3/2)),x)
```

```
[Out] (322*11^(1/2)*log(x + 1/2))/14641 - (322*11^(1/2)*log(x - (3^(1/2)*11^(1/2)
*(x^2 + 2/3)^(1/2))/3 - 4/3))/14641 + (117*3^(1/2)*(x^2 + 2/3)^(1/2))/(5324
*(x - (6^(1/2)*1i)/3)) + (117*3^(1/2)*(x^2 + 2/3)^(1/2))/(5324*(x + (6^(1/2
)*1i)/3)) - (3^(1/2)*(x^2 + 2/3)^(1/2))/(242*(x + x^2 + 1/4)) + (3^(1/2)*(x
^2 + 2/3)^(1/2))/(1331*(x + 1/2)) - (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)*179i
)/(15972*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)*179i)/(
15972*(x + (6^(1/2)*1i)/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(3/2),x)
```

```
[Out] Timed out
```

3.130 $\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{279x+398}{162\left(3x^2+2\right)^{3/2}}+\frac{32}{27}\sqrt{3x^2+2}-\frac{465x+152}{54\sqrt{3x^2+2}}+\frac{8\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{3}}$$

```
[Out] 1/162*(398+279*x)/(3*x^2+2)^(3/2)+8/3*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/54*(
-152-465*x)/(3*x^2+2)^(1/2)+32/27*(3*x^2+2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1814, 641, 215}

$$\frac{279x+398}{162\left(3x^2+2\right)^{3/2}}+\frac{32}{27}\sqrt{3x^2+2}-\frac{465x+152}{54\sqrt{3x^2+2}}+\frac{8\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (398 + 279*x)/(162*(2 + 3*x^2)^(3/2)) - (152 + 465*x)/(54*Sqrt[2 + 3*x^2])
+ (32*Sqrt[2 + 3*x^2])/27 + (8*ArcSinh[Sqrt[3/2]*x])/Sqrt[3]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{398+279x}{162\left(2+3x^2\right)^{3/2}} - \frac{1}{6}\int \frac{\frac{22}{3}-\frac{280x}{3}-144x^2-64x^3}{\left(2+3x^2\right)^{3/2}}\,dx$$
$$= \frac{398+279x}{162\left(2+3x^2\right)^{3/2}} - \frac{152+465x}{54\sqrt{2+3x^2}} + \frac{1}{12}\int \frac{96+\frac{128x}{3}}{\sqrt{2+3x^2}}\,dx$$
$$= \frac{398+279x}{162\left(2+3x^2\right)^{3/2}} - \frac{152+465x}{54\sqrt{2+3x^2}} + \frac{32}{27}\sqrt{2+3x^2} + 8\int \frac{1}{\sqrt{2+3x^2}}\,dx$$
$$= \frac{398+279x}{162\left(2+3x^2\right)^{3/2}} - \frac{152+465x}{54\sqrt{2+3x^2}} + \frac{32}{27}\sqrt{2+3x^2} + \frac{8\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.07, size = 63, normalized size = 0.86

$$\frac{1728x^4-4185x^3+936x^2+432\sqrt{3}\left(3x^2+2\right)^{3/2}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)-2511x+254}{162\left(3x^2+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (254 - 2511*x + 936*x^2 - 4185*x^3 + 1728*x^4 + 432*Sqrt[3]*(2 + 3*x^2)^(3/
2)*ArcSinh[Sqrt[3/2]*x])/(162*(2 + 3*x^2)^(3/2))
```

fricas [A] time = 0.85, size = 87, normalized size = 1.19

$$\frac{216\sqrt{3}\left(9x^4+12x^2+4\right)\log\left(-\sqrt{3}\sqrt{3x^2+2}\,x-3x^2-1\right)+\left(1728x^4-4185x^3+936x^2-2511x+254\right)\sqrt{3}}{162\left(9x^4+12x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/162*(216*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*
x^2 - 1) + (1728*x^4 - 4185*x^3 + 936*x^2 - 2511*x + 254)*sqrt(3*x^2 + 2))/
(9*x^4 + 12*x^2 + 4)
```

giac [A] time = 0.18, size = 53, normalized size = 0.73

$$-\frac{8}{3}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)+\frac{9((3(64x-155)x+104)x-279)x+254}{162\left(3x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] -8/3*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/162*(9*((3*(64*x - 155)*
x + 104)*x - 279)*x + 254)/(3*x^2 + 2)^(3/2)
```

maple [A] time = 0.01, size = 91, normalized size = 1.25

$$\frac{32x^4}{3\left(3x^2+2\right)^{\frac{3}{2}}}-\frac{8x^3}{\left(3x^2+2\right)^{\frac{3}{2}}}+\frac{52x^2}{9\left(3x^2+2\right)^{\frac{3}{2}}}-\frac{107x}{18\sqrt{3x^2+2}}-\frac{65x}{18\left(3x^2+2\right)^{\frac{3}{2}}}+\frac{8\sqrt{3}\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{3}+\frac{127}{81\left(3x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x)
```

```
[Out] 32/3*x^4/(3*x^2+2)^(3/2)+52/9*x^2/(3*x^2+2)^(3/2)+127/81/(3*x^2+2)^(3/2)-8*
x^3/(3*x^2+2)^(3/2)-107/18/(3*x^2+2)^(1/2)*x+8/3*arcsinh(1/2*6^(1/2)*x)*3^(
1/2)-65/18*x/(3*x^2+2)^(3/2)
```

maxima [A] time = 0.95, size = 105, normalized size = 1.44

$$\frac{32\,x^4}{3\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{8}{3}\,x\left(\frac{9\,x^2}{\left(3\,x^2+2\right)^{\frac{3}{2}}}+\frac{4}{\left(3\,x^2+2\right)^{\frac{3}{2}}}\right)+\frac{8}{3}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{6}\,x\right)-\frac{11\,x}{18\,\sqrt{3\,x^2+2}}+\frac{52\,x^2}{9\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{65\,x}{18\left(3\,x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/3*x^4/(3*x^2 + 2)^(3/2) - 8/3*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2)
^(3/2)) + 8/3*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 11/18*x/sqrt(3*x^2 + 2) + 52
/9*x^2/(3*x^2 + 2)^(3/2) - 65/18*x/(3*x^2 + 2)^(3/2) + 127/81/(3*x^2 + 2)^(
3/2)
```

mupad [B] time = 0.06, size = 212, normalized size = 2.90

$$\frac{32\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{27}+\frac{8\,\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{3}-\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{31}{16}+\frac{\sqrt{6}\,199\mathrm{i}}{144}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(-\frac{31}{24}+\frac{\sqrt{6}\,199\mathrm{i}}{216}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}+\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{31}{16}+\frac{\sqrt{6}\,19}{144}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}\right.}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

```
[Out] (32*3^(1/2)*(x^2 + 2/3)^(1/2))/27 + (8*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2)
)/3 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*199i)/144 - 31/16)/(x - (6^(1/2
)*1i)/3) - (6^(1/2)*((6^(1/2)*199i)/216 - 31/24)*1i)/(2*(x - (6^(1/2)*1i)/3
)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*199i)/144 + 31/16)/(x + (
6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*199i)/216 + 31/24)*1i)/(2*(x + (6^(1/2)
*1i)/3)^2)))/27 + (3^(1/2)*6^(1/2)*(6^(1/2)*1953i - 1824)*(x^2 + 2/3)^(1/2)
*1i)/(7776*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*1953i + 1824)*
(x^2 + 2/3)^(1/2)*1i)/(7776*(x - (6^(1/2)*1i)/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(5/2), x)
```

3.131 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx$

Optimal. Leaf size=60

$$\frac{70-47x}{54\left(3x^2+2\right)^{3/2}}-\frac{59x+168}{54\sqrt{3x^2+2}}+\frac{16\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{9\sqrt{3}}$$

```
[Out] 1/54*(70-47*x)/(3*x^2+2)^(3/2)+16/27*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/54*(-
168-59*x)/(3*x^2+2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1814, 12, 215}

$$\frac{70-47x}{54\left(3x^2+2\right)^{3/2}}-\frac{59x+168}{54\sqrt{3x^2+2}}+\frac{16\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (70 - 47*x)/(54*(2 + 3*x^2)^(3/2)) - (168 + 59*x)/(54*Sqrt[2 + 3*x^2]) + (1
6*ArcSinh[Sqrt[3/2]*x])/(9*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx &= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{1}{6}\int \frac{-\frac{74}{9}-56x-32x^2}{\left(2+3x^2\right)^{3/2}}\,dx\\ &= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{1}{12}\int \frac{64}{3\sqrt{2+3x^2}}\,dx\\ &= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{16}{9}\int \frac{1}{\sqrt{2+3x^2}}\,dx\\ &= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{16\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{9\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 0.97

$$\frac{-177x^3-504x^2+32\sqrt{3}\left(3x^2+2\right)^{3/2}\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)-165x-266}{54\left(3x^2+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (-266 - 165*x - 504*x^2 - 177*x^3 + 32*Sqrt[3]*(2 + 3*x^2)^(3/2)*ArcSinh[Sq
rt[3/2]*x])/(54*(2 + 3*x^2)^(3/2))
```

fricas [A] time = 0.78, size = 83, normalized size = 1.38

$$\frac{16\sqrt{3}\left(9x^4+12x^2+4\right)\log\left(-\sqrt{3}\sqrt{3x^2+2}\,x-3x^2-1\right)-\left(177x^3+504x^2+165x+266\right)\sqrt{3x^2+2}}{54\left(9x^4+12x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/54*(16*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^
2 - 1) - (177*x^3 + 504*x^2 + 165*x + 266)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x^2
 + 4)
```

giac [A] time = 0.19, size = 48, normalized size = 0.80

$$-\frac{16}{27}\sqrt{3}\log\left(-\sqrt{3}\,x+\sqrt{3x^2+2}\right)-\frac{3\left(\left(59x+168\right)x+55\right)x+266}{54\left(3x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] -16/27*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) - 1/54*(3*((59*x + 168)*x
+ 55)*x + 266)/(3*x^2 + 2)^(3/2)
```

maple [A] time = 0.01, size = 77, normalized size = 1.28

$$-\frac{16x^3}{9\left(3x^2+2\right)^{\frac{3}{2}}}-\frac{28x^2}{3\left(3x^2+2\right)^{\frac{3}{2}}}-\frac{x}{2\sqrt{3x^2+2}}-\frac{37x}{18\left(3x^2+2\right)^{\frac{3}{2}}}+\frac{16\sqrt{3}\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{27}-\frac{133}{27\left(3x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x)
```

```
[Out] -16/9/(3*x^2+2)^(3/2)*x^3-1/2/(3*x^2+2)^(1/2)*x+16/27*arcsinh(1/2*6^(1/2)*x
)*3^(1/2)-28/3/(3*x^2+2)^(3/2)*x^2-133/27/(3*x^2+2)^(3/2)-37/18/(3*x^2+2)^(
3/2)*x
```

maxima [B] time = 0.96, size = 91, normalized size = 1.52

$$-\frac{16}{27}\,x\left(\frac{9\,x^2}{\left(3\,x^2+2\right)^{\frac{3}{2}}}+\frac{4}{\left(3\,x^2+2\right)^{\frac{3}{2}}}\right)+\frac{16}{27}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}\,x\right)+\frac{37\,x}{54\,\sqrt{3\,x^2+2}}-\frac{28\,x^2}{3\,\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{37\,x}{18\,\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -16/27*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2)^(3/2)) + 16/27*sqrt(3)*ar
csinh(1/2*sqrt(6)*x) + 37/54*x/sqrt(3*x^2 + 2) - 28/3*x^2/(3*x^2 + 2)^(3/2)
 - 37/18*x/(3*x^2 + 2)^(3/2) - 133/27/(3*x^2 + 2)^(3/2)
```

mupad [B] time = 0.05, size = 200, normalized size = 3.33

$$\frac{16\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{27}+\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{-\frac{47}{48}+\frac{\sqrt{6}\,35\mathrm{i}}{48}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\,\left(-\frac{47}{72}+\frac{\sqrt{6}\,35\mathrm{i}}{72}\right)\,1\mathrm{i}}{2\,{\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}^2}\right)}{27}-\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{\frac{47}{48}+\frac{\sqrt{6}\,35\mathrm{i}}{48}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\,\left(\frac{47}{72}+\frac{\sqrt{6}\,35}{72}\right.}{2\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}\right.}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

```
[Out] (16*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(
((6^(1/2)*35i)/48 - 47/48)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*35i)/7
2 - 47/72)*1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*(x^2 + 2/3)^(1/2)
*(((6^(1/2)*35i)/48 + 47/48)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*35i)
/72 + 47/72)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*6^(1/2)*(6^(1/2
)*63i - 672)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6
^(1/2)*(6^(1/2)*63i + 672)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x - (6^(1/2)*1i)/3)
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x+1\right)^{2}\left(4x^{2}+3x+1\right)}{\left(3x^{2}+2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(5/2), x)
```

3.132 $\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}} dx$

Optimal. Leaf size=41

$$\frac{2-51x}{54(3x^2+2)^{3/2}}-\frac{16-13x}{18\sqrt{3x^2+2}}$$

[Out] 1/54*(2-51*x)/(3*x^2+2)^(3/2)+1/18*(-16+13*x)/(3*x^2+2)^(1/2)

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1814, 637}

$$\frac{2-51x}{54(3x^2+2)^{3/2}}-\frac{16-13x}{18\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]

[Out] (2 - 51*x)/(54*(2 + 3*x^2)^(3/2)) - (16 - 13*x)/(18*Sqrt[2 + 3*x^2])

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}} dx &= \frac{2-51x}{54(2+3x^2)^{3/2}}-\frac{1}{6}\int \frac{-\frac{26}{3}-16x}{(2+3x^2)^{3/2}} dx\\ &= \frac{2-51x}{54(2+3x^2)^{3/2}}-\frac{16-13x}{18\sqrt{2+3x^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.73

$$\frac{117x^3-144x^2+27x-94}{54(3x^2+2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]

[Out] (-94 + 27*x - 144*x^2 + 117*x^3)/(54*(2 + 3*x^2)^(3/2))

fricas [A] time = 0.77, size = 40, normalized size = 0.98

$$\frac{\left(117\,x^3-144\,x^2+27\,x-94\right)\sqrt{3\,x^2+2}}{54\left(9\,x^4+12\,x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/54*(117*x^3 - 144*x^2 + 27*x - 94)*sqrt(3*x^2 + 2)/(9*x^4 + 12*x^2 + 4)
```

giac [A] time = 0.34, size = 25, normalized size = 0.61

$$\frac{9\,((13\,x-16)x+3)x-94}{54\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/54*(9*((13*x - 16)*x + 3)*x - 94)/(3*x^2 + 2)^(3/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.66

$$\frac{117x^3-144x^2+27x-94}{54\left(3x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x)
```

```
[Out] 1/54*(117*x^3-144*x^2+27*x-94)/(3*x^2+2)^(3/2)
```

maxima [A] time = 0.42, size = 50, normalized size = 1.22

$$\frac{13\,x}{18\sqrt{3\,x^2+2}}-\frac{8\,x^2}{3\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{17\,x}{18\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{47}{27\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 13/18*x/sqrt(3*x^2 + 2) - 8/3*x^2/(3*x^2 + 2)^(3/2) - 17/18*x/(3*x^2 + 2)^(
3/2) - 47/27/(3*x^2 + 2)^(3/2)
```

mupad [B] time = 4.11, size = 185, normalized size = 4.51

$$\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{17}{16}+\frac{\sqrt{6}\,1i}{48}}{x+\frac{\sqrt{6}\,1i}{3}}+\frac{\sqrt{6}\left(-\frac{17}{24}+\frac{\sqrt{6}\,1i}{72}\right)1i}{2\left(x+\frac{\sqrt{6}\,1i}{3}\right)^2}\right)}{27}-\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{17}{16}+\frac{\sqrt{6}\,1i}{48}}{x-\frac{\sqrt{6}\,1i}{3}}-\frac{\sqrt{6}\left(\frac{17}{24}+\frac{\sqrt{6}\,1i}{72}\right)1i}{2\left(x-\frac{\sqrt{6}\,1i}{3}\right)^2}\right)}{27}-\frac{\sqrt{3}\,\sqrt{6}\,\left(-192+\sqrt{6}\cdots\right.}{2592\left(x-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

```
[Out] (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*1i)/48 - 17/16)/(x + (6^(1/2)*1i)/3)
+ (6^(1/2)*((6^(1/2)*1i)/72 - 17/24)*1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27 -
```

```
(3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*1i)/48 + 17/16)/(x - (6^(1/2)*1i)/3)
- (6^(1/2)*((6^(1/2)*1i)/72 + 17/24)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 -
(3^(1/2)*6^(1/2)*(6^(1/2)*69i - 192)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x - (6^(1
/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*69i + 192)*(x^2 + 2/3)^(1/2)*1i)/(2
592*(x + (6^(1/2)*1i)/3))
```

sympy [**B**] time = 77.50, size = 180, normalized size = 4.39

$$\frac{10x^3}{18x^2\sqrt{3x^2+2}+12\sqrt{3x^2+2}}+\frac{x^3}{6x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}-\frac{72x^2}{81x^2\sqrt{3x^2+2}+54\sqrt{3x^2+2}}+\frac{x}{6x^2\sqrt{3x^2+2}+4\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

```
[Out] 10*x**3/(18*x**2*sqrt(3*x**2 + 2) + 12*sqrt(3*x**2 + 2)) + x**3/(6*x**2*sqr
t(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) - 72*x**2/(81*x**2*sqrt(3*x**2 + 2) + 5
4*sqrt(3*x**2 + 2)) + x/(6*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) - 32
/(81*x**2*sqrt(3*x**2 + 2) + 54*sqrt(3*x**2 + 2)) - 5/(27*x**2*sqrt(3*x**2
+ 2) + 18*sqrt(3*x**2 + 2))
```

3.133 $\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{21x-38}{198\left(3x^2+2\right)^{3/2}}+\frac{95x+24}{726\sqrt{3x^2+2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

```
[Out] 1/198*(-38+21*x)/(3*x^2+2)^(3/2)-8/1331*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^
2+2)^(1/2))*11^(1/2)+1/726*(24+95*x)/(3*x^2+2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1647, 823, 12, 725, 206}

$$-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}+\frac{95x+24}{726\sqrt{3x^2+2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] -(38 - 21*x)/(198*(2 + 3*x^2)^(3/2)) + (24 + 95*x)/(726*Sqrt[2 + 3*x^2]) -
(8*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
```

```
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx &= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}-\frac{1}{18}\int \frac{-\frac{78}{11}-\frac{84x}{11}}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx\\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}+\frac{\int \frac{864}{11(1+2x)\sqrt{2+3x^2}}\,dx}{1188}\\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}+\frac{8}{121}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}-\frac{8}{121}\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 0.79

$$\frac{855x^3+216x^2+801x-274}{2178\left(3x^2+2\right)^{3/2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] (-274 + 801*x + 216*x^2 + 855*x^3)/(2178*(2 + 3*x^2)^(3/2)) - (8*ArcTanh[(4
 - 3*x)/Sqrt[22 + 33*x^2]])/(121*Sqrt[11])
```

fricas [A] time = 0.58, size = 103, normalized size = 1.41

$$\frac{72\sqrt{11}\left(9x^4+12x^2+4\right)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+11\left(855x^3+216x^2+801x-274\right)\sqrt{3x^2+2}}{23958\left(9x^4+12x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/23958*(72*sqrt(11)*(9*x^4 + 12*x^2 + 4)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3
*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*(855*x^3 + 216*x^2 +
801*x - 274)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x^2 + 4)
```

giac [A] time = 0.21, size = 91, normalized size = 1.25

$$\frac{8}{1331}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{9\left(\left(95x+24\right)x+89\right)x-274}{2178\left(3x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] 8/1331*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 +
 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/2178*(9*((
95*x + 24)*x + 89)*x - 274)/(3*x^2 + 2)^(3/2)
```

maple [B] time = 0.01, size = 133, normalized size = 1.82

$$\frac{x}{12\left(3x^2+2\right)^{\frac{3}{2}}}+\frac{x}{12\sqrt{3x^2+2}}+\frac{x}{44\left(-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{23x}{484\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}-\frac{8\sqrt{11}\ \operatorname{arctanh}\left(\frac{\ }{11\sqrt{\ }}\right.}{133}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x)
```

```
[Out] -2/9/(3*x^2+2)^(3/2)+1/12/(3*x^2+2)^(3/2)*x+1/12/(3*x^2+2)^(1/2)*x+1/33/(-3
*x+3*(x+1/2)^2+5/4)^(3/2)+1/44*x/(-3*x+3*(x+1/2)^2+5/4)^(3/2)+23/484/(-3*x+
3*(x+1/2)^2+5/4)^(1/2)*x+4/121/(-3*x+3*(x+1/2)^2+5/4)^(1/2)-8/1331*11^(1/2)
*arctanh(2/11*(-3*x+4)*11^(1/2)/(-12*x+12*(x+1/2)^2+5)^(1/2))
```

maxima [A] time = 0.97, size = 81, normalized size = 1.11

$$\frac{8}{1331}\sqrt{11}\ \operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\left|2\,x+1\right|}-\frac{2\,\sqrt{6}}{3\left|2\,x+1\right|}\right)+\frac{95\,x}{726\,\sqrt{3\,x^2+2}}+\frac{4}{121\,\sqrt{3\,x^2+2}}+\frac{7\,x}{66\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{19}{99\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x +
1)) + 95/726*x/sqrt(3*x^2 + 2) + 4/121/sqrt(3*x^2 + 2) + 7/66*x/(3*x^2 + 2)
^(3/2) - 19/99/(3*x^2 + 2)^(3/2)
```

mupad [B] time = 0.13, size = 218, normalized size = 2.99

$$\frac{\sqrt{11}\left(8\ln\left(x+\frac{1}{2}\right)-8\ln\left(x-\frac{\sqrt{3}\ \sqrt{11}\ \sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{1331}-\frac{\sqrt{3}\ \sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{21}{176}+\frac{\sqrt{6}\ 19\mathrm{i}}{176}}{x+\frac{\sqrt{6}\ 1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(-\frac{7}{88}+\frac{\sqrt{6}\ 19\mathrm{i}}{264}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\ 1\mathrm{i}}{3}\right)^2}\right)}{27}+\frac{\sqrt{3}\ \sqrt{x^2\ }}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(5/2)),x)
```

```
[Out] (11^(1/2)*(8*log(x + 1/2) - 8*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/
3 - 4/3)))/1331 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*19i)/176 - 21/176)/
(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*19i)/264 - 7/88)*1i)/(2*(x + (6^(
1/2)*1i)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*19i)/176 + 21/1
76)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*19i)/264 + 7/88)*1i)/(2*(x -
(6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*303i - 288)*(x^2 + 2/3)
^(1/2)*1i)/(104544*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*303i +
 288)*(x^2 + 2/3)^(1/2)*1i)/(104544*(x - (6^(1/2)*1i)/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(5/2),x)
```

```
[Out] Timed out
```

3.134 $\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=95

$$\frac{97x-10}{726\left(3x^2+2\right)^{3/2}}-\frac{16\sqrt{3x^2+2}}{1331(2x+1)}+\frac{887x+24}{7986\sqrt{3x^2+2}}-\frac{32\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

```
[Out] 1/726*(-10+97*x)/(3*x^2+2)^(3/2)-32/14641*arctanh(1/11*(4-3*x)*11^(1/2)/(3*
x^2+2)^(1/2))*11^(1/2)+1/7986*(24+887*x)/(3*x^2+2)^(1/2)-16/1331*(3*x^2+2)^
(1/2)/(1+2*x)
```

Rubi [A] time = 0.17, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1647, 807, 725, 206}

$$-\frac{10-97x}{726\left(3x^2+2\right)^{3/2}}-\frac{16\sqrt{3x^2+2}}{1331(2x+1)}+\frac{887x+24}{7986\sqrt{3x^2+2}}-\frac{32\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] -(10 - 97*x)/(726*(2 + 3*x^2)^(3/2)) + (24 + 887*x)/(7986*Sqrt[2 + 3*x^2])
 - (16*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)) - (32*ArcTanh[(4 - 3*x)/(Sqrt[11]*S
qrt[2 + 3*x^2])])/(1331*Sqrt[11])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{5/2}}\,dx = -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}-\frac{1}{18}\int \frac{-\frac{798}{121}-\frac{1968x}{121}-\frac{2328x^2}{121}}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx$$

$$= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}+\frac{1}{108}\int \frac{\frac{10368}{1331}+\frac{1728x}{1331}}{(1+2x)^2\sqrt{2+3x^2}}\,dx$$

$$= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}-\frac{16\sqrt{2+3x^2}}{1331(1+2x)}+\frac{32\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx}{1331}$$

$$= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}-\frac{16\sqrt{2+3x^2}}{1331(1+2x)}-\frac{32\,\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-}{\sqrt{2-}}\right.}{1331}$$

$$= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}-\frac{16\sqrt{2+3x^2}}{1331(1+2x)}-\frac{32\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.96

$$\frac{11\left(4458x^4+2805x^3+4602x^2+2717x-446\right)-192\sqrt{33x^2+22}\left(6x^3+3x^2+4x+2\right)\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{87846(2x+1)\left(3x^2+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] (11*(-446 + 2717*x + 4602*x^2 + 2805*x^3 + 4458*x^4) - 192*Sqrt[22 + 33*x^2
]*(2 + 4*x + 3*x^2 + 6*x^3)*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])/(87846*(1
 + 2*x)*(2 + 3*x^2)^(3/2))
```

fricas [A] time = 0.59, size = 134, normalized size = 1.41

$$\frac{96\sqrt{11}\left(18x^5+9x^4+24x^3+12x^2+8x+4\right)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+11\left(4458x^4+2805x^3+46\right.}{87846\left(18x^5+9x^4+24x^3+12x^2+8x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/87846*(96*sqrt(11)*(18*x^5 + 9*x^4 + 24*x^3 + 12*x^2 + 8*x + 4)*log(-(sqr
t(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) +
11*(4458*x^4 + 2805*x^3 + 4602*x^2 + 2717*x - 446)*sqrt(3*x^2 + 2))/(18*x^5
 + 9*x^4 + 24*x^3 + 12*x^2 + 8*x + 4)
```

giac [B] time = 0.52, size = 233, normalized size = 2.45

$$-\frac{1}{263538}\sqrt{11}\left(743\sqrt{11}\sqrt{3}-576\log\left(\sqrt{11}\sqrt{3}-3\right)\right)\text{sgn}\left(\frac{1}{2x+1}\right)-\frac{32\sqrt{11}\log\left(\sqrt{11}\left(\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}+\right.\right.}{14641\,\text{sgn}\left(\frac{1}{2x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/263538*sqrt(11)*(743*sqrt(11)*sqrt(3) - 576*log(sqrt(11)*sqrt(3) - 3))*s
gn(1/(2*x + 1)) - 32/14641*sqrt(11)*log(sqrt(11)*(sqrt(-6/(2*x + 1) + 11/(2
*x + 1)^2 + 3) + sqrt(11)/(2*x + 1)) - 3)/sgn(1/(2*x + 1)) + 1/7986*(((11*(
731/sgn(1/(2*x + 1)) + 528/((2*x + 1)*sgn(1/(2*x + 1))))/(2*x + 1) - 14163/
sgn(1/(2*x + 1)))/(2*x + 1) + 6111/sgn(1/(2*x + 1)))/(2*x + 1) - 2229/sgn(1
/(2*x + 1)))/((6/(2*x + 1) - 11/(2*x + 1)^2 - 3)*sqrt(-6/(2*x + 1) + 11/(2*
x + 1)^2 + 3))
```

maple [A] time = 0.01, size = 143, normalized size = 1.51

$$\frac{x}{6\left(3x^2+2\right)^{\frac{3}{2}}}+\frac{x}{6\sqrt{3x^2+2}}-\frac{10x}{121\left(-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}\right)^{\frac{3}{2}}}-\frac{98x}{1331\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}-\frac{32\sqrt{11}\ \mathrm{arctanh}\left(\frac{}{11}\right.}{1464}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x)
```

```
[Out] 1/6/(3*x^2+2)^(3/2)*x+1/6/(3*x^2+2)^(1/2)*x+4/363/(-3*x+3*(x+1/2)^2+5/4)^(3
/2)-10/121/(-3*x+3*(x+1/2)^2+5/4)^(3/2)*x-98/1331/(-3*x+3*(x+1/2)^2+5/4)^(1
/2)*x+16/1331/(-3*x+3*(x+1/2)^2+5/4)^(1/2)-32/14641*11^(1/2)*arctanh(2/11*(
-3*x+4)*11^(1/2)/(-12*x+12*(x+1/2)^2+5)^(1/2))-1/22/(x+1/2)/(-3*x+3*(x+1/2)
^2+5/4)^(3/2)
```

maxima [A] time = 0.98, size = 107, normalized size = 1.13

$$\frac{32}{14641}\sqrt{11}\ \mathrm{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{743\,x}{7986\,\sqrt{3\,x^2+2}}+\frac{16}{1331\,\sqrt{3\,x^2+2}}+\frac{61\,x}{726\,\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{}{11\left(2\left(3\,x^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/14641*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
+ 1)) + 743/7986*x/sqrt(3*x^2 + 2) + 16/1331/sqrt(3*x^2 + 2) + 61/726*x/(3*
x^2 + 2)^(3/2) - 1/11/(2*(3*x^2 + 2)^(3/2)*x + (3*x^2 + 2)^(3/2)) + 4/363/(
3*x^2 + 2)^(3/2)
```

mupad [B] time = 4.31, size = 270, normalized size = 2.84

$$\frac{\sqrt{11}\left(8\ln\left(x+\frac{1}{2}\right)-8\ln\left(x-\frac{\sqrt{3}\ \sqrt{11}\ \sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{14641}+\frac{\sqrt{11}\left(\frac{48\ln\left(x+\frac{1}{2}\right)}{1331}-\frac{48\ln\left(x-\frac{\sqrt{3}\ \sqrt{11}\ \sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331}\right)}{22}-\frac{8\sqrt{3}\ \sqrt{x^2+}}{1331\left(x+\frac{1}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(5/2)),x)
```

```
[Out] (11^(1/2)*(8*log(x + 1/2) - 8*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/
3 - 4/3)))/14641 + (11^(1/2)*((48*log(x + 1/2))/1331 - (48*log(x - (3^(1/2)
```

```
*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/1331))/22 - (8*3^(1/2)*(x^2 + 2/3)^(
1/2))/(1331*(x + 1/2)) - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*15i)/1936 -
291/1936)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*5i)/968 - 97/968)*1i)/(
2*(x + (6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*15i)
/1936 + 291/1936)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*5i)/968 + 97/96
8)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*2481i -
288)*(x^2 + 2/3)^(1/2)*1i)/(1149984*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2
)*(6^(1/2)*2481i + 288)*(x^2 + 2/3)^(1/2)*1i)/(1149984*(x - (6^(1/2)*1i)/3)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(5/2),x)
```

```
[Out] Timed out
```

3.135 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{351x+358}{7986\left(3x^2+2\right)^{3/2}}-\frac{8\sqrt{3x^2+2}}{1331(2x+1)}-\frac{8\sqrt{3x^2+2}}{1331(2x+1)^2}+\frac{2133x+1216}{29282\sqrt{3x^2+2}}-\frac{1216\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{14641\sqrt{11}}$$

```
[Out] 1/7986*(358+351*x)/(3*x^2+2)^(3/2)-1216/161051*arctanh(1/11*(4-3*x)*11^(1/2
)/(3*x^2+2)^(1/2))*11^(1/2)+1/29282*(1216+2133*x)/(3*x^2+2)^(1/2)-8/1331*(3
*x^2+2)^(1/2)/(1+2*x)^2-8/1331*(3*x^2+2)^(1/2)/(1+2*x)
```

Rubi [A] time = 0.21, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1647, 1651, 807, 725, 206}

$$\frac{351x+358}{7986\left(3x^2+2\right)^{3/2}}-\frac{8\sqrt{3x^2+2}}{1331(2x+1)}-\frac{8\sqrt{3x^2+2}}{1331(2x+1)^2}+\frac{2133x+1216}{29282\sqrt{3x^2+2}}-\frac{1216\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{14641\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] (358 + 351*x)/(7986*(2 + 3*x^2)^(3/2)) + (1216 + 2133*x)/(29282*Sqrt[2 + 3*
x^2]) - (8*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)^2) - (8*Sqrt[2 + 3*x^2])/(1331*
(1 + 2*x)) - (1216*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(14641*Sq
rt[11])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
```

```
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx = \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} - \frac{1}{18}\int \frac{-\frac{10926}{1331}-\frac{3132x}{121}-\frac{51048x^2}{1331}-\frac{16848x^3}{1331}}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx$$

$$= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} + \frac{1}{108}\int \frac{\frac{245376}{14641}+\frac{544320x}{14641}+\frac{525312x^2}{14641}}{(1+2x)^3\sqrt{2+3x^2}}\,dx$$

$$= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{\int \frac{-\frac{338688}{1331}-\frac{468288x}{1331}}{(1+2x)^2\sqrt{2+3x^2}}\,dx}{2376}$$

$$= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)} + \frac{1216\int \ldots}{1\ldots}$$

$$= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)} - \frac{1216\,\mathrm{Sub}\ldots}{\ldots}$$

$$= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)} - \frac{1216\tanh\ldots}{14\ldots}$$

Mathematica [A] time = 0.10, size = 75, normalized size = 0.64

$$\frac{\frac{11\left(67284x^5+111060x^4+116937x^3+109844x^2+57371x+7010\right)}{(2x+1)^2\left(3x^2+2\right)^{3/2}} - 7296\sqrt{11}\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{966306}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] ((11*(7010 + 57371*x + 109844*x^2 + 116937*x^3 + 111060*x^4 + 67284*x^5))/(
(1 + 2*x)^2*(2 + 3*x^2)^(3/2)) - 7296*Sqrt[11]*ArcTanh[(4 - 3*x)/Sqrt[22 +
33*x^2]])/966306
```

fricas [A] time = 0.90, size = 149, normalized size = 1.27

$$\frac{3648\sqrt{11}\left(36x^6+36x^5+57x^4+48x^3+28x^2+16x+4\right)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+11\left(67284x^5\ldots\right.}{966306\left(36x^6+36x^5+57x^4+48x^3+28x^2+16\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/966306*(3648*sqrt(11)*(36*x^6 + 36*x^5 + 57*x^4 + 48*x^3 + 28*x^2 + 16*x
+ 4)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2
+ 4*x + 1)) + 11*(67284*x^5 + 111060*x^4 + 116937*x^3 + 109844*x^2 + 57371*
x + 7010)*sqrt(3*x^2 + 2))/(36*x^6 + 36*x^5 + 57*x^4 + 48*x^3 + 28*x^2 + 16
*x + 4)
```

giac [A] time = 0.29, size = 183, normalized size = 1.56

$$\frac{1216}{161051}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{9\left((2133x+1216)x+1851\right)x+11234}{87846\left(3x^2+2\right)^{\frac{3}{2}}}+\frac{4\left(\sqrt{3}\left(\sqrt{3}\right.\right.}{1331\left(\left(\sqrt{}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1216/161051*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*
x^2 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/87846
*(9*((2133*x + 1216)*x + 1851)*x + 11234)/(3*x^2 + 2)^(3/2) + 4/1331*(sqrt(
3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 24*sqrt(3)*x - 8*sqrt(3) - 24*sqrt(3*x
^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(3*x^2
 + 2)) - 2)^2
```

maple [A] time = 0.01, size = 140, normalized size = 1.20

$$\frac{87x}{2662\left(-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{1869x}{29282\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}-\frac{1216\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{161051}+\frac{}{39}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x)
```

```
[Out] 152/3993/(-3*x+3*(x+1/2)^2+5/4)^(3/2)+87/2662/(-3*x+3*(x+1/2)^2+5/4)^(3/2)*
x+1869/29282/(-3*x+3*(x+1/2)^2+5/4)^(1/2)*x+608/14641/(-3*x+3*(x+1/2)^2+5/4
)^(1/2)-1216/161051*11^(1/2)*arctanh(2/11*(-3*x+4)*11^(1/2)/(-12*x+12*(x+1/
2)^2+5)^(1/2))+1/484/(x+1/2)/(-3*x+3*(x+1/2)^2+5/4)^(3/2)-1/88/(x+1/2)^2/(-
3*x+3*(x+1/2)^2+5/4)^(3/2)
```

maxima [A] time = 0.99, size = 147, normalized size = 1.26

$$\frac{1216}{161051}\sqrt{11}\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2|2x+1|}-\frac{2\sqrt{6}}{3|2x+1|}\right)+\frac{1869x}{29282\sqrt{3x^2+2}}+\frac{608}{14641\sqrt{3x^2+2}}+\frac{87x}{2662\left(3x^2+2\right)^{\frac{3}{2}}}-\frac{}{22\left(4\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1216/161051*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2
*x + 1)) + 1869/29282*x/sqrt(3*x^2 + 2) + 608/14641/sqrt(3*x^2 + 2) + 87/26
62*x/(3*x^2 + 2)^(3/2) - 1/22/(4*(3*x^2 + 2)^(3/2)*x^2 + 4*(3*x^2 + 2)^(3/2
)*x + (3*x^2 + 2)^(3/2)) + 1/242/(2*(3*x^2 + 2)^(3/2)*x + (3*x^2 + 2)^(3/2)
) + 152/3993/(3*x^2 + 2)^(3/2)
```

mupad [B] time = 4.19, size = 301, normalized size = 2.57

$$\frac{1216\sqrt{11}\ \ln\left(x+\frac{1}{2}\right)}{161051}-\frac{1216\sqrt{11}\ \ln\left(x-\frac{\sqrt{3}\ \sqrt{11}\ \sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{161051}-\frac{179\sqrt{3}\ \sqrt{x^2+\frac{2}{3}}}{95832\left(x^2+\frac{2\mathrm{i}\sqrt{6}\ x}{3}-\frac{2}{3}\right)}+\frac{711\sqrt{3}\ \sqrt{x^2+\frac{2}{3}}}{58564\left(x-\frac{\sqrt{6}\ 1\mathrm{i}}{3}\right)}+\frac{711}{5856}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(5/2)),x)
```

```
[Out] (1216*11^(1/2)*log(x + 1/2))/161051 - (1216*11^(1/2)*log(x - (3^(1/2)*11^(1
/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/161051 - (179*3^(1/2)*(x^2 + 2/3)^(1/2))/(
95832*((6^(1/2)*x*2i)/3 + x^2 - 2/3)) + (711*3^(1/2)*(x^2 + 2/3)^(1/2))/(58
564*(x - (6^(1/2)*1i)/3)) + (711*3^(1/2)*(x^2 + 2/3)^(1/2))/(58564*(x + (6^
(1/2)*1i)/3)) - (2*3^(1/2)*(x^2 + 2/3)^(1/2))/(1331*(x + x^2 + 1/4)) + (179
*3^(1/2)*(x^2 + 2/3)^(1/2))/(95832*((6^(1/2)*x*2i)/3 - x^2 + 2/3)) - (4*3^(
1/2)*(x^2 + 2/3)^(1/2))/(1331*(x + 1/2)) + (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/
2)*13i)/(21296*((6^(1/2)*x*2i)/3 + x^2 - 2/3)) - (3^(1/2)*6^(1/2)*(x^2 + 2/
3)^(1/2)*9265i)/(2108304*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(x^2 + 2/
3)^(1/2)*9265i)/(2108304*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(x^2 + 2/
3)^(1/2)*13i)/(21296*((6^(1/2)*x*2i)/3 - x^2 + 2/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(5/2),x)
```

```
[Out] Timed out
```

3.136 $\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=420

$$-\frac{\left(a+cx^2\right)^p (g+hx)^{m+1}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}\,h}{\sqrt{c}}+g}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)\left(afh^2(m+1\right.}{ch^3(m+1)(m+2p+3)}$$

```
[Out] f*(h*x+g)^(1+m)*(c*x^2+a)^(1+p)/c/h/(3+m+2*p)-(a*f*h^2*(1+m)-c*(2*f*g^2*(1+
p)-h*(-d*h+e*g)*(3+m+2*p)))*(h*x+g)^(1+m)*(c*x^2+a)^p*AppellF1(1+m,-p,-p,2+
m,(h*x+g)/(g-h*(-a)^(1/2)/c^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))/c/h^3/
(1+m)/(3+m+2*p)/((1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)))^p)/((1+(-h*x-g)/(g+h
*(-a)^(1/2)/c^(1/2)))^p)-(2*f*g*(1+p)-e*h*(3+m+2*p))*(h*x+g)^(2+m)*(c*x^2+a
)^p*AppellF1(2+m,-p,-p,3+m,(h*x+g)/(g-h*(-a)^(1/2)/c^(1/2)),(h*x+g)/(g+h*(-
a)^(1/2)/c^(1/2)))/h^3/(2+m)/(3+m+2*p)/((1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)
))^p)/((1+(-h*x-g)/(g+h*(-a)^(1/2)/c^(1/2)))^p)
```

Rubi [A] time = 0.60, antiderivative size = 417, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1654, 844, 760, 133}

$$-\frac{\left(a+cx^2\right)^p (g+hx)^{m+1}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}\,h}{\sqrt{c}}+g}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)\left(afh^2(m+1\right.}{ch^3(m+1)(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*h*(3 + m + 2*p)) - ((a*f*h^2*(
1 + m) - 2*c*f*g^2*(1 + p) + c*h*(e*g - d*h)*(3 + m + 2*p))*(g + h*x)^(1 +
m)*(a + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (g + h*x)/(g - (Sqrt[-a]*h)
/Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(c*h^3*(1 + m)*(3 + m + 2
*p)*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1 - (g + h*x)/(g + (Sqrt[
-a]*h)/Sqrt[c]))^p) - ((2*f*g*(1 + p) - e*h*(3 + m + 2*p))*(g + h*x)^(2 + m
)*(a + c*x^2)^p*AppellF1[2 + m, -p, -p, 3 + m, (g + h*x)/(g - (Sqrt[-a]*h)/
Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(h^3*(2 + m)*(3 + m + 2*p)
*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1 - (g + h*x)/(g + (Sqrt[-a]
*h)/Sqrt[c]))^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) dx = \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\int (g+hx)^m \left(-h^2(af(1+m)-cd(3+m \right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\left(eh-\frac{2fg(1+p)}{3+m+2p}\right)\int (g+hx)^{1+m}\left(a+cx^2\right)^{p}}{h^2}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\left(\left(eh-\frac{2fg(1+p)}{3+m+2p}\right)\left(a+cx^2\right)^p\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right.\right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} - \frac{\left(afh^2(1+m)-2cfg^2(1+p)+ch(eg-\right.}{}$$

Mathematica [F] time = 1.15, size = 0, normalized size = 0.00

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2), x]
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx^2+ex+d\right)\left(cx^2+a\right)^p\left(hx+g\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
[Out] integral((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx^2+ex+d\right)\left(cx^2+a\right)^p\left(hx+g\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(f\,x^{2}+e x +d \right) \left(c\,x^{2}+a \right)^{p} \left(h x +g \right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x)
[Out] int((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(f x^{2} + e x + d\right)} {\left(c x^{2} + a\right)}^{p} {\left(h x + g\right)}^{m}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g+h\,x\right)}^m\,{\left(c\,x^2+a\right)}^p\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**m*(c*x**2+a)**p*(f*x**2+e*x+d),x)
[Out] Timed out
```

3.137 $\int (g+hx)^m \sqrt{a+cx^2}\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=403

$$-\frac{\sqrt{a+cx^2}(g+hx)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)\left(afh^2(m+1)-c\left(3fg^2-h(m+4)(eg-dh)\right)\right)}{ch^3(m+1)(m+4)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}}}$$

```
[Out] f*(h*x+g)^(1+m)*(c*x^2+a)^(3/2)/c/h/(4+m)-(a*f*h^2*(1+m)-c*(3*f*g^2-h*(-d*h
+e*g)*(4+m)))*(h*x+g)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,(h*x+g)/(g-h*(-a)^(1
/2)/c^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))*(c*x^2+a)^(1/2)/c/h^3/(1+m)/
(4+m)/(1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)))^(1/2)/(1+(-h*x-g)/(g+h*(-a)^(1/
2)/c^(1/2)))^(1/2)-(3*f*g-e*h*(4+m))*(h*x+g)^(2+m)*AppellF1(2+m,-1/2,-1/2,3
+m,(h*x+g)/(g-h*(-a)^(1/2)/c^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))*(c*x^
2+a)^(1/2)/h^3/(2+m)/(4+m)/(1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)))^(1/2)/(1+(
-h*x-g)/(g+h*(-a)^(1/2)/c^(1/2)))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1654, 844, 760, 133}

$$\frac{\sqrt{a+cx^2}(g+hx)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)\left(-afh^2(m+1)-ch(m+4)(eg-dh)+3cfg^2\right)}{ch^3(m+1)(m+4)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + c*x^2)^(3/2))/(c*h*(4 + m)) + ((3*c*f*g^2 - a*f*h
^2*(1 + m) - c*h*(e*g - d*h)*(4 + m))*(g + h*x)^(1 + m)*Sqrt[a + c*x^2]*App
ellF1[1 + m, -1/2, -1/2, 2 + m, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g +
h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])]])/(c*h^3*(1 + m)*(4 + m)*Sqrt[1 - (g + h*x)
/(g - (Sqrt[-a]*h)/Sqrt[c])]*Sqrt[1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])]
) - ((3*f*g - e*h*(4 + m))*(g + h*x)^(2 + m)*Sqrt[a + c*x^2]*AppellF1[2 + m
, -1/2, -1/2, 3 + m, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g + h*x)/(g + (
Sqrt[-a]*h)/Sqrt[c])]])/(h^3*(2 + m)*(4 + m)*Sqrt[1 - (g + h*x)/(g - (Sqrt[-
a]*h)/Sqrt[c])]*Sqrt[1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\int (g+hx)^m\left(-h^2(af(1+m)-cd(4+\ldots\right.}{ch^2(\ldots}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)} - \frac{(3fg-eh(4+m))\int (g+hx)^{1+m}\sqrt{a+\ldots}}{h^2(4+m)}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)} - \frac{\left((3fg-eh(4+m))\sqrt{a+cx^2}\right)\text{Subst}\Big(\ldots}{h^3(4+m)\sqrt{1\ldots}}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\left(3cfg^2-afh^2(1+m)-ch(eg-dh)(4\ldots\right.}{ch^3(1\ldots}$$

Mathematica [F] time = 0.69, size = 0, normalized size = 0.00

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2), x]
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^2+a}\left(fx^2+ex+d\right)\left(hx+g\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2+a}\left(fx^2+ex+d\right)\left(hx+g\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \left(f\,x^2+ex+d\right)\sqrt{c\,x^2+a}\,\left(hx+g\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
[Out] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2+a}\left(fx^2+ex+d\right)\left(hx+g\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g+h\,x\right)^m\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^m*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^m*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+cx^2}\,\left(g+hx\right)^m\left(d+ex+fx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**m*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)*(g + h*x)**m*(d + e*x + f*x**2), x)
```

3.138 $\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=474

$$-\frac{f\left(a+cx^2\right)^p(g+hx)^{-2p}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)}{2h^3p}-\frac{\left(a+cx^2\right)^{p+}}{}$$

```
[Out] -1/2*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1+p)/h/(a*h^2+c*g^2)/(1+p)/((h*x+g)^(2+
2*p))-1/2*f*(c*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(h*x+g)/(g-h*(-a)^(1/2)/c
^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))/h^3/p/((h*x+g)^(2*p))/((1+(-h*x-g
)/(g-h*(-a)^(1/2)/c^(1/2)))^p)/((1+(-h*x-g)/(g+h*(-a)^(1/2)/c^(1/2)))^p)+(a
*h^2*(-e*h+2*f*g)+c*(-d*g*h^2+f*g^3))*(h*x+g)^(-1-2*p)*(c*x^2+a)^p*hypergeo
m([-p, -1-2*p],[-2*p],2*(h*x+g)*(-a)^(1/2)*c^(1/2)/(-h*(-a)^(1/2)+g*c^(1/2)
)/((-a)^(1/2)-x*c^(1/2)))*((-a)^(1/2)-x*c^(1/2))/h^2/(a*h^2+c*g^2)/(1+2*p)/
(h*(-a)^(1/2)+g*c^(1/2))/((-(h*(-a)^(1/2)+g*c^(1/2))*((-a)^(1/2)+x*c^(1/2))
/(-h*(-a)^(1/2)+g*c^(1/2))/((-a)^(1/2)-x*c^(1/2)))^p)
```

Rubi [A] time = 0.52, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1656, 760, 133, 807, 727}

$$-\frac{f\left(a+cx^2\right)^p(g+hx)^{-2p}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)}{2h^3p}+\frac{\left(\sqrt{-a}-\sqrt{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] -((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(1 + p))/(2*h*(c*g^2 + a*h^2)*(1 + p)
*(g + h*x)^(2*(1 + p))) - (f*(a + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p,
(g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])]
)/(2*h^3*p*(g + h*x)^(2*p)*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1
- (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c]))^p) + ((a*h^2*(2*f*g - e*h) + c*(f*g
^3 - d*g*h^2))*(Sqrt[-a] - Sqrt[c]*x)*(g + h*x)^(-1 - 2*p)*(a + c*x^2)^p*Hy
pergeometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[c]*(g + h*x))/((Sqrt[c
]*g - Sqrt[-a]*h)*(Sqrt[-a] - Sqrt[c]*x))])/(h^2*(Sqrt[c]*g + Sqrt[-a]*h)*(
c*g^2 + a*h^2)*(1 + 2*p)*(-(((Sqrt[c]*g + Sqrt[-a]*h)*(Sqrt[-a] + Sqrt[c]*x
))/((Sqrt[c]*g - Sqrt[-a]*h)*(Sqrt[-a] - Sqrt[c]*x))))^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 727

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((Rt[-(a*c), 2] - c*x)*(d + e*x)^(m + 1)*(a + c*x^2)^p*Hypergeometric2F1[m
+ 1, -p, m + 2, (2*c*Rt[-(a*c), 2]*(d + e*x))/((c*d - e*Rt[-(a*c), 2])*(Rt[
-(a*c), 2] - c*x))])/((m + 1)*(c*d + e*Rt[-(a*c), 2])*(((c*d + e*Rt[-(a*c),
 2])*(Rt[-(a*c), 2] + c*x))/((c*d - e*Rt[-(a*c), 2])*(-Rt[-(a*c), 2] + c*x)
))^p), x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Int
egerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1656

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> With[{q = Expon[Pq, x]}, Dist[Coeff[Pq, x, q]/e^q, Int[(d + e*x)^(m + q)
*(a + c*x^2)^p, x], x] + Dist[1/e^q, Int[(d + e*x)^m*(a + c*x^2)^p*ExpandTo
Sum[e^q*Pq - Coeff[Pq, x, q]*(d + e*x)^q, x], x], x]] /; FreeQ[{a, c, d, e,
 m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(IGtQ[m, 0] && Rat
ionalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)dx=\frac{\int (g+hx)^{-3-2p}\left(-fg^2+dh^2-h(2fg-eh)x\right)\left(a+cx^2\right)^p\,dx}{h^2}+\frac{f\int$$

$$=-\frac{\left(fg^2-egh+dh^2\right)(g+hx)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{2h\left(cg^2+ah^2\right)(1+p)}-\frac{\left(ah^2(2fg-eh\right.}{}$$

$$=-\frac{\left(fg^2-egh+dh^2\right)(g+hx)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{2h\left(cg^2+ah^2\right)(1+p)}-\frac{f(g+hx)^{-2p}\left(\right.}{}$$

Mathematica [F] time = 2.85, size = 0, normalized size = 0.00

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2), x]
```

fricas [F] time = 1.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx^2+ex+d\right)\left(cx^2+a\right)^p\left(hx+g\right)^{-2p-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] integral((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx^2 + ex + d\right)\left(cx^2 + a\right)^p\left(hx + g\right)^{-2\,p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(f\,x^{2}+ex+d\right)\left(c\,x^{2}+a\right)^{p}\left(hx+g\right)^{-2p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x)
[Out] int((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx^2 + ex + d\right)\left(cx^2 + a\right)^p\left(hx + g\right)^{-2\,p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^p\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^{2\,p+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3),x)
[Out] int(((a + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**(-3-2*p)*(c*x**2+a)**p*(f*x**2+e*x+d),x)
[Out] Timed out
```

3.139 $\int (d+ex)^m \left(-cd^2 + bde + be^2x + ce^2x^2\right)^p \left((-cd + be)f +\right.$

Optimal. Leaf size=222

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)} - \frac{(d+ex)^m(-be+cd-cex)^2\left(-d(cd-be)+be^2x+ce^2x^2\right)^p\left(\frac{c(d+ex)}{2cd-be}\right)}{}$$

[Out] g*(e*x+d)^(-1+m)*(-d*(-b*e+c*d)+b*e^2*x+c*e^2*x^2)^(2+p)/c/e^2/(3+m+2*p)-(b*e*g*(1+m+p)+c*(d*g*(1-m)-e*f*(3+m+2*p)))*(e*x+d)^m*(c*(e*x+d)/(-b*e+2*c*d))^(-m-p)*(-c*e*x-b*e+c*d)^2*(-d*(-b*e+c*d)+b*e^2*x+c*e^2*x^2)^p*hypergeom([-m-p, 2+p],[3+p],(-c*e*x-b*e+c*d)/(-b*e+2*c*d))/c^2/e^2/(2+p)/(3+m+2*p)

Rubi [A] time = 0.41, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 70, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1632, 794, 679, 677, 70, 69}

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)} - \frac{(d+ex)^m(-be+cd-cex)^2\left(-d(cd-be)+be^2x+ce^2x^2\right)^p\left(\frac{c(d+ex)}{2cd-be}\right)}{}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^m*(-(c*d^2) + b*d*e + b*e^2*x + c*e^2*x^2)^p*(-((c*d - b*e)*f) + (c*e*f - c*d*g + b*e*g)*x + c*e*g*x^2),x]

[Out] (g*(d + e*x)^(-1 + m)*(-(d*(c*d - b*e)) + b*e^2*x + c*e^2*x^2)^(2 + p))/(c*e^2*(3 + m + 2*p)) - ((b*e*g*(1 + m + p) + c*(d*g*(1 - m) - e*f*(3 + m + 2*p)))*(d + e*x)^m*((c*(d + e*x))/(2*c*d - b*e))^(-m - p)*(c*d - b*e - c*e*x)^2*(-(d*(c*d - b*e)) + b*e^2*x + c*e^2*x^2)^p*Hypergeometric2F1[-m - p, 2 + p, 3 + p, (c*d - b*e - c*e*x)/(2*c*d - b*e)])/(c^2*e^2*(2 + p)*(3 + m + 2*p))

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c - a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^m*(a + b*x + c*x^2)^FracPart[p])/((1 + (e*x)/d)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) && !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m
], Int[(1 + (e*x)/d)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !Intege
rQ[p] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 1632

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e
 + c*d*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rubi steps

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left(-(cd-be)f+(cef-cdg+beg)x+cegx^2\right) dx = (de)\int (d+ex)^{-1+m}$$

$$= \frac{g(d+ex)^{-1+m}\left(-d\right.}{ce^2}$$

$$= \frac{g(d+ex)^{-1+m}\left(-d\right.}{ce^2}$$

$$= \frac{g(d+ex)^{-1+m}\left(-d\right.}{ce^2}$$

$$= \frac{g(d+ex)^{-1+m}\left(-d\right.}{ce^2}$$

$$= \frac{g(d+ex)^{-1+m}\left(-d\right.}{ce^2}$$

Mathematica [A] time = 0.32, size = 165, normalized size = 0.74

$$\frac{(d+ex)^m(be-cd+cex)^2(-((d+ex)(c(d-ex)-be)))^p\left(\frac{e\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-p}(-beg(m+p+1)+cdg(m-1)+cef(m+2p+3))\,{}_2F_1\left(-m-p,p+\right.}{p+2}\right.}{c^2e^3(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(-(c*d^2) + b*d*e + b*e^2*x + c*e^2*x^2)^p*((-(c*d) +
 b*e)*f + (c*e*f - c*d*g + b*e*g)*x + c*e*g*x^2),x]
```

```
[Out] ((d + e*x)^m*(-(c*d) + b*e + c*e*x)^2*(-((d + e*x)*(-(b*e) + c*(d - e*x))))
^p*(c*e*g*(d + e*x) + (e*(c*d*g*(-1 + m) - b*e*g*(1 + m + p) + c*e*f*(3 + m
 + 2*p))*((c*(d + e*x))/(2*c*d - b*e))^(-m - p)*Hypergeometric2F1[-m - p, 2
 + p, 3 + p, (-(c*d) + b*e + c*e*x)/(-2*c*d + b*e)])/(2 + p)))/(c^2*e^3*(3
+ m + 2*p))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cegx^2-(cd-be)f+\left(cef-(cd-be)g\right)x\right)\left(ce^2x^2+be^2x-cd^2+bde\right)^p(ex+d)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g
-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="fricas")
```

```
[Out] integral((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - (c*d - b*e)*g)*x)*(c*e^2*x^2
 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cegx^2-(cd-be)f+\left(cef-cdg+beg\right)x\right)\left(ce^2x^2+be^2x-cd^2+bde\right)^p(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g
-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="giac")
```

```
[Out] integrate((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - c*d*g + b*e*g)*x)*(c*e^2*x^
2 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int\left(ceg\,x^2-(-be+cd)\,f+\left(beg-cdg+cef\right)x\right)(ex+d)^m\left(c\,e^2x^2+b\,e^2x+bde-c\,d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g-c*d*g
+c*e*f)*x+c*e*g*x^2),x)
```

```
[Out] int((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g-c*d*g
+c*e*f)*x+c*e*g*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cegx^2-(cd-be)f+\left(cef-cdg+beg\right)x\right)\left(ce^2x^2+be^2x-cd^2+bde\right)^p(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g
-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="maxima")
```

```
[Out] integrate((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - c*d*g + b*e*g)*x)*(c*e^2*x^
2 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d+e\,x)^m\,\left(c\,e\,g\,x^2+\left(b\,e\,g-c\,d\,g+c\,e\,f\right)\,x+f\,\left(b\,e-c\,d\right)\right)\,\left(-c\,d^2+b\,d\,e+c\,e^2\,x^2+b\,e^2\,x\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(f*(b*e - c*d) + x*(b*e*g - c*d*g + c*e*f) + c*e*g*x^2)*(c*
e^2*x^2 - c*d^2 + b*d*e + b*e^2*x)^p,x)
```

```
[Out] int((d + e*x)^m*(f*(b*e - c*d) + x*(b*e*g - c*d*g + c*e*f) + c*e*g*x^2)*(c*
e^2*x^2 - c*d^2 + b*d*e + b*e^2*x)^p, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*e**2*x**2+b*e**2*x+b*d*e-c*d**2)**p*(-(-b*e+c*d)*f+
(b*e*g-c*d*g+c*e*f)*x+c*e*g*x**2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.140 $\int \left(a+bx+cx^2\right)^4 \left(A+Cx^2\right) dx$

Optimal. Leaf size=254

$$a^4Ax+2a^3Abx^2+abx^4\left(a^2C+A\left(3ac+b^2\right)\right)+\frac{1}{3}a^2x^3\left(a^2C+4aAc+6Ab^2\right)+\frac{1}{7}x^7\left(C\left(6a^2c^2+12ab^2c+b^4\right)+2Ac^2\right.$$

```
[Out] a^4*A*x+2*a^3*A*b*x^2+1/3*a^2*(4*A*a*c+6*A*b^2+C*a^2)*x^3+a*b*(A*(3*a*c+b^2
)+a^2*C)*x^4+1/5*(A*(6*a^2*c^2+12*a*b^2*c+b^4)+2*a^2*(2*a*c+3*b^2)*C)*x^5+2
/3*b*(3*a*c+b^2)*(A*c+C*a)*x^6+1/7*(2*A*c^2*(2*a*c+3*b^2)+(6*a^2*c^2+12*a*b
^2*c+b^4)*C)*x^7+1/2*b*c*(A*c^2+(3*a*c+b^2)*C)*x^8+1/9*c^2*(A*c^2+4*C*a*c+6
*C*b^2)*x^9+2/5*b*c^3*C*x^10+1/11*c^4*C*x^11
```

Rubi [A] time = 0.33, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1657}

$$\frac{1}{7}x^7\left(C\left(6a^2c^2+12ab^2c+b^4\right)+2Ac^2\left(2ac+3b^2\right)\right)+\frac{1}{5}x^5\left(A\left(6a^2c^2+12ab^2c+b^4\right)+2a^2C\left(2ac+3b^2\right)\right)+abx^4\left(a^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4*(A + C*x^2),x]
```

```
[Out] a^4*A*x + 2*a^3*A*b*x^2 + (a^2*(6*A*b^2 + 4*a*A*c + a^2*C)*x^3)/3 + a*b*(A*
(b^2 + 3*a*c) + a^2*C)*x^4 + ((A*(b^4 + 12*a*b^2*c + 6*a^2*c^2) + 2*a^2*(3*
b^2 + 2*a*c)*C)*x^5)/5 + (2*b*(b^2 + 3*a*c)*(A*c + a*C)*x^6)/3 + ((2*A*c^2*
(3*b^2 + 2*a*c) + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*C)*x^7)/7 + (b*c*(A*c^2 +
(b^2 + 3*a*c)*C)*x^8)/2 + (c^2*(A*c^2 + 6*b^2*C + 4*a*c*C)*x^9)/9 + (2*b*c^
3*C*x^10)/5 + (c^4*C*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)^4\left(A+Cx^2\right)dx &= \int \left(a^4A+4a^3Abx+a^2\left(6Ab^2+4aAc+a^2C\right)x^2+4ab\left(A\left(b^2+3ac\right)+a^2C\right)\right.\\ &= a^4Ax+2a^3Abx^2+\frac{1}{3}a^2\left(6Ab^2+4aAc+a^2C\right)x^3+ab\left(A\left(b^2+3ac\right)+a^2C\right)x\end{aligned}$$

Mathematica [A] time = 0.09, size = 256, normalized size = 1.01

$$a^4Ax+2a^3Abx^2+abx^4\left(a^2C+3aAc+Ab^2\right)+\frac{1}{3}a^2x^3\left(a^2C+4aAc+6Ab^2\right)+\frac{1}{7}x^7\left(6a^2c^2C+4aAc^3+12ab^2cC+6A\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4*(A + C*x^2),x]
```

```
[Out] a^4*A*x + 2*a^3*A*b*x^2 + (a^2*(6*A*b^2 + 4*a*A*c + a^2*C)*x^3)/3 + a*b*(A*
b^2 + 3*a*A*c + a^2*C)*x^4 + ((A*b^4 + 12*a*A*b^2*c + 6*a^2*A*c^2 + 6*a^2*b
^2*C + 4*a^3*c*C)*x^5)/5 + (2*b*(b^2 + 3*a*c)*(A*c + a*C)*x^6)/3 + ((6*A*b^
2*c^2 + 4*a*A*c^3 + b^4*C + 12*a*b^2*c*C + 6*a^2*c^2*C)*x^7)/7 + (b*c*(A*c^
2 + b^2*C + 3*a*c*C)*x^8)/2 + (c^2*(A*c^2 + 6*b^2*C + 4*a*c*C)*x^9)/9 + (2*
b*c^3*C*x^10)/5 + (c^4*C*x^11)/11
```

fricas [A] time = 0.71, size = 308, normalized size = 1.21

$$\frac{1}{11}x^{11}c^4C+\frac{2}{5}x^{10}c^3bC+\frac{2}{3}x^9c^2b^2C+\frac{4}{9}x^9c^3aC+\frac{1}{9}x^9c^4A+\frac{1}{2}x^8cb^3C+\frac{3}{2}x^8c^2baC+\frac{1}{2}x^8c^3bA+\frac{1}{7}x^7b^4C+\frac{12}{7}x^7cb^2aC+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^4*C + 2/5*x^10*c^3*b*C + 2/3*x^9*c^2*b^2*C + 4/9*x^9*c^3*a*C +
1/9*x^9*c^4*A + 1/2*x^8*c*b^3*C + 3/2*x^8*c^2*b*a*C + 1/2*x^8*c^3*b*A + 1/7
*x^7*b^4*C + 12/7*x^7*c*b^2*a*C + 6/7*x^7*c^2*a^2*C + 6/7*x^7*c^2*b^2*A + 4
/7*x^7*c^3*a*A + 2/3*x^6*b^3*a*C + 2*x^6*c*b*a^2*C + 2/3*x^6*c*b^3*A + 2*x^
6*c^2*b*a*A + 6/5*x^5*b^2*a^2*C + 4/5*x^5*c*a^3*C + 1/5*x^5*b^4*A + 12/5*x^
5*c*b^2*a*A + 6/5*x^5*c^2*a^2*A + x^4*b*a^3*C + x^4*b^3*a*A + 3*x^4*c*b*a^2
*A + 1/3*x^3*a^4*C + 2*x^3*b^2*a^2*A + 4/3*x^3*c*a^3*A + 2*x^2*b*a^3*A + x*
a^4*A
```

giac [A] time = 0.15, size = 308, normalized size = 1.21

$$\frac{1}{11}\,Cc^4x^{11}+\frac{2}{5}\,Cbc^3x^{10}+\frac{2}{3}\,Cb^2c^2x^9+\frac{4}{9}\,Cac^3x^9+\frac{1}{9}\,Ac^4x^9+\frac{1}{2}\,Cb^3cx^8+\frac{3}{2}\,Cabc^2x^8+\frac{1}{2}\,Abc^3x^8+\frac{1}{7}\,Cb^4x^7+\frac{12}{7}\,Cab^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/11*C*c^4*x^11 + 2/5*C*b*c^3*x^10 + 2/3*C*b^2*c^2*x^9 + 4/9*C*a*c^3*x^9 +
1/9*A*c^4*x^9 + 1/2*C*b^3*c*x^8 + 3/2*C*a*b*c^2*x^8 + 1/2*A*b*c^3*x^8 + 1/7
*C*b^4*x^7 + 12/7*C*a*b^2*c*x^7 + 6/7*C*a^2*c^2*x^7 + 6/7*A*b^2*c^2*x^7 + 4
/7*A*a*c^3*x^7 + 2/3*C*a*b^3*x^6 + 2*C*a^2*b*c*x^6 + 2/3*A*b^3*c*x^6 + 2*A*
a*b*c^2*x^6 + 6/5*C*a^2*b^2*x^5 + 1/5*A*b^4*x^5 + 4/5*C*a^3*c*x^5 + 12/5*A*
a*b^2*c*x^5 + 6/5*A*a^2*c^2*x^5 + C*a^3*b*x^4 + A*a*b^3*x^4 + 3*A*a^2*b*c*x
^4 + 1/3*C*a^4*x^3 + 2*A*a^2*b^2*x^3 + 4/3*A*a^3*c*x^3 + 2*A*a^3*b*x^2 + A*
a^4*x
```

maple [A] time = 0.00, size = 343, normalized size = 1.35

$$\frac{C\,c^4x^{11}}{11}+\frac{2Cb\,c^3x^{10}}{5}+\frac{\left(A\,c^4+\left(4b^2c^2+2\left(2ac+b^2\right)c^2\right)C\right)x^9}{9}+\frac{\left(4Ab\,c^3+\left(4ab\,c^2+4\left(2ac+b^2\right)bc\right)C\right)x^8}{8}+2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4*(C*x^2+A),x)
```

```
[Out] 1/11*c^4*C*x^11+2/5*b*c^3*C*x^10+1/9*((2*(2*a*c+b^2)*c^2+4*b^2*c^2)*C+c^4*A
)*x^9+1/8*((4*a*b*c^2+4*(2*a*c+b^2)*b*c)*C+4*b*c^3*A)*x^8+1/7*((2*a^2*c^2+8
*a*b^2*c+(2*a*c+b^2)^2)*C+(2*(2*a*c+b^2)*c^2+4*b^2*c^2)*A)*x^7+1/6*((4*a^2*
b*c+4*a*b*(2*a*c+b^2))*C+(4*a*b*c^2+4*(2*a*c+b^2)*b*c)*A)*x^6+1/5*((2*a^2*(
2*a*c+b^2)+4*a^2*b^2)*C+(2*a^2*c^2+8*a*b^2*c+(2*a*c+b^2)^2)*A)*x^5+1/4*(4*a
^3*b*C+(4*a^2*b*c+4*a*b*(2*a*c+b^2))*A)*x^4+1/3*(a^4*C+(2*a^2*(2*a*c+b^2)+4
*a^2*b^2)*A)*x^3+2*a^3*A*b*x^2+a^4*A*x
```

maxima [A] time = 0.44, size = 263, normalized size = 1.04

$$\frac{1}{11}\,Cc^4x^{11}+\frac{2}{5}\,Cbc^3x^{10}+\frac{1}{9}\left(6\,Cb^2c^2+4\,Cac^3+Ac^4\right)x^9+\frac{1}{2}\left(Cb^3c+3\,Cabc^2+Abc^3\right)x^8+\frac{1}{7}\left(Cb^4+12\,Cab^2c+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/11*C*c^4*x^11 + 2/5*C*b*c^3*x^10 + 1/9*(6*C*b^2*c^2 + 4*C*a*c^3 + A*c^4)*
x^9 + 1/2*(C*b^3*c + 3*C*a*b*c^2 + A*b*c^3)*x^8 + 1/7*(C*b^4 + 12*C*a*b^2*c
 + 4*A*a*c^3 + 6*(C*a^2 + A*b^2)*c^2)*x^7 + 2*A*a^3*b*x^2 + 2/3*(C*a*b^3 +
3*A*a*b*c^2 + (3*C*a^2*b + A*b^3)*c)*x^6 + A*a^4*x + 1/5*(6*C*a^2*b^2 + A*b
^4 + 6*A*a^2*c^2 + 4*(C*a^3 + 3*A*a*b^2)*c)*x^5 + (C*a^3*b + A*a*b^3 + 3*A*
a^2*b*c)*x^4 + 1/3*(C*a^4 + 6*A*a^2*b^2 + 4*A*a^3*c)*x^3
```

mupad [B] time = 0.13, size = 244, normalized size = 0.96

$$x^5 \left(\frac{4\,C\,a^3\,c}{5}+\frac{6\,C\,a^2\,b^2}{5}+\frac{6\,A\,a^2\,c^2}{5}+\frac{12\,A\,a\,b^2\,c}{5}+\frac{A\,b^4}{5}\right)+x^7 \left(\frac{6\,C\,a^2\,c^2}{7}+\frac{12\,C\,a\,b^2\,c}{7}+\frac{4\,A\,a\,c^3}{7}+\frac{C\,b^4}{7}+\frac{6\,A\,b}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^4,x)
```

```
[Out] x^5*((A*b^4)/5 + (6*A*a^2*c^2)/5 + (6*C*a^2*b^2)/5 + (4*C*a^3*c)/5 + (12*A*
a*b^2*c)/5) + x^7*((C*b^4)/7 + (6*A*b^2*c^2)/7 + (6*C*a^2*c^2)/7 + (4*A*a*c
^3)/7 + (12*C*a*b^2*c)/7) + x^3*((C*a^4)/3 + 2*A*a^2*b^2 + (4*A*a^3*c)/3) +
 x^9*((A*c^4)/9 + (2*C*b^2*c^2)/3 + (4*C*a*c^3)/9) + (C*c^4*x^11)/11 + A*a^
4*x + (2*b*x^6*(3*a*c + b^2)*(A*c + C*a))/3 + a*b*x^4*(A*b^2 + C*a^2 + 3*A*
a*c) + (b*c*x^8*(A*c^2 + C*b^2 + 3*C*a*c))/2 + 2*A*a^3*b*x^2 + (2*C*b*c^3*x
^10)/5
```

sympy [A] time = 0.14, size = 320, normalized size = 1.26

$$Aa^4x+2Aa^3bx^2+\frac{2Cbc^3x^{10}}{5}+\frac{Cc^4x^{11}}{11}+x^9\left(\frac{Ac^4}{9}+\frac{4Cac^3}{9}+\frac{2Cb^2c^2}{3}\right)+x^8\left(\frac{Abc^3}{2}+\frac{3Cabc^2}{2}+\frac{Cb^3c}{2}\right)+x^7\left(\frac{4Aac^3}{7}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4*(C*x**2+A),x)
```

```
[Out] A*a**4*x + 2*A*a**3*b*x**2 + 2*C*b*c**3*x**10/5 + C*c**4*x**11/11 + x**9*(A
*c**4/9 + 4*C*a*c**3/9 + 2*C*b**2*c**2/3) + x**8*(A*b*c**3/2 + 3*C*a*b*c**2
/2 + C*b**3*c/2) + x**7*(4*A*a*c**3/7 + 6*A*b**2*c**2/7 + 6*C*a**2*c**2/7 +
 12*C*a*b**2*c/7 + C*b**4/7) + x**6*(2*A*a*b*c**2 + 2*A*b**3*c/3 + 2*C*a**2
*b*c + 2*C*a*b**3/3) + x**5*(6*A*a**2*c**2/5 + 12*A*a*b**2*c/5 + A*b**4/5 +
 4*C*a**3*c/5 + 6*C*a**2*b**2/5) + x**4*(3*A*a**2*b*c + A*a*b**3 + C*a**3*b
) + x**3*(4*A*a**3*c/3 + 2*A*a**2*b**2 + C*a**4/3)
```

3.141 $\int \left(a+bx+cx^2\right)^3 \left(A+Cx^2\right) dx$

Optimal. Leaf size=161

$$a^3Ax+\frac{1}{4}bx^4\left(3a^2C+A\left(6ac+b^2\right)\right)+\frac{1}{3}ax^3\left(a^2C+3A\left(ac+b^2\right)\right)+\frac{3}{2}a^2Abx^2+\frac{1}{7}cx^7\left(3C\left(ac+b^2\right)+Ac^2\right)+\frac{1}{6}bx^6$$

```
[Out] a^3*A*x+3/2*a^2*A*b*x^2+1/3*a*(3*A*(a*c+b^2)+a^2*C)*x^3+1/4*b*(A*(6*a*c+b^2
)+3*a^2*C)*x^4+3/5*(a*c+b^2)*(A*c+C*a)*x^5+1/6*b*(3*A*c^2+(6*a*c+b^2)*C)*x^
6+1/7*c*(A*c^2+3*(a*c+b^2)*C)*x^7+3/8*b*c^2*C*x^8+1/9*c^3*C*x^9
```

Rubi [A] time = 0.19, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1657}

$$\frac{1}{4}bx^4\left(3a^2C+A\left(6ac+b^2\right)\right)+\frac{1}{3}ax^3\left(a^2C+3A\left(ac+b^2\right)\right)+\frac{3}{2}a^2Abx^2+a^3Ax+\frac{1}{7}cx^7\left(3C\left(ac+b^2\right)+Ac^2\right)+\frac{1}{6}bx^6$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3*(A + C*x^2),x]
```

```
[Out] a^3*A*x + (3*a^2*A*b*x^2)/2 + (a*(3*A*(b^2 + a*c) + a^2*C)*x^3)/3 + (b*(A*(
b^2 + 6*a*c) + 3*a^2*C)*x^4)/4 + (3*(b^2 + a*c)*(A*c + a*C)*x^5)/5 + (b*(3*
A*c^2 + (b^2 + 6*a*c)*C)*x^6)/6 + (c*(A*c^2 + 3*(b^2 + a*c)*C)*x^7)/7 + (3*
b*c^2*C*x^8)/8 + (c^3*C*x^9)/9
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^3\left(A+Cx^2\right)dx=\int \left(a^3A+3a^2Abx+a\left(3A\left(b^2+ac\right)+a^2C\right)x^2+b\left(A\left(b^2+6ac\right)+3a^2C\right)\right.$$
$$=a^3Ax+\frac{3}{2}a^2Abx^2+\frac{1}{3}a\left(3A\left(b^2+ac\right)+a^2C\right)x^3+\frac{1}{4}b\left(A\left(b^2+6ac\right)+3a^2C\right.$$

Mathematica [A] time = 0.05, size = 163, normalized size = 1.01

$$a^3Ax+\frac{1}{4}bx^4\left(3a^2C+6aAc+Ab^2\right)+\frac{1}{3}ax^3\left(a^2C+3aAc+3Ab^2\right)+\frac{3}{2}a^2Abx^2+\frac{1}{7}cx^7\left(3acC+Ac^2+3b^2C\right)+\frac{1}{6}bx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3*(A + C*x^2),x]
```

```
[Out] a^3*A*x + (3*a^2*A*b*x^2)/2 + (a*(3*A*b^2 + 3*a*A*c + a^2*C)*x^3)/3 + (b*(A
*b^2 + 6*a*A*c + 3*a^2*C)*x^4)/4 + (3*(b^2 + a*c)*(A*c + a*C)*x^5)/5 + (b*(
3*A*c^2 + b^2*C + 6*a*c*C)*x^6)/6 + (c*(A*c^2 + 3*b^2*C + 3*a*c*C)*x^7)/7 +
 (3*b*c^2*C*x^8)/8 + (c^3*C*x^9)/9
```

fricas [A] time = 0.65, size = 187, normalized size = 1.16

$$\frac{1}{9}x^9c^3C+\frac{3}{8}x^8c^2bC+\frac{3}{7}x^7cb^2C+\frac{3}{7}x^7c^2aC+\frac{1}{7}x^7c^3A+\frac{1}{6}x^6b^3C+x^6cbaC+\frac{1}{2}x^6c^2bA+\frac{3}{5}x^5b^2aC+\frac{3}{5}x^5ca^2C+\frac{3}{5}x^5cb^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^3*C + 3/8*x^8*c^2*b*C + 3/7*x^7*c*b^2*C + 3/7*x^7*c^2*a*C + 1/7*x
^7*c^3*A + 1/6*x^6*b^3*C + x^6*c*b*a*C + 1/2*x^6*c^2*b*A + 3/5*x^5*b^2*a*C
+ 3/5*x^5*c*a^2*C + 3/5*x^5*c*b^2*A + 3/5*x^5*c^2*a*A + 3/4*x^4*b*a^2*C + 1
/4*x^4*b^3*A + 3/2*x^4*c*b*a*A + 1/3*x^3*a^3*C + x^3*b^2*a*A + x^3*c*a^2*A
+ 3/2*x^2*b*a^2*A + x*a^3*A
```

giac [A] time = 0.17, size = 187, normalized size = 1.16

$$\frac{1}{9}Cc^3x^9+\frac{3}{8}Cbc^2x^8+\frac{3}{7}Cb^2cx^7+\frac{3}{7}Cac^2x^7+\frac{1}{7}Ac^3x^7+\frac{1}{6}Cb^3x^6+Cabcx^6+\frac{1}{2}Abc^2x^6+\frac{3}{5}Cab^2x^5+\frac{3}{5}Ca^2cx^5+\frac{3}{5}Ab^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 3/7*C*b^2*c*x^7 + 3/7*C*a*c^2*x^7 + 1/7*A
*c^3*x^7 + 1/6*C*b^3*x^6 + C*a*b*c*x^6 + 1/2*A*b*c^2*x^6 + 3/5*C*a*b^2*x^5
+ 3/5*C*a^2*c*x^5 + 3/5*A*b^2*c*x^5 + 3/5*A*a*c^2*x^5 + 3/4*C*a^2*b*x^4 + 1
/4*A*b^3*x^4 + 3/2*A*a*b*c*x^4 + 1/3*C*a^3*x^3 + A*a*b^2*x^3 + A*a^2*c*x^3
+ 3/2*A*a^2*b*x^2 + A*a^3*x
```

maple [A] time = 0.00, size = 223, normalized size = 1.39

$$\frac{Cc^3x^9}{9}+\frac{3Cbc^2x^8}{8}+\frac{\left(Ac^3+\left(ac^2+2b^2c+\left(2ac+b^2\right)c\right)C\right)x^7}{7}+\frac{3Aa^2bx^2}{2}+\frac{\left(3Abc^2+\left(4abc+\left(2ac+b^2\right)b\right)C\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3*(C*x^2+A),x)
```

```
[Out] 1/9*C*c^3*x^9+3/8*b*c^2*C*x^8+1/7*((a*c^2+2*b^2*c+c*(2*a*c+b^2))*C+c^3*A)*x
^7+1/6*((4*a*b*c+b*(2*a*c+b^2))*C+3*b*c^2*A)*x^6+1/5*((a*(2*a*c+b^2)+2*b^2*
a+c*a^2)*C+(a*c^2+2*b^2*c+c*(2*a*c+b^2))*A)*x^5+1/4*(3*a^2*b*C+(4*a*b*c+b*(
2*a*c+b^2))*A)*x^4+1/3*(a^3*C+(a*(2*a*c+b^2)+2*b^2*a+c*a^2)*A)*x^3+3/2*a^2*
A*b*x^2+A*a^3*x
```

maxima [A] time = 0.43, size = 165, normalized size = 1.02

$$\frac{1}{9}Cc^3x^9+\frac{3}{8}Cbc^2x^8+\frac{1}{7}\left(3Cb^2c+3Cac^2+Ac^3\right)x^7+\frac{1}{6}\left(Cb^3+6Cabc+3Abc^2\right)x^6+\frac{3}{2}Aa^2bx^2+\frac{3}{5}\left(Cab^2+Aac^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 1/7*(3*C*b^2*c + 3*C*a*c^2 + A*c^3)*x^7 +
 1/6*(C*b^3 + 6*C*a*b*c + 3*A*b*c^2)*x^6 + 3/2*A*a^2*b*x^2 + 3/5*(C*a*b^2 +
 A*a*c^2 + (C*a^2 + A*b^2)*c)*x^5 + A*a^3*x + 1/4*(3*C*a^2*b + A*b^3 + 6*A*
a*b*c)*x^4 + 1/3*(C*a^3 + 3*A*a*b^2 + 3*A*a^2*c)*x^3
```

mupad [B] time = 0.07, size = 149, normalized size = 0.93

$$x^3\left(\frac{C\,a^3}{3}+A\,c\,a^2+A\,a\,b^2\right)+x^7\left(\frac{3\,C\,b^2\,c}{7}+\frac{A\,c^3}{7}+\frac{3\,C\,a\,c^2}{7}\right)+\frac{b\,x^4\,\left(3\,C\,a^2+6\,A\,c\,a+A\,b^2\right)}{4}+\frac{b\,x^6\,\left(C\,b^2+3\,A\,c^2\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^3,x)
```

```
[Out] x^3*((C*a^3)/3 + A*a*b^2 + A*a^2*c) + x^7*((A*c^3)/7 + (3*C*a*c^2)/7 + (3*C
*b^2*c)/7) + (b*x^4*(A*b^2 + 3*C*a^2 + 6*A*a*c))/4 + (b*x^6*(3*A*c^2 + C*b^
```

```
2 + 6*C*a*c))/6 + (C*c^3*x^9)/9 + A*a^3*x + (3*x^5*(a*c + b^2)*(A*c + C*a))
/5 + (3*A*a^2*b*x^2)/2 + (3*C*b*c^2*x^8)/8
```

sympy [A] time = 0.11, size = 197, normalized size = 1.22

$$Aa^3x+\frac{3Aa^2bx^2}{2}+\frac{3Cbc^2x^8}{8}+\frac{Cc^3x^9}{9}+x^7\left(\frac{Ac^3}{7}+\frac{3Cac^2}{7}+\frac{3Cb^2c}{7}\right)+x^6\left(\frac{Abc^2}{2}+Cabc+\frac{Cb^3}{6}\right)+x^5\left(\frac{3Aac^2}{5}+\frac{3A}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3*(C*x**2+A),x)
```

```
[Out] A*a**3*x + 3*A*a**2*b*x**2/2 + 3*C*b*c**2*x**8/8 + C*c**3*x**9/9 + x**7*(A*
c**3/7 + 3*C*a*c**2/7 + 3*C*b**2*c/7) + x**6*(A*b*c**2/2 + C*a*b*c + C*b**3
/6) + x**5*(3*A*a*c**2/5 + 3*A*b**2*c/5 + 3*C*a**2*c/5 + 3*C*a*b**2/5) + x*
*4*(3*A*a*b*c/2 + A*b**3/4 + 3*C*a**2*b/4) + x**3*(A*a**2*c + A*a*b**2 + C*
a**3/3)
```

3.142 $\int \left(a+bx+cx^2\right)^2 \left(A+Cx^2\right) dx$

Optimal. Leaf size=96

$$\frac{1}{3}x^3\left(a^2C+A\left(2ac+b^2\right)\right)+a^2Ax+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+Ac^2\right)+\frac{1}{2}bx^4(aC+Ac)+aAbx^2+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

```
[Out] a^2*A*x+a*A*b*x^2+1/3*(A*(2*a*c+b^2)+a^2*C)*x^3+1/2*b*(A*c+C*a)*x^4+1/5*(A*
c^2+(2*a*c+b^2)*C)*x^5+1/3*b*c*C*x^6+1/7*c^2*C*x^7
```

Rubi [A] time = 0.10, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1657}

$$\frac{1}{3}x^3\left(a^2C+A\left(2ac+b^2\right)\right)+a^2Ax+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+Ac^2\right)+\frac{1}{2}bx^4(aC+Ac)+aAbx^2+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2*(A + C*x^2),x]
```

```
[Out] a^2*A*x + a*A*b*x^2 + ((A*(b^2 + 2*a*c) + a^2*C)*x^3)/3 + (b*(A*c + a*C)*x^
4)/2 + ((A*c^2 + (b^2 + 2*a*c)*C)*x^5)/5 + (b*c*C*x^6)/3 + (c^2*C*x^7)/7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)^2 \left(A+Cx^2\right) dx &= \int \left(a^2A+2aAbx+\left(A\left(b^2+2ac\right)+a^2C\right)x^2+2b(Ac+aC)x^3+\left(Ac^2+\left(b^2+\right.\right.\right.\\ &= a^2Ax+aAbx^2+\frac{1}{3}\left(A\left(b^2+2ac\right)+a^2C\right)x^3+\frac{1}{2}b(Ac+aC)x^4+\frac{1}{5}\left(Ac^2+\left(b^2\right.\right.\end{aligned}$$

Mathematica [A] time = 0.02, size = 96, normalized size = 1.00

$$\frac{1}{3}x^3\left(a^2C+2aAc+Ab^2\right)+a^2Ax+\frac{1}{5}x^5\left(2acC+Ac^2+b^2C\right)+\frac{1}{2}bx^4(aC+Ac)+aAbx^2+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2*(A + C*x^2),x]
```

```
[Out] a^2*A*x + a*A*b*x^2 + ((A*b^2 + 2*a*A*c + a^2*C)*x^3)/3 + (b*(A*c + a*C)*x^
4)/2 + ((A*c^2 + b^2*C + 2*a*c*C)*x^5)/5 + (b*c*C*x^6)/3 + (c^2*C*x^7)/7
```

fricas [A] time = 0.83, size = 99, normalized size = 1.03

$$\frac{1}{7}x^7c^2C+\frac{1}{3}x^6cbC+\frac{1}{5}x^5b^2C+\frac{2}{5}x^5caC+\frac{1}{5}x^5c^2A+\frac{1}{2}x^4baC+\frac{1}{2}x^4cbA+\frac{1}{3}x^3a^2C+\frac{1}{3}x^3b^2A+\frac{2}{3}x^3caA+x^2baA+xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c^2*C + 1/3*x^6*c*b*C + 1/5*x^5*b^2*C + 2/5*x^5*c*a*C + 1/5*x^5*c^2
*A + 1/2*x^4*b*a*C + 1/2*x^4*c*b*A + 1/3*x^3*a^2*C + 1/3*x^3*b^2*A + 2/3*x^
3*c*a*A + x^2*b*a*A + x*a^2*A
```

giac [A] time = 0.15, size = 99, normalized size = 1.03

$$\frac{1}{7}Cc^2x^7+\frac{1}{3}Cbcx^6+\frac{1}{5}Cb^2x^5+\frac{2}{5}Cacx^5+\frac{1}{5}Ac^2x^5+\frac{1}{2}Cabx^4+\frac{1}{2}Abcx^4+\frac{1}{3}Ca^2x^3+\frac{1}{3}Ab^2x^3+\frac{2}{3}Aacx^3+Aabx^2+A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/7*C*c^2*x^7 + 1/3*C*b*c*x^6 + 1/5*C*b^2*x^5 + 2/5*C*a*c*x^5 + 1/5*A*c^2*x
^5 + 1/2*C*a*b*x^4 + 1/2*A*b*c*x^4 + 1/3*C*a^2*x^3 + 1/3*A*b^2*x^3 + 2/3*A*
a*c*x^3 + A*a*b*x^2 + A*a^2*x
```

maple [A] time = 0.00, size = 90, normalized size = 0.94

$$\frac{C\,c^2x^7}{7}+\frac{Cbc\,x^6}{3}+Aab\,x^2+\frac{\left(A\,c^2+\left(2ac+b^2\right)C\right)x^5}{5}+A\,a^2x+\frac{(2bcA+2abC)\,x^4}{4}+\frac{\left(C\,a^2+\left(2ac+b^2\right)A\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2*(C*x^2+A),x)
```

```
[Out] 1/7*C*c^2*x^7+1/3*b*c*C*x^6+1/5*(A*c^2+(2*a*c+b^2)*C)*x^5+1/4*(2*A*b*c+2*C*
a*b)*x^4+1/3*(A*(2*a*c+b^2)+a^2*C)*x^3+a*A*b*x^2+A*a^2*x
```

maxima [A] time = 0.43, size = 87, normalized size = 0.91

$$\frac{1}{7}Cc^2x^7+\frac{1}{3}Cbcx^6+\frac{1}{5}\left(Cb^2+2\,Cac+Ac^2\right)x^5+Aabx^2+\frac{1}{2}\left(Cab+Abc\right)x^4+Aa^2x+\frac{1}{3}\left(Ca^2+Ab^2+2\,Aac\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/7*C*c^2*x^7 + 1/3*C*b*c*x^6 + 1/5*(C*b^2 + 2*C*a*c + A*c^2)*x^5 + A*a*b*x
^2 + 1/2*(C*a*b + A*b*c)*x^4 + A*a^2*x + 1/3*(C*a^2 + A*b^2 + 2*A*a*c)*x^3
```

mupad [B] time = 4.09, size = 88, normalized size = 0.92

$$x^3\left(\frac{C\,a^2}{3}+\frac{2\,A\,c\,a}{3}+\frac{A\,b^2}{3}\right)+x^5\left(\frac{C\,b^2}{5}+\frac{A\,c^2}{5}+\frac{2\,C\,a\,c}{5}\right)+\frac{C\,c^2\,x^7}{7}+A\,a^2\,x+\frac{b\,x^4\,\left(A\,c+C\,a\right)}{2}+A\,a\,b\,x^2+\frac{C\,b\,c\,x^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^2,x)
```

```
[Out] x^3*((A*b^2)/3 + (C*a^2)/3 + (2*A*a*c)/3) + x^5*((A*c^2)/5 + (C*b^2)/5 + (2
*C*a*c)/5) + (C*c^2*x^7)/7 + A*a^2*x + (b*x^4*(A*c + C*a))/2 + A*a*b*x^2 +
(C*b*c*x^6)/3
```

sympy [A] time = 0.09, size = 102, normalized size = 1.06

$$Aa^2x+Aabx^2+\frac{Cbcx^6}{3}+\frac{Cc^2x^7}{7}+x^5\left(\frac{Ac^2}{5}+\frac{2Cac}{5}+\frac{Cb^2}{5}\right)+x^4\left(\frac{Abc}{2}+\frac{Cab}{2}\right)+x^3\left(\frac{2Aac}{3}+\frac{Ab^2}{3}+\frac{Ca^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2*(C*x**2+A),x)
```

```
[Out] A*a**2*x + A*a*b*x**2 + C*b*c*x**6/3 + C*c**2*x**7/7 + x**5*(A*c**2/5 + 2*C
*a*c/5 + C*b**2/5) + x**4*(A*b*c/2 + C*a*b/2) + x**3*(2*A*a*c/3 + A*b**2/3
+ C*a**2/3)
```

3.143 $\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx$

Optimal. Leaf size=46

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}Abx^2+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

```
[Out] a*A*x+1/2*A*b*x^2+1/3*(A*c+C*a)*x^3+1/4*b*C*x^4+1/5*c*C*x^5
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1657}

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}Abx^2+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(A + C*x^2),x]
```

```
[Out] a*A*x + (A*b*x^2)/2 + ((A*c + a*C)*x^3)/3 + (b*C*x^4)/4 + (c*C*x^5)/5
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx &= \int \left(aA+Abx+(Ac+aC)x^2+bCx^3+cCx^4\right)\,dx\\ &= aAx+\frac{1}{2}Abx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.00

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}Abx^2+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(A + C*x^2),x]
```

```
[Out] a*A*x + (A*b*x^2)/2 + ((A*c + a*C)*x^3)/3 + (b*C*x^4)/4 + (c*C*x^5)/5
```

fricas [A] time = 0.73, size = 40, normalized size = 0.87

$$\frac{1}{5}x^5cC+\frac{1}{4}x^4bC+\frac{1}{3}x^3aC+\frac{1}{3}x^3cA+\frac{1}{2}x^2bA+xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c*C + 1/4*x^4*b*C + 1/3*x^3*a*C + 1/3*x^3*c*A + 1/2*x^2*b*A + x*a*A
```

giac [A] time = 0.18, size = 40, normalized size = 0.87

$$\frac{1}{5}Ccx^5+\frac{1}{4}Cbx^4+\frac{1}{3}Cax^3+\frac{1}{3}Acx^3+\frac{1}{2}Abx^2+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/5*C*c*x^5 + 1/4*C*b*x^4 + 1/3*C*a*x^3 + 1/3*A*c*x^3 + 1/2*A*b*x^2 + A*a*x
```

maple [A] time = 0.00, size = 39, normalized size = 0.85

$$\frac{Cc\,x^5}{5}+\frac{Cb\,x^4}{4}+\frac{Ab\,x^2}{2}+Aax+\frac{(Ac+aC)\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(C*x^2+A),x)
```

```
[Out] A*a*x+1/2*A*b*x^2+1/3*(A*c+C*a)*x^3+1/4*b*C*x^4+1/5*C*c*x^5
```

maxima [A] time = 0.43, size = 38, normalized size = 0.83

$$\frac{1}{5}\,Ccx^5+\frac{1}{4}\,Cbx^4+\frac{1}{2}\,Abx^2+\frac{1}{3}\,(Ca+Ac)x^3+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/5*C*c*x^5 + 1/4*C*b*x^4 + 1/2*A*b*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

mupad [B] time = 0.03, size = 39, normalized size = 0.85

$$\frac{C\,c\,x^5}{5}+\frac{C\,b\,x^4}{4}+\left(\frac{A\,c}{3}+\frac{C\,a}{3}\right)\,x^3+\frac{A\,b\,x^2}{2}+A\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2),x)
```

```
[Out] x^3*((A*c)/3 + (C*a)/3) + A*a*x + (A*b*x^2)/2 + (C*b*x^4)/4 + (C*c*x^5)/5
```

sympy [A] time = 0.07, size = 42, normalized size = 0.91

$$Aax+\frac{Abx^2}{2}+\frac{Cbx^4}{4}+\frac{Ccx^5}{5}+x^3\left(\frac{Ac}{3}+\frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(C*x**2+A),x)
```

```
[Out] A*a*x + A*b*x**2/2 + C*b*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 + C*a/3)
```

3.144 $\int \frac{A+Cx^2}{a+bx+cx^2}\,dx$

Optimal. Leaf size=81

$$-\frac{\left(C\left(b^2-2ac\right)+2Ac^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}+\frac{Cx}{c}$$

```
[Out] C*x/c-1/2*b*C*ln(c*x^2+b*x+a)/c^2-(2*A*c^2+(-2*a*c+b^2)*C)*arctanh((2*c*x+b
)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1657, 634, 618, 206, 628}

$$-\frac{\left(C\left(b^2-2ac\right)+2Ac^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (C*x)/c - ((2*A*c^2 + (b^2 - 2*a*c)*C)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(c^2*Sqrt[b^2 - 4*a*c]) - (b*C*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Cx^2}{a+bx+cx^2}\,dx &= \int \left(\frac{C}{c}+\frac{Ac-aC-bCx}{c\left(a+bx+cx^2\right)}\right)dx\\ &= \frac{Cx}{c}+\frac{\int \frac{Ac-aC-bCx}{a+bx+cx^2}\,dx}{c}\\ &= \frac{Cx}{c}-\frac{(bC)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{1}{2}\left(2A+\frac{\left(b^2-2ac\right)C}{c^2}\right)\int \frac{1}{a+bx+cx^2}\,dx\\ &= \frac{Cx}{c}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}+\left(-2A-\frac{\left(b^2-2ac\right)C}{c^2}\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+\right.\\ &= \frac{Cx}{c}-\frac{\left(2A+\frac{\left(b^2-2ac\right)C}{c^2}\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}\end{aligned}$$

Mathematica [A] time = 0.09, size = 84, normalized size = 1.04

$$\frac{\left(-2acC+2Ac^2+b^2C\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (C*x)/c + ((2*A*c^2 + b^2*C - 2*a*c*C)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c
]])/(c^2*Sqrt[-b^2 + 4*a*c]) - (b*C*Log[a + b*x + c*x^2])/(2*c^2)
```

fricas [A] time = 0.77, size = 265, normalized size = 3.27

$$\left[\frac{\left(Cb^2-2\,Cac+2\,Ac^2\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+2\left(Cb^2c-4\,Cac^2\right)x-\left(Cb^3-4\,Cab\right.}{2\left(b^2c^2-4\,ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*((C*b^2 - 2*C*a*c + 2*A*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(C*
b^2*c - 4*C*a*c^2)*x - (C*b^3 - 4*C*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 -
 4*a*c^3), -1/2*(2*(C*b^2 - 2*C*a*c + 2*A*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(C*b^2*c - 4*C*a*c^2)*x +
(C*b^3 - 4*C*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 0.15, size = 78, normalized size = 0.96

$$\frac{Cx}{c}-\frac{Cb\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{\left(Cb^2-2\,Cac+2\,Ac^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] C*x/c - 1/2*C*b*log(c*x^2 + b*x + a)/c^2 + (C*b^2 - 2*C*a*c + 2*A*c^2)*arct
an((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [A] time = 0.01, size = 140, normalized size = 1.73

$$\frac{2A\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{2Ca\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c}+\frac{C\,b^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^2}-\frac{Cb\ln\left(c\,x^2+bx+a\right)}{2c^2}+\frac{Cx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a),x)
```

```
[Out] C/c*x-1/2*b*C*ln(c*x^2+b*x+a)/c^2+2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))*A-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*
a*C+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*C
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.19, size = 224, normalized size = 2.77

$$\frac{2\,A\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}+\frac{C\,x}{c}+\frac{C\,b^3\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)}-\frac{2\,C\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}}+\frac{C\,b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,}}\right.}{c^2\,\sqrt{4\,a\,}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2),x)
```

```
[Out] (2*A*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^
2)^(1/2) + (C*x)/c + (C*b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) -
 (2*C*a*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*
c - b^2)^(1/2)) + (C*b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)
^(1/2)))/(c^2*(4*a*c - b^2)^(1/2)) - (2*C*a*b*c*log(a + b*x + c*x^2))/(4*a*
c^3 - b^2*c^2)
```

sympy [B] time = 1.21, size = 413, normalized size = 5.10

$$\frac{Cx}{c}+\left(-\frac{Cb}{2c^2}-\frac{\sqrt{-4ac+b^2}\left(-2Ac^2+2Cac-Cb^2\right)}{2c^2\left(4ac-b^2\right)}\right)\log\left(x+\frac{-Abc-Cab-4ac^2\left(-\frac{Cb}{2c^2}-\frac{\sqrt{-4ac+b^2}\left(-2Ac^2+2Cac-Cb^2\right)}{2c^2\left(4ac-b^2\right)}\right.}{-2Ac^2+2Cac\,-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a),x)
```

```
[Out] C*x/c + (-C*b/(2*c**2) - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)
/(2*c**2*(4*a*c - b**2)))*log(x + (-A*b*c - C*a*b - 4*a*c**2*(-C*b/(2*c**2)
 - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c - b**2
))) + b**2*c*(-C*b/(2*c**2) - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*
b**2)/(2*c**2*(4*a*c - b**2))))/(-2*A*c**2 + 2*C*a*c - C*b**2)) + (-C*b/(2*
c**2) + sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c -
 b**2)))*log(x + (-A*b*c - C*a*b - 4*a*c**2*(-C*b/(2*c**2) + sqrt(-4*a*c +
b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c - b**2))) + b**2*c*(-C*
b/(2*c**2) + sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*
a*c - b**2))))/(-2*A*c**2 + 2*C*a*c - C*b**2))
```

3.145 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=100

$$\frac{4(aC+Ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] (-b*c*(A+a*C/c)-(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*
(A*c+C*a)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1660, 12, 618, 206}

$$\frac{4(aC+Ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] -((b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(c*(b^2 - 4*a*c)*(a +
 b*x + c*x^2))) + (4*(A*c + a*C)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b
^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{2(Ac+aC)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2(Ac+aC))\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{(4(Ac+aC))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2c\right.}{b^2-4ac}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4(Ac+aC)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.08, size = 98, normalized size = 0.98

$$\frac{aC(b-2cx)+Ac(b+2cx)+b^2Cx}{c\left(4ac-b^2\right)(a+x(b+cx))}+\frac{4(aC+Ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b +
 c*x))) + (4*(A*c + a*C)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*
a*c)^(3/2)
```

fricas [B] time = 1.26, size = 511, normalized size = 5.11

$$\left[-\frac{Cab^3-4\,Aabc^2+2\left(Ca^2c+Aac^2+\left(Cac^2+Ac^3\right)x^2+\left(Cabc+Abc^2\right)x\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-}}{cx^2+bx+a}\right.}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-(C*a*b^3 - 4*A*a*b*c^2 + 2*(C*a^2*c + A*a*c^2 + (C*a*c^2 + A*c^3)*x^2 + (
C*a*b*c + A*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*
a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (4*C*a^2*b - A*b^
3)*c + (C*b^4 - 6*C*a*b^2*c - 8*A*a*c^3 + 2*(4*C*a^2 + A*b^2)*c^2)*x)/(a*b^
4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2
 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), -(C*a*b^3 - 4*A*a*b*c^2 - 4*(C*
a^2*c + A*a*c^2 + (C*a*c^2 + A*c^3)*x^2 + (C*a*b*c + A*b*c^2)*x)*sqrt(-b^2
+ 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (4*C*a^2*b
 - A*b^3)*c + (C*b^4 - 6*C*a*b^2*c - 8*A*a*c^3 + 2*(4*C*a^2 + A*b^2)*c^2)*x
)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x)]
```

giac [A] time = 0.16, size = 108, normalized size = 1.08

$$-\frac{4\,(Ca+Ac)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{Cb^2x-2\,Cacx+2\,Ac^2x+Cab+Abc}{\left(b^2c-4\,ac^2\right)\left(cx^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4*(C*a + A*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-
b^2 + 4*a*c)) - (C*b^2*x - 2*C*a*c*x + 2*A*c^2*x + C*a*b + A*b*c)/((b^2*c -
 4*a*c^2)*(c*x^2 + b*x + a))
```

maple [A] time = 0.01, size = 146, normalized size = 1.46

$$\frac{4Ac\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{4Ca\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{\frac{(Ac+aC)b}{\left(4ac-b^2\right)c}+\frac{\left(2Ac^2-2Cac+Cb^2\right)x}{\left(4ac-b^2\right)c}}{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^2,x)
```

```
[Out] ((2*A*c^2-2*C*a*c+C*b^2)/c/(4*a*c-b^2)*x+b/c*(A*c+C*a)/(4*a*c-b^2))/(c*x^2+
b*x+a)+4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*c+4/(4*a*c
-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*C
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.53, size = 172, normalized size = 1.72

$$\frac{\frac{A\,b\,c+C\,a\,b}{c\,\left(4\,a\,c-b^2\right)}+\frac{x\,\left(C\,b^2+2\,A\,c^2-2\,C\,a\,c\right)}{c\,\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a}-\frac{4\,\mathrm{atan}\left(\frac{\left(\frac{2\,\left(A\,c+C\,a\right)\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{4\,c\,x\,\left(A\,c+C\,a\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,A\,c+2\,C\,a}\right)\,\left(A\,c+C\,a\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^2,x)
```

```
[Out] ((A*b*c + C*a*b)/(c*(4*a*c - b^2)) + (x*(2*A*c^2 + C*b^2 - 2*C*a*c))/(c*(4*
a*c - b^2)))/(a + b*x + c*x^2) - (4*atan((((2*(A*c + C*a)*(b^3 - 4*a*b*c))/
(4*a*c - b^2)^(5/2) - (4*c*x*(A*c + C*a))/(4*a*c - b^2)^(3/2))*(4*a*c - b^2
))/(2*A*c + 2*C*a))*(A*c + C*a))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 1.21, size = 376, normalized size = 3.76

$$-2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(Ac+Ca)\log\left(x+\frac{2Abc+2Cab-32a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(Ac+Ca)+16ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(A}{4Ac^2+4Cac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**2,x)
```

```
[Out] -2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a)*log(x + (2*A*b*c + 2*C*a*b - 32*a
**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) + 16*a*b**2*c*sqrt(-1/(4*a*
c - b**2)**3)*(A*c + C*a) - 2*b**4*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a))/
(4*A*c**2 + 4*C*a*c)) + 2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a)*log(x + (2
*A*b*c + 2*C*a*b + 32*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) - 16
*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) + 2*b**4*sqrt(-1/(4*a*c -
b**2)**3)*(A*c + C*a))/(4*A*c**2 + 4*C*a*c)) + (A*b*c + C*a*b + x*(2*A*c**2
 - 2*C*a*c + C*b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2)
 + x*(4*a*b*c**2 - b**3*c))
```

3.146 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=161

$$-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{2\left(C\left(2ac+b^2\right)+6Ac^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{(b+2cx)\left(2aC+6Ac+\right.}{2\left(b^2-4ac\right)^2\left(a+bx+\right.}$$

```
[Out] 1/2*(-b*c*(A+a*C/c)-(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^2+1/2*(6*A*c+2*a*C+b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)-2*(6*A
*c^2+(2*a*c+b^2)*C)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2
)
```

Rubi [A] time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1660, 12, 614, 618, 206}

$$-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{2\left(C\left(2ac+b^2\right)+6Ac^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{(b+2cx)\left(2aC+6Ac+\right.}{2\left(b^2-4ac\right)^2\left(a+bx+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(2*c*(b^2 - 4*a*c)*(a
+ b*x + c*x^2)^2) + ((6*A*c + 2*a*C + (b^2*C)/c)*(b + 2*c*x))/(2*(b^2 - 4*a
*c)^2*(a + b*x + c*x^2)) - (2*(6*A*c^2 + (b^2 + 2*a*c)*C)*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\int \frac{6Ac+2aC+\frac{b^2C}{c}}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{\left(6Ac^2+\left(b^2\right.\right.}{}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{\left(2\left(6Ac^2+\left(\right.\right.\right.}{}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{2\left(6Ac^2+\left(\right.\right.}{}$$

Mathematica [A] time = 0.20, size = 160, normalized size = 0.99

$$\frac{1}{2}\left(\frac{(b+2cx)\left(C\left(2ac+b^2\right)+6Ac^2\right)}{c\left(b^2-4ac\right)^2(a+x(b+cx))}+\frac{4\left(C\left(2ac+b^2\right)+6Ac^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}+\frac{aC(b-2cx)+Ac(b+2cx)+b}{c\left(4ac-b^2\right)(a+x(b+cx))^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] (((6*A*c^2 + (b^2 + 2*a*c)*C)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c
*x))) + (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c)*(a
+ x*(b + c*x))^2) + (4*(6*A*c^2 + (b^2 + 2*a*c)*C)*ArcTan[(b + 2*c*x)/Sqrt[
-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

fricas [B] time = 0.89, size = 1199, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(6*C*a^2*b^3 - A*b^5 - 40*A*a^2*b*c^2 + 2*(C*b^4*c - 2*C*a*b^2*c^2 - 2
4*A*a*c^4 - 2*(4*C*a^2 - 3*A*b^2)*c^3)*x^3 + 3*(C*b^5 - 2*C*a*b^3*c - 24*A*
a*b*c^3 - 2*(4*C*a^2*b - 3*A*b^3)*c^2)*x^2 + 2*(C*a^2*b^2 + 2*C*a^3*c + 6*A
*a^2*c^2 + (C*b^2*c^2 + 2*C*a*c^3 + 6*A*c^4)*x^4 + 2*(C*b^3*c + 2*C*a*b*c^2
 + 6*A*b*c^3)*x^3 + (C*b^4 + 4*C*a*b^2*c + 12*A*a*c^3 + 2*(2*C*a^2 + 3*A*b^
```

```
2)*c^2)*x^2 + 2*(C*a*b^3 + 2*C*a^2*b*c + 6*A*a*b*c^2)*x)*sqrt(b^2 - 4*a*c)*
log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) - 2*(12*C*a^3*b - 7*A*a*b^3)*c + 2*(5*C*a*b^4 - 40*A*a^2*c^
3 + 2*(4*C*a^3 + A*a*b^2)*c^2 - 2*(11*C*a^2*b^2 - A*b^4)*c)*x)/(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128
*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)
, 1/2*(6*C*a^2*b^3 - A*b^5 - 40*A*a^2*b*c^2 + 2*(C*b^4*c - 2*C*a*b^2*c^2 -
24*A*a*c^4 - 2*(4*C*a^2 - 3*A*b^2)*c^3)*x^3 + 3*(C*b^5 - 2*C*a*b^3*c - 24*A
*a*b*c^3 - 2*(4*C*a^2*b - 3*A*b^3)*c^2)*x^2 - 4*(C*a^2*b^2 + 2*C*a^3*c + 6*
A*a^2*c^2 + (C*b^2*c^2 + 2*C*a*c^3 + 6*A*c^4)*x^4 + 2*(C*b^3*c + 2*C*a*b*c^
2 + 6*A*b*c^3)*x^3 + (C*b^4 + 4*C*a*b^2*c + 12*A*a*c^3 + 2*(2*C*a^2 + 3*A*b
^2)*c^2)*x^2 + 2*(C*a*b^3 + 2*C*a^2*b*c + 6*A*a*b*c^2)*x)*sqrt(-b^2 + 4*a*c
)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(12*C*a^3*b - 7
*A*a*b^3)*c + 2*(5*C*a*b^4 - 40*A*a^2*c^3 + 2*(4*C*a^3 + A*a*b^2)*c^2 - 2*(
11*C*a^2*b^2 - A*b^4)*c)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

giac [A] time = 0.18, size = 217, normalized size = 1.35

$$\frac{2\left(Cb^2+2\,Cac+6\,Ac^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{2\,Cb^2cx^3+4\,Cac^2x^3+12\,Ac^3x^3+3\,Cb^3x^2+6\,Cabcx^2+18\,Abc}{2\left(b^4-8\,ab^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2*(C*b^2 + 2*C*a*c + 6*A*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4
- 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*C*b^2*c*x^3 + 4*C*a*
c^2*x^3 + 12*A*c^3*x^3 + 3*C*b^3*x^2 + 6*C*a*b*c*x^2 + 18*A*b*c^2*x^2 + 10*
C*a*b^2*x - 4*C*a^2*c*x + 4*A*b^2*c*x + 20*A*a*c^2*x + 6*C*a^2*b - A*b^3 +
10*A*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

maple [B] time = 0.01, size = 373, normalized size = 2.32

$$\frac{12A\,c^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}+\frac{4Cac\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}+\frac{2C\,b^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}+\frac{(6A}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^3,x)
```

```
[Out] (c*(6*A*c^2+2*C*a*c+C*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(6*A*c^2+2*
C*a*c+C*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+(10*A*a*c^2+2*A*b^2*c-2*C*a^2*c
+5*C*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/2*b*(10*A*a*c-A*b^2+6*C*a^2)/(16
*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+12/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*c^2+4/(16*a^2*c^2-8*a*b
^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*C*a*c+2/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))*C*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.17, size = 401, normalized size = 2.49

$$\frac{\frac{6\,C\,a^2\,b+10\,A\,c\,a\,b-A\,b^3}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x\,\left(-2\,C\,a^2\,c+5\,C\,a\,b^2+10\,A\,a\,c^2+2\,A\,b^2\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{3\,b\,x^2\,\left(C\,b^2+6\,A\,c^2+2\,C\,a\,c\right)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{c\,x^3\,\left(C\,b^2+6\,A\,c^2+2\,C\,a\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}+\frac{2\,\mathrm{atan}\left(\left(\left(16\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^3,x)
```

```
[Out] ((6*C*a^2*b - A*b^3 + 10*A*a*b*c)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(
10*A*a*c^2 + 2*A*b^2*c + 5*C*a*b^2 - 2*C*a^2*c))/(b^4 + 16*a^2*c^2 - 8*a*b^
2*c) + (3*b*x^2*(6*A*c^2 + C*b^2 + 2*C*a*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2
*c)) + (c*x^3*(6*A*c^2 + C*b^2 + 2*C*a*c))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/
(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (2*atan(((((b^5
 + 16*a^2*b*c^2 - 8*a*b^3*c)*(6*A*c^2 + C*b^2 + 2*C*a*c))/((4*a*c - b^2)^(5
/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (2*c*x*(6*A*c^2 + C*b^2 + 2*C*a*c))/(
4*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*A*c^2 + C*b^2 + 2*C*
a*c))*(6*A*c^2 + C*b^2 + 2*C*a*c))/(4*a*c - b^2)^(5/2)
```

sympy [B] time = 2.36, size = 774, normalized size = 4.81

$$-\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(6Ac^2+2Cac+Cb^2\right)\log\left(x+\frac{6Abc^2+2Cabc+Cb^3-64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(6Ac^2+2Cac+Cb\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**3,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2)*log(x + (6*A*b*c*
*2 + 2*C*a*b*c + C*b**3 - 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2
 + 2*C*a*c + C*b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c*
*2 + 2*C*a*c + C*b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 +
 2*C*a*c + C*b**2) + b**6*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c +
C*b**2))/(12*A*c**3 + 4*C*a*c**2 + 2*C*b**2*c)) + sqrt(-1/(4*a*c - b**2)**5
)*(6*A*c**2 + 2*C*a*c + C*b**2)*log(x + (6*A*b*c**2 + 2*C*a*b*c + C*b**3 +
64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2) - 48*
a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2) + 1
2*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2) - b**6*
sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2))/(12*A*c**3 + 4*C*
a*c**2 + 2*C*b**2*c)) + (10*A*a*b*c - A*b**3 + 6*C*a**2*b + x**3*(12*A*c**3
 + 4*C*a*c**2 + 2*C*b**2*c) + x**2*(18*A*b*c**2 + 6*C*a*b*c + 3*C*b**3) + x
*(20*A*a*c**2 + 4*A*b**2*c - 4*C*a**2*c + 10*C*a*b**2))/(32*a**4*c**2 - 16*
a**3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c*
*2) + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**
3 - 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

3.147 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^4}\,dx$

Optimal. Leaf size=206

$$-\frac{2(b+2cx)\left(C\left(ac+b^2\right)+5Ac^2\right)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{8c\left(C\left(ac+b^2\right)+5Ac^2\right)\tanh^{-1}\left(\right.}{\left(b^2-4ac\right)^{7/2}}$$

```
[Out] 1/3*(-b*c*(A+a*C/c)-(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^3+1/3*(5*A*c+(a+1/c*b^2)*C)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^2-2*(5
*A*c^2+(a*c+b^2)*C)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b*x+a)+8*c*(5*A*c^2+(a*
c+b^2)*C)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(7/2)
```

Rubi [A] time = 0.19, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1660, 12, 614, 618, 206}

$$-\frac{2(b+2cx)\left(C\left(ac+b^2\right)+5Ac^2\right)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{8c\left(C\left(ac+b^2\right)+5Ac^2\right)\tanh^{-1}\left(\right.}{\left(b^2-4ac\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^4,x]
```

```
[Out] -(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(3*c*(b^2 - 4*a*c)*(a
+ b*x + c*x^2)^3) + ((5*A*c + (a + b^2/c)*C)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^
2*(a + b*x + c*x^2)^2) - (2*(5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/((b^2 -
4*a*c)^3*(a + b*x + c*x^2)) + (8*c*(5*A*c^2 + (b^2 + a*c)*C)*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(7/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^4}\,dx = -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{\int \frac{2\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)}{\left(a+bx+cx^2\right)^3}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{\left(2\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)\right)\int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} + \frac{\left(2\left(5Ac^2+\left(\right.\right.\right.}{}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{2\left(5Ac^2+\left(b\right.\right.}{\left(b^2-4ac\right.}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{2\left(5Ac^2+\left(b\right.\right.}{\left(b^2-4ac\right.}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{2\left(5Ac^2+\left(b\right.\right.}{\left(b^2-4ac\right.}$$

Mathematica [A] time = 0.41, size = 204, normalized size = 0.99

$$\frac{1}{3}\left(-\frac{6(b+2cx)\left(C\left(ac+b^2\right)+5Ac^2\right)}{\left(b^2-4ac\right)^3(a+x(b+cx))} + \frac{(b+2cx)\left(C\left(ac+b^2\right)+5Ac^2\right)}{c\left(b^2-4ac\right)^2(a+x(b+cx))^2} + \frac{24c\left(C\left(ac+b^2\right)+5Ac^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b}}\right.}{\left(4ac-b^2\right)^{7/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^4,x]
```

```
[Out] (((5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x
))^2) - (6*(5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/((b^2 - 4*a*c)^3*(a + x*(
b + c*x))) + (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c
)*(a + x*(b + c*x))^3) + (24*c*(5*A*c^2 + (b^2 + a*c)*C)*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2))/3
```

fricas [B] time = 0.97, size = 2103, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/3*(C*a^2*b^5 + A*b^7 - 264*A*a^3*b*c^3 + 12*(C*b^4*c^3 - 3*C*a*b^2*c^4
- 20*A*a*c^6 - (4*C*a^2 - 5*A*b^2)*c^5)*x^5 + 30*(C*b^5*c^2 - 3*C*a*b^3*c^3
 - 20*A*a*b*c^5 - (4*C*a^2*b - 5*A*b^3)*c^4)*x^4 + 2*(11*C*b^6*c - 17*C*a*b
^4*c^2 - 320*A*a^2*c^5 - 4*(16*C*a^3 + 35*A*a*b^2)*c^4 - (92*C*a^2*b^2 - 55
*A*b^4)*c^3)*x^3 - 2*(52*C*a^4*b - 59*A*a^2*b^3)*c^2 + 3*(C*b^7 + 13*C*a*b^
5*c - 320*A*a^2*b*c^4 - 4*(16*C*a^3*b - 15*A*a*b^3)*c^3 - (52*C*a^2*b^3 - 5
*A*b^5)*c^2)*x^2 + 12*(C*a^3*b^2*c + C*a^4*c^2 + 5*A*a^3*c^3 + (C*b^2*c^4 +
 C*a*c^5 + 5*A*c^6)*x^6 + 3*(C*b^3*c^3 + C*a*b*c^4 + 5*A*b*c^5)*x^5 + 3*(C*
b^4*c^2 + 2*C*a*b^2*c^3 + 5*A*a*c^5 + (C*a^2 + 5*A*b^2)*c^4)*x^4 + (C*b^5*c
 + 7*C*a*b^3*c^2 + 30*A*a*b*c^4 + (6*C*a^2*b + 5*A*b^3)*c^3)*x^3 + 3*(C*a*b
^4*c + 2*C*a^2*b^2*c^2 + 5*A*a^2*c^4 + (C*a^3 + 5*A*a*b^2)*c^3)*x^2 + 3*(C*
a^2*b^3*c + C*a^3*b*c^2 + 5*A*a^2*b*c^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^
2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a
)) + (22*C*a^3*b^3 - 17*A*a*b^5)*c + 3*(C*a*b^6 - 176*A*a^3*c^4 + 4*(4*C*a^
4 - 7*A*a^2*b^2)*c^3 - 2*(46*C*a^3*b^2 - 11*A*a*b^4)*c^2 + (18*C*a^2*b^4 -
A*b^6)*c)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3 + 2
56*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6 + 2
56*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a^3*b^3*
c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3 - 160*
a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3*b^5*c^3 - 128
0*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*c + 80*a^3*b^6
*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a^3*b^7*c + 96*
a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x), -1/3*(C*a^2*b^5 + A*b^7
- 264*A*a^3*b*c^3 + 12*(C*b^4*c^3 - 3*C*a*b^2*c^4 - 20*A*a*c^6 - (4*C*a^2 -
 5*A*b^2)*c^5)*x^5 + 30*(C*b^5*c^2 - 3*C*a*b^3*c^3 - 20*A*a*b*c^5 - (4*C*a^
2*b - 5*A*b^3)*c^4)*x^4 + 2*(11*C*b^6*c - 17*C*a*b^4*c^2 - 320*A*a^2*c^5 -
4*(16*C*a^3 + 35*A*a*b^2)*c^4 - (92*C*a^2*b^2 - 55*A*b^4)*c^3)*x^3 - 2*(52*
C*a^4*b - 59*A*a^2*b^3)*c^2 + 3*(C*b^7 + 13*C*a*b^5*c - 320*A*a^2*b*c^4 - 4
*(16*C*a^3*b - 15*A*a*b^3)*c^3 - (52*C*a^2*b^3 - 5*A*b^5)*c^2)*x^2 - 24*(C*
a^3*b^2*c + C*a^4*c^2 + 5*A*a^3*c^3 + (C*b^2*c^4 + C*a*c^5 + 5*A*c^6)*x^6 +
 3*(C*b^3*c^3 + C*a*b*c^4 + 5*A*b*c^5)*x^5 + 3*(C*b^4*c^2 + 2*C*a*b^2*c^3 +
 5*A*a*c^5 + (C*a^2 + 5*A*b^2)*c^4)*x^4 + (C*b^5*c + 7*C*a*b^3*c^2 + 30*A*a
*b*c^4 + (6*C*a^2*b + 5*A*b^3)*c^3)*x^3 + 3*(C*a*b^4*c + 2*C*a^2*b^2*c^2 +
5*A*a^2*c^4 + (C*a^3 + 5*A*a*b^2)*c^3)*x^2 + 3*(C*a^2*b^3*c + C*a^3*b*c^2 +
 5*A*a^2*b*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b
)/(b^2 - 4*a*c)) + (22*C*a^3*b^3 - 17*A*a*b^5)*c + 3*(C*a*b^6 - 176*A*a^3*c
^4 + 4*(4*C*a^4 - 7*A*a^2*b^2)*c^3 - 2*(46*C*a^3*b^2 - 11*A*a*b^4)*c^2 + (1
8*C*a^2*b^4 - A*b^6)*c)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a
^6*b^2*c^3 + 256*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a
^3*b^2*c^6 + 256*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4
- 256*a^3*b^3*c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*
b^6*c^3 - 160*a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3
*b^5*c^3 - 1280*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*
c + 80*a^3*b^6*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a
^3*b^7*c + 96*a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x)]
```

giac [B] time = 0.17, size = 407, normalized size = 1.98

$$-\frac{8\left(Cb^2c + Cac^2 + 5\,Ac^3\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^6 - 12\,ab^4c + 48\,a^2b^2c^2 - 64\,a^3c^3\right)\sqrt{-b^2+4\,ac}} - \frac{12\,Cb^2c^3x^5 + 12\,Cac^4x^5 + 60\,Ac^5x^5 + 30\,Cb^3c^2x^4 + 30\,Ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] -8*(C*b^2*c + C*a*c^2 + 5*A*c^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b
^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) - 1/3*(1
2*C*b^2*c^3*x^5 + 12*C*a*c^4*x^5 + 60*A*c^5*x^5 + 30*C*b^3*c^2*x^4 + 30*C*a
```

```
*b*c^3*x^4 + 150*A*b*c^4*x^4 + 22*C*b^4*c*x^3 + 54*C*a*b^2*c^2*x^3 + 32*C*a
^2*c^3*x^3 + 110*A*b^2*c^3*x^3 + 160*A*a*c^4*x^3 + 3*C*b^5*x^2 + 51*C*a*b^3
*c*x^2 + 48*C*a^2*b*c^2*x^2 + 15*A*b^3*c^2*x^2 + 240*A*a*b*c^3*x^2 + 3*C*a*
b^4*x + 66*C*a^2*b^2*c*x - 3*A*b^4*c*x - 12*C*a^3*c^2*x + 54*A*a*b^2*c^2*x
+ 132*A*a^2*c^3*x + C*a^2*b^3 + A*b^5 + 26*C*a^3*b*c - 13*A*a*b^3*c + 66*A*
a^2*b*c^2)/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*(c*x^2 + b*x +
 a)^3)
```

maple [B] time = 0.02, size = 643, normalized size = 3.12

$$\frac{40A\,c^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(64c^3a^3-48a^2b^2c^2+12a\,b^4c-b^6\right)\sqrt{4ac-b^2}}+\frac{8Ca\,c^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(64c^3a^3-48a^2b^2c^2+12a\,b^4c-b^6\right)\sqrt{4ac-b^2}}+\frac{8C\,b^2}{\left(64c^3a^3-48a^2b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^4,x)
```

```
[Out] (4*c^3*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^5
+10*c^2*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*b*
x^4+2/3*(16*a*c+11*b^2)*c*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+
12*a*b^4*c-b^6)*x^3+b*(16*a*c+b^2)*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2
*b^2*c^2+12*a*b^4*c-b^6)*x^2+(44*A*a^2*c^3+18*A*a*b^2*c^2-A*b^4*c-4*C*a^3*c
^2+22*C*a^2*b^2*c+C*a*b^4)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x+1/3
*(66*A*a^2*c^2-13*A*a*b^2*c+A*b^4+26*C*a^3*c+C*a^2*b^2)*b/(64*a^3*c^3-48*a^
2*b^2*c^2+12*a*b^4*c-b^6))/(c*x^2+b*x+a)^3+40*c^3/(64*a^3*c^3-48*a^2*b^2*c^
2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A+8
*c^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*C*a+8*c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^
6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*C*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.36, size = 698, normalized size = 3.39

$$-\frac{\frac{26\,C\,a^3\,b\,c+C\,a^2\,b^3+66\,A\,a^2\,b\,c^2-13\,A\,a\,b^3\,c+A\,b^5}{3\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}+\frac{x\,\left(-4\,C\,a^3\,c^2+22\,C\,a^2\,b^2\,c+44\,A\,a^2\,c^3+C\,a\,b^4+18\,A\,a\,b^2\,c^2-A\,b^4\,c\right)}{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}+\frac{2\,x^3\,\left(11\,b^2\,c+16\,a\,c^2\right)\left(C\right.}{3\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c\right.}}{x^2\,\left(3\,c\,a^2+3\,a\,b^2\right)+x^4\,\left(3\,b^2\,c+3\,a\,c^2\right)+a^3+x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^4,x)
```

```
[Out] - ((A*b^5 + C*a^2*b^3 - 13*A*a*b^3*c + 26*C*a^3*b*c + 66*A*a^2*b*c^2)/(3*(b
^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(44*A*a^2*c^3 - 4*C*a^
3*c^2 - A*b^4*c + C*a*b^4 + 18*A*a*b^2*c^2 + 22*C*a^2*b^2*c))/(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (2*x^3*(16*a*c^2 + 11*b^2*c)*(5*A*c^2
 + C*b^2 + C*a*c))/(3*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (
```

```
x^2*(b^3 + 16*a*b*c)*(5*A*c^2 + C*b^2 + C*a*c))/(b^6 - 64*a^3*c^3 + 48*a^2*
b^2*c^2 - 12*a*b^4*c) + (4*c^3*x^5*(5*A*c^2 + C*b^2 + C*a*c))/(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (10*b*c^2*x^4*(5*A*c^2 + C*b^2 + C*a*
c))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(x^2*(3*a*b^2 + 3*a^2
*c) + x^4*(3*a*c^2 + 3*b^2*c) + a^3 + x^3*(b^3 + 6*a*b*c) + c^3*x^6 + 3*b*c
^2*x^5 + 3*a^2*b*x) - (8*c*atan((((8*c^2*x*(5*A*c^2 + C*b^2 + C*a*c))/(4*a*
c - b^2)^(7/2) + (4*c*(5*A*c^2 + C*b^2 + C*a*c)*(b^7 - 64*a^3*b*c^3 + 48*a^
2*b^3*c^2 - 12*a*b^5*c))/((4*a*c - b^2)^(7/2)*(b^6 - 64*a^3*c^3 + 48*a^2*b^
2*c^2 - 12*a*b^4*c)))*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(20
*A*c^3 + 4*C*a*c^2 + 4*C*b^2*c))*(5*A*c^2 + C*b^2 + C*a*c))/(4*a*c - b^2)^(
7/2)
```

sympy [B] time = 4.22, size = 1224, normalized size = 5.94

$$-4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^7}}\left(5Ac^2+Cac+Cb^2\right)\log\left(x+\frac{20Abc^3+4Cabc^2+4Cb^3c-1024a^4c^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^7}}\left(5Ac^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**4,x)
```

```
[Out] -4*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2)*log(x + (20*A*b
*c**3 + 4*C*a*b*c**2 + 4*C*b**3*c - 1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)**
7)*(5*A*c**2 + C*a*c + C*b**2) + 1024*a**3*b**2*c**4*sqrt(-1/(4*a*c - b**2)
**7)*(5*A*c**2 + C*a*c + C*b**2) - 384*a**2*b**4*c**3*sqrt(-1/(4*a*c - b**2
)**7)*(5*A*c**2 + C*a*c + C*b**2) + 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2)**
7)*(5*A*c**2 + C*a*c + C*b**2) - 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c
**2 + C*a*c + C*b**2))/(40*A*c**4 + 8*C*a*c**3 + 8*C*b**2*c**2)) + 4*c*sqrt
(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2)*log(x + (20*A*b*c**3 + 4
*C*a*b*c**2 + 4*C*b**3*c + 1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c
**2 + C*a*c + C*b**2) - 1024*a**3*b**2*c**4*sqrt(-1/(4*a*c - b**2)**7)*(5*A
*c**2 + C*a*c + C*b**2) + 384*a**2*b**4*c**3*sqrt(-1/(4*a*c - b**2)**7)*(5*
A*c**2 + C*a*c + C*b**2) - 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c
**2 + C*a*c + C*b**2) + 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a
*c + C*b**2))/(40*A*c**4 + 8*C*a*c**3 + 8*C*b**2*c**2)) + (66*A*a**2*b*c**2
 - 13*A*a*b**3*c + A*b**5 + 26*C*a**3*b*c + C*a**2*b**3 + x**5*(60*A*c**5 +
 12*C*a*c**4 + 12*C*b**2*c**3) + x**4*(150*A*b*c**4 + 30*C*a*b*c**3 + 30*C*
b**3*c**2) + x**3*(160*A*a*c**4 + 110*A*b**2*c**3 + 32*C*a**2*c**3 + 54*C*a
*b**2*c**2 + 22*C*b**4*c) + x**2*(240*A*a*b*c**3 + 15*A*b**3*c**2 + 48*C*a*
*2*b*c**2 + 51*C*a*b**3*c + 3*C*b**5) + x*(132*A*a**2*c**3 + 54*A*a*b**2*c*
*2 - 3*A*b**4*c - 12*C*a**3*c**2 + 66*C*a**2*b**2*c + 3*C*a*b**4))/(192*a**
6*c**3 - 144*a**5*b**2*c**2 + 36*a**4*b**4*c - 3*a**3*b**6 + x**6*(192*a**3
*c**6 - 144*a**2*b**2*c**5 + 36*a*b**4*c**4 - 3*b**6*c**3) + x**5*(576*a**3
*b*c**5 - 432*a**2*b**3*c**4 + 108*a*b**5*c**3 - 9*b**7*c**2) + x**4*(576*a
**4*c**5 + 144*a**3*b**2*c**4 - 324*a**2*b**4*c**3 + 99*a*b**6*c**2 - 9*b**
8*c) + x**3*(1152*a**4*b*c**4 - 672*a**3*b**3*c**3 + 72*a**2*b**5*c**2 + 18
*a*b**7*c - 3*b**9) + x**2*(576*a**5*c**4 + 144*a**4*b**2*c**3 - 324*a**3*b
**4*c**2 + 99*a**2*b**6*c - 9*a*b**8) + x*(576*a**5*b*c**3 - 432*a**4*b**3*
c**2 + 108*a**3*b**5*c - 9*a**2*b**7))
```

3.148 $\int \frac{(d+ex)^3\left(f+gx+hx^2\right)}{a+bx+cx^2} dx$

Optimal. Leaf size=591

$$\frac{\log\left(a+bx+cx^2\right)\left(c^2e\left(a^2e^2h+2abe(3dh+eg)+b^2\left(3d^2h+3deg+e^2f\right)\right)-b^2ce^2(3aeh+3bdh+beg)-c^3\left(ae\left(3\right.\right.\right.}{2c^5}$$

```
[Out] -(b^3*e^3*h-c^3*d*(d^2*h+3*d*e*g+3*e^2*f)-b*c*e^2*(2*a*e*h+3*b*d*h+b*e*g)+c
^2*e*(a*e*(3*d*h+e*g)+b*(3*d^2*h+3*d*e*g+e^2*f)))*x/c^4+1/2*e*(b^2*e^2*h+c^
2*(3*d^2*h+3*d*e*g+e^2*f)-c*e*(a*e*h+3*b*d*h+b*e*g))*x^2/c^3+1/3*e^2*(-b*e*
h+3*c*d*h+c*e*g)*x^3/c^2+1/4*e^3*h*x^4/c+1/2*(c^4*d^2*(d*g+3*e*f)+b^4*e^3*h
-b^2*c*e^2*(3*a*e*h+3*b*d*h+b*e*g)+c^2*e*(a^2*e^2*h+2*a*b*e*(3*d*h+e*g)+b^2
*(3*d^2*h+3*d*e*g+e^2*f))-c^3*(b*d*(d^2*h+3*d*e*g+3*e^2*f)+a*e*(3*d^2*h+3*d
*e*g+e^2*f)))*ln(c*x^2+b*x+a)/c^5-(2*c^5*d^3*f-b^5*e^3*h+b^3*c*e^2*(5*a*e*h
+3*b*d*h+b*e*g)-c^4*d*(b*d*(d*g+3*e*f)+2*a*(d^2*h+3*d*e*g+3*e^2*f))-b*c^2*e
*(5*a^2*e^2*h+4*a*b*e*(3*d*h+e*g)+b^2*(3*d^2*h+3*d*e*g+e^2*f))+c^3*(2*a^2*e
^2*(3*d*h+e*g)+b^2*d*(d^2*h+3*d*e*g+3*e^2*f)+3*a*b*e*(3*d^2*h+3*d*e*g+e^2*f
)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 1.43, antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1628, 634, 618, 206, 628}

$$\frac{\log\left(a+bx+cx^2\right)\left(c^2e\left(a^2e^2h+2abe(3dh+eg)+b^2\left(3d^2h+3deg+e^2f\right)\right)-b^2ce^2(3aeh+3bdh+beg)-c^3\left(ae\left(3\right.\right.\right.}{2c^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] -(((b^3*e^3*h - c^3*d*(3*e^2*f + 3*d*e*g + d^2*h) - b*c*e^2*(b*e*g + 3*b*d*
h + 2*a*e*h) + c^2*e*(a*e*(e*g + 3*d*h) + b*(e^2*f + 3*d*e*g + 3*d^2*h)))*x
)/c^4) + (e*(b^2*e^2*h + c^2*(e^2*f + 3*d*e*g + 3*d^2*h) - c*e*(b*e*g + 3*b
*d*h + a*e*h))*x^2)/(2*c^3) + (e^2*(c*e*g + 3*c*d*h - b*e*h)*x^3)/(3*c^2) +
 (e^3*h*x^4)/(4*c) - ((2*c^5*d^3*f - b^5*e^3*h + b^3*c*e^2*(b*e*g + 3*b*d*h
 + 5*a*e*h) - c^4*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f + 3*d*e*g + d^2*h)) -
 b*c^2*e*(5*a^2*e^2*h + 4*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f + 3*d*e*g + 3*d^
2*h)) + c^3*(2*a^2*e^2*(e*g + 3*d*h) + b^2*d*(3*e^2*f + 3*d*e*g + d^2*h) +
3*a*b*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
])/(c^5*Sqrt[b^2 - 4*a*c]) + ((c^4*d^2*(3*e*f + d*g) + b^4*e^3*h - b^2*c*e^
2*(b*e*g + 3*b*d*h + 3*a*e*h) + c^2*e*(a^2*e^2*h + 2*a*b*e*(e*g + 3*d*h) +
b^2*(e^2*f + 3*d*e*g + 3*d^2*h)) - c^3*(b*d*(3*e^2*f + 3*d*e*g + d^2*h) + a
*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*Log[a + b*x + c*x^2])/(2*c^5)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = \int\left(-\frac{b^3e^3h-c^3d\left(3e^2f+3deg+d^2h\right)-bce^2(beg+3bdh+2aeh)+c^2e\left(ae(e\right.}{c^4}\right.$$

$$= -\frac{\left(b^3e^3h-c^3d\left(3e^2f+3deg+d^2h\right)-bce^2(beg+3bdh+2aeh)+c^2e\left(ae(eg+\right.\right.}{c^4}$$

$$= -\frac{\left(b^3e^3h-c^3d\left(3e^2f+3deg+d^2h\right)-bce^2(beg+3bdh+2aeh)+c^2e\left(ae(eg+\right.\right.}{c^4}$$

$$= -\frac{\left(b^3e^3h-c^3d\left(3e^2f+3deg+d^2h\right)-bce^2(beg+3bdh+2aeh)+c^2e\left(ae(eg+\right.\right.}{c^4}$$

$$= -\frac{\left(b^3e^3h-c^3d\left(3e^2f+3deg+d^2h\right)-bce^2(beg+3bdh+2aeh)+c^2e\left(ae(eg+\right.\right.}{c^4}$$

Mathematica [A] time = 0.65, size = 585, normalized size = 0.99

$$\frac{6\log(a+x(b+cx))\left(c^2e\left(a^2e^2h+2abe(3dh+eg)+b^2\left(3d^2h+3deg+e^2f\right)\right)-b^2ce^2(3aeh+3bdh+beg)-c^3\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] (12*c*(-(b^3*e^3*h) + c^3*d*(3*e^2*f + 3*d*e*g + d^2*h) + b*c*e^2*(b*e*g +
3*b*d*h + 2*a*e*h) - c^2*e*(a*e*(e*g + 3*d*h) + b*(e^2*f + 3*d*e*g + 3*d^2*
h)))*x + 6*c^2*e*(b^2*e^2*h + c^2*(e^2*f + 3*d*e*g + 3*d^2*h) - c*e*(b*e*g
+ 3*b*d*h + a*e*h))*x^2 + 4*c^3*e^2*(c*e*g + 3*c*d*h - b*e*h)*x^3 + 3*c^4*e
^3*h*x^4 + (12*(2*c^5*d^3*f - b^5*e^3*h + b^3*c*e^2*(b*e*g + 3*b*d*h + 5*a*
e*h) - c^4*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f + 3*d*e*g + d^2*h)) - b*c^2*
e*(5*a^2*e^2*h + 4*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f + 3*d*e*g + 3*d^2*h)) +
 c^3*(2*a^2*e^2*(e*g + 3*d*h) + b^2*d*(3*e^2*f + 3*d*e*g + d^2*h) + 3*a*b*e
*(e^2*f + 3*d*e*g + 3*d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] + 6*(c^4*d^2*(3*e*f + d*g) + b^4*e^3*h - b^2*c*e^2*(b*e*g +
3*b*d*h + 3*a*e*h) + c^2*e*(a^2*e^2*h + 2*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f
+ 3*d*e*g + 3*d^2*h)) - c^3*(b*d*(3*e^2*f + 3*d*e*g + d^2*h) + a*e*(e^2*f +
 3*d*e*g + 3*d^2*h)))*Log[a + x*(b + c*x)])/(12*c^5)
```

fricas [A] time = 2.91, size = 2150, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*c^4 - 4*a*c^5)*e^3*h*x^4 + 4*((b^2*c^4 - 4*a*c^5)*e^3*g + (3*
(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*h)*x^3 + 6*((b^2*c^4
 - 4*a*c^5)*e^3*f + (3*(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^
3)*g + (3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*
c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*h)*x^2 - 6*sqrt(b^2 - 4*a*c)*((2*c^5*d^
3 - 3*b*c^4*d^2*e + 3*(b^2*c^3 - 2*a*c^4)*d*e^2 - (b^3*c^2 - 3*a*b*c^3)*e^3
)*f - (b*c^4*d^3 - 3*(b^2*c^3 - 2*a*c^4)*d^2*e + 3*(b^3*c^2 - 3*a*b*c^3)*d*
e^2 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^3)*g + ((b^2*c^3 - 2*a*c^4)*d^3 -
 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2
- (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 12*((3*(b^2*c^4
 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*f + (3*(b^2*c^4 - 4*a*c^5)*d
^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*
e^3)*g + ((b^2*c^4 - 4*a*c^5)*d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3*(b^4*
c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*e^2 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*
e^3)*h)*x + 6*((3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2
 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*f + ((b^2*c^4 - 4*a*c^5)*d^3 -
3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*e^2
 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^3)*g - ((b^3*c^3 - 4*a*b*c^4)*d^3
- 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e + 3*(b^5*c - 6*a*b^3*c^2 + 8*
a^2*b*c^3)*d*e^2 - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^3)*h)*l
og(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6), 1/12*(3*(b^2*c^4 - 4*a*c^5)*e^3*h
*x^4 + 4*((b^2*c^4 - 4*a*c^5)*e^3*g + (3*(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c
^3 - 4*a*b*c^4)*e^3)*h)*x^3 + 6*((b^2*c^4 - 4*a*c^5)*e^3*f + (3*(b^2*c^4 -
4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*g + (3*(b^2*c^4 - 4*a*c^5)*d^2*
e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3
)*h)*x^2 - 12*sqrt(-b^2 + 4*a*c)*((2*c^5*d^3 - 3*b*c^4*d^2*e + 3*(b^2*c^3 -
 2*a*c^4)*d*e^2 - (b^3*c^2 - 3*a*b*c^3)*e^3)*f - (b*c^4*d^3 - 3*(b^2*c^3 -
2*a*c^4)*d^2*e + 3*(b^3*c^2 - 3*a*b*c^3)*d*e^2 - (b^4*c - 4*a*b^2*c^2 + 2*a
^2*c^3)*e^3)*g + ((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e +
 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2
)*e^3)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 12*((3*(b
^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*f + (3*(b^2*c^4 - 4*a*
c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2
*c^4)*e^3)*g + ((b^2*c^4 - 4*a*c^5)*d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3
*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*e^2 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b
*c^3)*e^3)*h)*x + 6*((3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)
*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*f + ((b^2*c^4 - 4*a*c^5)*
d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)
*d*e^2 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^3)*g - ((b^3*c^3 - 4*a*b*c^4
)*d^3 - 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e + 3*(b^5*c - 6*a*b^3*c^
2 + 8*a^2*b*c^3)*d*e^2 - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^3
)*h)*log(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6)]
```

giac [A] time = 0.17, size = 771, normalized size = 1.30

$$\frac{3\,c^3hx^4e^3 + 12\,c^3dhx^3e^2 + 18\,c^3d^2hx^2e + 12\,c^3d^3hx + 4\,c^3gx^3e^3 - 4\,bc^2hx^3e^3 + 18\,c^3dgx^2e^2 - 18\,bc^2dhx^2e^2 + 36\,c^3\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*(3*c^3*h*x^4*e^3 + 12*c^3*d*h*x^3*e^2 + 18*c^3*d^2*h*x^2*e + 12*c^3*d^
3*h*x + 4*c^3*g*x^3*e^3 - 4*b*c^2*h*x^3*e^3 + 18*c^3*d*g*x^2*e^2 - 18*b*c^2
*d*h*x^2*e^2 + 36*c^3*d^2*g*x*e - 36*b*c^2*d^2*h*x*e + 6*c^3*f*x^2*e^3 - 6*
b*c^2*g*x^2*e^3 + 6*b^2*c*h*x^2*e^3 - 6*a*c^2*h*x^2*e^3 + 36*c^3*d*f*x*e^2
- 36*b*c^2*d*g*x*e^2 + 36*b^2*c*d*h*x*e^2 - 36*a*c^2*d*h*x*e^2 - 12*b*c^2*f
*x*e^3 + 12*b^2*c*g*x*e^3 - 12*a*c^2*g*x*e^3 - 12*b^3*h*x*e^3 + 24*a*b*c*h*
x*e^3)/c^4 + 1/2*(c^4*d^3*g - b*c^3*d^3*h + 3*c^4*d^2*f*e - 3*b*c^3*d^2*g*e
 + 3*b^2*c^2*d^2*h*e - 3*a*c^3*d^2*h*e - 3*b*c^3*d*f*e^2 + 3*b^2*c^2*d*g*e^
2 - 3*a*c^3*d*g*e^2 - 3*b^3*c*d*h*e^2 + 6*a*b*c^2*d*h*e^2 + b^2*c^2*f*e^3 -
 a*c^3*f*e^3 - b^3*c*g*e^3 + 2*a*b*c^2*g*e^3 + b^4*h*e^3 - 3*a*b^2*c*h*e^3
+ a^2*c^2*h*e^3)*log(c*x^2 + b*x + a)/c^5 + (2*c^5*d^3*f - b*c^4*d^3*g + b^
2*c^3*d^3*h - 2*a*c^4*d^3*h - 3*b*c^4*d^2*f*e + 3*b^2*c^3*d^2*g*e - 6*a*c^4
*d^2*g*e - 3*b^3*c^2*d^2*h*e + 9*a*b*c^3*d^2*h*e + 3*b^2*c^3*d*f*e^2 - 6*a*
c^4*d*f*e^2 - 3*b^3*c^2*d*g*e^2 + 9*a*b*c^3*d*g*e^2 + 3*b^4*c*d*h*e^2 - 12*
a*b^2*c^2*d*h*e^2 + 6*a^2*c^3*d*h*e^2 - b^3*c^2*f*e^3 + 3*a*b*c^3*f*e^3 + b
^4*c*g*e^3 - 4*a*b^2*c^2*g*e^3 + 2*a^2*c^3*g*e^3 - b^5*h*e^3 + 5*a*b^3*c*h*
e^3 - 5*a^2*b*c^2*h*e^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
+ 4*a*c)*c^5)
```

maple [B] time = 0.01, size = 1738, normalized size = 2.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x)
```

```
[Out] 1/4*e^3*h*x^4/c+1/2/c*ln(c*x^2+b*x+a)*d^3*g+2/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*d^3*f+1/3/c*x^3*e^3*g+1/2/c*x^2*e^3*f+1/c*d^3*h*x+
9/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*d*e^2*g+9/c
^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*d^2*e*h-12/c^3
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*d*e^2*h-1/2/c^
4*ln(c*x^2+b*x+a)*b^3*e^3*g+1/2/c^3*ln(c*x^2+b*x+a)*b^2*e^3*f-1/2/c^2*ln(c*
x^2+b*x+a)*b*d^3*h+3/2/c*ln(c*x^2+b*x+a)*d^2*e*f-1/3/c^2*x^3*b*e^3*h+1/c*x^
3*d*e^2*h-1/2/c^2*x^2*a*e^3*h+1/2/c^3*x^2*b^2*e^3*h-1/2/c^2*x^2*b*e^3*g+3/2
/c*x^2*d^2*e*h+3/2/c*x^2*d*e^2*g+1/c^3*ln(c*x^2+b*x+a)*a*b*e^3*g-3/2/c^2*ln
(c*x^2+b*x+a)*a*d^2*e*h-3/2/c^2*ln(c*x^2+b*x+a)*a*d*e^2*g-3/2/c^4*ln(c*x^2+
b*x+a)*b^3*d*e^2*h+3/2/c^3*ln(c*x^2+b*x+a)*b^2*d^2*e*h+3/2/c^3*ln(c*x^2+b*x
+a)*b^2*d*e^2*g-3/2/c^2*ln(c*x^2+b*x+a)*b*d^2*e*g-3/2/c^2*ln(c*x^2+b*x+a)*b
*d*e^2*f+1/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*e^
3*g+5/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*e^3*h
+3/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*d*e^2*h-3/
c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^2*e*f+3/c^3*ln(
c*x^2+b*x+a)*a*b*d*e^2*h+6/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*a^2*d*e^2*h-6/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))*a*d^2*e*g-6/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*
d*e^2*f-4/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e
^3*g+3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*e^3*f-
5/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*e^3*h-3/c
^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*d^2*e*h-3/c^3/
(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*d*e^2*g+3/c^2/(4*
a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d^2*e*g+3/c^2/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d*e^2*f-1/c^2*a*e^3*g*x
-1/c^4*b^3*e^3*h*x+1/c^3*b^2*e^3*g*x-1/c^2*b*e^3*f*x+3/c*d^2*e*g*x+3/c*d*e^
2*f*x+1/2/c^3*ln(c*x^2+b*x+a)*a^2*e^3*h-1/2/c^2*ln(c*x^2+b*x+a)*a*e^3*f+1/2
/c^5*ln(c*x^2+b*x+a)*b^4*e^3*h-3/2/c^2*x^2*b*d*e^2*h+2/c^3*a*b*e^3*h*x-3/c^
2*a*d*e^2*h*x+3/c^3*b^2*d*e^2*h*x-3/c^2*b*d^2*e*h*x-3/c^2*b*d*e^2*g*x-1/c^3
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^3*f+1/c^2/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d^3*h-1/c/(4*a*c-b^2)
^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^3*g-1/c^5/(4*a*c-b^2)^(1/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5*e^3*h+2/c^2/(4*a*c-b^2)^(1/2)*arcta
```

```
n((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*e^3*g-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*a*d^3*h-3/2/c^4*ln(c*x^2+b*x+a)*a*b^2*e^3*h
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.46, size = 967, normalized size = 1.64

$$x^3\left(\frac{g\,e^3+3\,d\,h\,e^2}{3\,c}-\frac{b\,e^3\,h}{3\,c^2}\right)+x\left(\frac{h\,d^3+3\,g\,d^2\,e+3\,f\,d\,e^2}{c}+\frac{b\left(\frac{b\left(\frac{g\,e^3+3\,d\,h\,e^2}{c}-\frac{b\,e^3\,h}{c^2}\right)}{c}-\frac{3\,h\,d^2\,e+3\,g\,d\,e^2+f\,e^3}{c}+\frac{a\,e^3\,h}{c^2}\right)}{c}-\frac{a\,(\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

```
[Out] x^3*((e^3*g + 3*d*e^2*h)/(3*c) - (b*e^3*h)/(3*c^2)) + x*((d^3*h + 3*d*e^2*f
 + 3*d^2*e*g)/c + (b*((b*((e^3*g + 3*d*e^2*h)/c - (b*e^3*h)/c^2))/c - (e^3*
f + 3*d*e^2*g + 3*d^2*e*h)/c + (a*e^3*h)/c^2))/c - (a*((e^3*g + 3*d*e^2*h)/
c - (b*e^3*h)/c^2))/c) - x^2*((b*((e^3*g + 3*d*e^2*h)/c - (b*e^3*h)/c^2))/(
2*c) - (e^3*f + 3*d*e^2*g + 3*d^2*e*h)/(2*c) + (a*e^3*h)/(2*c^2)) - (log(a
+ b*x + c*x^2)*(b^6*e^3*h + 4*a^2*c^4*e^3*f + b^2*c^4*d^3*g + b^4*c^2*e^3*f
 - 4*a^3*c^3*e^3*h - b^3*c^3*d^3*h - 4*a*c^5*d^3*g - b^5*c*e^3*g + 4*a*b*c^
4*d^3*h - 7*a*b^4*c*e^3*h - 12*a*c^5*d^2*e*f - 3*b^5*c*d*e^2*h - 5*a*b^2*c^
3*e^3*f + 6*a*b^3*c^2*e^3*g - 8*a^2*b*c^3*e^3*g + 12*a^2*c^4*d*e^2*g + 3*b^
2*c^4*d^2*e*f - 3*b^3*c^3*d*e^2*f + 12*a^2*c^4*d^2*e*h - 3*b^3*c^3*d^2*e*g
+ 3*b^4*c^2*d*e^2*g + 3*b^4*c^2*d^2*e*h + 13*a^2*b^2*c^2*e^3*h + 12*a*b*c^4
*d*e^2*f + 12*a*b*c^4*d^2*e*g - 15*a*b^2*c^3*d*e^2*g - 15*a*b^2*c^3*d^2*e*h
 + 18*a*b^3*c^2*d*e^2*h - 24*a^2*b*c^3*d*e^2*h))/(2*(4*a*c^6 - b^2*c^5)) +
(e^3*h*x^4)/(4*c) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/
2))*(2*c^5*d^3*f - b^5*e^3*h + 2*a^2*c^3*e^3*g - b^3*c^2*e^3*f + b^2*c^3*d^
3*h - 2*a*c^4*d^3*h - b*c^4*d^3*g + b^4*c*e^3*g + 3*a*b*c^3*e^3*f + 5*a*b^3
*c*e^3*h - 6*a*c^4*d*e^2*f - 6*a*c^4*d^2*e*g - 3*b*c^4*d^2*e*f + 3*b^4*c*d*
e^2*h - 4*a*b^2*c^2*e^3*g - 5*a^2*b*c^2*e^3*h + 3*b^2*c^3*d*e^2*f + 6*a^2*c
^3*d*e^2*h + 3*b^2*c^3*d^2*e*g - 3*b^3*c^2*d*e^2*g - 3*b^3*c^2*d^2*e*h + 9*
a*b*c^3*d*e^2*g + 9*a*b*c^3*d^2*e*h - 12*a*b^2*c^2*d*e^2*h))/(c^5*(4*a*c -
b^2)^(1/2))
```

sympy [B] time = 118.42, size = 4972, normalized size = 8.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] x**3*(-b*e**3*h/(3*c**2) + d*e**2*h/c + e**3*g/(3*c)) + x**2*(-a*e**3*h/(2*
c**2) + b**2*e**3*h/(2*c**3) - 3*b*d*e**2*h/(2*c**2) - b*e**3*g/(2*c**2) +
```

```
3*d**2*e*h/(2*c) + 3*d*e**2*g/(2*c) + e**3*f/(2*c)) + x*(2*a*b*e**3*h/c**3
- 3*a*d*e**2*h/c**2 - a*e**3*g/c**2 - b**3*e**3*h/c**4 + 3*b**2*d*e**2*h/c*
*3 + b**2*e**3*g/c**3 - 3*b*d**2*e*h/c**2 - 3*b*d*e**2*g/c**2 - b*e**3*f/c*
*2 + d**3*h/c + 3*d**2*e*g/c + 3*d*e**2*f/c) + (-sqrt(-4*a*c + b**2)*(5*a**
2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e*
*3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h
 - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**
2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g
 + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c*
*3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3
*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3
*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3
*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e*
*2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c
**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d
**3*g + 3*c**4*d**2*e*f)/(2*c**5))*log(x + (2*a**3*c**2*e**3*h - 4*a**2*b**
2*c*e**3*h + 9*a**2*b*c**2*d*e**2*h + 3*a**2*b*c**2*e**3*g - 6*a**2*c**3*d*
*2*e*h - 6*a**2*c**3*d*e**2*g - 2*a**2*c**3*e**3*f + a*b**4*e**3*h - 3*a*b*
*3*c*d*e**2*h - a*b**3*c*e**3*g + 3*a*b**2*c**2*d**2*e*h + 3*a*b**2*c**2*d*
e**2*g + a*b**2*c**2*e**3*f - a*b*c**3*d**3*h - 3*a*b*c**3*d**2*e*g - 3*a*b
*c**3*d*e**2*f - 4*a*c**5*(-sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e**3*h - 6*a
**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a*b**2*c**2
*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*
g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e*
*2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c**2*d**2*e
*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c**3*d**3*h - 3*b**2*c*
*3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*
c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3*h - 3*a*b**2*c*e**3*
h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*
d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e**2*h - b**3*c*e**3*g
+ 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c**2*e**3*f - b*c**3*d
**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e
*f)/(2*c**5)) + 2*a*c**4*d**3*g + 6*a*c**4*d**2*e*f + b**2*c**4*(-sqrt(-4*a
*c + b**2)*(5*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*
g - 5*a*b**3*c*e**3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*
a*b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3
*h + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*
h - b**4*c*e**3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2
*e**3*f - b**2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f +
b*c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2))
+ (a**2*c**2*e**3*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*
e**3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*
h - 3*b**3*c*d*e**2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*
d*e**2*g + b**2*c**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*
d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e*f)/(2*c**5)) - b*c**4*d**3*f)/(5*a**
2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e*
*3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h
 - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**
2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g
 + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c*
*3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3
*b*c**4*d**2*e*f - 2*c**5*d**3*f)) + (sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e*
*3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a
*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c*
*3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a
*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c
**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c**3*d**3*h -
 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**
2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3*h - 3*a*b*
```

```
*2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3*d**2*e*h -
 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e**2*h - b**3
*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c**2*e**3*f
- b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d**3*g + 3*c
**4*d**2*e*f)/(2*c**5))*log(x + (2*a**3*c**2*e**3*h - 4*a**2*b**2*c*e**3*h
+ 9*a**2*b*c**2*d*e**2*h + 3*a**2*b*c**2*e**3*g - 6*a**2*c**3*d**2*e*h - 6*
a**2*c**3*d*e**2*g - 2*a**2*c**3*e**3*f + a*b**4*e**3*h - 3*a*b**3*c*d*e**2
*h - a*b**3*c*e**3*g + 3*a*b**2*c**2*d**2*e*h + 3*a*b**2*c**2*d*e**2*g + a*
b**2*c**2*e**3*f - a*b*c**3*d**3*h - 3*a*b*c**3*d**2*e*g - 3*a*b*c**3*d*e**
2*f - 4*a*c**5*(sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e
**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a*b**2*c**2*d*e**2*h +
4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c*
*3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*
e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*
c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g
- 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)
/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3*h - 3*a*b**2*c*e**3*h + 6*a*b*c*
*2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*d*e**2*g - a
*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e**2*h - b**3*c*e**3*g + 3*b**2*c**
2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c**2*e**3*f - b*c**3*d**3*h - 3*b*
c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e*f)/(2*c**5)
) + 2*a*c**4*d**3*g + 6*a*c**4*d**2*e*f + b**2*c**4*(sqrt(-4*a*c + b**2)*(5
*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*
c*e**3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2
*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4
*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e*
*3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**
2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g
 + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*
e**3*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*
c**3*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*
d*e**2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b*
*2*c**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c*
*4*d**3*g + 3*c**4*d**2*e*f)/(2*c**5)) - b*c**4*d**3*f)/(5*a**2*b*c**2*e**3
*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a*b
**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c**3
*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a*c
**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c**
2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c**3*d**3*h - 3
*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**2*
e*f - 2*c**5*d**3*f)) + e**3*h*x**4/(4*c)
```

3.149 $\int \frac{(d+ex)^2(f+gx+hx^2)}{a+bx+cx^2}\,dx$

Optimal. Leaf size=348

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^2\left(2a^2e^2h+3abe(2dh+eg)+b^2\left(d^2h+2deg+e^2f\right)\right)-b^2ce(4aeh+2bdh+beg)-c^3\left(2a\right.\right.}{c^4\sqrt{b^2-4ac}}$$

```
[Out] (b^2*e^2*h+c^2*(d^2*h+2*d*e*g+e^2*f)-c*e*(a*e*h+2*b*d*h+b*e*g))*x/c^3+1/2*e
*(-b*e*h+2*c*d*h+c*e*g)*x^2/c^2+1/3*e^2*h*x^3/c+1/2*(c^3*d*(d*g+2*e*f)-b^3*
e^2*h+b*c*e*(2*a*e*h+2*b*d*h+b*e*g)-c^2*(a*e*(2*d*h+e*g)+b*(d^2*h+2*d*e*g+e
^2*f)))*ln(c*x^2+b*x+a)/c^4-(2*c^4*d^2*f+b^4*e^2*h-b^2*c*e*(4*a*e*h+2*b*d*h
+b*e*g)-c^3*(b*d*(d*g+2*e*f)+2*a*(d^2*h+2*d*e*g+e^2*f))+c^2*(2*a^2*e^2*h+3*
a*b*e*(2*d*h+e*g)+b^2*(d^2*h+2*d*e*g+e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2
)^(1/2))/c^4/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1628, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^2\left(2a^2e^2h+3abe(2dh+eg)+b^2\left(d^2h+2deg+e^2f\right)\right)-b^2ce(4aeh+2bdh+beg)-c^3\left(2a\right.\right.}{c^4\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] ((b^2*e^2*h + c^2*(e^2*f + 2*d*e*g + d^2*h) - c*e*(b*e*g + 2*b*d*h + a*e*h)
)*x)/c^3 + (e*(c*e*g + 2*c*d*h - b*e*h)*x^2)/(2*c^2) + (e^2*h*x^3)/(3*c) -
((2*c^4*d^2*f + b^4*e^2*h - b^2*c*e*(b*e*g + 2*b*d*h + 4*a*e*h) - c^3*(b*d*
(2*e*f + d*g) + 2*a*(e^2*f + 2*d*e*g + d^2*h)) + c^2*(2*a^2*e^2*h + 3*a*b*e
*(e*g + 2*d*h) + b^2*(e^2*f + 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) + ((c^3*d*(2*e*f + d*g) - b^3*e^2*h +
 b*c*e*(b*e*g + 2*b*d*h + 2*a*e*h) - c^2*(a*e*(e*g + 2*d*h) + b*(e^2*f + 2*
d*e*g + d^2*h)))*Log[a + b*x + c*x^2])/(2*c^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = \int \left(\frac{b^2e^2h+c^2\left(e^2f+2deg+d^2h\right)-ce(beg+2bdh+aeh)}{c^3}+\frac{e(ceg+2cdh-beh}{c^2}\right.$$

$$= \frac{\left(b^2e^2h+c^2\left(e^2f+2deg+d^2h\right)-ce(beg+2bdh+aeh)\right)x}{c^3}+\frac{e(ceg+2cdh-beh}{2c^2}$$

$$= \frac{\left(b^2e^2h+c^2\left(e^2f+2deg+d^2h\right)-ce(beg+2bdh+aeh)\right)x}{c^3}+\frac{e(ceg+2cdh-beh}{2c^2}$$

$$= \frac{\left(b^2e^2h+c^2\left(e^2f+2deg+d^2h\right)-ce(beg+2bdh+aeh)\right)x}{c^3}+\frac{e(ceg+2cdh-beh}{2c^2}$$

$$= \frac{\left(b^2e^2h+c^2\left(e^2f+2deg+d^2h\right)-ce(beg+2bdh+aeh)\right)x}{c^3}+\frac{e(ceg+2cdh-beh}{2c^2}$$

Mathematica [A] time = 0.38, size = 345, normalized size = 0.99

$$\frac{\frac{6\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(c^2\left(2a^2e^2h+3abe(2dh+eg)+b^2\left(d^2h+2deg+e^2f\right)\right)-b^2ce(4aeh+2bdh+beg)-c^3\left(2a\left(d^2h+2deg+e^2f\right)+bd(dg+2ef)\right)+b^4e^2h+2c^4d^2f\right)}{\sqrt{4ac-b^2}}+3\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] (6*c*(b^2*e^2*h + c^2*(e^2*f + 2*d*e*g + d^2*h) - c*e*(b*e*g + 2*b*d*h + a*
e*h))*x + 3*c^2*e*(c*e*g + 2*c*d*h - b*e*h)*x^2 + 2*c^3*e^2*h*x^3 + (6*(2*c
^4*d^2*f + b^4*e^2*h - b^2*c*e*(b*e*g + 2*b*d*h + 4*a*e*h) - c^3*(b*d*(2*e*
f + d*g) + 2*a*(e^2*f + 2*d*e*g + d^2*h)) + c^2*(2*a^2*e^2*h + 3*a*b*e*(e*g
 + 2*d*h) + b^2*(e^2*f + 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
4*a*c]])/Sqrt[-b^2 + 4*a*c] + 3*(c^3*d*(2*e*f + d*g) - b^3*e^2*h + b*c*e*(b
*e*g + 2*b*d*h + 2*a*e*h) - c^2*(a*e*(e*g + 2*d*h) + b*(e^2*f + 2*d*e*g + d
^2*h)))*Log[a + x*(b + c*x)])/(6*c^4)
```

fricas [A] time = 1.11, size = 1273, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c^3 - 4*a*c^4)*e^2*h*x^3 + 3*((b^2*c^3 - 4*a*c^4)*e^2*g + (2*(
b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*h)*x^2 + 3*sqrt(b^2 - 4
*a*c)*((2*c^4*d^2 - 2*b*c^3*d*e + (b^2*c^2 - 2*a*c^3)*e^2)*f - (b*c^3*d^2 -
 2*(b^2*c^2 - 2*a*c^3)*d*e + (b^3*c - 3*a*b*c^2)*e^2)*g + ((b^2*c^2 - 2*a*c
^3)*d^2 - 2*(b^3*c - 3*a*b*c^2)*d*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^2)*h)
```

```
*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a)) + 6*((b^2*c^3 - 4*a*c^4)*e^2*f + (2*(b^2*c^3 - 4*a*c^4)*d*
e - (b^3*c^2 - 4*a*b*c^3)*e^2)*g + ((b^2*c^3 - 4*a*c^4)*d^2 - 2*(b^3*c^2 -
4*a*b*c^3)*d*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^2)*h)*x + 3*((2*(b^2*c
^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*f + ((b^2*c^3 - 4*a*c^4)*d^2
 - 2*(b^3*c^2 - 4*a*b*c^3)*d*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^2)*g -
 ((b^3*c^2 - 4*a*b*c^3)*d^2 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e + (b^
5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2)*h)*log(c*x^2 + b*x + a))/(b^2*c^4 - 4*a*c
^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*e^2*h*x^3 + 3*((b^2*c^3 - 4*a*c^4)*e^2*g +
(2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*h)*x^2 - 6*sqrt(-b^
2 + 4*a*c)*((2*c^4*d^2 - 2*b*c^3*d*e + (b^2*c^2 - 2*a*c^3)*e^2)*f - (b*c^3*
d^2 - 2*(b^2*c^2 - 2*a*c^3)*d*e + (b^3*c - 3*a*b*c^2)*e^2)*g + ((b^2*c^2 -
2*a*c^3)*d^2 - 2*(b^3*c - 3*a*b*c^2)*d*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^
2)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*((b^2*c^3 -
 4*a*c^4)*e^2*f + (2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*g
 + ((b^2*c^3 - 4*a*c^4)*d^2 - 2*(b^3*c^2 - 4*a*b*c^3)*d*e + (b^4*c - 5*a*b^
2*c^2 + 4*a^2*c^3)*e^2)*h)*x + 3*((2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4
*a*b*c^3)*e^2)*f + ((b^2*c^3 - 4*a*c^4)*d^2 - 2*(b^3*c^2 - 4*a*b*c^3)*d*e +
 (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^2)*g - ((b^3*c^2 - 4*a*b*c^3)*d^2 - 2*
(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2
)*h)*log(c*x^2 + b*x + a))/(b^2*c^4 - 4*a*c^5)]
```

giac [A] time = 0.16, size = 426, normalized size = 1.22

$$\frac{2c^2hx^3e^2+6c^2dhx^2e+6c^2d^2hx+3c^2gx^2e^2-3bchx^2e^2+12c^2dgxe-12bcdhxe+6c^2fxe^2-6bcgxe^2+6b^2}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(2*c^2*h*x^3*e^2 + 6*c^2*d*h*x^2*e + 6*c^2*d^2*h*x + 3*c^2*g*x^2*e^2 -
3*b*c*h*x^2*e^2 + 12*c^2*d*g*x*e - 12*b*c*d*h*x*e + 6*c^2*f*x*e^2 - 6*b*c*g
*x*e^2 + 6*b^2*h*x*e^2 - 6*a*c*h*x*e^2)/c^3 + 1/2*(c^3*d^2*g - b*c^2*d^2*h
+ 2*c^3*d*f*e - 2*b*c^2*d*g*e + 2*b^2*c*d*h*e - 2*a*c^2*d*h*e - b*c^2*f*e^2
 + b^2*c*g*e^2 - a*c^2*g*e^2 - b^3*h*e^2 + 2*a*b*c*h*e^2)*log(c*x^2 + b*x +
 a)/c^4 + (2*c^4*d^2*f - b*c^3*d^2*g + b^2*c^2*d^2*h - 2*a*c^3*d^2*h - 2*b*
c^3*d*f*e + 2*b^2*c^2*d*g*e - 4*a*c^3*d*g*e - 2*b^3*c*d*h*e + 6*a*b*c^2*d*h
*e + b^2*c^2*f*e^2 - 2*a*c^3*f*e^2 - b^3*c*g*e^2 + 3*a*b*c^2*g*e^2 + b^4*h*
e^2 - 4*a*b^2*c*h*e^2 + 2*a^2*c^2*h*e^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a
*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

maple [B] time = 0.01, size = 1028, normalized size = 2.95

$$\frac{e^2hx^3}{3c}+\frac{2a^2e^2h\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^2}-\frac{4ab^2e^2h\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^3}+\frac{6abdeh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^2}+\frac{3abe^2g\arctan\left(\frac{2cx+}{\sqrt{4ac-}}\right)}{\sqrt{4ac-b^2}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x)
```

```
[Out] 1/3*e^2*h*x^3/c+1/2/c*ln(c*x^2+b*x+a)*d^2*g+2/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*d^2*f+1/2/c*x^2*e^2*g+1/c*d^2*h*x+1/c*e^2*f*x+6/c^
2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*d*e*h-2/c^2*b*d
*e*h*x-1/c^2*ln(c*x^2+b*x+a)*b*d*e*g+2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*a^2*e^2*h-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))*a*d^2*h-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*a*e^2*f+1/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b
^4*e^2*h+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d^
```

```
2*h+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*e^2*f-1
/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^2*g-1/c^3/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^2*g+1/c^3*ln(c*x^2+
b*x+a)*a*b*e^2*h-1/c^2*ln(c*x^2+b*x+a)*a*d*e*h+1/c^3*ln(c*x^2+b*x+a)*b^2*d*
e*h-1/2/c^4*ln(c*x^2+b*x+a)*b^3*e^2*h+1/2/c^3*ln(c*x^2+b*x+a)*b^2*e^2*g-1/2
/c^2*ln(c*x^2+b*x+a)*b*d^2*h-1/2/c^2*ln(c*x^2+b*x+a)*a*e^2*g-1/2/c^2*x^2*b*
e^2*h+1/c*x^2*d*e*h-1/c^2*a*e^2*h*x+1/c^3*b^2*e^2*h*x-1/c^2*b*e^2*g*x+2/c*d
*e*g*x-4/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e^
2*h-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d*e*f+3/c^2
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*e^2*g-4/c/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*e*g-1/2/c^2*ln(c*x^2+b*
x+a)*b*e^2*f+1/c*ln(c*x^2+b*x+a)*d*e*f-2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*b^3*d*e*h+2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))*b^2*d*e*g
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.68, size = 557, normalized size = 1.60

$$x^2\left(\frac{g\,e^2+2\,d\,h\,e}{2\,c}-\frac{b\,e^2\,h}{2\,c^2}\right)-x\left(\frac{b\left(\frac{g\,e^2+2\,d\,h\,e}{c}-\frac{b\,e^2\,h}{c^2}\right)}{c}-\frac{h\,d^2+2\,g\,d\,e+f\,e^2}{c}+\frac{a\,e^2\,h}{c^2}\right)-\frac{\ln\left(c\,x^2+b\,x+a\right)\left(-8\,h\,a^2\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

```
[Out] x^2*((e^2*g + 2*d*e*h)/(2*c) - (b*e^2*h)/(2*c^2)) - x*((b*((e^2*g + 2*d*e*h
)/c - (b*e^2*h)/c^2))/c - (e^2*f + d^2*h + 2*d*e*g)/c + (a*e^2*h)/c^2) - (l
og(a + b*x + c*x^2)*(4*a^2*c^3*e^2*g - b^5*e^2*h + b^2*c^3*d^2*g - b^3*c^2*
e^2*f - b^3*c^2*d^2*h - 4*a*c^4*d^2*g + b^4*c*e^2*g + 4*a*b*c^3*e^2*f + 4*a
*b*c^3*d^2*h + 6*a*b^3*c*e^2*h + 2*b^2*c^3*d*e*f + 8*a^2*c^3*d*e*h - 2*b^3*
c^2*d*e*g - 5*a*b^2*c^2*e^2*g - 8*a^2*b*c^2*e^2*h - 8*a*c^4*d*e*f + 2*b^4*c
*d*e*h + 8*a*b*c^3*d*e*g - 10*a*b^2*c^2*d*e*h))/(2*(4*a*c^5 - b^2*c^4)) + (
e^2*h*x^3)/(3*c) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2
))*(2*c^4*d^2*f + b^4*e^2*h + b^2*c^2*e^2*f + 2*a^2*c^2*e^2*h + b^2*c^2*d^2
*h - 2*a*c^3*e^2*f - 2*a*c^3*d^2*h - b*c^3*d^2*g - b^3*c*e^2*g + 3*a*b*c^2*
e^2*g - 4*a*b^2*c*e^2*h + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - 2*b*c^3*d*e*f -
 2*b^3*c*d*e*h + 6*a*b*c^2*d*e*h))/(c^4*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 47.19, size = 2839, normalized size = 8.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] x**2*(-b*e**2*h/(2*c**2) + d*e*h/c + e**2*g/(2*c)) + x*(-a*e**2*h/c**2 + b*
*2*e**2*h/c**3 - 2*b*d*e*h/c**2 - b*e**2*g/c**2 + d**2*h/c + 2*d*e*g/c + e*
*2*f/c) + (-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6
```

```
*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*
a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d*
*2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*
f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e
*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*
d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c*
*4))*log(x + (-3*a**2*b*c*e**2*h + 4*a**2*c**2*d*e*h + 2*a**2*c**2*e**2*g +
 a*b**3*e**2*h - 2*a*b**2*c*d*e*h - a*b**2*c*e**2*g + a*b*c**2*d**2*h + 2*a
*b*c**2*d*e*g + a*b*c**2*e**2*f + 4*a*c**4*(-sqrt(-4*a*c + b**2)*(2*a**2*c*
*2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*
c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e
*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*
f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2))
 + (2*a*b*c*e**2*h - 2*a*c**2*d*e*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*
c*d*e*h + b**2*c*e**2*g - b*c**2*d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f +
c**3*d**2*g + 2*c**3*d*e*f)/(2*c**4)) - 2*a*c**3*d**2*g - 4*a*c**3*d*e*f -
b**2*c**3*(-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6
*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*
a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d*
*2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*
f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e
*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*
d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c*
*4)) + b*c**3*d**2*f)/(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*
d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**
2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b
**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4
*d**2*f)) + (sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h +
6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2
*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d
**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e
*f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*
e*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2
*d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c
**4))*log(x + (-3*a**2*b*c*e**2*h + 4*a**2*c**2*d*e*h + 2*a**2*c**2*e**2*g
+ a*b**3*e**2*h - 2*a*b**2*c*d*e*h - a*b**2*c*e**2*g + a*b*c**2*d**2*h + 2*
a*b*c**2*d*e*g + a*b*c**2*e**2*f + 4*a*c**4*(sqrt(-4*a*c + b**2)*(2*a**2*c*
*2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*
c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e
*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*
f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2))
 + (2*a*b*c*e**2*h - 2*a*c**2*d*e*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*
c*d*e*h + b**2*c*e**2*g - b*c**2*d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f +
c**3*d**2*g + 2*c**3*d*e*f)/(2*c**4)) - 2*a*c**3*d**2*g - 4*a*c**3*d*e*f -
b**2*c**3*(sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*
a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a
*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**
2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f
 + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e*
h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*d
**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c**
4)) + b*c**3*d**2*f)/(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d
*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2
*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b*
*2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*
d**2*f)) + e**2*h*x**3/(3*c)
```

3.150 $\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx$

Optimal. Leaf size=177

$$\frac{\log\left(a+bx+cx^2\right)\left(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef)\right)}{2c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c^2(2adh+2aeg+bdg+bef\right.}{c^3\sqrt{b^2-}}$$

```
[Out] (-b*e*h+c*d*h+c*e*g)*x/c^2+1/2*e*h*x^2/c+1/2*(c^2*(d*g+e*f)+b^2*e*h-c*(a*e*
h+b*d*h+b*e*g))*ln(c*x^2+b*x+a)/c^3-(2*c^3*d*f-b^3*e*h-c^2*(2*a*d*h+2*a*e*g
+b*d*g+b*e*f)+b*c*(3*a*e*h+b*d*h+b*e*g))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/
2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1628, 634, 618, 206, 628}

$$\frac{\log\left(a+bx+cx^2\right)\left(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef)\right)}{2c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c^2(2adh+2aeg+bdg+bef\right.}{c^3\sqrt{b^2-}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] ((c*e*g + c*d*h - b*e*h)*x)/c^2 + (e*h*x^2)/(2*c) - ((2*c^3*d*f - b^3*e*h -
 c^2*(b*e*f + b*d*g + 2*a*e*g + 2*a*d*h) + b*c*(b*e*g + b*d*h + 3*a*e*h))*A
rcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*(e*f
 + d*g) + b^2*e*h - c*(b*e*g + b*d*h + a*e*h))*Log[a + b*x + c*x^2])/(2*c^3
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = \int \left(\frac{ceg+cdh-beh}{c^2}+\frac{ehx}{c}+\frac{c^2df+abeh-ac(eg+dh)+\left(c^2(ef+dg)+b^2e\right.}{c^2\left(a+bx+cx^2\right)}\right.$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}+\frac{\int \frac{c^2df+abeh-ac(eg+dh)+\left(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)\right)x}{a+bx+cx^2}}{c^2}$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}+\frac{\left(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)\right)\int \frac{b-}{a+b}}{2c^3}$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}+\frac{\left(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)\right)\log\left(\right.}{2c^3}$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}-\frac{\left(2c^3df-b^3eh-c^2(bef+bdg+2aeg+2adh)+\right.}{c^3\sqrt{b^2-4}}$$

Mathematica [A] time = 0.20, size = 173, normalized size = 0.98

$$\frac{\log(a+x(b+cx))\left(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef)\right)-\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(c^2(2adh+2aeg+bdg+bef)-bc(3aeh+bdh+\right.}{\sqrt{4ac-b^2}}}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*c*(c*e*g + c*d*h - b*e*h)*x + c^2*e*h*x^2 - (2*(-2*c^3*d*f + b^3*e*h + c
^2*(b*e*f + b*d*g + 2*a*e*g + 2*a*d*h) - b*c*(b*e*g + b*d*h + 3*a*e*h))*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c^2*(e*f + d*g)
+ b^2*e*h - c*(b*e*g + b*d*h + a*e*h))*Log[a + x*(b + c*x)])/(2*c^3)
```

fricas [A] time = 0.93, size = 654, normalized size = 3.69

$$\left[\frac{\left(b^2c^2-4\,ac^3\right)ehx^2+\sqrt{b^2-4\,ac}\left(\left(2\,c^3d-bc^2e\right)f-\left(bc^2d-\left(b^2c-2\,ac^2\right)e\right)g+\left(\left(b^2c-2\,ac^2\right)d-\left(b^3-3\,abc\right)e\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2 - 4*a*c^3)*e*h*x^2 + sqrt(b^2 - 4*a*c)*((2*c^3*d - b*c^2*e)*
f - (b*c^2*d - (b^2*c - 2*a*c^2)*e)*g + ((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b
*c)*e)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x
 + b))/(c*x^2 + b*x + a)) + 2*((b^2*c^2 - 4*a*c^3)*e*g + ((b^2*c^2 - 4*a*c^
3)*d - (b^3*c - 4*a*b*c^2)*e)*h)*x + ((b^2*c^2 - 4*a*c^3)*e*f + ((b^2*c^2 -
 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*g - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*
a*b^2*c + 4*a^2*c^2)*e)*h)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*(
(b^2*c^2 - 4*a*c^3)*e*h*x^2 - 2*sqrt(-b^2 + 4*a*c)*((2*c^3*d - b*c^2*e)*f -
 (b*c^2*d - (b^2*c - 2*a*c^2)*e)*g + ((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)
*e)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*((b^2*c^2
- 4*a*c^3)*e*g + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*h)*x + ((b
^2*c^2 - 4*a*c^3)*e*f + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*g -
```

```
((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*h)*log(c*x^2 + b
*x + a))/(b^2*c^3 - 4*a*c^4)]
```

giac [A] time = 0.19, size = 201, normalized size = 1.14

$$\frac{chx^2e + 2\,cdhx + 2\,cgxe - 2\,bhxe}{2\,c^2} + \frac{\left(c^2dg - bcdh + c^2fe - bcge + b^2he - ache\right)\log\left(cx^2 + bx + a\right)}{2\,c^3} + \frac{\left(2\,c^3df - bc^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(c*h*x^2*e + 2*c*d*h*x + 2*c*g*x*e - 2*b*h*x*e)/c^2 + 1/2*(c^2*d*g - b*
c*d*h + c^2*f*e - b*c*g*e + b^2*h*e - a*c*h*e)*log(c*x^2 + b*x + a)/c^3 + (
2*c^3*d*f - b*c^2*d*g + b^2*c*d*h - 2*a*c^2*d*h - b*c^2*f*e + b^2*c*g*e - 2
*a*c^2*g*e - b^3*h*e + 3*a*b*c*h*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
(sqrt(-b^2 + 4*a*c)*c^3)
```

maple [B] time = 0.01, size = 510, normalized size = 2.88

$$\frac{3abeh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^2} - \frac{2adh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c} - \frac{2aeg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c} - \frac{b^3eh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^3} + \frac{b^2dh\arctan\left(\cdots\right)}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x)
```

```
[Out] 1/2*e*h*x^2/c-1/c^2*b*e*h*x+1/c*d*h*x+1/c*e*g*x-1/2/c^2*ln(c*x^2+b*x+a)*a*e
*h+1/2/c^3*ln(c*x^2+b*x+a)*b^2*e*h-1/2/c^2*ln(c*x^2+b*x+a)*b*d*h-1/2/c^2*ln
(c*x^2+b*x+a)*b*e*g+1/2/c*ln(c*x^2+b*x+a)*d*g+1/2/c*ln(c*x^2+b*x+a)*e*f+3/c
^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*e*h-2/c/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*h-2/c/(4*a*c-b^2)^(1/2)
*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*e*g+2/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*d*f-1/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*b^3*e*h+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*b^2*d*h+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b
^2*e*g-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d*g-1/c/
(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*e*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.53, size = 273, normalized size = 1.54

$$x\left(\frac{d\,h+e\,g}{c} - \frac{b\,e\,h}{c^2}\right) - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(b^4\,e\,h - 4\,a\,c^3\,d\,g - 4\,a\,c^3\,e\,f - b^3\,c\,d\,h - b^3\,c\,e\,g + b^2\,c^2\,d\,g + b^2\,c^2\,e\,f + \cdots\right.}{2\,\left(4\,a\,c^4 - b^2\,c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

```
[Out] x*((d*h + e*g)/c - (b*e*h)/c^2) - (log(a + b*x + c*x^2)*(b^4*e*h - 4*a*c^3*
d*g - 4*a*c^3*e*f - b^3*c*d*h - b^3*c*e*g + b^2*c^2*d*g + b^2*c^2*e*f + 4*a
^2*c^2*e*h + 4*a*b*c^2*d*h + 4*a*b*c^2*e*g - 5*a*b^2*c*e*h))/(2*(4*a*c^4 -
b^2*c^3)) - (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(b^3
*e*h - 2*c^3*d*f + 2*a*c^2*d*h + 2*a*c^2*e*g + b*c^2*d*g + b*c^2*e*f - b^2*
c*d*h - b^2*c*e*g - 3*a*b*c*e*h))/(c^3*(4*a*c - b^2)^(1/2)) + (e*h*x^2)/(2*
c)
```

sympy [B] time = 14.46, size = 1265, normalized size = 7.15

$$x\left(-\frac{beh}{c^2}+\frac{dh}{c}+\frac{eg}{c}\right)+\left(-\frac{\sqrt{-4ac+b^2}\left(3abceh-2ac^2dh-2ac^2eg-b^3eh+b^2cdh+b^2ceg-bc^2dg-bc^2ef+2c^3\right.}{2c^3\left(4ac-b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] x*(-b*e*h/c**2 + d*h/c + e*g/c) + (-sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*
c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g -
 b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b
*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3))*log(x + (2*a**2*c*e*h - a
*b**2*e*h + a*b*c*d*h + a*b*c*e*g + 4*a*c**3*(-sqrt(-4*a*c + b**2)*(3*a*b*c
*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b
*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b
**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) - 2*a*c**2*d*g
 - 2*a*c**2*e*f - b**2*c**2*(-sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2*d
*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**
2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h
 + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) + b*c**2*d*f)/(3*a*b*c*e*h - 2*
a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g
 - b*c**2*e*f + 2*c**3*d*f)) + (sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2
*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c
**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d
*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3))*log(x + (2*a**2*c*e*h - a*b**
2*e*h + a*b*c*d*h + a*b*c*e*g + 4*a*c**3*(sqrt(-4*a*c + b**2)*(3*a*b*c*e*h
- 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2
*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e
*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) - 2*a*c**2*d*g - 2*
a*c**2*e*f - b**2*c**2*(sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2*d*h - 2
*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f
+ 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h + b*c
*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) + b*c**2*d*f)/(3*a*b*c*e*h - 2*a*c**2
*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c
**2*e*f + 2*c**3*d*f)) + e*h*x**2/(2*c)
```

3.151 $\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx$

Optimal. Leaf size=92

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2ach+b^2h-bcg+2c^2f\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{hx}{c}$$

```
[Out] h*x/c+1/2*(-b*h+c*g)*ln(c*x^2+b*x+a)/c^2-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*arc
tanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1657, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2ach+b^2h-bcg+2c^2f\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{hx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (h*x)/c - ((2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2
 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + ((c*g - b*h)*Log[a + b*x + c*x^2])/(2
*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \int \left(\frac{h}{c}+\frac{cf-ah+(cg-bh)x}{c\left(a+bx+cx^2\right)}\right)dx$$
$$= \frac{hx}{c}+\frac{\int \frac{cf-ah+(cg-bh)x}{a+bx+cx^2}\,dx}{c}$$
$$= \frac{hx}{c}+\frac{(cg-bh)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{\left(2c^2f-bcg+b^2h-2ach\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2}$$
$$= \frac{hx}{c}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}-\frac{\left(2c^2f-bcg+b^2h-2ach\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,\right.}{c^2}$$
$$= \frac{hx}{c}-\frac{\left(2c^2f-bcg+b^2h-2ach\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}$$

Mathematica [A] time = 0.07, size = 95, normalized size = 1.03

$$\frac{\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(-2ach+b^2h-bcg+2c^2f\right)}{c^2\sqrt{4ac-b^2}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{hx}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (h*x)/c + ((2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]) + ((c*g - b*h)*Log[a + b*x + c*x^2])/(
2*c^2)
```

fricas [A] time = 0.77, size = 302, normalized size = 3.28

$$\left[\frac{2\left(b^2c-4\,ac^2\right)hx-\left(2\,c^2f-bcg+\left(b^2-2\,ac\right)h\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+\left(\left(b^2c-4\right.\right.}{2\left(b^2c^2-4\,ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c - 4*a*c^2)*h*x - (2*c^2*f - b*c*g + (b^2 - 2*a*c)*h)*sqrt(b^
2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*
x + b))/(c*x^2 + b*x + a)) + ((b^2*c - 4*a*c^2)*g - (b^3 - 4*a*b*c)*h)*log(
c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), 1/2*(2*(b^2*c - 4*a*c^2)*h*x - 2*(2*
c^2*f - b*c*g + (b^2 - 2*a*c)*h)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*g - (b^3 - 4*a*b*c)*h)*
log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 0.16, size = 89, normalized size = 0.97

$$\frac{hx}{c}+\frac{\left(cg-bh\right)\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{\left(2\,c^2f-bcg+b^2h-2\,ach\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] h*x/c + 1/2*(c*g - b*h)*log(c*x^2 + b*x + a)/c^2 + (2*c^2*f - b*c*g + b^2*h
 - 2*a*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [B] time = 0.00, size = 196, normalized size = 2.13

$$-\frac{2ah\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c}+\frac{b^2h\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^2}-\frac{bg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c}+\frac{2f\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{bh\ln\left(cx^2+bx+a\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(c*x^2+b*x+a),x)
```

```
[Out] h*x/c-1/2/c^2*ln(c*x^2+b*x+a)*h*b+1/2/c*ln(c*x^2+b*x+a)*g-2/c/(4*a*c-b^2)^(
1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*h+2/(4*a*c-b^2)^(1/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*f+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*b^2*h-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*b*g
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.25, size = 132, normalized size = 1.43

$$\frac{h\,x}{c}+\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(h\,b^3-g\,b^2\,c-4\,a\,h\,b\,c+4\,a\,g\,c^2\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)}+\frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(h\,b^2-g\,b\,c+2\,f\,c^2-2\,a\right.}{c^2\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/(a + b*x + c*x^2),x)
```

```
[Out] (h*x)/c + (log(a + b*x + c*x^2)*(b^3*h + 4*a*c^2*g - b^2*c*g - 4*a*b*c*h))/
(2*(4*a*c^3 - b^2*c^2)) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^
2)^(1/2))*(2*c^2*f + b^2*h - 2*a*c*h - b*c*g))/(c^2*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 2.14, size = 488, normalized size = 5.30

$$\left(-\frac{\sqrt{-4ac+b^2}\,\left(2ach-b^2h+bcg-2c^2f\right)}{2c^2\left(4ac-b^2\right)}-\frac{bh-cg}{2c^2}\right)\log\left(x+\frac{-abh-4ac^2\left(-\frac{\sqrt{-4ac+b^2}\left(2ach-b^2h+bcg-2c^2f\right)}{2c^2\left(4ac-b^2\right)}-\frac{bh-cg}{2c^2}\right)}{2ach-b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c
- b**2)) - (b*h - c*g)/(2*c**2))*log(x + (-a*b*h - 4*a*c**2*(-sqrt(-4*a*c +
 b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h
 - c*g)/(2*c**2)) + 2*a*c*g + b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*
h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)) - b*c
```

```
*f)/(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)) + (sqrt(-4*a*c + b**2)*(2*a*c*h
- b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)
)*log(x + (-a*b*h - 4*a*c**2*(sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g
 - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)) + 2*a*c*g + b*
*2*c*(sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*
a*c - b**2)) - (b*h - c*g)/(2*c**2)) - b*c*f)/(2*a*c*h - b**2*h + b*c*g - 2
*c**2*f)) + h*x/c
```

3.152 $\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)} dx$

Optimal. Leaf size=196

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df\right)}{c\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{\log\left(a+bx+cx^2\right)(-aeh+bdh-cdg}{2c\left(ae^2-bde+cd^2\right)}$$

```
[Out] (d^2*h-d*e*g+e^2*f)*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)-1/2*(-a*e*h+b*d*h-c*d*g
+c*e*f)*ln(c*x^2+b*x+a)/c/(a*e^2-b*d*e+c*d^2)-(2*c^2*d*f+b*(-a*e+b*d)*h-c*(
2*a*d*h-2*a*e*g+b*d*g+b*e*f))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(a*e^
2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1628, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df\right)}{c\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{\log\left(a+bx+cx^2\right)(-aeh+bdh-cdg}{2c\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x]
```

```
[Out] -(((2*c^2*d*f + b*(b*d - a*e)*h - c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*Ar
cTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
 a*e^2))) + ((e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(e*(c*d^2 - b*d*e + a*e^
2)) - ((c*e*f - c*d*g + b*d*h - a*e*h)*Log[a + b*x + c*x^2])/(2*c*(c*d^2 -
b*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{e^2f-deg+d^2h}{\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{cdf-bef+aeg-adh-(cef-cdg+bdh-$$

$$= \frac{\left(e^2f-deg+d^2h\right)\log(d+ex)}{e\left(cd^2-bde+ae^2\right)}+\frac{\int \frac{cdf-bef+aeg-adh-(cef-cdg+bdh-aeh)x}{a+bx+cx^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{\left(e^2f-deg+d^2h\right)\log(d+ex)}{e\left(cd^2-bde+ae^2\right)}-\frac{(cef-cdg+bdh-aeh)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c\left(cd^2-bde+ae^2\right)}+\frac{\left(2c^2\right.}{}$$

$$= \frac{\left(e^2f-deg+d^2h\right)\log(d+ex)}{e\left(cd^2-bde+ae^2\right)}-\frac{(cef-cdg+bdh-aeh)\log\left(a+bx+cx^2\right)}{2c\left(cd^2-bde+ae^2\right)}-$$

$$= -\frac{\left(2c^2df+b(bd-ae)h-c(bef+bdg-2aeg+2adh)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}+\frac{\left(e^2\right.}{}$$

Mathematica [A] time = 0.21, size = 193, normalized size = 0.98

$$\frac{-2e\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(c(2adh-2aeg+bdg+bef)+bh(ae-bd)-2c^2df\right)+2c\sqrt{4ac-b^2}\log(d+ex)\left(d^2h-de\right.}{2ce\sqrt{4ac-b^2}\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*e*(-2*c^2*d*f + b*(-(b*d) + a*e)*h + c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d
*h))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + 2*c*Sqrt[-b^2 + 4*a*c]*(e^2*f
 - d*e*g + d^2*h)*Log[d + e*x] - Sqrt[-b^2 + 4*a*c]*e*(c*e*f - c*d*g + b*d*
h - a*e*h)*Log[a + x*(b + c*x)])/(2*c*Sqrt[-b^2 + 4*a*c]*e*(c*d^2 + e*(-(b*
d) + a*e)))
```

fricas [A] time = 83.61, size = 625, normalized size = 3.19

$$\left[-\frac{\sqrt{b^2-4\,ac}\left(\left(2\,c^2de-bce^2\right)f-\left(bcde-2\,ace^2\right)g-\left(abe^2-\left(b^2-2\,ac\right)de\right)h\right)\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx}{cx^2+bx+a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(b^2 - 4*a*c)*((2*c^2*d*e - b*c*e^2)*f - (b*c*d*e - 2*a*c*e^2)*g
 - (a*b*e^2 - (b^2 - 2*a*c)*d*e)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c
+ sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + ((b^2*c - 4*a*c^2)*e^
2*f - (b^2*c - 4*a*c^2)*d*e*g + ((b^3 - 4*a*b*c)*d*e - (a*b^2 - 4*a^2*c)*e^
2)*h)*log(c*x^2 + b*x + a) - 2*((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)
*d*e*g + (b^2*c - 4*a*c^2)*d^2*h)*log(e*x + d))/((b^2*c^2 - 4*a*c^3)*d^2*e
- (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), -1/2*(2*sqrt(-b^2
 + 4*a*c)*((2*c^2*d*e - b*c*e^2)*f - (b*c*d*e - 2*a*c*e^2)*g - (a*b*e^2 - (
b^2 - 2*a*c)*d*e)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
+ ((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g + ((b^3 - 4*a*b*c)*d*e
```

```
 - (a*b^2 - 4*a^2*c)*e^2)*h)*log(c*x^2 + b*x + a) - 2*((b^2*c - 4*a*c^2)*e^
2*f - (b^2*c - 4*a*c^2)*d*e*g + (b^2*c - 4*a*c^2)*d^2*h)*log(e*x + d))/((b^
2*c^2 - 4*a*c^3)*d^2*e - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*
e^3)]
```

giac [A] time = 0.16, size = 204, normalized size = 1.04

$$\frac{\left(cdg-bdh-cfe+ahe\right)\log\left(cx^2+bx+a\right)}{2\left(c^2d^2-bcde+ace^2\right)}+\frac{\left(d^2h-dge+fe^2\right)\log\left(|xe+d|\right)}{cd^2e-bde^2+ae^3}+\frac{\left(2\,c^2df-bcdg+b^2dh-2\,acdh-b\right.}{\left(c^2d^2-bcde+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(c*d*g - b*d*h - c*f*e + a*h*e)*log(c*x^2 + b*x + a)/(c^2*d^2 - b*c*d*e
 + a*c*e^2) + (d^2*h - d*g*e + f*e^2)*log(abs(x*e + d))/(c*d^2*e - b*d*e^2
+ a*e^3) + (2*c^2*d*f - b*c*d*g + b^2*d*h - 2*a*c*d*h - b*c*f*e + 2*a*c*g*e
 - a*b*h*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^2 - b*c*d*e + a*
c*e^2)*sqrt(-b^2 + 4*a*c))
```

maple [B] time = 0.01, size = 622, normalized size = 3.17

$$-\frac{abeh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-bde+c\,d^2\right)\sqrt{4ac-b^2}\;c}-\frac{2adh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-bde+c\,d^2\right)\sqrt{4ac-b^2}}+\frac{2aeg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-bde+c\,d^2\right)\sqrt{4ac-b^2}}+\frac{b^2dh\arctan}{\left(a\,e^2-bde+c\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)/c*ln(c*x^2+b*x+a)*a*e*h-1/2/(a*e^2-b*d*e+c*d^2)/c*l
n(c*x^2+b*x+a)*b*d*h+1/2/(a*e^2-b*d*e+c*d^2)*ln(c*x^2+b*x+a)*d*g-1/2/(a*e^2
-b*d*e+c*d^2)*ln(c*x^2+b*x+a)*e*f-2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*h+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*e*g-1/(a*e^2-b*d*e+c*d^2)/(4*a*
c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*e*f+2/(a*e^2-b*d*e+c*d^2
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c*d*f-1/(a*e^2-b*d*
e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))/c*b*a*e*h+1/
(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))/c
*b^2*d*h-1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*b*d*g+1/(a*e^2-b*d*e+c*d^2)/e*ln(e*x+d)*d^2*h-1/(a*e^2-b*d*e+c*d^
2)*ln(e*x+d)*d*g+1/(a*e^2-b*d*e+c*d^2)*e*ln(e*x+d)*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 10.45, size = 2467, normalized size = 12.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x)

[Out] (log(a^2*b*e^4*g - 2*a*b^2*e^4*f - 2*a^3*e^4*h + 6*a^2*c*e^4*f - 4*a*c^2*d^
4*h + b^2*c*d^4*h + b^3*d^3*e*h - 2*b^3*e^4*f*x + a^2*e^4*g*(b^2 - 4*a*c)^(
1/2) + a*b^2*d*e^3*g + 6*a*c^2*d^3*e*g + b*c^2*d^3*e*f + 3*a^2*b*d*e^3*h -
10*a^2*c*d*e^3*g - 2*b^2*c*d^3*e*g + a*b^2*e^4*g*x - a^2*b*e^4*h*x - 2*a^2*
c*e^4*g*x + b^3*d*e^3*g*x + 2*c^3*d^3*e*f*x - 3*a^2*d*e^3*h*(b^2 - 4*a*c)^(
1/2) - c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) - b^2*d^3*e*h*(b^2 - 4*a*c)^(1/2) -
2*b^2*e^4*f*x*(b^2 - 4*a*c)^(1/2) - a^2*e^4*h*x*(b^2 - 4*a*c)^(1/2) - 2*c^2
*d^4*h*x*(b^2 - 4*a*c)^(1/2) - 10*a*c^2*d^2*e^2*f - 4*a*b^2*d^2*e^2*h + b^2
*c*d^2*e^2*f + 10*a^2*c*d^2*e^2*h - b^3*d^2*e^2*h*x - 2*a*b*e^4*f*(b^2 - 4*
a*c)^(1/2) - b*c*d^4*h*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d*e^3*f - 3*a*b*c*d^3*
e*h + 7*a*b*c*e^4*f*x - 5*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) - b^2*d^2*e^2
*h*x*(b^2 - 4*a*c)^(1/2) + a*b*d*e^3*g*(b^2 - 4*a*c)^(1/2) + 7*a*c*d*e^3*f*
(b^2 - 4*a*c)^(1/2) + 5*a*c*d^3*e*h*(b^2 - 4*a*c)^(1/2) + 2*b*c*d^3*e*g*(b^
2 - 4*a*c)^(1/2) + a*b*e^4*g*x*(b^2 - 4*a*c)^(1/2) + 3*a*c*e^4*f*x*(b^2 - 4
*a*c)^(1/2) + 3*a*b*c*d^2*e^2*g - 14*a*c^2*d*e^3*f*x + 5*b^2*c*d*e^3*f*x -
10*a*c^2*d^3*e*h*x - b*c^2*d^3*e*g*x + 6*a^2*c*d*e^3*h*x + 3*b^2*c*d^3*e*h*
x + 2*a*b*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) - 7*a*c*d^2*e^2*g*(b^2 - 4*a*c)^(1/
2) - b*c*d^2*e^2*f*(b^2 - 4*a*c)^(1/2) + b^2*d*e^3*g*x*(b^2 - 4*a*c)^(1/2)
+ 3*c^2*d^3*e*g*x*(b^2 - 4*a*c)^(1/2) + 14*a*c^2*d^2*e^2*g*x - 3*b*c^2*d^2*
e^2*f*x - 2*b^2*c*d^2*e^2*g*x + 5*a*c*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2) - 2*b
*c*d^2*e^2*g*x*(b^2 - 4*a*c)^(1/2) - 7*a*b*c*d*e^3*g*x - 5*a*c*d*e^3*g*x*(b
^2 - 4*a*c)^(1/2) + 5*b*c*d*e^3*f*x*(b^2 - 4*a*c)^(1/2) + b*c*d^3*e*h*x*(b^
2 - 4*a*c)^(1/2) + a*b*c*d^2*e^2*h*x)*(b^3*d*h + 4*a*c^2*d*g - 4*a*c^2*e*f
- a*b^2*e*h - b^2*c*d*g + b^2*c*e*f + 4*a^2*c*e*h - 2*c^2*d*f*(b^2 - 4*a*c)
^(1/2) - b^2*d*h*(b^2 - 4*a*c)^(1/2) - 4*a*b*c*d*h + a*b*e*h*(b^2 - 4*a*c)^
(1/2) + 2*a*c*d*h*(b^2 - 4*a*c)^(1/2) - 2*a*c*e*g*(b^2 - 4*a*c)^(1/2) + b*c
*d*g*(b^2 - 4*a*c)^(1/2) + b*c*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^2 +
4*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a*b*c^2*d*e)) - (
log(a^2*b*e^4*g - 2*a*b^2*e^4*f - 2*a^3*e^4*h + 6*a^2*c*e^4*f - 4*a*c^2*d^4
*h + b^2*c*d^4*h + b^3*d^3*e*h - 2*b^3*e^4*f*x - a^2*e^4*g*(b^2 - 4*a*c)^(1
/2) + a*b^2*d*e^3*g + 6*a*c^2*d^3*e*g + b*c^2*d^3*e*f + 3*a^2*b*d*e^3*h - 1
0*a^2*c*d*e^3*g - 2*b^2*c*d^3*e*g + a*b^2*e^4*g*x - a^2*b*e^4*h*x - 2*a^2*c
*e^4*g*x + b^3*d*e^3*g*x + 2*c^3*d^3*e*f*x + 3*a^2*d*e^3*h*(b^2 - 4*a*c)^(1
/2) + c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) + b^2*d^3*e*h*(b^2 - 4*a*c)^(1/2) + 2
*b^2*e^4*f*x*(b^2 - 4*a*c)^(1/2) + a^2*e^4*h*x*(b^2 - 4*a*c)^(1/2) + 2*c^2*
d^4*h*x*(b^2 - 4*a*c)^(1/2) - 10*a*c^2*d^2*e^2*f - 4*a*b^2*d^2*e^2*h + b^2*
c*d^2*e^2*f + 10*a^2*c*d^2*e^2*h - b^3*d^2*e^2*h*x + 2*a*b*e^4*f*(b^2 - 4*a
*c)^(1/2) + b*c*d^4*h*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d*e^3*f - 3*a*b*c*d^3*e
*h + 7*a*b*c*e^4*f*x + 5*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) + b^2*d^2*e^2*
h*x*(b^2 - 4*a*c)^(1/2) - a*b*d*e^3*g*(b^2 - 4*a*c)^(1/2) - 7*a*c*d*e^3*f*(
b^2 - 4*a*c)^(1/2) - 5*a*c*d^3*e*h*(b^2 - 4*a*c)^(1/2) - 2*b*c*d^3*e*g*(b^2
 - 4*a*c)^(1/2) - a*b*e^4*g*x*(b^2 - 4*a*c)^(1/2) - 3*a*c*e^4*f*x*(b^2 - 4*
a*c)^(1/2) + 3*a*b*c*d^2*e^2*g - 14*a*c^2*d*e^3*f*x + 5*b^2*c*d*e^3*f*x - 1
0*a*c^2*d^3*e*h*x - b*c^2*d^3*e*g*x + 6*a^2*c*d*e^3*h*x + 3*b^2*c*d^3*e*h*x
 - 2*a*b*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) + 7*a*c*d^2*e^2*g*(b^2 - 4*a*c)^(1/2
) + b*c*d^2*e^2*f*(b^2 - 4*a*c)^(1/2) - b^2*d*e^3*g*x*(b^2 - 4*a*c)^(1/2) -
 3*c^2*d^3*e*g*x*(b^2 - 4*a*c)^(1/2) + 14*a*c^2*d^2*e^2*g*x - 3*b*c^2*d^2*e
^2*f*x - 2*b^2*c*d^2*e^2*g*x - 5*a*c*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2) + 2*b*
c*d^2*e^2*g*x*(b^2 - 4*a*c)^(1/2) - 7*a*b*c*d*e^3*g*x + 5*a*c*d*e^3*g*x*(b^
2 - 4*a*c)^(1/2) - 5*b*c*d*e^3*f*x*(b^2 - 4*a*c)^(1/2) - b*c*d^3*e*h*x*(b^2
 - 4*a*c)^(1/2) + a*b*c*d^2*e^2*h*x)*(4*a*c^2*e*f - 4*a*c^2*d*g - b^3*d*h +
 a*b^2*e*h + b^2*c*d*g - b^2*c*e*f - 4*a^2*c*e*h - 2*c^2*d*f*(b^2 - 4*a*c)^
(1/2) - b^2*d*h*(b^2 - 4*a*c)^(1/2) + 4*a*b*c*d*h + a*b*e*h*(b^2 - 4*a*c)^(
1/2) + 2*a*c*d*h*(b^2 - 4*a*c)^(1/2) - 2*a*c*e*g*(b^2 - 4*a*c)^(1/2) + b*c*
d*g*(b^2 - 4*a*c)^(1/2) + b*c*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^2 + 4
*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a*b*c^2*d*e)) + (l
og(d + e*x)*(e^2*f + d^2*h - d*e*g))/(a*e^3 - b*d*e^2 + c*d^2*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)/(c*x**2+b*x+a),x)
[Out] Timed out
```

3.153 $\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=316

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2}$$

```
[Out] (-d^2*h+d*e*g-e^2*f)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)+(c*d*(-d*g+2*e*f)+a*e*(-
2*d*h+e*g)-b*(-d^2*h+e^2*f))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2-1/2*(c*d*(-d*g
+2*e*f)+a*e*(-2*d*h+e*g)-b*(-d^2*h+e^2*f))*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d
^2)^2-(2*c^2*d^2*f+2*a^2*e^2*h-a*b*e*(2*d*h+e*g)+b^2*(d^2*h+e^2*f)-c*(b*d*(
d*g+2*e*f)+2*a*(d^2*h-2*d*e*g+e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)/(a*e^2-b*d*e+c*d^2)^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1628, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x]
```

```
[Out] -((e^2*f - d*e*g + d^2*h)/(e*(c*d^2 - b*d*e + a*e^2)*(d + e*x))) - ((2*c^2*
d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c*(b*d*(2
*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2
- 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2) + ((c*d*(2*e*f - d
*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))*Log[d + e*x])/(c*d^2 - b*d*e +
 a*e^2)^2 - ((c*d*(2*e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))*Lo
g[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)}\,dx = \int\left(\frac{e^2f-deg+d^2h}{\left(cd^2-bde+ae^2\right)(d+ex)^2}+\frac{e\left(cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)\right.}{\left(cd^2-bde+ae^2\right)^2(d+ex)}\right.$$

$$=-\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{\left(cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)\right)\mathrm{lo}}{\left(cd^2-bde+ae^2\right)^2}$$

$$=-\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{\left(cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)\right)\mathrm{lo}}{\left(cd^2-bde+ae^2\right)^2}$$

$$=-\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{\left(cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)\right)\mathrm{lo}}{\left(cd^2-bde+ae^2\right)^2}$$

$$=-\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}-\frac{\left(2c^2d^2f+2a^2e^2h-abe(eg+2dh)+b^2\left(e^2f+d^2\right.\right.}{\sqrt{b^2-}}$$

Mathematica [A] time = 0.54, size = 281, normalized size = 0.89

$$\frac{\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)}{\sqrt{4ac-b^2}}-\frac{2\left(e(ae-bd)+cd^2\right)\left(d^2h-deg+e^2f\right)}{e(d+ex)}+2\log(d}{2\left(e(ae\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x]
```

```
[Out] ((-2*(c*d^2 + e*(-(b*d) + a*e))*(e^2*f - d*e*g + d^2*h))/(e*(d + e*x)) + (2
*(2*c^2*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c
*(b*d*(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(c*d*(2*e*f - d*g) + a*e*(e*g - 2
*d*h) + b*(-(e^2*f) + d^2*h))*Log[d + e*x] + (c*d*(-2*e*f + d*g) + a*e*(-(e
*g) + 2*d*h) + b*(e^2*f - d^2*h))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*
d) + a*e))^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.20, size = 449, normalized size = 1.42

$$\frac{\left(2\,c^2d^2fe^2 - bcd^2ge^2 + b^2d^2he^2 - 2\,acd^2he^2 - 2\,bcdfe^3 + 4\,acdge^3 - 2\,abdhe^3 + b^2fe^4 - 2\,acfe^4 - abge^4 + 2\,a^2\right.}{\left(c^2d^4 - 2\,bcd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,abde^3 + a^2e^4\right)\sqrt{-b^2 + 4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] (2*c^2*d^2*f*e^2 - b*c*d^2*g*e^2 + b^2*d^2*h*e^2 - 2*a*c*d^2*h*e^2 - 2*b*c*
d*f*e^3 + 4*a*c*d*g*e^3 - 2*a*b*d*h*e^3 + b^2*f*e^4 - 2*a*c*f*e^4 - a*b*g*e
^4 + 2*a^2*h*e^4)*arctan((2*c*d - 2*c*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e +
d) - 2*a*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((c^2*d^4 - 2*b*c
*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-b^2 + 4
*a*c)) + 1/2*(c*d^2*g - b*d^2*h - 2*c*d*f*e + 2*a*d*h*e + b*f*e^2 - a*g*e^2
)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e
+ d)^2 + a*e^2/(x*e + d)^2)/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^
2*e^2 - 2*a*b*d*e^3 + a^2*e^4) - (d^2*h*e/(x*e + d) - d*g*e^2/(x*e + d) + f
*e^3/(x*e + d))/(c*d^2*e^2 - b*d*e^3 + a*e^4)
```

maple [B] time = 0.01, size = 1125, normalized size = 3.56

$$\frac{2a^2e^2h\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-bde+c\,d^2\right)^2\sqrt{4ac-b^2}}-\frac{2abdeh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-bde+c\,d^2\right)^2\sqrt{4ac-b^2}}-\frac{ab\,e^2g\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-bde+c\,d^2\right)^2\sqrt{4ac-b^2}}-\frac{2ac\,d^2h\,\mathrm{arc}}{\left(a\,e^2-bde+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x)
```

```
[Out] -1/(a*e^2-b*d*e+c*d^2)*e/(e*x+d)*f+1/(a*e^2-b*d*e+c*d^2)/(e*x+d)*d*g-2/(a*e
^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c
*e^2*f-1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*b*c*d^2*g-1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))*a*b*e^2*g-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c*d^2*h-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*
c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c*d*e*f-2/(a*e^2-b*d*e+c
*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*d*e*h+4/(
a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*
a*c*d*e*g-1/2/(a*e^2-b*d*e+c*d^2)^2*ln(c*x^2+b*x+a)*g*e^2*a-1/2/(a*e^2-b*d*
e+c*d^2)^2*ln(c*x^2+b*x+a)*b*d^2*h+1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2
)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*e^2*f+2/(a*e^2-b*d*e+c*d^2)^2/(4*
a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2*d^2*f-2/(a*e^2-b*d*e
+c*d^2)^2*ln(e*x+d)*a*d*e*h+2/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*c*d*f*e+1/(a*
e^2-b*d*e+c*d^2)^2*ln(c*x^2+b*x+a)*a*d*e*h-1/(a*e^2-b*d*e+c*d^2)^2*c*ln(c*x
^2+b*x+a)*d*f*e+2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))*a^2*e^2*h+1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d^2*h+1/2/(a*e^2-b*d*e+c*d^2)^2*ln(c*x^
2+b*x+a)*f*e^2*b+1/2/(a*e^2-b*d*e+c*d^2)^2*c*ln(c*x^2+b*x+a)*g*d^2-1/(a*e^2
-b*d*e+c*d^2)/e/(e*x+d)*d^2*h+1/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*a*e^2*g+1/(
a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*b*d^2*h-1/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*b*
e^2*f-1/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*c*d^2*g
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 14.71, size = 3991, normalized size = 12.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x)
```

```
[Out] (log(d + e*x)*(e^2*(a*g - b*f) + d^2*(b*h - c*g) - d*e*(2*a*h - 2*c*f)))/(a
^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2)
 + (log(2*a*b^3*e^4*f - 2*b^2*c^2*d^4*g - 2*a^2*b^2*e^4*g + 6*a*c^3*d^4*g +
 b*c^3*d^4*f + a^3*b*e^4*h + 6*a^3*c*e^4*g + 2*b^3*c*d^4*h + 2*b^4*e^4*f*x
+ 2*c^4*d^4*f*x - c^3*d^4*f*(b^2 - 4*a*c)^(1/2) + a^3*e^4*h*(b^2 - 4*a*c)^(
1/2) - 7*a^2*b*c*e^4*f - 7*a*b*c^2*d^4*h - 16*a*c^3*d^3*e*f - 16*a^3*c*d*e^
3*h - 2*a*b^3*e^4*g*x - 2*a*c^3*d^4*h*x - b*c^3*d^4*g*x - 2*a^3*c*e^4*h*x +
 2*a*b^2*e^4*f*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*e^4*g*(b^2 - 4*a*c)^(1/2) - a^
2*c*e^4*f*(b^2 - 4*a*c)^(1/2) + a*c^2*d^4*h*(b^2 - 4*a*c)^(1/2) + 2*b*c^2*d
^4*g*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d^4*h*(b^2 - 4*a*c)^(1/2) + 2*b^3*e^4*f*
x*(b^2 - 4*a*c)^(1/2) + 3*c^3*d^4*g*x*(b^2 - 4*a*c)^(1/2) + 16*a^2*c^2*d*e^
3*f - a*b^3*d^2*e^2*h + 2*a^2*b^2*d*e^3*h + 2*b^2*c^2*d^3*e*f - b^3*c*d^2*e
^2*f + 16*a^2*c^2*d^3*e*h + 2*a^2*c^2*e^4*f*x + a^2*b^2*e^4*h*x + b^2*c^2*d
^4*h*x - b^4*d^2*e^2*h*x - 20*a^2*c^2*d^2*e^2*g + 14*a*c^2*d^2*e^2*f*(b^2 -
 4*a*c)^(1/2) - a*b^2*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) + b^2*c*d^2*e^2*f*(b^2
- 4*a*c)^(1/2) - 14*a^2*c*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) - b^3*d^2*e^2*h*x*(
b^2 - 4*a*c)^(1/2) + 10*b^2*c^2*d^2*e^2*f*x + 28*a^2*c^2*d^2*e^2*h*x - 6*a*
b^2*c*d*e^3*f + 4*a*b*c^2*d^3*e*g + 4*a^2*b*c*d*e^3*g - 6*a*b^2*c*d^3*e*h -
 8*a*b^2*c*e^4*f*x + 7*a^2*b*c*e^4*g*x + 2*a*b^3*d*e^3*h*x + 16*a*c^3*d^3*e
*g*x - 4*b*c^3*d^3*e*f*x - 8*b^3*c*d*e^3*f*x + 2*b^3*c*d^3*e*h*x - 8*a*c^2*
d^3*e*g*(b^2 - 4*a*c)^(1/2) - 2*b*c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) + 2*a^2*b
*d*e^3*h*(b^2 - 4*a*c)^(1/2) + 8*a^2*c*d*e^3*g*(b^2 - 4*a*c)^(1/2) - 2*a*b^
2*e^4*g*x*(b^2 - 4*a*c)^(1/2) + a^2*b*e^4*h*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*c
*e^4*g*x*(b^2 - 4*a*c)^(1/2) - 3*b*c^2*d^4*h*x*(b^2 - 4*a*c)^(1/2) - 8*c^3*
d^3*e*f*x*(b^2 - 4*a*c)^(1/2) + 10*a*b*c^2*d^2*e^2*f + 2*a*b^2*c*d^2*e^2*g
+ 10*a^2*b*c*d^2*e^2*h - 28*a*c^3*d^2*e^2*f*x - 16*a^2*c^2*d*e^3*g*x - 2*b^
2*c^2*d^3*e*g*x + b^3*c*d^2*e^2*g*x + 8*a*c^2*d*e^3*f*x*(b^2 - 4*a*c)^(1/2)
 + 2*a*b^2*d*e^3*h*x*(b^2 - 4*a*c)^(1/2) - 8*b^2*c*d*e^3*f*x*(b^2 - 4*a*c)^
(1/2) + 8*a*c^2*d^3*e*h*x*(b^2 - 4*a*c)^(1/2) - 2*b*c^2*d^3*e*g*x*(b^2 - 4*
a*c)^(1/2) - 8*a^2*c*d*e^3*h*x*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d^3*e*h*x*(b^2
 - 4*a*c)^(1/2) - 10*a*b*c^2*d^2*e^2*g*x - 10*a*c^2*d^2*e^2*g*x*(b^2 - 4*a*
c)^(1/2) + 12*b*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) + b^2*c*d^2*e^2*g*x*(b^
2 - 4*a*c)^(1/2) - 10*a*b*c*d*e^3*f*(b^2 - 4*a*c)^(1/2) + 10*a*b*c*d^3*e*h*
(b^2 - 4*a*c)^(1/2) - 4*a*b*c*e^4*f*x*(b^2 - 4*a*c)^(1/2) + 28*a*b*c^2*d*e^
3*f*x + 6*a*b^2*c*d*e^3*g*x - 12*a*b*c^2*d^3*e*h*x - 12*a^2*b*c*d*e^3*h*x +
 6*a*b*c*d*e^3*g*x*(b^2 - 4*a*c)^(1/2) - 2*a*b*c*d^2*e^2*h*x*(b^2 - 4*a*c)^
(1/2))*(b^3*d^2*h - b^3*e^2*f + a*b^2*e^2*g + 4*a*c^2*d^2*g - 4*a^2*c*e^2*g
 - b^2*c*d^2*g - b^2*e^2*f*(b^2 - 4*a*c)^(1/2) - 2*c^2*d^2*f*(b^2 - 4*a*c)^
(1/2) - 2*a^2*e^2*h*(b^2 - 4*a*c)^(1/2) - b^2*d^2*h*(b^2 - 4*a*c)^(1/2) + 4
*a*b*c*e^2*f - 4*a*b*c*d^2*h - 8*a*c^2*d*e*f - 2*a*b^2*d*e*h + 2*b^2*c*d*e*
f + 8*a^2*c*d*e*h + a*b*e^2*g*(b^2 - 4*a*c)^(1/2) + 2*a*c*e^2*f*(b^2 - 4*a*
c)^(1/2) + 2*a*c*d^2*h*(b^2 - 4*a*c)^(1/2) + b*c*d^2*g*(b^2 - 4*a*c)^(1/2)
+ 2*a*b*d*e*h*(b^2 - 4*a*c)^(1/2) - 4*a*c*d*e*g*(b^2 - 4*a*c)^(1/2) + 2*b*c
*d*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 -
b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3
*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2)) - (log(2*a*b^3
```

```
*e^4*f - 2*b^2*c^2*d^4*g - 2*a^2*b^2*e^4*g + 6*a*c^3*d^4*g + b*c^3*d^4*f +
a^3*b*e^4*h + 6*a^3*c*e^4*g + 2*b^3*c*d^4*h + 2*b^4*e^4*f*x + 2*c^4*d^4*f*x
 + c^3*d^4*f*(b^2 - 4*a*c)^(1/2) - a^3*e^4*h*(b^2 - 4*a*c)^(1/2) - 7*a^2*b*
c*e^4*f - 7*a*b*c^2*d^4*h - 16*a*c^3*d^3*e*f - 16*a^3*c*d*e^3*h - 2*a*b^3*e
^4*g*x - 2*a*c^3*d^4*h*x - b*c^3*d^4*g*x - 2*a^3*c*e^4*h*x - 2*a*b^2*e^4*f*
(b^2 - 4*a*c)^(1/2) + 2*a^2*b*e^4*g*(b^2 - 4*a*c)^(1/2) + a^2*c*e^4*f*(b^2
- 4*a*c)^(1/2) - a*c^2*d^4*h*(b^2 - 4*a*c)^(1/2) - 2*b*c^2*d^4*g*(b^2 - 4*a
*c)^(1/2) + 2*b^2*c*d^4*h*(b^2 - 4*a*c)^(1/2) - 2*b^3*e^4*f*x*(b^2 - 4*a*c)
^(1/2) - 3*c^3*d^4*g*x*(b^2 - 4*a*c)^(1/2) + 16*a^2*c^2*d*e^3*f - a*b^3*d^2
*e^2*h + 2*a^2*b^2*d*e^3*h + 2*b^2*c^2*d^3*e*f - b^3*c*d^2*e^2*f + 16*a^2*c
^2*d^3*e*h + 2*a^2*c^2*e^4*f*x + a^2*b^2*e^4*h*x + b^2*c^2*d^4*h*x - b^4*d^
2*e^2*h*x - 20*a^2*c^2*d^2*e^2*g - 14*a*c^2*d^2*e^2*f*(b^2 - 4*a*c)^(1/2) +
 a*b^2*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) - b^2*c*d^2*e^2*f*(b^2 - 4*a*c)^(1/2)
+ 14*a^2*c*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) + b^3*d^2*e^2*h*x*(b^2 - 4*a*c)^(1
/2) + 10*b^2*c^2*d^2*e^2*f*x + 28*a^2*c^2*d^2*e^2*h*x - 6*a*b^2*c*d*e^3*f +
 4*a*b*c^2*d^3*e*g + 4*a^2*b*c*d*e^3*g - 6*a*b^2*c*d^3*e*h - 8*a*b^2*c*e^4*
f*x + 7*a^2*b*c*e^4*g*x + 2*a*b^3*d*e^3*h*x + 16*a*c^3*d^3*e*g*x - 4*b*c^3*
d^3*e*f*x - 8*b^3*c*d*e^3*f*x + 2*b^3*c*d^3*e*h*x + 8*a*c^2*d^3*e*g*(b^2 -
4*a*c)^(1/2) + 2*b*c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*d*e^3*h*(b^2 -
 4*a*c)^(1/2) - 8*a^2*c*d*e^3*g*(b^2 - 4*a*c)^(1/2) + 2*a*b^2*e^4*g*x*(b^2
- 4*a*c)^(1/2) - a^2*b*e^4*h*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*c*e^4*g*x*(b^2 -
 4*a*c)^(1/2) + 3*b*c^2*d^4*h*x*(b^2 - 4*a*c)^(1/2) + 8*c^3*d^3*e*f*x*(b^2
- 4*a*c)^(1/2) + 10*a*b*c^2*d^2*e^2*f + 2*a*b^2*c*d^2*e^2*g + 10*a^2*b*c*d^
2*e^2*h - 28*a*c^3*d^2*e^2*f*x - 16*a^2*c^2*d*e^3*g*x - 2*b^2*c^2*d^3*e*g*x
 + b^3*c*d^2*e^2*g*x - 8*a*c^2*d*e^3*f*x*(b^2 - 4*a*c)^(1/2) - 2*a*b^2*d*e^
3*h*x*(b^2 - 4*a*c)^(1/2) + 8*b^2*c*d*e^3*f*x*(b^2 - 4*a*c)^(1/2) - 8*a*c^2
*d^3*e*h*x*(b^2 - 4*a*c)^(1/2) + 2*b*c^2*d^3*e*g*x*(b^2 - 4*a*c)^(1/2) + 8*
a^2*c*d*e^3*h*x*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d^3*e*h*x*(b^2 - 4*a*c)^(1/2)
 - 10*a*b*c^2*d^2*e^2*g*x + 10*a*c^2*d^2*e^2*g*x*(b^2 - 4*a*c)^(1/2) - 12*b
*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) - b^2*c*d^2*e^2*g*x*(b^2 - 4*a*c)^(1/2
) + 10*a*b*c*d*e^3*f*(b^2 - 4*a*c)^(1/2) - 10*a*b*c*d^3*e*h*(b^2 - 4*a*c)^(
1/2) + 4*a*b*c*e^4*f*x*(b^2 - 4*a*c)^(1/2) + 28*a*b*c^2*d*e^3*f*x + 6*a*b^2
*c*d*e^3*g*x - 12*a*b*c^2*d^3*e*h*x - 12*a^2*b*c*d*e^3*h*x - 6*a*b*c*d*e^3*
g*x*(b^2 - 4*a*c)^(1/2) + 2*a*b*c*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2))*(b^3*e^2
*f - b^3*d^2*h - a*b^2*e^2*g - 4*a*c^2*d^2*g + 4*a^2*c*e^2*g + b^2*c*d^2*g
- b^2*e^2*f*(b^2 - 4*a*c)^(1/2) - 2*c^2*d^2*f*(b^2 - 4*a*c)^(1/2) - 2*a^2*e
^2*h*(b^2 - 4*a*c)^(1/2) - b^2*d^2*h*(b^2 - 4*a*c)^(1/2) - 4*a*b*c*e^2*f +
4*a*b*c*d^2*h + 8*a*c^2*d*e*f + 2*a*b^2*d*e*h - 2*b^2*c*d*e*f - 8*a^2*c*d*e
*h + a*b*e^2*g*(b^2 - 4*a*c)^(1/2) + 2*a*c*e^2*f*(b^2 - 4*a*c)^(1/2) + 2*a*
c*d^2*h*(b^2 - 4*a*c)^(1/2) + b*c*d^2*g*(b^2 - 4*a*c)^(1/2) + 2*a*b*d*e*h*(
b^2 - 4*a*c)^(1/2) - 4*a*c*d*e*g*(b^2 - 4*a*c)^(1/2) + 2*b*c*d*e*f*(b^2 - 4
*a*c)^(1/2)))/(2*(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^4 - b
^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2*
d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2)) - (e^2*f + d^2*h - d*e*g)/(e*
(d + e*x)*(a*e^2 + c*d^2 - b*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)**2/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.154 $\int \frac{f+gx+hx^2}{(d+ex)^3\left(a+bx+cx^2\right)}\,dx$

<u>**Optimal**</u>. Leaf size=509

$$-\frac{\log\left(a+bx+cx^2\right)\left(e^3\left(a^2h-abg+b^2f\right)-c\left(ae\left(3d^2h-3deg+e^2f\right)+b\left(3de^2f-d^3h\right)\right)+c^2d^2(3ef-dg)\right)}{2\left(ae^2-bde+cd^2\right)^3}+\frac{\log\ldots}{\ldots}$$

```
[Out] 1/2*(-d^2*h+d*e*g-e^2*f)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^2+(-c*d*(-d*g+2*e*f)
-a*e*(-2*d*h+e*g)+b*(-d^2*h+e^2*f))/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)+(c^2*d^2*
(-d*g+3*e*f)+e^3*(a^2*h-a*b*g+b^2*f)-c*(a*e*(3*d^2*h-3*d*e*g+e^2*f)+b*(-d^3
*h+3*d*e^2*f)))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^3-1/2*(c^2*d^2*(-d*g+3*e*f)+e
^3*(a^2*h-a*b*g+b^2*f)-c*(a*e*(3*d^2*h-3*d*e*g+e^2*f)+b*(-d^3*h+3*d*e^2*f))
)*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d^2)^3-(2*c^3*d^3*f-b*e^3*(a^2*h-a*b*g+b^2
*f)-c^2*d*(b*d*(d*g+3*e*f)+2*a*(d^2*h-3*d*e*g+3*e^2*f))-c*(2*a^2*e^2*(-3*d*
h+e*g)-3*a*b*e*(-d^2*h-d*e*g+e^2*f)-b^2*(d^3*h+3*d*e^2*f)))*arctanh((2*c*x+
b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)^3/(-4*a*c+b^2)^(1/2)
```

<u>**Rubi**</u> **[A]** time = 1.25, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1628, 634, 618, 206, 628}

$$-\frac{\log\left(a+bx+cx^2\right)\left(e^3\left(a^2h-abg+b^2f\right)-ace\left(3d^2h-3deg+e^2f\right)-bc\left(3de^2f-d^3h\right)+c^2d^2(3ef-dg)\right)}{2\left(ae^2-bde+cd^2\right)^3}+\frac{\log(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x]
```

```
[Out] -(e^2*f - d*e*g + d^2*h)/(2*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (c*d*(
2*e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))/((c*d^2 - b*d*e + a*e
^2)^2*(d + e*x)) - ((2*c^3*d^3*f - b*e^3*(b^2*f - a*b*g + a^2*h) - c^2*d*(b
*d*(3*e*f + d*g) + 2*a*(3*e^2*f - 3*d*e*g + d^2*h)) - c*(2*a^2*e^2*(e*g - 3
*d*h) - 3*a*b*e*(e^2*f - d*e*g - d^2*h) - b^2*(3*d*e^2*f + d^3*h)))*ArcTanh
[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)
^3) + ((c^2*d^2*(3*e*f - d*g) + e^3*(b^2*f - a*b*g + a^2*h) - a*c*e*(e^2*f
- 3*d*e*g + 3*d^2*h) - b*c*(3*d*e^2*f - d^3*h))*Log[d + e*x])/(c*d^2 - b*d*
e + a*e^2)^3 - ((c^2*d^2*(3*e*f - d*g) + e^3*(b^2*f - a*b*g + a^2*h) - a*c*
e*(e^2*f - 3*d*e*g + 3*d^2*h) - b*c*(3*d*e^2*f - d^3*h))*Log[a + b*x + c*x^
2])/(2*(c*d^2 - b*d*e + a*e^2)^3)
```

<u>Rule 206</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 618</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

<u>Rule 628</u>

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)^3\left(a+bx+cx^2\right)}dx = \int \left(\frac{e^2f-deg+d^2h}{\left(cd^2-bde+ae^2\right)(d+ex)^3}+\frac{e\left(cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d\right.\right.}{\left(cd^2-bde+ae^2\right)^2(d+ex)^2}\right.$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2}-\frac{cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right.}{\left(cd^2-bde+ae^2\right)^2(d+ex)}$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2}-\frac{cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right.}{\left(cd^2-bde+ae^2\right)^2(d+ex)}$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2}-\frac{cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right.}{\left(cd^2-bde+ae^2\right)^2(d+ex)}$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2}-\frac{cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right.}{\left(cd^2-bde+ae^2\right)^2(d+ex)}$$

Mathematica [A] time = 0.77, size = 504, normalized size = 0.99

$$-\frac{\log(d+ex)\left(-\left(e^3\left(a^2h-abg+b^2f\right)\right)+ace\left(3d^2h-3deg+e^2f\right)+bc\left(3de^2f-d^3h\right)+c^2d^2(dg-3ef)\right)}{\left(e(ae-bd)+cd^2\right)^3}+\frac{\log(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x]
```

```
[Out] -1/2*(e^2*f - d*e*g + d^2*h)/(e*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) + (
c*d*(-2*e*f + d*g) + a*e*(-(e*g) + 2*d*h) + b*(e^2*f - d^2*h))/((c*d^2 + e*
(-(b*d) + a*e))^2*(d + e*x)) + ((-2*c^3*d^3*f + b*e^3*(b^2*f - a*b*g + a^2*
h) + c^2*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f - 3*d*e*g + d^2*h)) - c*(-2*a^
2*e^2*(e*g - 3*d*h) + 3*a*b*e*(e^2*f - d*e*g - d^2*h) + b^2*(3*d*e^2*f + d^
3*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*(-(c*d^2
) + e*(b*d - a*e))^3) - ((c^2*d^2*(-3*e*f + d*g) - e^3*(b^2*f - a*b*g + a^2
*h) + a*c*e*(e^2*f - 3*d*e*g + 3*d^2*h) + b*c*(3*d*e^2*f - d^3*h))*Log[d +
e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 + ((c^2*d^2*(-3*e*f + d*g) - e^3*(b^2*f
- a*b*g + a^2*h) + a*c*e*(e^2*f - 3*d*e*g + 3*d^2*h) + b*c*(3*d*e^2*f - d^3
*h))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.21, size = 1002, normalized size = 1.97

$$\frac{\left(c^2d^3g - bcd^3h - 3\,c^2d^2fe + 3\,acd^2he + 3\,bcdfe^2 - 3\,acdge^2 - b^2fe^3 + acfe^3 + abge^3 - a^2he^3\right)\log\left(cx^2 + bx + a\right)}{2\left(c^3d^6 - 3\,bc^2d^5e + 3\,b^2cd^4e^2 + 3\,ac^2d^4e^2 - b^3d^3e^3 - 6\,abcd^3e^3 + 3\,ab^2d^2e^4 + 3\,a^2cd^2e^4 - 3\,a^2bde^5 + a^3e^6\right)}.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(c^2*d^3*g - b*c*d^3*h - 3*c^2*d^2*f*e + 3*a*c*d^2*h*e + 3*b*c*d*f*e^2
- 3*a*c*d*g*e^2 - b^2*f*e^3 + a*c*f*e^3 + a*b*g*e^3 - a^2*h*e^3)*log(c*x^2
+ b*x + a)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b
^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*
d*e^5 + a^3*e^6) - (c^2*d^3*g*e - b*c*d^3*h*e - 3*c^2*d^2*f*e^2 + 3*a*c*d^2
*h*e^2 + 3*b*c*d*f*e^3 - 3*a*c*d*g*e^3 - b^2*f*e^4 + a*c*f*e^4 + a*b*g*e^4
- a^2*h*e^4)*log(abs(x*e + d))/(c^3*d^6*e - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e
^3 + 3*a*c^2*d^4*e^3 - b^3*d^3*e^4 - 6*a*b*c*d^3*e^4 + 3*a*b^2*d^2*e^5 + 3*
a^2*c*d^2*e^5 - 3*a^2*b*d*e^6 + a^3*e^7) + (2*c^3*d^3*f - b*c^2*d^3*g + b^2
*c*d^3*h - 2*a*c^2*d^3*h - 3*b*c^2*d^2*f*e + 6*a*c^2*d^2*g*e - 3*a*b*c*d^2*
h*e + 3*b^2*c*d*f*e^2 - 6*a*c^2*d*f*e^2 - 3*a*b*c*d*g*e^2 + 6*a^2*c*d*h*e^2
 - b^3*f*e^3 + 3*a*b*c*f*e^3 + a*b^2*g*e^3 - 2*a^2*c*g*e^3 - a^2*b*h*e^3)*a
rctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d
^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4
+ 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*sqrt(-b^2 + 4*a*c)) - 1/2*(c^2
*d^6*h - 3*c^2*d^5*g*e + 5*c^2*d^4*f*e^2 + 4*b*c*d^4*g*e^2 - b^2*d^4*h*e^2
- 2*a*c*d^4*h*e^2 - 8*b*c*d^3*f*e^3 - b^2*d^3*g*e^3 - 2*a*c*d^3*g*e^3 + 4*a
*b*d^3*h*e^3 + 3*b^2*d^2*f*e^4 + 6*a*c*d^2*f*e^4 - 3*a^2*d^2*h*e^4 - 4*a*b*
d*f*e^5 + a^2*d*g*e^5 + a^2*f*e^6 - 2*(c^2*d^4*g*e^2 - b*c*d^4*h*e^2 - 2*c^
2*d^3*f*e^3 - b*c*d^3*g*e^3 + b^2*d^3*h*e^3 + 2*a*c*d^3*h*e^3 + 3*b*c*d^2*f
*e^4 - 3*a*b*d^2*h*e^4 - b^2*d*f*e^5 - 2*a*c*d*f*e^5 + a*b*d*g*e^5 + 2*a^2*
d*h*e^5 + a*b*f*e^6 - a^2*g*e^6)*x)*e^(-1)/((c*d^2 - b*d*e + a*e^2)^3*(x*e
+ d)^2)
```

maple [B] time = 0.02, size = 1945, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x)
```

```
[Out] -3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))*b*c^2*d^2*e*f+6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*a^2*c*d*e^2*h+3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2
)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*e^3*f+6/(a*e^2-b*d*e+c*d^2)^3/(
4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^2*d^2*e*g-6/(a*e^2
-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^2
*d*e^2*f+3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))*b^2*c*d*e^2*f-1/2/(a*e^2-b*d*e+c*d^2)*e/(e*x+d)^2*f+1/2/(a*e^2-
b*d*e+c*d^2)/(e*x+d)^2*d*g+1/2/(a*e^2-b*d*e+c*d^2)^3*ln(c*x^2+b*x+a)*g*e^3*
```

```
b*a+1/2/(a*e^2-b*d*e+c*d^2)^3*c*ln(c*x^2+b*x+a)*f*e^3*a-1/2/(a*e^2-b*d*e+c*
d^2)^3*c*ln(c*x^2+b*x+a)*b*d^3*h-3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*d*e^2*g-3/(a*e^2-b*d*e+c*d^2)^3/(
4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*d^2*e*h+1/(a*e^2
-b*d*e+c*d^2)^3*ln(e*x+d)*a^2*e^3*h+1/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*b^2*e
^3*f-1/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*c^2*d^3*g-1/(a*e^2-b*d*e+c*d^2)^2/(e
*x+d)*a*e^2*g-1/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*b*d^2*h+1/(a*e^2-b*d*e+c*d^2)
^2/(e*x+d)*b*e^2*f+1/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*c*g*d^2-3/2/(a*e^2-b*d*e
+c*d^2)^3*c*ln(c*x^2+b*x+a)*d*g*e^2*a+3/2/(a*e^2-b*d*e+c*d^2)^3*c*ln(c*x^2+
b*x+a)*d*f*e^2*b-1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))*a^2*b*e^3*h-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c*e^3*g+1/(a*e^2-b*d*e+c*d^2)^3/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e^3*g-2/(a*e^2-b*d*
e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^2*d^3*
h+1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))*b^2*c*d^3*h-1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))*b*c^2*d^3*g+3/2/(a*e^2-b*d*e+c*d^2)^3*c*ln(c*x^2+b*x+a)
*a*d^2*e*h-3/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*a*c*d^2*e*h+3/(a*e^2-b*d*e+c*d
^2)^3*ln(e*x+d)*a*c*d*e^2*g-3/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*b*c*d*e^2*f-3
/2/(a*e^2-b*d*e+c*d^2)^3*c^2*ln(c*x^2+b*x+a)*d^2*f*e-1/(a*e^2-b*d*e+c*d^2)^
3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^3*f+2/(a*e^2-
b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^3*d^
3*f-1/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*a*b*e^3*g-1/(a*e^2-b*d*e+c*d^2)^3*ln(
e*x+d)*a*c*e^3*f+1/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*b*c*d^3*h+3/(a*e^2-b*d*e
+c*d^2)^3*ln(e*x+d)*c^2*d^2*f*e+2/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*a*d*e*h-2/(
a*e^2-b*d*e+c*d^2)^2/(e*x+d)*c*d*e*f+1/2/(a*e^2-b*d*e+c*d^2)^3*c^2*ln(c*x^2
+b*x+a)*g*d^3-1/2/(a*e^2-b*d*e+c*d^2)^3*ln(c*x^2+b*x+a)*a^2*e^3*h-1/2/(a*e^
2-b*d*e+c*d^2)^3*ln(c*x^2+b*x+a)*f*e^3*b^2-1/2/(a*e^2-b*d*e+c*d^2)/e/(e*x+d
)^2*d^2*h
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 6.82, size = 12784, normalized size = 25.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x)
```

```
[Out] symsum(log(root(24*a^6*b*c*d*e^11*z^3 + 24*a*b*c^6*d^11*e*z^3 + 240*a^4*b*c
^3*d^5*e^7*z^3 + 240*a^3*b*c^4*d^7*e^5*z^3 + 120*a^5*b*c^2*d^3*e^9*z^3 + 12
0*a^2*b*c^5*d^9*e^3*z^3 - 54*a^5*b^2*c*d^2*e^10*z^3 - 54*a*b^2*c^5*d^10*e^2
*z^3 + 50*a^4*b^3*c*d^3*e^9*z^3 + 50*a*b^3*c^4*d^9*e^3*z^3 - 36*a^2*b^5*c*d
^5*e^7*z^3 - 36*a*b^5*c^2*d^7*e^5*z^3 + 26*a*b^6*c*d^6*e^6*z^3 - 340*a^3*b^
2*c^3*d^6*e^6*z^3 - 225*a^4*b^2*c^2*d^4*e^8*z^3 - 225*a^2*b^2*c^4*d^8*e^4*z
^3 + 180*a^3*b^3*c^2*d^5*e^7*z^3 + 180*a^2*b^3*c^3*d^7*e^5*z^3 - 30*a^2*b^4
*c^2*d^6*e^6*z^3 - 6*b^7*c*d^7*e^5*z^3 - 6*b^3*c^5*d^11*e*z^3 - 6*a^5*b^3*d
*e^11*z^3 - 6*a*b^7*d^5*e^7*z^3 - 20*b^5*c^3*d^9*e^3*z^3 + 15*b^6*c^2*d^8*e
^4*z^3 + 15*b^4*c^4*d^10*e^2*z^3 - 80*a^4*c^4*d^6*e^6*z^3 - 60*a^5*c^3*d^4*
e^8*z^3 - 60*a^3*c^5*d^8*e^4*z^3 - 24*a^6*c^2*d^2*e^10*z^3 - 24*a^2*c^6*d^1
```

0*e^2*z^3 - 20*a^3*b^5*d^3*e^9*z^3 + 15*a^4*b^4*d^2*e^10*z^3 + 15*a^2*b^6*d^4*e^8*z^3 - 4*a^7*c*e^12*z^3 - 4*a*c^7*d^12*z^3 + b^8*d^6*e^6*z^3 + b^2*c^6*d^12*z^3 + a^6*b^2*e^12*z^3 - 9*a^3*b^2*c*d*e^5*g*h*z - 9*a*b^2*c^3*d^5*e*g*h*z - 30*a^3*b*c^2*d*e^5*f*h*z + 9*a^2*b^3*c*d*e^5*f*h*z + 3*a*b^4*c*d^2*e^4*f*h*z + 27*a*b*c^4*d^4*e^2*f*g*z + 6*a^2*b^2*c^2*d^3*e^3*g*h*z - 33*a^2*b^2*c^2*d^2*e^4*f*h*z + 18*a*b*c^4*d^5*e*f*h*z - 12*a*b^4*c*d*e^5*f*g*z + 27*a^3*b*c^2*d^2*e^4*g*h*z + 27*a^2*b*c^3*d^4*e^2*g*h*z - 3*a^2*b^3*c*d^2*e^4*g*h*z - 3*a*b^3*c^2*d^4*e^2*g*h*z + 52*a^2*b*c^3*d^3*e^3*f*h*z - 4*a*b^3*c^2*d^3*e^3*f*h*z - 3*a*b^2*c^3*d^4*e^2*f*h*z - 93*a^2*b*c^3*d^2*e^4*f*g*z + 51*a^2*b^2*c^2*d*e^5*f*g*z - 34*a*b^2*c^3*d^3*e^3*f*g*z + 27*a*b^3*c^2*d^2*e^4*f*g*z - 24*a*c^5*d^5*e*f*g*z - 7*a^4*b*c*e^6*g*h*z - 7*a*b*c^4*d^6*g*h*z + a*b^4*c*d^3*e^3*g*h*z - 80*a^3*c^3*d^3*e^3*g*h*z + 3*b^4*c^2*d^4*e^2*f*h*z - 66*a^2*c^4*d^4*e^2*f*h*z + 54*a^3*c^3*d^2*e^4*f*h*z - 3*b^3*c^3*d^4*e^2*f*g*z + 80*a^2*c^4*d^3*e^3*f*g*z - 21*a^2*b*c^3*d^5*e*h^2*z + 6*a*b^3*c^2*d^5*e*h^2*z - 21*a^3*b*c^2*d*e^5*g^2*z + 6*a^2*b^3*c*d*e^5*g^2*z - 66*a*b*c^4*d^3*e^3*f^2*z - 30*a*b^3*c^2*d*e^5*f^2*z + 27*a^2*b*c^3*d*e^5*f^2*z - 12*a^2*b^2*c^2*d^4*e^2*h^2*z - 12*a^2*b^2*c^2*d^2*e^4*g^2*z + 24*a^4*c^2*d*e^5*g*h*z + 24*a^2*c^4*d^5*e*g*h*z - 3*b^3*c^3*d^5*e*f*h*z - b^5*c*d^3*e^3*f*h*z + 3*b^2*c^4*d^5*e*f*g*z - 24*a^3*c^3*d*e^5*f*g*z + 9*a^3*b^2*c*e^6*f*h*z - 10*a^2*b^3*c*e^6*f*g*z + 9*a^3*b*c^2*e^6*f*g*z + 3*a^4*b*c*d*e^5*h^2*z + 3*a*b*c^4*d^5*e*g^2*z + 14*a^3*b*c^2*d^3*e^3*h^2*z + 3*a^3*b^2*c*d^2*e^4*h^2*z - a^2*b^3*c*d^3*e^3*h^2*z + 14*a^2*b*c^3*d^3*e^3*g^2*z + 3*a*b^2*c^3*d^4*e^2*g^2*z - a*b^3*c^2*d^3*e^3*g^2*z + 63*a*b^2*c^3*d^2*e^4*f^2*z + 2*b^3*c^3*d^6*g*h*z - 6*a^4*c^2*e^6*f*h*z + 2*a^3*b^3*e^6*g*h*z - b^2*c^4*d^6*f*h*z - 2*a^2*b^4*e^6*f*h*z + 6*b^5*c*d*e^5*f^2*z + 3*b*c^5*d^5*e*f^2*z + 6*a*b^4*c*e^6*f^2*z + b^4*c^2*d^3*e^3*f*g*z + 33*a^3*c^3*d^4*e^2*h^2*z - 27*a^4*c^2*d^2*e^4*h^2*z + 33*a^3*c^3*d^2*e^4*g^2*z - 27*a^2*c^4*d^4*e^2*g^2*z + 19*b^3*c^3*d^3*e^3*f^2*z - 15*b^4*c^2*d^2*e^4*f^2*z - 12*b^2*c^4*d^4*e^2*f^2*z - 27*a^2*c^4*d^2*e^4*f^2*z - 9*a^2*b^2*c^2*e^6*f^2*z + 2*a*c^5*d^6*f*h*z + 2*a*b^5*e^6*f*g*z + 33*a*c^5*d^4*e^2*f^2*z + 4*a^3*b^2*c*e^6*g^2*z + 4*a*b^2*c^3*d^6*h^2*z - b^4*c^2*d^6*h^2*z - b^2*c^4*d^6*g^2*z - a^4*c^2*e^6*g^2*z - a^4*b^2*e^6*h^2*z - a^2*c^4*d^6*h^2*z + 3*a^3*c^3*e^6*f^2*z - a^2*b^4*e^6*g^2*z + b*c^5*d^6*f*g*z + 3*a^5*c*e^6*h^2*z + 3*a*c^5*d^6*g^2*z - c^6*d^6*f^2*z - b^6*e^6*f^2*z + 6*a*b^2*c^2*d*e^2*f*g*h - 2*a*b^3*c*e^3*f*g*h + 3*a^2*b*c^2*d^2*e*g*h^2 - 3*a^2*b*c^2*d*e^2*g^2*h - 3*a^2*b*c^2*d*e^2*f*h^2 - 3*a*b^2*c^2*d^2*e*f*h^2 - 6*a^2*c^3*d*e^2*f*g*h + 2*a^2*b*c^2*e^3*f*g*h + 6*a*b*c^3*d*e^2*f^2*h - 6*a*b*c^3*d*e^2*f*g^2 - 2*b^2*c^3*d^3*f*g*h - 9*a*c^4*d^2*e*f^2*h - 3*b*c^4*d^2*e*f^2*g + 3*a*c^4*d^2*e*f*g^2 + 3*a*c^4*d*e^2*f^2*g - 2*a^3*b*c*e^3*g*h^2 + 2*a*b*c^3*d^3*g^2*h - 2*a*b*c^3*d^3*f*h^2 + 2*a*c^4*d^3*f*g*h - 3*b^3*c^2*d*e^2*f^2*h + 3*b^2*c^3*d^2*e*f^2*h + 3*a^3*c^2*d*e^2*g*h^2 - 3*a^2*c^3*d^2*e*g^2*h + 9*a^2*c^3*d^2*e*f*h^2 + 3*b^2*c^3*d*e^2*f^2*g - 3*a*b^2*c^2*e^3*f^2*h + 2*a^2*b^2*c*e^3*f*h^2 - a*b^2*c^2*d^3*g*h^2 + 2*a*b^2*c^2*e^3*f*g^2 - 3*a^3*c^2*e^3*f*h^2 + 3*a^2*c^3*e^3*f^2*h - b^3*c^2*e^3*f^2*g - a^2*c^3*d^3*g*h^2 - a^2*c^3*e^3*f*g^2 - 3*a^3*c^2*d^2*e*h^3 + 3*a^2*c^3*d*e^2*g^3 - a^2*b*c^2*e^3*g^3 - 3*b*c^4*d*e^2*f^3 + a^2*b^2*c*e^3*g^2*h + a^3*c^2*e^3*g^2*h + b^3*c^2*d^3*f*h^2 + a^2*b*c^2*d^3*h^3 + b^4*c*e^3*f^2*h + b*c^4*d^3*f^2*h + b*c^4*d^3*f*g^2 - c^5*d^3*f^2*g + 3*c^5*d^2*e*f^3 - a*c^4*e^3*f^3 - a*c^4*d^3*g^3 + b^2*c^3*e^3*f^3 + a^4*c*e^3*h^3, z, k)*(root(24*a^6*b*c*d*e^11*z^3 + 24*a*b*c^6*d^11*e*z^3 + 240*a^4*b*c^3*d^5*e^7*z^3 + 240*a^3*b*c^4*d^7*e^5*z^3 + 120*a^5*b*c^2*d^3*e^9*z^3 + 120*a^2*b*c^5*d^9*e^3*z^3 - 54*a^5*b^2*c*d^2*e^10*z^3 - 54*a*b^2*c^5*d^10*e^2*z^3 + 50*a^4*b^3*c*d^3*e^9*z^3 + 50*a*b^3*c^4*d^9*e^3*z^3 - 36*a^2*b^5*c*d^5*e^7*z^3 - 36*a*b^5*c^2*d^7*e^5*z^3 + 26*a*b^6*c*d^6*e^6*z^3 - 340*a^3*b^2*c^3*d^6*e^6*z^3 - 225*a^4*b^2*c^2*d^4*e^8*z^3 - 225*a^2*b^2*c^4*d^8*e^4*z^3 + 180*a^3*b^3*c^2*d^5*e^7*z^3 + 180*a^2*b^3*c^3*d^7*e^5*z^3 - 30*a^2*b^4*c^2*d^6*e^6*z^3 - 6*b^7*c*d^7*e^5*z^3 - 6*b^3*c^5*d^11*e*z^3 - 6*a^5*b^3*d*e^11*z^3 - 6*a*b^7*d^5*e^7*z^3 - 20*b^5*c^3*d^9*e^3*z^3 + 15*b^6*c^2*d^8*e^4*z^3 + 15*b^4*c^4*d^10*e^2*z^3 - 80*a^4*c^4*d^6*e^6*z^3 - 60*a^5*c^3*d^4*e^8*z^3 - 60*a^3*c^5*d^8*e^4*z^3 - 24*a^6*c^2*d^2*e^10*z^3 - 2

4*a^2*c^6*d^10*e^2*z^3 - 20*a^3*b^5*d^3*e^9*z^3 + 15*a^4*b^4*d^2*e^10*z^3 +
15*a^2*b^6*d^4*e^8*z^3 - 4*a^7*c*e^12*z^3 - 4*a*c^7*d^12*z^3 + b^8*d^6*e^6
*z^3 + b^2*c^6*d^12*z^3 + a^6*b^2*e^12*z^3 - 9*a^3*b^2*c*d*e^5*g*h*z - 9*a*
b^2*c^3*d^5*e*g*h*z - 30*a^3*b*c^2*d*e^5*f*h*z + 9*a^2*b^3*c*d*e^5*f*h*z +
3*a*b^4*c*d^2*e^4*f*h*z + 27*a*b*c^4*d^4*e^2*f*g*z + 6*a^2*b^2*c^2*d^3*e^3*
g*h*z - 33*a^2*b^2*c^2*d^2*e^4*f*h*z + 18*a*b*c^4*d^5*e*f*h*z - 12*a*b^4*c*
d*e^5*f*g*z + 27*a^3*b*c^2*d^2*e^4*g*h*z + 27*a^2*b*c^3*d^4*e^2*g*h*z - 3*a
^2*b^3*c*d^2*e^4*g*h*z - 3*a*b^3*c^2*d^4*e^2*g*h*z + 52*a^2*b*c^3*d^3*e^3*f
*h*z - 4*a*b^3*c^2*d^3*e^3*f*h*z - 3*a*b^2*c^3*d^4*e^2*f*h*z - 93*a^2*b*c^3
*d^2*e^4*f*g*z + 51*a^2*b^2*c^2*d*e^5*f*g*z - 34*a*b^2*c^3*d^3*e^3*f*g*z +
27*a*b^3*c^2*d^2*e^4*f*g*z - 24*a*c^5*d^5*e*f*g*z - 7*a^4*b*c*e^6*g*h*z - 7
*a*b*c^4*d^6*g*h*z + a*b^4*c*d^3*e^3*g*h*z - 80*a^3*c^3*d^3*e^3*g*h*z + 3*b
^4*c^2*d^4*e^2*f*h*z - 66*a^2*c^4*d^4*e^2*f*h*z + 54*a^3*c^3*d^2*e^4*f*h*z
- 3*b^3*c^3*d^4*e^2*f*g*z + 80*a^2*c^4*d^3*e^3*f*g*z - 21*a^2*b*c^3*d^5*e*h
^2*z + 6*a*b^3*c^2*d^5*e*h^2*z - 21*a^3*b*c^2*d*e^5*g^2*z + 6*a^2*b^3*c*d*e
^5*g^2*z - 66*a*b*c^4*d^3*e^3*f^2*z - 30*a*b^3*c^2*d*e^5*f^2*z + 27*a^2*b*c
^3*d*e^5*f^2*z - 12*a^2*b^2*c^2*d^4*e^2*h^2*z - 12*a^2*b^2*c^2*d^2*e^4*g^2*
z + 24*a^4*c^2*d*e^5*g*h*z + 24*a^2*c^4*d^5*e*g*h*z - 3*b^3*c^3*d^5*e*f*h*z
- b^5*c*d^3*e^3*f*h*z + 3*b^2*c^4*d^5*e*f*g*z - 24*a^3*c^3*d*e^5*f*g*z + 9
*a^3*b^2*c*e^6*f*h*z - 10*a^2*b^3*c*e^6*f*g*z + 9*a^3*b*c^2*e^6*f*g*z + 3*a
^4*b*c*d*e^5*h^2*z + 3*a*b*c^4*d^5*e*g^2*z + 14*a^3*b*c^2*d^3*e^3*h^2*z + 3
*a^3*b^2*c*d^2*e^4*h^2*z - a^2*b^3*c*d^3*e^3*h^2*z + 14*a^2*b*c^3*d^3*e^3*g
^2*z + 3*a*b^2*c^3*d^4*e^2*g^2*z - a*b^3*c^2*d^3*e^3*g^2*z + 63*a*b^2*c^3*d
^2*e^4*f^2*z + 2*b^3*c^3*d^6*g*h*z - 6*a^4*c^2*e^6*f*h*z + 2*a^3*b^3*e^6*g*
h*z - b^2*c^4*d^6*f*h*z - 2*a^2*b^4*e^6*f*h*z + 6*b^5*c*d*e^5*f^2*z + 3*b*c
^5*d^5*e*f^2*z + 6*a*b^4*c*e^6*f^2*z + b^4*c^2*d^3*e^3*f*g*z + 33*a^3*c^3*d
^4*e^2*h^2*z - 27*a^4*c^2*d^2*e^4*h^2*z + 33*a^3*c^3*d^2*e^4*g^2*z - 27*a^2
*c^4*d^4*e^2*g^2*z + 19*b^3*c^3*d^3*e^3*f^2*z - 15*b^4*c^2*d^2*e^4*f^2*z -
12*b^2*c^4*d^4*e^2*f^2*z - 27*a^2*c^4*d^2*e^4*f^2*z - 9*a^2*b^2*c^2*e^6*f^2
*z + 2*a*c^5*d^6*f*h*z + 2*a*b^5*e^6*f*g*z + 33*a*c^5*d^4*e^2*f^2*z + 4*a^3
*b^2*c*e^6*g^2*z + 4*a*b^2*c^3*d^6*h^2*z - b^4*c^2*d^6*h^2*z - b^2*c^4*d^6*
g^2*z - a^4*c^2*e^6*g^2*z - a^4*b^2*e^6*h^2*z - a^2*c^4*d^6*h^2*z + 3*a^3*c
^3*e^6*f^2*z - a^2*b^4*e^6*g^2*z + b*c^5*d^6*f*g*z + 3*a^5*c*e^6*h^2*z + 3*
a*c^5*d^6*g^2*z - c^6*d^6*f^2*z - b^6*e^6*f^2*z + 6*a*b^2*c^2*d*e^2*f*g*h -
2*a*b^3*c*e^3*f*g*h + 3*a^2*b*c^2*d^2*e*g*h^2 - 3*a^2*b*c^2*d*e^2*g^2*h -
3*a^2*b*c^2*d*e^2*f*h^2 - 3*a*b^2*c^2*d^2*e*f*h^2 - 6*a^2*c^3*d*e^2*f*g*h +
2*a^2*b*c^2*e^3*f*g*h + 6*a*b*c^3*d*e^2*f^2*h - 6*a*b*c^3*d*e^2*f*g^2 - 2*
b^2*c^3*d^3*f*g*h - 9*a*c^4*d^2*e*f^2*h - 3*b*c^4*d^2*e*f^2*g + 3*a*c^4*d^2
*e*f*g^2 + 3*a*c^4*d*e^2*f^2*g - 2*a^3*b*c*e^3*g*h^2 + 2*a*b*c^3*d^3*g^2*h
- 2*a*b*c^3*d^3*f*h^2 + 2*a*c^4*d^3*f*g*h - 3*b^3*c^2*d*e^2*f^2*h + 3*b^2*c
^3*d^2*e*f^2*h + 3*a^3*c^2*d*e^2*g*h^2 - 3*a^2*c^3*d^2*e*g^2*h + 9*a^2*c^3*
d^2*e*f*h^2 + 3*b^2*c^3*d*e^2*f^2*g - 3*a*b^2*c^2*e^3*f^2*h + 2*a^2*b^2*c*e
^3*f*h^2 - a*b^2*c^2*d^3*g*h^2 + 2*a*b^2*c^2*e^3*f*g^2 - 3*a^3*c^2*e^3*f*h^
2 + 3*a^2*c^3*e^3*f^2*h - b^3*c^2*e^3*f^2*g - a^2*c^3*d^3*g*h^2 - a^2*c^3*e
^3*f*g^2 - 3*a^3*c^2*d^2*e*h^3 + 3*a^2*c^3*d*e^2*g^3 - a^2*b*c^2*e^3*g^3 -
3*b*c^4*d*e^2*f^3 + a^2*b^2*c*e^3*g^2*h + a^3*c^2*e^3*g^2*h + b^3*c^2*d^3*f
*h^2 + a^2*b*c^2*d^3*h^3 + b^4*c*e^3*f^2*h + b*c^4*d^3*f^2*h + b*c^4*d^3*f*
g^2 - c^5*d^3*f^2*g + 3*c^5*d^2*e*f^3 - a*c^4*e^3*f^3 - a*c^4*d^3*g^3 + b^2
*c^3*e^3*f^3 + a^4*c*e^3*h^3, z, k)*((8*a*c^6*d^9*e^2 + 8*a^5*c^2*d*e^10 -
b^6*c*d^5*e^6 + 32*a^2*c^5*d^7*e^4 + 48*a^3*c^4*d^5*e^6 + 32*a^4*c^3*d^3*e^
8 + 3*b^2*c^5*d^9*e^2 - 2*b^3*c^4*d^8*e^3 - 2*b^4*c^3*d^7*e^4 + 3*b^5*c^2*d
^6*e^5 - a^5*b*c*e^11 - b*c^6*d^10*e + 114*a^2*b^2*c^3*d^5*e^6 - 38*a^2*b^3
*c^2*d^4*e^7 + 60*a^3*b^2*c^2*d^3*e^8 - 37*a*b*c^5*d^8*e^3 + 3*a*b^5*c*d^4*
e^7 + 3*a^4*b^2*c*d*e^10 + 60*a*b^2*c^4*d^7*e^4 - 38*a*b^3*c^3*d^6*e^5 + 4*
a*b^4*c^2*d^5*e^6 - 106*a^2*b*c^4*d^6*e^5 - 2*a^2*b^4*c*d^3*e^8 - 106*a^3*b
*c^3*d^4*e^7 - 2*a^3*b^3*c*d^2*e^9 - 37*a^4*b*c^2*d^2*e^9)/(a^4*e^8 + c^4*d
^8 + b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 - 4*
b^3*c*d^5*e^3 + 6*a^2*b^2*d^2*e^6 + 6*a^2*c^2*d^4*e^4 + 6*b^2*c^2*d^6*e^2 -
4*a^3*b*d*e^7 - 4*b*c^3*d^7*e - 12*a*b*c^2*d^5*e^3 + 12*a*b^2*c*d^4*e^4 -

12*a^2*b*c*d^3*e^5) + (x*(6*a^5*c^2*e^11 - 2*c^7*d^10*e - 2*a^4*b^2*c*e^11 - 2*a*c^6*d^8*e^3 + 10*b*c^6*d^9*e^2 - 2*b^6*c*d^4*e^7 + 12*a^2*c^5*d^6*e^5 + 28*a^3*c^4*d^4*e^7 + 22*a^4*c^3*d^2*e^9 - 22*b^2*c^5*d^8*e^3 + 28*b^3*c^4*d^7*e^4 - 22*b^4*c^3*d^6*e^5 + 10*b^5*c^2*d^5*e^6 + 24*a^2*b^2*c^3*d^4*e^7 + 12*a^2*b^3*c^2*d^3*e^8 + 20*a^3*b^2*c^2*d^2*e^9 + 8*a*b*c^5*d^7*e^4 + 8*a*b^5*c*d^3*e^8 + 8*a^3*b^3*c*d*e^10 - 22*a^4*b*c^2*d*e^10 - 20*a*b^2*c^4*d^6*e^5 + 32*a*b^3*c^3*d^5*e^6 - 26*a*b^4*c^2*d^4*e^7 - 36*a^2*b*c^4*d^5*e^6 - 12*a^2*b^4*c*d^2*e^9 - 56*a^3*b*c^3*d^3*e^8))/(a^4*e^8 + c^4*d^8 + b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 - 4*b^3*c*d^5*e^3 + 6*a^2*b^2*d^2*e^6 + 6*a^2*c^2*d^4*e^4 + 6*b^2*c^2*d^6*e^2 - 4*a^3*b*d*e^7 - 4*b*c^3*d^7*e - 12*a*b*c^2*d^5*e^3 + 12*a*b^2*c*d^4*e^4 - 12*a^2*b*c*d^3*e^5)) + (a^4*c^2*e^8*g + c^6*d^7*e*f + a^4*b*c*e^8*h - a*c^5*d^7*e*h - b*c^5*d^7*e*g + a^2*b^3*c*e^8*f - 2*a^3*b*c^2*e^8*f - a^3*b^2*c*e^8*g + 3*a*c^5*d^5*e^3*f + a^3*c^3*d*e^7*f + a*c^5*d^6*e^2*g - b*c^5*d^6*e^2*f + b^5*c*d^2*e^6*f - a^4*c^2*d*e^7*h + b^2*c^4*d^7*e*h + 3*a^2*c^4*d^3*e^5*f + 3*a^2*c^4*d^4*e^4*g + 3*a^3*c^3*d^2*e^6*g - 3*b^2*c^4*d^5*e^3*f + 6*b^3*c^3*d^4*e^4*f - 4*b^4*c^2*d^3*e^5*f - 3*a^2*c^4*d^5*e^3*h - 3*a^3*c^3*d^3*e^5*h + 2*b^2*c^4*d^6*e^2*g - b^3*c^3*d^5*e^3*g - 2*b^3*c^3*d^6*e^2*h + b^4*c^2*d^5*e^3*h - a*b^2*c^3*d^3*e^5*f + 4*a*b^3*c^2*d^2*e^6*f - 5*a^2*b*c^3*d^2*e^6*f + 2*a^2*b^2*c^2*d*e^7*f - 2*a*b^2*c^3*d^4*e^4*g + 4*a*b^3*c^2*d^3*e^5*g - a^2*b*c^3*d^3*e^5*g + 5*a*b^2*c^3*d^5*e^3*h - 4*a*b^3*c^2*d^4*e^4*h + a^2*b*c^3*d^4*e^4*h + a^2*b^3*c*d^2*e^6*h + 2*a^3*b*c^2*d^2*e^6*h - 2*a*b^4*c*d*e^7*f - 5*a^2*b^2*c^2*d^2*e^6*g + 2*a^2*b^2*c^2*d^3*e^5*h - 4*a*b*c^4*d^4*e^4*f - 2*a*b*c^4*d^5*e^3*g - a*b^4*c*d^2*e^6*g + 2*a^2*b^3*c*d*e^7*g - 2*a^3*b^2*c*d*e^7*h)/(a^4*e^8 + c^4*d^8 + b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 - 4*b^3*c*d^5*e^3 + 6*a^2*b^2*d^2*e^6 + 6*a^2*c^2*d^4*e^4 + 6*b^2*c^2*d^6*e^2 - 4*a^3*b*d*e^7 - 4*b*c^3*d^7*e - 12*a*b*c^2*d^5*e^3 + 12*a*b^2*c*d^4*e^4 - 12*a^2*b*c*d^3*e^5) + (x*(3*a^4*c^2*e^8*h - 3*a^3*c^3*e^8*f + 5*c^6*d^6*e^2*f - c^6*d^7*e*g + b*c^5*d^7*e*h - 2*a^3*b*c^2*e^8*g + 7*a*c^5*d^4*e^4*f + 5*a*c^5*d^5*e^3*g - 15*b*c^5*d^5*e^3*f + 7*a^3*c^3*d*e^7*g - 5*a*c^5*d^6*e^2*h + b*c^5*d^6*e^2*g + 2*a^2*b^2*c^2*e^8*f - a^2*c^4*d^2*e^6*f + 13*a^2*c^4*d^3*e^5*g + 17*b^2*c^4*d^4*e^4*f - 9*b^3*c^3*d^3*e^5*f + 2*b^4*c^2*d^2*e^6*f - 7*a^2*c^4*d^4*e^4*h + a^3*c^3*d^2*e^6*h + b^2*c^4*d^5*e^3*g - b^3*c^3*d^4*e^4*g - b^2*c^4*d^6*e^2*h - b^3*c^3*d^5*e^3*h + b^4*c^2*d^4*e^4*h + 11*a*b^2*c^3*d^2*e^6*f + 13*a*b^2*c^3*d^3*e^5*g - 2*a*b^3*c^2*d^2*e^6*g - 19*a^2*b*c^3*d^2*e^6*g + 4*a^2*b^2*c^2*d*e^7*g - a*b^2*c^3*d^4*e^4*h - 4*a*b^3*c^2*d^3*e^5*h + a^2*b*c^3*d^3*e^5*h + 8*a^2*b^2*c^2*d^2*e^6*h - 14*a*b*c^4*d^3*e^5*f - 4*a*b^3*c^2*d*e^7*f + a^2*b*c^3*d*e^7*f - 16*a*b*c^4*d^4*e^4*g + 10*a*b*c^4*d^5*e^3*h - 8*a^3*b*c^2*d*e^7*h))/(a^4*e^8 + c^4*d^8 + b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 - 4*b^3*c*d^5*e^3 + 6*a^2*b^2*d^2*e^6 + 6*a^2*c^2*d^4*e^4 + 6*b^2*c^2*d^6*e^2 - 4*a^3*b*d*e^7 - 4*b*c^3*d^7*e - 12*a*b*c^2*d^5*e^3 + 12*a*b^2*c*d^4*e^4 - 12*a^2*b*c*d^3*e^5)) - (2*c^5*d^3*e^2*f^2 - b^3*c^2*e^5*f^2 - c^5*d^4*e*f*g + 2*a^2*c^3*d^3*e^2*h^2 + a*b*c^3*e^5*f^2 - 2*a*c^4*d*e^4*f^2 - a^2*c^3*e^5*f*g + a^3*c^2*e^5*g*h - a^2*b*c^2*e^5*g^2 - 2*a*c^4*d^3*e^2*g^2 - 5*b*c^4*d^2*e^3*f^2 + 2*a^2*c^3*d*e^4*g^2 + 4*b^2*c^3*d*e^4*f^2 - 2*a^3*c^2*d*e^4*h^2 + a*b*c^3*d^2*e^3*g^2 - b^2*c^3*d^2*e^3*f*g - 6*a^2*c^3*d^2*e^3*g*h - 2*b^2*c^3*d^3*e^2*f*h + b^3*c^2*d^2*e^3*f*h + a*c^4*d^4*e*g*h + b*c^4*d^4*e*f*h + a^2*b*c^2*d^2*e^3*h^2 - a*b*c^3*d^4*e*h^2 + 2*a*b^2*c^2*e^5*f*g - a^2*b*c^2*e^5*f*h + 6*a*c^4*d^2*e^3*f*g - 4*a*c^4*d^3*e^2*f*h + 2*b*c^4*d^3*e^2*f*g + 4*a^2*c^3*d*e^4*f*h + 4*a*b*c^3*d^2*e^3*f*h - 2*a*b^2*c^2*d*e^4*f*h + 2*a*b*c^3*d^3*e^2*g*h + 2*a^2*b*c^2*d*e^4*g*h - a*b^2*c^2*d^2*e^3*g*h - 6*a*b*c^3*d*e^4*f*g)/(a^4*e^8 + c^4*d^8 + b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 - 4*b^3*c*d^5*e^3 + 6*a^2*b^2*d^2*e^6 + 6*a^2*c^2*d^4*e^4 + 6*b^2*c^2*d^6*e^2 - 4*a^3*b*d*e^7 - 4*b*c^3*d^7*e - 12*a*b*c^2*d^5*e^3 + 12*a*b^2*c*d^4*e^4 - 12*a^2*b*c*d^3*e^5) + (x*(c^5*d^4*e*g^2 + a^2*c^3*e^5*g^2 + b^2*c^3*e^5*f^2 + 4*c^5*d^2*e^3*f^2 + 4*a^2*c^3*d^2*e^3*h^2 - 4*b*c^4*d*e^4*f^2 - 4*c^5*d^3*e^2*f*g - 2*a*c^4*d^2*e^3*g^2 + b^2*c^3*d^4*e*h^2 - 4*a*b*c^3*d^3*e^2*h^2 - 2*b^2*c^3 *d^2*e^3*f*h - 2*a*b*c^3*e^5*f*g + 4*a*c^4*d*e^4*f*g - 2*b*c^4*d^4*e*g*h - 8*a*c^4*d^2*e^3*f*h + 2*b*c^4*d^2*e^3*f*g + 4*a*c^4*d^3*e^2*g*h + 4*b*c^4*d^3*e^2*f*h - 4*a^2*c^3*d*e^4*g*h + 2*a*b*c^3*d^2*e^3*g*h + 4*a*b*c^3*d*e^4*f*h))/(a^4*e^8 + c^4*d^8 + b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 - 4*b^3*c*d^5*e^3 + 6*a^2*b^2*d^2*e^6 + 6*a^2*c^2*d^4*e^4 + 6*b^2*c^2*d^6*e^2 - 4*a^3*b*d*e^7 - 4*b*c^3*d^7*e - 12*a*b*c^2*d^5*e^3 + 12*a*b^2*c*d^4*e^4 - 12*a^2*b*c*d^3*e^5))*root(24*a^6*b*c*d*e^11*z^3 + 24*a*b*c^6*d^11*e*z^3 + 240*a^4*b*c^3*d^5*e^7*z^3 + 240*a^3*b*c^4*d^7*e^5*z^3 + 120*a^5*b*c^2*d^3*e^9*z^3 + 120*a^2*b*c^5*d^9*e^3*z^3 - 54*a^5*b^2*c*d^2*e^10*z^3 - 54*a*b^2*c^5*d^10*e^2*z^3 + 50*a^4*b^3*c*d^3*e^9*z^3 + 50*a*b^3*c^4*d^9*e^3*z^3 - 36*a^2*b^5*c*d^5*e^7*z^3 - 36*a*b^5*c^2*d^7*e^5*z^3 + 26*a*b^6*c*d^6*e^6*z^3 - 340*a^3*b^2*c^3*d^6*e^6*z^3 - 225*a^4*b^2*c^2*d^4*e^8*z^3 - 225*a^2*b^2*c^4*d^8*e^4*z^3 + 180*a^3*b^3*c^2*d^5*e^7*z^3 + 180*a^2*b^3*c^3*d^7*e^5*z^3 - 30*a^2*b^4*c^2*d^6*e^6*z^3 - 6*b^7*c*d^7*e^5*z^3 - 6*b^3*c^5*d^11*e*z^3 - 6*a^5*b^3*d*e^11*z^3 - 6*a*b^7*d^5*e^7*z^3 - 20*b^5*c^3*d^9*e^3*z^3 + 15*b^6*c^2*d^8*e^4*z^3 + 15*b^4*c^4*d^10*e^2*z^3 - 80*a^4*c^4*d^6*e^6*z^3 - 60*a^5*c^3*d^4*e^8*z^3 - 60*a^3*c^5*d^8*e^4*z^3 - 24*a^6*c^2*d^2*e^10*z^3 - 24*a^2*c^6*d^10*e^2*z^3 - 20*a^3*b^5*d^3*e^9*z^3 + 15*a^4*b^4*d^2*e^10*z^3 + 15*a^2*b^6*d^4*e^8*z^3 - 4*a^7*c*e^12*z^3 - 4*a*c^7*d^12*z^3 + b^8*d^6*e^6*z^3 + b^2*c^6*d^12*z^3 + a^6*b^2*e^12*z^3 - 9*a^3*b^2*c*d*e^5*g*h*z - 9*a*b^2*c^3*d^5*e*g*h*z - 30*a^3*b*c^2*d*e^5*f*h*z + 9*a^2*b^3*c*d*e^5*f*h*z + 3*a*b^4*c*d^2*e^4*f*h*z + 27*a*b*c^4*d^4*e^2*f*g*z + 6*a^2*b^2*c^2*d^3*e^3*g*h*z - 33*a^2*b^2*c^2*d^2*e^4*f*h*z + 18*a*b*c^4*d^5*e*f*h*z - 12*a*b^4*c*d*e^5*f*g*z + 27*a^3*b*c^2*d^2*e^4*g*h*z + 27*a^2*b*c^3*d^4*e^2*g*h*z - 3*a^2*b^3*c*d^2*e^4*g*h*z - 3*a*b^3*c^2*d^4*e^2*g*h*z + 52*a^2*b*c^3*d^3*e^3*f*h*z - 4*a*b^3*c^2*d^3*e^3*f*h*z - 3*a*b^2*c^3*d^4*e^2*f*h*z - 93*a^2*b*c^3*d^2*e^4*f*g*z + 51*a^2*b^2*c^2*d*e^5*f*g*z - 34*a*b^2*c^3*d^3*e^3*f*g*z + 27*a*b^3*c^2*d^2*e^4*f*g*z - 24*a*c^5*d^5*e*f*g*z - 7*a^4*b*c*e^6*g*h*z - 7*a*b*c^4*d^6*g*h*z + a*b^4*c*d^3*e^3*g*h*z - 80*a^3*c^3*d^3*e^3*g*h*z + 3*b^4*c^2*d^4*e^2*f*h*z - 66*a^2*c^4*d^4*e^2*f*h*z + 54*a^3*c^3*d^2*e^4*f*h*z - 3*b^3*c^3*d^4*e^2*f*g*z + 80*a^2*c^4*d^3*e^3*f*g*z - 21*a^2*b*c^3*d^5*e*h^2*z + 6*a*b^3*c^2*d^5*e*h^2*z - 21*a^3*b*c^2*d*e^5*g^2*z + 6*a^2*b^3*c*d*e^5*g^2*z - 66*a*b*c^4*d^3*e^3*f^2*z - 30*a*b^3*c^2*d*e^5*f^2*z + 27*a^2*b*c^3*d*e^5*f^2*z - 12*a^2*b^2*c^2*d^4*e^2*h^2*z - 12*a^2*b^2*c^2*d^2*e^4*g^2*z + 24*a^4*c^2*d*e^5*g*h*z + 24*a^2*c^4*d^5*e*g*h*z - 3*b^3*c^3*d^5*e*f*h*z - b^5*c*d^3*e^3*f*h*z + 3*b^2*c^4*d^5*e*f*g*z - 24*a^3*c^3*d*e^5*f*g*z + 9*a^3*b^2*c*e^6*f*h*z - 10*a^2*b^3*c*e^6*f*g*z + 9*a^3*b*c^2*e^6*f*g*z + 3*a^4*b*c*d*e^5*h^2*z + 3*a*b*c^4*d^5*e*g^2*z + 14*a^3*b*c^2*d^3*e^3*h^2*z + 3*a^3*b^2*c*d^2*e^4*h^2*z - a^2*b^3*c*d^3*e^3*h^2*z + 14*a^2*b*c^3*d^3*e^3*g^2*z + 3*a*b^2*c^3*d^4*e^2*g^2*z - a*b^3*c^2*d^3*e^3*g^2*z + 63*a*b^2*c^3*d^2*e^4*f^2*z + 2*b^3*c^3*d^6*g*h*z - 6*a^4*c^2*e^6*f*h*z + 2*a^3*b^3*e^6*g*h*z - b^2*c^4*d^6*f*h*z - 2*a^2*b^4*e^6*f*h*z + 6*b^5*c*d*e^5*f^2*z + 3*b*c^5*d^5*e*f^2*z + 6*a*b^4*c*e^6*f^2*z + b^4*c^2*d^3*e^3*f*g*z + 33*a^3*c^3*d^4*e^2*h^2*z - 27*a^4*c^2*d^2*e^4*h^2*z + 33*a^3*c^3*d^2*e^4*g^2*z - 27*a^2*c^4*d^4*e^2*g^2*z + 19*b^3*c^3*d^3*e^3*f^2*z - 15*b^4*c^2*d^2*e^4*f^2*z - 12*b^2*c^4*d^4*e^2*f^2*z - 27*a^2*c^4*d^2*e^4*f^2*z - 9*a^2*b^2*c^2*e^6*f^2*z + 2*a*c^5*d^6*f*h*z + 2*a*b^5*e^6*f*g*z + 33*a*c^5*d^4*e^2*f^2*z + 4*a^3*b^2*c*e^6*g^2*z + 4*a*b^2*c^3*d^6*h^2*z - b^4*c^2*d^6*h^2*z - b^2*c^4*d^6*g^2*z - a^4*c^2*e^6*g^2*z - a^4*b^2*e^6*h^2*z - a^2*c^4*d^6*h^2*z + 3*a^3*c^3*e^6*f^2*z - a^2*b^4*e^6*g^2*z + b*c^5*d^6*f*g*z + 3*a^5*c*e^6*h^2*z + 3*a*c^5*d^6*g^2*z - c^6*d^6*f^2*z - b^6*e^6*f^2*z + 6*a*b^2*c^2*d*e^2*f*g*h - 2*a*b^3*c*e^3*f*g*h + 3*a^2*b*c^2*d^2*e*g*h^2 - 3*a^2*b*c^2*d*e^2*g^2*h - 3*a^2*b*c^2*d*e^2*f*h^2 - 3*a*b^2*c^2*d^2*e*f*h^2 - 6*a^2*c^3*d*e^2*f*g*h + 2*a^2*b*c^2*e^3*f*g*h + 6*a*b*c^3*d*e^2*f^2*h - 6*a*b*c^3*d*e^2*f*g^2 - 2*b^2*c^3*d^3*f*g*h - 9*a*c^4*d^2*e*f^2*h - 3*b*c^4*d^2*e*f^2*g + 3*a*c^4*d^2*e*f*g^2 + 3*a*c^4*d*e^2*f^2*g - 2*a^3*b*c*e^3*g*h^2 + 2*a*b*c^3*d^3*g^2*h - 2*a*b*c^3*d^3*f*h^2 + 2*a*c^4*d^3*f*g*h - 3*b^3*c^2*d*e^2*f^2*h + 3*b^2*c^3*d^2*e*f^2*h + 3*a^3*c^2*d*e^2*g*h^2 - 3*a^2*c^3*d^2*e*g^2*h + 9*a^2*c^3*d^2*e*f*h^2 + 3*b^2*c^3*d*e^2*f^2*g - 3*a*b^2*c^2*e^3*f^

```
2*h + 2*a^2*b^2*c*e^3*f*h^2 - a*b^2*c^2*d^3*g*h^2 + 2*a*b^2*c^2*e^3*f*g^2 -
 3*a^3*c^2*e^3*f*h^2 + 3*a^2*c^3*e^3*f^2*h - b^3*c^2*e^3*f^2*g - a^2*c^3*d^
3*g*h^2 - a^2*c^3*e^3*f*g^2 - 3*a^3*c^2*d^2*e*h^3 + 3*a^2*c^3*d*e^2*g^3 - a
^2*b*c^2*e^3*g^3 - 3*b*c^4*d*e^2*f^3 + a^2*b^2*c*e^3*g^2*h + a^3*c^2*e^3*g^
2*h + b^3*c^2*d^3*f*h^2 + a^2*b*c^2*d^3*h^3 + b^4*c*e^3*f^2*h + b*c^4*d^3*f
^2*h + b*c^4*d^3*f*g^2 - c^5*d^3*f^2*g + 3*c^5*d^2*e*f^3 - a*c^4*e^3*f^3 -
a*c^4*d^3*g^3 + b^2*c^3*e^3*f^3 + a^4*c*e^3*h^3, z, k), k, 1, 3) - ((a*e^4*
f + c*d^4*h + a*d*e^3*g - 3*b*d*e^3*f + b*d^3*e*h - 3*c*d^3*e*g - 3*a*d^2*e
^2*h + b*d^2*e^2*g + 5*c*d^2*e^2*f)/(2*e*(a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 -
 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2)) + (x*(a*e^3*g - b*e^3*f - 2*a*
d*e^2*h + 2*c*d*e^2*f + b*d^2*e*h - c*d^2*e*g))/(a^2*e^4 + c^2*d^4 + b^2*d^
2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2))/(d^2 + e^2*x^2 + 2*d*e*
x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)**3/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.155 $\int \frac{(d+ex)^2\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=288

$$\frac{(d+ex)^2\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{e^2x\left(-6ach+2b^2h-bcg+2c^2f\right)}{c^2\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\right.}{}$$

```
[Out] e^2*(2*c^2*f+2*b^2*h-c*(6*a*h+b*g))*x/c^2/(-4*a*c+b^2)+(e*x+d)^2*(c*(2*a*g-
b*(f+a*h/c))-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)
+(4*c^4*d^2*f-2*b^4*e^2*h-6*a*c^2*e*(2*a*e*h+2*b*d*h+b*e*g)+b^2*c*e*(12*a*e
*h+2*b*d*h+b*e*g)-c^3*(2*b*d*(d*g+2*e*f)-4*a*(d^2*h+2*d*e*g+e^2*f)))*arctan
h((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)+1/2*e*(-2*b*e*h+2*c*
d*h+c*e*g)*ln(c*x^2+b*x+a)/c^3
```

Rubi [A] time = 0.70, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1644, 773, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(b^2ce(12aeh+2bdh+beg)-c^3\left(2bd(dg+2ef)-4a\left(d^2h+2deg+e^2f\right)\right)-6ac^2e(2aeh+2bd\right.}{c^3\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] (e^2*(2*c^2*f - b*c*g + 2*b^2*h - 6*a*c*h)*x)/(c^2*(b^2 - 4*a*c)) + ((d + e
*x)^2*(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*x)
)/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((4*c^4*d^2*f - 2*b^4*e^2*h - 6*a*c
^2*e*(b*e*g + 2*b*d*h + 2*a*e*h) + b^2*c*e*(b*e*g + 2*b*d*h + 12*a*e*h) - c
^3*(2*b*d*(2*e*f + d*g) - 4*a*(e^2*f + 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) + (e*(c*e*g + 2*c*d*h - 2*
b*e*h)*Log[a + b*x + c*x^2])/(2*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\int\frac{(d+ex)\left(2cdf-2\right.}{}$$

$$= \frac{e^2\left(2c^2f-bcg+2b^2h-6ach\right)x}{c^2\left(b^2-4ac\right)}+\frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bc\right.\right.}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$= \frac{e^2\left(2c^2f-bcg+2b^2h-6ach\right)x}{c^2\left(b^2-4ac\right)}+\frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bc\right.\right.}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$= \frac{e^2\left(2c^2f-bcg+2b^2h-6ach\right)x}{c^2\left(b^2-4ac\right)}+\frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bc\right.\right.}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$= \frac{e^2\left(2c^2f-bcg+2b^2h-6ach\right)x}{c^2\left(b^2-4ac\right)}+\frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bc\right.\right.}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Mathematica [A] time = 0.81, size = 398, normalized size = 1.38

$$-\frac{\frac{2\left(bc\left(-3a^2e^2h+ac\left(d^2h+2de(g+3hx)+e^2(f+3gx)\right)+c^2d(d(f-gx)-2efx)\right)+2c^2\left(a^2e(2dh+e(g+hx))-ac\left(d^2(g+hx)+2de(f+gx)+e^2fx\right)+c^2d^2fx\right)+b^3e(aeh-c\right.}{\left(b^2-4ac\right)(a+x(b+cx))}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] (2*c*e^2*h*x - (2*(b^4*e^2*h*x + b^3*e*(a*e*h - c*(e*g + 2*d*h)*x) + b^2*c*
(c*(e^2*f + 2*d*e*g + d^2*h)*x - a*e*(e*g + 2*d*h + 4*e*h*x)) + 2*c^2*(c^2*
```

```
d^2*f*x - a*c*(e^2*f*x + 2*d*e*(f + g*x) + d^2*(g + h*x)) + a^2*e*(2*d*h +
e*(g + h*x))) + b*c*(-3*a^2*e^2*h + c^2*d*(-2*e*f*x + d*(f - g*x)) + a*c*(d
^2*h + e^2*(f + 3*g*x) + 2*d*e*(g + 3*h*x)))))/((b^2 - 4*a*c)*(a + x*(b + c
*x))) + (2*(4*c^4*d^2*f - 2*b^4*e^2*h - 6*a*c^2*e*(b*e*g + 2*b*d*h + 2*a*e*
h) + b^2*c*e*(b*e*g + 2*b*d*h + 12*a*e*h) + c^3*(-2*b*d*(2*e*f + d*g) + 4*a
*(e^2*f + 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2
+ 4*a*c)^(3/2) + e*(c*e*g + 2*c*d*h - 2*b*e*h)*Log[a + x*(b + c*x)])/(2*c^3
)
```

fricas [**B**] time = 2.48, size = 2771, normalized size = 9.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*h*x^3 + 2*(b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*e^2*h*x^2 + ((4*(c^5*d^2 - b*c^4*d*e + a*c^4*e^2)*f -
(2*b*c^4*d^2 - 8*a*c^4*d*e - (b^3*c^2 - 6*a*b*c^3)*e^2)*g + 2*(2*a*c^4*d^2
+ (b^3*c^2 - 6*a*b*c^3)*d*e - (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^2)*h)*x^2
 + 4*(a*c^4*d^2 - a*b*c^3*d*e + a^2*c^3*e^2)*f - (2*a*b*c^3*d^2 - 8*a^2*c^3
*d*e - (a*b^3*c - 6*a^2*b*c^2)*e^2)*g + 2*(2*a^2*c^3*d^2 + (a*b^3*c - 6*a^2
*b*c^2)*d*e - (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*e^2)*h + (4*(b*c^4*d^2 - b^
2*c^3*d*e + a*b*c^3*e^2)*f - (2*b^2*c^3*d^2 - 8*a*b*c^3*d*e - (b^4*c - 6*a*
b^2*c^2)*e^2)*g + 2*(2*a*b*c^3*d^2 + (b^4*c - 6*a*b^2*c^2)*d*e - (b^5 - 6*a
*b^3*c + 6*a^2*b*c^2)*e^2)*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x
 + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*((b^
3*c^3 - 4*a*b*c^4)*d^2 - 4*(a*b^2*c^3 - 4*a^2*c^4)*d*e + (a*b^3*c^2 - 4*a^2
*b*c^3)*e^2)*f + 2*(2*(a*b^2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*
c^3)*d*e + (a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*e^2)*g - 2*((a*b^3*c^2 - 4
*a^2*b*c^3)*d^2 - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*d*e + (a*b^5 - 7*
a^2*b^3*c + 12*a^3*b*c^2)*e^2)*h - 2*((2*(b^2*c^4 - 4*a*c^5)*d^2 - 2*(b^3*c
^3 - 4*a*b*c^4)*d*e + (b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*e^2)*f - ((b^3*c^
3 - 4*a*b*c^4)*d^2 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d*e + (b^5*c - 7
*a*b^3*c^2 + 12*a^2*b*c^3)*e^2)*g + ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^
2 - 2*(b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*d*e + (b^6 - 9*a*b^4*c + 26*a^2*
b^2*c^2 - 24*a^3*c^3)*e^2)*h)*x + ((a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e
^2*g + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*g + 2*((b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*d*e - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2)*h)*x^2 +
2*((a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*d*e - (a*b^5 - 8*a^2*b^3*c + 16*a
^3*b*c^2)*e^2)*h + ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2*g + 2*((b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e - (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*e^2)
*h)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4
*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^
5)*x), 1/2*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*h*x^3 + 2*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*e^2*h*x^2 + 2*((4*(c^5*d^2 - b*c^4*d*e + a*c^4*e
^2)*f - (2*b*c^4*d^2 - 8*a*c^4*d*e - (b^3*c^2 - 6*a*b*c^3)*e^2)*g + 2*(2*a*
c^4*d^2 + (b^3*c^2 - 6*a*b*c^3)*d*e - (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^2
)*h)*x^2 + 4*(a*c^4*d^2 - a*b*c^3*d*e + a^2*c^3*e^2)*f - (2*a*b*c^3*d^2 - 8
*a^2*c^3*d*e - (a*b^3*c - 6*a^2*b*c^2)*e^2)*g + 2*(2*a^2*c^3*d^2 + (a*b^3*c
 - 6*a^2*b*c^2)*d*e - (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*e^2)*h + (4*(b*c^4*
d^2 - b^2*c^3*d*e + a*b*c^3*e^2)*f - (2*b^2*c^3*d^2 - 8*a*b*c^3*d*e - (b^4*
c - 6*a*b^2*c^2)*e^2)*g + 2*(2*a*b*c^3*d^2 + (b^4*c - 6*a*b^2*c^2)*d*e - (b
^5 - 6*a*b^3*c + 6*a^2*b*c^2)*e^2)*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*((b^3*c^3 - 4*a*b*c^4)*d^2 - 4*(
a*b^2*c^3 - 4*a^2*c^4)*d*e + (a*b^3*c^2 - 4*a^2*b*c^3)*e^2)*f + 2*(2*(a*b^2
*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e + (a*b^4*c - 6*a^2*
b^2*c^2 + 8*a^3*c^3)*e^2)*g - 2*((a*b^3*c^2 - 4*a^2*b*c^3)*d^2 - 2*(a*b^4*c
 - 6*a^2*b^2*c^2 + 8*a^3*c^3)*d*e + (a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*e^
2)*h - 2*((2*(b^2*c^4 - 4*a*c^5)*d^2 - 2*(b^3*c^3 - 4*a*b*c^4)*d*e + (b^4*c
```

```
^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*e^2)*f - ((b^3*c^3 - 4*a*b*c^4)*d^2 - 2*(b^4*
c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d*e + (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e
^2)*g + ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2 - 2*(b^5*c - 7*a*b^3*c^2 +
 12*a^2*b*c^3)*d*e + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*e^2)*h
)*x + ((a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e^2*g + ((b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*e^2*g + 2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e - (b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2)*h)*x^2 + 2*((a*b^4*c - 8*a^2*b^2*c^2
+ 16*a^3*c^3)*d*e - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e^2)*h + ((b^5*c -
 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2*g + 2*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*d*e - (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*e^2)*h)*x)*log(c*x^2 + b*x + a))
/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*
c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x)]
```

giac [A] time = 0.18, size = 540, normalized size = 1.88

$$\frac{hxe^2}{c^2} - \frac{\left(4\,c^4d^2f - 2\,bc^3d^2g + 4\,ac^3d^2h - 4\,bc^3dfe + 8\,ac^3dge + 2\,b^3cdhe - 12\,abc^2dhe + 4\,ac^3fe^2 + b^3cge^2 - 6\,abc^2\right.}{\left(b^2c^3 - 4\,ac^4\right)\sqrt{-b^2 + 4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] h*x*e^2/c^2 - (4*c^4*d^2*f - 2*b*c^3*d^2*g + 4*a*c^3*d^2*h - 4*b*c^3*d*f*e
+ 8*a*c^3*d*g*e + 2*b^3*c*d*h*e - 12*a*b*c^2*d*h*e + 4*a*c^3*f*e^2 + b^3*c*
g*e^2 - 6*a*b*c^2*g*e^2 - 2*b^4*h*e^2 + 12*a*b^2*c*h*e^2 - 12*a^2*c^2*h*e^2
)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4
*a*c)) + 1/2*(2*c*d*h*e + c*g*e^2 - 2*b*h*e^2)*log(c*x^2 + b*x + a)/c^3 - (
(2*c^4*d^2*f - b*c^3*d^2*g + b^2*c^2*d^2*h - 2*a*c^3*d^2*h - 2*b*c^3*d*f*e
+ 2*b^2*c^2*d*g*e - 4*a*c^3*d*g*e - 2*b^3*c*d*h*e + 6*a*b*c^2*d*h*e + b^2*c
^2*f*e^2 - 2*a*c^3*f*e^2 - b^3*c*g*e^2 + 3*a*b*c^2*g*e^2 + b^4*h*e^2 - 4*a*
b^2*c*h*e^2 + 2*a^2*c^2*h*e^2)*x/c + (b*c^3*d^2*f - 2*a*c^3*d^2*g + a*b*c^2
*d^2*h - 4*a*c^3*d*f*e + 2*a*b*c^2*d*g*e - 2*a*b^2*c*d*h*e + 4*a^2*c^2*d*h*
e + a*b*c^2*f*e^2 - a*b^2*c*g*e^2 + 2*a^2*c^2*g*e^2 + a*b^3*h*e^2 - 3*a^2*b
*c*h*e^2)/c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

maple [B] time = 0.02, size = 1712, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x)
```

```
[Out] e^2*h/c^2*x+6/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*d*e*h-4/(c*x^2+b*x+a)/(4*a*
c-b^2)*x*a*d*e*g+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^4*e^2*h+1/c/(c*x^2+b*x
+a)/(4*a*c-b^2)*x*b^2*d^2*h+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2*e^2*h+1/c/(
c*x^2+b*x+a)/(4*a*c-b^2)*x*b^2*e^2*f+4/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*d*e*
h+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^3*e^2*h+1/c/(c*x^2+b*x+a)/(4*a*c-b^2)
*a*b*d^2*h+1/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*e^2*f-1/c^2/(c*x^2+b*x+a)/(4*a
*c-b^2)*x*b^3*e^2*g-3/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*e^2*h-1/c^2/(c*x^
2+b*x+a)/(4*a*c-b^2)*a*b^2*e^2*g-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2))*a*b*e^2*g+4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*a*e^2*f-2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^
2*g-2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*d^2*g+1/(c*x^2+b*x+a)/(4*a*c-b^2)*b*d^2*f
+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d^2*f+4/(4*a*c-b
^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d^2*h-4/(4*a*c-b^2)^(3/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d*e*f-12/c/(4*a*c-b^2)^(3/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*e^2*h-2/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*b^4*e^2*h-1/2/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2*e^2
*g+1/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^2*g+2/
```

```
c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*e^2*g-2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*e^2*f
-2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*d^2*h-1/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b*d^2*
g-4/(c*x^2+b*x+a)/(4*a*c-b^2)*a*d*e*f+1/c^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3
*e^2*h+2*c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*d^2*f+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*
a^2*e^2*g+8/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*e*g+3
/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*e^2*g+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^
2*d*e*g+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*d*e*g-4/c^2/(c*x^2+b*x+a)/(4*a*c-
b^2)*x*a*b^2*e^2*h-2/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^3*d*e*h-2/c^2/(c*x^2
+b*x+a)/(4*a*c-b^2)*a*b^2*d*e*h-12/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2))*a*b*d*e*h+4/c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*d*e*h+12/c^2/(4
*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e^2*h+2/c^2/(4*a*
c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*d*e*h-4/c^2/(4*a*c-b^2
)*ln(c*x^2+b*x+a)*a*b*e^2*h-1/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2*d*e*h-2/(
c*x^2+b*x+a)/(4*a*c-b^2)*x*b*d*e*f
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 5.78, size = 742, normalized size = 2.58

$$\frac{\frac{-3\,h\,a^2\,b\,c\,e^2+4\,h\,a^2\,c^2\,d\,e+2\,g\,a^2\,c^2\,e^2+h\,a\,b^3\,e^2-2\,h\,a\,b^2\,c\,d\,e-g\,a\,b^2\,c\,e^2+h\,a\,b\,c^2\,d^2+2\,g\,a\,b\,c^2\,d\,e+f\,a\,b\,c^2\,e^2-2\,g\,a\,c^3\,d^2-4\,f\,a\,c^3\,d\,e+f\,b\,c^3\,d^2}{c\,\left(4\,a\,c-b^2\right)}+\frac{x\,\left(2\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x)
```

```
[Out] ((2*a^2*c^2*e^2*g - 2*a*c^3*d^2*g + b*c^3*d^2*f + a*b^3*e^2*h + a*b*c^2*e^2
*f + a*b*c^2*d^2*h - a*b^2*c*e^2*g - 3*a^2*b*c*e^2*h + 4*a^2*c^2*d*e*h - 4*
a*c^3*d*e*f + 2*a*b*c^2*d*e*g - 2*a*b^2*c*d*e*h)/(c*(4*a*c - b^2)) + (x*(2*
c^4*d^2*f + b^4*e^2*h + b^2*c^2*e^2*f + 2*a^2*c^2*e^2*h + b^2*c^2*d^2*h - 2
*a*c^3*e^2*f - 2*a*c^3*d^2*h - b*c^3*d^2*g - b^3*c*e^2*g + 3*a*b*c^2*e^2*g
- 4*a*b^2*c*e^2*h + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - 2*b*c^3*d*e*f - 2*b^3
*c*d*e*h + 6*a*b*c^2*d*e*h))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x)
 + (log(a + b*x + c*x^2)*(2*b^7*e^2*h + 64*a^3*c^4*e^2*g - b^6*c*e^2*g - 24
*a*b^5*c*e^2*h + 128*a^3*c^4*d*e*h + 12*a*b^4*c^2*e^2*g - 128*a^3*b*c^3*e^2
*h - 2*b^6*c*d*e*h - 48*a^2*b^2*c^3*e^2*g + 96*a^2*b^3*c^2*e^2*h + 24*a*b^4
*c^2*d*e*h - 96*a^2*b^2*c^3*d*e*h))/(2*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4
 - 48*a^2*b^2*c^5)) + (atan((2*c*x)/(4*a*c - b^2)^(1/2) - (b^3*c^2 - 4*a*b*
c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(4*c^4*d^2*f - 2*b^4*e^2*h - 12*a^2*c^2*e^2
*h + 4*a*c^3*e^2*f + 4*a*c^3*d^2*h - 2*b*c^3*d^2*g + b^3*c*e^2*g - 6*a*b*c^
2*e^2*g + 12*a*b^2*c*e^2*h + 8*a*c^3*d*e*g - 4*b*c^3*d*e*f + 2*b^3*c*d*e*h
- 12*a*b*c^2*d*e*h))/(c^3*(4*a*c - b^2)^(3/2)) + (e^2*h*x)/c^2
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

3.156 $\int \frac{(d+ex)(f+gx+hx^2)}{(a+bx+cx^2)^2} dx$

Optimal. Leaf size=178

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2c^2(b(dg+ef)-2a(dh+eg\right.}{c^2\left(b^2-4ac\right)^{3/2}}$$

[Out] (e*x+d)*(c*(2*a*g-b*(f+a*h/c))-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)+(4*c^3*d*f+b^3*e*h-6*a*b*c*e*h-2*c^2*(b*(d*g+e*f)-2*a*(d*h+e*g)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/2*e*h*ln(c*x^2+b*x+a)/c^2

Rubi [A] time = 0.27, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1644, 634, 618, 206, 628}

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2c^2(b(dg+ef)-2a(dh+eg\right.}{c^2\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]

[Out] ((d + e*x)*(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((4*c^3*d*f + b^3*e*h - 6*a*b*c*e*h - 2*c^2*(b*(e*f + d*g) - 2*a*(e*g + d*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*(b^2 - 4*a*c)^(3/2)) + (e*h*Log[a + b*x + c*x^2])/(2*c^2)

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \int \frac{2cdf-b(ef+d\cdots}{\cdots}$$

$$= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{(eh)\int \frac{b+2cx}{a+bx+c\cdots}}{2c^2}\cdots$$

$$= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{eh\log\left(a+b\cdots\right.}{2c^2}\cdots$$

$$= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\left(4c^3df+b^3e\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.46, size = 225, normalized size = 1.26

$$\frac{-\frac{2\left(2c\left(a^2eh-ac(d(g+hx)+e(f+gx))+c^2dfx\right)+b^2(cx(dh+eg)-aeh)+bc(adh+ae(g+3hx)+cd(f-gx)-cefx)+b^3(-e)hx\right)}{\left(b^2-4ac\right)(a+x(b+cx))}+\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(-2c^2(b(dg\cdots\right.}{\cdots}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] ((-2*(-(b^3*e*h*x) + b^2*(-(a*e*h) + c*(e*g + d*h)*x) + b*c*(a*d*h - c*e*f*
x + c*d*(f - g*x) + a*e*(g + 3*h*x)) + 2*c*(a^2*e*h + c^2*d*f*x - a*c*(e*(f
 + g*x) + d*(g + h*x)))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*(4*c^3*d*f
 + b^3*e*h - 6*a*b*c*e*h - 2*c^2*(b*(e*f + d*g) - 2*a*(e*g + d*h)))*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + e*h*Log[a + x*(b +
c*x)])/(2*c^2)
```

fricas [B] time = 1.27, size = 1413, normalized size = 7.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(((2*(2*c^4*d - b*c^3*e)*f - 2*(b*c^3*d - 2*a*c^3*e)*g + (4*a*c^3*d +
(b^3*c - 6*a*b*c^2)*e)*h)*x^2 + 2*(2*a*c^3*d - a*b*c^2*e)*f - 2*(a*b*c^2*d
```

```
- 2*a^2*c^2*e)*g + (4*a^2*c^2*d + (a*b^3 - 6*a^2*b*c)*e)*h + (2*(2*b*c^3*d
- b^2*c^2*e)*f - 2*(b^2*c^2*d - 2*a*b*c^2*e)*g + (4*a*b*c^2*d + (b^4 - 6*a*
b^2*c)*e)*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c +
sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*((b^3*c^2 - 4*a*b*c^3
)*d - 2*(a*b^2*c^2 - 4*a^2*c^3)*e)*f + 2*(2*(a*b^2*c^2 - 4*a^2*c^3)*d - (a*
b^3*c - 4*a^2*b*c^2)*e)*g - 2*((a*b^3*c - 4*a^2*b*c^2)*d - (a*b^4 - 6*a^2*b
^2*c + 8*a^3*c^2)*e)*h - 2*((2*(b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*b*c^3
)*e)*f - ((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*e)*g
+ ((b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d - (b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*e
)*h)*x + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e*h*x^2 + (b^5 - 8*a*b^3*c + 1
6*a^2*b*c^2)*e*h*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*e*h)*log(c*x^2 + b*
x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 +
16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x), 1/2*(2*((2*(2*
c^4*d - b*c^3*e)*f - 2*(b*c^3*d - 2*a*c^3*e)*g + (4*a*c^3*d + (b^3*c - 6*a*
b*c^2)*e)*h)*x^2 + 2*(2*a*c^3*d - a*b*c^2*e)*f - 2*(a*b*c^2*d - 2*a^2*c^2*e
)*g + (4*a^2*c^2*d + (a*b^3 - 6*a^2*b*c)*e)*h + (2*(2*b*c^3*d - b^2*c^2*e)*
f - 2*(b^2*c^2*d - 2*a*b*c^2*e)*g + (4*a*b*c^2*d + (b^4 - 6*a*b^2*c)*e)*h)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 - 2*((b^3*c^2 - 4*a*b*c^3)*d - 2*(a*b^2*c^2 - 4*a^2*c^3)*e)*f + 2*(2*(a*b^
2*c^2 - 4*a^2*c^3)*d - (a*b^3*c - 4*a^2*b*c^2)*e)*g - 2*((a*b^3*c - 4*a^2*b
*c^2)*d - (a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*e)*h - 2*((2*(b^2*c^3 - 4*a*c^4
)*d - (b^3*c^2 - 4*a*b*c^3)*e)*f - ((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*
b^2*c^2 + 8*a^2*c^3)*e)*g + ((b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d - (b^5 - 7
*a*b^3*c + 12*a^2*b*c^2)*e)*h)*x + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e*h*
x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*h*x + (a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2)*e*h)*log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 +
 (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2
*b*c^4)*x)]
```

giac [A] time = 0.17, size = 285, normalized size = 1.60

$$\frac{he\log\left(cx^2+bx+a\right)}{2\,c^2}-\frac{\left(4\,c^3df-2\,bc^2dg+4\,ac^2dh-2\,bc^2fe+4\,ac^2ge+b^3he-6\,abche\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^2-4\,ac^3\right)\sqrt{-b^2+4\,ac}}-\frac{bc^2 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*h*e*log(c*x^2 + b*x + a)/c^2 - (4*c^3*d*f - 2*b*c^2*d*g + 4*a*c^2*d*h -
 2*b*c^2*f*e + 4*a*c^2*g*e + b^3*h*e - 6*a*b*c*h*e)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) - (b*c^2*d*f - 2*a
*c^2*d*g + a*b*c*d*h - 2*a*c^2*f*e + a*b*c*g*e - a*b^2*h*e + 2*a^2*c*h*e +
(2*c^3*d*f - b*c^2*d*g + b^2*c*d*h - 2*a*c^2*d*h - b*c^2*f*e + b^2*c*g*e -
2*a*c^2*g*e - b^3*h*e + 3*a*b*c*h*e)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^
2)
```

maple [B] time = 0.01, size = 500, normalized size = 2.81

$$-\frac{6abeh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c}+\frac{4adh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{4aeg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{b^3eh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c^2}-\frac{2bdg\arctan\left(\cdots\right)}{\left(4ac-\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x)
```

```
[Out] ((3*a*b*c*e*h-2*a*c^2*d*h-2*a*c^2*e*g-b^3*e*h+b^2*c*d*h+b^2*c*e*g-b*c^2*d*g
-b*c^2*e*f+2*c^3*d*f)/c^2/(4*a*c-b^2)*x+(2*a^2*c*e*h-a*b^2*e*h+a*b*c*d*h+a*
b*c*e*g-2*a*c^2*d*g-2*a*c^2*e*f+b*c^2*d*f)/(4*a*c-b^2)/c^2)/(c*x^2+b*x+a)+2
```

```
/c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*e*h-1/2/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^
2*e*h-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*e*h+4/(
4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*h+4/(4*a*c-b^2)^(3
/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*e*g-2/(4*a*c-b^2)^(3/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*b*d*g-2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*
c-b^2)^(1/2))*b*e*f+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))*d*f+1/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e*h
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.04, size = 376, normalized size = 2.11

$$\frac{\frac{b c^2 d f-2 a c^2 e f-2 a c^2 d g-a b^2 e h+2 a^2 c e h+a b c d h+a b c e g}{c^2 (4 a c-b^2)}-\frac{x (b^3 e h-2 c^3 d f+2 a c^2 d h+2 a c^2 e g+b c^2 d g+b c^2 e f-b^2 c d h-b^2 c e g-3 a b c e h}{c^2 (4 a c-b^2)}}{c x^2+b x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x)
```

```
[Out] ((b*c^2*d*f - 2*a*c^2*e*f - 2*a*c^2*d*g - a*b^2*e*h + 2*a^2*c*e*h + a*b*c*d
*h + a*b*c*e*g)/(c^2*(4*a*c - b^2)) - (x*(b^3*e*h - 2*c^3*d*f + 2*a*c^2*d*h
 + 2*a*c^2*e*g + b*c^2*d*g + b*c^2*e*f - b^2*c*d*h - b^2*c*e*g - 3*a*b*c*e*
h))/(c^2*(4*a*c - b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6*e*h
 - 64*a^3*c^3*e*h + 48*a^2*b^2*c^2*e*h - 12*a*b^4*c*e*h))/(2*(64*a^3*c^5 -
b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)) + (atan((2*c*x)/(4*a*c - b^2)^(1/
2) - (b^3*c - 4*a*b*c^2)/(c*(4*a*c - b^2)^(3/2)))*(4*c^3*d*f + b^3*e*h + 4*
a*c^2*d*h + 4*a*c^2*e*g - 2*b*c^2*d*g - 2*b*c^2*e*f - 6*a*b*c*e*h))/(c^2*(4
*a*c - b^2)^(3/2))
```

sympy [B] time = 60.26, size = 1535, normalized size = 8.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] (e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*
c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-16*a**2*c*
*3*(e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4
*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 8*a**2*c*e*h +
8*a*b**2*c**2*(e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c
**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*
f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - a*b*
*2*e*h - 2*a*b*c*d*h - 2*a*b*c*e*g - b**4*c*(e*h/(2*c**2) - sqrt(-(4*a*c -
b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*
d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2
```

```
+ 12*a*b**4*c - b**6))) + b**2*c*d*g + b**2*c*e*f - 2*b*c**2*d*f)/(6*a*b*c*
e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f
- 4*c**3*d*f)) + (e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*
a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3
*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log
(x + (-16*a**2*c**3*(e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h -
 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c
**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))
+ 8*a**2*c*e*h + 8*a*b**2*c**2*(e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*
a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**
2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6))) - a*b**2*e*h - 2*a*b*c*d*h - 2*a*b*c*e*g - b**4*c*(e*h/(2*c**2)
+ sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**
3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 4
8*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + b**2*c*d*g + b**2*c*e*f - 2*b*c*
*2*d*f)/(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*
g + 2*b*c**2*e*f - 4*c**3*d*f)) + (2*a**2*c*e*h - a*b**2*e*h + a*b*c*d*h +
a*b*c*e*g - 2*a*c**2*d*g - 2*a*c**2*e*f + b*c**2*d*f + x*(3*a*b*c*e*h - 2*a
*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g
- b*c**2*e*f + 2*c**3*d*f))/(4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b
**2*c**3) + x*(4*a*b*c**3 - b**3*c**2))
```

3.157 $\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx$

Optimal. Leaf size=118

$$\frac{c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2ah-bg+2cf)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (c*(2*a*g-b*(f+a*h/c))-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*x)/c/(-4*a*c+b^2)/(c*
x^2+b*x+a)+2*(2*a*h-b*g+2*c*f)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1660, 12, 618, 206}

$$\frac{c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2ah-bg+2cf)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*x)/(c*(b
^2 - 4*a*c)*(a + b*x + c*x^2)) + (2*(2*c*f - b*g + 2*a*h)*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{\left(a+bx+cx^2\right)^2}\,dx = \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{2cf-bg+2ah}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2cf-bg+2ah)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$

$$= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{(2(2cf-bg+2ah))\,\mathrm{Subst}\left(\int \frac{1}{b^2-4\ldots}\right.}{b^2-4ac}$$

$$= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{2(2cf-bg+2ah)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right.}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.11, size = 114, normalized size = 0.97

$$\frac{abh-2ac(g+hx)+b^2hx+bc(f-gx)+2c^2fx}{c\left(4ac-b^2\right)(a+x(b+cx))}-\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)(-2ah+bg-2cf)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (a*b*h + 2*c^2*f*x + b^2*h*x + b*c*(f - g*x) - 2*a*c*(g + h*x))/(c*(-b^2 +
4*a*c)*(a + x*(b + c*x))) - (2*(-2*c*f + b*g - 2*a*h)*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

fricas [B] time = 0.94, size = 632, normalized size = 5.36

$$\left[-\frac{\left(2\,ac^2f-abcg+2\,a^2ch+\left(2\,c^3f-bc^2g+2\,ac^2h\right)x^2+\left(2\,bc^2f-b^2cg+2\,abch\right)x\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2\ldots}{c\ldots}\right.}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-((2*a*c^2*f - a*b*c*g + 2*a^2*c*h + (2*c^3*f - b*c^2*g + 2*a*c^2*h)*x^2 +
 (2*b*c^2*f - b^2*c*g + 2*a*b*c*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*
b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (
b^3*c - 4*a*b*c^2)*f - 2*(a*b^2*c - 4*a^2*c^2)*g + (a*b^3 - 4*a^2*b*c)*h +
(2*(b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b*c^2)*g + (b^4 - 6*a*b^2*c + 8*a^2
*c^2)*h)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), (2*(2*a*c^2*f
- a*b*c*g + 2*a^2*c*h + (2*c^3*f - b*c^2*g + 2*a*c^2*h)*x^2 + (2*b*c^2*f -
b^2*c*g + 2*a*b*c*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*
x + b)/(b^2 - 4*a*c)) - (b^3*c - 4*a*b*c^2)*f + 2*(a*b^2*c - 4*a^2*c^2)*g -
 (a*b^3 - 4*a^2*b*c)*h - (2*(b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b*c^2)*g +
 (b^4 - 6*a*b^2*c + 8*a^2*c^2)*h)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3
+ (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3)*x)]
```

giac [A] time = 0.16, size = 125, normalized size = 1.06

$$-\frac{2\left(2\,cf-bg+2\,ah\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{2\,c^2fx-bcgx+b^2hx-2\,achx+bcf-2\,acg+abh}{\left(b^2c-4\,ac^2\right)\left(cx^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*(2*c*f - b*g + 2*a*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a
*c)*sqrt(-b^2 + 4*a*c)) - (2*c^2*f*x - b*c*g*x + b^2*h*x - 2*a*c*h*x + b*c*
f - 2*a*c*g + a*b*h)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

maple [A] time = 0.01, size = 194, normalized size = 1.64

$$\frac{4ah\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}-\frac{2bg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{4cf\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{-\frac{\left(2ach-b^2h+bcg-2c^2f\right)x}{\left(4ac-b^2\right)c}+\frac{abh-2acg+bcf}{\left(4ac-b^2\right)c}}{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x)
```

```
[Out] (-(2*a*c*h-b^2*h+b*c*g-2*c^2*f)/c/(4*a*c-b^2)*x+1/c*(a*b*h-2*a*c*g+b*c*f)/(
4*a*c-b^2))/(c*x^2+b*x+a)+4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*a*h-2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*g+4/(4
*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 3.90, size = 203, normalized size = 1.72

$$\frac{\frac{a\,b\,h-2\,a\,c\,g+b\,c\,f}{c\,\left(4\,a\,c-b^2\right)}+\frac{x\,\left(h\,b^2-g\,b\,c+2\,f\,c^2-2\,a\,h\,c\right)}{c\,\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a}-\frac{2\,\mathrm{atan}\left(\frac{\left(\frac{\left(b^3-4\,a\,b\,c\right)\,\left(2\,a\,h-b\,g+2\,c\,f\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{2\,c\,x\,\left(2\,a\,h-b\,g+2\,c\,f\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,a\,h-b\,g+2\,c\,f}\right)\,\left(2\,a\,h-b\,g+2\,c\,f\right.}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x)
```

```
[Out] ((a*b*h - 2*a*c*g + b*c*f)/(c*(4*a*c - b^2)) + (x*(2*c^2*f + b^2*h - 2*a*c*
h - b*c*g))/(c*(4*a*c - b^2)))/(a + b*x + c*x^2) - (2*atan(((((b^3 - 4*a*b*
c)*(2*a*h - b*g + 2*c*f))/(4*a*c - b^2)^(5/2) - (2*c*x*(2*a*h - b*g + 2*c*f
))/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a*h - b*g + 2*c*f))*(2*a*h - b*g
+ 2*c*f))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 2.24, size = 459, normalized size = 3.89

$$-\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(2ah-bg+2cf\right)\log\left(x+\frac{-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(2ah-bg+2cf\right)+8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(2ah\ldots\right.}{4ach-2\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f)*log(x + (-16*a**2*c**2*sq
rt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f) + 8*a*b**2*c*sqrt(-1/(4*a*c
- b**2)**3)*(2*a*h - b*g + 2*c*f) + 2*a*b*h - b**4*sqrt(-1/(4*a*c - b**2)**
3)*(2*a*h - b*g + 2*c*f) - b**2*g + 2*b*c*f)/(4*a*c*h - 2*b*c*g + 4*c**2*f)
) + sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f)*log(x + (16*a**2*c**2*
sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f) - 8*a*b**2*c*sqrt(-1/(4*a*
c - b**2)**3)*(2*a*h - b*g + 2*c*f) + 2*a*b*h + b**4*sqrt(-1/(4*a*c - b**2)
**3)*(2*a*h - b*g + 2*c*f) - b**2*g + 2*b*c*f)/(4*a*c*h - 2*b*c*g + 4*c**2*
f)) + (a*b*h - 2*a*c*g + b*c*f + x*(-2*a*c*h + b**2*h - b*c*g + 2*c**2*f))/
(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*(4*a*b*c**2 - b**
3*c))
```

3.158 $\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=407

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ce\left(2a^2e(eg-dh)-ab\left(d^2h+deg+3e^2f\right)+2b^2d^2g\right)+be\left(-2a^2e^2h+4abdeh+b^2\left(d^2(-h)\right.\right.\right.}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)^2}$$

```
[Out] (b^2*e*f-b*(a*d*h+a*e*g+c*d*f)-2*a*(-a*e*h-c*d*g+c*e*f)-(2*c^2*d*f+b*(-a*e+
b*d)*h-c*(2*a*d*h-2*a*e*g+b*d*g+b*e*f))*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)
/(c*x^2+b*x+a)+(4*c^3*d^3*f+b*e*(4*a*b*d*e*h-2*a^2*e^2*h+b^2*(-d^2*h-d*e*g+
e^2*f))-2*c^2*d*(b*d*(d*g+3*e*f)-2*a*(d^2*h-d*e*g+3*e^2*f))+2*c*e*(2*b^2*d^
2*g+2*a^2*e*(-d*h+e*g)-a*b*(d^2*h+d*e*g+3*e^2*f)))*arctanh((2*c*x+b)/(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e+c*d^2)^2+e*(d^2*h-d*e*g+e^2*f
)*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2-1/2*e*(d^2*h-d*e*g+e^2*f)*ln(c*x^2+b*x+a)
/(a*e^2-b*d*e+c*d^2)^2
```

Rubi [A] time = 1.09, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1646, 800, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ce\left(2a^2e(eg-dh)-ab\left(d^2h+deg+3e^2f\right)+2b^2d^2g\right)+be\left(-2a^2e^2h+4abdeh+b^2\left(d^2(-h)\right.\right.\right.}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (b^2*e*f - b*(c*d*f + a*e*g + a*d*h) - 2*a*(c*e*f - c*d*g - a*e*h) - (2*c^2
*d*f + b*(b*d - a*e)*h - c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*x)/((b^2 -
4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)) + ((4*c^3*d^3*f + b*e*(4*
a*b*d*e*h - 2*a^2*e^2*h + b^2*(e^2*f - d*e*g - d^2*h)) - 2*c^2*d*(b*d*(3*e*
f + d*g) - 2*a*(3*e^2*f - d*e*g + d^2*h)) + 2*c*e*(2*b^2*d^2*g + 2*a^2*e*(e
*g - d*h) - a*b*(3*e^2*f + d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^2) + (e*(e^2*f - d*e*
g + d^2*h)*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^2 - (e*(e^2*f - d*e*g + d^
2*h)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx = \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(be\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(be\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(be\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(be\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(be\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(be\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

Mathematica [A] time = 0.90, size = 405, normalized size = 1.00

$$\frac{-2a^2eh+ab(dh+e(g-hx))+2ac(e(f+gx)-d(g+hx))+b^2(dhx-ef)+bc(d(f-gx)-efx)+2c^2dfx}{\left(b^2-4ac\right)(a+x(b+cx))\left(e(bd-ae)-cd^2\right)}-\frac{\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-2*a^2*e*h + 2*c^2*d*f*x + b^2*(-(e*f) + d*h*x) + b*c*(-(e*f*x) + d*(f - g
*x)) + a*b*(d*h + e*(g - h*x)) + 2*a*c*(e*(f + g*x) - d*(g + h*x)))/((b^2 -
 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*(a + x*(b + c*x))) - ((-4*c^3*d^3*f + 2*
c^2*d*(b*d*(3*e*f + d*g) - 2*a*(3*e^2*f - d*e*g + d^2*h)) + b*e*(-4*a*b*d*e
*h + 2*a^2*e^2*h + b^2*(-(e^2*f) + d*e*g + d^2*h)) + 2*c*e*(-2*b^2*d^2*g +
2*a^2*e*(-(e*g) + d*h) + a*b*(3*e^2*f + d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^2) +
 (e*(e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^2 - (e
*(e^2*f - d*e*g + d^2*h)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e)
)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.19, size = 860, normalized size = 2.11

$$-\frac{\left(d^2he - dge^2 + fe^3\right)\log\left(cx^2 + bx + a\right)}{2\left(c^2d^4 - 2\,bcd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,abde^3 + a^2e^4\right)} + \frac{\left(d^2he^2 - dge^3 + fe^4\right)\log\left(|xe + d|\right)}{c^2d^4e - 2\,bcd^3e^2 + b^2d^2e^3 + 2\,acd^2e^3 - 2\,abde^4 + a^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(d^2*h*e - d*g*e^2 + f*e^3)*log(c*x^2 + b*x + a)/(c^2*d^4 - 2*b*c*d^3*
e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4) + (d^2*h*e^2 - d*g
*e^3 + f*e^4)*log(abs(x*e + d))/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 +
2*a*c*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5) - (4*c^3*d^3*f - 2*b*c^2*d^3*g + 4*a
*c^2*d^3*h - 6*b*c^2*d^2*f*e + 4*b^2*c*d^2*g*e - 4*a*c^2*d^2*g*e - b^3*d^2*
h*e - 2*a*b*c*d^2*h*e + 12*a*c^2*d*f*e^2 - b^3*d*g*e^2 - 2*a*b*c*d*g*e^2 +
4*a*b^2*d*h*e^2 - 4*a^2*c*d*h*e^2 + b^3*f*e^3 - 6*a*b*c*f*e^3 + 4*a^2*c*g*e
^3 - 2*a^2*b*h*e^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2*d^4 -
4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2
*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 -
4*a^3*c*e^4)*sqrt(-b^2 + 4*a*c)) - (b*c^2*d^3*f - 2*a*c^2*d^3*g + a*b*c*d^3
*h - 2*b^2*c*d^2*f*e + 2*a*c^2*d^2*f*e + 3*a*b*c*d^2*g*e - a*b^2*d^2*h*e -
2*a^2*c*d^2*h*e + b^3*d*f*e^2 - a*b*c*d*f*e^2 - a*b^2*d*g*e^2 - 2*a^2*c*d*g
*e^2 + 3*a^2*b*d*h*e^2 - a*b^2*f*e^3 + 2*a^2*c*f*e^3 + a^2*b*g*e^3 - 2*a^3*
h*e^3 + (2*c^3*d^3*f - b*c^2*d^3*g + b^2*c*d^3*h - 2*a*c^2*d^3*h - 3*b*c^2*
d^2*f*e + b^2*c*d^2*g*e + 2*a*c^2*d^2*g*e - b^3*d^2*h*e + a*b*c*d^2*h*e + b
^2*c*d*f*e^2 + 2*a*c^2*d*f*e^2 - 3*a*b*c*d*g*e^2 + 2*a*b^2*d*h*e^2 - 2*a^2*
c*d*h*e^2 - a*b*c*f*e^3 + 2*a^2*c*g*e^3 - a^2*b*h*e^3)*x)/((c*d^2 - b*d*e +
 a*e^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

maple [B] time = 0.02, size = 3202, normalized size = 7.87

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x)
```

```
[Out] 2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*a*d*e^2*g-4/(a*e^2-b*
d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c*d*
e^2*h+4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2
)^(1/2))*a*b^2*d*e^2*h-6/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*e^3*f-4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(
3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^2*d^2*e*g+12/(a*e^2-b*d*e+c*d^
2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^2*d*e^2*f+4/
(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
*b^2*c*d^2*e*g-6/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(
4*a*c-b^2)^(1/2))*b*c^2*d^2*e*f-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c*ln(c*
x^2+b*x+a)*a*d^2*e*h-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^
2*b*e^3*h+2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2*c*e^3*g-2
/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*c^2*d^3*h-1/(a*e^2-b*d
*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^3*d^2*e*h+1/(a*e^2-b*d*e+c*d^2)^2
/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^2*c*d^3*h-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x
+a)/(4*a*c-b^2)*x*b*c^2*d^3*g+e^3/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*f+3/(a*e^
2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*d*e^2*h-2/(a*e^2-b*d*e+c*d
^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*c*d^2*e*h-2/(a*e^2-b*d*e+c*d^2)^2/(c*x^
2+b*x+a)/(4*a*c-b^2)*a^2*c*d*e^2*g-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4
*a*c-b^2)*a*b^2*d^2*e*h-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a
*b^2*d*e^2*g+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c*d^3*h+
2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*c^2*d^2*e*f-2/(a*e^2-b*
d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2*c*d^2*e*f+2/(a*e^2-b*d*e+c*d^2)^
2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*c^2*d*e^2*f+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+
b*x+a)/(4*a*c-b^2)*x*b^2*c*d^2*e*g+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4
*a*c-b^2)*x*b^2*c*d*e^2*f-3/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)
*x*b*c^2*d^2*e*f+3/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c*d^
2*e*g-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2
)^(1/2))*a*b*c*d^2*e*h-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*d*e^2*g-2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+
a)/(4*a*c-b^2)*x*a^2*c*d*e^2*h+2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c
-b^2)*x*a*b^2*d*e^2*h-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b
*c*d*e^2*f-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*c*e^3*f+
2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*c^2*d^2*e*g+1/(a*e^2-
b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*c*d^2*e*h-3/(a*e^2-b*d*e+c*d
^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*c*d*e^2*g+1/2/(a*e^2-b*d*e+c*d^2)^2/(
4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2*d^2*e*h-1/2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2
)*ln(c*x^2+b*x+a)*b^2*d*e^2*g-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*e^3*h+4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c
-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c*e^3*g+2/(a*e^2-b*d*e+
c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*c^3*d^3*f+1/(a*e^2-b*d*e+c*d^2)^2/(c*x
^2+b*x+a)/(4*a*c-b^2)*a^2*b*e^3*g+2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*
a*c-b^2)*a^2*c*e^3*f-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^
2*e^3*f-2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*c^2*d^3*g+4/(a*
e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*
c^2*d^3*h-1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*b^3*d^2*e*h-1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*d*e^2*g+e/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*d
^2*h-e^2/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*d*g-2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2
+b*x+a)/(4*a*c-b^2)*a^3*e^3*h+4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^3*d^3*f+1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b
^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^3*f+1/2/(a*e^2-b*d*e+c*
d^2)^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2*e^3*f+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2
+b*x+a)/(4*a*c-b^2)*b^3*d*e^2*f+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*
c-b^2)*b*c^2*d^3*f-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*b*c^2*d^3*g-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c*ln(
c*x^2+b*x+a)*a*e^3*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 6.70, size = 13698, normalized size = 33.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x)
```

```
[Out] symsum(log(root(768*a^5*b*c^4*d^3*e^5*z^3 + 768*a^4*b*c^5*d^5*e^3*z^3 - 192
*a^5*b^3*c^2*d*e^7*z^3 - 192*a^2*b^3*c^5*d^7*e*z^3 - 68*a^3*b^6*c*d^2*e^6*z
^3 - 68*a*b^6*c^3*d^6*e^2*z^3 + 36*a^2*b^7*c*d^3*e^5*z^3 + 36*a*b^7*c^2*d^5
*e^3*z^3 + 256*a^6*b*c^3*d*e^7*z^3 + 256*a^3*b*c^6*d^7*e*z^3 + 48*a^4*b^5*c
*d*e^7*z^3 + 48*a*b^5*c^4*d^7*e*z^3 - 480*a^4*b^2*c^4*d^4*e^4*z^3 + 440*a^3
*b^4*c^3*d^4*e^4*z^3 - 320*a^4*b^3*c^3*d^3*e^5*z^3 - 320*a^3*b^3*c^4*d^5*e^
3*z^3 + 240*a^4*b^4*c^2*d^2*e^6*z^3 + 240*a^2*b^4*c^4*d^6*e^2*z^3 - 192*a^5
*b^2*c^3*d^2*e^6*z^3 - 192*a^3*b^2*c^5*d^6*e^2*z^3 - 90*a^2*b^6*c^2*d^4*e^4
*z^3 - 48*a^3*b^5*c^2*d^3*e^5*z^3 - 48*a^2*b^5*c^3*d^5*e^3*z^3 - 4*b^9*c*d^
5*e^3*z^3 - 4*b^7*c^3*d^7*e*z^3 - 4*a^3*b^7*d*e^7*z^3 - 4*a*b^9*d^3*e^5*z^3
 - 12*a^5*b^4*c*e^8*z^3 - 12*a*b^4*c^5*d^8*z^3 + 6*b^8*c^2*d^6*e^2*z^3 - 38
4*a^5*c^5*d^4*e^4*z^3 - 256*a^6*c^4*d^2*e^6*z^3 - 256*a^4*c^6*d^6*e^2*z^3 +
 6*a^2*b^8*d^2*e^6*z^3 + 48*a^6*b^2*c^2*e^8*z^3 + 48*a^2*b^2*c^6*d^8*z^3 -
64*a^7*c^3*e^8*z^3 - 64*a^3*c^7*d^8*z^3 + b^10*d^4*e^4*z^3 + b^6*c^4*d^8*z^
3 + a^4*b^6*e^8*z^3 - 28*a*b^4*c*d^3*e^3*g*h*z - 10*a^3*b^2*c*d*e^5*g*h*z -
 10*a*b^2*c^3*d^5*e*g*h*z + 16*a*b^4*c*d^2*e^4*f*h*z + 14*a^2*b^3*c*d*e^5*f
*h*z + 4*a*b*c^4*d^4*e^2*f*g*z + 84*a^2*b^2*c^2*d^3*e^3*g*h*z - 108*a^2*b^2
*c^2*d^2*e^4*f*h*z + 16*a*b*c^4*d^5*e*f*h*z - 20*a*b^4*c*d*e^5*f*g*z + 8*a^
2*b^3*c*d^2*e^4*g*h*z + 8*a*b^3*c^2*d^4*e^2*g*h*z - 4*a^3*b*c^2*d^2*e^4*g*h
*z - 4*a^2*b*c^3*d^4*e^2*g*h*z + 16*a^2*b*c^3*d^3*e^3*f*h*z + 16*a*b^3*c^2*
d^3*e^3*f*h*z - 14*a*b^2*c^3*d^4*e^2*f*h*z + 66*a^2*b^2*c^2*d*e^5*f*g*z - 3
6*a*b^2*c^3*d^3*e^3*f*g*z + 20*a*b^3*c^2*d^2*e^4*f*g*z + 12*a^2*b*c^3*d^2*e
^4*f*g*z + 8*a*c^5*d^5*e*f*g*z + 4*a^4*b*c*e^6*g*h*z - 2*a*b^5*d*e^5*f*h*z
+ 4*a*b*c^4*d^6*g*h*z - 112*a^3*c^3*d^3*e^3*g*h*z - 3*b^4*c^2*d^4*e^2*f*h*z
 + 120*a^3*c^3*d^2*e^4*f*h*z - 16*a^2*c^4*d^4*e^2*f*h*z + 14*b^3*c^3*d^4*e^
2*f*g*z - 2*b^4*c^2*d^3*e^3*f*g*z + 16*a^2*c^4*d^3*e^3*f*g*z + 8*a*b^4*c*d^
4*e^2*h^2*z + 4*a^2*b*c^3*d^5*e*h^2*z + 2*a*b^3*c^2*d^5*e*h^2*z + 8*a*b^4*c
*d^2*e^4*g^2*z + 4*a^3*b*c^2*d*e^5*g^2*z + 2*a^2*b^3*c*d*e^5*g^2*z + 48*a*b
*c^4*d^3*e^3*f^2*z + 36*a^2*b*c^3*d*e^5*f^2*z - 6*a*b^3*c^2*d*e^5*f^2*z - 4
5*a^2*b^2*c^2*d^4*e^2*h^2*z - 45*a^2*b^2*c^2*d^2*e^4*g^2*z + 2*b^5*c*d^4*e^
2*g*h*z - b^4*c^2*d^5*e*g*h*z + 8*a^4*c^2*d*e^5*g*h*z + 8*a^2*c^4*d^5*e*g*h
*z + 2*b^3*c^3*d^5*e*f*h*z - 14*b^2*c^4*d^5*e*f*g*z - 2*b^5*c*d^2*e^4*f*g*z
 + 2*a*b^5*d^2*e^4*g*h*z - a^2*b^4*d*e^5*g*h*z - 120*a^3*c^3*d*e^5*f*g*z -
6*a^3*b^2*c*e^6*f*h*z + 12*a^3*b*c^2*e^6*f*g*z - 2*a^2*b^3*c*e^6*f*g*z - 4*
a^4*b*c*d*e^5*h^2*z - 4*a*b*c^4*d^5*e*g^2*z + 6*a^3*b^2*c*d^2*e^4*h^2*z + 2
*a^2*b^3*c*d^3*e^3*h^2*z + 6*a*b^2*c^3*d^4*e^2*g^2*z + 2*a*b^3*c^2*d^3*e^3*
g^2*z - 18*a*b^2*c^3*d^2*e^4*f^2*z - b^6*d^2*e^4*f*h*z + 12*b*c^5*d^5*e*f^2
*z + 12*a*b^4*c*e^6*f^2*z + 56*a^3*c^3*d^4*e^2*h^2*z - 5*b^4*c^2*d^4*e^2*g^
2*z - 4*a^4*c^2*d^2*e^4*h^2*z + 56*a^3*c^3*d^2*e^4*g^2*z - 9*b^2*c^4*d^4*e^
2*f^2*z - 5*a^2*b^4*d^2*e^4*h^2*z - 4*a^2*c^4*d^4*e^2*g^2*z + 3*b^4*c^2*d^2
*e^4*f^2*z - 2*b^3*c^3*d^3*e^3*f^2*z - 36*a^2*c^4*d^2*e^4*f^2*z - 45*a^2*b^
2*c^2*e^6*f^2*z + 2*b^6*d*e^5*f*g*z - 8*a*c^5*d^6*f*h*z + 4*b*c^5*d^6*f*g*z
 + 4*b^3*c^3*d^5*e*g^2*z + 2*b^5*c*d^3*e^3*g^2*z + 4*a^3*b^3*d*e^5*h^2*z +
2*a*b^5*d^3*e^3*h^2*z - 24*a*c^5*d^4*e^2*f^2*z + b^6*d^3*e^3*g*h*z + a^2*b^
```

4*e^6*f*h*z - b^6*d^4*e^2*h^2*z - b^6*d^2*e^4*g^2*z - 4*a^4*c^2*e^6*g^2*z - 4*a^2*c^4*d^6*h^2*z - b^2*c^4*d^6*g^2*z - a^4*b^2*e^6*h^2*z + 48*a^3*c^3*e ^6*f^2*z - 4*c^6*d^6*f^2*z - b^6*e^6*f^2*z - 16*a*b*c^2*d^2*e^3*f*g*h - 4*a *b^2*c*d*e^4*f*g*h - 4*b*c^3*d^4*e*f*g*h - 4*a^2*b*c*e^5*f*g*h + 6*b^2*c^2* d^3*e^2*f*g*h - 8*a^2*b*c*d^2*e^3*g*h^2 + 8*a*b*c^2*d^3*e^2*g^2*h + 2*a*b^2 *c*d^3*e^2*g*h^2 - 2*a*b^2*c*d^2*e^3*g^2*h + 6*a*b^2*c*d^2*e^3*f*h^2 + 4*b^ 3*c*d^2*e^3*f*g*h - 16*a*c^3*d^3*e^2*f*g*h - 8*a^2*c^2*d*e^4*f*g*h + 4*a^2* b*c*d*e^4*g^2*h - 4*a*b*c^2*d^4*e*g*h^2 + 4*a^2*b*c*d*e^4*f*h^2 + 16*a*b*c^ 2*d*e^4*f*g^2 - 2*b^3*c*d*e^4*f^2*h + 8*a*c^3*d^4*e*f*h^2 - 4*b^3*c*d*e^4*f *g^2 - 24*a*c^3*d*e^4*f^2*g - 2*a*b^3*d*e^4*f*h^2 + 6*a*b^2*c*e^5*f^2*h - 1 2*a*b*c^2*e^5*f^2*g - 12*a^2*c^2*d^3*e^2*g*h^2 + 12*a^2*c^2*d^2*e^3*g^2*h - 3*b^2*c^2*d^2*e^3*f^2*h - 5*b^2*c^2*d^2*e^3*f*g^2 + 4*a^2*c^2*d^2*e^3*f*h^ 2 + 2*b^4*d*e^4*f*g*h - 2*b^3*c*d^3*e^2*g^2*h - 4*b*c^3*d^3*e^2*f^2*h - 2*b ^3*c*d^3*e^2*f*h^2 + 24*a*c^3*d^2*e^3*f^2*h + 9*b^2*c^2*d*e^4*f^2*g + 4*b*c ^3*d^3*e^2*f*g^2 + 2*a*b^3*d^2*e^3*g*h^2 - a^2*b^2*d*e^4*g*h^2 + 8*a*c^3*d^ 2*e^3*f*g^2 + 4*a^2*b*c*d^3*e^2*h^3 - 4*a*b*c^2*d^2*e^3*g^3 - b^4*d^2*e^3*g ^2*h - 4*c^4*d^3*e^2*f^2*g - b^4*d^2*e^3*f*h^2 + 4*a^2*c^2*e^5*f*g^2 + 4*a^ 2*c^2*d^4*e*h^3 + 2*b^3*c*d^2*e^3*g^3 - 4*a^2*c^2*d*e^4*g^3 - 2*a*b^3*d^3*e ^2*h^3 + 4*c^4*d^4*e*f^2*h + 2*b^3*c*e^5*f^2*g - 4*b*c^3*d*e^4*f^3 + b^2*c^ 2*d^4*e*g^2*h - b^2*c^2*d^3*e^2*g^3 + b^4*d^3*e^2*g*h^2 + a^2*b^2*e^5*f*h^2 + 4*c^4*d^2*e^3*f^3 - 3*b^2*c^2*e^5*f^3 + a^2*b^2*d^2*e^3*h^3 - b^4*e^5*f^ 2*h + 16*a*c^3*e^5*f^3, z, k)*((a*b^5*c*e^6*f - 8*a^4*c^3*e^6*g + 8*a*c^6*d ^5*e*f - b^6*c*d*e^5*f + 20*a^3*b*c^3*e^6*f - a^3*b^3*c*e^6*h + 8*a^3*c^4*d *e^5*f + 4*a^4*b*c^2*e^6*h - 2*b^2*c^5*d^5*e*f + 8*a^2*c^5*d^5*e*h + 8*a^4* c^3*d*e^5*h + b^3*c^4*d^5*e*g + b^6*c*d^2*e^4*g - b^6*c*d^3*e^3*h - 9*a^2*b ^3*c^2*e^6*f + 2*a^3*b^2*c^2*e^6*g + 16*a^2*c^5*d^3*e^3*f - 8*a^2*c^5*d^4*e ^2*g - 16*a^3*c^4*d^2*e^4*g + 3*b^3*c^4*d^4*e^2*f + 16*a^3*c^4*d^3*e^3*h - 2*b^4*c^3*d^4*e^2*g + b^5*c^2*d^4*e^2*h - 4*a*b^2*c^4*d^3*e^3*f - 2*a*b^3*c ^3*d^2*e^4*f + 8*a^2*b*c^4*d^2*e^4*f - 26*a^2*b^2*c^3*d*e^5*f + 10*a*b^2*c^ 4*d^4*e^2*g + 2*a*b^3*c^3*d^3*e^3*g - 8*a*b^4*c^2*d^2*e^4*g - 8*a^2*b*c^4*d ^3*e^3*g + 5*a^2*b^3*c^2*d*e^5*g - 5*a*b^3*c^3*d^4*e^2*h + 8*a*b^4*c^2*d^3* e^3*h + 4*a^2*b*c^4*d^4*e^2*h + 8*a^3*b*c^3*d^2*e^4*h - 10*a^3*b^2*c^2*d*e^ 5*h - 4*a*b*c^5*d^5*e*g - a*b^5*c*d*e^5*g + 20*a^2*b^2*c^3*d^2*e^4*g - 20*a ^2*b^2*c^3*d^3*e^3*h - 2*a^2*b^3*c^2*d^2*e^4*h - 12*a*b*c^5*d^4*e^2*f + 10* a*b^4*c^2*d*e^5*f - 4*a^3*b*c^3*d*e^5*g - 2*a*b^2*c^4*d^5*e*h + 2*a^2*b^4*c *d*e^5*h)/(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^ 6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^ 5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b *c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3) + root(768*a^5*b*c^4* d^3*e^5*z^3 + 768*a^4*b*c^5*d^5*e^3*z^3 - 192*a^5*b^3*c^2*d*e^7*z^3 - 192*a ^2*b^3*c^5*d^7*e*z^3 - 68*a^3*b^6*c*d^2*e^6*z^3 - 68*a*b^6*c^3*d^6*e^2*z^3 + 36*a^2*b^7*c*d^3*e^5*z^3 + 36*a*b^7*c^2*d^5*e^3*z^3 + 256*a^6*b*c^3*d*e^7 *z^3 + 256*a^3*b*c^6*d^7*e*z^3 + 48*a^4*b^5*c*d*e^7*z^3 + 48*a*b^5*c^4*d^7* e*z^3 - 480*a^4*b^2*c^4*d^4*e^4*z^3 + 440*a^3*b^4*c^3*d^4*e^4*z^3 - 320*a^4 *b^3*c^3*d^3*e^5*z^3 - 320*a^3*b^3*c^4*d^5*e^3*z^3 + 240*a^4*b^4*c^2*d^2*e^ 6*z^3 + 240*a^2*b^4*c^4*d^6*e^2*z^3 - 192*a^5*b^2*c^3*d^2*e^6*z^3 - 192*a^3 *b^2*c^5*d^6*e^2*z^3 - 90*a^2*b^6*c^2*d^4*e^4*z^3 - 48*a^3*b^5*c^2*d^3*e^5* z^3 - 48*a^2*b^5*c^3*d^5*e^3*z^3 - 4*b^9*c*d^5*e^3*z^3 - 4*b^7*c^3*d^7*e*z^ 3 - 4*a^3*b^7*d*e^7*z^3 - 4*a*b^9*d^3*e^5*z^3 - 12*a^5*b^4*c*e^8*z^3 - 12*a *b^4*c^5*d^8*z^3 + 6*b^8*c^2*d^6*e^2*z^3 - 384*a^5*c^5*d^4*e^4*z^3 - 256*a^ 6*c^4*d^2*e^6*z^3 - 256*a^4*c^6*d^6*e^2*z^3 + 6*a^2*b^8*d^2*e^6*z^3 + 48*a^ 6*b^2*c^2*e^8*z^3 + 48*a^2*b^2*c^6*d^8*z^3 - 64*a^7*c^3*e^8*z^3 - 64*a^3*c^ 7*d^8*z^3 + b^10*d^4*e^4*z^3 + b^6*c^4*d^8*z^3 + a^4*b^6*e^8*z^3 - 28*a*b^4 *c*d^3*e^3*g*h*z - 10*a^3*b^2*c*d*e^5*g*h*z - 10*a*b^2*c^3*d^5*e*g*h*z + 16 *a*b^4*c*d^2*e^4*f*h*z + 14*a^2*b^3*c*d*e^5*f*h*z + 4*a*b*c^4*d^4*e^2*f*g*z + 84*a^2*b^2*c^2*d^3*e^3*g*h*z - 108*a^2*b^2*c^2*d^2*e^4*f*h*z + 16*a*b*c^ 4*d^5*e*f*h*z - 20*a*b^4*c*d*e^5*f*g*z + 8*a^2*b^3*c*d^2*e^4*g*h*z + 8*a*b^ 3*c^2*d^4*e^2*g*h*z - 4*a^3*b*c^2*d^2*e^4*g*h*z - 4*a^2*b*c^3*d^4*e^2*g*h*z + 16*a^2*b*c^3*d^3*e^3*f*h*z + 16*a*b^3*c^2*d^3*e^3*f*h*z - 14*a*b^2*c^3*d

^4*e^2*f*h*z + 66*a^2*b^2*c^2*d*e^5*f*g*z - 36*a*b^2*c^3*d^3*e^3*f*g*z + 20
*a*b^3*c^2*d^2*e^4*f*g*z + 12*a^2*b*c^3*d^2*e^4*f*g*z + 8*a*c^5*d^5*e*f*g*z
+ 4*a^4*b*c*e^6*g*h*z - 2*a*b^5*d*e^5*f*h*z + 4*a*b*c^4*d^6*g*h*z - 112*a^
3*c^3*d^3*e^3*g*h*z - 3*b^4*c^2*d^4*e^2*f*h*z + 120*a^3*c^3*d^2*e^4*f*h*z -
16*a^2*c^4*d^4*e^2*f*h*z + 14*b^3*c^3*d^4*e^2*f*g*z - 2*b^4*c^2*d^3*e^3*f*
g*z + 16*a^2*c^4*d^3*e^3*f*g*z + 8*a*b^4*c*d^4*e^2*h^2*z + 4*a^2*b*c^3*d^5*
e*h^2*z + 2*a*b^3*c^2*d^5*e*h^2*z + 8*a*b^4*c*d^2*e^4*g^2*z + 4*a^3*b*c^2*d
*e^5*g^2*z + 2*a^2*b^3*c*d*e^5*g^2*z + 48*a*b*c^4*d^3*e^3*f^2*z + 36*a^2*b*
c^3*d*e^5*f^2*z - 6*a*b^3*c^2*d*e^5*f^2*z - 45*a^2*b^2*c^2*d^4*e^2*h^2*z -
45*a^2*b^2*c^2*d^2*e^4*g^2*z + 2*b^5*c*d^4*e^2*g*h*z - b^4*c^2*d^5*e*g*h*z
+ 8*a^4*c^2*d*e^5*g*h*z + 8*a^2*c^4*d^5*e*g*h*z + 2*b^3*c^3*d^5*e*f*h*z - 1
4*b^2*c^4*d^5*e*f*g*z - 2*b^5*c*d^2*e^4*f*g*z + 2*a*b^5*d^2*e^4*g*h*z - a^2
*b^4*d*e^5*g*h*z - 120*a^3*c^3*d*e^5*f*g*z - 6*a^3*b^2*c*e^6*f*h*z + 12*a^3
*b*c^2*e^6*f*g*z - 2*a^2*b^3*c*e^6*f*g*z - 4*a^4*b*c*d*e^5*h^2*z - 4*a*b*c^
4*d^5*e*g^2*z + 6*a^3*b^2*c*d^2*e^4*h^2*z + 2*a^2*b^3*c*d^3*e^3*h^2*z + 6*a
*b^2*c^3*d^4*e^2*g^2*z + 2*a*b^3*c^2*d^3*e^3*g^2*z - 18*a*b^2*c^3*d^2*e^4*f
^2*z - b^6*d^2*e^4*f*h*z + 12*b*c^5*d^5*e*f^2*z + 12*a*b^4*c*e^6*f^2*z + 56
*a^3*c^3*d^4*e^2*h^2*z - 5*b^4*c^2*d^4*e^2*g^2*z - 4*a^4*c^2*d^2*e^4*h^2*z
+ 56*a^3*c^3*d^2*e^4*g^2*z - 9*b^2*c^4*d^4*e^2*f^2*z - 5*a^2*b^4*d^2*e^4*h^
2*z - 4*a^2*c^4*d^4*e^2*g^2*z + 3*b^4*c^2*d^2*e^4*f^2*z - 2*b^3*c^3*d^3*e^3
*f^2*z - 36*a^2*c^4*d^2*e^4*f^2*z - 45*a^2*b^2*c^2*e^6*f^2*z + 2*b^6*d*e^5*
f*g*z - 8*a*c^5*d^6*f*h*z + 4*b*c^5*d^6*f*g*z + 4*b^3*c^3*d^5*e*g^2*z + 2*b
^5*c*d^3*e^3*g^2*z + 4*a^3*b^3*d*e^5*h^2*z + 2*a*b^5*d^3*e^3*h^2*z - 24*a*c
^5*d^4*e^2*f^2*z + b^6*d^3*e^3*g*h*z + a^2*b^4*e^6*f*h*z - b^6*d^4*e^2*h^2*
z - b^6*d^2*e^4*g^2*z - 4*a^4*c^2*e^6*g^2*z - 4*a^2*c^4*d^6*h^2*z - b^2*c^4
*d^6*g^2*z - a^4*b^2*e^6*h^2*z + 48*a^3*c^3*e^6*f^2*z - 4*c^6*d^6*f^2*z - b
^6*e^6*f^2*z - 16*a*b*c^2*d^2*e^3*f*g*h - 4*a*b^2*c*d*e^4*f*g*h - 4*b*c^3*d
^4*e*f*g*h - 4*a^2*b*c*e^5*f*g*h + 6*b^2*c^2*d^3*e^2*f*g*h - 8*a^2*b*c*d^2*
e^3*g*h^2 + 8*a*b*c^2*d^3*e^2*g^2*h + 2*a*b^2*c*d^3*e^2*g*h^2 - 2*a*b^2*c*d
^2*e^3*g^2*h + 6*a*b^2*c*d^2*e^3*f*h^2 + 4*b^3*c*d^2*e^3*f*g*h - 16*a*c^3*d
^3*e^2*f*g*h - 8*a^2*c^2*d*e^4*f*g*h + 4*a^2*b*c*d*e^4*g^2*h - 4*a*b*c^2*d^
4*e*g*h^2 + 4*a^2*b*c*d*e^4*f*h^2 + 16*a*b*c^2*d*e^4*f*g^2 - 2*b^3*c*d*e^4*
f^2*h + 8*a*c^3*d^4*e*f*h^2 - 4*b^3*c*d*e^4*f*g^2 - 24*a*c^3*d*e^4*f^2*g -
2*a*b^3*d*e^4*f*h^2 + 6*a*b^2*c*e^5*f^2*h - 12*a*b*c^2*e^5*f^2*g - 12*a^2*c
^2*d^3*e^2*g*h^2 + 12*a^2*c^2*d^2*e^3*g^2*h - 3*b^2*c^2*d^2*e^3*f^2*h - 5*b
^2*c^2*d^2*e^3*f*g^2 + 4*a^2*c^2*d^2*e^3*f*h^2 + 2*b^4*d*e^4*f*g*h - 2*b^3*
c*d^3*e^2*g^2*h - 4*b*c^3*d^3*e^2*f^2*h - 2*b^3*c*d^3*e^2*f*h^2 + 24*a*c^3*
d^2*e^3*f^2*h + 9*b^2*c^2*d*e^4*f^2*g + 4*b*c^3*d^3*e^2*f*g^2 + 2*a*b^3*d^2
*e^3*g*h^2 - a^2*b^2*d*e^4*g*h^2 + 8*a*c^3*d^2*e^3*f*g^2 + 4*a^2*b*c*d^3*e^
2*h^3 - 4*a*b*c^2*d^2*e^3*g^3 - b^4*d^2*e^3*g^2*h - 4*c^4*d^3*e^2*f^2*g - b
^4*d^2*e^3*f*h^2 + 4*a^2*c^2*e^5*f*g^2 + 4*a^2*c^2*d^4*e*h^3 + 2*b^3*c*d^2*
e^3*g^3 - 4*a^2*c^2*d*e^4*g^3 - 2*a*b^3*d^3*e^2*h^3 + 4*c^4*d^4*e*f^2*h + 2
*b^3*c*e^5*f^2*g - 4*b*c^3*d*e^4*f^3 + b^2*c^2*d^4*e*g^2*h - b^2*c^2*d^3*e^
2*g^3 + b^4*d^3*e^2*g*h^2 + a^2*b^2*e^5*f*h^2 + 4*c^4*d^2*e^3*f^3 - 3*b^2*c
^2*e^5*f^3 + a^2*b^2*d^2*e^3*h^3 - b^4*e^5*f^2*h + 16*a*c^3*e^5*f^3, z, k)*
((128*a^5*c^4*d*e^6 - 16*a^5*b*c^3*e^7 - a^3*b^5*c*e^7 - b^5*c^4*d^6*e - b^
8*c*d^3*e^4 + 8*a^4*b^3*c^2*e^7 + 128*a^3*c^6*d^5*e^2 + 256*a^4*c^5*d^3*e^4
+ b^6*c^3*d^5*e^2 + b^7*c^2*d^4*e^3 - 48*a^2*b^2*c^5*d^5*e^2 + 168*a^2*b^3
*c^4*d^4*e^3 - 80*a^2*b^4*c^3*d^3*e^4 - 27*a^2*b^5*c^2*d^2*e^5 + 32*a^3*b^2
*c^4*d^3*e^4 + 168*a^3*b^3*c^3*d^2*e^5 + 8*a*b^3*c^5*d^6*e + a*b^7*c*d^2*e^
5 - 16*a^2*b*c^6*d^6*e + a^2*b^6*c*d*e^6 - 27*a*b^5*c^3*d^4*e^3 + 18*a*b^6*
c^2*d^3*e^4 - 304*a^3*b*c^5*d^4*e^3 - 304*a^4*b*c^4*d^2*e^5 - 48*a^4*b^2*c^
3*d*e^6)/(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6
*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5
*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*
c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3) - (x*(2*a^2*b^6*c*e^7
- 96*a^5*c^4*e^7 + 32*a^2*c^7*d^6*e + 2*b^4*c^5*d^6*e + 2*b^8*c*d^2*e^5 - 2
2*a^3*b^4*c^2*e^7 + 80*a^4*b^2*c^3*e^7 - 32*a^3*c^6*d^4*e^3 - 160*a^4*c^5*d
^2*e^5 - 6*b^5*c^4*d^5*e^2 + 8*b^6*c^3*d^4*e^3 - 6*b^7*c^2*d^3*e^4 - 4*a*b^

7*c*d*e^6 + 144*a^2*b^2*c^5*d^4*e^3 - 128*a^2*b^3*c^4*d^3*e^4 + 6*a^2*b^4*c^3*d^2*e^5 + 112*a^3*b^2*c^4*d^2*e^5 - 16*a*b^2*c^6*d^6*e + 160*a^4*b*c^4*d*e^6 + 48*a*b^3*c^5*d^5*e^2 - 66*a*b^4*c^4*d^4*e^3 + 52*a*b^5*c^3*d^3*e^4 - 14*a*b^6*c^2*d^2*e^5 - 96*a^2*b*c^6*d^5*e^2 + 42*a^2*b^5*c^2*d*e^6 + 64*a^3*b*c^5*d^3*e^4 - 144*a^3*b^3*c^3*d*e^6))/(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)) - (x*(8*a^3*b*c^3*e^6*g - 2*a*b^4*c^2*e^6*f - 48*a^3*c^4*e^6*f - 16*a*c^6*d^4*e^2*f + a^2*b^4*c*e^6*h + 32*a^3*c^4*d*e^5*g + 2*b^5*c^2*d*e^5*f + b^6*c*d^2*e^4*h + 20*a^2*b^2*c^3*e^6*f - 2*a^2*b^3*c^2*e^6*g - 64*a^2*c^5*d^2*e^4*f - 4*a^3*b^2*c^2*e^6*h + 32*a^2*c^5*d^3*e^3*g + 4*b^2*c^5*d^4*e^2*f - 8*b^3*c^4*d^3*e^3*f + 2*b^4*c^3*d^2*e^4*f - 32*a^2*c^5*d^4*e^2*h - 32*a^3*c^4*d^2*e^4*h - 2*b^3*c^4*d^4*e^2*g + 6*b^4*c^3*d^3*e^3*g - 4*b^5*c^2*d^2*e^4*g - b^4*c^3*d^4*e^2*h + 8*a*b^2*c^4*d^2*e^4*f - 32*a*b^2*c^4*d^3*e^3*g + 20*a*b^3*c^3*d^2*e^4*g - 16*a^2*b*c^4*d^2*e^4*g - 32*a^2*b^2*c^3*d*e^5*g + 12*a*b^2*c^4*d^4*e^2*h - 8*a*b^3*c^3*d^3*e^3*h - 4*a*b^4*c^2*d^2*e^4*h + 32*a^2*b*c^4*d^3*e^3*h + 8*a^2*b^3*c^2*d*e^5*h - 2*a*b^5*c*d*e^5*h + 8*a^2*b^2*c^3*d^2*e^4*h + 32*a*b*c^5*d^3*e^3*f - 24*a*b^3*c^3*d*e^5*f + 64*a^2*b*c^4*d*e^5*f + 8*a*b*c^5*d^4*e^2*g + 6*a*b^4*c^2*d*e^5*g))/(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)) - (4*a^2*c^3*d^3*e^2*h^2 - 4*c^5*d^3*e^2*f^2 - b^3*c^2*e^5*f^2 - b^2*c^3*d^3*e^2*g^2 + b^3*c^2*d^2*e^3*g^2 + 4*a*b*c^3*e^5*f^2 - 8*a*c^4*d*e^4*f^2 - 8*a^2*c^3*e^5*f*g + 4*b*c^4*d^2*e^3*f^2 + 4*a^2*c^3*d*e^4*g^2 + b^2*c^3*d*e^4*f^2 - 2*a*b^2*c^2*d*e^4*g^2 + a*b^3*c*d^2*e^3*h^2 - a^2*b^2*c*d*e^4*h^2 - 4*b^2*c^3*d^2*e^3*f*g - 8*a^2*c^3*d^2*e^3*g*h - 2*b^2*c^3*d^3*e^2*f*h + b^3*c^2*d^2*e^3*f*h + b^3*c^2*d^3*e^2*g*h - a*b^3*c*e^5*f*h + b^4*c*d*e^4*f*h - 2*a*b^2*c^2*d^3*e^2*h^2 + 2*a*b^2*c^2*e^5*f*g + 4*a^2*b*c^2*e^5*f*h + 4*b*c^4*d^3*e^2*f*g + 8*a^2*c^3*d*e^4*f*h - b^4*c*d^2*e^3*g*h + 4*a*b*c^3*d^2*e^3*f*h - 8*a*b^2*c^2*d*e^4*f*h + 2*a*b^2*c^2*d^2*e^3*g*h + 4*a*b*c^3*d*e^4*f*g + a*b^3*c*d*e^4*g*h)/(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3) + (x*(4*a^2*c^3*e^5*g^2 + b^2*c^3*e^5*f^2 + 4*c^5*d^2*e^3*f^2 + 4*a^2*c^3*d^2*e^3*h^2 + b^2*c^3*d^2*e^3*g^2 - 4*b*c^4*d*e^4*f^2 + a^2*b^2*c*e^5*h^2 + b^4*c*d^2*e^3*h^2 + 4*a^2*b*c^2*d*e^4*h^2 + 4*b^2*c^3*d^2*e^3*f*h - 2*b^3*c^2*d^2*e^3*g*h - 4*a*b*c^3*e^5*f*g + 8*a*c^4*d*e^4*f*g - 4*a*b^2*c^2*d^2*e^3*h^2 - 4*a*b*c^3*d*e^4*g^2 - 2*a*b^3*c*d*e^4*h^2 + 2*a*b^2*c^2*e^5*f*h - 4*a^2*b*c^2*e^5*g*h - 8*a*c^4*d^2*e^3*f*h - 4*b*c^4*d^2*e^3*f*g + 2*b^2*c^3*d*e^4*f*g - 8*a^2*c^3*d*e^4*g*h - 2*b^3*c^2*d*e^4*f*h + 4*a*b*c^3*d^2*e^3*g*h + 6*a*b^2*c^2*d*e^4*g*h))/(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3))*root(768*a^5*b*c^4*d^3*e^5*z^3 + 768*a^4*b*c^5*d^5*e^3*z^3 - 192*a^5*b^3*c^2*d*e^7*z^3 - 192*a^2*b^3*c^5*d^7*e*z^3 - 68*a^3*b^6*c*d^2*e^6*z^3 - 68*a*b^6*c^3*d^6*e^2*z^3 + 36*a^2*b^7*c*d^3*e^5*z^3 + 36*a*b^7*c^2*d^5*e^3*z^3 + 256*a^6*b*c^3*d*e^7*z^3 + 256*a^3*b*c^6*d^7*e*z^3 + 48*a^4*b^5*c*d*e^7*z^3 + 48*a*b^5*c^4*d^7*e*z^3 - 480*a^4*b^2*c^4*d^4*e^4*z^3 + 440*a^3*b^4*c^3*d^4*e^4*z^3 - 320*a^4*b^3*c^3*d^3*e^5*z^3 - 320*a^3*b^3*c^4*d^5*e^3*z^3 + 240*a^4*b^4*c^2*d^2*e^6*z^3 + 240*a^2*b^4*c^4*d^6*e^2*z^3 - 192*a^5*b^2*c^3*d^2*e^6*z^3 - 192*a^3*b^2*c^5*d^6*e^2*z^3 - 90*a^2*b^6*c^2*d^4*e^4*z^3 - 48*a^3*b^5*c^2*d^3*e^5*z^3 - 48*a^2*b^5*c^3*d^5*e^3*z^3 - 4*b^9*c*d^5*e^3*z^3 - 4*b^7*c^3*d^7*e*z^3 - 4*a^3*b^7*d*e^7*z^3 - 4*a*b^9*d^3*e^5*z^3 - 12*a^5*b^4*c*e^8*z^3 - 12*a*b^4*c^5*d^8*z^3 + 6*b^8*c^2*d^6*e^2*z^3 - 384*a^5*c^5*d^4*e^4*z^3 - 256*a^6*c^4*d^2*e^6*z^3 - 256*a

^4*c^6*d^6*e^2*z^3 + 6*a^2*b^8*d^2*e^6*z^3 + 48*a^6*b^2*c^2*e^8*z^3 + 48*a^2*b^2*c^6*d^8*z^3 - 64*a^7*c^3*e^8*z^3 - 64*a^3*c^7*d^8*z^3 + b^10*d^4*e^4*z^3 + b^6*c^4*d^8*z^3 + a^4*b^6*e^8*z^3 - 28*a*b^4*c*d^3*e^3*g*h*z - 10*a^3*b^2*c*d*e^5*g*h*z - 10*a*b^2*c^3*d^5*e*g*h*z + 16*a*b^4*c*d^2*e^4*f*h*z + 14*a^2*b^3*c*d*e^5*f*h*z + 4*a*b*c^4*d^4*e^2*f*g*z + 84*a^2*b^2*c^2*d^3*e^3*g*h*z - 108*a^2*b^2*c^2*d^2*e^4*f*h*z + 16*a*b*c^4*d^5*e*f*h*z - 20*a*b^4*c*d*e^5*f*g*z + 8*a^2*b^3*c*d^2*e^4*g*h*z + 8*a*b^3*c^2*d^4*e^2*g*h*z - 4*a^3*b*c^2*d^2*e^4*g*h*z - 4*a^2*b*c^3*d^4*e^2*g*h*z + 16*a^2*b*c^3*d^3*e^3*f*h*z + 16*a*b^3*c^2*d^3*e^3*f*h*z - 14*a*b^2*c^3*d^4*e^2*f*h*z + 66*a^2*b^2*c^2*d*e^5*f*g*z - 36*a*b^2*c^3*d^3*e^3*f*g*z + 20*a*b^3*c^2*d^2*e^4*f*g*z + 12*a^2*b*c^3*d^2*e^4*f*g*z + 8*a*c^5*d^5*e*f*g*z + 4*a^4*b*c*e^6*g*h*z - 2*a*b^5*d*e^5*f*h*z + 4*a*b*c^4*d^6*g*h*z - 112*a^3*c^3*d^3*e^3*g*h*z - 3*b^4*c^2*d^4*e^2*f*h*z + 120*a^3*c^3*d^2*e^4*f*h*z - 16*a^2*c^4*d^4*e^2*f*h*z + 14*b^3*c^3*d^4*e^2*f*g*z - 2*b^4*c^2*d^3*e^3*f*g*z + 16*a^2*c^4*d^3*e^3*f*g*z + 8*a*b^4*c*d^4*e^2*h^2*z + 4*a^2*b*c^3*d^5*e*h^2*z + 2*a*b^3*c^2*d^5*e*h^2*z + 8*a*b^4*c*d^2*e^4*g^2*z + 4*a^3*b*c^2*d*e^5*g^2*z + 2*a^2*b^3*c*d*e^5*g^2*z + 48*a*b*c^4*d^3*e^3*f^2*z + 36*a^2*b*c^3*d*e^5*f^2*z - 6*a*b^3*c^2*d*e^5*f^2*z - 45*a^2*b^2*c^2*d^4*e^2*h^2*z - 45*a^2*b^2*c^2*d^2*e^4*g^2*z + 2*b^5*c*d^4*e^2*g*h*z - b^4*c^2*d^5*e*g*h*z + 8*a^4*c^2*d*e^5*g*h*z + 8*a^2*c^4*d^5*e*g*h*z + 2*b^3*c^3*d^5*e*f*h*z - 14*b^2*c^4*d^5*e*f*g*z - 2*b^5*c*d^2*e^4*f*g*z + 2*a*b^5*d^2*e^4*g*h*z - a^2*b^4*d*e^5*g*h*z - 120*a^3*c^3*d*e^5*f*g*z - 6*a^3*b^2*c*e^6*f*h*z + 12*a^3*b*c^2*e^6*f*g*z - 2*a^2*b^3*c*e^6*f*g*z - 4*a^4*b*c*d*e^5*h^2*z - 4*a*b*c^4*d^5*e*g^2*z + 6*a^3*b^2*c*d^2*e^4*h^2*z + 2*a^2*b^3*c*d^3*e^3*h^2*z + 6*a*b^2*c^3*d^4*e^2*g^2*z + 2*a*b^3*c^2*d^3*e^3*g^2*z - 18*a*b^2*c^3*d^2*e^4*f^2*z - b^6*d^2*e^4*f*h*z + 12*b*c^5*d^5*e*f^2*z + 12*a*b^4*c*e^6*f^2*z + 56*a^3*c^3*d^4*e^2*h^2*z - 5*b^4*c^2*d^4*e^2*g^2*z - 4*a^4*c^2*d^2*e^4*h^2*z + 56*a^3*c^3*d^2*e^4*g^2*z - 9*b^2*c^4*d^4*e^2*f^2*z - 5*a^2*b^4*d^2*e^4*h^2*z - 4*a^2*c^4*d^4*e^2*g^2*z + 3*b^4*c^2*d^2*e^4*f^2*z - 2*b^3*c^3*d^3*e^3*f^2*z - 36*a^2*c^4*d^2*e^4*f^2*z - 45*a^2*b^2*c^2*e^6*f^2*z + 2*b^6*d*e^5*f*g*z - 8*a*c^5*d^6*f*h*z + 4*b*c^5*d^6*f*g*z + 4*b^3*c^3*d^5*e*g^2*z + 2*b^5*c*d^3*e^3*g^2*z + 4*a^3*b^3*d*e^5*h^2*z + 2*a*b^5*d^3*e^3*h^2*z - 24*a*c^5*d^4*e^2*f^2*z + b^6*d^3*e^3*g*h*z + a^2*b^4*e^6*f*h*z - b^6*d^4*e^2*h^2*z - b^6*d^2*e^4*g^2*z - 4*a^4*c^2*e^6*g^2*z - 4*a^2*c^4*d^6*h^2*z - b^2*c^4*d^6*g^2*z - a^4*b^2*e^6*h^2*z + 48*a^3*c^3*e^6*f^2*z - 4*c^6*d^6*f^2*z - b^6*e^6*f^2*z - 16*a*b*c^2*d^2*e^3*f*g*h - 4*a*b^2*c*d*e^4*f*g*h - 4*b*c^3*d^4*e*f*g*h - 4*a^2*b*c*e^5*f*g*h + 6*b^2*c^2*d^3*e^2*f*g*h - 8*a^2*b*c*d^2*e^3*g*h^2 + 8*a*b*c^2*d^3*e^2*g^2*h + 2*a*b^2*c*d^3*e^2*g*h^2 - 2*a*b^2*c*d^2*e^3*g^2*h + 6*a*b^2*c*d^2*e^3*f*h^2 + 4*b^3*c*d^2*e^3*f*g*h - 16*a*c^3*d^3*e^2*f*g*h - 8*a^2*c^2*d*e^4*f*g*h + 4*a^2*b*c*d*e^4*g^2*h - 4*a*b*c^2*d^4*e*g*h^2 + 4*a^2*b*c*d*e^4*f*h^2 + 16*a*b*c^2*d*e^4*f*g^2 - 2*b^3*c*d*e^4*f^2*h + 8*a*c^3*d^4*e*f*h^2 - 4*b^3*c*d*e^4*f*g^2 - 24*a*c^3*d*e^4*f^2*g - 2*a*b^3*d*e^4*f*h^2 + 6*a*b^2*c*e^5*f^2*h - 12*a*b*c^2*e^5*f^2*g - 12*a^2*c^2*d^3*e^2*g*h^2 + 12*a^2*c^2*d^2*e^3*g^2*h - 3*b^2*c^2*d^2*e^3*f^2*h - 5*b^2*c^2*d^2*e^3*f*g^2 + 4*a^2*c^2*d^2*e^3*f*h^2 + 2*b^4*d*e^4*f*g*h - 2*b^3*c*d^3*e^2*g^2*h - 4*b*c^3*d^3*e^2*f^2*h - 2*b^3*c*d^3*e^2*f*h^2 + 24*a*c^3*d^2*e^3*f^2*h + 9*b^2*c^2*d*e^4*f^2*g + 4*b*c^3*d^3*e^2*f*g^2 + 2*a*b^3*d^2*e^3*g*h^2 - a^2*b^2*d*e^4*g*h^2 + 8*a*c^3*d^2*e^3*f*g^2 + 4*a^2*b*c*d^3*e^2*h^3 - 4*a*b*c^2*d^2*e^3*g^3 - b^4*d^2*e^3*g^2*h - 4*c^4*d^3*e^2*f^2*g - b^4*d^2*e^3*f*h^2 + 4*a^2*c^2*e^5*f*g^2 + 4*a^2*c^2*d^4*e*h^3 + 2*b^3*c*d^2*e^3*g^3 - 4*a^2*c^2*d*e^4*g^3 - 2*a*b^3*d^3*e^2*h^3 + 4*c^4*d^4*e*f^2*h + 2*b^3*c*e^5*f^2*g - 4*b*c^3*d*e^4*f^3 + b^2*c^2*d^4*e*g^2*h - b^2*c^2*d^3*e^2*g^3 + b^4*d^3*e^2*g*h^2 + a^2*b^2*e^5*f*h^2 + 4*c^4*d^2*e^3*f^3 - 3*b^2*c^2*e^5*f^3 + a^2*b^2*d^2*e^3*h^3 - b^4*e^5*f^2*h + 16*a*c^3*e^5*f^3, z, k), k, 1, 3) - ((a*b*d*h - 2*a^2*e*h - b^2*e*f + a*b*e*g - 2*a*c*d*g + 2*a*c*e*f + b*c*d*f)/(a*b^2*e^2 - 4*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e) - (x*(a*b*e*h - b^2*d*h - 2*c^2*d*f + 2*a*c*d*h - 2*a*c*e*g + b*c*d*g + b*c*e*f))/(a*b^2*e^2 - 4*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e))/(a + b*x + c*x^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

3.159 $\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=673

$$-\frac{cx\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)+b\left(a^2e^2h\right.}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] -e*(d^2*h-d*e*g+e^2*f)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)+(-b^3*e^2*f+b^2*e*(a*e
*g+2*c*d*f)-2*a*c*(c*d*(-d*g+2*e*f)+a*e*(-2*d*h+e*g))-b*(c^2*d^2*f+a^2*e^2*
h-a*c*(-d^2*h-2*d*e*g+3*e^2*f))-c*(2*c^2*d^2*f+2*a^2*e^2*h-a*b*e*(2*d*h+e*g
)+b^2*(d^2*h+e^2*f)-c*(b*d*(d*g+2*e*f)+2*a*(d^2*h-2*d*e*g+e^2*f)))*x)/(-4*a
*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)+(4*c^4*d^4*f-b^3*e^3*(2*a*d*h-a
*e*g-b*d*g+2*b*e*f)-2*c^3*d^2*(b*d*(d*g+4*e*f)-2*a*(d^2*h-2*d*e*g+6*e^2*f))
-6*c^2*e*(4*a*b*d*e^2*f-b^2*d^3*g+2*a^2*e*(2*d^2*h-2*d*e*g+e^2*f))-c*e*(6*a
^2*b*e^3*g-4*a^3*e^3*h-b^3*d*(-2*d^2*h-3*d*e*g+4*e^2*f)-6*a*b^2*e*(2*d^2*h-
d*e*g+2*e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(
a*e^2-b*d*e+c*d^2)^3-e*(e^2*(2*a*d*h-a*e*g-b*d*g+2*b*e*f)-c*d*(2*d^2*h-3*d*
e*g+4*e^2*f))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^3+1/2*e*(e^2*(2*a*d*h-a*e*g-b*d
*g+2*b*e*f)-c*d*(2*d^2*h-3*d*e*g+4*e^2*f))*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d
^2)^3
```

Rubi [A] time = 2.56, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1646, 1628, 634, 618, 206, 628}

$$-\frac{cx\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)+b\left(a^2e^2h\right.}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((e*(e^2*f - d*e*g + d^2*h))/((c*d^2 - b*d*e + a*e^2)^2*(d + e*x))) - (b^3
*e^2*f - b^2*e*(2*c*d*f + a*e*g) + 2*a*c*(c*d*(2*e*f - d*g) + a*e*(e*g - 2*
d*h)) + b*(c^2*d^2*f + a^2*e^2*h - a*c*(3*e^2*f - 2*d*e*g - d^2*h)) + c*(2*
c^2*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c*(b*
d*(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*x)/((b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)^2*(a + b*x + c*x^2)) + ((4*c^4*d^4*f - b^3*e^3*(2*b*e*f -
b*d*g - a*e*g + 2*a*d*h) - 2*c^3*d^2*(b*d*(4*e*f + d*g) - 2*a*(6*e^2*f - 2*
d*e*g + d^2*h)) - 6*c^2*e*(4*a*b*d*e^2*f - b^2*d^3*g + 2*a^2*e*(e^2*f - 2*d
*e*g + 2*d^2*h)) - c*e*(6*a^2*b*e^3*g - 4*a^3*e^3*h - b^3*d*(4*e^2*f - 3*d*
e*g - 2*d^2*h) - 6*a*b^2*e*(2*e^2*f - d*e*g + 2*d^2*h)))*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^3) - (e*
(e^2*(2*b*e*f - b*d*g - a*e*g + 2*a*d*h) - c*d*(4*e^2*f - 3*d*e*g + 2*d^2*h
))*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^3 + (e*(e^2*(2*b*e*f - b*d*g - a*e
*g + 2*a*d*h) - c*d*(4*e^2*f - 3*d*e*g + 2*d^2*h))*Log[a + b*x + c*x^2])/(2
*(c*d^2 - b*d*e + a*e^2)^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx = -\frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+ae(eg-2dh))+b\left(c^2d^2f+\right.}{}$$

$$= -\frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+ae(eg-2dh))+b\left(c^2d^2f+\right.}{}$$

$$= -\frac{e\left(e^2f-deg+d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)}{}$$

$$= -\frac{e\left(e^2f-deg+d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)}{}$$

$$= -\frac{e\left(e^2f-deg+d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)}{}$$

$$= -\frac{e\left(e^2f-deg+d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)}{}$$

Mathematica [A] time = 2.21, size = 650, normalized size = 0.97

$$\frac{b\left(-a^2e^2h+ac\left(d^2(-h)-2de(g-hx)+e^2(3f+gx)\right)+c^2d(-df+dgx+2efx)\right)+2c\left(a^2(-e)(e(g+hx)-2dh)\right.}{\left(b^2-4ac\right)(a+x(b+cx))\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((e*(e^2*f - d*e*g + d^2*h))/((c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x))) + (
-(b^3*e^2*f) + b^2*(a*e^2*g - c*(-2*d*e*f + e^2*f*x + d^2*h*x)) + b*(-(a^2*
e^2*h) + c^2*d*(-(d*f) + 2*e*f*x + d*g*x) + a*c*(-(d^2*h) + e^2*(3*f + g*x)
 - 2*d*e*(g - h*x))) + 2*c*(-(c^2*d^2*f*x) + a*c*(e^2*f*x - 2*d*e*(f + g*x)
 + d^2*(g + h*x)) - a^2*e*(-2*d*h + e*(g + h*x))))/((b^2 - 4*a*c)*(c*d^2 +
e*(-(b*d) + a*e))^2*(a + x*(b + c*x))) - ((4*c^4*d^4*f + b^3*e^3*(-2*b*e*f
+ b*d*g + a*e*g - 2*a*d*h) - 2*c^3*d^2*(b*d*(4*e*f + d*g) - 2*a*(6*e^2*f -
2*d*e*g + d^2*h)) - 6*c^2*e*(4*a*b*d*e^2*f - b^2*d^3*g + 2*a^2*e*(e^2*f - 2
*d*e*g + 2*d^2*h)) + c*e*(-6*a^2*b*e^3*g + 4*a^3*e^3*h + b^3*d*(4*e^2*f - 3
*d*e*g - 2*d^2*h) + 6*a*b^2*e*(2*e^2*f - d*e*g + 2*d^2*h)))*ArcTan[(b + 2*c
*x)/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(-(c*d^2) + e*(b*d - a*e))^3
) + ((e^3*(-2*b*e*f + b*d*g + a*e*g - 2*a*d*h) + c*d*e*(4*e^2*f - 3*d*e*g +
 2*d^2*h))*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 - ((e^3*(-2*b*e*f + b
*d*g + a*e*g - 2*a*d*h) + c*d*e*(4*e^2*f - 3*d*e*g + 2*d^2*h))*Log[a + x*(b
 + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

[Out] Timed out

giac [B] time = 0.33, size = 1437, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(4*c^4*d^4*f*e^2 - 2*b*c^3*d^4*g*e^2 + 4*a*c^3*d^4*h*e^2 - 8*b*c^3*d^3*f*e
^3 + 6*b^2*c^2*d^3*g*e^3 - 8*a*c^3*d^3*g*e^3 - 2*b^3*c*d^3*h*e^3 + 24*a*c^3
*d^2*f*e^4 - 3*b^3*c*d^2*g*e^4 + 12*a*b^2*c*d^2*h*e^4 - 24*a^2*c^2*d^2*h*e^
4 + 4*b^3*c*d*f*e^5 - 24*a*b*c^2*d*f*e^5 + b^4*d*g*e^5 - 6*a*b^2*c*d*g*e^5
+ 24*a^2*c^2*d*g*e^5 - 2*a*b^3*d*h*e^5 - 2*b^4*f*e^6 + 12*a*b^2*c*f*e^6 - 1
2*a^2*c^2*f*e^6 + a*b^3*g*e^6 - 6*a^2*b*c*g*e^6 + 4*a^3*c*h*e^6)*arctan((2*
c*d - 2*c*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*a*e^2/(x*e + d))*e^(-
1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((b^2*c^3*d^6 - 4*a*c^4*d^6 - 3*b^3*c^2*d^5*e
 + 12*a*b*c^3*d^5*e + 3*b^4*c*d^4*e^2 - 9*a*b^2*c^2*d^4*e^2 - 12*a^2*c^3*d^
4*e^2 - b^5*d^3*e^3 - 2*a*b^3*c*d^3*e^3 + 24*a^2*b*c^2*d^3*e^3 + 3*a*b^4*d^
2*e^4 - 9*a^2*b^2*c*d^2*e^4 - 12*a^3*c^2*d^2*e^4 - 3*a^2*b^3*d*e^5 + 12*a^3
*b*c*d*e^5 + a^3*b^2*e^6 - 4*a^4*c*e^6)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c*d^3*
h*e - 3*c*d^2*g*e^2 + 4*c*d*f*e^3 + b*d*g*e^3 - 2*a*d*h*e^3 - 2*b*f*e^4 + a
*g*e^4)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e
/(x*e + d)^2 + a*e^2/(x*e + d)^2)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^
2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a
^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6) - (d^2*h*e^5/(x*e + d) - d*g*e^6/(x
*e + d) + f*e^7/(x*e + d))/(c^2*d^4*e^4 - 2*b*c*d^3*e^5 + b^2*d^2*e^6 + 2*a
*c*d^2*e^6 - 2*a*b*d*e^7 + a^2*e^8) - ((2*c^4*d^3*f*e - b*c^3*d^3*g*e + b^2
*c^2*d^3*h*e - 2*a*c^3*d^3*h*e - 3*b*c^3*d^2*f*e^2 + 6*a*c^3*d^2*g*e^2 - 3*
a*b*c^2*d^2*h*e^2 + 3*b^2*c^2*d*f*e^3 - 6*a*c^3*d*f*e^3 - 3*a*b*c^2*d*g*e^3
 + 6*a^2*c^2*d*h*e^3 - b^3*c*f*e^4 + 3*a*b*c^2*f*e^4 + a*b^2*c*g*e^4 - 2*a^
2*c^2*g*e^4 - a^2*b*c*h*e^4)/(c*d^2 - b*d*e + a*e^2) - (2*c^4*d^4*f*e^2 - b
*c^3*d^4*g*e^2 + b^2*c^2*d^4*h*e^2 - 2*a*c^3*d^4*h*e^2 - 4*b*c^3*d^3*f*e^3
+ 8*a*c^3*d^3*g*e^3 - 4*a*b*c^2*d^3*h*e^3 + 6*b^2*c^2*d^2*f*e^4 - 12*a*c^3*
d^2*f*e^4 - 6*a*b*c^2*d^2*g*e^4 + 12*a^2*c^2*d^2*h*e^4 - 4*b^3*c*d*f*e^5 +
12*a*b*c^2*d*f*e^5 + 4*a*b^2*c*d*g*e^5 - 8*a^2*c^2*d*g*e^5 - 4*a^2*b*c*d*h*
e^5 + b^4*f*e^6 - 4*a*b^2*c*f*e^6 + 2*a^2*c^2*f*e^6 - a*b^3*g*e^6 + 3*a^2*b
*c*g*e^6 + a^2*b^2*h*e^6 - 2*a^3*c*h*e^6)*e^(-1)/((c*d^2 - b*d*e + a*e^2)*(
x*e + d)))/((c*d^2 - b*d*e + a*e^2)^2*(b^2 - 4*a*c)*(c - 2*c*d/(x*e + d) +
c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + a*e^2/(x*e + d)^2))
```

maple [B] time = 0.04, size = 4716, normalized size = 7.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x)
```

```
[Out] -e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*f-6/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/
(4*a*c-b^2)*a*b*c^2*d^2*e^2*f-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^
2+b*x+a)*a*b*d*e^3*g+12/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*c*d^2*e^2*h-6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b
^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*c*d*e^3*g-24/(a*e^2-b*d
*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c^2*d
*e^3*f-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*a^2*b*e^4*g+4/
(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x*a^2*d*e^3*g+1/(a*e^2-
b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*a*b^2*e^4*f+4/(a*e^2-b*d*e+c*d
^2)^3/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x*a*d^3*e*g-1/(a*e^2-b*d*e+c*d^2)^3/(c*
x^2+b*x+a)*c/(4*a*c-b^2)*x*b^3*d^3*e*h-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a
```

```
)*c/(4*a*c-b^2)*x*b^3*d*e^3*f+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*
a*c-b^2)*x*b^2*d^3*e*g+3/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*a*c-b^2
)*x*b^2*d^2*e^2*f-4/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x*b
*d^3*e*f+6/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*c*d^2*e^2*
h-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*c*d^3*e*h-3/(a*e^
2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*c*d^2*e^2*g+1/(a*e^2-b*d*e
+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*c*d*e^3*f+4/(a*e^2-b*d*e+c*d^2)^3
/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c^2*d^3*e*g-6/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b
*x+a)*c^2/(4*a*c-b^2)*x*a*b*d^2*e^2*g-4/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)
*c/(4*a*c-b^2)*x*a^2*b*d*e^3*h+3/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c/(4*a
*c-b^2)*x*a*b^2*d^2*e^2*h+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c/(4*a*c-b^
2)*x*a*b^2*d*e^3*g-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*b^4*e^4*f-1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*ln(c*x^
2+b*x+a)*b^3*e^4*f+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*c^4*d^4*f-2*e^3/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*a*d*h
+e^3/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*b*d*g+2*e/(a*e^2-b*d*e+c*d^2)^3*ln(e*x
+d)*c*d^3*h-3*e^2/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*c*d^2*g+4*e^3/(a*e^2-b*d*
e+c*d^2)^3*ln(e*x+d)*c*d*f+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^2+b
*x+a)*a^2*d*e^3*h+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*a*b
*e^4*f+6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c^2*ln(c*x^2+b*x+a)*a*d^2*e^2*g+
1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c^2*d^4*h+4/(a*e^2-b*
d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*c^3*d^3*e*f+3/(a*e^2-b*d*e+c*d^2)^
3/(c*x^2+b*x+a)/(4*a*c-b^2)*b^3*c*d^2*e^2*f-8/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-
b^2)*c^2*ln(c*x^2+b*x+a)*a*d*e^3*f+1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln
(c*x^2+b*x+a)*b^2*d^3*e*h-3/2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^2+
b*x+a)*b^2*d^2*e^2*g+2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*
b^2*d*e^3*f-1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*b^2*d*e^3
*h-8/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*b*c^3*d^3*e*f-6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))*a^2*b*c*e^4*g-24/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(
3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c^2*d^2*e^2*h+24/(a*e^2-b*d*e+
c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c^2*d*e^
3*g-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*a*b^3*d*e^3*h+12/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*c*e^4*f-8/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^
(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^3*d^3*e*g+24/(a*e^2-b*d*e+c*d
^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^3*d^2*e^2*f
-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))*b^3*c*d^3*e*h-3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*b^3*c*d^2*e^2*g+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3
/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*c*d*e^3*f-3/(a*e^2-b*d*e+c*d^2)
^3/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2*c^2*d^3*e*f-4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c
-b^2)*c^2*ln(c*x^2+b*x+a)*a*d^3*e*h+6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/
2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2*d^3*e*g-4/(a*e^2-b*d*e+c*d^2
)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*c^2*d^3*e*h+4/(a*e^2-b*d*e+c*d^2)^3/(c*x^
2+b*x+a)/(4*a*c-b^2)*a^2*c^2*d*e^3*f+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/
(4*a*c-b^2)*a*b^3*d*e^3*g-4/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)
*a^3*c*d*e^3*h-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b^2*d*
e^3*h-3/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*c*e^4*f+1/(a*
e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^3*e^4*f-2/(a*e^2-b*d*e+c*d
^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*c^3*d^4*g-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+
b*x+a)/(4*a*c-b^2)*b^4*d*e^3*f+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c
-b^2)*b*c^3*d^4*f+2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^4/(4*a*c-b^2)*x*d
^4*f+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^3*b*e^4*h+2/(a*e^2
-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^3*c*e^4*g-1/(a*e^2-b*d*e+c*d^2)
^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b^2*e^4*g+1/2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c
-b^2)*ln(c*x^2+b*x+a)*a*b^2*e^4*g+1/2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*ln(
c*x^2+b*x+a)*b^3*d*e^3*g-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^2+b*x
+a)*a^2*e^4*g+1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4
```

```
*a*c-b^2)^(1/2))*a*b^3*e^4*g+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3*c*e^4*h-12/(a*e^2-b*d*e+c*d^2)^3/(4*a*c
-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c^2*e^4*f-2/(a*e^2-b*d*
e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^3*d^4*
g+1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))*b^4*d*e^3*g+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))*a*c^3*d^4*h-e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*d^2*h+e^2/(
a*e^2-b*d*e+c*d^2)^2/(e*x+d)*d*g+e^4/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*a*g-2*
e^4/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*b*f+2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+
a)*c/(4*a*c-b^2)*x*a^3*e^4*h-2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*a
*c-b^2)*x*a^2*e^4*f-2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x
*a*d^4*h+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x*b^2*d^4*h-
1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x*b*d^4*g
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 8.93, size = 26278, normalized size = 39.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x)
```

```
[Out] ((a*b^2*e^3*f - 2*a*c^2*d^3*g + b*c^2*d^3*f - 4*a^2*c*e^3*f + b^3*d*e^2*f -
 2*a*b^2*d*e^2*g + 4*a*c^2*d^2*e*f + a*b^2*d^2*e*h + a^2*b*d*e^2*h + 6*a^2*
c*d*e^2*g - 2*b^2*c*d^2*e*f - 8*a^2*c*d^2*e*h + a*b*c*d^3*h - 3*a*b*c*d*e^2
*f + 2*a*b*c*d^2*e*g)/(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^
4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b
*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2) + (x*(2*b^3*e^3*f + 2*c^3
*d^3*f - a*b^2*e^3*g - 2*a*c^2*d^3*h - b*c^2*d^3*g + a^2*b*e^3*h + 2*a^2*c*
e^3*g + b^2*c*d^3*h - b^3*d*e^2*g + b^3*d^2*e*h + 2*a*c^2*d*e^2*f + 2*a*c^2
*d^2*e*g - b*c^2*d^2*e*f - b^2*c*d*e^2*f - 2*a^2*c*d*e^2*h - 7*a*b*c*e^3*f
+ 5*a*b*c*d*e^2*g - 5*a*b*c*d^2*e*h))/(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*
e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3
*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2) - (x^2*(6
*a*c^2*e^3*f - 2*b^2*c*e^3*f - 2*a^2*c*e^3*h - 2*c^3*d^2*e*f - 8*a*c^2*d*e^
2*g + 2*b*c^2*d*e^2*f + 6*a*c^2*d^2*e*h + b*c^2*d^2*e*g + b^2*c*d*e^2*g - 2
*b^2*c*d^2*e*h + a*b*c*e^3*g + 2*a*b*c*d*e^2*h))/(4*a*c^3*d^4 + 4*a^3*c*e^4
 - a^2*b^2*e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*
e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2
))/(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3) + symsum(log((x*(36*a^
2*c^5*e^7*f^2 + 4*b^4*c^3*e^7*f^2 + 4*a^4*c^3*e^7*h^2 + 4*c^7*d^4*e^3*f^2 +
 a^2*b^2*c^3*e^7*g^2 + 64*a^2*c^5*d^2*e^5*g^2 + 12*b^2*c^5*d^2*e^5*f^2 + 36
*a^2*c^5*d^4*e^3*h^2 - 24*a^3*c^4*d^2*e^5*h^2 + b^2*c^5*d^4*e^3*g^2 + 2*b^3
*c^4*d^3*e^4*g^2 + b^4*c^3*d^2*e^5*g^2 + 4*b^4*c^3*d^4*e^3*h^2 - 24*a^3*c^4
*e^7*f*h - 24*a*b^2*c^4*e^7*f^2 - 24*a*c^6*d^2*e^5*f^2 - 8*b*c^6*d^3*e^4*f^
2 - 8*b^3*c^4*d*e^6*f^2 - 16*a*b*c^5*d^3*e^4*g^2 + 2*a*b^3*c^3*d*e^6*g^2 -
16*a^2*b*c^4*d*e^6*g^2 - 8*a^3*b*c^3*d*e^6*h^2 + 8*a^2*b^2*c^3*e^7*f*h + 80
*a^2*c^5*d^2*e^5*f*h - 96*a^2*c^5*d^3*e^4*g*h + 8*b^2*c^5*d^4*e^3*f*h - 8*b
^3*c^4*d^3*e^4*f*h + 8*b^4*c^3*d^2*e^5*f*h - 4*b^3*c^4*d^4*e^3*g*h - 4*b^4*
```

c^3*d^3*e^4*g*h - 14*a*b^2*c^4*d^2*e^5*g^2 - 24*a*b^2*c^4*d^4*e^3*h^2 - 8*a
*b^3*c^3*d^3*e^4*h^2 + 24*a^2*b*c^4*d^3*e^4*h^2 + 24*a*b*c^5*d*e^6*f^2 - 4*
a*b^3*c^3*e^7*f*g + 12*a^2*b*c^4*e^7*f*g + 32*a*c^6*d^3*e^4*f*g - 96*a^2*c^
5*d*e^6*f*g - 4*a^3*b*c^3*e^7*g*h - 24*a*c^6*d^4*e^3*f*h - 4*b*c^6*d^4*e^3*
f*g - 4*b^4*c^3*d*e^6*f*g + 32*a^3*c^4*d*e^6*g*h + 12*a^2*b^2*c^3*d^2*e^5*h
^2 - 24*a*b*c^5*d^2*e^5*f*g + 48*a*b^2*c^4*d*e^6*f*g + 16*a*b*c^5*d^3*e^4*f
*h - 8*a*b^3*c^3*d*e^6*f*h + 16*a^2*b*c^4*d*e^6*f*h + 12*a*b*c^5*d^4*e^3*g*
h - 40*a*b^2*c^4*d^2*e^5*f*h + 48*a*b^2*c^4*d^3*e^4*g*h - 24*a^2*b*c^4*d^2*
e^5*g*h))/(16*a^2*c^6*d^8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^
8*d^4*e^4 - 8*a*b^2*c^5*d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5
*d*e^7 - 4*b^5*c^3*d^7*e - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5
*d^6*e^2 + 96*a^4*c^4*d^4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64
*a^2*b^2*c^4*d^6*e^2 + 32*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 14
4*a^3*b^2*c^3*d^4*e^4 + 32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 3
2*a*b^3*c^4*d^7*e + 4*a*b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d
*e^7 - 64*a^5*b*c^2*d*e^7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 2
0*a^2*b^5*c*d^3*e^5 - 192*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^
4*b*c^3*d^3*e^5) - root(3840*a^6*b*c^5*d^5*e^7*z^3 + 3840*a^5*b*c^6*d^7*e^5
*z^3 + 1920*a^7*b*c^4*d^3*e^9*z^3 + 1920*a^4*b*c^7*d^9*e^3*z^3 - 288*a^7*b^
3*c^2*d*e^11*z^3 - 288*a^2*b^3*c^7*d^11*e*z^3 + 210*a^4*b^7*c*d^3*e^9*z^3 +
 210*a*b^7*c^4*d^9*e^3*z^3 - 174*a^5*b^6*c*d^2*e^10*z^3 - 174*a*b^6*c^5*d^1
0*e^2*z^3 - 120*a^3*b^8*c*d^4*e^8*z^3 - 120*a*b^8*c^3*d^8*e^4*z^3 + 12*a^2*
b^9*c*d^5*e^7*z^3 + 12*a*b^9*c^2*d^7*e^5*z^3 + 384*a^8*b*c^3*d*e^11*z^3 + 3
84*a^3*b*c^8*d^11*e*z^3 + 72*a^6*b^5*c*d*e^11*z^3 + 72*a*b^5*c^6*d^11*e*z^3
 + 18*a*b^10*c*d^6*e^6*z^3 - 4800*a^5*b^2*c^5*d^6*e^6*z^3 - 3120*a^6*b^2*c^
4*d^4*e^8*z^3 - 3120*a^4*b^2*c^6*d^8*e^4*z^3 + 2160*a^4*b^4*c^4*d^6*e^6*z^3
 - 1776*a^4*b^5*c^3*d^5*e^7*z^3 - 1776*a^3*b^5*c^4*d^7*e^5*z^3 + 1740*a^5*b
^4*c^3*d^4*e^8*z^3 + 1740*a^3*b^4*c^5*d^8*e^4*z^3 + 960*a^5*b^3*c^4*d^5*e^7
*z^3 + 960*a^4*b^3*c^5*d^7*e^5*z^3 - 672*a^7*b^2*c^3*d^2*e^10*z^3 - 672*a^3
*b^2*c^7*d^10*e^2*z^3 + 648*a^6*b^4*c^2*d^2*e^10*z^3 + 648*a^2*b^4*c^6*d^10
*e^2*z^3 - 600*a^5*b^5*c^2*d^3*e^9*z^3 - 600*a^2*b^5*c^5*d^9*e^3*z^3 + 372*
a^3*b^7*c^2*d^5*e^7*z^3 + 372*a^2*b^7*c^3*d^7*e^5*z^3 + 316*a^3*b^6*c^3*d^6
*e^6*z^3 - 222*a^2*b^8*c^2*d^6*e^6*z^3 - 160*a^6*b^3*c^3*d^3*e^9*z^3 - 160*
a^3*b^3*c^6*d^9*e^3*z^3 + 15*a^4*b^6*c^2*d^4*e^8*z^3 + 15*a^2*b^6*c^4*d^8*e
^4*z^3 - 6*b^11*c*d^7*e^5*z^3 - 6*b^7*c^5*d^11*e*z^3 - 6*a^5*b^7*d*e^11*z^3
 - 6*a*b^11*d^5*e^7*z^3 - 12*a^7*b^4*c*e^12*z^3 - 12*a*b^4*c^7*d^12*z^3 - 2
0*b^9*c^3*d^9*e^3*z^3 + 15*b^10*c^2*d^8*e^4*z^3 + 15*b^8*c^4*d^10*e^2*z^3 -
 1280*a^6*c^6*d^6*e^6*z^3 - 960*a^7*c^5*d^4*e^8*z^3 - 960*a^5*c^7*d^8*e^4*z
^3 - 384*a^8*c^4*d^2*e^10*z^3 - 384*a^4*c^8*d^10*e^2*z^3 - 20*a^3*b^9*d^3*e
^9*z^3 + 15*a^4*b^8*d^2*e^10*z^3 + 15*a^2*b^10*d^4*e^8*z^3 + 48*a^8*b^2*c^2
*e^12*z^3 + 48*a^2*b^2*c^8*d^12*z^3 - 64*a^9*c^3*e^12*z^3 - 64*a^3*c^9*d^12
*z^3 + b^12*d^6*e^6*z^3 + b^6*c^6*d^12*z^3 + a^6*b^6*e^12*z^3 - 44*a^3*b^4*
c*d*e^7*g*h*z - 20*a*b^6*c*d^3*e^5*g*h*z - 12*a*b^2*c^5*d^7*e*g*h*z + 432*a
^4*b*c^3*d*e^7*f*h*z + 84*a^2*b^5*c*d*e^7*f*h*z + 28*a*b^6*c*d^2*e^6*f*h*z
 - 8*a*b*c^6*d^6*e^2*f*g*z - 804*a^3*b^2*c^3*d^3*e^5*g*h*z + 564*a^2*b^2*c^4
*d^5*e^3*g*h*z + 222*a^3*b^3*c^2*d^2*e^6*g*h*z + 186*a^2*b^4*c^2*d^3*e^5*g*
h*z - 166*a^2*b^3*c^3*d^4*e^4*g*h*z + 792*a^3*b^2*c^3*d^2*e^6*f*h*z - 744*a
^2*b^2*c^4*d^4*e^4*f*h*z + 492*a^2*b^3*c^3*d^3*e^5*f*h*z - 264*a^2*b^4*c^2*
d^2*e^6*f*h*z + 996*a^2*b^2*c^4*d^3*e^5*f*g*z - 870*a^2*b^3*c^3*d^2*e^6*f*g
*z + 16*a*b*c^6*d^7*e*f*h*z - 56*a*b^6*c*d*e^7*f*g*z - 264*a^4*b*c^3*d^2*e^
6*g*h*z + 208*a^3*b*c^4*d^4*e^4*g*h*z + 156*a^4*b^2*c^2*d*e^7*g*h*z - 148*a
*b^4*c^3*d^5*e^3*g*h*z + 54*a*b^5*c^2*d^4*e^4*g*h*z - 48*a^2*b^5*c*d^2*e^6*
g*h*z - 24*a^2*b*c^5*d^6*e^2*g*h*z + 10*a*b^3*c^4*d^6*e^2*g*h*z - 656*a^3*b
*c^4*d^3*e^5*f*h*z - 308*a^3*b^3*c^2*d*e^7*f*h*z + 116*a*b^4*c^3*d^4*e^4*f*
h*z - 84*a*b^5*c^2*d^3*e^5*f*h*z + 68*a*b^3*c^4*d^5*e^3*f*h*z - 48*a^2*b*c^
5*d^5*e^3*f*h*z - 24*a*b^2*c^5*d^6*e^2*f*h*z + 1320*a^3*b*c^4*d^2*e^6*f*g*z
 - 732*a^3*b^2*c^3*d*e^7*f*g*z + 306*a^2*b^4*c^2*d*e^7*f*g*z - 304*a*b^4*c^
3*d^3*e^5*f*g*z + 222*a*b^5*c^2*d^2*e^6*f*g*z + 110*a*b^3*c^4*d^4*e^4*f*g*z
 - 84*a*b^2*c^5*d^5*e^3*f*g*z + 16*a*c^7*d^7*e*f*g*z - 8*a*b^7*d*e^7*f*h*z

+ 4*a*b*c^6*d^8*g*h*z + 6*b^6*c^2*d^5*e^3*g*h*z + 6*b^5*c^3*d^6*e^2*g*h*z +
1072*a^4*c^4*d^3*e^5*g*h*z - 720*a^3*c^5*d^5*e^3*g*h*z - 8*b^6*c^2*d^4*e^4
*f*h*z - 8*b^4*c^4*d^6*e^2*f*h*z + 1072*a^3*c^5*d^4*e^4*f*h*z - 960*a^4*c^4
*d^2*e^6*f*h*z + 30*b^6*c^2*d^3*e^5*f*g*z + 30*b^3*c^5*d^6*e^2*f*g*z - 10*b
^5*c^3*d^4*e^4*f*g*z - 10*b^4*c^4*d^5*e^3*f*g*z - 1488*a^3*c^5*d^3*e^5*f*g*
z + 48*a^2*c^6*d^5*e^3*f*g*z - 24*a^4*b^2*c^2*e^8*f*h*z + 186*a^3*b^3*c^2*e
^8*f*g*z + 4*a^4*b^3*c*d*e^7*h^2*z + 4*a*b^6*c*d^4*e^4*h^2*z + 4*a*b^3*c^4*
d^7*e*h^2*z + 168*a^4*b*c^3*d*e^7*g^2*z + 24*a^2*b^5*c*d*e^7*g^2*z + 18*a*b
^6*c*d^2*e^6*g^2*z - 912*a^3*b*c^4*d*e^7*f^2*z - 192*a*b^5*c^2*d*e^7*f^2*z
+ 144*a*b*c^6*d^5*e^3*f^2*z + 432*a^3*b^2*c^3*d^4*e^4*h^2*z - 168*a^4*b^2*c
^2*d^2*e^6*h^2*z - 168*a^2*b^2*c^4*d^6*e^2*h^2*z - 108*a^2*b^4*c^2*d^4*e^4*
h^2*z - 20*a^3*b^3*c^2*d^3*e^5*h^2*z - 20*a^2*b^3*c^3*d^5*e^3*h^2*z - 426*a
^2*b^2*c^4*d^4*e^4*g^2*z + 336*a^3*b^2*c^3*d^2*e^6*g^2*z + 274*a^2*b^3*c^3*
d^3*e^5*g^2*z - 120*a^2*b^4*c^2*d^2*e^6*g^2*z - 864*a^2*b^2*c^4*d^2*e^6*f^2
*z - 2*b^7*c*d^4*e^4*g*h*z - 2*b^4*c^4*d^7*e*g*h*z - 240*a^5*c^3*d*e^7*g*h*
z + 16*a^2*c^6*d^7*e*g*h*z + 4*b^7*c*d^3*e^5*f*h*z + 4*b^3*c^5*d^7*e*f*h*z
- 20*b^7*c*d^2*e^6*f*g*z - 20*b^2*c^6*d^7*e*f*g*z + 4*a^2*b^6*d*e^7*g*h*z +
4*a*b^7*d^2*e^6*g*h*z + 528*a^4*c^4*d*e^7*f*g*z + 12*a^5*b*c^2*e^8*g*h*z -
2*a^4*b^3*c*e^8*g*h*z + 4*a^3*b^4*c*e^8*f*h*z - 228*a^4*b*c^3*e^8*f*g*z -
48*a^2*b^5*c*e^8*f*g*z - 8*a*b*c^6*d^7*e*g^2*z + 36*a^3*b^4*c*d^2*e^6*h^2*z
+ 36*a*b^4*c^3*d^6*e^2*h^2*z + 12*a^2*b^5*c*d^3*e^5*h^2*z + 12*a*b^5*c^2*d
^5*e^3*h^2*z - 312*a^3*b*c^4*d^3*e^5*g^2*z + 104*a*b^4*c^3*d^4*e^4*g^2*z -
102*a^3*b^3*c^2*d*e^7*g^2*z - 66*a*b^5*c^2*d^3*e^5*g^2*z + 24*a^2*b*c^5*d^5
*e^3*g^2*z + 24*a*b^2*c^5*d^6*e^2*g^2*z - 18*a*b^3*c^4*d^5*e^3*g^2*z + 744*
a^2*b^3*c^3*d*e^7*f^2*z + 240*a^2*b*c^5*d^3*e^5*f^2*z + 216*a*b^4*c^3*d^2*e
^6*f^2*z - 120*a*b^2*c^5*d^4*e^4*f^2*z + 24*a^5*c^3*e^8*f*h*z + 16*b^7*c*d*
e^7*f^2*z + 16*b*c^7*d^7*e*f^2*z - 2*a*b^7*d*e^7*g^2*z + 48*a*b^6*c*e^8*f^2
*z - 4*b^6*c^2*d^6*e^2*h^2*z - 536*a^4*c^4*d^4*e^4*h^2*z + 240*a^5*c^3*d^2*
e^6*h^2*z + 240*a^3*c^5*d^6*e^2*h^2*z - 12*b^6*c^2*d^4*e^4*g^2*z - 12*b^4*c
^4*d^6*e^2*g^2*z + 10*b^5*c^3*d^5*e^3*g^2*z + 528*a^3*c^5*d^4*e^4*g^2*z - 4
32*a^4*c^4*d^2*e^6*g^2*z + 20*b^4*c^4*d^4*e^4*f^2*z - 16*b^6*c^2*d^2*e^6*f^
2*z - 16*b^2*c^6*d^6*e^2*f^2*z - 16*a^2*c^6*d^6*e^2*g^2*z - 8*b^5*c^3*d^3*e
^5*f^2*z - 8*b^3*c^5*d^5*e^3*f^2*z - 4*a^2*b^6*d^2*e^6*h^2*z + 912*a^3*c^5*
d^2*e^6*f^2*z - 120*a^2*c^6*d^4*e^4*f^2*z - 45*a^4*b^2*c^2*e^8*g^2*z + 264*
a^3*b^2*c^3*e^8*f^2*z - 192*a^2*b^4*c^2*e^8*f^2*z + 4*b^8*d*e^7*f*g*z - 8*a
*c^7*d^8*f*h*z + 4*b*c^7*d^8*f*g*z + 4*a*b^7*e^8*f*g*z + 6*b^7*c*d^3*e^5*g^
2*z + 6*b^3*c^5*d^7*e*g^2*z - 48*a*c^7*d^6*e^2*f^2*z + 12*a^3*b^4*c*e^8*g^2
*z - b^8*d^2*e^6*g^2*z - 4*a^6*c^2*e^8*h^2*z + 48*a^5*c^3*e^8*g^2*z - 4*a^2
*c^6*d^8*h^2*z - b^2*c^6*d^8*g^2*z - 36*a^4*c^4*e^8*f^2*z - a^2*b^6*e^8*g^2
*z - 4*c^8*d^8*f^2*z - 4*b^8*e^8*f^2*z - 80*a*b*c^4*d^3*e^3*f*g*h + 24*a^2*
b*c^3*d*e^5*f*g*h + 16*a*b^3*c^2*d*e^5*f*g*h - 72*a*b^2*c^3*d^2*e^4*f*g*h -
48*a^2*b*c^3*d^3*e^3*g*h^2 + 16*a*b^3*c^2*d^3*e^3*g*h^2 - 12*a*b^2*c^3*d^3
*e^3*g^2*h - 6*a^2*b^2*c^2*d*e^5*g^2*h - 72*a^2*b^2*c^2*d*e^5*f*h^2 + 48*a*
b^2*c^3*d^3*e^3*f*h^2 + 24*a^2*b*c^3*d^2*e^4*f*h^2 - 8*a*b^3*c^2*d^2*e^4*f*
h^2 - 8*b^5*c*d*e^5*f*g*h - 8*b*c^5*d^5*e*f*g*h - 8*a*b^4*c*e^6*f*g*h + 24*
b^3*c^3*d^3*e^3*f*g*h + 16*b^4*c^2*d^2*e^4*f*g*h + 16*b^2*c^4*d^4*e^2*f*g*h
+ 48*a^2*c^4*d^2*e^4*f*g*h + 48*a^2*b^2*c^2*e^6*f*g*h + 40*a^3*b*c^2*d*e^5
*g*h^2 + 28*a*b*c^4*d^4*e^2*g^2*h - 8*a^2*b^3*c*d*e^5*g*h^2 - 8*a*b^4*c*d^2
*e^4*g*h^2 + 96*a*b^2*c^3*d*e^5*f^2*h + 24*a*b*c^4*d^2*e^4*f^2*h + 16*a*b*c
^4*d^4*e^2*f*h^2 + 96*a*b*c^4*d^2*e^4*f*g^2 - 48*a*b^2*c^3*d*e^5*f*g^2 + 12
*a^2*b^2*c^2*d^2*e^4*g*h^2 - 56*a*c^5*d^4*e^2*f*g*h - 8*a*b*c^4*d^5*e*g*h^2
+ 4*a*b^4*c*d*e^5*g^2*h + 16*a*b^4*c*d*e^5*f*h^2 - 48*a*b*c^4*d*e^5*f^2*g
- 24*a^3*c^3*e^6*f*g*h + 16*a*c^5*d^5*e*f*h^2 - 6*b^4*c^2*d^3*e^3*g^2*h - 6
*b^3*c^3*d^4*e^2*g^2*h + 4*b^4*c^2*d^4*e^2*g*h^2 + 80*a^2*c^4*d^3*e^3*g^2*h
- 44*a^2*c^4*d^4*e^2*g*h^2 + 24*a^3*c^3*d^2*e^4*g*h^2 - 16*b^3*c^3*d^2*e^4
*f^2*h - 16*b^2*c^4*d^3*e^3*f^2*h - 8*b^4*c^2*d^3*e^3*f*h^2 - 8*b^3*c^3*d^4
*e^2*f*h^2 + 60*b^2*c^4*d^2*e^4*f^2*g - 48*a^2*c^4*d^3*e^3*f*h^2 - 24*b^3*c
^3*d^2*e^4*f*g^2 - 24*b^2*c^4*d^3*e^3*f*g^2 - 24*a^3*b*c^2*d^2*e^4*h^3 + 24
*a^2*b*c^3*d^4*e^2*h^3 + 8*a^2*b^3*c*d^2*e^4*h^3 - 8*a*b^3*c^2*d^4*e^2*h^3

+ 18*a*b^2*c^3*d^2*e^4*g^3 + 2*b^5*c*d^2*e^4*g^2*h + 2*b^2*c^4*d^5*e*g^2*h
- 48*a^3*c^3*d*e^5*g^2*h - 8*b^4*c^2*d*e^5*f^2*h - 8*b*c^5*d^4*e^2*f^2*h -
168*a^2*c^4*d*e^5*f^2*h + 96*a*c^5*d^3*e^3*f^2*h + 64*a^3*c^3*d*e^5*f*h^2 +
12*b^4*c^2*d*e^5*f*g^2 + 12*b*c^5*d^4*e^2*f*g^2 - 168*a*c^5*d^2*e^4*f^2*g
+ 48*a^2*c^4*d*e^5*f*g^2 + 48*a*c^5*d^3*e^3*f*g^2 - 12*a^3*b*c^2*e^6*g^2*h
+ 2*a^2*b^3*c*e^6*g^2*h + 48*a^2*b*c^3*e^6*f^2*h - 48*a*b^3*c^2*e^6*f^2*h -
8*a^3*b*c^2*e^6*f*h^2 - 60*a^2*b*c^3*e^6*f*g^2 + 48*a*b^2*c^3*e^6*f^2*g +
12*a*b^3*c^2*e^6*f*g^2 + 24*a^2*b*c^3*d*e^5*g^3 - 24*a*b*c^4*d^3*e^3*g^3 -
6*a*b^3*c^2*d*e^5*g^3 - 12*c^6*d^4*e^2*f^2*g + 4*a^4*c^2*e^6*g*h^2 - 12*b^4
*c^2*e^6*f^2*g + 36*a^2*c^4*e^6*f^2*g - 8*a^4*c^2*d*e^5*h^3 + 8*a^2*c^4*d^5
*e*h^3 - 24*b^2*c^4*d*e^5*f^3 - 24*b*c^5*d^2*e^4*f^3 + 8*c^6*d^5*e*f^2*h +
8*b^5*c*e^6*f^2*h + 144*a*c^5*d*e^5*f^3 - 72*a*b*c^4*e^6*f^3 + 10*b^3*c^3*d
^3*e^3*g^3 - 3*b^4*c^2*d^2*e^4*g^3 - 3*b^2*c^4*d^4*e^2*g^3 - 48*a^2*c^4*d^2
*e^4*g^3 - 3*a^2*b^2*c^2*e^6*g^3 + 16*c^6*d^3*e^3*f^3 + 16*b^3*c^3*e^6*f^3
+ 16*a^3*c^3*e^6*g^3, z, k)*((8*a^6*c^3*e^9*h - 24*a^5*c^4*e^9*f - 8*a*c^8*
d^8*e*f + 2*a^2*b^6*c*e^9*f - a^3*b^5*c*e^9*g - 20*a^5*b*c^3*e^9*g + 16*a^5
*c^4*d*e^8*g + 2*b^2*c^7*d^8*e*f + 2*b^8*c*d^2*e^7*f - 8*a^2*c^7*d^8*e*h -
b^3*c^6*d^8*e*g - b^8*c*d^3*e^6*g - 18*a^3*b^4*c^2*e^9*f + 46*a^4*b^2*c^3*e
^9*f + 9*a^4*b^3*c^2*e^9*g - 48*a^2*c^7*d^6*e^3*f - 96*a^3*c^6*d^4*e^5*f -
80*a^4*c^5*d^2*e^7*f - 2*a^5*b^2*c^2*e^9*h + 16*a^2*c^7*d^7*e^2*g + 48*a^3*
c^6*d^5*e^4*g + 48*a^4*c^5*d^3*e^6*g - 6*b^3*c^6*d^7*e^2*f + 4*b^4*c^5*d^6*
e^3*f + 4*b^6*c^3*d^4*e^5*f - 6*b^7*c^2*d^3*e^6*f - 16*a^3*c^6*d^6*e^3*h +
16*a^5*c^4*d^2*e^7*h + 4*b^4*c^5*d^7*e^2*g - 3*b^5*c^4*d^6*e^3*g - 3*b^6*c^
3*d^5*e^4*g + 4*b^7*c^2*d^4*e^5*g - 2*b^5*c^4*d^7*e^2*h + 4*b^6*c^3*d^6*e^3
*h - 2*b^7*c^2*d^5*e^4*h - 4*a*b^2*c^6*d^6*e^3*f - 14*a*b^3*c^5*d^5*e^4*f -
38*a*b^4*c^4*d^4*e^5*f + 54*a*b^5*c^3*d^3*e^6*f - 10*a*b^6*c^2*d^2*e^7*f +
56*a^2*b*c^6*d^5*e^4*f + 34*a^2*b^5*c^2*d*e^8*f + 40*a^3*b*c^5*d^3*e^6*f -
74*a^3*b^3*c^3*d*e^8*f - 20*a*b^2*c^6*d^7*e^2*g + 10*a*b^3*c^5*d^6*e^3*g +
34*a*b^4*c^4*d^5*e^4*g - 33*a*b^5*c^3*d^4*e^5*g + 4*a*b^6*c^2*d^3*e^6*g +
8*a^2*b*c^6*d^6*e^3*g - 16*a^3*b*c^5*d^4*e^5*g - 10*a^3*b^4*c^2*d*e^8*g - 4
0*a^4*b*c^4*d^2*e^7*g + 20*a^4*b^2*c^3*d*e^8*g + 10*a*b^3*c^5*d^7*e^2*h - 2
6*a*b^4*c^4*d^6*e^3*h + 12*a*b^5*c^3*d^5*e^4*h - 8*a^2*b*c^6*d^7*e^2*h - 4*
a^2*b^6*c*d^2*e^7*h - 8*a^3*b*c^5*d^5*e^4*h + 8*a^4*b*c^4*d^3*e^6*h - 10*a^
4*b^3*c^2*d*e^8*h - 4*a*b^7*c*d*e^8*f + 4*a*b*c^7*d^8*e*g + 112*a^2*b^2*c^5
*d^4*e^5*f - 130*a^2*b^3*c^4*d^3*e^6*f - 28*a^2*b^4*c^3*d^2*e^7*f + 164*a^3
*b^2*c^4*d^2*e^7*f - 100*a^2*b^2*c^5*d^5*e^4*g + 72*a^2*b^3*c^4*d^4*e^5*g +
12*a^2*b^4*c^3*d^3*e^6*g - 7*a^2*b^5*c^2*d^2*e^7*g - 60*a^3*b^2*c^4*d^3*e^
6*g + 22*a^3*b^3*c^3*d^2*e^7*g + 44*a^2*b^2*c^5*d^6*e^3*h - 14*a^2*b^3*c^4*
d^5*e^4*h - 12*a^2*b^5*c^2*d^3*e^6*h + 14*a^3*b^3*c^3*d^3*e^6*h + 26*a^3*b^
4*c^2*d^2*e^7*h - 44*a^4*b^2*c^3*d^2*e^7*h + 24*a*b*c^7*d^7*e^2*f + 8*a^4*b
*c^4*d*e^8*f + a*b^7*c*d^2*e^7*g + a^2*b^6*c*d*e^8*g + 2*a*b^2*c^6*d^8*e*h
+ 2*a*b^7*c*d^3*e^6*h + 2*a^3*b^5*c*d*e^8*h + 8*a^5*b*c^3*d*e^8*h)/(16*a^2*
c^6*d^8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*a*b^
2*c^5*d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5*c^3
*d^7*e - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96*a^4*
c^4*d^4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d^6*e
^2 + 32*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3*d^4*
e^4 + 32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^7*e
+ 4*a*b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5*b*c^
2*d*e^7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^3*e^
5 - 192*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^5) +
root(3840*a^6*b*c^5*d^5*e^7*z^3 + 3840*a^5*b*c^6*d^7*e^5*z^3 + 1920*a^7*b*
c^4*d^3*e^9*z^3 + 1920*a^4*b*c^7*d^9*e^3*z^3 - 288*a^7*b^3*c^2*d*e^11*z^3 -
288*a^2*b^3*c^7*d^11*e*z^3 + 210*a^4*b^7*c*d^3*e^9*z^3 + 210*a*b^7*c^4*d^9
*e^3*z^3 - 174*a^5*b^6*c*d^2*e^10*z^3 - 174*a*b^6*c^5*d^10*e^2*z^3 - 120*a^
3*b^8*c*d^4*e^8*z^3 - 120*a*b^8*c^3*d^8*e^4*z^3 + 12*a^2*b^9*c*d^5*e^7*z^3
+ 12*a*b^9*c^2*d^7*e^5*z^3 + 384*a^8*b*c^3*d*e^11*z^3 + 384*a^3*b*c^8*d^11*
e*z^3 + 72*a^6*b^5*c*d*e^11*z^3 + 72*a*b^5*c^6*d^11*e*z^3 + 18*a*b^10*c*d^6
*e^6*z^3 - 4800*a^5*b^2*c^5*d^6*e^6*z^3 - 3120*a^6*b^2*c^4*d^4*e^8*z^3 - 31

20*a^4*b^2*c^6*d^8*e^4*z^3 + 2160*a^4*b^4*c^4*d^6*e^6*z^3 - 1776*a^4*b^5*c^3*d^5*e^7*z^3 - 1776*a^3*b^5*c^4*d^7*e^5*z^3 + 1740*a^5*b^4*c^3*d^4*e^8*z^3 + 1740*a^3*b^4*c^5*d^8*e^4*z^3 + 960*a^5*b^3*c^4*d^5*e^7*z^3 + 960*a^4*b^3*c^5*d^7*e^5*z^3 - 672*a^7*b^2*c^3*d^2*e^10*z^3 - 672*a^3*b^2*c^7*d^10*e^2*z^3 + 648*a^6*b^4*c^2*d^2*e^10*z^3 + 648*a^2*b^4*c^6*d^10*e^2*z^3 - 600*a^5*b^5*c^2*d^3*e^9*z^3 - 600*a^2*b^5*c^5*d^9*e^3*z^3 + 372*a^3*b^7*c^2*d^5*e^7*z^3 + 372*a^2*b^7*c^3*d^7*e^5*z^3 + 316*a^3*b^6*c^3*d^6*e^6*z^3 - 222*a^2*b^8*c^2*d^6*e^6*z^3 - 160*a^6*b^3*c^3*d^3*e^9*z^3 - 160*a^3*b^3*c^6*d^9*e^3*z^3 + 15*a^4*b^6*c^2*d^4*e^8*z^3 + 15*a^2*b^6*c^4*d^8*e^4*z^3 - 6*b^11*c*d^7*e^5*z^3 - 6*b^7*c^5*d^11*e*z^3 - 6*a^5*b^7*d*e^11*z^3 - 6*a*b^11*d^5*e^7*z^3 - 12*a^7*b^4*c*e^12*z^3 - 12*a*b^4*c^7*d^12*z^3 - 20*b^9*c^3*d^9*e^3*z^3 + 15*b^10*c^2*d^8*e^4*z^3 + 15*b^8*c^4*d^10*e^2*z^3 - 1280*a^6*c^6*d^6*e^6*z^3 - 960*a^7*c^5*d^4*e^8*z^3 - 960*a^5*c^7*d^8*e^4*z^3 - 384*a^8*c^4*d^2*e^10*z^3 - 384*a^4*c^8*d^10*e^2*z^3 - 20*a^3*b^9*d^3*e^9*z^3 + 15*a^4*b^8*d^2*e^10*z^3 + 15*a^2*b^10*d^4*e^8*z^3 + 48*a^8*b^2*c^2*e^12*z^3 + 48*a^2*b^2*c^8*d^12*z^3 - 64*a^9*c^3*e^12*z^3 - 64*a^3*c^9*d^12*z^3 + b^12*d^6*e^6*z^3 + b^6*c^6*d^12*z^3 + a^6*b^6*e^12*z^3 - 44*a^3*b^4*c*d*e^7*g*h*z - 20*a*b^6*c*d^3*e^5*g*h*z - 12*a*b^2*c^5*d^7*e*g*h*z + 432*a^4*b*c^3*d*e^7*f*h*z + 84*a^2*b^5*c*d*e^7*f*h*z + 28*a*b^6*c*d^2*e^6*f*h*z - 8*a*b*c^6*d^6*e^2*f*g*z - 804*a^3*b^2*c^3*d^3*e^5*g*h*z + 564*a^2*b^2*c^4*d^5*e^3*g*h*z + 222*a^3*b^3*c^2*d^2*e^6*g*h*z + 186*a^2*b^4*c^2*d^3*e^5*g*h*z - 166*a^2*b^3*c^3*d^4*e^4*g*h*z + 792*a^3*b^2*c^3*d^2*e^6*f*h*z - 744*a^2*b^2*c^4*d^4*e^4*f*h*z + 492*a^2*b^3*c^3*d^3*e^5*f*h*z - 264*a^2*b^4*c^2*d^2*e^6*f*h*z + 996*a^2*b^2*c^4*d^3*e^5*f*g*z - 870*a^2*b^3*c^3*d^2*e^6*f*g*z + 16*a*b*c^6*d^7*e*f*h*z - 56*a*b^6*c*d*e^7*f*g*z - 264*a^4*b*c^3*d^2*e^6*g*h*z + 208*a^3*b*c^4*d^4*e^4*g*h*z + 156*a^4*b^2*c^2*d*e^7*g*h*z - 148*a*b^4*c^3*d^5*e^3*g*h*z + 54*a*b^5*c^2*d^4*e^4*g*h*z - 48*a^2*b^5*c*d^2*e^6*g*h*z - 24*a^2*b*c^5*d^6*e^2*g*h*z + 10*a*b^3*c^4*d^6*e^2*g*h*z - 656*a^3*b*c^4*d^3*e^5*f*h*z - 308*a^3*b^3*c^2*d*e^7*f*h*z + 116*a*b^4*c^3*d^4*e^4*f*h*z - 84*a*b^5*c^2*d^3*e^5*f*h*z + 68*a*b^3*c^4*d^5*e^3*f*h*z - 48*a^2*b*c^5*d^5*e^3*f*h*z - 24*a*b^2*c^5*d^6*e^2*f*h*z + 1320*a^3*b*c^4*d^2*e^6*f*g*z - 732*a^3*b^2*c^3*d*e^7*f*g*z + 306*a^2*b^4*c^2*d*e^7*f*g*z - 304*a*b^4*c^3*d^3*e^5*f*g*z + 222*a*b^5*c^2*d^2*e^6*f*g*z + 110*a*b^3*c^4*d^4*e^4*f*g*z - 84*a*b^2*c^5*d^5*e^3*f*g*z + 16*a*c^7*d^7*e*f*g*z - 8*a*b^7*d*e^7*f*h*z + 4*a*b*c^6*d^8*g*h*z + 6*b^6*c^2*d^5*e^3*g*h*z + 6*b^5*c^3*d^6*e^2*g*h*z + 1072*a^4*c^4*d^3*e^5*g*h*z - 720*a^3*c^5*d^5*e^3*g*h*z - 8*b^6*c^2*d^4*e^4*f*h*z - 8*b^4*c^4*d^6*e^2*f*h*z + 1072*a^3*c^5*d^4*e^4*f*h*z - 960*a^4*c^4*d^2*e^6*f*h*z + 30*b^6*c^2*d^3*e^5*f*g*z + 30*b^3*c^5*d^6*e^2*f*g*z - 10*b^5*c^3*d^4*e^4*f*g*z - 10*b^4*c^4*d^5*e^3*f*g*z - 1488*a^3*c^5*d^3*e^5*f*g*z + 48*a^2*c^6*d^5*e^3*f*g*z - 24*a^4*b^2*c^2*e^8*f*h*z + 186*a^3*b^3*c^2*e^8*f*g*z + 4*a^4*b^3*c*d*e^7*h^2*z + 4*a*b^6*c*d^4*e^4*h^2*z + 4*a*b^3*c^4*d^7*e*h^2*z + 168*a^4*b*c^3*d*e^7*g^2*z + 24*a^2*b^5*c*d*e^7*g^2*z + 18*a*b^6*c*d^2*e^6*g^2*z - 912*a^3*b*c^4*d*e^7*f^2*z - 192*a*b^5*c^2*d*e^7*f^2*z + 144*a*b*c^6*d^5*e^3*f^2*z + 432*a^3*b^2*c^3*d^4*e^4*h^2*z - 168*a^4*b^2*c^2*d^2*e^6*h^2*z - 168*a^2*b^2*c^4*d^6*e^2*h^2*z - 108*a^2*b^4*c^2*d^4*e^4*h^2*z - 20*a^3*b^3*c^2*d^3*e^5*h^2*z - 20*a^2*b^3*c^3*d^5*e^3*h^2*z - 426*a^2*b^2*c^4*d^4*e^4*g^2*z + 336*a^3*b^2*c^3*d^2*e^6*g^2*z + 274*a^2*b^3*c^3*d^3*e^5*g^2*z - 120*a^2*b^4*c^2*d^2*e^6*g^2*z - 864*a^2*b^2*c^4*d^2*e^6*f^2*z - 2*b^7*c*d^4*e^4*g*h*z - 2*b^4*c^4*d^7*e*g*h*z - 240*a^5*c^3*d*e^7*g*h*z + 16*a^2*c^6*d^7*e*g*h*z + 4*b^7*c*d^3*e^5*f*h*z + 4*b^3*c^5*d^7*e*f*h*z - 20*b^7*c*d^2*e^6*f*g*z - 20*b^2*c^6*d^7*e*f*g*z + 4*a^2*b^6*d*e^7*g*h*z + 4*a*b^7*d^2*e^6*g*h*z + 528*a^4*c^4*d*e^7*f*g*z + 12*a^5*b*c^2*e^8*g*h*z - 2*a^4*b^3*c*e^8*g*h*z + 4*a^3*b^4*c*e^8*f*h*z - 228*a^4*b*c^3*e^8*f*g*z - 48*a^2*b^5*c*e^8*f*g*z - 8*a*b*c^6*d^7*e*g^2*z + 36*a^3*b^4*c*d^2*e^6*h^2*z + 36*a*b^4*c^3*d^6*e^2*h^2*z + 12*a^2*b^5*c*d^3*e^5*h^2*z + 12*a*b^5*c^2*d^5*e^3*h^2*z - 312*a^3*b*c^4*d^3*e^5*g^2*z + 104*a*b^4*c^3*d^4*e^4*g^2*z - 102*a^3*b^3*c^2*d*e^7*g^2*z - 66*a*b^5*c^2*d^3*e^5*g^2*z + 24*a^2*b*c^5*d^5*e^3*g^2*z + 24*a*b^2*c^5*d^6*e^2*g^2*z - 18*a*b^3*c^4*d^5*e^3*g^2*z + 744*a^2*b^3*c^3*d*e^7*f^2*z + 240*a^2*b*c^5*d^3*e^5*f^2*z + 216*a*b^4*c^3*d^2*e^6*f^2*z - 120*a*b

^2*c^5*d^4*e^4*f^2*z + 24*a^5*c^3*e^8*f*h*z + 16*b^7*c*d*e^7*f^2*z + 16*b*c
^7*d^7*e*f^2*z - 2*a*b^7*d*e^7*g^2*z + 48*a*b^6*c*e^8*f^2*z - 4*b^6*c^2*d^6
*e^2*h^2*z - 536*a^4*c^4*d^4*e^4*h^2*z + 240*a^5*c^3*d^2*e^6*h^2*z + 240*a^
3*c^5*d^6*e^2*h^2*z - 12*b^6*c^2*d^4*e^4*g^2*z - 12*b^4*c^4*d^6*e^2*g^2*z +
 10*b^5*c^3*d^5*e^3*g^2*z + 528*a^3*c^5*d^4*e^4*g^2*z - 432*a^4*c^4*d^2*e^6
*g^2*z + 20*b^4*c^4*d^4*e^4*f^2*z - 16*b^6*c^2*d^2*e^6*f^2*z - 16*b^2*c^6*d
^6*e^2*f^2*z - 16*a^2*c^6*d^6*e^2*g^2*z - 8*b^5*c^3*d^3*e^5*f^2*z - 8*b^3*c
^5*d^5*e^3*f^2*z - 4*a^2*b^6*d^2*e^6*h^2*z + 912*a^3*c^5*d^2*e^6*f^2*z - 12
0*a^2*c^6*d^4*e^4*f^2*z - 45*a^4*b^2*c^2*e^8*g^2*z + 264*a^3*b^2*c^3*e^8*f^
2*z - 192*a^2*b^4*c^2*e^8*f^2*z + 4*b^8*d*e^7*f*g*z - 8*a*c^7*d^8*f*h*z + 4
*b*c^7*d^8*f*g*z + 4*a*b^7*e^8*f*g*z + 6*b^7*c*d^3*e^5*g^2*z + 6*b^3*c^5*d^
7*e*g^2*z - 48*a*c^7*d^6*e^2*f^2*z + 12*a^3*b^4*c*e^8*g^2*z - b^8*d^2*e^6*g
^2*z - 4*a^6*c^2*e^8*h^2*z + 48*a^5*c^3*e^8*g^2*z - 4*a^2*c^6*d^8*h^2*z - b
^2*c^6*d^8*g^2*z - 36*a^4*c^4*e^8*f^2*z - a^2*b^6*e^8*g^2*z - 4*c^8*d^8*f^2
*z - 4*b^8*e^8*f^2*z - 80*a*b*c^4*d^3*e^3*f*g*h + 24*a^2*b*c^3*d*e^5*f*g*h
+ 16*a*b^3*c^2*d*e^5*f*g*h - 72*a*b^2*c^3*d^2*e^4*f*g*h - 48*a^2*b*c^3*d^3*
e^3*g*h^2 + 16*a*b^3*c^2*d^3*e^3*g*h^2 - 12*a*b^2*c^3*d^3*e^3*g^2*h - 6*a^2
*b^2*c^2*d*e^5*g^2*h - 72*a^2*b^2*c^2*d*e^5*f*h^2 + 48*a*b^2*c^3*d^3*e^3*f*
h^2 + 24*a^2*b*c^3*d^2*e^4*f*h^2 - 8*a*b^3*c^2*d^2*e^4*f*h^2 - 8*b^5*c*d*e^
5*f*g*h - 8*b*c^5*d^5*e*f*g*h - 8*a*b^4*c*e^6*f*g*h + 24*b^3*c^3*d^3*e^3*f*
g*h + 16*b^4*c^2*d^2*e^4*f*g*h + 16*b^2*c^4*d^4*e^2*f*g*h + 48*a^2*c^4*d^2*
e^4*f*g*h + 48*a^2*b^2*c^2*e^6*f*g*h + 40*a^3*b*c^2*d*e^5*g*h^2 + 28*a*b*c^
4*d^4*e^2*g^2*h - 8*a^2*b^3*c*d*e^5*g*h^2 - 8*a*b^4*c*d^2*e^4*g*h^2 + 96*a*
b^2*c^3*d*e^5*f^2*h + 24*a*b*c^4*d^2*e^4*f^2*h + 16*a*b*c^4*d^4*e^2*f*h^2 +
 96*a*b*c^4*d^2*e^4*f*g^2 - 48*a*b^2*c^3*d*e^5*f*g^2 + 12*a^2*b^2*c^2*d^2*e
^4*g*h^2 - 56*a*c^5*d^4*e^2*f*g*h - 8*a*b*c^4*d^5*e*g*h^2 + 4*a*b^4*c*d*e^5
*g^2*h + 16*a*b^4*c*d*e^5*f*h^2 - 48*a*b*c^4*d*e^5*f^2*g - 24*a^3*c^3*e^6*f
*g*h + 16*a*c^5*d^5*e*f*h^2 - 6*b^4*c^2*d^3*e^3*g^2*h - 6*b^3*c^3*d^4*e^2*g
^2*h + 4*b^4*c^2*d^4*e^2*g*h^2 + 80*a^2*c^4*d^3*e^3*g^2*h - 44*a^2*c^4*d^4*
e^2*g*h^2 + 24*a^3*c^3*d^2*e^4*g*h^2 - 16*b^3*c^3*d^2*e^4*f^2*h - 16*b^2*c^
4*d^3*e^3*f^2*h - 8*b^4*c^2*d^3*e^3*f*h^2 - 8*b^3*c^3*d^4*e^2*f*h^2 + 60*b^
2*c^4*d^2*e^4*f^2*g - 48*a^2*c^4*d^3*e^3*f*h^2 - 24*b^3*c^3*d^2*e^4*f*g^2 -
 24*b^2*c^4*d^3*e^3*f*g^2 - 24*a^3*b*c^2*d^2*e^4*h^3 + 24*a^2*b*c^3*d^4*e^2
*h^3 + 8*a^2*b^3*c*d^2*e^4*h^3 - 8*a*b^3*c^2*d^4*e^2*h^3 + 18*a*b^2*c^3*d^2
*e^4*g^3 + 2*b^5*c*d^2*e^4*g^2*h + 2*b^2*c^4*d^5*e*g^2*h - 48*a^3*c^3*d*e^5
*g^2*h - 8*b^4*c^2*d*e^5*f^2*h - 8*b*c^5*d^4*e^2*f^2*h - 168*a^2*c^4*d*e^5*
f^2*h + 96*a*c^5*d^3*e^3*f^2*h + 64*a^3*c^3*d*e^5*f*h^2 + 12*b^4*c^2*d*e^5*
f*g^2 + 12*b*c^5*d^4*e^2*f*g^2 - 168*a*c^5*d^2*e^4*f^2*g + 48*a^2*c^4*d*e^5
*f*g^2 + 48*a*c^5*d^3*e^3*f*g^2 - 12*a^3*b*c^2*e^6*g^2*h + 2*a^2*b^3*c*e^6*
g^2*h + 48*a^2*b*c^3*e^6*f^2*h - 48*a*b^3*c^2*e^6*f^2*h - 8*a^3*b*c^2*e^6*f
*h^2 - 60*a^2*b*c^3*e^6*f*g^2 + 48*a*b^2*c^3*e^6*f^2*g + 12*a*b^3*c^2*e^6*f
*g^2 + 24*a^2*b*c^3*d*e^5*g^3 - 24*a*b*c^4*d^3*e^3*g^3 - 6*a*b^3*c^2*d*e^5*
g^3 - 12*c^6*d^4*e^2*f^2*g + 4*a^4*c^2*e^6*g*h^2 - 12*b^4*c^2*e^6*f^2*g + 3
6*a^2*c^4*e^6*f^2*g - 8*a^4*c^2*d*e^5*h^3 + 8*a^2*c^4*d^5*e*h^3 - 24*b^2*c^
4*d*e^5*f^3 - 24*b*c^5*d^2*e^4*f^3 + 8*c^6*d^5*e*f^2*h + 8*b^5*c*e^6*f^2*h
+ 144*a*c^5*d*e^5*f^3 - 72*a*b*c^4*e^6*f^3 + 10*b^3*c^3*d^3*e^3*g^3 - 3*b^4
*c^2*d^2*e^4*g^3 - 3*b^2*c^4*d^4*e^2*g^3 - 48*a^2*c^4*d^2*e^4*g^3 - 3*a^2*b
^2*c^2*e^6*g^3 + 16*c^6*d^3*e^3*f^3 + 16*b^3*c^3*e^6*f^3 + 16*a^3*c^3*e^6*g
^3, z, k)*((a^5*b^5*c*e^11 + 16*a^7*b*c^3*e^11 - 128*a^7*c^4*d*e^10 + b^5*c
^6*d^10*e + b^10*c*d^5*e^6 - 8*a^6*b^3*c^2*e^11 - 128*a^3*c^8*d^9*e^2 - 512
*a^4*c^7*d^7*e^4 - 768*a^5*c^6*d^5*e^6 - 512*a^6*c^5*d^3*e^8 - 3*b^6*c^5*d^
9*e^2 + 2*b^7*c^4*d^8*e^3 + 2*b^8*c^3*d^7*e^4 - 3*b^9*c^2*d^6*e^5 + 16*a^2*
b^2*c^7*d^9*e^2 - 264*a^2*b^3*c^6*d^8*e^3 + 480*a^2*b^4*c^5*d^7*e^4 - 246*a
^2*b^5*c^4*d^6*e^5 - 66*a^2*b^6*c^3*d^5*e^6 + 62*a^2*b^7*c^2*d^4*e^7 - 704*
a^3*b^2*c^6*d^7*e^4 - 240*a^3*b^3*c^5*d^6*e^5 + 800*a^3*b^4*c^4*d^5*e^6 - 2
46*a^3*b^5*c^3*d^4*e^7 - 76*a^3*b^6*c^2*d^3*e^8 - 1440*a^4*b^2*c^5*d^5*e^6
- 240*a^4*b^3*c^4*d^4*e^7 + 480*a^4*b^4*c^3*d^3*e^8 + 21*a^4*b^5*c^2*d^2*e^
9 - 704*a^5*b^2*c^4*d^3*e^8 - 264*a^5*b^3*c^3*d^2*e^9 - 8*a*b^3*c^7*d^10*e
- 3*a*b^9*c*d^4*e^7 + 16*a^2*b*c^8*d^10*e - 3*a^4*b^6*c*d*e^10 + 16*a*b^4*c

^6*d^9*e^2 + 21*a*b^5*c^5*d^8*e^3 - 76*a*b^6*c^4*d^7*e^4 + 62*a*b^7*c^3*d^6 *e^5 - 12*a*b^8*c^2*d^5*e^6 + 2*a^2*b^8*c*d^3*e^8 + 592*a^3*b*c^7*d^8*e^3 + 2*a^3*b^7*c*d^2*e^9 + 1696*a^4*b*c^6*d^6*e^5 + 1696*a^5*b*c^5*d^4*e^7 + 16 *a^5*b^4*c^2*d*e^10 + 592*a^6*b*c^4*d^2*e^9 + 16*a^6*b^2*c^3*d*e^10)/(16*a^ 2*c^6*d^8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*a* b^2*c^5*d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5*c ^3*d^7*e - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96*a^ 4*c^4*d^4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d^6 *e^2 + 32*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3*d^ 4*e^4 + 32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^7* e + 4*a*b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5*b* c^2*d*e^7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^3* e^5 - 192*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^5) + (x*(2*a^4*b^6*c*e^11 - 96*a^7*c^4*e^11 + 32*a^2*c^9*d^10*e + 2*b^4*c^7*d ^10*e + 2*b^10*c*d^4*e^7 - 22*a^5*b^4*c^2*e^11 + 80*a^6*b^2*c^3*e^11 + 32*a ^3*c^8*d^8*e^3 - 192*a^4*c^7*d^6*e^5 - 448*a^5*c^6*d^4*e^7 - 352*a^6*c^5*d^ 2*e^9 - 10*b^5*c^6*d^9*e^2 + 22*b^6*c^5*d^8*e^3 - 28*b^7*c^4*d^7*e^4 + 22*b ^8*c^3*d^6*e^5 - 10*b^9*c^2*d^5*e^6 + 336*a^2*b^2*c^7*d^8*e^3 - 384*a^2*b^3 *c^6*d^7*e^4 + 180*a^2*b^4*c^5*d^6*e^5 + 132*a^2*b^5*c^4*d^5*e^6 - 200*a^2* b^6*c^3*d^4*e^7 + 52*a^2*b^7*c^2*d^3*e^8 + 416*a^3*b^2*c^6*d^6*e^5 - 800*a^ 3*b^3*c^5*d^5*e^6 + 580*a^3*b^4*c^4*d^4*e^7 + 24*a^3*b^5*c^3*d^3*e^8 - 116* a^3*b^6*c^2*d^2*e^9 - 160*a^4*b^2*c^5*d^4*e^7 - 640*a^4*b^3*c^4*d^3*e^8 + 3 30*a^4*b^4*c^3*d^2*e^9 - 144*a^5*b^2*c^4*d^2*e^9 - 16*a*b^2*c^8*d^10*e - 8* a*b^9*c*d^3*e^8 - 8*a^3*b^7*c*d*e^10 + 352*a^6*b*c^4*d*e^10 + 80*a*b^3*c^7* d^9*e^2 - 174*a*b^4*c^6*d^8*e^3 + 216*a*b^5*c^5*d^7*e^4 - 156*a*b^6*c^4*d^6 *e^5 + 48*a*b^7*c^3*d^5*e^6 + 10*a*b^8*c^2*d^4*e^7 - 160*a^2*b*c^8*d^9*e^2 + 12*a^2*b^8*c*d^2*e^9 - 128*a^3*b*c^7*d^7*e^4 + 576*a^4*b*c^6*d^5*e^6 + 86 *a^4*b^5*c^2*d*e^10 + 896*a^5*b*c^5*d^3*e^8 - 304*a^5*b^3*c^3*d*e^10))/(16* a^2*c^6*d^8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8* a*b^2*c^5*d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5 *c^3*d^7*e - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96* a^4*c^4*d^4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d ^6*e^2 + 32*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3* d^4*e^4 + 32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^ 7*e + 4*a*b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5* b*c^2*d*e^7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^ 3*e^5 - 192*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^ 5)) - (x*(48*a^5*c^4*e^9*g - 72*a^4*b*c^4*e^9*f + 16*a*c^8*d^7*e^2*f + 144* a^4*c^5*d*e^8*f - 8*a^5*b*c^3*e^9*h - 80*a^5*c^4*d*e^8*h - 4*a^2*b^5*c^2*e^ 9*f + 34*a^3*b^3*c^3*e^9*f + 2*a^3*b^4*c^2*e^9*g - 20*a^4*b^2*c^3*e^9*g + 1 76*a^2*c^7*d^5*e^4*f + 304*a^3*c^6*d^3*e^6*f + 2*a^4*b^3*c^2*e^9*h - 80*a^2 *c^7*d^6*e^3*g - 112*a^3*c^6*d^4*e^5*g + 16*a^4*c^5*d^2*e^7*g - 4*b^2*c^7*d ^7*e^2*f + 14*b^3*c^6*d^6*e^3*f - 10*b^4*c^5*d^5*e^4*f - 10*b^5*c^4*d^4*e^5 *f + 14*b^6*c^3*d^3*e^6*f - 4*b^7*c^2*d^2*e^7*f + 48*a^2*c^7*d^7*e^2*h + 16 *a^3*c^6*d^5*e^4*h - 112*a^4*c^5*d^3*e^6*h + 2*b^3*c^6*d^7*e^2*g - 12*b^4*c ^5*d^6*e^3*g + 20*b^5*c^4*d^5*e^4*g - 12*b^6*c^3*d^4*e^5*g + 2*b^7*c^2*d^3* e^6*g + 2*b^4*c^5*d^7*e^2*h - 2*b^5*c^4*d^6*e^3*h - 2*b^6*c^3*d^5*e^4*h + 2 *b^7*c^2*d^4*e^5*h - 4*a*b^2*c^6*d^5*e^4*f + 150*a*b^3*c^5*d^4*e^5*f - 128* a*b^4*c^4*d^3*e^6*f + 14*a*b^5*c^3*d^2*e^7*f - 440*a^2*b*c^6*d^4*e^5*f - 62 *a^2*b^4*c^3*d*e^8*f - 456*a^3*b*c^5*d^2*e^7*f + 84*a^3*b^2*c^4*d*e^8*f + 6 8*a*b^2*c^6*d^6*e^3*g - 118*a*b^3*c^5*d^5*e^4*g + 54*a*b^4*c^4*d^4*e^5*g + 6*a*b^5*c^3*d^3*e^6*g - 2*a*b^6*c^2*d^2*e^7*g + 152*a^2*b*c^6*d^5*e^4*g - 2 *a^2*b^5*c^2*d*e^8*g + 72*a^3*b*c^5*d^3*e^6*g + 30*a^3*b^3*c^3*d*e^8*g - 20 *a*b^2*c^6*d^7*e^2*h + 30*a*b^3*c^5*d^6*e^3*h - 4*a*b^4*c^4*d^5*e^4*h + 6*a *b^5*c^3*d^4*e^5*h - 12*a*b^6*c^2*d^3*e^6*h - 88*a^2*b*c^6*d^6*e^3*h + 72*a ^3*b*c^5*d^4*e^5*h - 12*a^3*b^4*c^2*d*e^8*h + 152*a^4*b*c^4*d^2*e^7*h + 68* a^4*b^2*c^3*d*e^8*h + 212*a^2*b^2*c^5*d^3*e^6*f + 122*a^2*b^3*c^4*d^2*e^7*f + 4*a^2*b^2*c^5*d^4*e^5*g - 74*a^2*b^3*c^4*d^3*e^6*g - 4*a^2*b^4*c^3*d^2*e ^7*g + 44*a^3*b^2*c^4*d^2*e^7*g + 44*a^2*b^2*c^5*d^5*e^4*h - 74*a^2*b^3*c^4

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*d^4*e^5*h + 54*a^2*b^4*c^3*d^3*e^6*h + 20*a^2*b^5*c^2*d^2*e^7*h + 4*a^3*b^
2*c^4*d^3*e^6*h - 118*a^3*b^3*c^3*d^2*e^7*h - 56*a*b*c^7*d^6*e^3*f + 8*a*b^
6*c^2*d*e^8*f - 8*a*b*c^7*d^7*e^2*g - 88*a^4*b*c^4*d*e^8*g))/(16*a^2*c^6*d^
8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*a*b^2*c^5*
d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5*c^3*d^7*e
 - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96*a^4*c^4*d^
4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d^6*e^2 + 3
2*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3*d^4*e^4 +
32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^7*e + 4*a*
b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5*b*c^2*d*e^
7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^3*e^5 - 19
2*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^5)) - (32*
a^2*c^5*d^3*e^4*g^2 - 4*c^7*d^5*e^2*f^2 - a^2*b^3*c^2*e^7*g^2 - 4*b^5*c^2*e
^7*f^2 - 4*b^2*c^5*d^3*e^4*f^2 - 4*b^3*c^4*d^2*e^5*f^2 + 12*a^2*c^5*d^5*e^2
*h^2 - 40*a^3*c^4*d^3*e^4*h^2 - b^2*c^5*d^5*e^2*g^2 + b^3*c^4*d^4*e^3*g^2 +
 b^4*c^3*d^3*e^4*g^2 - b^5*c^2*d^2*e^5*g^2 + 24*a^3*c^4*e^7*f*g - 8*a^4*c^3
*e^7*g*h + 28*a*b^3*c^3*e^7*f^2 - 48*a^2*b*c^4*e^7*f^2 + 4*a^3*b*c^3*e^7*g^
2 - 8*a*c^6*d^3*e^4*f^2 + 60*a^2*c^5*d*e^6*f^2 + 8*b*c^6*d^4*e^3*f^2 - 32*a
^3*c^4*d*e^6*g^2 + 8*b^4*c^3*d*e^6*f^2 + 12*a^4*c^3*d*e^6*h^2 + 24*a*b*c^5*
d^2*e^5*f^2 - 48*a*b^2*c^4*d*e^6*f^2 + 4*a*b*c^5*d^4*e^3*g^2 - 2*a*b^4*c^2*
d*e^6*g^2 - 22*a^2*b^2*c^3*e^7*f*g - 4*a^2*b^3*c^2*e^7*f*h - 112*a^2*c^5*d^
2*e^5*f*g + 2*a^3*b^2*c^2*e^7*g*h + 80*a^2*c^5*d^3*e^4*f*h - 6*b^2*c^5*d^4*
e^3*f*g + 4*b^3*c^4*d^3*e^4*f*g - 6*b^4*c^3*d^2*e^5*f*g - 40*a^2*c^5*d^4*e^
3*g*h + 80*a^3*c^4*d^2*e^5*g*h - 4*b^2*c^5*d^5*e^2*f*h + 4*b^3*c^4*d^4*e^3*
f*h + 4*b^4*c^3*d^3*e^4*f*h - 4*b^5*c^2*d^2*e^5*f*h + 2*b^3*c^4*d^5*e^2*g*h
 - 4*b^4*c^3*d^4*e^3*g*h + 2*b^5*c^2*d^3*e^4*g*h - 18*a*b^2*c^4*d^3*e^4*g^2
 + 12*a*b^3*c^3*d^2*e^5*g^2 - 24*a^2*b*c^4*d^2*e^5*g^2 + 15*a^2*b^2*c^3*d*e
^6*g^2 - 4*a*b^2*c^4*d^5*e^2*h^2 + 4*a*b^3*c^3*d^4*e^3*h^2 - 4*a*b^4*c^2*d^
3*e^4*h^2 - 8*a^2*b*c^4*d^4*e^3*h^2 - 8*a^3*b*c^3*d^2*e^5*h^2 - 4*a^3*b^2*c
^2*d*e^6*h^2 + 4*a*b^4*c^2*e^7*f*g + 16*a^3*b*c^3*e^7*f*h - 8*a*c^6*d^4*e^3
*f*g + 8*a*c^6*d^5*e^2*f*h + 4*b*c^6*d^5*e^2*f*g - 56*a^3*c^4*d*e^6*f*h + 4
*b^5*c^2*d*e^6*f*g + 20*a^2*b^2*c^3*d^3*e^4*h^2 + 4*a^2*b^3*c^2*d^2*e^5*h^2
 + 8*a*b*c^5*d^3*e^4*f*g - 40*a*b^3*c^3*d*e^6*f*g + 100*a^2*b*c^4*d*e^6*f*g
 - 4*a*b*c^5*d^5*e^2*g*h - 4*a^3*b*c^3*d*e^6*g*h + 44*a*b^2*c^4*d^2*e^5*f*g
 - 48*a*b^2*c^4*d^3*e^4*f*h + 32*a*b^3*c^3*d^2*e^5*f*h - 48*a^2*b*c^4*d^2*e
^5*f*h + 12*a^2*b^2*c^3*d*e^6*f*h + 18*a*b^2*c^4*d^4*e^3*g*h - 8*a*b^3*c^3*
d^3*e^4*g*h + 2*a*b^4*c^2*d^2*e^5*g*h + 24*a^2*b*c^4*d^3*e^4*g*h + 2*a^2*b^
3*c^2*d*e^6*g*h - 36*a^2*b^2*c^3*d^2*e^5*g*h)/(16*a^2*c^6*d^8 + a^4*b^4*e^8
 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*a*b^2*c^5*d^8 - 8*a^5*b^2
*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5*c^3*d^7*e - 4*b^7*c*d^5*
e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96*a^4*c^4*d^4*e^4 + 64*a^5*
c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d^6*e^2 + 32*a^2*b^3*c^3*d
^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3*d^4*e^4 + 32*a^3*b^3*c^2*
d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^7*e + 4*a*b^6*c*d^4*e^4 -
 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5*b*c^2*d*e^7 - 44*a*b^4*c^
3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^3*e^5 - 192*a^3*b*c^4*d^5
*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^5))*root(3840*a^6*b*c^5*d
^5*e^7*z^3 + 3840*a^5*b*c^6*d^7*e^5*z^3 + 1920*a^7*b*c^4*d^3*e^9*z^3 + 1920
*a^4*b*c^7*d^9*e^3*z^3 - 288*a^7*b^3*c^2*d*e^11*z^3 - 288*a^2*b^3*c^7*d^11*
e*z^3 + 210*a^4*b^7*c*d^3*e^9*z^3 + 210*a*b^7*c^4*d^9*e^3*z^3 - 174*a^5*b^6
*c*d^2*e^10*z^3 - 174*a*b^6*c^5*d^10*e^2*z^3 - 120*a^3*b^8*c*d^4*e^8*z^3 -
120*a*b^8*c^3*d^8*e^4*z^3 + 12*a^2*b^9*c*d^5*e^7*z^3 + 12*a*b^9*c^2*d^7*e^5
*z^3 + 384*a^8*b*c^3*d*e^11*z^3 + 384*a^3*b*c^8*d^11*e*z^3 + 72*a^6*b^5*c*d
*e^11*z^3 + 72*a*b^5*c^6*d^11*e*z^3 + 18*a*b^10*c*d^6*e^6*z^3 - 4800*a^5*b^
2*c^5*d^6*e^6*z^3 - 3120*a^6*b^2*c^4*d^4*e^8*z^3 - 3120*a^4*b^2*c^6*d^8*e^4
*z^3 + 2160*a^4*b^4*c^4*d^6*e^6*z^3 - 1776*a^4*b^5*c^3*d^5*e^7*z^3 - 1776*a
^3*b^5*c^4*d^7*e^5*z^3 + 1740*a^5*b^4*c^3*d^4*e^8*z^3 + 1740*a^3*b^4*c^5*d^
8*e^4*z^3 + 960*a^5*b^3*c^4*d^5*e^7*z^3 + 960*a^4*b^3*c^5*d^7*e^5*z^3 - 672
*a^7*b^2*c^3*d^2*e^10*z^3 - 672*a^3*b^2*c^7*d^10*e^2*z^3 + 648*a^6*b^4*c^2*
```

d^2*e^10*z^3 + 648*a^2*b^4*c^6*d^10*e^2*z^3 - 600*a^5*b^5*c^2*d^3*e^9*z^3 -
 600*a^2*b^5*c^5*d^9*e^3*z^3 + 372*a^3*b^7*c^2*d^5*e^7*z^3 + 372*a^2*b^7*c^
3*d^7*e^5*z^3 + 316*a^3*b^6*c^3*d^6*e^6*z^3 - 222*a^2*b^8*c^2*d^6*e^6*z^3 -
 160*a^6*b^3*c^3*d^3*e^9*z^3 - 160*a^3*b^3*c^6*d^9*e^3*z^3 + 15*a^4*b^6*c^2
*d^4*e^8*z^3 + 15*a^2*b^6*c^4*d^8*e^4*z^3 - 6*b^11*c*d^7*e^5*z^3 - 6*b^7*c^
5*d^11*e*z^3 - 6*a^5*b^7*d*e^11*z^3 - 6*a*b^11*d^5*e^7*z^3 - 12*a^7*b^4*c*e
^12*z^3 - 12*a*b^4*c^7*d^12*z^3 - 20*b^9*c^3*d^9*e^3*z^3 + 15*b^10*c^2*d^8*
e^4*z^3 + 15*b^8*c^4*d^10*e^2*z^3 - 1280*a^6*c^6*d^6*e^6*z^3 - 960*a^7*c^5*
d^4*e^8*z^3 - 960*a^5*c^7*d^8*e^4*z^3 - 384*a^8*c^4*d^2*e^10*z^3 - 384*a^4*
c^8*d^10*e^2*z^3 - 20*a^3*b^9*d^3*e^9*z^3 + 15*a^4*b^8*d^2*e^10*z^3 + 15*a^
2*b^10*d^4*e^8*z^3 + 48*a^8*b^2*c^2*e^12*z^3 + 48*a^2*b^2*c^8*d^12*z^3 - 64
*a^9*c^3*e^12*z^3 - 64*a^3*c^9*d^12*z^3 + b^12*d^6*e^6*z^3 + b^6*c^6*d^12*z
^3 + a^6*b^6*e^12*z^3 - 44*a^3*b^4*c*d*e^7*g*h*z - 20*a*b^6*c*d^3*e^5*g*h*z
 - 12*a*b^2*c^5*d^7*e*g*h*z + 432*a^4*b*c^3*d*e^7*f*h*z + 84*a^2*b^5*c*d*e^
7*f*h*z + 28*a*b^6*c*d^2*e^6*f*h*z - 8*a*b*c^6*d^6*e^2*f*g*z - 804*a^3*b^2*
c^3*d^3*e^5*g*h*z + 564*a^2*b^2*c^4*d^5*e^3*g*h*z + 222*a^3*b^3*c^2*d^2*e^6
*g*h*z + 186*a^2*b^4*c^2*d^3*e^5*g*h*z - 166*a^2*b^3*c^3*d^4*e^4*g*h*z + 79
2*a^3*b^2*c^3*d^2*e^6*f*h*z - 744*a^2*b^2*c^4*d^4*e^4*f*h*z + 492*a^2*b^3*c
^3*d^3*e^5*f*h*z - 264*a^2*b^4*c^2*d^2*e^6*f*h*z + 996*a^2*b^2*c^4*d^3*e^5*
f*g*z - 870*a^2*b^3*c^3*d^2*e^6*f*g*z + 16*a*b*c^6*d^7*e*f*h*z - 56*a*b^6*c
*d*e^7*f*g*z - 264*a^4*b*c^3*d^2*e^6*g*h*z + 208*a^3*b*c^4*d^4*e^4*g*h*z +
156*a^4*b^2*c^2*d*e^7*g*h*z - 148*a*b^4*c^3*d^5*e^3*g*h*z + 54*a*b^5*c^2*d^
4*e^4*g*h*z - 48*a^2*b^5*c*d^2*e^6*g*h*z - 24*a^2*b*c^5*d^6*e^2*g*h*z + 10*
a*b^3*c^4*d^6*e^2*g*h*z - 656*a^3*b*c^4*d^3*e^5*f*h*z - 308*a^3*b^3*c^2*d*e
^7*f*h*z + 116*a*b^4*c^3*d^4*e^4*f*h*z - 84*a*b^5*c^2*d^3*e^5*f*h*z + 68*a*
b^3*c^4*d^5*e^3*f*h*z - 48*a^2*b*c^5*d^5*e^3*f*h*z - 24*a*b^2*c^5*d^6*e^2*f
*h*z + 1320*a^3*b*c^4*d^2*e^6*f*g*z - 732*a^3*b^2*c^3*d*e^7*f*g*z + 306*a^2
*b^4*c^2*d*e^7*f*g*z - 304*a*b^4*c^3*d^3*e^5*f*g*z + 222*a*b^5*c^2*d^2*e^6*
f*g*z + 110*a*b^3*c^4*d^4*e^4*f*g*z - 84*a*b^2*c^5*d^5*e^3*f*g*z + 16*a*c^7
*d^7*e*f*g*z - 8*a*b^7*d*e^7*f*h*z + 4*a*b*c^6*d^8*g*h*z + 6*b^6*c^2*d^5*e^
3*g*h*z + 6*b^5*c^3*d^6*e^2*g*h*z + 1072*a^4*c^4*d^3*e^5*g*h*z - 720*a^3*c^
5*d^5*e^3*g*h*z - 8*b^6*c^2*d^4*e^4*f*h*z - 8*b^4*c^4*d^6*e^2*f*h*z + 1072*
a^3*c^5*d^4*e^4*f*h*z - 960*a^4*c^4*d^2*e^6*f*h*z + 30*b^6*c^2*d^3*e^5*f*g*
z + 30*b^3*c^5*d^6*e^2*f*g*z - 10*b^5*c^3*d^4*e^4*f*g*z - 10*b^4*c^4*d^5*e^
3*f*g*z - 1488*a^3*c^5*d^3*e^5*f*g*z + 48*a^2*c^6*d^5*e^3*f*g*z - 24*a^4*b^
2*c^2*e^8*f*h*z + 186*a^3*b^3*c^2*e^8*f*g*z + 4*a^4*b^3*c*d*e^7*h^2*z + 4*a
*b^6*c*d^4*e^4*h^2*z + 4*a*b^3*c^4*d^7*e*h^2*z + 168*a^4*b*c^3*d*e^7*g^2*z
+ 24*a^2*b^5*c*d*e^7*g^2*z + 18*a*b^6*c*d^2*e^6*g^2*z - 912*a^3*b*c^4*d*e^7
*f^2*z - 192*a*b^5*c^2*d*e^7*f^2*z + 144*a*b*c^6*d^5*e^3*f^2*z + 432*a^3*b^
2*c^3*d^4*e^4*h^2*z - 168*a^4*b^2*c^2*d^2*e^6*h^2*z - 168*a^2*b^2*c^4*d^6*e
^2*h^2*z - 108*a^2*b^4*c^2*d^4*e^4*h^2*z - 20*a^3*b^3*c^2*d^3*e^5*h^2*z - 2
0*a^2*b^3*c^3*d^5*e^3*h^2*z - 426*a^2*b^2*c^4*d^4*e^4*g^2*z + 336*a^3*b^2*c
^3*d^2*e^6*g^2*z + 274*a^2*b^3*c^3*d^3*e^5*g^2*z - 120*a^2*b^4*c^2*d^2*e^6*
g^2*z - 864*a^2*b^2*c^4*d^2*e^6*f^2*z - 2*b^7*c*d^4*e^4*g*h*z - 2*b^4*c^4*d
^7*e*g*h*z - 240*a^5*c^3*d*e^7*g*h*z + 16*a^2*c^6*d^7*e*g*h*z + 4*b^7*c*d^3
*e^5*f*h*z + 4*b^3*c^5*d^7*e*f*h*z - 20*b^7*c*d^2*e^6*f*g*z - 20*b^2*c^6*d^
7*e*f*g*z + 4*a^2*b^6*d*e^7*g*h*z + 4*a*b^7*d^2*e^6*g*h*z + 528*a^4*c^4*d*e
^7*f*g*z + 12*a^5*b*c^2*e^8*g*h*z - 2*a^4*b^3*c*e^8*g*h*z + 4*a^3*b^4*c*e^8
*f*h*z - 228*a^4*b*c^3*e^8*f*g*z - 48*a^2*b^5*c*e^8*f*g*z - 8*a*b*c^6*d^7*e
*g^2*z + 36*a^3*b^4*c*d^2*e^6*h^2*z + 36*a*b^4*c^3*d^6*e^2*h^2*z + 12*a^2*b
^5*c*d^3*e^5*h^2*z + 12*a*b^5*c^2*d^5*e^3*h^2*z - 312*a^3*b*c^4*d^3*e^5*g^2
*z + 104*a*b^4*c^3*d^4*e^4*g^2*z - 102*a^3*b^3*c^2*d*e^7*g^2*z - 66*a*b^5*c
^2*d^3*e^5*g^2*z + 24*a^2*b*c^5*d^5*e^3*g^2*z + 24*a*b^2*c^5*d^6*e^2*g^2*z
- 18*a*b^3*c^4*d^5*e^3*g^2*z + 744*a^2*b^3*c^3*d*e^7*f^2*z + 240*a^2*b*c^5*
d^3*e^5*f^2*z + 216*a*b^4*c^3*d^2*e^6*f^2*z - 120*a*b^2*c^5*d^4*e^4*f^2*z +
 24*a^5*c^3*e^8*f*h*z + 16*b^7*c*d*e^7*f^2*z + 16*b*c^7*d^7*e*f^2*z - 2*a*b
^7*d*e^7*g^2*z + 48*a*b^6*c*e^8*f^2*z - 4*b^6*c^2*d^6*e^2*h^2*z - 536*a^4*c
^4*d^4*e^4*h^2*z + 240*a^5*c^3*d^2*e^6*h^2*z + 240*a^3*c^5*d^6*e^2*h^2*z -
12*b^6*c^2*d^4*e^4*g^2*z - 12*b^4*c^4*d^6*e^2*g^2*z + 10*b^5*c^3*d^5*e^3*g^

```
2*z + 528*a^3*c^5*d^4*e^4*g^2*z - 432*a^4*c^4*d^2*e^6*g^2*z + 20*b^4*c^4*d^
4*e^4*f^2*z - 16*b^6*c^2*d^2*e^6*f^2*z - 16*b^2*c^6*d^6*e^2*f^2*z - 16*a^2*
c^6*d^6*e^2*g^2*z - 8*b^5*c^3*d^3*e^5*f^2*z - 8*b^3*c^5*d^5*e^3*f^2*z - 4*a
^2*b^6*d^2*e^6*h^2*z + 912*a^3*c^5*d^2*e^6*f^2*z - 120*a^2*c^6*d^4*e^4*f^2*
z - 45*a^4*b^2*c^2*e^8*g^2*z + 264*a^3*b^2*c^3*e^8*f^2*z - 192*a^2*b^4*c^2*
e^8*f^2*z + 4*b^8*d*e^7*f*g*z - 8*a*c^7*d^8*f*h*z + 4*b*c^7*d^8*f*g*z + 4*a
*b^7*e^8*f*g*z + 6*b^7*c*d^3*e^5*g^2*z + 6*b^3*c^5*d^7*e*g^2*z - 48*a*c^7*d
^6*e^2*f^2*z + 12*a^3*b^4*c*e^8*g^2*z - b^8*d^2*e^6*g^2*z - 4*a^6*c^2*e^8*h
^2*z + 48*a^5*c^3*e^8*g^2*z - 4*a^2*c^6*d^8*h^2*z - b^2*c^6*d^8*g^2*z - 36*
a^4*c^4*e^8*f^2*z - a^2*b^6*e^8*g^2*z - 4*c^8*d^8*f^2*z - 4*b^8*e^8*f^2*z -
 80*a*b*c^4*d^3*e^3*f*g*h + 24*a^2*b*c^3*d*e^5*f*g*h + 16*a*b^3*c^2*d*e^5*f
*g*h - 72*a*b^2*c^3*d^2*e^4*f*g*h - 48*a^2*b*c^3*d^3*e^3*g*h^2 + 16*a*b^3*c
^2*d^3*e^3*g*h^2 - 12*a*b^2*c^3*d^3*e^3*g^2*h - 6*a^2*b^2*c^2*d*e^5*g^2*h -
 72*a^2*b^2*c^2*d*e^5*f*h^2 + 48*a*b^2*c^3*d^3*e^3*f*h^2 + 24*a^2*b*c^3*d^2
*e^4*f*h^2 - 8*a*b^3*c^2*d^2*e^4*f*h^2 - 8*b^5*c*d*e^5*f*g*h - 8*b*c^5*d^5*
e*f*g*h - 8*a*b^4*c*e^6*f*g*h + 24*b^3*c^3*d^3*e^3*f*g*h + 16*b^4*c^2*d^2*e
^4*f*g*h + 16*b^2*c^4*d^4*e^2*f*g*h + 48*a^2*c^4*d^2*e^4*f*g*h + 48*a^2*b^2
*c^2*e^6*f*g*h + 40*a^3*b*c^2*d*e^5*g*h^2 + 28*a*b*c^4*d^4*e^2*g^2*h - 8*a^
2*b^3*c*d*e^5*g*h^2 - 8*a*b^4*c*d^2*e^4*g*h^2 + 96*a*b^2*c^3*d*e^5*f^2*h +
24*a*b*c^4*d^2*e^4*f^2*h + 16*a*b*c^4*d^4*e^2*f*h^2 + 96*a*b*c^4*d^2*e^4*f*
g^2 - 48*a*b^2*c^3*d*e^5*f*g^2 + 12*a^2*b^2*c^2*d^2*e^4*g*h^2 - 56*a*c^5*d^
4*e^2*f*g*h - 8*a*b*c^4*d^5*e*g*h^2 + 4*a*b^4*c*d*e^5*g^2*h + 16*a*b^4*c*d*
e^5*f*h^2 - 48*a*b*c^4*d*e^5*f^2*g - 24*a^3*c^3*e^6*f*g*h + 16*a*c^5*d^5*e*
f*h^2 - 6*b^4*c^2*d^3*e^3*g^2*h - 6*b^3*c^3*d^4*e^2*g^2*h + 4*b^4*c^2*d^4*e
^2*g*h^2 + 80*a^2*c^4*d^3*e^3*g^2*h - 44*a^2*c^4*d^4*e^2*g*h^2 + 24*a^3*c^3
*d^2*e^4*g*h^2 - 16*b^3*c^3*d^2*e^4*f^2*h - 16*b^2*c^4*d^3*e^3*f^2*h - 8*b^
4*c^2*d^3*e^3*f*h^2 - 8*b^3*c^3*d^4*e^2*f*h^2 + 60*b^2*c^4*d^2*e^4*f^2*g -
48*a^2*c^4*d^3*e^3*f*h^2 - 24*b^3*c^3*d^2*e^4*f*g^2 - 24*b^2*c^4*d^3*e^3*f*
g^2 - 24*a^3*b*c^2*d^2*e^4*h^3 + 24*a^2*b*c^3*d^4*e^2*h^3 + 8*a^2*b^3*c*d^2
*e^4*h^3 - 8*a*b^3*c^2*d^4*e^2*h^3 + 18*a*b^2*c^3*d^2*e^4*g^3 + 2*b^5*c*d^2
*e^4*g^2*h + 2*b^2*c^4*d^5*e*g^2*h - 48*a^3*c^3*d*e^5*g^2*h - 8*b^4*c^2*d*e
^5*f^2*h - 8*b*c^5*d^4*e^2*f^2*h - 168*a^2*c^4*d*e^5*f^2*h + 96*a*c^5*d^3*e
^3*f^2*h + 64*a^3*c^3*d*e^5*f*h^2 + 12*b^4*c^2*d*e^5*f*g^2 + 12*b*c^5*d^4*e
^2*f*g^2 - 168*a*c^5*d^2*e^4*f^2*g + 48*a^2*c^4*d*e^5*f*g^2 + 48*a*c^5*d^3*
e^3*f*g^2 - 12*a^3*b*c^2*e^6*g^2*h + 2*a^2*b^3*c*e^6*g^2*h + 48*a^2*b*c^3*e
^6*f^2*h - 48*a*b^3*c^2*e^6*f^2*h - 8*a^3*b*c^2*e^6*f*h^2 - 60*a^2*b*c^3*e^
6*f*g^2 + 48*a*b^2*c^3*e^6*f^2*g + 12*a*b^3*c^2*e^6*f*g^2 + 24*a^2*b*c^3*d*
e^5*g^3 - 24*a*b*c^4*d^3*e^3*g^3 - 6*a*b^3*c^2*d*e^5*g^3 - 12*c^6*d^4*e^2*f
^2*g + 4*a^4*c^2*e^6*g*h^2 - 12*b^4*c^2*e^6*f^2*g + 36*a^2*c^4*e^6*f^2*g -
8*a^4*c^2*d*e^5*h^3 + 8*a^2*c^4*d^5*e*h^3 - 24*b^2*c^4*d*e^5*f^3 - 24*b*c^5
*d^2*e^4*f^3 + 8*c^6*d^5*e*f^2*h + 8*b^5*c*e^6*f^2*h + 144*a*c^5*d*e^5*f^3
- 72*a*b*c^4*e^6*f^3 + 10*b^3*c^3*d^3*e^3*g^3 - 3*b^4*c^2*d^2*e^4*g^3 - 3*b
^2*c^4*d^4*e^2*g^3 - 48*a^2*c^4*d^2*e^4*g^3 - 3*a^2*b^2*c^2*e^6*g^3 + 16*c^
6*d^3*e^3*f^3 + 16*b^3*c^3*e^6*f^3 + 16*a^3*c^3*e^6*g^3, z, k), k, 1, 3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)**2/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

3.160 $\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx$

Optimal. Leaf size=62

$$\frac{x^2}{2}+\frac{2(2-x)}{3(x^2-x+1)}+2\log\left(x^2-x+1\right)+3x+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 3*x+1/2*x^2+2/3*(2-x)/(x^2-x+1)+2*ln(x^2-x+1)+10/9*arctan(1/3*(1-2*x)*3^(1/
2))*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{x^2}{2}+\frac{2(2-x)}{3(x^2-x+1)}+2\log\left(x^2-x+1\right)+3x+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] 3*x + x^2/2 + (2*(2 - x))/(3*(1 - x + x^2)) + (10*ArcTan[(1 - 2*x)/Sqrt[3]]
)/(3*Sqrt[3]) + 2*Log[1 - x + x^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(1+x+x^2\right)}{\left(1-x+x^2\right)^2}\,dx &= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{-2+6x+6x^2+3x^3}{1-x+x^2}\,dx\\
&= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(9+3x-\frac{11-12x}{1-x+x^2}\right)dx\\
&= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{1}{3}\int \frac{11-12x}{1-x+x^2}\,dx\\
&= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{5}{3}\int \frac{1}{1-x+x^2}\,dx+2\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+2\log\left(1-x+x^2\right)+\frac{10}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+2\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.97

$$\frac{x^2}{2}-\frac{2(x-2)}{3\left(x^2-x+1\right)}+2\log\left(x^2-x+1\right)+3x-\frac{10\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + x + x^2))/(1 - x + x^2)^2,x]
[Out] 3*x + x^2/2 - (2*(-2 + x))/(3*(1 - x + x^2)) - (10*ArcTan[(-1 + 2*x)/Sqrt[3
]])/(3*Sqrt[3]) + 2*Log[1 - x + x^2]
```

fricas [A] time = 1.36, size = 75, normalized size = 1.21

$$\frac{9\,x^4+45\,x^3-20\,\sqrt{3}\left(x^2-x+1\right)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-45\,x^2+36\left(x^2-x+1\right)\log\left(x^2-x+1\right)+42\,x+2}{18\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
[Out] 1/18*(9*x^4 + 45*x^3 - 20*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1
)) - 45*x^2 + 36*(x^2 - x + 1)*log(x^2 - x + 1) + 42*x + 24)/(x^2 - x + 1)
```

giac [A] time = 0.16, size = 51, normalized size = 0.82

$$\frac{1}{2}\,x^2-\frac{10}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+3\,x-\frac{2\,(x-2)}{3\left(x^2-x+1\right)}+2\,\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 2/3*(x - 2)/(x
^2 - x + 1) + 2*log(x^2 - x + 1)
```

maple [A] time = 0.01, size = 53, normalized size = 0.85

$$\frac{x^2}{2}+3x-\frac{10\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+2\ln\left(x^2-x+1\right)+\frac{-\frac{2x}{3}+\frac{4}{3}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+x+1)/(x^2-x+1)^2,x)
```

```
[Out] 1/2*x^2+3*x+(-2/3*x+4/3)/(x^2-x+1)+2*ln(x^2-x+1)-10/9*3^(1/2)*arctan(1/3*(2
*x-1)*3^(1/2))
```

maxima [A] time = 0.96, size = 51, normalized size = 0.82

$$\frac{1}{2}x^2-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+3x-\frac{2(x-2)}{3\left(x^2-x+1\right)}+2\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 2/3*(x - 2)/(x
^2 - x + 1) + 2*log(x^2 - x + 1)
```

mupad [B] time = 0.04, size = 55, normalized size = 0.89

$$3x+2\ln\left(x^2-x+1\right)-\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}-\frac{10\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

```
[Out] 3*x + 2*log(x^2 - x + 1) - ((2*x)/3 - 4/3)/(x^2 - x + 1) - (10*3^(1/2)*atan
((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9 + x^2/2
```

sympy [A] time = 0.16, size = 60, normalized size = 0.97

$$\frac{x^2}{2}+3x+\frac{4-2x}{3x^2-3x+3}+2\log\left(x^2-x+1\right)-\frac{10\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] x**2/2 + 3*x + (4 - 2*x)/(3*x**2 - 3*x + 3) + 2*log(x**2 - x + 1) - 10*sqrt
(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

3.161 $\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2} dx$

Optimal. Leaf size=55

$$\frac{2(1-2x)}{3(x^2-x+1)}+\frac{3}{2}\log\left(x^2-x+1\right)+x-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] x+2/3*(1-2*x)/(x^2-x+1)+3/2*ln(x^2-x+1)-7/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(
1/2)
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{2(1-2x)}{3(x^2-x+1)}+\frac{3}{2}\log\left(x^2-x+1\right)+x-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] x + (2*(1 - 2*x))/(3*(1 - x + x^2)) - (7*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt
[3]) + (3*Log[1 - x + x^2])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(1+x+x^2\right)}{\left(1-x+x^2\right)^2}\,dx &= \frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{2+6x+3x^2}{1-x+x^2}\,dx\\
&= \frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(3-\frac{1-9x}{1-x+x^2}\right)dx\\
&= x+\frac{2(1-2x)}{3\left(1-x+x^2\right)}-\frac{1}{3}\int \frac{1-9x}{1-x+x^2}\,dx\\
&= x+\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{7}{6}\int \frac{1}{1-x+x^2}\,dx+\frac{3}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= x+\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{3}{2}\log\left(1-x+x^2\right)-\frac{7}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= x+\frac{2(1-2x)}{3\left(1-x+x^2\right)}-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{3}{2}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 1.00

$$-\frac{2(2x-1)}{3\left(x^2-x+1\right)}+\frac{3}{2}\log\left(x^2-x+1\right)+x+\frac{7\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] x - (2*(-1 + 2*x))/(3*(1 - x + x^2)) + (7*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sq
rt[3]) + (3*Log[1 - x + x^2])/2
```

fricas [A] time = 0.81, size = 70, normalized size = 1.27

$$\frac{18\,x^3+14\,\sqrt{3}\left(x^2-x+1\right)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-18\,x^2+27\left(x^2-x+1\right)\log\left(x^2-x+1\right)-6\,x+12}{18\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(18*x^3 + 14*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) - 18*
x^2 + 27*(x^2 - x + 1)*log(x^2 - x + 1) - 6*x + 12)/(x^2 - x + 1)
```

giac [A] time = 0.16, size = 46, normalized size = 0.84

$$\frac{7}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+x-\frac{2\,(2\,x-1)}{3\left(x^2-x+1\right)}+\frac{3}{2}\,\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 2/3*(2*x - 1)/(x^2 - x + 1)
 + 3/2*log(x^2 - x + 1)
```

maple [A] time = 0.01, size = 46, normalized size = 0.84

$$x+\frac{7\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+\frac{3\ln\left(x^2-x+1\right)}{2}+\frac{-\frac{4x}{3}+\frac{2}{3}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2+x+1)/(x^2-x+1)^2,x)
```

```
[Out] x+(-4/3*x+2/3)/(x^2-x+1)+3/2*ln(x^2-x+1)+7/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(
1/2))
```

maxima [A] time = 0.95, size = 46, normalized size = 0.84

$$\frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{2(2x-1)}{3\left(x^2-x+1\right)}+\frac{3}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 2/3*(2*x - 1)/(x^2 - x + 1)
 + 3/2*log(x^2 - x + 1)
```

mupad [B] time = 0.04, size = 48, normalized size = 0.87

$$x+\frac{3\ln\left(x^2-x+1\right)}{2}-\frac{\frac{4x}{3}-\frac{2}{3}}{x^2-x+1}+\frac{7\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

```
[Out] x + (3*log(x^2 - x + 1))/2 - ((4*x)/3 - 2/3)/(x^2 - x + 1) + (7*3^(1/2)*ata
n((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9
```

sympy [A] time = 0.15, size = 54, normalized size = 0.98

$$x+\frac{2-4x}{3x^2-3x+3}+\frac{3\log\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] x + (2 - 4*x)/(3*x**2 - 3*x + 3) + 3*log(x**2 - x + 1)/2 + 7*sqrt(3)*atan(2
*sqrt(3)*x/3 - sqrt(3)/3)/9
```

3.162 $\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx$

Optimal. Leaf size=52

$$-\frac{2(x+1)}{3\left(x^2-x+1\right)}+\frac{1}{2}\log\left(x^2-x+1\right)-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -2/3*(1+x)/(x^2-x+1)+1/2*ln(x^2-x+1)-11/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/
2)
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1660, 634, 618, 204, 628}

$$-\frac{2(x+1)}{3\left(x^2-x+1\right)}+\frac{1}{2}\log\left(x^2-x+1\right)-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] (-2*(1 + x))/(3*(1 - x + x^2)) - (11*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3])
 + Log[1 - x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
```

```
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(1+x+x^2\right)}{\left(1-x+x^2\right)^2}\,dx &= -\frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{4+3x}{1-x+x^2}\,dx\\
&= -\frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{11}{6}\int \frac{1}{1-x+x^2}\,dx\\
&= -\frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{2}\log\left(1-x+x^2\right)-\frac{11}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= -\frac{2(1+x)}{3\left(1-x+x^2\right)}-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 1.00

$$-\frac{2(x+1)}{3\left(x^2-x+1\right)}+\frac{1}{2}\log\left(x^2-x+1\right)+\frac{11\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] (-2*(1 + x))/(3*(1 - x + x^2)) + (11*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]
) + Log[1 - x + x^2]/2
```

fricas [A] time = 1.35, size = 60, normalized size = 1.15

$$\frac{22\sqrt{3}\left(x^2-x+1\right)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+9\left(x^2-x+1\right)\log\left(x^2-x+1\right)-12x-12}{18\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(22*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9*(x^2 - x +
 1)*log(x^2 - x + 1) - 12*x - 12)/(x^2 - x + 1)
```

giac [A] time = 0.15, size = 43, normalized size = 0.83

$$\frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-\frac{2(x+1)}{3\left(x^2-x+1\right)}+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 2/3*(x + 1)/(x^2 - x + 1) + 1/
2*log(x^2 - x + 1)
```

maple [A] time = 0.00, size = 45, normalized size = 0.87

$$\frac{11\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+\frac{\ln\left(x^2-x+1\right)}{2}+\frac{-\frac{2x}{3}-\frac{2}{3}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+x+1)/(x^2-x+1)^2,x)
```

```
[Out] (-2/3*x-2/3)/(x^2-x+1)+1/2*ln(x^2-x+1)+11/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))
```

maxima [A] time = 0.96, size = 43, normalized size = 0.83

$$\frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{2(x+1)}{3\left(x^2-x+1\right)}+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 2/3*(x + 1)/(x^2 - x + 1) + 1/
2*log(x^2 - x + 1)
```

mupad [B] time = 3.84, size = 59, normalized size = 1.13

$$\frac{\ln\left(x^2-x+1\right)}{2}-\frac{2x}{3\left(x^2-x+1\right)}-\frac{2}{3\left(x^2-x+1\right)}+\frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

```
[Out] log(x^2 - x + 1)/2 - (2*x)/(3*(x^2 - x + 1)) - 2/(3*(x^2 - x + 1)) + (11*3^
(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9
```

sympy [A] time = 0.15, size = 53, normalized size = 1.02

$$\frac{-2x-2}{3x^2-3x+3}+\frac{\log\left(x^2-x+1\right)}{2}+\frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] (-2*x - 2)/(3*x**2 - 3*x + 3) + log(x**2 - x + 1)/2 + 11*sqrt(3)*atan(2*sqr
t(3)*x/3 - sqrt(3)/3)/9
```

3.163 $\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2} \, dx$

Optimal. Leaf size=41

$$-\frac{2(2-x)}{3\left(x^2-x+1\right)}-\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -2/3*(2-x)/(x^2-x+1)-10/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1660, 12, 618, 204}

$$-\frac{2(2-x)}{3\left(x^2-x+1\right)}-\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(1 - x + x^2)^2,x]
```

```
[Out] (-2*(2 - x))/(3*(1 - x + x^2)) - (10*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2}\,dx &= -\frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{5}{1-x+x^2}\,dx\\ &= -\frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{5}{3}\int \frac{1}{1-x+x^2}\,dx\\ &= -\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{10}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= -\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.95

$$\frac{2(x-2)}{3\left(x^2-x+1\right)}+\frac{10\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(1 - x + x^2)^2,x]
```

```
[Out] (2*(-2 + x))/(3*(1 - x + x^2)) + (10*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]
)
```

fricas [A] time = 1.27, size = 41, normalized size = 1.00

$$\frac{2\left(5\sqrt{3}\left(x^2-x+1\right)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+3x-6\right)}{9\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 2/9*(5*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 6)/(x^2
- x + 1)
```

giac [A] time = 0.16, size = 32, normalized size = 0.78

$$\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{2(x-2)}{3\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(x - 2)/(x^2 - x + 1)
```

maple [A] time = 0.00, size = 34, normalized size = 0.83

$$\frac{10\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/(x^2-x+1)^2,x)
```

```
[Out] (2/3*x-4/3)/(x^2-x+1)+10/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 0.95, size = 32, normalized size = 0.78

$$\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{2(x-2)}{3(x^2-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
[Out] 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(x - 2)/(x^2 - x + 1)
```

mupad [B] time = 3.83, size = 35, normalized size = 0.85

$$\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}+\frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^2 - x + 1)^2,x)
[Out] ((2*x)/3 - 4/3)/(x^2 - x + 1) + (10*3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/
3))/9
```

sympy [A] time = 0.14, size = 41, normalized size = 1.00

$$\frac{2x-4}{3x^2-3x+3}+\frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(x**2-x+1)**2,x)
[Out] (2*x - 4)/(3*x**2 - 3*x + 3) + 10*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

3.164 $\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{2(1-2x)}{3\left(x^2-x+1\right)}-\frac{1}{2}\log\left(x^2-x+1\right)+\log(x)-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -2/3*(1-2*x)/(x^2-x+1)+ln(x)-1/2*ln(x^2-x+1)-11/9*arctan(1/3*(1-2*x)*3^(1/2
))*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1646, 800, 634, 618, 204, 628}

$$-\frac{2(1-2x)}{3\left(x^2-x+1\right)}-\frac{1}{2}\log\left(x^2-x+1\right)+\log(x)-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x*(1 - x + x^2)^2),x]
```

```
[Out] (-2*(1 - 2*x))/(3*(1 - x + x^2)) - (11*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3
]) + Log[x] - Log[1 - x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx &= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{3+4x}{x\left(1-x+x^2\right)}\,dx \\
&= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(\frac{3}{x}+\frac{7-3x}{1-x+x^2}\right)dx \\
&= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\log(x)+\frac{1}{3}\int \frac{7-3x}{1-x+x^2}\,dx \\
&= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\log(x)-\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{11}{6}\int \frac{1}{1-x+x^2}\,dx \\
&= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\log(x)-\frac{1}{2}\log\left(1-x+x^2\right)-\frac{11}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right) \\
&= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{2}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 1.00

$$\frac{2(2x-1)}{3\left(x^2-x+1\right)}-\frac{1}{2}\log\left(x^2-x+1\right)+\log(x)+\frac{11\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x*(1 - x + x^2)^2),x]
```

```
[Out] (2*(-1 + 2*x))/(3*(1 - x + x^2)) + (11*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[
3]) + Log[x] - Log[1 - x + x^2]/2
```

fricas [A] time = 1.09, size = 72, normalized size = 1.29

$$\frac{22\sqrt{3}\left(x^2-x+1\right)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-9\left(x^2-x+1\right)\log\left(x^2-x+1\right)+18\left(x^2-x+1\right)\log(x)+24x-1}{18\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(22*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) - 9*(x^2 - x +
 1)*log(x^2 - x + 1) + 18*(x^2 - x + 1)*log(x) + 24*x - 12)/(x^2 - x + 1)
```

giac [A] time = 0.15, size = 48, normalized size = 0.86

$$\frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{2(2x-1)}{3\left(x^2-x+1\right)}-\frac{1}{2}\log\left(x^2-x+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(2*x - 1)/(x^2 - x + 1) -
1/2*log(x^2 - x + 1) + log(abs(x))
```

maple [A] time = 0.01, size = 48, normalized size = 0.86

$$\frac{11\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+\ln(x)-\frac{\ln\left(x^2-x+1\right)}{2}-\frac{-\frac{4x}{3}+\frac{2}{3}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x/(x^2-x+1)^2,x)
```

```
[Out] -(-4/3*x+2/3)/(x^2-x+1)-1/2*ln(x^2-x+1)+11/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(
1/2))+ln(x)
```

maxima [A] time = 0.95, size = 47, normalized size = 0.84

$$\frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{2(2x-1)}{3\left(x^2-x+1\right)}-\frac{1}{2}\log\left(x^2-x+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(2*x - 1)/(x^2 - x + 1) -
1/2*log(x^2 - x + 1) + log(x)
```

mupad [B] time = 0.10, size = 58, normalized size = 1.04

$$\ln(x)+\frac{\frac{4x}{3}-\frac{2}{3}}{x^2-x+1}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,11\mathrm{i}}{18}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,11\mathrm{i}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x*(x^2 - x + 1)^2),x)
```

```
[Out] log(x) + ((4*x)/3 - 2/3)/(x^2 - x + 1) - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*11i)/18 + 1/2) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*11i)/18 - 1/
2)
```

sympy [A] time = 0.18, size = 54, normalized size = 0.96

$$\frac{4x-2}{3x^2-3x+3}+\log\left(x\right)-\frac{\log\left(x^2-x+1\right)}{2}+\frac{11\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x/(x**2-x+1)**2,x)
```

```
[Out] (4*x - 2)/(3*x**2 - 3*x + 3) + log(x) - log(x**2 - x + 1)/2 + 11*sqrt(3)*at
an(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

3.165 $\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2} dx$

Optimal. Leaf size=61

$$\frac{2(x+1)}{3\left(x^2-x+1\right)}-\frac{3}{2}\log\left(x^2-x+1\right)-\frac{1}{x}+3\log(x)-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -1/x+2/3*(1+x)/(x^2-x+1)+3*ln(x)-3/2*ln(x^2-x+1)-7/9*arctan(1/3*(1-2*x)*3^(
1/2))*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1646, 1628, 634, 618, 204, 628}

$$\frac{2(x+1)}{3\left(x^2-x+1\right)}-\frac{3}{2}\log\left(x^2-x+1\right)-\frac{1}{x}+3\log(x)-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x^2*(1 - x + x^2)^2),x]
```

```
[Out] -x^(-1) + (2*(1 + x))/(3*(1 - x + x^2)) - (7*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*
Sqrt[3]) + 3*Log[x] - (3*Log[1 - x + x^2])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx &= \frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{3+6x+2x^2}{x^2\left(1-x+x^2\right)}\,dx\\
&= \frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(\frac{3}{x^2}+\frac{9}{x}+\frac{8-9x}{1-x+x^2}\right)dx\\
&= -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+3\log(x)+\frac{1}{3}\int \frac{8-9x}{1-x+x^2}\,dx\\
&= -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+3\log(x)+\frac{7}{6}\int \frac{1}{1-x+x^2}\,dx-\frac{3}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)-\frac{7}{3}\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2\right.\\
&= -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 61, normalized size = 1.00

$$
\frac{2(x+1)}{3\left(x^2-x+1\right)}-\frac{3}{2}\log\left(x^2-x+1\right)-\frac{1}{x}+3\log(x)+\frac{7\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x^2*(1 - x + x^2)^2),x]
```

```
[Out] -x^(-1) + (2*(1 + x))/(3*(1 - x + x^2)) + (7*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3
*Sqrt[3]) + 3*Log[x] - (3*Log[1 - x + x^2])/2
```

fricas [A] time = 1.20, size = 85, normalized size = 1.39

$$
\frac{14\sqrt{3}\left(x^3-x^2+x\right)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-6x^2-27\left(x^3-x^2+x\right)\log\left(x^2-x+1\right)+54\left(x^3-x^2+x\right)\log(x)+}{18\left(x^3-x^2+x\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(14*sqrt(3)*(x^3 - x^2 + x)*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*x^2 - 27
*(x^3 - x^2 + x)*log(x^2 - x + 1) + 54*(x^3 - x^2 + x)*log(x) + 30*x - 18)/
(x^3 - x^2 + x)
```

giac [A] time = 0.15, size = 55, normalized size = 0.90

$$\frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x^2-5x+3}{3\left(x^3-x^2+x\right)}-\frac{3}{2}\log\left(x^2-x+1\right)+3\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/3*(x^2 - 5*x + 3)/(x^3 - x^2
+ x) - 3/2*log(x^2 - x + 1) + 3*log(abs(x))
```

maple [A] time = 0.01, size = 55, normalized size = 0.90

$$\frac{7\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+3\ln(x)-\frac{3\ln\left(x^2-x+1\right)}{2}-\frac{1}{x}-\frac{-\frac{2x}{3}-\frac{2}{3}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x^2/(x^2-x+1)^2,x)
```

```
[Out] -(-2/3*x-2/3)/(x^2-x+1)-3/2*ln(x^2-x+1)+7/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))-1/x+3*ln(x)
```

maxima [A] time = 0.96, size = 54, normalized size = 0.89

$$\frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x^2-5x+3}{3\left(x^3-x^2+x\right)}-\frac{3}{2}\log\left(x^2-x+1\right)+3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/3*(x^2 - 5*x + 3)/(x^3 - x^2
+ x) - 3/2*log(x^2 - x + 1) + 3*log(x)
```

mupad [B] time = 4.13, size = 68, normalized size = 1.11

$$3\ln(x)-\frac{\frac{x^2}{3}-\frac{5x}{3}+1}{x^3-x^2+x}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,7\mathrm{i}}{18}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{3}{2}+\frac{\sqrt{3}\,7\mathrm{i}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^2*(x^2 - x + 1)^2),x)
```

```
[Out] 3*log(x) - (x^2/3 - (5*x)/3 + 1)/(x - x^2 + x^3) - log(x - (3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*7i)/18 + 3/2) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*7i)
/18 - 3/2)
```

sympy [A] time = 0.20, size = 65, normalized size = 1.07

$$\frac{-x^2+5x-3}{3x^3-3x^2+3x}+3\log(x)-\frac{3\log\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**2/(x**2-x+1)**2,x)
```

```
[Out] (-x**2 + 5*x - 3)/(3*x**3 - 3*x**2 + 3*x) + 3*log(x) - 3*log(x**2 - x + 1)/
2 + 7*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

3.166 $\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx$

Optimal. Leaf size=68

$$\frac{2(2-x)}{3\left(x^2-x+1\right)}-\frac{1}{2x^2}-2\log\left(x^2-x+1\right)-\frac{3}{x}+4\log(x)+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -1/2/x^2-3/x+2/3*(2-x)/(x^2-x+1)+4*ln(x)-2*ln(x^2-x+1)+10/9*arctan(1/3*(1-2
*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1646, 1628, 634, 618, 204, 628}

$$\frac{2(2-x)}{3\left(x^2-x+1\right)}-\frac{1}{2x^2}-2\log\left(x^2-x+1\right)-\frac{3}{x}+4\log(x)+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x^3*(1 - x + x^2)^2),x]
```

```
[Out] -1/(2*x^2) - 3/x + (2*(2 - x))/(3*(1 - x + x^2)) + (10*ArcTan[(1 - 2*x)/Sqr
t[3]])/(3*Sqrt[3]) + 4*Log[x] - 2*Log[1 - x + x^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx &= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{3+6x+6x^2-2x^3}{x^3\left(1-x+x^2\right)}\,dx \\
&= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(\frac{3}{x^3}+\frac{9}{x^2}+\frac{12}{x}+\frac{1-12x}{1-x+x^2}\right)dx \\
&= -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+4\log(x)+\frac{1}{3}\int \frac{1-12x}{1-x+x^2}\,dx \\
&= -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+4\log(x)-\frac{5}{3}\int \frac{1}{1-x+x^2}\,dx-2\int \frac{-1+2x}{1-x+x^2}\,dx \\
&= -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+4\log(x)-2\log\left(1-x+x^2\right)+\frac{10}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx \right. \\
&= -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+4\log(x)-2\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.97

$$
-\frac{2(x-2)}{3\left(x^2-x+1\right)}-\frac{1}{2x^2}-2\log\left(x^2-x+1\right)-\frac{3}{x}+4\log(x)-\frac{10\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x^3*(1 - x + x^2)^2),x]
```

```
[Out] -1/2*1/x^2 - 3/x - (2*(-2 + x))/(3*(1 - x + x^2)) - (10*ArcTan[(-1 + 2*x)/S
qrt[3]])/(3*Sqrt[3]) + 4*Log[x] - 2*Log[1 - x + x^2]
```

fricas [A] time = 0.92, size = 98, normalized size = 1.44

$$
-\frac{66\,x^3+20\,\sqrt{3}\left(x^4-x^3+x^2\right)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-69\,x^2+36\left(x^4-x^3+x^2\right)\log\left(x^2-x+1\right)-72\left(x^4-x^3+x^2\right)\log(x)+45\,x+9}{18\left(x^4-x^3+x^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(66*x^3 + 20*sqrt(3)*(x^4 - x^3 + x^2)*arctan(1/3*sqrt(3)*(2*x - 1))
- 69*x^2 + 36*(x^4 - x^3 + x^2)*log(x^2 - x + 1) - 72*(x^4 - x^3 + x^2)*log
(x) + 45*x + 9)/(x^4 - x^3 + x^2)
```

giac [A] time = 0.16, size = 63, normalized size = 0.93

$$-\frac{10}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-\frac{22\,x^3-23\,x^2+15\,x+3}{6\left(x^2-x+1\right)x^2}-2\,\log\left(x^2-x+1\right)+4\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] -10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*(22*x^3 - 23*x^2 + 15*x +
 3)/((x^2 - x + 1)*x^2) - 2*log(x^2 - x + 1) + 4*log(abs(x))
```

maple [A] time = 0.01, size = 60, normalized size = 0.88

$$-\frac{10\sqrt{3}\;\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+4\ln(x)-2\ln\left(x^2-x+1\right)-\frac{3}{x}-\frac{1}{2x^2}-\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x^3/(x^2-x+1)^2,x)
```

```
[Out] -(2/3*x-4/3)/(x^2-x+1)-2*ln(x^2-x+1)-10/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2
))-1/2/x^2-3/x+4*ln(x)
```

maxima [A] time = 0.95, size = 63, normalized size = 0.93

$$-\frac{10}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-\frac{22\,x^3-23\,x^2+15\,x+3}{6\left(x^4-x^3+x^2\right)}-2\,\log\left(x^2-x+1\right)+4\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] -10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*(22*x^3 - 23*x^2 + 15*x +
 3)/(x^4 - x^3 + x^2) - 2*log(x^2 - x + 1) + 4*log(x)
```

mupad [B] time = 0.10, size = 75, normalized size = 1.10

$$4\,\ln(x)+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-2+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(2+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right)-\frac{\frac{11\,x^3}{3}-\frac{23\,x^2}{6}+\frac{5\,x}{2}+\frac{1}{2}}{x^4-x^3+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^3*(x^2 - x + 1)^2),x)
```

```
[Out] 4*log(x) + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*5i)/9 - 2) - log(x + (3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*5i)/9 + 2) - ((5*x)/2 - (23*x^2)/6 + (11*x^3)/
3 + 1/2)/(x^2 - x^3 + x^4)
```

sympy [A] time = 0.22, size = 71, normalized size = 1.04

$$4\log(x)-2\log\left(x^2-x+1\right)-\frac{10\sqrt{3}\;\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}+\frac{-22x^3+23x^2-15x-3}{6x^4-6x^3+6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**3/(x**2-x+1)**2,x)
```

```
[Out] 4*log(x) - 2*log(x**2 - x + 1) - 10*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)
/9 + (-22*x**3 + 23*x**2 - 15*x - 3)/(6*x**4 - 6*x**3 + 6*x**2)
```

3.167 $\int \frac{1-x^2}{\left(1+x+x^2\right)^2} \, dx$

Optimal. Leaf size=10

$$\frac{x}{x^2+x+1}$$

[Out] x/(x^2+x+1)

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1588}

$$\frac{x}{x^2+x+1}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + x + x^2)^2,x]

[Out] x/(1 + x + x^2)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1-x^2}{\left(1+x+x^2\right)^2} \, dx = \frac{x}{1+x+x^2}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{x}{x^2+x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + x + x^2)^2,x]

[Out] x/(1 + x + x^2)

fricas [A] time = 0.82, size = 10, normalized size = 1.00

$$\frac{x}{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="fricas")

[Out] x/(x^2 + x + 1)

giac [A] time = 0.15, size = 8, normalized size = 0.80

$$\frac{1}{x+\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="giac")
```

```
[Out] 1/(x + 1/x + 1)
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{x}{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2+x+1)^2,x)
```

```
[Out] x/(x^2+x+1)
```

maxima [A] time = 0.43, size = 10, normalized size = 1.00

$$\frac{x}{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="maxima")
```

```
[Out] x/(x^2 + x + 1)
```

mupad [B] time = 0.05, size = 10, normalized size = 1.00

$$\frac{x}{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x + x^2 + 1)^2,x)
```

```
[Out] x/(x + x^2 + 1)
```

sympy [A] time = 0.10, size = 7, normalized size = 0.70

$$\frac{x}{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2+x+1)**2,x)
```

```
[Out] x/(x**2 + x + 1)
```

3.168 $\int \frac{1+x^2}{1+x+x^2}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{2}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] x-1/2*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1657, 634, 618, 204, 628}

$$-\frac{1}{2}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x + x^2),x]
```

```
[Out] x + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^2}{1+x+x^2}\,dx &= \int \left(1-\frac{x}{1+x+x^2}\right)dx\\
&= x-\int \frac{x}{1+x+x^2}\,dx\\
&= x+\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx\\
&= x-\frac{1}{2}\log\left(1+x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= x+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$
-\frac{1}{2}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + x + x^2),x]
[Out] x + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

fricas [A] time = 1.03, size = 27, normalized size = 0.87

$$
\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+x-\frac{1}{2}\log\left(x^2+x+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

giac [A] time = 0.17, size = 27, normalized size = 0.87

$$
\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+x-\frac{1}{2}\log\left(x^2+x+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

maple [A] time = 0.00, size = 28, normalized size = 0.90

$$
x+\frac{\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}-\frac{\ln\left(x^2+x+1\right)}{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2+x+1),x)
[Out] x-1/2*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

maxima [A] time = 0.95, size = 27, normalized size = 0.87

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x-\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

mupad [B] time = 0.03, size = 29, normalized size = 0.94

$$x-\frac{\ln\left(x^2+x+1\right)}{2}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x + x^2 + 1),x)
```

```
[Out] x - log(x + x^2 + 1)/2 + (3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3))/3
```

sympy [A] time = 0.12, size = 36, normalized size = 1.16

$$x-\frac{\log\left(x^2+x+1\right)}{2}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2+x+1),x)
```

```
[Out] x - log(x**2 + x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

3.169 $\int \frac{-1+x^2}{25-6x+x^2} \, dx$

Optimal. Leaf size=23

$$3 \log \left(x^2-6x+25\right)+x-2 \tan^{-1}\left(\frac{x-3}{4}\right)$$

[Out] x-2*arctan(-3/4+1/4*x)+3*ln(x^2-6*x+25)

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1657, 634, 618, 204, 628}

$$3 \log \left(x^2-6x+25\right)+x-2 \tan^{-1}\left(\frac{x-3}{4}\right)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)/(25 - 6*x + x^2),x]

[Out] x - 2*ArcTan[(-3 + x)/4] + 3*Log[25 - 6*x + x^2]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{-1+x^2}{25-6x+x^2}\,dx = \int \left(1-\frac{2(13-3x)}{25-6x+x^2}\right)dx$$
$$= x-2\int \frac{13-3x}{25-6x+x^2}\,dx$$
$$= x+3\int \frac{-6+2x}{25-6x+x^2}\,dx-8\int \frac{1}{25-6x+x^2}\,dx$$
$$= x+3\log\left(25-6x+x^2\right)+16\,\text{Subst}\left(\int \frac{1}{-64-x^2}\,dx,x,-6+2x\right)$$
$$= x-2\tan^{-1}\left(\frac{1}{4}(-3+x)\right)+3\log\left(25-6x+x^2\right)$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$3\log\left(x^2-6x+25\right)+x-2\tan^{-1}\left(\frac{x-3}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(25 - 6*x + x^2),x]
[Out] x - 2*ArcTan[(-3 + x)/4] + 3*Log[25 - 6*x + x^2]
```

fricas [A] time = 0.66, size = 21, normalized size = 0.91

$$x-2\,\arctan\left(\frac{1}{4}\,x-\frac{3}{4}\right)+3\,\log\left(x^2-6\,x+25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2-6*x+25),x, algorithm="fricas")
[Out] x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

giac [A] time = 0.15, size = 21, normalized size = 0.91

$$x-2\,\arctan\left(\frac{1}{4}\,x-\frac{3}{4}\right)+3\,\log\left(x^2-6\,x+25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2-6*x+25),x, algorithm="giac")
[Out] x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$x-2\arctan\left(\frac{x}{4}-\frac{3}{4}\right)+3\ln\left(x^2-6x+25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2-6*x+25),x)
[Out] x-2*arctan(-3/4+1/4*x)+3*ln(x^2-6*x+25)
```

maxima [A] time = 0.95, size = 21, normalized size = 0.91

$$x-2\,\arctan\left(\frac{1}{4}\,x-\frac{3}{4}\right)+3\,\log\left(x^2-6\,x+25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2-6*x+25),x, algorithm="maxima")
```

```
[Out] x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

mupad [B] time = 0.04, size = 21, normalized size = 0.91

$$x+3\ \ln\left(x^2-6\,x+25\right)-2\,\mathrm{atan}\left(\frac{x}{4}-\frac{3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^2 - 6*x + 25),x)
```

```
[Out] x + 3*log(x^2 - 6*x + 25) - 2*atan(x/4 - 3/4)
```

sympy [A] time = 0.11, size = 22, normalized size = 0.96

$$x+3\log\left(x^2-6x+25\right)-2\,\mathrm{atan}\left(\frac{x}{4}-\frac{3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2-6*x+25),x)
```

```
[Out] x + 3*log(x**2 - 6*x + 25) - 2*atan(x/4 - 3/4)
```

3.170 $\int \frac{-10+3x^2}{4-4x+x^2}\,dx$

Optimal. Leaf size=21

$$3x+\frac{2}{2-x}+12\log(2-x)$$

```
[Out] 2/(2-x)+3*x+12*ln(2-x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {27, 697}

$$3x+\frac{2}{2-x}+12\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(-10 + 3*x^2)/(4 - 4*x + x^2),x]
```

```
[Out] 2/(2 - x) + 3*x + 12*Log[2 - x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-10+3x^2}{4-4x+x^2}\,dx &= \int \frac{-10+3x^2}{(-2+x)^2}\,dx\\ &= \int \left(3+\frac{2}{(-2+x)^2}+\frac{12}{-2+x}\right)dx\\ &= \frac{2}{2-x}+3x+12\log(2-x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.90

$$3(x-2)-\frac{2}{x-2}+12\log(x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-10 + 3*x^2)/(4 - 4*x + x^2),x]
```

```
[Out] -2/(-2 + x) + 3*(-2 + x) + 12*Log[-2 + x]
```

fricas [A] time = 0.78, size = 25, normalized size = 1.19

$$\frac{3\,x^2+12\,(x-2)\log(x-2)-6\,x-2}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="fricas")
[Out] (3*x^2 + 12*(x - 2)*log(x - 2) - 6*x - 2)/(x - 2)
```

giac [A] time = 0.17, size = 18, normalized size = 0.86

$$3x - \frac{2}{x-2} + 12 \log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="giac")
[Out] 3*x - 2/(x - 2) + 12*log(abs(x - 2))
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$3x + 12\ln(x-2) - \frac{2}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-10)/(x^2-4*x+4),x)
[Out] 3*x+12*ln(x-2)-2/(x-2)
```

maxima [A] time = 0.42, size = 17, normalized size = 0.81

$$3x - \frac{2}{x-2} + 12 \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="maxima")
[Out] 3*x - 2/(x - 2) + 12*log(x - 2)
```

mupad [B] time = 0.04, size = 17, normalized size = 0.81

$$3x + 12 \ln(x-2) - \frac{2}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 10)/(x^2 - 4*x + 4),x)
[Out] 3*x + 12*log(x - 2) - 2/(x - 2)
```

sympy [A] time = 0.09, size = 14, normalized size = 0.67

$$3x + 12\log(x-2) - \frac{2}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-10)/(x**2-4*x+4),x)
[Out] 3*x + 12*log(x - 2) - 2/(x - 2)
```

3.171 $\int \frac{8+x^2}{6-5x+x^2} \, dx$

Optimal. Leaf size=18

$$x-12\log(2-x)+17\log(3-x)$$

```
[Out] x-12*ln(2-x)+17*ln(3-x)
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1657, 632, 31}

$$x-12\log(2-x)+17\log(3-x)$$

Antiderivative was successfully verified.

```
[In] Int[(8 + x^2)/(6 - 5*x + x^2),x]
```

```
[Out] x - 12*Log[2 - x] + 17*Log[3 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{8+x^2}{6-5x+x^2} \, dx &= \int \left(1+\frac{2+5x}{6-5x+x^2}\right) \, dx \\
&= x+\int \frac{2+5x}{6-5x+x^2} \, dx \\
&= x-12 \int \frac{1}{-2+x} \, dx+17 \int \frac{1}{-3+x} \, dx \\
&= x-12\log(2-x)+17\log(3-x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$x-12\log(2-x)+17\log(3-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + x^2)/(6 - 5*x + x^2),x]
```

```
[Out] x - 12*Log[2 - x] + 17*Log[3 - x]
```

fricas [**A**] time = 0.69, size = 14, normalized size = 0.78

$$x - 12 \log(x - 2) + 17 \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+8)/(x^2-5*x+6),x, algorithm="fricas")
[Out] x - 12*log(x - 2) + 17*log(x - 3)
```

giac [**A**] time = 0.15, size = 16, normalized size = 0.89

$$x - 12 \log(|x - 2|) + 17 \log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+8)/(x^2-5*x+6),x, algorithm="giac")
[Out] x - 12*log(abs(x - 2)) + 17*log(abs(x - 3))
```

maple [**A**] time = 0.01, size = 15, normalized size = 0.83

$$x + 17 \ln(x - 3) - 12 \ln(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+8)/(x^2-5*x+6),x)
[Out] x-12*ln(x-2)+17*ln(x-3)
```

maxima [**A**] time = 0.43, size = 14, normalized size = 0.78

$$x - 12 \log(x - 2) + 17 \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+8)/(x^2-5*x+6),x, algorithm="maxima")
[Out] x - 12*log(x - 2) + 17*log(x - 3)
```

mupad [**B**] time = 3.92, size = 14, normalized size = 0.78

$$x - 12 \ln(x - 2) + 17 \ln(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 8)/(x^2 - 5*x + 6),x)
[Out] x - 12*log(x - 2) + 17*log(x - 3)
```

sympy [**A**] time = 0.11, size = 14, normalized size = 0.78

$$x + 17 \log(x - 3) - 12 \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+8)/(x**2-5*x+6),x)
[Out] x + 17*log(x - 3) - 12*log(x - 2)
```

3.172 $\int \frac{-4+3x+x^2}{-8-2x+x^2} \, dx$

Optimal. Leaf size=14

$$x + 4\log(4-x) + \log(x+2)$$

```
[Out] x+4*ln(4-x)+ln(2+x)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1657, 632, 31}

$$x + 4\log(4-x) + \log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 3*x + x^2)/(-8 - 2*x + x^2),x]
```

```
[Out] x + 4*Log[4 - x] + Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-4+3x+x^2}{-8-2x+x^2} \, dx &= \int \left(1+\frac{4+5x}{-8-2x+x^2}\right) dx \\
&= x + \int \frac{4+5x}{-8-2x+x^2} \, dx \\
&= x + 4\int \frac{1}{-4+x} \, dx + \int \frac{1}{2+x} \, dx \\
&= x + 4\log(4-x) + \log(2+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$x + 4\log(4-x) + \log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 3*x + x^2)/(-8 - 2*x + x^2),x]
```

```
[Out] x + 4*Log[4 - x] + Log[2 + x]
```

fricas [A] time = 0.86, size = 12, normalized size = 0.86

$$x + \log(x + 2) + 4 \log(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="fricas")
[Out] x + log(x + 2) + 4*log(x - 4)
```

giac [A] time = 0.16, size = 14, normalized size = 1.00

$$x + \log(|x + 2|) + 4 \log(|x - 4|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="giac")
[Out] x + log(abs(x + 2)) + 4*log(abs(x - 4))
```

maple [A] time = 0.01, size = 13, normalized size = 0.93

$$x + \ln(x + 2) + 4 \ln(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x-4)/(x^2-2*x-8),x)
[Out] x+ln(2+x)+4*ln(x-4)
```

maxima [A] time = 0.43, size = 12, normalized size = 0.86

$$x + \log(x + 2) + 4 \log(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="maxima")
[Out] x + log(x + 2) + 4*log(x - 4)
```

mupad [B] time = 3.85, size = 12, normalized size = 0.86

$$x + \ln(x + 2) + 4 \ln(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x + x^2 - 4)/(2*x - x^2 + 8),x)
[Out] x + log(x + 2) + 4*log(x - 4)
```

sympy [A] time = 0.11, size = 12, normalized size = 0.86

$$x + 4 \log(x - 4) + \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x-4)/(x**2-2*x-8),x)
[Out] x + 4*log(x - 4) + log(x + 2)
```

3.173 $\int \frac{7+5x+4x^2}{5+4x+4x^2} \, dx$

Optimal. Leaf size=27

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(x+\frac{1}{2}\right)$$

```
[Out] x+3/8*arctan(1/2+x)+1/8*ln(4*x^2+4*x+5)
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1657, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(x+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
```

```
[Out] x + (3*ArcTan[1/2 + x])/8 + Log[5 + 4*x + 4*x^2]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx &= \int \left(1+\frac{2+x}{5+4x+4x^2}\right)dx \\
&= x+\int \frac{2+x}{5+4x+4x^2}\,dx \\
&= x+\frac{1}{8}\int \frac{4+8x}{5+4x+4x^2}\,dx+\frac{3}{2}\int \frac{1}{5+4x+4x^2}\,dx \\
&= x+\frac{1}{8}\log\left(5+4x+4x^2\right)-3\,\text{Subst}\left(\int \frac{1}{-64-x^2}\,dx,x,4+8x\right) \\
&= x+\frac{3}{8}\tan^{-1}\left(\frac{1}{2}+x\right)+\frac{1}{8}\log\left(5+4x+4x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.15

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(\frac{1}{2}(2x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
[Out] x + (3*ArcTan[(1 + 2*x)/2])/8 + Log[5 + 4*x + 4*x^2]/8
```

fricas [A] time = 0.61, size = 21, normalized size = 0.78

$$x+\frac{3}{8}\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\log\left(4x^2+4x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="fricas")
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

giac [A] time = 0.15, size = 21, normalized size = 0.78

$$x+\frac{3}{8}\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\log\left(4x^2+4x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="giac")
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

maple [A] time = 0.00, size = 22, normalized size = 0.81

$$x+\frac{3\arctan\left(x+\frac{1}{2}\right)}{8}+\frac{\ln\left(4x^2+4x+5\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+5*x+7)/(4*x^2+4*x+5),x)
[Out] x+3/8*arctan(x+1/2)+1/8*ln(4*x^2+4*x+5)
```

maxima [A] time = 0.94, size = 21, normalized size = 0.78

$$x+\frac{3}{8}\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\log\left(4x^2+4x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="maxima")
```

```
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

mupad [B] time = 3.80, size = 17, normalized size = 0.63

$$x+\frac{\ln\left(x^2+x+\frac{5}{4}\right)}{8}+\frac{3\,\mathrm{atan}\left(x+\frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 4*x^2 + 7)/(4*x + 4*x^2 + 5),x)
```

```
[Out] x + log(x + x^2 + 5/4)/8 + (3*atan(x + 1/2))/8
```

sympy [A] time = 0.12, size = 22, normalized size = 0.81

$$x+\frac{\log\left(x^2+x+\frac{5}{4}\right)}{8}+\frac{3\,\mathrm{atan}\left(x+\frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+5*x+7)/(4*x**2+4*x+5),x)
```

```
[Out] x + log(x**2 + x + 5/4)/8 + 3*atan(x + 1/2)/8
```

3.174 $\int \frac{2-x+x^2}{-5+2x+x^2}\,dx$

Optimal. Leaf size=48

$$x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(x-\sqrt{6}+1\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(x+\sqrt{6}+1\right)$$

```
[Out] x-1/6*ln(1+x-6^(1/2))*(9-5*6^(1/2))-1/6*ln(1+x+6^(1/2))*(9+5*6^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1657, 632, 31}

$$x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(x-\sqrt{6}+1\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(x+\sqrt{6}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^2)/(-5 + 2*x + x^2),x]
```

```
[Out] x - ((9 - 5*Sqrt[6])*Log[1 - Sqrt[6] + x])/6 - ((9 + 5*Sqrt[6])*Log[1 + Sqr
t[6] + x])/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2-x+x^2}{-5+2x+x^2}\,dx &= \int \left(1+\frac{7-3x}{-5+2x+x^2}\right)dx\\
&= x+\int \frac{7-3x}{-5+2x+x^2}\,dx\\
&= x+\frac{1}{6}\left(-9+5\sqrt{6}\right)\int \frac{1}{1-\sqrt{6}+x}\,dx-\frac{1}{6}\left(9+5\sqrt{6}\right)\int \frac{1}{1+\sqrt{6}+x}\,dx\\
&= x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(1-\sqrt{6}+x\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(1+\sqrt{6}+x\right)
\end{aligned}$$

Mathematica [A] time = 0.08, size = 48, normalized size = 1.00

$$x+\frac{1}{6}\left(5\sqrt{6}-9\right)\log\left(-x+\sqrt{6}-1\right)+\frac{1}{6}\left(-9-5\sqrt{6}\right)\log\left(x+\sqrt{6}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^2)/(-5 + 2*x + x^2),x]
```

```
[Out] x + ((-9 + 5*Sqrt[6])*Log[-1 + Sqrt[6] - x])/6 + ((-9 - 5*Sqrt[6])*Log[1 +
Sqrt[6] + x])/6
```

fricas [A] time = 0.90, size = 55, normalized size = 1.15

$$\frac{5}{6}\sqrt{3}\sqrt{2}\log\left(-\frac{2\sqrt{3}\sqrt{2}(x+1)-x^2-2x-7}{x^2+2x-5}\right)+x-\frac{3}{2}\log\left(x^2+2x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="fricas")
```

```
[Out] 5/6*sqrt(3)*sqrt(2)*log(-(2*sqrt(3)*sqrt(2)*(x + 1) - x^2 - 2*x - 7)/(x^2 +
 2*x - 5)) + x - 3/2*log(x^2 + 2*x - 5)
```

giac [A] time = 0.19, size = 45, normalized size = 0.94

$$\frac{5}{6}\sqrt{6}\log\left(\frac{\left|2x-2\sqrt{6}+2\right|}{\left|2x+2\sqrt{6}+2\right|}\right)+x-\frac{3}{2}\log\left(\left|x^2+2x-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="giac")
```

```
[Out] 5/6*sqrt(6)*log(abs(2*x - 2*sqrt(6) + 2)/abs(2*x + 2*sqrt(6) + 2)) + x - 3/
2*log(abs(x^2 + 2*x - 5))
```

maple [A] time = 0.00, size = 30, normalized size = 0.62

$$x-\frac{5\sqrt{6}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{6}}{12}\right)}{3}-\frac{3\ln\left(x^2+2x-5\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+2)/(x^2+2*x-5),x)
```

```
[Out] x-3/2*ln(x^2+2*x-5)-5/3*6^(1/2)*arctanh(1/12*(2*x+2)*6^(1/2))
```

maxima [A] time = 0.96, size = 36, normalized size = 0.75

$$\frac{5}{6}\sqrt{6}\log\left(\frac{x-\sqrt{6}+1}{x+\sqrt{6}+1}\right)+x-\frac{3}{2}\log\left(x^2+2x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="maxima")
```

```
[Out] 5/6*sqrt(6)*log((x - sqrt(6) + 1)/(x + sqrt(6) + 1)) + x - 3/2*log(x^2 + 2*
x - 5)
```

mupad [B] time = 0.11, size = 35, normalized size = 0.73

$$x-\ln\left(x+\sqrt{6}+1\right)\left(\frac{5\sqrt{6}}{6}+\frac{3}{2}\right)+\ln\left(x-\sqrt{6}+1\right)\left(\frac{5\sqrt{6}}{6}-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x + 2)/(2*x + x^2 - 5),x)
```

```
[Out] x - log(x + 6^(1/2) + 1)*((5*6^(1/2))/6 + 3/2) + log(x - 6^(1/2) + 1)*((5*6
^(1/2))/6 - 3/2)
```

sympy [A] time = 0.12, size = 46, normalized size = 0.96

$$x+\left(-\frac{5\sqrt{6}}{6}-\frac{3}{2}\right)\log\left(x+1+\sqrt{6}\right)+\left(-\frac{3}{2}+\frac{5\sqrt{6}}{6}\right)\log\left(x-\sqrt{6}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+2)/(x**2+2*x-5),x)
```

```
[Out] x + (-5*sqrt(6)/6 - 3/2)*log(x + 1 + sqrt(6)) + (-3/2 + 5*sqrt(6)/6)*log(x
- sqrt(6) + 1)
```

3.175 $\int \frac{1+4x+3x^2}{\left(4+7x+2x^2\right)^2} \, dx$

Optimal. Leaf size=21

$$-\frac{3x+2}{2\left(2x^2+7x+4\right)}$$

```
[Out] 1/2*(-2-3*x)/(2*x^2+7*x+4)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1660, 8}

$$-\frac{3x+2}{2\left(2x^2+7x+4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 3*x^2)/(4 + 7*x + 2*x^2)^2,x]
```

```
[Out] -(2 + 3*x)/(2*(4 + 7*x + 2*x^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+4x+3x^2}{\left(4+7x+2x^2\right)^2} \, dx &= -\frac{2+3x}{2\left(4+7x+2x^2\right)}-\frac{\int 0 \, dx}{17}\\
&= -\frac{2+3x}{2\left(4+7x+2x^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{-3x-2}{2\left(2x^2+7x+4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 3*x^2)/(4 + 7*x + 2*x^2)^2,x]
```

```
[Out] (-2 - 3*x)/(2*(4 + 7*x + 2*x^2))
```

fricas [A] time = 0.78, size = 19, normalized size = 0.90

$$-\frac{3\,x+2}{2\,\left(2\,x^2+7\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="fricas")
[Out] -1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

giac [A] time = 0.16, size = 19, normalized size = 0.90

$$-\frac{3\,x+2}{2\,\left(2\,x^2+7\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="giac")
[Out] -1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

maple [A] time = 0.00, size = 17, normalized size = 0.81

$$\frac{-\frac{3x}{4}-\frac{1}{2}}{x^2+\frac{7}{2}x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x)
[Out] (-3/4*x-1/2)/(x^2+7/2*x+2)
```

maxima [A] time = 0.43, size = 19, normalized size = 0.90

$$-\frac{3\,x+2}{2\,\left(2\,x^2+7\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="maxima")
[Out] -1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

mupad [B] time = 3.84, size = 17, normalized size = 0.81

$$-\frac{\frac{3\,x}{4}+\frac{1}{2}}{x^2+\frac{7\,x}{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 3*x^2 + 1)/(7*x + 2*x^2 + 4)^2,x)
[Out] -((3*x)/4 + 1/2)/((7*x)/2 + x^2 + 2)
```

sympy [A] time = 0.12, size = 15, normalized size = 0.71

$$\frac{-3x-2}{4x^2+14x+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+4*x+1)/(2*x**2+7*x+4)**2,x)
[Out] (-3*x - 2)/(4*x**2 + 14*x + 8)
```

3.176 $\int \frac{1+x+x^2}{\left(3+2x+x^2\right)^2} \, dx$

Optimal. Leaf size=39

$$\frac{1-x}{4\left(x^2+2x+3\right)}+\frac{3\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] 1/4*(1-x)/(x^2+2*x+3)+3/8*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1660, 12, 618, 204}

$$\frac{1-x}{4\left(x^2+2x+3\right)}+\frac{3\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(3 + 2*x + x^2)^2,x]
```

```
[Out] (1 - x)/(4*(3 + 2*x + x^2)) + (3*ArcTan[(1 + x)/Sqrt[2]])/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2}{\left(3+2x+x^2\right)^2}\,dx &= \frac{1-x}{4\left(3+2x+x^2\right)}+\frac{1}{8}\int \frac{6}{3+2x+x^2}\,dx\\ &= \frac{1-x}{4\left(3+2x+x^2\right)}+\frac{3}{4}\int \frac{1}{3+2x+x^2}\,dx\\ &= \frac{1-x}{4\left(3+2x+x^2\right)}-\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2+2x\right)\\ &= \frac{1-x}{4\left(3+2x+x^2\right)}+\frac{3\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.00

$$\frac{1-x}{4\left(x^2+2x+3\right)}+\frac{3\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(3 + 2*x + x^2)^2,x]
[Out] (1 - x)/(4*(3 + 2*x + x^2)) + (3*ArcTan[(1 + x)/Sqrt[2]])/(4*Sqrt[2])
```

fricas [A] time = 0.64, size = 39, normalized size = 1.00

$$\frac{3\sqrt{2}\left(x^2+2x+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\,(x+1)\right)-2x+2}{8\left(x^2+2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="fricas")
[Out] 1/8*(3*sqrt(2)*(x^2 + 2*x + 3)*arctan(1/2*sqrt(2)*(x + 1)) - 2*x + 2)/(x^2
+ 2*x + 3)
```

giac [A] time = 0.16, size = 30, normalized size = 0.77

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,(x+1)\right)-\frac{x-1}{4\left(x^2+2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="giac")
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - 1/4*(x - 1)/(x^2 + 2*x + 3)
```

maple [A] time = 0.01, size = 34, normalized size = 0.87

$$\frac{3\sqrt{2}\,\arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{8}+\frac{-\frac{x}{4}+\frac{1}{4}}{x^2+2x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/(x^2+2*x+3)^2,x)
[Out] (-1/4*x+1/4)/(x^2+2*x+3)+3/8*2^(1/2)*arctan(1/4*(2*x+2)*2^(1/2))
```

maxima [A] time = 0.96, size = 30, normalized size = 0.77

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\frac{x-1}{4\left(x^2+2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="maxima")
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - 1/4*(x - 1)/(x^2 + 2*x + 3)
```

mupad [B] time = 3.84, size = 36, normalized size = 0.92

$$\frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}+\frac{\sqrt{2}}{2}\right)}{8}-\frac{\frac{x}{4}-\frac{1}{4}}{x^2+2x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(2*x + x^2 + 3)^2,x)
[Out] (3*2^(1/2)*atan((2^(1/2)*x)/2 + 2^(1/2)/2))/8 - (x/4 - 1/4)/(2*x + x^2 + 3)
```

sympy [A] time = 0.14, size = 37, normalized size = 0.95

$$\frac{1-x}{4x^2+8x+12}+\frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}+\frac{\sqrt{2}}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(x**2+2*x+3)**2,x)
[Out] (1 - x)/(4*x**2 + 8*x + 12) + 3*sqrt(2)*atan(sqrt(2)*x/2 + sqrt(2)/2)/8
```

3.177 $\int \frac{-1+2x+5x^2}{(1+x+x^2)^4} \, dx$

Optimal. Leaf size=11

$$-\frac{x}{(x^2+x+1)^3}$$

[Out] -x/(x^2+x+1)^3

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$-\frac{x}{(x^2+x+1)^3}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 2*x + 5*x^2)/(1 + x + x^2)^4,x]

[Out] -(x/(1 + x + x^2)^3)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4} \, dx = -\frac{x}{(1+x+x^2)^3}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{x}{(x^2+x+1)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 2*x + 5*x^2)/(1 + x + x^2)^4,x]

[Out] -(x/(1 + x + x^2)^3)

fricas [B] time = 0.83, size = 33, normalized size = 3.00

$$-\frac{x}{x^6+3x^5+6x^4+7x^3+6x^2+3x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="fricas")

[Out] -x/(x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)

giac [A] time = 0.15, size = 11, normalized size = 1.00

$$-\frac{x}{\left(x^2+x+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="giac")
[Out] -x/(x^2 + x + 1)^3
```

maple [A] time = 0.00, size = 12, normalized size = 1.09

$$-\frac{x}{\left(x^2+x+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x-1)/(x^2+x+1)^4,x)
[Out] -x/(x^2+x+1)^3
```

maxima [B] time = 0.44, size = 33, normalized size = 3.00

$$-\frac{x}{x^6+3\,x^5+6\,x^4+7\,x^3+6\,x^2+3\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="maxima")
[Out] -x/(x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

mupad [B] time = 3.80, size = 11, normalized size = 1.00

$$-\frac{x}{\left(x^2+x+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5*x^2 - 1)/(x + x^2 + 1)^4,x)
[Out] -x/(x + x^2 + 1)^3
```

sympy [B] time = 0.13, size = 31, normalized size = 2.82

$$-\frac{x}{x^6+3x^5+6x^4+7x^3+6x^2+3x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x-1)/(x**2+x+1)**4,x)
[Out] -x/(x**6 + 3*x**5 + 6*x**4 + 7*x**3 + 6*x**2 + 3*x + 1)
```

3.178 $\int \left(a+bx+cx^2\right)^{5/2} \left(A+Cx^2\right) dx$

Optimal. Leaf size=267

$$-\frac{5\left(b^2-4ac\right)^3\left(-4acC+32Ac^2+9b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{11/2}}+\frac{5\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+bx+cx^2}\left(-4acC\right.}{16384c^5}$$

```
[Out] -5/6144*(-4*a*c+b^2)*(32*A*c^2-4*C*a*c+9*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(3/
2)/c^4+1/384*(32*A*c^2-4*C*a*c+9*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(5/2)/c^3-9
/112*b*C*(c*x^2+b*x+a)^(7/2)/c^2+1/8*C*x*(c*x^2+b*x+a)^(7/2)/c-5/32768*(-4*
a*c+b^2)^3*(32*A*c^2-4*C*a*c+9*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+
b*x+a)^(1/2))/c^(11/2)+5/16384*(-4*a*c+b^2)^2*(32*A*c^2-4*C*a*c+9*C*b^2)*(2
*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

Rubi [A] time = 0.24, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}\left(-4acC+32Ac^2+9b^2C\right)}{384c^3}-\frac{5\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acC+32Ac^2+\right.}{6144c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)*(A + C*x^2),x]
```

```
[Out] (5*(b^2 - 4*a*c)^2*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b*x
+ c*x^2])/(16384*c^5) - (5*(b^2 - 4*a*c)*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(6144*c^4) + ((32*A*c^2 + 9*b^2*C - 4*a*c
*C)*(b + 2*c*x)*(a + b*x + c*x^2)^(5/2))/(384*c^3) - (9*b*C*(a + b*x + c*x^
2)^(7/2))/(112*c^2) + (C*x*(a + b*x + c*x^2)^(7/2))/(8*c) - (5*(b^2 - 4*a*c
)^3*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(32768*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{5/2}\left(A+Cx^2\right)dx = \frac{Cx\left(a+bx+cx^2\right)^{7/2}}{8c} + \frac{\int \left(8Ac-aC-\frac{9bCx}{2}\right)\left(a+bx+cx^2\right)^{5/2}dx}{8c}$$

$$= -\frac{9bC\left(a+bx+cx^2\right)^{7/2}}{112c^2} + \frac{Cx\left(a+bx+cx^2\right)^{7/2}}{8c} + \frac{\left(\frac{9b^2C}{2}+2c(8Ac-aC)\right)}{16 \ldots}$$

$$= \frac{\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{384c^3} - \frac{9bC\left(a+bx+cx^2\right) \ldots}{112c^2}$$

$$= -\frac{5\left(b^2-4ac\right)\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{6144c^4} + \frac{(32A \ldots}{}$$

$$= \frac{5\left(b^2-4ac\right)^2\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5} - \frac{5\left(b^2- \ldots\right.}{}$$

$$= \frac{5\left(b^2-4ac\right)^2\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5} - \frac{5\left(b^2- \ldots\right.}{}$$

$$= \frac{5\left(b^2-4ac\right)^2\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5} - \frac{5\left(b^2- \ldots\right.}{}$$

Mathematica [A] time = 0.86, size = 344, normalized size = 1.29

$$\frac{-\frac{1120A\left(b^2-4ac\right)\left(16c^{3/2}(b+2cx)(a+x(b+cx))^{3/2}-3\left(b^2-4ac\right)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)\right)}{c^{5/2}} + 57344A(b+2 \ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)*(A + C*x^2),x]
```

```
[Out] (57344*A*(b + 2*c*x)*(a + x*(b + c*x))^(5/2) - (55296*b*C*(a + x*(b + c*x))
^(7/2))/c + 86016*C*x*(a + x*(b + c*x))^(7/2) - (1120*A*(b^2 - 4*a*c)*(16*c
^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b
+ 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])])))/c^(5/2) + (7*(9*b^2 - 4*a*c)*C*(256*c^(5/2)*(
b + 2*c*x)*(a + x*(b + c*x))^(5/2) - 5*(b^2 - 4*a*c)*(16*c^(3/2)*(b + 2*c*x
)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a +
 x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
+ c*x)])]))))/c^(9/2))/(688128*c)
```

fricas [B] time = 1.20, size = 953, normalized size = 3.57

$$\left[\frac{105\left(9\,Cb^8 - 112\,Cab^6c - 2048\,Aa^3c^5 + 256\left(Ca^4 + 6\,Aa^2b^2\right)c^4 - 384\left(2\,Ca^3b^2 + Aab^4\right)c^3 + 32\left(15\,Ca^2b^4 + Ab^6\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] [1/1376256*(105*(9*C*b^8 - 112*C*a*b^6*c - 2048*A*a^3*c^5 + 256*(C*a^4 + 6*
A*a^2*b^2)*c^4 - 384*(2*C*a^3*b^2 + A*a*b^4)*c^3 + 32*(15*C*a^2*b^4 + A*b^6
)*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(c) - 4*a*c) + 4*(43008*C*c^8*x^7 + 101376*C*b*c^7*x^6 + 945*C
*b^7*c - 10500*C*a*b^5*c^2 + 118272*A*a^2*b*c^5 + 256*(243*C*b^2*c^6 + 476*
C*a*c^7 + 224*A*c^8)*x^5 - 64*(663*C*a^3*b + 560*A*a*b^3)*c^4 + 128*(3*C*b^
3*c^5 + 1228*C*a*b*c^6 + 1120*A*b*c^7)*x^4 + 112*(337*C*a^2*b^3 + 30*A*b^5)
*c^3 - 16*(27*C*b^4*c^4 - 216*C*a*b^2*c^5 - 11648*A*a*c^7 - 112*(59*C*a^2 +
 54*A*b^2)*c^6)*x^3 + 8*(63*C*b^5*c^3 - 568*C*a*b^3*c^4 + 34944*A*a*b*c^6 +
 16*(87*C*a^2*b + 14*A*b^3)*c^5)*x^2 - 2*(315*C*b^6*c^2 - 3164*C*a*b^4*c^3
- 118272*A*a^2*c^6 - 1344*(5*C*a^3 + 8*A*a*b^2)*c^5 + 16*(597*C*a^2*b^2 + 7
0*A*b^4)*c^4)*x)*sqrt(c*x^2 + b*x + a))/c^6, 1/688128*(105*(9*C*b^8 - 112*C
*a*b^6*c - 2048*A*a^3*c^5 + 256*(C*a^4 + 6*A*a^2*b^2)*c^4 - 384*(2*C*a^3*b^
2 + A*a*b^4)*c^3 + 32*(15*C*a^2*b^4 + A*b^6)*c^2)*sqrt(-c)*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(43008*C
*c^8*x^7 + 101376*C*b*c^7*x^6 + 945*C*b^7*c - 10500*C*a*b^5*c^2 + 118272*A*
a^2*b*c^5 + 256*(243*C*b^2*c^6 + 476*C*a*c^7 + 224*A*c^8)*x^5 - 64*(663*C*a
^3*b + 560*A*a*b^3)*c^4 + 128*(3*C*b^3*c^5 + 1228*C*a*b*c^6 + 1120*A*b*c^7)
*x^4 + 112*(337*C*a^2*b^3 + 30*A*b^5)*c^3 - 16*(27*C*b^4*c^4 - 216*C*a*b^2*
c^5 - 11648*A*a*c^7 - 112*(59*C*a^2 + 54*A*b^2)*c^6)*x^3 + 8*(63*C*b^5*c^3
- 568*C*a*b^3*c^4 + 34944*A*a*b*c^6 + 16*(87*C*a^2*b + 14*A*b^3)*c^5)*x^2 -
 2*(315*C*b^6*c^2 - 3164*C*a*b^4*c^3 - 118272*A*a^2*c^6 - 1344*(5*C*a^3 + 8
*A*a*b^2)*c^5 + 16*(597*C*a^2*b^2 + 70*A*b^4)*c^4)*x)*sqrt(c*x^2 + b*x + a)
)/c^6]
```

giac [B] time = 0.27, size = 482, normalized size = 1.81

$$\frac{1}{344064}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(2\left(12\left(14\,Cc^2x+33\,Cbc\right)x+\frac{243\,Cb^2c^7+476\,Cac^8+224\,Ac^9}{c^7}\right)x+\frac{3\,Cb^3c^6+}{}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/344064*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(12*(14*C*c^2*x + 33*C*b*c)*x
 + (243*C*b^2*c^7 + 476*C*a*c^8 + 224*A*c^9)/c^7)*x + (3*C*b^3*c^6 + 1228*C
*a*b*c^7 + 1120*A*b*c^8)/c^7)*x - (27*C*b^4*c^5 - 216*C*a*b^2*c^6 - 6608*C*
a^2*c^7 - 6048*A*b^2*c^7 - 11648*A*a*c^8)/c^7)*x + (63*C*b^5*c^4 - 568*C*a*
b^3*c^5 + 1392*C*a^2*b*c^6 + 224*A*b^3*c^6 + 34944*A*a*b*c^7)/c^7)*x - (315
*C*b^6*c^3 - 3164*C*a*b^4*c^4 + 9552*C*a^2*b^2*c^5 + 1120*A*b^4*c^5 - 6720*
C*a^3*c^6 - 10752*A*a*b^2*c^6 - 118272*A*a^2*c^7)/c^7)*x + (945*C*b^7*c^2 -
 10500*C*a*b^5*c^3 + 37744*C*a^2*b^3*c^4 + 3360*A*b^5*c^4 - 42432*C*a^3*b*c
^5 - 35840*A*a*b^3*c^5 + 118272*A*a^2*b*c^6)/c^7) + 5/32768*(9*C*b^8 - 112*
C*a*b^6*c + 480*C*a^2*b^4*c^2 + 32*A*b^6*c^2 - 768*C*a^3*b^2*c^3 - 384*A*a*
b^4*c^3 + 256*C*a^4*c^4 + 1536*A*a^2*b^2*c^4 - 2048*A*a^3*c^5)*log(abs(-2*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

maple [B] time = 0.01, size = 997, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x)
```

```
[Out] -9/112*b*C*(c*x^2+b*x+a)^(7/2)/c^2+1/8*C*x*(c*x^2+b*x+a)^(7/2)/c-95/2048*C/
c^3*b^4*(c*x^2+b*x+a)^(1/2)*x*a-5/32*A/c*(c*x^2+b*x+a)^(1/2)*x*a*b^2+55/512
*C/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*a^2+25/384*C/c^2*b^2*(c*x^2+b*x+a)^(3/2)*x
*a+5/16*A/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^3-5/1024*A/
c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^6-5/128*C*a^4/c^(3/2)
*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-45/32768*C/c^(11/2)*b^8*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/128*C/c^3*b^3*(c*x^2+b*x+a)^(5/2)-1
5/2048*C/c^4*b^5*(c*x^2+b*x+a)^(3/2)+45/16384*C/c^5*b^7*(c*x^2+b*x+a)^(1/2)
+1/12*A/c*(c*x^2+b*x+a)^(5/2)*b+5/24*A*(c*x^2+b*x+a)^(3/2)*x*a-5/192*A/c^2*
(c*x^2+b*x+a)^(3/2)*b^3+5/16*A*(c*x^2+b*x+a)^(1/2)*x*a^2+5/512*A/c^3*(c*x^2
+b*x+a)^(1/2)*b^5+1/6*A*(c*x^2+b*x+a)^(5/2)*x-15/1024*C/c^3*b^4*(c*x^2+b*x+
a)^(3/2)*x+25/768*C/c^3*b^3*(c*x^2+b*x+a)^(3/2)*a-5/96*A/c*(c*x^2+b*x+a)^(3
/2)*x*b^2+5/48*A/c*(c*x^2+b*x+a)^(3/2)*b*a+5/256*A/c^2*(c*x^2+b*x+a)^(1/2)*
x*b^4+35/2048*C/c^(9/2)*b^6*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1
/48*C*a/c*(c*x^2+b*x+a)^(5/2)*x-1/96*C*a/c^2*(c*x^2+b*x+a)^(5/2)*b-5/192*C*
a^2/c*(c*x^2+b*x+a)^(3/2)*x+45/8192*C/c^4*b^6*(c*x^2+b*x+a)^(1/2)*x+55/1024
*C/c^3*b^3*(c*x^2+b*x+a)^(1/2)*a^2-95/4096*C/c^4*b^5*(c*x^2+b*x+a)^(1/2)*a-
5/384*C*a^2/c^2*(c*x^2+b*x+a)^(3/2)*b-5/128*C*a^3/c*(c*x^2+b*x+a)^(1/2)*x-5
/256*C*a^3/c^2*(c*x^2+b*x+a)^(1/2)*b+3/64*C/c^2*b^2*(c*x^2+b*x+a)^(5/2)*x+1
5/128*C/c^(5/2)*b^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^3-75/1024
*C/c^(7/2)*b^4*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+5/32*A/c*(c*
x^2+b*x+a)^(1/2)*b*a^2-5/64*A/c^2*(c*x^2+b*x+a)^(1/2)*b^3*a-15/64*A/c^(3/2)
*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a^2+15/256*A/c^(5/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(C\,x^2+A\right)\,{\left(c\,x^2+b\,x+a\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] int((A + C*x^2)*(a + b*x + c*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + C x^{2}\right) \left(a + b x + c x^{2}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)*(C*x**2+A),x)
```

```
[Out] Integral((A + C*x**2)*(a + b*x + c*x**2)**(5/2), x)
```

3.179 $\int \left(a+bx+cx^2\right)^{3/2} \left(A+Cx^2\right) dx$

Optimal. Leaf size=212

$$\frac{\left(b^2-4ac\right)^2 \left(-4acC+24Ac^2+7b^2C\right) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4acC+24A\right.}{512c^4}$$

```
[Out] 1/192*(24*A*c^2-4*C*a*c+7*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^3-7/60*b*C
*(c*x^2+b*x+a)^(5/2)/c^2+1/6*C*x*(c*x^2+b*x+a)^(5/2)/c+1/1024*(-4*a*c+b^2)^
2*(24*A*c^2-4*C*a*c+7*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/c^(9/2)-1/512*(-4*a*c+b^2)*(24*A*c^2-4*C*a*c+7*C*b^2)*(2*c*x+b)*(c*x^2
+b*x+a)^(1/2)/c^4
```

Rubi [A] time = 0.18, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acC+24Ac^2+7b^2C\right)}{192c^3}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4acC+24Ac^2+7b^2\right.}{512c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(A + C*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(24*A*c^2 + 7*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b*x + c
*x^2])/(512*c^4) + ((24*A*c^2 + 7*b^2*C - 4*a*c*C)*(b + 2*c*x)*(a + b*x + c
*x^2)^(3/2))/(192*c^3) - (7*b*C*(a + b*x + c*x^2)^(5/2))/(60*c^2) + (C*x*(a
 + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*c)^2*(24*A*c^2 + 7*b^2*C - 4*a*c
*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeqQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{3/2}\left(A+Cx^2\right)dx = \frac{Cx\left(a+bx+cx^2\right)^{5/2}}{6c}+\frac{\int\left(6Ac-aC-\frac{7bCx}{2}\right)\left(a+bx+cx^2\right)^{3/2}dx}{6c}$$

$$= -\frac{7bC\left(a+bx+cx^2\right)^{5/2}}{60c^2}+\frac{Cx\left(a+bx+cx^2\right)^{5/2}}{6c}+\frac{\left(\frac{7b^2C}{2}+2c(6Ac-aC)\right)}{12\ldots}$$

$$= \frac{\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3}-\frac{7bC\left(a+bx+cx^2\right)\ldots}{60c^2}$$

$$= -\frac{\left(b^2-4ac\right)\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{\left(24Ac^2\ldots\right.}{\ldots}$$

$$= -\frac{\left(b^2-4ac\right)\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{\left(24Ac^2\ldots\right.}{\ldots}$$

$$= -\frac{\left(b^2-4ac\right)\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{\left(24Ac^2\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.58, size = 267, normalized size = 1.26

$$\frac{\frac{360A\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{3/2}}+1920A(b+2cx)(a+x(b+cx))^{3/2}+\frac{C\left(5\left(7b^2-4ac\right)\left(\ldots\right.\right.}{\ldots}}{15\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(A + C*x^2),x]
```

```
[Out] (1920*A*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 2560*C*x*(a + x*(b + c*x))^(5
/2) + (360*A*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] +
(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(3
/2) + (C*(-1792*b*(a + x*(b + c*x))^(5/2) + 5*(7*b^2 - 4*a*c)*((16*(b + 2*c
*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*S
qrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)])]))/c^(5/2))))/c)/(15360*c)
```

fricas [A] time = 1.09, size = 605, normalized size = 2.85

$$\left[\frac{15\left(7Cb^6-60Cab^4c+384Aa^2c^4-64\left(Ca^3+3Aab^2\right)c^3+24\left(6Ca^2b^2+Ab^4\right)c^2\right)\sqrt{c}\log\left(-8c^2x^2-8bcx-\ldots\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] [1/30720*(15*(7*C*b^6 - 60*C*a*b^4*c + 384*A*a^2*c^4 - 64*(C*a^3 + 3*A*a*b^
2)*c^3 + 24*(6*C*a^2*b^2 + A*b^4)*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b
^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(1280*C*c^6*x
^5 + 1664*C*b*c^5*x^4 - 105*C*b^5*c + 760*C*a*b^3*c^2 + 2400*A*a*b*c^4 - 72
*(18*C*a^2*b + 5*A*b^3)*c^3 + 16*(3*C*b^2*c^4 + 140*C*a*c^5 + 120*A*c^6)*x^
3 - 8*(7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*b*c^5)*x^2 + 2*(35*C*b^4*c^2 - 21
6*C*a*b^2*c^3 + 2400*A*a*c^5 + 120*(2*C*a^2 + A*b^2)*c^4)*x)*sqrt(c*x^2 + b
*x + a))/c^5, -1/15360*(15*(7*C*b^6 - 60*C*a*b^4*c + 384*A*a^2*c^4 - 64*(C*
a^3 + 3*A*a*b^2)*c^3 + 24*(6*C*a^2*b^2 + A*b^4)*c^2)*sqrt(-c)*arctan(1/2*sq
rt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1280
*C*c^6*x^5 + 1664*C*b*c^5*x^4 - 105*C*b^5*c + 760*C*a*b^3*c^2 + 2400*A*a*b*
c^4 - 72*(18*C*a^2*b + 5*A*b^3)*c^3 + 16*(3*C*b^2*c^4 + 140*C*a*c^5 + 120*A
*c^6)*x^3 - 8*(7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*b*c^5)*x^2 + 2*(35*C*b^4*
c^2 - 216*C*a*b^2*c^3 + 2400*A*a*c^5 + 120*(2*C*a^2 + A*b^2)*c^4)*x)*sqrt(c
*x^2 + b*x + a))/c^5]
```

giac [A] time = 0.28, size = 297, normalized size = 1.40

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\,(10\,Ccx+13\,Cb)x+\frac{3\,Cb^2c^4+140\,Cac^5+120\,Ac^6}{c^5}\right)x-\frac{7\,Cb^3c^3-36\,Cabc^4-36\right.\right.\right.}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*C*c*x + 13*C*b)*x + (3*C*b^2*c
^4 + 140*C*a*c^5 + 120*A*c^6)/c^5)*x - (7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*
b*c^5)/c^5)*x + (35*C*b^4*c^2 - 216*C*a*b^2*c^3 + 240*C*a^2*c^4 + 120*A*b^2
*c^4 + 2400*A*a*c^5)/c^5)*x - (105*C*b^5*c - 760*C*a*b^3*c^2 + 1296*C*a^2*b
*c^3 + 360*A*b^3*c^3 - 2400*A*a*b*c^4)/c^5) - 1/1024*(7*C*b^6 - 60*C*a*b^4*
c + 144*C*a^2*b^2*c^2 + 24*A*b^4*c^2 - 64*C*a^3*c^3 - 192*A*a*b^2*c^3 + 384
*A*a^2*c^4)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^
(9/2)
```

maple [B] time = 0.01, size = 613, normalized size = 2.89

$$\frac{3A\,a^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8\sqrt{c}}-\frac{3Aa\,b^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{16c^{\frac{3}{2}}}+\frac{3A\,b^4\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{128c^{\frac{5}{2}}}-\frac{C\,a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x)
```

```
[Out] 1/8*C/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*a-7/60*b*C*(c*x^2+b*x+a)^(5/2)/c^2+1/6*
C*x*(c*x^2+b*x+a)^(5/2)/c+3/8*A/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))*a^2+3/128*A/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^
4+7/192*C/c^3*b^3*(c*x^2+b*x+a)^(3/2)-7/512*C/c^4*b^5*(c*x^2+b*x+a)^(1/2)+7
/1024*C/c^(9/2)*b^6*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/16*C*a^3/
c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/8*A/c*(c*x^2+b*x+a)^(
3/2)*b+3/8*A*(c*x^2+b*x+a)^(1/2)*x*a-3/64*A/c^2*(c*x^2+b*x+a)^(1/2)*b^3-3/3
2*A/c*(c*x^2+b*x+a)^(1/2)*x*b^2+3/16*A/c*(c*x^2+b*x+a)^(1/2)*b*a+1/4*A*(c*x
^2+b*x+a)^(3/2)*x-1/32*C*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b+7/96*C/c^2*b^2*(c*x^
2+b*x+a)^(3/2)*x-7/256*C/c^3*b^4*(c*x^2+b*x+a)^(1/2)*x+1/16*C/c^3*b^3*(c*x^
2+b*x+a)^(1/2)*a-3/16*A/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*b^2*a+9/64*C/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-1
5/256*C/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/24*C*a/
c*(c*x^2+b*x+a)^(3/2)*x-1/48*C*a/c^2*(c*x^2+b*x+a)^(3/2)*b-1/16*C*a^2/c*(c*
x^2+b*x+a)^(1/2)*x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(C\,x^2+A\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int((A + C*x^2)*(a + b*x + c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + C x^{2}\right) \left(a + b x + c x^{2}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(C*x**2+A),x)
```

```
[Out] Integral((A + C*x**2)*(a + b*x + c*x**2)**(3/2), x)
```

3.180 $\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)dx$

Optimal. Leaf size=157

$$-\frac{\left(b^2-4ac\right)\left(-4acC+16Ac^2+5b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}+\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acC+16Ac^2+5b^2C\right)}{64c^3}$$

```
[Out] -5/24*b*C*(c*x^2+b*x+a)^(3/2)/c^2+1/4*C*x*(c*x^2+b*x+a)^(3/2)/c-1/128*(-4*a
*c+b^2)*(16*A*c^2-4*C*a*c+5*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x
+a)^(1/2))/c^(7/2)+1/64*(16*A*c^2-4*C*a*c+5*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^
(1/2)/c^3
```

Rubi [A] time = 0.12, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acC+16Ac^2+5b^2C\right)}{64c^3}-\frac{\left(b^2-4ac\right)\left(-4acC+16Ac^2+5b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(A + C*x^2),x]
```

```
[Out] ((16*A*c^2 + 5*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(64*c^3)
 - (5*b*C*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (C*x*(a + b*x + c*x^2)^(3/2))
/(4*c) - ((b^2 - 4*a*c)*(16*A*c^2 + 5*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
```

```
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)dx = \frac{Cx\left(a+bx+cx^2\right)^{3/2}}{4c}+\frac{\int\left(4Ac-aC-\frac{5bCx}{2}\right)\sqrt{a+bx+cx^2}\,dx}{4c}$$

$$= -\frac{5bC\left(a+bx+cx^2\right)^{3/2}}{24c^2}+\frac{Cx\left(a+bx+cx^2\right)^{3/2}}{4c}+\frac{\left(\frac{5b^2C}{2}+2c(4Ac-aC)\right)\int}{8c^2}$$

$$= \frac{\left(16Ac^2+5b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}-\frac{5bC\left(a+bx+cx^2\right)^{3/2}}{24c^2}+$$

$$= \frac{\left(16Ac^2+5b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}-\frac{5bC\left(a+bx+cx^2\right)^{3/2}}{24c^2}+$$

$$= \frac{\left(16Ac^2+5b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}-\frac{5bC\left(a+bx+cx^2\right)^{3/2}}{24c^2}+$$

Mathematica [A] time = 0.21, size = 144, normalized size = 0.92

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(C\left(b\left(8c^2x^2-52ac\right)+24c^2x\left(a+2cx^2\right)+15b^3-10b^2cx\right)+48Ac^2(b+2cx)\right)-3\left(b^2-4a\right.}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(A + C*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(48*A*c^2*(b + 2*c*x) + C*(15*b^3 - 10*b^2
*c*x + 24*c^2*x*(a + 2*c*x^2) + b*(-52*a*c + 8*c^2*x^2))) - 3*(b^2 - 4*a*c)
*(16*A*c^2 + 5*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(
b + c*x)])])/(384*c^(7/2))
```

fricas [A] time = 0.88, size = 355, normalized size = 2.26

$$\left[-\frac{3\left(5\,Cb^4-24\,Cab^2c-64\,Aac^3+16\left(Ca^2+Ab^2\right)c^2\right)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}\,(2\,cx+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(5*C*b^4 - 24*C*a*b^2*c - 64*A*a*c^3 + 16*(C*a^2 + A*b^2)*c^2)*s
qrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)
*sqrt(c) - 4*a*c) - 4*(48*C*c^4*x^3 + 8*C*b*c^3*x^2 + 15*C*b^3*c - 52*C*a*b
*c^2 + 48*A*b*c^3 - 2*(5*C*b^2*c^2 - 12*C*a*c^3 - 48*A*c^4)*x)*sqrt(c*x^2 +
 b*x + a))/c^4, 1/384*(3*(5*C*b^4 - 24*C*a*b^2*c - 64*A*a*c^3 + 16*(C*a^2 +
 A*b^2)*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)
/(c^2*x^2 + b*c*x + a*c)) + 2*(48*C*c^4*x^3 + 8*C*b*c^3*x^2 + 15*C*b^3*c -
52*C*a*b*c^2 + 48*A*b*c^3 - 2*(5*C*b^2*c^2 - 12*C*a*c^3 - 48*A*c^4)*x)*sqrt
(c*x^2 + b*x + a))/c^4]
```

giac [A] time = 0.22, size = 160, normalized size = 1.02

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,Cx+\frac{Cb}{c}\right)x-\frac{5\,Cb^2c-12\,Cac^2-48\,Ac^3}{c^3}\right)x+\frac{15\,Cb^3-52\,Cabc+48\,Abc^2}{c^3}\right)+\frac{\left(5\,Cb^4-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*C*x + C*b/c)*x - (5*C*b^2*c - 12*C*a*c
^2 - 48*A*c^3)/c^3)*x + (15*C*b^3 - 52*C*a*b*c + 48*A*b*c^2)/c^3) + 1/128*(
5*C*b^4 - 24*C*a*b^2*c + 16*C*a^2*c^2 + 16*A*b^2*c^2 - 64*A*a*c^3)*log(abs(
-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

maple [B] time = 0.01, size = 327, normalized size = 2.08

$$\frac{Aa\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2\sqrt{c}}-\frac{A\,b^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}-\frac{C\,a^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}+\frac{3Ca\,b^2\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x)
```

```
[Out] 1/4*C*x*(c*x^2+b*x+a)^(3/2)/c-5/24*b*C*(c*x^2+b*x+a)^(3/2)/c^2+5/32*C/c^2*b
^2*(c*x^2+b*x+a)^(1/2)*x+5/64*C/c^3*b^3*(c*x^2+b*x+a)^(1/2)+3/16*C/c^(5/2)*
b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-5/128*C/c^(7/2)*b^4*ln((c
*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/8*C*a/c*(c*x^2+b*x+a)^(1/2)*x-1/16
*C*a/c^2*(c*x^2+b*x+a)^(1/2)*b-1/8*C*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))+1/2*A*(c*x^2+b*x+a)^(1/2)*x+1/4*A/c*(c*x^2+b*x+a)^(1/2)*b
+1/2*A/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*A/c^(3/2)*
ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.26, size = 240, normalized size = 1.53

$$A\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a}-\frac{C\,a\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}+\frac{A\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] A*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (C*a*((x/2 + b/(4*c))*(a + b*x
+ c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c -
```

```
 b^2/4))/(2*c^(3/2))))/(4*c) + (A*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^
2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) - (5*C*b*((log((b + 2*c*x)/c^(1/2) + 2
*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2)
 - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (C*x*(a + b
*x + c*x^2)^(3/2))/(4*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + Cx^2\right) \sqrt{a + bx + cx^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(C*x**2+A),x)
```

```
[Out] Integral((A + C*x**2)*sqrt(a + b*x + c*x**2), x)
```

3.181 $\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=104

$$\frac{\left(-4acC+8Ac^2+3b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}-\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}$$

```
[Out] 1/8*(8*A*c^2-4*C*a*c+3*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(5/2)-3/4*b*C*(c*x^2+b*x+a)^(1/2)/c^2+1/2*C*x*(c*x^2+b*x+a)^(1/2)/c
```

Rubi [A] time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1661, 640, 621, 206}

$$\frac{\left(-4acC+8Ac^2+3b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}-\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-3*b*C*Sqrt[a + b*x + c*x^2])/(4*c^2) + (C*x*Sqrt[a + b*x + c*x^2])/(2*c)
+ ((8*A*c^2 + 3*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(8*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{Cx\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{2Ac-aC-\frac{3bCx}{2}}{\sqrt{a+bx+cx^2}}\,dx}{2c}$$

$$= -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2} + \frac{Cx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(\frac{3b^2C}{2}+2c(2Ac-aC)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{4c^2}$$

$$= -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2} + \frac{Cx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(\frac{3b^2C}{2}+2c(2Ac-aC)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx$$

$$= -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2} + \frac{Cx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(8Ac^2+3b^2C-4acC\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx}}\right.}{8c^{5/2}}$$

Mathematica [A] time = 0.15, size = 86, normalized size = 0.83

$$\frac{\left(-4acC+8Ac^2+3b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{8c^{5/2}} + \frac{C(2cx-3b)\sqrt{a+x(b+cx)}}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (C*(-3*b + 2*c*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) + ((8*A*c^2 + 3*b^2*C - 4*
a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(8*c^(5/2))
```

fricas [A] time = 1.02, size = 203, normalized size = 1.95

$$\left[\frac{\left(3Cb^2-4Cac+8Ac^2\right)\sqrt{c}\log\left(-8c^2x^2-8bcx-b^2-4\sqrt{cx^2+bx+a}\,(2cx+b)\sqrt{c}-4ac\right)+4\left(2Cc^2x-3\right.}{16c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*C*b^2 - 4*C*a*c + 8*A*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2
 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*C*c^2*x - 3*
C*b*c)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((3*C*b^2 - 4*C*a*c + 8*A*c^2)*sqrt
(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x
 + a*c)) - 2*(2*C*c^2*x - 3*C*b*c)*sqrt(c*x^2 + b*x + a))/c^3]
```

giac [A] time = 0.25, size = 84, normalized size = 0.81

$$\frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2Cx}{c}-\frac{3Cb}{c^2}\right)-\frac{\left(3Cb^2-4Cac+8Ac^2\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{8c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*C*x/c - 3*C*b/c^2) - 1/8*(3*C*b^2 - 4*C*a*c +
8*A*c^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/
2)
```

maple [A] time = 0.01, size = 136, normalized size = 1.31

$$\frac{A\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}-\frac{Ca\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}+\frac{3C\,b^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{5}{2}}}+\frac{\sqrt{c\,x^2+bx+}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*C*x*(c*x^2+b*x+a)^(1/2)/c-3/4*b*C*(c*x^2+b*x+a)^(1/2)/c^2+3/8*C/c^(5/2)
*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*C*a/c^(3/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+A*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))/c^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\,x^2+A}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((A + C*x^2)/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

3.182 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{C\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

```
[Out] C*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)-2*(b*c*(A+a*C/
c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1660, 12, 621, 206}

$$\frac{C\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(c*(b^2 - 4*a*c)*S
qrt[a + b*x + c*x^2]) + (C*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int -\frac{\left(b^2-4ac\right)C}{2c\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{C\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{(2C)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{C\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.73, size = 104, normalized size = 1.06

$$\frac{\frac{2\sqrt{c}\left(aC(b-2cx)+Ac(b+2cx)+b^2Cx\right)}{\sqrt{a+x(b+cx)}} - C\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x)))/Sqrt[a + x*(b +
c*x)] - (b^2 - 4*a*c)*C*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)
])])/(c^(3/2)*(-b^2 + 4*a*c))
```

fricas [B] time = 1.22, size = 403, normalized size = 4.11

$$\left[\frac{\left(Cab^2-4\,Ca^2c+\left(Cb^2c-4\,Cac^2\right)x^2+\left(Cb^3-4\,Cabc\right)x\right)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}\,(2\,cx+\right.}{2\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^2+\left(b^3c^2-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((C*a*b^2 - 4*C*a^2*c + (C*b^2*c - 4*C*a*c^2)*x^2 + (C*b^3 - 4*C*a*b*c
)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*
x + b)*sqrt(c) - 4*a*c) - 4*(C*a*b*c + A*b*c^2 + (C*b^2*c - 2*C*a*c^2 + 2*A
*c^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4
)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -((C*a*b^2 - 4*C*a^2*c + (C*b^2*c - 4*C*a
*c^2)*x^2 + (C*b^3 - 4*C*a*b*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a
)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(C*a*b*c + A*b*c^2 + (C
*b^2*c - 2*C*a*c^2 + 2*A*c^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*
c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x)]
```

giac [A] time = 0.27, size = 110, normalized size = 1.12

$$-\frac{2\left(\frac{\left(Cb^2-2\,Cac+2\,Ac^2\right)x}{b^2c-4\,ac^2}+\frac{Cab+Abc}{b^2c-4\,ac^2}\right)}{\sqrt{cx^2+bx+a}} - \frac{C\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((C*b^2 - 2*C*a*c + 2*A*c^2)*x/(b^2*c - 4*a*c^2) + (C*a*b + A*b*c)/(b^2*
c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - C*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*sqrt(c) - b))/c^(3/2)
```

maple [A] time = 0.01, size = 169, normalized size = 1.72

$$\frac{C b^2 x}{\left(4ac-b^2\right)\sqrt{c x^2+bx+a}\, c}+\frac{C b^3}{2\left(4ac-b^2\right)\sqrt{c x^2+bx+a}\, c^2}+\frac{2\left(2cx+b\right) A}{\left(4ac-b^2\right)\sqrt{c x^2+bx+a}}-\frac{Cx}{\sqrt{c x^2+bx+a}\, c}+-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -C*x/c/(c*x^2+b*x+a)^(1/2)+1/2*C/c^2*b/(c*x^2+b*x+a)^(1/2)+C/c*b^2/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*x+1/2*C/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+C/c
^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+2*A*(2*c*x+b)/(4*a*c-b^2
)/(c*x^2+b*x+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 4.21, size = 108, normalized size = 1.10

$$\frac{C\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}}+\frac{A\,\left(\frac{b}{2}+c\,x\right)}{\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}+\frac{C\,\left(\frac{a\,b}{2}-x\,\left(a\,c-\frac{b^2}{2}\right)\right)}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] (C*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) + (A*(b/2 +
c*x))/((a*c - b^2/4)*(a + b*x + c*x^2)^(1/2)) + (C*((a*b)/2 - x*(a*c - b^2/
2)))/(c*(a*c - b^2/4)*(a + b*x + c*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((A + C*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

3.183 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2(b+2cx)\left(4aC+8Ac+\frac{b^2C}{c}\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] -2/3*(b*c*(A+a*C/c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^(3/2)+2/3*(8*A*c+4*a*C+b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(1
/2)
```

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1660, 12, 613}

$$\frac{2(b+2cx)\left(4aC+8Ac+\frac{b^2C}{c}\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(3*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(3/2)) + (2*(8*A*c + 4*a*C + (b^2*C)/c)*(b + 2*c*x))/(3*
(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\int \frac{8Ac+4aC+\frac{b^2C}{c}}{2\left(a+bx+cx^2\right)^{3/2}}dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{\left(8Ac+4aC+\frac{b^2C}{c}\right)\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}}dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\left(8Ac+4aC+\frac{b^2C}{c}\right)(b+2cx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 0.89, size = 107, normalized size = 0.94

$$\frac{2C\left(8a^2b+4ax\left(3b^2+3bcx+2c^2x^2\right)+b^2x^2(3b+2cx)\right)-2A(b+2cx)\left(-4c\left(3a+2cx^2\right)+b^2-8bcx\right)}{3\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*A*(b + 2*c*x)*(b^2 - 8*b*c*x - 4*c*(3*a + 2*c*x^2)) + 2*C*(8*a^2*b + b^
2*x^2*(3*b + 2*c*x) + 4*a*x*(3*b^2 + 3*b*c*x + 2*c^2*x^2)))/(3*(b^2 - 4*a*c
)^2*(a + x*(b + c*x))^(3/2))
```

fricas [B] time = 2.34, size = 242, normalized size = 2.12

$$\frac{2\left(8\,Ca^2b-Ab^3+12\,Aabc+2\left(Cb^2c+4\,Cac^2+8\,Ac^3\right)x^3+3\left(Cb^3+4\,Cabc+8\,Abc^2\right)x^2+6\left(2\,Cab^2\right.\right.}{3\left(a^2b^4-8\,a^3b^2c+16\,a^4c^2+\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^4+2\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x^3+\left(b^6-6\,ab^4c+32\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*C*a^2*b - A*b^3 + 12*A*a*b*c + 2*(C*b^2*c + 4*C*a*c^2 + 8*A*c^3)*x^3
 + 3*(C*b^3 + 4*C*a*b*c + 8*A*b*c^2)*x^2 + 6*(2*C*a*b^2 + A*b^2*c + 4*A*a*c
^2)*x)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x
^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b
*c^2)*x)
```

giac [A] time = 0.26, size = 193, normalized size = 1.69

$$\frac{2\left(\left(\left(\frac{2\left(Cb^2c+4\,Cac^2+8\,Ac^3\right)x}{b^4-8\,ab^2c+16\,a^2c^2}+\frac{3\left(Cb^3+4\,Cabc+8\,Abc^2\right)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{6\left(2\,Cab^2+Ab^2c+4\,Aac^2\right)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{8\,Ca^2b-Ab^3+12\,Aabc}{b^4-8\,ab^2c+16\,a^2c^2}\right)}{3\left(cx^2+bx+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(((2*(C*b^2*c + 4*C*a*c^2 + 8*A*c^3)*x/(b^4 - 8*a*b^2*c + 16*a^2*c^2) +
 3*(C*b^3 + 4*C*a*b*c + 8*A*b*c^2)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + 6*(2
*C*a*b^2 + A*b^2*c + 4*A*a*c^2)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + (8*C*a^
```

```
2*b - A*b^3 + 12*A*a*b*c)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c*x^2 + b*x + a)
^(3/2)
```

maple [A] time = 0.01, size = 137, normalized size = 1.20

$$\frac{\frac{32}{3}A\,c^3x^3+\frac{16}{3}Ca\,c^2x^3+\frac{4}{3}C\,b^2c\,x^3+16Ab\,c^2x^2+8Cabc\,x^2+2C\,b^3x^2+16Aa\,c^2x+4A\,b^2cx+8Ca\,b^2x+8Aabc\cdots}{\left(c\,x^2+bx+a\right)^{\frac{3}{2}}\left(16a^2c^2-8a\,b^2c+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/3/(c*x^2+b*x+a)^(3/2)*(16*A*c^3*x^3+8*C*a*c^2*x^3+2*C*b^2*c*x^3+24*A*b*c^
2*x^2+12*C*a*b*c*x^2+3*C*b^3*x^2+24*A*a*c^2*x+6*A*b^2*c*x+12*C*a*b^2*x+12*A
*a*b*c-A*b^3+8*C*a^2*b)/(16*a^2*c^2-8*a*b^2*c+b^4)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 4.14, size = 127, normalized size = 1.11

$$\frac{2\,\left(8\,C\,a^2\,b+12\,C\,a\,b^2\,x+12\,C\,a\,b\,c\,x^2+12\,A\,a\,b\,c+8\,C\,a\,c^2\,x^3+24\,A\,a\,c^2\,x+3\,C\,b^3\,x^2-A\,b^3+2\,C\,b^2\,c\,x^3+6\cdots\right.}{3\,{\left(4\,a\,c-b^2\right)}^2\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] (2*(16*A*c^3*x^3 - A*b^3 + 3*C*b^3*x^2 + 8*C*a^2*b + 24*A*a*c^2*x + 6*A*b^2
*c*x + 12*C*a*b^2*x + 24*A*b*c^2*x^2 + 8*C*a*c^2*x^3 + 2*C*b^2*c*x^3 + 12*A
*a*b*c + 12*C*a*b*c*x^2))/(3*(4*a*c - b^2)^2*(a + b*x + c*x^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

3.184 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{16(b+2cx)\left(4acC+16Ac^2+3b^2C\right)}{15\left(b^2-4ac\right)^3\sqrt{a+bx+cx^2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}+\frac{2(b+2cx)\left(4aC+16Ac+\right.}{15\left(b^2-4ac\right)^2\left(a+bx+c\right.}$$

```
[Out] -2/5*(b*c*(A+a*C/c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^(5/2)+2/15*(16*A*c+4*a*C+3*b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a
)^(3/2)-16/15*(16*A*c^2+4*C*a*c+3*C*b^2)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b*
x+a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1660, 12, 614, 613}

$$-\frac{16(b+2cx)\left(4acC+16Ac^2+3b^2C\right)}{15\left(b^2-4ac\right)^3\sqrt{a+bx+cx^2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}+\frac{2(b+2cx)\left(4aC+16Ac+\right.}{15\left(b^2-4ac\right)^2\left(a+bx+c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(7/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(5*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(5/2)) + (2*(16*A*c + 4*a*C + (3*b^2*C)/c)*(b + 2*c*x))/
(15*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(3/2)) - (16*(16*A*c^2 + 3*b^2*C + 4*
a*c*C)*(b + 2*c*x))/(15*(b^2 - 4*a*c)^3*Sqrt[a + b*x + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
```

```
- 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{7/2}}\,dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}} - \frac{2\int \frac{16Ac+4aC+\frac{3b^2C}{c}}{2\left(a+bx+cx^2\right)^{5/2}}\,dx}{5\left(b^2-4ac\right)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}} - \frac{\left(16Ac+4aC+\frac{3b^2C}{c}\right)\int \frac{1}{\left(a+bx+cx^2\right)^{5/2}}\,dx}{5\left(b^2-4ac\right)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}} + \frac{2\left(16Ac+4aC+\frac{3b^2C}{c}\right)(b+2cx)}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{3/2}} + \frac{(8(\cdots}{\cdots}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}} + \frac{2\left(16Ac+4aC+\frac{3b^2C}{c}\right)(b+2cx)}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{3/2}} - \frac{16(\cdots}{\cdots}$$

Mathematica [A] time = 1.93, size = 148, normalized size = 0.89

$$\frac{2\left(\left(b^2-4ac\right)(b+2cx)(a+x(b+cx))\left(4acC+16Ac^2+3b^2C\right)-8c(b+2cx)(a+x(b+cx))^2\left(4acC+16Ac^2+3b^{\cdots}\right.\right.}{15c\left(b^2-4ac\right)^3(a+x(b+cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(7/2),x]
```

```
[Out] (2*((b^2 - 4*a*c)*(16*A*c^2 + 3*b^2*C + 4*a*c*C)*(b + 2*c*x)*(a + x*(b + c*
x)) - 8*c*(16*A*c^2 + 3*b^2*C + 4*a*c*C)*(b + 2*c*x)*(a + x*(b + c*x))^2 -
3*(b^2 - 4*a*c)^2*(b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))))/(15*c*(b^
2 - 4*a*c)^3*(a + x*(b + c*x))^(5/2))
```

fricas [B] time = 17.77, size = 563, normalized size = 3.37

$$-\frac{2\left(8\,Ca^2b^3+3\,Ab^5+240\,Aa^2bc^2+16\left(3\,Cb^2c^3+4\,Cac^4+16\,Ac^5\right)x^5+40\left(3\,Cb^3c^2+4\,Cabc^3+16\,Abc^4\right)\cdots\right.}{15\left(a^3b^6-12\,a^4b^4c+48\,a^5b^2c^2-64\,a^6c^3+\left(b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6\right)x^6+3\left(b^7c^2-12\,ab^5c^3+48\,\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*C*a^2*b^3 + 3*A*b^5 + 240*A*a^2*b*c^2 + 16*(3*C*b^2*c^3 + 4*C*a*c^
4 + 16*A*c^5)*x^5 + 40*(3*C*b^3*c^2 + 4*C*a*b*c^3 + 16*A*b*c^4)*x^4 + 10*(9
*C*b^4*c + 24*C*a*b^2*c^2 + 64*A*a*c^4 + 16*(C*a^2 + 3*A*b^2)*c^3)*x^3 + 5*
(3*C*b^5 + 40*C*a*b^3*c + 192*A*a*b*c^3 + 16*(3*C*a^2*b + A*b^3)*c^2)*x^2 +
 8*(12*C*a^3*b - 5*A*a*b^3)*c + 10*(2*C*a*b^4 + 24*A*a*b^2*c^2 + 48*A*a^2*c
^3 + (24*C*a^2*b^2 - A*b^4)*c)*x)*sqrt(c*x^2 + b*x + a)/(a^3*b^6 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c
^5 - 64*a^3*c^6)*x^6 + 3*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*
b*c^5)*x^5 + 3*(b^8*c - 11*a*b^6*c^2 + 36*a^2*b^4*c^3 - 16*a^3*b^2*c^4 - 64
*a^4*c^5)*x^4 + (b^9 - 6*a*b^7*c - 24*a^2*b^5*c^2 + 224*a^3*b^3*c^3 - 384*a
^4*b*c^4)*x^3 + 3*(a*b^8 - 11*a^2*b^6*c + 36*a^3*b^4*c^2 - 16*a^4*b^2*c^3 -
```

```
 64*a^5*c^4)*x^2 + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^
3)*x)
```

giac [B] time = 0.28, size = 452, normalized size = 2.71

$$-\frac{2\left(\left(\left(2\left(4\left(\frac{2\left(3Cb^2c^3+4Cac^4+16Ac^5\right)x}{b^6-12ab^4c+48a^2b^2c^2-64a^3c^3}+\frac{5\left(3Cb^3c^2+4Cabc^3+16Abc^4\right)}{b^6-12ab^4c+48a^2b^2c^2-64a^3c^3}\right)x+\frac{5\left(9Cb^4c+24Cab^2c^2+16Ca^2c^3+48Ab^2c^3+64Aac^4\right)}{b^6-12ab^4c+48a^2b^2c^2-64a^3c^3}\right)x\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(((2*(4*(2*(3*C*b^2*c^3 + 4*C*a*c^4 + 16*A*c^5)*x/(b^6 - 12*a*b^4*c +
 48*a^2*b^2*c^2 - 64*a^3*c^3) + 5*(3*C*b^3*c^2 + 4*C*a*b*c^3 + 16*A*b*c^4)/
(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))*x + 5*(9*C*b^4*c + 24*C*a
*b^2*c^2 + 16*C*a^2*c^3 + 48*A*b^2*c^3 + 64*A*a*c^4)/(b^6 - 12*a*b^4*c + 48
*a^2*b^2*c^2 - 64*a^3*c^3))*x + 5*(3*C*b^5 + 40*C*a*b^3*c + 48*C*a^2*b*c^2
+ 16*A*b^3*c^2 + 192*A*a*b*c^3)/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3
*c^3))*x + 10*(2*C*a*b^4 + 24*C*a^2*b^2*c - A*b^4*c + 24*A*a*b^2*c^2 + 48*A
*a^2*c^3)/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))*x + (8*C*a^2*b^
3 + 3*A*b^5 + 96*C*a^3*b*c - 40*A*a*b^3*c + 240*A*a^2*b*c^2)/(b^6 - 12*a*b^
4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))/(c*x^2 + b*x + a)^(5/2)
```

maple [B] time = 0.01, size = 316, normalized size = 1.89

$$\frac{\frac{128}{15}Cac^4x^5+\frac{32}{5}Cb^2c^3x^5+\frac{256}{3}Abc^4x^4+16Cb^3c^2x^4+\frac{256}{3}Aac^4x^3+64Ab^2c^3x^3+\frac{64}{3}Ca^2c^3x^3+12Cb^4cx^3+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x)
```

```
[Out] 2/15/(c*x^2+b*x+a)^(5/2)*(256*A*c^5*x^5+64*C*a*c^4*x^5+48*C*b^2*c^3*x^5+640
*A*b*c^4*x^4+160*C*a*b*c^3*x^4+120*C*b^3*c^2*x^4+640*A*a*c^4*x^3+480*A*b^2*
c^3*x^3+160*C*a^2*c^3*x^3+240*C*a*b^2*c^2*x^3+90*C*b^4*c*x^3+960*A*a*b*c^3*
x^2+80*A*b^3*c^2*x^2+240*C*a^2*b*c^2*x^2+200*C*a*b^3*c*x^2+15*C*b^5*x^2+480
*A*a^2*c^3*x+240*A*a*b^2*c^2*x-10*A*b^4*c*x+240*C*a^2*b^2*c*x+20*C*a*b^4*x+
240*A*a^2*b*c^2-40*A*a*b^3*c+3*A*b^5+96*C*a^3*b*c+8*C*a^2*b^3)/(64*a^3*c^3-
48*a^2*b^2*c^2+12*a*b^4*c-b^6)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 4.53, size = 578, normalized size = 3.46

$$\frac{\frac{bc\left(56Cb^2+256Ac^2+32Cac\right)}{15\left(4ac^2-b^2c\right)\left(4ac-b^2\right)^2}+\frac{2c^2x\left(56Cb^2+256Ac^2+32Cac\right)}{15\left(4ac^2-b^2c\right)\left(4ac-b^2\right)^2}}{\sqrt{cx^2+bx+a}}+\frac{\frac{8Cbc}{15\left(4ac^2-b^2c\right)\left(4ac-b^2\right)}+\frac{16Cc^2x}{15\left(4ac^2-b^2c\right)\left(4ac-b^2\right)}}{\sqrt{cx^2+bx+a}}-\frac{\frac{4Cx}{15\left(4ac-b^2\right)}\cdots}{\left(cx^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(7/2),x)
```

```
[Out] ((b*c*(256*A*c^2 + 56*C*b^2 + 32*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2
)^2) + (2*c^2*x*(256*A*c^2 + 56*C*b^2 + 32*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4
*a*c - b^2)^2))/(a + b*x + c*x^2)^(1/2) + ((8*C*b*c)/(15*(4*a*c^2 - b^2*c)*
(4*a*c - b^2)) + (16*C*c^2*x)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*
x + c*x^2)^(1/2) - ((4*C*x)/(15*(4*a*c - b^2)) - (2*C*b)/(15*c*(4*a*c - b^2
)))/(a + b*x + c*x^2)^(3/2) + (x*((4*A*c^2)/(5*(4*a*c^2 - b^2*c)) + (2*C*b^
2)/(5*(4*a*c^2 - b^2*c)) - (4*C*a*c)/(5*(4*a*c^2 - b^2*c))) + (2*A*b*c)/(5*
(4*a*c^2 - b^2*c)) + (2*C*a*b)/(5*(4*a*c^2 - b^2*c)))/(a + b*x + c*x^2)^(5/
2) + (x*((2*c*(32*A*c^2 + 8*C*b^2 + 8*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4*a*c
- b^2)) + (16*C*a*c^2)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*b^2*c)/(
15*(4*a*c^2 - b^2*c)*(4*a*c - b^2))) + (b*(32*A*c^2 + 8*C*b^2 + 8*C*a*c))/(
15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*a*b*c)/(15*(4*a*c^2 - b^2*c)*(4*
a*c - b^2)))/(a + b*x + c*x^2)^(3/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

3.185 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=220

$$\frac{256c(b+2cx)\left(4acC+24Ac^2+5b^2C\right)}{105\left(b^2-4ac\right)^4\sqrt{a+bx+cx^2}}-\frac{32(b+2cx)\left(4acC+24Ac^2+5b^2C\right)}{105\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^{3/2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+b\right.}{7c\left(b^2-4ac\right)\left(a+bx+c\right.}$$

```
[Out] -2/7*(b*c*(A+a*C/c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^(7/2)+2/35*(24*A*c+4*a*C+5*b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a
)^(5/2)-32/105*(24*A*c^2+4*C*a*c+5*C*b^2)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b
*x+a)^(3/2)+256/105*c*(24*A*c^2+4*C*a*c+5*C*b^2)*(2*c*x+b)/(-4*a*c+b^2)^4/(
c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1660, 12, 614, 613}

$$\frac{256c(b+2cx)\left(4acC+24Ac^2+5b^2C\right)}{105\left(b^2-4ac\right)^4\sqrt{a+bx+cx^2}}-\frac{32(b+2cx)\left(4acC+24Ac^2+5b^2C\right)}{105\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^{3/2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+b\right.}{7c\left(b^2-4ac\right)\left(a+bx+c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(9/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(7*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(7/2)) + (2*(24*A*c + 4*a*C + (5*b^2*C)/c)*(b + 2*c*x))/
(35*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(5/2)) - (32*(24*A*c^2 + 5*b^2*C + 4*
a*c*C)*(b + 2*c*x))/(105*(b^2 - 4*a*c)^3*(a + b*x + c*x^2)^(3/2)) + (256*c*
(24*A*c^2 + 5*b^2*C + 4*a*c*C)*(b + 2*c*x))/(105*(b^2 - 4*a*c)^4*Sqrt[a + b
*x + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
```

```
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{9/2}}\,dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}-\frac{2\int \frac{24Ac+4aC+\frac{5b^2C}{c}}{2\left(a+bx+cx^2\right)^{7/2}}\,dx}{7\left(b^2-4ac\right)}$$

$$=-\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}-\frac{\left(24Ac+4aC+\frac{5b^2C}{c}\right)\int \frac{1}{\left(a+bx+cx^2\right)^{7/2}}\,dx}{7\left(b^2-4ac\right)}$$

$$=-\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}+\frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}}+\frac{(16\cdots}{\cdots}$$

$$=-\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}+\frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}}-\frac{32\cdots}{1\cdots}$$

$$=-\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}+\frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}}-\frac{32\cdots}{1\cdots}$$

Mathematica [A] time = 1.74, size = 199, normalized size = 0.90

$$\frac{2\left(3\left(b^2-4ac\right)^2(b+2cx)(a+x(b+cx))\left(4acC+24Ac^2+5b^2C\right)-16c\left(b^2-4ac\right)(b+2cx)(a+x(b+cx))^2\left(4acC\cdots\right.\right.}{105c\left(b^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(9/2),x]
```

```
[Out] (2*(3*(b^2 - 4*a*c)^2*(24*A*c^2 + 5*b^2*C + 4*a*c*C)*(b + 2*c*x)*(a + x*(b
+ c*x)) - 16*c*(b^2 - 4*a*c)*(24*A*c^2 + 5*b^2*C + 4*a*c*C)*(b + 2*c*x)*(a
+ x*(b + c*x))^2 + 128*c^2*(24*A*c^2 + 5*b^2*C + 4*a*c*C)*(b + 2*c*x)*(a +
x*(b + c*x))^3 - 15*(b^2 - 4*a*c)^3*(b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2
*c*x))))/(105*c*(b^2 - 4*a*c)^4*(a + x*(b + c*x))^(7/2))
```

fricas [B] time = 52.07, size = 978, normalized size = 4.45

$$-\frac{2\left(8\,Ca^2b^5+15\,Ab^7-6720\,Aa^3bc^3-256\left(5\,Cb^2c^5+4\,Cac^6+24\,Ac^7\right)x^7-896\left(5\,Cb^3c^4+4\,Cab\cdots\right.\right.}{105\left(a^4b^8-16\,a^5b^6c+96\,a^6b^4c^2-256\,a^7b^2c^3+256\,a^8c^4+\left(b^8c^4-16\,ab^6c^5+96\,a^2b^4c^6-256\,a^3b^2c^7+256\,a^4c^8\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(8*C*a^2*b^5 + 15*A*b^7 - 6720*A*a^3*b*c^3 - 256*(5*C*b^2*c^5 + 4*C*
a*c^6 + 24*A*c^7)*x^7 - 896*(5*C*b^3*c^4 + 4*C*a*b*c^5 + 24*A*b*c^6)*x^6 -
224*(25*C*b^4*c^3 + 40*C*a*b^2*c^4 + 96*A*a*c^6 + 8*(2*C*a^2 + 15*A*b^2)*c^
5)*x^5 - 560*(5*C*b^5*c^2 + 24*C*a*b^3*c^3 + 96*A*a*b*c^5 + 8*(2*C*a^2*b +
```

```
3*A*b^3)*c^4)*x^4 - 70*(5*C*b^6*c + 124*C*a*b^4*c^2 + 384*A*a^2*c^5 + 64*(C
*a^3 + 9*A*a*b^2)*c^4 + 8*(22*C*a^2*b^2 + 3*A*b^4)*c^3)*x^3 - 240*(8*C*a^4*
b - 7*A*a^2*b^3)*c^2 + 7*(5*C*b^7 - 196*C*a*b^5*c - 5760*A*a^2*b*c^4 - 960*
(C*a^3*b + A*a*b^3)*c^3 - 8*(170*C*a^2*b^3 - 3*A*b^5)*c^2)*x^2 - 4*(80*C*a^
3*b^3 + 63*A*a*b^5)*c + 14*(2*C*a*b^6 - 720*A*a^2*b^2*c^3 - 960*A*a^3*c^4 -
 60*(8*C*a^3*b^2 - A*a*b^4)*c^2 - (80*C*a^2*b^4 + 3*A*b^6)*c)*x)*sqrt(c*x^2
 + b*x + a)/(a^4*b^8 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3 + 25
6*a^8*c^4 + (b^8*c^4 - 16*a*b^6*c^5 + 96*a^2*b^4*c^6 - 256*a^3*b^2*c^7 + 25
6*a^4*c^8)*x^8 + 4*(b^9*c^3 - 16*a*b^7*c^4 + 96*a^2*b^5*c^5 - 256*a^3*b^3*c
^6 + 256*a^4*b*c^7)*x^7 + 2*(3*b^10*c^2 - 46*a*b^8*c^3 + 256*a^2*b^6*c^4 -
576*a^3*b^4*c^5 + 256*a^4*b^2*c^6 + 512*a^5*c^7)*x^6 + 4*(b^11*c - 13*a*b^9
*c^2 + 48*a^2*b^7*c^3 + 32*a^3*b^5*c^4 - 512*a^4*b^3*c^5 + 768*a^5*b*c^6)*x
^5 + (b^12 - 4*a*b^10*c - 90*a^2*b^8*c^2 + 800*a^3*b^6*c^3 - 2240*a^4*b^4*c
^4 + 1536*a^5*b^2*c^5 + 1536*a^6*c^6)*x^4 + 4*(a*b^11 - 13*a^2*b^9*c + 48*a
^3*b^7*c^2 + 32*a^4*b^5*c^3 - 512*a^5*b^3*c^4 + 768*a^6*b*c^5)*x^3 + 2*(3*a
^2*b^10 - 46*a^3*b^8*c + 256*a^4*b^6*c^2 - 576*a^5*b^4*c^3 + 256*a^6*b^2*c^
4 + 512*a^7*c^5)*x^2 + 4*(a^3*b^9 - 16*a^4*b^7*c + 96*a^5*b^5*c^2 - 256*a^6
*b^3*c^3 + 256*a^7*b*c^4)*x)
```

giac [B] time = 0.31, size = 805, normalized size = 3.66

$$2\left(\left(\left(2\left(8\left(2\left(4\left(\frac{2\left(5Cb^2c^5+4Cac^6+24Ac^7\right)x}{b^8-16ab^6c+96a^2b^4c^2-256a^3b^2c^3+256a^4c^4}+\frac{7\left(5Cb^3c^4+4Cabc^5+24Abc^6\right)}{b^8-16ab^6c+96a^2b^4c^2-256a^3b^2c^3+256a^4c^4}\right)x+\frac{7\left(25Cb^4c^3+40Cab^2c^4+\right.}{b^8-16ab^6c+96a^2b}\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/105*(((2*(8*(2*(4*(2*(5*C*b^2*c^5 + 4*C*a*c^6 + 24*A*c^7)*x/(b^8 - 16*a*b
^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4) + 7*(5*C*b^3*c^4 + 4
*C*a*b*c^5 + 24*A*b*c^6)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c
^3 + 256*a^4*c^4))*x + 7*(25*C*b^4*c^3 + 40*C*a*b^2*c^4 + 16*C*a^2*c^5 + 12
0*A*b^2*c^5 + 96*A*a*c^6)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*
c^3 + 256*a^4*c^4))*x + 35*(5*C*b^5*c^2 + 24*C*a*b^3*c^3 + 16*C*a^2*b*c^4 +
 24*A*b^3*c^4 + 96*A*a*b*c^5)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*
b^2*c^3 + 256*a^4*c^4))*x + 35*(5*C*b^6*c + 124*C*a*b^4*c^2 + 176*C*a^2*b^2
*c^3 + 24*A*b^4*c^3 + 64*C*a^3*c^4 + 576*A*a*b^2*c^4 + 384*A*a^2*c^5)/(b^8
- 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x - 7*(5*C*
b^7 - 196*C*a*b^5*c - 1360*C*a^2*b^3*c^2 + 24*A*b^5*c^2 - 960*C*a^3*b*c^3 -
 960*A*a*b^3*c^3 - 5760*A*a^2*b*c^4)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 2
56*a^3*b^2*c^3 + 256*a^4*c^4))*x - 14*(2*C*a*b^6 - 80*C*a^2*b^4*c - 3*A*b^6
*c - 480*C*a^3*b^2*c^2 + 60*A*a*b^4*c^2 - 720*A*a^2*b^2*c^3 - 960*A*a^3*c^4
)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x -
(8*C*a^2*b^5 + 15*A*b^7 - 320*C*a^3*b^3*c - 252*A*a*b^5*c - 1920*C*a^4*b*c^
2 + 1680*A*a^2*b^3*c^2 - 6720*A*a^3*b*c^3)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c
^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))/(c*x^2 + b*x + a)^(7/2)
```

maple [B] time = 0.01, size = 555, normalized size = 2.52

$$\frac{2048}{105}Cac^6x^7+\frac{512}{21}Cb^2c^5x^7+\frac{2048}{5}Abc^6x^6+\frac{256}{3}Cb^3c^4x^6+\frac{2048}{5}Aac^6x^5+512Ab^2c^5x^5+\frac{1024}{15}Ca^2c^5x^5+\frac{320}{3}C\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x)
```

```
[Out] 2/105/(c*x^2+b*x+a)^(7/2)*(6144*A*c^7*x^7+1024*C*a*c^6*x^7+1280*C*b^2*c^5*x
^7+21504*A*b*c^6*x^6+3584*C*a*b*c^5*x^6+4480*C*b^3*c^4*x^6+21504*A*a*c^6*x^
```

```
5+26880*A*b^2*c^5*x^5+3584*C*a^2*c^5*x^5+8960*C*a*b^2*c^4*x^5+5600*C*b^4*c^
3*x^5+53760*A*a*b*c^5*x^4+13440*A*b^3*c^4*x^4+8960*C*a^2*b*c^4*x^4+13440*C*
a*b^3*c^3*x^4+2800*C*b^5*c^2*x^4+26880*A*a^2*c^5*x^3+40320*A*a*b^2*c^4*x^3+
1680*A*b^4*c^3*x^3+4480*C*a^3*c^4*x^3+12320*C*a^2*b^2*c^3*x^3+8680*C*a*b^4*
c^2*x^3+350*C*b^6*c*x^3+40320*A*a^2*b*c^4*x^2+6720*A*a*b^3*c^3*x^2-168*A*b^
5*c^2*x^2+6720*C*a^3*b*c^3*x^2+9520*C*a^2*b^3*c^2*x^2+1372*C*a*b^5*c*x^2-35
*C*b^7*x^2+13440*A*a^3*c^4*x+10080*A*a^2*b^2*c^3*x-840*A*a*b^4*c^2*x+42*A*b
^6*c*x+6720*C*a^3*b^2*c^2*x+1120*C*a^2*b^4*c*x-28*C*a*b^6*x+6720*A*a^3*b*c^
3-1680*A*a^2*b^3*c^2+252*A*a*b^5*c-15*A*b^7+1920*C*a^4*b*c^2+320*C*a^3*b^3*
c-8*C*a^2*b^5)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [**B**] time = 5.06, size = 1018, normalized size = 4.63

$$\frac{x\left(\frac{2\,c^2\,\left(160\,C\,b^2+768\,A\,c^2+96\,C\,a\,c\right)}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}-\frac{64\,C\,a\,c^3}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}+\frac{32\,C\,b^2\,c^2}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}\right)+\frac{b\,c\,\left(160\,C\,b^2+768\,A\,c^2+96\,C\,a\,c\right)}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}+\cdots}{{\left(c\,x^2+b\,x+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(9/2),x)
```

```
[Out] (x*((2*c^2*(768*A*c^2 + 160*C*b^2 + 96*C*a*c))/(105*(4*a*c^2 - b^2*c)*(4*a*
c - b^2)^2) - (64*C*a*c^3)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (32*C*
b^2*c^2)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2)) + (b*c*(768*A*c^2 + 160*C
*b^2 + 96*C*a*c))/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (32*C*a*b*c^2)/
(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(3/2) - ((8*C*b)
/(105*(4*a*c - b^2)^2) - (16*C*c*x)/(105*(4*a*c - b^2)^2))/(a + b*x + c*x^2
)^(3/2) + ((8*C*b*c)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) + (16*C*c^2*x)/(
105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x + c*x^2)^(3/2) - ((4*C*x)/(3
5*(4*a*c - b^2)) - (2*C*b)/(35*c*(4*a*c - b^2)))/(a + b*x + c*x^2)^(5/2) +
((b*c*(6144*A*c^3 + 896*C*a*c^2 + 1312*C*b^2*c))/(105*(4*a*c^2 - b^2*c)*(4*
a*c - b^2)^3) + (2*c^2*x*(6144*A*c^3 + 896*C*a*c^2 + 1312*C*b^2*c))/(105*(4
*a*c^2 - b^2*c)*(4*a*c - b^2)^3))/(a + b*x + c*x^2)^(1/2) + (x*((4*A*c^2)/(
7*(4*a*c^2 - b^2*c)) + (2*C*b^2)/(7*(4*a*c^2 - b^2*c)) - (4*C*a*c)/(7*(4*a*
c^2 - b^2*c))) + (2*A*b*c)/(7*(4*a*c^2 - b^2*c)) + (2*C*a*b)/(7*(4*a*c^2 -
b^2*c)))/(a + b*x + c*x^2)^(7/2) + (x*((2*c*(48*A*c^2 + 12*C*b^2 + 8*C*a*c)
)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) + (16*C*a*c^2)/(35*(4*a*c^2 - b^2*c)
*(4*a*c - b^2)) - (8*C*b^2*c)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2))) + (b*(4
8*A*c^2 + 12*C*b^2 + 8*C*a*c))/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*
a*b*c)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x + c*x^2)^(5/2) - ((32
*C*b*c^2)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (64*C*c^3*x)/(105*(4*a*
c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(1/2) + ((64*C*b*c^2)/(105
*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (128*C*c^3*x)/(105*(4*a*c^2 - b^2*c)*
(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(1/2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

3.186 $\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=930

$$\frac{f\left(cx^2+bx+a\right)^{3/2}(g+hx)^4}{7ch}-\frac{(6cfg-14ceh+11bfh)\left(cx^2+bx+a\right)^{3/2}(g+hx)^3}{84c^2h}+\frac{\left(-4\left(3fg^2-7h(eg+2dh)\right)c^2\right.}{}$$

```
[Out] 1/280*(33*b^2*f*h^2-2*c*h*(16*a*f*h+21*b*e*h+8*b*f*g)-4*c^2*(3*f*g^2-7*h*(2
*d*h+e*g)))*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c^3/h-1/84*(11*b*f*h-14*c*e*h+6*c
*f*g)*(h*x+g)^3*(c*x^2+b*x+a)^(3/2)/c^2/h+1/7*f*(h*x+g)^4*(c*x^2+b*x+a)^(3/
2)/c/h+1/13440*(1155*b^4*f*h^4-128*c^4*g^2*(3*f*g^2-7*h*(12*d*h+e*g))-42*b^
2*c*h^3*(78*a*f*h+35*b*(e*h+3*f*g))+8*c^2*h^2*(128*a^2*f*h^2+343*a*b*h*(e*h
+3*f*g)+b^2*(537*f*g^2+245*h*(d*h+3*e*g)))-16*c^3*h*(16*a*h*(15*f*g^2+7*h*(
d*h+3*e*g))+b*g*(17*f*g^2+21*h*(25*d*h+19*e*g)))-6*c*h*(231*b^3*f*h^3-6*b*c
*h^2*(74*a*f*h+49*b*e*h+59*b*f*g)+16*c^3*g*(3*f*g^2-7*h*(7*d*h+e*g))+8*c^2*
h*(a*h*(35*e*h+41*f*g)+b*(5*f*g^2+7*h*(7*d*h+9*e*g))))*x)*(c*x^2+b*x+a)^(3/
2)/c^5/h-1/2048*(-4*a*c+b^2)*(256*c^5*d*g^3-33*b^5*f*h^3+6*b^3*c*h^2*(20*a*
f*h+7*b*(e*h+3*f*g))-8*b*c^2*h*(10*a^2*f*h^2+14*a*b*h*(e*h+3*f*g)+7*b^2*(d*
h^2+3*e*g*h+3*f*g^2))-64*c^4*g*(2*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+
16*c^3*(2*a^2*h^2*(e*h+3*f*g)+5*b^2*g*(f*g^2+3*h*(d*h+e*g))+6*a*b*h*(3*f*g^
2+h*(d*h+3*e*g))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13
/2)+1/1024*(256*c^5*d*g^3-33*b^5*f*h^3+6*b^3*c*h^2*(20*a*f*h+7*b*(e*h+3*f*g
))-8*b*c^2*h*(10*a^2*f*h^2+14*a*b*h*(e*h+3*f*g)+7*b^2*(d*h^2+3*e*g*h+3*f*g^
2))-64*c^4*g*(2*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+16*c^3*(2*a^2*h^2*
(e*h+3*f*g)+5*b^2*g*(f*g^2+3*h*(d*h+e*g))+6*a*b*h*(3*f*g^2+h*(d*h+3*e*g))))
*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^6
```

Rubi [A] time = 3.01, antiderivative size = 927, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 621, 206}

$$\frac{f\left(cx^2+bx+a\right)^{3/2}(g+hx)^4}{7ch}-\frac{(6cfg-14ceh+11bfh)\left(cx^2+bx+a\right)^{3/2}(g+hx)^3}{84c^2h}+\frac{\left(-4\left(3fg^2-7h(eg+2dh)\right)c^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((256*c^5*d*g^3 - 33*b^5*f*h^3 - 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2*
b*g*(e*g + 3*d*h)) + 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) - 8*b*c^2*h
*(10*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)
) + 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 6
*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(10
24*c^6) + ((33*b^2*f*h^2 - 2*c*h*(8*b*f*g + 21*b*e*h + 16*a*f*h) - 4*c^2*(3
*f*g^2 - 7*h*(e*g + 2*d*h)))*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(280*c^3*
h) - ((6*c*f*g - 14*c*e*h + 11*b*f*h)*(g + h*x)^3*(a + b*x + c*x^2)^(3/2))/
(84*c^2*h) + (f*(g + h*x)^4*(a + b*x + c*x^2)^(3/2))/(7*c*h) + ((1155*b^4*f
*h^4 - 128*c^4*(3*f*g^4 - 7*g^2*h*(e*g + 12*d*h)) - 42*b^2*c*h^3*(78*a*f*h
+ 35*b*(3*f*g + e*h)) + 8*c^2*h^2*(128*a^2*f*h^2 + 343*a*b*h*(3*f*g + e*h)
+ b^2*(537*f*g^2 + 245*h*(3*e*g + d*h))) - 16*c^3*h*(16*a*h*(15*f*g^2 + 7*h
*(3*e*g + d*h)) + b*g*(17*f*g^2 + 21*h*(19*e*g + 25*d*h))) - 6*c*h*(231*b^3
*f*h^3 - 6*b*c*h^2*(59*b*f*g + 49*b*e*h + 74*a*f*h) + 16*c^3*(3*f*g^3 - 7*g
*h*(e*g + 7*d*h)) + 8*c^2*h*(5*b*f*g^2 + 7*b*h*(9*e*g + 7*d*h) + a*h*(41*f*
g + 35*e*h)))*x)*(a + b*x + c*x^2)^(3/2))/(13440*c^5*h) - ((b^2 - 4*a*c)*(2
56*c^5*d*g^3 - 33*b^5*f*h^3 - 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2*b*g
*(e*g + 3*d*h)) + 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) - 8*b*c^2*h*(1
0*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) +
```

```
 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 6*a*
b*h*(3*f*g^2 + h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(2048*c^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch} + \frac{\int (g+hx)^3\left(-\frac{1}{2}h(3bfg-14cdh+\right.}{}$$

$$= -\frac{(6cfg-14ceh+11bfh)(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{84c^2h} + \frac{f(g+hx)}{}$$

$$= \frac{\left(33b^2fh^2-2ch(8bfg+21beh+16afh)-4c^2\left(3fg^2-7h(eg+2\right.\right.}{280c^3h}$$

$$= \frac{\left(33b^2fh^2-2ch(8bfg+21beh+16afh)-4c^2\left(3fg^2-7h(eg+2\right.\right.}{280c^3h}$$

$$= \frac{\left(256c^5dg^3-33b^5fh^3-64c^4g\left(afg^2+3ah(eg+dh)+2bg(eg+3\right.\right.}{}$$

$$= \frac{\left(256c^5dg^3-33b^5fh^3-64c^4g\left(afg^2+3ah(eg+dh)+2bg(eg+3\right.\right.}{}$$

$$= \frac{\left(256c^5dg^3-33b^5fh^3-64c^4g\left(afg^2+3ah(eg+dh)+2bg(eg+3\right.\right.}{}$$

Mathematica [A] time = 2.42, size = 1093, normalized size = 1.18

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(-3465fh^3b^6+210ch^2(63fg+21eh+11fhx)b^5-84ch\left(-260afh^2+35c(6eg+2dh+ehx)h\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]

[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^6*f*h^3 + 210*b^5*c*h^2*(63*f*g +
 21*e*h + 11*f*h*x) - 84*b^4*c*h*(-260*a*f*h^2 + 35*c*h*(6*e*g + 2*d*h + e*
h*x) + c*f*(210*g^2 + 105*g*h*x + 22*h^2*x^2)) - 16*b^2*c^2*(2163*a^2*f*h^3
 - 2*a*c*h*(7*h*(345*e*g + 115*d*h + 56*e*h*x) + 3*f*(805*g^2 + 392*g*h*x +
 81*h^2*x^2)) + 2*c^2*(7*d*h*(180*g^2 + 75*g*h*x + 14*h^2*x^2) + 21*e*(20*g
^3 + 25*g^2*h*x + 14*g*h^2*x^2 + 3*h^3*x^3) + f*x*(175*g^3 + 294*g^2*h*x +
189*g*h^2*x^2 + 44*h^3*x^3))) + 16*b^3*c^2*(-42*a*h^2*(35*e*h + 3*f*(35*g +
 6*h*x)) + c*(f*(525*g^3 + 735*g^2*h*x + 441*g*h^2*x^2 + 99*h^3*x^3) + 7*h*
(5*d*h*(45*g + 7*h*x) + 3*e*(75*g^2 + 35*g*h*x + 7*h^2*x^2)))) + 32*b*c^3*(
a^2*h^2*(2373*f*g + 791*e*h + 397*f*h*x) - 2*a*c*(f*(455*g^3 + 609*g^2*h*x
+ 357*g*h^2*x^2 + 79*h^3*x^3) + 7*h*(d*h*(195*g + 29*h*x) + e*(195*g^2 + 87
*g*h*x + 17*h^2*x^2))) + 4*c^2*(21*d*(10*g^3 + 10*g^2*h*x + 5*g*h^2*x^2 + h
^3*x^3) + x*(7*e*(10*g^3 + 15*g^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^3) + f*x*(35*
g^3 + 63*g^2*h*x + 42*g*h^2*x^2 + 10*h^3*x^3)))) + 64*c^3*(128*a^3*f*h^3 -
a^2*c*h*(7*h*(96*e*g + 32*d*h + 15*e*h*x) + f*(672*g^2 + 315*g*h*x + 64*h^2
*x^2)) + 2*a*c^2*(7*d*h*(120*g^2 + 45*g*h*x + 8*h^2*x^2) + 7*e*(40*g^3 + 45
*g^2*h*x + 24*g*h^2*x^2 + 5*h^3*x^3) + 3*f*x*(35*g^3 + 56*g^2*h*x + 35*g*h^
2*x^2 + 8*h^3*x^3)) + 4*c^3*x*(21*d*(10*g^3 + 20*g^2*h*x + 15*g*h^2*x^2 + 4
*h^3*x^3) + x*(7*e*(20*g^3 + 45*g^2*h*x + 36*g*h^2*x^2 + 10*h^3*x^3) + 3*f*
x*(35*g^3 + 84*g^2*h*x + 70*g*h^2*x^2 + 20*h^3*x^3))))) + 105*(b^2 - 4*a*c)
*(-256*c^5*d*g^3 + 33*b^5*f*h^3 + 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2
*b*g*(e*g + 3*d*h)) - 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) + 8*b*c^2*
h*(10*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2
)) - 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) +
6*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + x*(b + c*x)])])/(215040*c^(13/2))
```

fricas [A] time = 2.45, size = 2817, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/430080*(105*(16*(16*(b^2*c^5 - 4*a*c^6)*d - 8*(b^3*c^4 - 4*a*b*c^5)*e +
(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*f)*g^3 - 24*(16*(b^3*c^4 - 4*a*b*c^
5)*d - 2*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*e + (7*b^5*c^2 - 40*a*b^3*
c^3 + 48*a^2*b*c^4)*f)*g^2*h + 6*(8*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)
*d - 4*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*e + (21*b^6*c - 140*a*b^4*
c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*f)*g*h^2 - (8*(7*b^5*c^2 - 40*a*b^3*c^3
 + 48*a^2*b*c^4)*d - 2*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3
*c^4)*e + (33*b^7 - 252*a*b^5*c + 560*a^2*b^3*c^2 - 320*a^3*b*c^3)*f)*h^3)*
sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b
)*sqrt(c) - 4*a*c) + 4*(15360*c^7*f*h^3*x^6 + 1280*(42*c^7*f*g*h^2 + (14*c^
7*e + b*c^6*f)*h^3)*x^5 + 128*(504*c^7*f*g^2*h + 42*(12*c^7*e + b*c^6*f)*g*
h^2 + (168*c^7*d + 14*b*c^6*e - (11*b^2*c^5 - 24*a*c^6)*f)*h^3)*x^4 + 560*(
48*b*c^6*d - 8*(3*b^2*c^5 - 8*a*c^6)*e + (15*b^3*c^4 - 52*a*b*c^5)*f)*g^3 -
 168*(80*(3*b^2*c^5 - 8*a*c^6)*d - 10*(15*b^3*c^4 - 52*a*b*c^5)*e + (105*b^
4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*f)*g^2*h + 42*(40*(15*b^3*c^4 - 52*a*b
*c^5)*d - 4*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*e + (315*b^5*c^2 -
1680*a*b^3*c^3 + 1808*a^2*b*c^4)*f)*g*h^2 - (56*(105*b^4*c^3 - 460*a*b^2*c^
4 + 256*a^2*c^5)*d - 14*(315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*e +
 (3465*b^6*c - 21840*a*b^4*c^2 + 34608*a^2*b^2*c^3 - 8192*a^3*c^4)*f)*h^3 +
 16*(1680*c^7*f*g^3 + 504*(10*c^7*e + b*c^6*f)*g^2*h + 42*(120*c^7*d + 12*b
*c^6*e - (9*b^2*c^5 - 20*a*c^6)*f)*g*h^2 + (168*b*c^6*d - 14*(9*b^2*c^5 - 2
0*a*c^6)*e + (99*b^3*c^4 - 316*a*b*c^5)*f)*h^3)*x^3 + 8*(560*(8*c^7*e + b*c
^6*f)*g^3 + 168*(80*c^7*d + 10*b*c^6*e - (7*b^2*c^5 - 16*a*c^6)*f)*g^2*h +
42*(40*b*c^6*d - 4*(7*b^2*c^5 - 16*a*c^6)*e + (21*b^3*c^4 - 68*a*b*c^5)*f)*
g*h^2 - (56*(7*b^2*c^5 - 16*a*c^6)*d - 14*(21*b^3*c^4 - 68*a*b*c^5)*e + (23
1*b^4*c^3 - 972*a*b^2*c^4 + 512*a^2*c^5)*f)*h^3)*x^2 + 2*(560*(48*c^7*d + 8
*b*c^6*e - (5*b^2*c^5 - 12*a*c^6)*f)*g^3 + 168*(80*b*c^6*d - 10*(5*b^2*c^5
- 12*a*c^6)*e + (35*b^3*c^4 - 116*a*b*c^5)*f)*g^2*h - 42*(40*(5*b^2*c^5 - 1
2*a*c^6)*d - 4*(35*b^3*c^4 - 116*a*b*c^5)*e + (105*b^4*c^3 - 448*a*b^2*c^4
+ 240*a^2*c^5)*f)*g*h^2 + (56*(35*b^3*c^4 - 116*a*b*c^5)*d - 14*(105*b^4*c^
3 - 448*a*b^2*c^4 + 240*a^2*c^5)*e + (1155*b^5*c^2 - 6048*a*b^3*c^3 + 6352*
a^2*b*c^4)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a))/c^7, 1/215040*(105*(16*(16*(b^
2*c^5 - 4*a*c^6)*d - 8*(b^3*c^4 - 4*a*b*c^5)*e + (5*b^4*c^3 - 24*a*b^2*c^4
+ 16*a^2*c^5)*f)*g^3 - 24*(16*(b^3*c^4 - 4*a*b*c^5)*d - 2*(5*b^4*c^3 - 24*a
*b^2*c^4 + 16*a^2*c^5)*e + (7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*f)*g^2
*h + 6*(8*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*d - 4*(7*b^5*c^2 - 40*a*b
^3*c^3 + 48*a^2*b*c^4)*e + (21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64
*a^3*c^4)*f)*g*h^2 - (8*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*d - 2*(21
*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*e + (33*b^7 - 252*a*
b^5*c + 560*a^2*b^3*c^2 - 320*a^3*b*c^3)*f)*h^3)*sqrt(-c)*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(15360*c^
7*f*h^3*x^6 + 1280*(42*c^7*f*g*h^2 + (14*c^7*e + b*c^6*f)*h^3)*x^5 + 128*(5
04*c^7*f*g^2*h + 42*(12*c^7*e + b*c^6*f)*g*h^2 + (168*c^7*d + 14*b*c^6*e -
(11*b^2*c^5 - 24*a*c^6)*f)*h^3)*x^4 + 560*(48*b*c^6*d - 8*(3*b^2*c^5 - 8*a*
c^6)*e + (15*b^3*c^4 - 52*a*b*c^5)*f)*g^3 - 168*(80*(3*b^2*c^5 - 8*a*c^6)*d
 - 10*(15*b^3*c^4 - 52*a*b*c^5)*e + (105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*
c^5)*f)*g^2*h + 42*(40*(15*b^3*c^4 - 52*a*b*c^5)*d - 4*(105*b^4*c^3 - 460*a
*b^2*c^4 + 256*a^2*c^5)*e + (315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)
*f)*g*h^2 - (56*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*d - 14*(315*b^5
*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*e + (3465*b^6*c - 21840*a*b^4*c^2 +
 34608*a^2*b^2*c^3 - 8192*a^3*c^4)*f)*h^3 + 16*(1680*c^7*f*g^3 + 504*(10*c^
```

```
7*e + b*c^6*f)*g^2*h + 42*(120*c^7*d + 12*b*c^6*e - (9*b^2*c^5 - 20*a*c^6)*
f)*g*h^2 + (168*b*c^6*d - 14*(9*b^2*c^5 - 20*a*c^6)*e + (99*b^3*c^4 - 316*a
*b*c^5)*f)*h^3)*x^3 + 8*(560*(8*c^7*e + b*c^6*f)*g^3 + 168*(80*c^7*d + 10*b
*c^6*e - (7*b^2*c^5 - 16*a*c^6)*f)*g^2*h + 42*(40*b*c^6*d - 4*(7*b^2*c^5 -
16*a*c^6)*e + (21*b^3*c^4 - 68*a*b*c^5)*f)*g*h^2 - (56*(7*b^2*c^5 - 16*a*c^
6)*d - 14*(21*b^3*c^4 - 68*a*b*c^5)*e + (231*b^4*c^3 - 972*a*b^2*c^4 + 512*
a^2*c^5)*f)*h^3)*x^2 + 2*(560*(48*c^7*d + 8*b*c^6*e - (5*b^2*c^5 - 12*a*c^6
)*f)*g^3 + 168*(80*b*c^6*d - 10*(5*b^2*c^5 - 12*a*c^6)*e + (35*b^3*c^4 - 11
6*a*b*c^5)*f)*g^2*h - 42*(40*(5*b^2*c^5 - 12*a*c^6)*d - 4*(35*b^3*c^4 - 116
*a*b*c^5)*e + (105*b^4*c^3 - 448*a*b^2*c^4 + 240*a^2*c^5)*f)*g*h^2 + (56*(3
5*b^3*c^4 - 116*a*b*c^5)*d - 14*(105*b^4*c^3 - 448*a*b^2*c^4 + 240*a^2*c^5)
*e + (1155*b^5*c^2 - 6048*a*b^3*c^3 + 6352*a^2*b*c^4)*f)*h^3)*x)*sqrt(c*x^2
 + b*x + a))/c^7]
```

giac [A] time = 0.31, size = 1702, normalized size = 1.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*f*h^3*x + (42*c^6*f*g*h^
2 + b*c^5*f*h^3 + 14*c^6*h^3*e)/c^6)*x + (504*c^6*f*g^2*h + 42*b*c^5*f*g*h^
2 + 168*c^6*d*h^3 - 11*b^2*c^4*f*h^3 + 24*a*c^5*f*h^3 + 504*c^6*g*h^2*e + 1
4*b*c^5*h^3*e)/c^6)*x + (1680*c^6*f*g^3 + 504*b*c^5*f*g^2*h + 5040*c^6*d*g*
h^2 - 378*b^2*c^4*f*g*h^2 + 840*a*c^5*f*g*h^2 + 168*b*c^5*d*h^3 + 99*b^3*c^
3*f*h^3 - 316*a*b*c^4*f*h^3 + 5040*c^6*g^2*h*e + 504*b*c^5*g*h^2*e - 126*b^
2*c^4*h^3*e + 280*a*c^5*h^3*e)/c^6)*x + (560*b*c^5*f*g^3 + 13440*c^6*d*g^2*
h - 1176*b^2*c^4*f*g^2*h + 2688*a*c^5*f*g^2*h + 1680*b*c^5*d*g*h^2 + 882*b^
3*c^3*f*g*h^2 - 2856*a*b*c^4*f*g*h^2 - 392*b^2*c^4*d*h^3 + 896*a*c^5*d*h^3
- 231*b^4*c^2*f*h^3 + 972*a*b^2*c^3*f*h^3 - 512*a^2*c^4*f*h^3 + 4480*c^6*g^
3*e + 1680*b*c^5*g^2*h*e - 1176*b^2*c^4*g*h^2*e + 2688*a*c^5*g*h^2*e + 294*
b^3*c^3*h^3*e - 952*a*b*c^4*h^3*e)/c^6)*x + (26880*c^6*d*g^3 - 2800*b^2*c^4
*f*g^3 + 6720*a*c^5*f*g^3 + 13440*b*c^5*d*g^2*h + 5880*b^3*c^3*f*g^2*h - 19
488*a*b*c^4*f*g^2*h - 8400*b^2*c^4*d*g*h^2 + 20160*a*c^5*d*g*h^2 - 4410*b^4
*c^2*f*g*h^2 + 18816*a*b^2*c^3*f*g*h^2 - 10080*a^2*c^4*f*g*h^2 + 1960*b^3*c
^3*d*h^3 - 6496*a*b*c^4*d*h^3 + 1155*b^5*c*f*h^3 - 6048*a*b^3*c^2*f*h^3 + 6
352*a^2*b*c^3*f*h^3 + 4480*b*c^5*g^3*e - 8400*b^2*c^4*g^2*h*e + 20160*a*c^5
*g^2*h*e + 5880*b^3*c^3*g*h^2*e - 19488*a*b*c^4*g*h^2*e - 1470*b^4*c^2*h^3*
e + 6272*a*b^2*c^3*h^3*e - 3360*a^2*c^4*h^3*e)/c^6)*x + (26880*b*c^5*d*g^3
+ 8400*b^3*c^3*f*g^3 - 29120*a*b*c^4*f*g^3 - 40320*b^2*c^4*d*g^2*h + 107520
*a*c^5*d*g^2*h - 17640*b^4*c^2*f*g^2*h + 77280*a*b^2*c^3*f*g^2*h - 43008*a^
2*c^4*f*g^2*h + 25200*b^3*c^3*d*g*h^2 - 87360*a*b*c^4*d*g*h^2 + 13230*b^5*c
*f*g*h^2 - 70560*a*b^3*c^2*f*g*h^2 + 75936*a^2*b*c^3*f*g*h^2 - 5880*b^4*c^2
*d*h^3 + 25760*a*b^2*c^3*d*h^3 - 14336*a^2*c^4*d*h^3 - 3465*b^6*f*h^3 + 218
40*a*b^4*c*f*h^3 - 34608*a^2*b^2*c^2*f*h^3 + 8192*a^3*c^3*f*h^3 - 13440*b^2
*c^4*g^3*e + 35840*a*c^5*g^3*e + 25200*b^3*c^3*g^2*h*e - 87360*a*b*c^4*g^2*
h*e - 17640*b^4*c^2*g*h^2*e + 77280*a*b^2*c^3*g*h^2*e - 43008*a^2*c^4*g*h^2
*e + 4410*b^5*c*h^3*e - 23520*a*b^3*c^2*h^3*e + 25312*a^2*b*c^3*h^3*e)/c^6)
 + 1/2048*(256*b^2*c^5*d*g^3 - 1024*a*c^6*d*g^3 + 80*b^4*c^3*f*g^3 - 384*a*
b^2*c^4*f*g^3 + 256*a^2*c^5*f*g^3 - 384*b^3*c^4*d*g^2*h + 1536*a*b*c^5*d*g^
2*h - 168*b^5*c^2*f*g^2*h + 960*a*b^3*c^3*f*g^2*h - 1152*a^2*b*c^4*f*g^2*h
+ 240*b^4*c^3*d*g*h^2 - 1152*a*b^2*c^4*d*g*h^2 + 768*a^2*c^5*d*g*h^2 + 126*
b^6*c*f*g*h^2 - 840*a*b^4*c^2*f*g*h^2 + 1440*a^2*b^2*c^3*f*g*h^2 - 384*a^3*
c^4*f*g*h^2 - 56*b^5*c^2*d*h^3 + 320*a*b^3*c^3*d*h^3 - 384*a^2*b*c^4*d*h^3
- 33*b^7*f*h^3 + 252*a*b^5*c*f*h^3 - 560*a^2*b^3*c^2*f*h^3 + 320*a^3*b*c^3*
f*h^3 - 128*b^3*c^4*g^3*e + 512*a*b*c^5*g^3*e + 240*b^4*c^3*g^2*h*e - 1152*
a*b^2*c^4*g^2*h*e + 768*a^2*c^5*g^2*h*e - 168*b^5*c^2*g*h^2*e + 960*a*b^3*c
^3*g*h^2*e - 1152*a^2*b*c^4*g*h^2*e + 42*b^6*c*h^3*e - 280*a*b^4*c^2*h^3*e
```

```
+ 480*a^2*b^2*c^3*h^3*e - 128*a^3*c^4*h^3*e)*log(abs(-2*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2)
```

maple [B] time = 0.02, size = 3543, normalized size = 3.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 3/5*x^2*(c*x^2+b*x+a)^(3/2)/c*f*g^2*h-7/40/c^2*b*x*(c*x^2+b*x+a)^(3/2)*d*h^
3-3/8*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g^2*h+3/4*x
*(c*x^2+b*x+a)^(3/2)/c*d*g*h^2+3/4*x*(c*x^2+b*x+a)^(3/2)/c*e*g^2*h-5/8/c^2*
b*(c*x^2+b*x+a)^(3/2)*d*g*h^2+15/64/c^3*b^3*(c*x^2+b*x+a)^(1/2)*d*g*h^2+15/
64/c^3*b^3*(c*x^2+b*x+a)^(1/2)*e*g^2*h+3/16/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))*a*f*g^3-15/128/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+
(c*x^2+b*x+a)^(1/2))*d*g*h^2-15/128/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*e*g^2*h-1/8*a/c*x*(c*x^2+b*x+a)^(1/2)*f*g^3-1/16*a/c^2*(c*
x^2+b*x+a)^(1/2)*b*f*g^3-3/8*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))*d*g*h^2-33/320*f*h^3/c^4*b^3*x*(c*x^2+b*x+a)^(3/2)-33/512*f*h^3/c
^5*b^5*x*(c*x^2+b*x+a)^(1/2)+15/128*f*h^3/c^5*b^4*a*(c*x^2+b*x+a)^(1/2)-39/
160*f*h^3/c^4*b^2*a*(c*x^2+b*x+a)^(3/2)-5/64*f*h^3/c^4*b^2*a^2*(c*x^2+b*x+a
)^(1/2)-63/512*f*h^3/c^(11/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))*a+35/128*f*h^3/c^(9/2)*b^3*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))-5/32*f*h^3/c^(7/2)*b*a^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/3
5*f*h^3*a/c^2*x^2*(c*x^2+b*x+a)^(3/2)-11/84*f*h^3/c^2*b*x^3*(c*x^2+b*x+a)^(
3/2)+33/280*f*h^3/c^3*b^2*x^2*(c*x^2+b*x+a)^(3/2)+7/16/c^3*b^2*(c*x^2+b*x+a
)^(3/2)*f*g^2*h-7/64/c^3*b^3*x*(c*x^2+b*x+a)^(1/2)*d*h^3-21/128/c^4*b^4*(c*
x^2+b*x+a)^(1/2)*e*g*h^2-21/128/c^4*b^4*(c*x^2+b*x+a)^(1/2)*f*g^2*h-2/5*a/c
^2*(c*x^2+b*x+a)^(3/2)*e*g*h^2-2/5*a/c^2*(c*x^2+b*x+a)^(3/2)*f*g^2*h-5/32/c
^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*h^3+21/256/c^(9/
2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g*h^2-3/8*a/c*x*(c*x^2
+b*x+a)^(1/2)*e*g^2*h-3/16*a/c^2*(c*x^2+b*x+a)^(1/2)*b*d*g*h^2-3/16*a/c^2*(
c*x^2+b*x+a)^(1/2)*b*e*g^2*h-3/4/c*b*x*(c*x^2+b*x+a)^(1/2)*d*g^2*h-3/4/c^(3
/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*g^2*h-3/8*a/c*x*(c*x^
2+b*x+a)^(1/2)*d*g*h^2+1/3*(c*x^2+b*x+a)^(3/2)/c*e*g^3+1/2*d*g^3*x*(c*x^2+b
*x+a)^(1/2)+21/256/c^(9/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
f*g^2*h+3/32/c^3*b^2*a*(c*x^2+b*x+a)^(1/2)*d*h^3+3/16/c^(5/2)*b*a^2*ln((c*x
+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^3+7/16/c^3*b^2*(c*x^2+b*x+a)^(3/2)
*e*g*h^2-5/8/c^2*b*(c*x^2+b*x+a)^(3/2)*e*g^2*h+5/32/c^2*b^2*x*(c*x^2+b*x+a)
^(1/2)*f*g^3-1/4/c*b*x*(c*x^2+b*x+a)^(1/2)*e*g^3-3/8/c^2*b^2*(c*x^2+b*x+a)^
(1/2)*d*g^2*h-1/4/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e
*g^3+3/16/c^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*g^2*h+2
1/512/c^5*b^5*(c*x^2+b*x+a)^(1/2)*e*h^3+15/64*f*h^3/c^4*b^3*a*x*(c*x^2+b*x+
a)^(1/2)+111/560*f*h^3/c^3*b*a*x*(c*x^2+b*x+a)^(3/2)-15/32/c^(7/2)*b^3*ln((
c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f*g^2*h+3/16/c^2*b*a*x*(c*x^2+b*x
+a)^(1/2)*d*h^3+9/32/c^3*b^2*a*(c*x^2+b*x+a)^(1/2)*e*g*h^2+9/32/c^3*b^2*a*(
c*x^2+b*x+a)^(1/2)*f*g^2*h+9/16/c^(5/2)*b*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*e*g*h^2+9/16/c^(5/2)*b*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))*f*g^2*h-21/40/c^2*b*x*(c*x^2+b*x+a)^(3/2)*e*g*h^2-21/40/c^2*b*x*
(c*x^2+b*x+a)^(3/2)*f*g^2*h-21/64/c^3*b^3*x*(c*x^2+b*x+a)^(1/2)*e*g*h^2-21/
64/c^3*b^3*x*(c*x^2+b*x+a)^(1/2)*f*g^2*h+7/48/c^3*b^2*(c*x^2+b*x+a)^(3/2)*d
*h^3-7/128/c^4*b^4*(c*x^2+b*x+a)^(1/2)*d*h^3+7/256/c^(9/2)*b^5*ln((c*x+1/2*
b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^3-2/15*a/c^2*(c*x^2+b*x+a)^(3/2)*d*h^3+
1/4*x*(c*x^2+b*x+a)^(3/2)/c*f*g^3-5/24/c^2*b*(c*x^2+b*x+a)^(3/2)*f*g^3+5/64
/c^3*b^3*(c*x^2+b*x+a)^(1/2)*f*g^3-5/128/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*f*g^3-1/8*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))*f*g^3+(c*x^2+b*x+a)^(3/2)/c*d*g^2*h-1/8/c^2*b^2*(c*x^2+b*x+a)^
(1/2)*e*g^3+1/16/c^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*
g^3+1/4*d*g^3/c*(c*x^2+b*x+a)^(1/2)*b+1/2*d*g^3/c^(1/2)*ln((c*x+1/2*b)/c^(1
```

```
/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*d*g^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))*b^2+3/32*a^2/c^3*(c*x^2+b*x+a)^(1/2)*b*f*g*h^2-3/8*a/c^2*x*(c
*x^2+b*x+a)^(3/2)*f*g*h^2+3/16*a^2/c^2*x*(c*x^2+b*x+a)^(1/2)*f*g*h^2+105/25
6/c^(9/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f*g*h^2-7/32/c^
3*b^2*a*x*(c*x^2+b*x+a)^(1/2)*e*h^3-21/64/c^4*b^3*a*(c*x^2+b*x+a)^(1/2)*f*g
*h^2-45/64/c^(7/2)*b^2*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g*
h^2+49/80/c^3*b*a*(c*x^2+b*x+a)^(3/2)*f*g*h^2+3/5*x^2*(c*x^2+b*x+a)^(3/2)/c
*e*g*h^2+63/160/c^3*b^2*x*(c*x^2+b*x+a)^(3/2)*f*g*h^2+63/256/c^4*b^4*x*(c*x
^2+b*x+a)^(1/2)*f*g*h^2-5/32*f*h^3/c^3*b*a^2*x*(c*x^2+b*x+a)^(1/2)-15/32/c^
(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g*h^2+15/32/c^2*b
^2*x*(c*x^2+b*x+a)^(1/2)*d*g*h^2+15/32/c^2*b^2*x*(c*x^2+b*x+a)^(1/2)*e*g^2*
h+9/16/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*g*h^2+9/
16/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g^2*h-21/32/
c^3*b^2*a*x*(c*x^2+b*x+a)^(1/2)*f*g*h^2+9/16/c^2*b*a*x*(c*x^2+b*x+a)^(1/2)*
e*g*h^2+9/16/c^2*b*a*x*(c*x^2+b*x+a)^(1/2)*f*g^2*h-7/64/c^4*b^3*(c*x^2+b*x+
a)^(3/2)*e*h^3-9/20/c^2*b*x^2*(c*x^2+b*x+a)^(3/2)*f*g*h^2-3/20/c^2*b*x^2*(c
*x^2+b*x+a)^(3/2)*e*h^3+21/160/c^3*b^2*x*(c*x^2+b*x+a)^(3/2)*e*h^3-21/64/c^
4*b^3*(c*x^2+b*x+a)^(3/2)*f*g*h^2+21/256/c^4*b^4*x*(c*x^2+b*x+a)^(1/2)*e*h^
3+63/512/c^5*b^5*(c*x^2+b*x+a)^(1/2)*f*g*h^2+35/256/c^(9/2)*b^4*ln((c*x+1/2
*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*h^3-63/1024/c^(11/2)*b^6*ln((c*x+1/2*b
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g*h^2-7/64/c^4*b^3*a*(c*x^2+b*x+a)^(1/2)*e
*h^3-15/64/c^(7/2)*b^2*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h^
3+49/240/c^3*b*a*(c*x^2+b*x+a)^(3/2)*e*h^3-1/8*a/c^2*x*(c*x^2+b*x+a)^(3/2)*
e*h^3+1/16*a^2/c^2*x*(c*x^2+b*x+a)^(1/2)*e*h^3+1/32*a^2/c^3*(c*x^2+b*x+a)^(
1/2)*b*e*h^3+3/16*a^3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f
*g*h^2+1/2*x^3*(c*x^2+b*x+a)^(3/2)/c*f*g*h^2-21/1024/c^(11/2)*b^6*ln((c*x+1
/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h^3+1/16*a^3/c^(5/2)*ln((c*x+1/2*b)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*e*h^3+1/6*x^3*(c*x^2+b*x+a)^(3/2)/c*e*h^3+33/204
8*f*h^3/c^(13/2)*b^7*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+8/105*f*h^
3*a^2/c^3*(c*x^2+b*x+a)^(3/2)+11/128*f*h^3/c^5*b^4*(c*x^2+b*x+a)^(3/2)-33/1
024*f*h^3/c^6*b^6*(c*x^2+b*x+a)^(1/2)+1/7*f*h^3*x^4*(c*x^2+b*x+a)^(3/2)/c+1
/5*x^2*(c*x^2+b*x+a)^(3/2)/c*d*h^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 14.70, size = 3262, normalized size = 3.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*g^3*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (8*a^3*f*h^3*(a + b*x + c*x
^2)^(1/2))/(105*c^3) - (33*b^6*f*h^3*(a + b*x + c*x^2)^(1/2))/(1024*c^6) +
(d*h^3*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) + (e*h^3*x^3*(a + b*x + c*x^2)^(3
/2))/(6*c) + (f*h^3*x^4*(a + b*x + c*x^2)^(3/2))/(7*c) - (a*f*g^3*((x/2 + b
/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x
^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) + (d*g^3*log((b/2 + c*x)/c^(1
/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*g^3*log((b +
```

```
2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))
- (2*a*d*h^3*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(5*c) - (5*b*f*g^3*((log((b + 2*c*x)/c^(1/2) + 2*(a
+ b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3
*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (e*g^3*(8*c*(a
+ c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2) + (33*b^7*f*h
^3*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1/2) + 2*c*x))/(2048*c^(13/2)) + (f
*g^3*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*e*h^3*((5*b*((log((b + 2*c*x)/c^
(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a
 + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x
*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/
2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*
c^(3/2))))/(4*c)))/(2*c) + (7*b*d*h^3*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(
a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) -
 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x +
 c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((
b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/
(4*c)))/(10*c) - (3*b*e*h^3*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a +
b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b
^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x
^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2
+ c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c
)))/(10*c) - (2*a*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b
^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x
 + c*x^2)^(1/2))/(24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(
4*c) + (3*d*g*h^2*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (3*e*g^2*h*x*(a + b*x
+ c*x^2)^(3/2))/(4*c) + (3*a*f*g*h^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a
 + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) -
3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x +
c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b
/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(
4*c)))/(2*c) + (21*b*e*g*h^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x +
 c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 +
2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(
3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x
)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(
10*c) + (21*b*f*g^2*h*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)
^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x
)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(
4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/
2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) -
 (9*b*f*g*h^2*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(
1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*
(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*
c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2)
 + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (
2*a*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))
/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2
))/(24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(4*c) + (35*a^2
*b^3*f*h^3*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1/2) + 2*c*x))/(128*c^(9/2)
) + (13*a*b^4*f*h^3*(a + b*x + c*x^2)^(1/2))/(64*c^5) - (4*a*f*h^3*x^2*(a +
 b*x + c*x^2)^(3/2))/(35*c^2) - (11*b*f*h^3*x^3*(a + b*x + c*x^2)^(3/2))/(8
4*c^2) - (33*b^3*f*h^3*x*(a + b*x + c*x^2)^(3/2))/(320*c^4) + (11*b^5*f*h^3
*x*(a + b*x + c*x^2)^(1/2))/(512*c^5) + (3*e*g*h^2*x^2*(a + b*x + c*x^2)^(3
/2))/(5*c) + (3*f*g^2*h*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) + (f*g*h^2*x^3*(
a + b*x + c*x^2)^(3/2))/(2*c) - (3*a*d*g*h^2*((x/2 + b/(4*c))*(a + b*x + c*
x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2
/4))/(2*c^(3/2))))/(4*c) - (3*a*e*g^2*h*((x/2 + b/(4*c))*(a + b*x + c*x^2)^
```

```
(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/
(2*c^(3/2))))/(4*c) + (3*d*g^2*h*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x
^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (103*a^2*b^2*f*h^3*(a + b*x + c*
x^2)^(1/2))/(320*c^4) - (6*a*e*g*h^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x
 + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2
+ 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (15*b*d*g*h^2*((log(
(b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/
2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2
)))/(8*c) - (6*a*f*g^2*h*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1
/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(
a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (15*b*e*g^2*h*((log((b + 2*c*x)/
c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*
(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) +
(8*a^2*f*h^3*x^2*(a + b*x + c*x^2)^(1/2))/(105*c^2) + (33*b^2*f*h^3*x^2*(a
+ b*x + c*x^2)^(3/2))/(280*c^3) + (11*b^4*f*h^3*x^2*(a + b*x + c*x^2)^(1/2)
)/(128*c^4) + (d*g^2*h*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2
)^(1/2))/(8*c^2) - (5*a^3*b*f*h^3*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1/2)
 + 2*c*x))/(32*c^(7/2)) - (63*a*b^5*f*h^3*log(b + 2*c^(1/2)*(a + b*x + c*x^
2)^(1/2) + 2*c*x))/(512*c^(11/2)) - (39*a*b^2*f*h^3*x^2*(a + b*x + c*x^2)^(
1/2))/(160*c^3) + (111*a*b*f*h^3*x*(a + b*x + c*x^2)^(3/2))/(560*c^3) - (26
9*a^2*b*f*h^3*x*(a + b*x + c*x^2)^(1/2))/(3360*c^3) - (3*a*b^3*f*h^3*x*(a +
 b*x + c*x^2)^(1/2))/(320*c^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g + hx\right)^3 \sqrt{a + bx + cx^2} \left(d + ex + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**3*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

3.187 $\int (g+hx)^2\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=584

$$-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b^2\left(dh^2+2egh+fg^2\right)\right)-28b^2ch(2afh}{1024c^{11/2}}$$

```
[Out] -1/20*(3*b*f*h-4*c*e*h+2*c*f*g)*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c^2/h+1/6*f*(
h*x+g)^3*(c*x^2+b*x+a)^(3/2)/c/h-1/960*(105*b^3*f*h^3+64*c^3*g*(f*g^2-2*h*(
5*d*h+e*g))-28*b*c*h^2*(7*a*f*h+5*b*(e*h+2*f*g))+8*c^2*h*(16*a*h*(e*h+2*f*g
)+b*(7*f*g^2+25*h*(d*h+2*e*g)))-6*c*h*(21*b^2*f*h^2-4*c*h*(5*a*f*h+7*b*e*h+
2*b*f*g)-8*c^2*(f*g^2-h*(5*d*h+2*e*g)))*x)*(c*x^2+b*x+a)^(3/2)/c^4/h-1/1024
*(-4*a*c+b^2)*(128*c^4*d*g^2+21*b^4*f*h^2-28*b^2*c*h*(2*a*f*h+b*e*h+2*b*f*g
)-32*c^3*(2*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+8*c^2*(2*a^2*f*h^2+6*a
*b*h*(e*h+2*f*g)+5*b^2*(d*h^2+2*e*g*h+f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2
)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/512*(128*c^4*d*g^2+21*b^4*f*h^2-28*b^2*c*
h*(2*a*f*h+b*e*h+2*b*f*g)-32*c^3*(2*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2)
)+8*c^2*(2*a^2*f*h^2+6*a*b*h*(e*h+2*f*g)+5*b^2*(d*h^2+2*e*g*h+f*g^2)))*(2*c
*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

Rubi [A] time = 1.44, antiderivative size = 581, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1653, 832, 779, 612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\,\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b^2\left(h(dh+2eg)+fg^2\right)\right)-28b^2ch(2afh+beh+2b}{512c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((128*c^4*d*g^2 + 21*b^4*f*h^2 - 28*b^2*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 3
2*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 2*b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*
h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^2*(f*g^2 + h*(2*e*g + d*h))))*(b + 2*c*x)
*Sqrt[a + b*x + c*x^2])/(512*c^5) - ((2*c*f*g - 4*c*e*h + 3*b*f*h)*(g + h*x
)^2*(a + b*x + c*x^2)^(3/2))/(20*c^2*h) + (f*(g + h*x)^3*(a + b*x + c*x^2)^
(3/2))/(6*c*h) - ((105*b^3*f*h^3 + 64*c^3*(f*g^3 - 2*g*h*(e*g + 5*d*h)) - 2
8*b*c*h^2*(7*a*f*h + 5*b*(2*f*g + e*h)) + 8*c^2*h*(7*b*f*g^2 + 25*b*h*(2*e*
g + d*h) + 16*a*h*(2*f*g + e*h)) - 6*c*h*(21*b^2*f*h^2 - 4*c*h*(2*b*f*g + 7
*b*e*h + 5*a*f*h) - 8*c^2*(f*g^2 - h*(2*e*g + 5*d*h)))*x)*(a + b*x + c*x^2)
^(3/2))/(960*c^4*h) - ((b^2 - 4*a*c)*(128*c^4*d*g^2 + 21*b^4*f*h^2 - 28*b^2
*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 32*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 2*
b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^2*(f*
g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(1024*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch} + \frac{\int (g+hx)^2\left(-\frac{3}{2}h(bfg-4cdh+\cdots\right.}{\cdots}$$
$$= -\frac{(2cfg-4ceh+3bfh)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{20c^2h} + \frac{f(g+hx\cdots}{\cdots}$$
$$= -\frac{(2cfg-4ceh+3bfh)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{20c^2h} + \frac{f(g+hx\cdots}{\cdots}$$
$$= \frac{\left(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3\left(a\cdots\right.\right.}{\cdots}$$
$$= \frac{\left(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3\left(a\cdots\right.\right.}{\cdots}$$
$$= \frac{\left(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3\left(a\cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 0.97, size = 436, normalized size = 0.75

$$\frac{\frac{3h\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b^2\left(h(dh+2eg)+fg^2\right)\right)-28b^2ch(2afh+beh+2bfg)-3\cdots\right.}{512c^{9/2}}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((-3*(2*c*f*g - 4*c*e*h + 3*b*f*h)*(g + h*x)^2*(a + x*(b + c*x))^(3/2))/(10
*c) + f*(g + h*x)^3*(a + x*(b + c*x))^(3/2) - ((a + x*(b + c*x))^(3/2)*(105
*b^3*f*h^3 - 14*b*c*h^2*(14*a*f*h + b*(20*f*g + 10*e*h + 9*f*h*x)) + 8*c^2*
h*(b*f*g*(7*g + 6*h*x) + b*h*(50*e*g + 25*d*h + 21*e*h*x) + a*h*(32*f*g + 1
6*e*h + 15*f*h*x)) + 16*c^3*(f*g^2*(4*g + 3*h*x) - h*(2*e*g*(4*g + 3*h*x) +
 5*d*h*(8*g + 3*h*x)))))/(160*c^3) + (3*h*(128*c^4*d*g^2 + 21*b^4*f*h^2 - 2
8*b^2*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 32*c^3*(a*f*g^2 + a*h*(2*e*g + d*h)
 + 2*b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^
2*(f*g^2 + h*(2*e*g + d*h))))*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]
- (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(5
12*c^(9/2)))/(6*c*h)
```

fricas [A] time = 1.75, size = 1791, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/30720*(15*(8*(16*(b^2*c^4 - 4*a*c^5)*d - 8*(b^3*c^3 - 4*a*b*c^4)*e + (5
*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g^2 - 8*(16*(b^3*c^3 - 4*a*b*c^4)*
d - 2*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + (7*b^5*c - 40*a*b^3*c^2 +
 48*a^2*b*c^3)*f)*g*h + (8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*d - 4*(7
*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*e + (21*b^6 - 140*a*b^4*c + 240*a^2*b
^2*c^2 - 64*a^3*c^3)*f)*h^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*h^2*x^5 + 1
28*(24*c^6*f*g*h + (12*c^6*e + b*c^5*f)*h^2)*x^4 + 16*(120*c^6*f*g^2 + 24*(
```

```
10*c^6*e + b*c^5*f)*g*h + (120*c^6*d + 12*b*c^5*e - (9*b^2*c^4 - 20*a*c^5)*
f)*h^2)*x^3 + 40*(48*b*c^5*d - 8*(3*b^2*c^4 - 8*a*c^5)*e + (15*b^3*c^3 - 52
*a*b*c^4)*f)*g^2 - 8*(80*(3*b^2*c^4 - 8*a*c^5)*d - 10*(15*b^3*c^3 - 52*a*b*
c^4)*e + (105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*f)*g*h + (40*(15*b^3*c
^3 - 52*a*b*c^4)*d - 4*(105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*e + (315
*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f)*h^2 + 8*(40*(8*c^6*e + b*c^5*f
)*g^2 + 8*(80*c^6*d + 10*b*c^5*e - (7*b^2*c^4 - 16*a*c^5)*f)*g*h + (40*b*c^
5*d - 4*(7*b^2*c^4 - 16*a*c^5)*e + (21*b^3*c^3 - 68*a*b*c^4)*f)*h^2)*x^2 +
2*(40*(48*c^6*d + 8*b*c^5*e - (5*b^2*c^4 - 12*a*c^5)*f)*g^2 + 8*(80*b*c^5*d
 - 10*(5*b^2*c^4 - 12*a*c^5)*e + (35*b^3*c^3 - 116*a*b*c^4)*f)*g*h - (40*(5
*b^2*c^4 - 12*a*c^5)*d - 4*(35*b^3*c^3 - 116*a*b*c^4)*e + (105*b^4*c^2 - 44
8*a*b^2*c^3 + 240*a^2*c^4)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^6, 1/15360*(
15*(8*(16*(b^2*c^4 - 4*a*c^5)*d - 8*(b^3*c^3 - 4*a*b*c^4)*e + (5*b^4*c^2 -
24*a*b^2*c^3 + 16*a^2*c^4)*f)*g^2 - 8*(16*(b^3*c^3 - 4*a*b*c^4)*d - 2*(5*b^
4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c
^3)*f)*g*h + (8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*d - 4*(7*b^5*c - 40
*a*b^3*c^2 + 48*a^2*b*c^3)*e + (21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64
*a^3*c^3)*f)*h^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqr
t(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(1280*c^6*f*h^2*x^5 + 128*(24*c^6*f*g*h
+ (12*c^6*e + b*c^5*f)*h^2)*x^4 + 16*(120*c^6*f*g^2 + 24*(10*c^6*e + b*c^5*
f)*g*h + (120*c^6*d + 12*b*c^5*e - (9*b^2*c^4 - 20*a*c^5)*f)*h^2)*x^3 + 40*
(48*b*c^5*d - 8*(3*b^2*c^4 - 8*a*c^5)*e + (15*b^3*c^3 - 52*a*b*c^4)*f)*g^2
- 8*(80*(3*b^2*c^4 - 8*a*c^5)*d - 10*(15*b^3*c^3 - 52*a*b*c^4)*e + (105*b^4
*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*f)*g*h + (40*(15*b^3*c^3 - 52*a*b*c^4)*
d - 4*(105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*e + (315*b^5*c - 1680*a*b
^3*c^2 + 1808*a^2*b*c^3)*f)*h^2 + 8*(40*(8*c^6*e + b*c^5*f)*g^2 + 8*(80*c^6
*d + 10*b*c^5*e - (7*b^2*c^4 - 16*a*c^5)*f)*g*h + (40*b*c^5*d - 4*(7*b^2*c^
4 - 16*a*c^5)*e + (21*b^3*c^3 - 68*a*b*c^4)*f)*h^2)*x^2 + 2*(40*(48*c^6*d +
 8*b*c^5*e - (5*b^2*c^4 - 12*a*c^5)*f)*g^2 + 8*(80*b*c^5*d - 10*(5*b^2*c^4
- 12*a*c^5)*e + (35*b^3*c^3 - 116*a*b*c^4)*f)*g*h - (40*(5*b^2*c^4 - 12*a*c
^5)*d - 4*(35*b^3*c^3 - 116*a*b*c^4)*e + (105*b^4*c^2 - 448*a*b^2*c^3 + 240
*a^2*c^4)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^6]
```

giac [A] time = 0.31, size = 1012, normalized size = 1.73

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,fh^2x+\frac{24\,c^5fgh+bc^4fh^2+12\,c^5h^2e}{c^5}\right)x+\frac{120\,c^5fg^2+24\,bc^4fgh+120\,c^5dh^2}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f*h^2*x + (24*c^5*f*g*h + b*c^
4*f*h^2 + 12*c^5*h^2*e)/c^5)*x + (120*c^5*f*g^2 + 24*b*c^4*f*g*h + 120*c^5*
d*h^2 - 9*b^2*c^3*f*h^2 + 20*a*c^4*f*h^2 + 240*c^5*g*h*e + 12*b*c^4*h^2*e)/
c^5)*x + (40*b*c^4*f*g^2 + 640*c^5*d*g*h - 56*b^2*c^3*f*g*h + 128*a*c^4*f*g
*h + 40*b*c^4*d*h^2 + 21*b^3*c^2*f*h^2 - 68*a*b*c^3*f*h^2 + 320*c^5*g^2*e +
 80*b*c^4*g*h*e - 28*b^2*c^3*h^2*e + 64*a*c^4*h^2*e)/c^5)*x + (1920*c^5*d*g
^2 - 200*b^2*c^3*f*g^2 + 480*a*c^4*f*g^2 + 640*b*c^4*d*g*h + 280*b^3*c^2*f*
g*h - 928*a*b*c^3*f*g*h - 200*b^2*c^3*d*h^2 + 480*a*c^4*d*h^2 - 105*b^4*c*f
*h^2 + 448*a*b^2*c^2*f*h^2 - 240*a^2*c^3*f*h^2 + 320*b*c^4*g^2*e - 400*b^2*
c^3*g*h*e + 960*a*c^4*g*h*e + 140*b^3*c^2*h^2*e - 464*a*b*c^3*h^2*e)/c^5)*x
 + (1920*b*c^4*d*g^2 + 600*b^3*c^2*f*g^2 - 2080*a*b*c^3*f*g^2 - 1920*b^2*c^
3*d*g*h + 5120*a*c^4*d*g*h - 840*b^4*c*f*g*h + 3680*a*b^2*c^2*f*g*h - 2048*
a^2*c^3*f*g*h + 600*b^3*c^2*d*h^2 - 2080*a*b*c^3*d*h^2 + 315*b^5*f*h^2 - 16
80*a*b^3*c*f*h^2 + 1808*a^2*b*c^2*f*h^2 - 960*b^2*c^3*g^2*e + 2560*a*c^4*g^
2*e + 1200*b^3*c^2*g*h*e - 4160*a*b*c^3*g*h*e - 420*b^4*c*h^2*e + 1840*a*b^
2*c^2*h^2*e - 1024*a^2*c^3*h^2*e)/c^5) + 1/1024*(128*b^2*c^4*d*g^2 - 512*a*
c^5*d*g^2 + 40*b^4*c^2*f*g^2 - 192*a*b^2*c^3*f*g^2 + 128*a^2*c^4*f*g^2 - 12
8*b^3*c^3*d*g*h + 512*a*b*c^4*d*g*h - 56*b^5*c*f*g*h + 320*a*b^3*c^2*f*g*h
```

```
- 384*a^2*b*c^3*f*g*h + 40*b^4*c^2*d*h^2 - 192*a*b^2*c^3*d*h^2 + 128*a^2*c^
4*d*h^2 + 21*b^6*f*h^2 - 140*a*b^4*c*f*h^2 + 240*a^2*b^2*c^2*f*h^2 - 64*a^3
*c^3*f*h^2 - 64*b^3*c^3*g^2*e + 256*a*b*c^4*g^2*e + 80*b^4*c^2*g*h*e - 384*
a*b^2*c^3*g*h*e + 256*a^2*c^4*g*h*e - 28*b^5*c*h^2*e + 160*a*b^3*c^2*h^2*e
- 192*a^2*b*c^3*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(
c) - b))/c^(11/2)
```

maple [B] time = 0.02, size = 2179, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/2/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*g*h-1/4*a/c*
(c*x^2+b*x+a)^(1/2)*x*e*g*h-1/8*a/c^2*(c*x^2+b*x+a)^(1/2)*b*e*g*h-7/32/c^3*
b^3*(c*x^2+b*x+a)^(1/2)*x*f*g*h-5/16/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*a*f*g*h+1/3*(c*x^2+b*x+a)^(3/2)/c*e*g^2+1/2*d*g^2*(c*x^2+
b*x+a)^(1/2)*x-1/4*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
e*g*h+1/16*f*h^2*a^2/c^2*(c*x^2+b*x+a)^(1/2)*x+1/32*f*h^2*a^2/c^3*(c*x^2+b*
x+a)^(1/2)*b+21/256*f*h^2/c^4*b^4*(c*x^2+b*x+a)^(1/2)*x+35/256*f*h^2/c^(9/2
)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-7/64*f*h^2/c^4*b^3*a*(c
*x^2+b*x+a)^(1/2)-15/64*f*h^2/c^(7/2)*b^2*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))+49/240*f*h^2/c^3*b*a*(c*x^2+b*x+a)^(3/2)-1/8*f*h^2*a/c^2*x*(
c*x^2+b*x+a)^(3/2)+5/32/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*f*g^2+5/32/c^3*b^3*(c
*x^2+b*x+a)^(1/2)*e*g*h+3/16/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))*a*d*h^2+2/5*x^2*(c*x^2+b*x+a)^(3/2)/c*f*g*h-7/40/c^2*b*x*(c*x^2+b
*x+a)^(3/2)*e*h^2+7/24/c^3*b^2*(c*x^2+b*x+a)^(3/2)*f*g*h-7/64/c^3*b^3*(c*x^
2+b*x+a)^(1/2)*x*e*h^2-7/64/c^4*b^4*(c*x^2+b*x+a)^(1/2)*f*g*h-5/32/c^(7/2)*
b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*h^2+7/128/c^(9/2)*b^5*l
n((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g*h+3/32/c^3*b^2*a*(c*x^2+b*x+
a)^(1/2)*e*h^2+3/16/c^(5/2)*b*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))*e*h^2-4/15*a/c^2*(c*x^2+b*x+a)^(3/2)*f*g*h-5/12/c^2*b*(c*x^2+b*x+a)^(3/2
)*e*g*h+5/32/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*d*h^2-3/20*f*h^2/c^2*b*x^2*(c*x^
2+b*x+a)^(3/2)+21/160*f*h^2/c^3*b^2*x*(c*x^2+b*x+a)^(3/2)-1/8*a/c*(c*x^2+b*
x+a)^(1/2)*x*d*h^2-1/8*a/c*(c*x^2+b*x+a)^(1/2)*x*f*g^2-1/16*a/c^2*(c*x^2+b*
x+a)^(1/2)*b*d*h^2-1/16*a/c^2*(c*x^2+b*x+a)^(1/2)*b*f*g^2-1/8*d*g^2/c^(3/2)
*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2+1/5*x^2*(c*x^2+b*x+a)^(3/2
)/c*e*h^2+7/48/c^3*b^2*(c*x^2+b*x+a)^(3/2)*e*h^2-7/128/c^4*b^4*(c*x^2+b*x+a
)^(1/2)*e*h^2+7/256/c^(9/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*e*h^2-2/15*a/c^2*(c*x^2+b*x+a)^(3/2)*e*h^2+1/6*f*h^2*x^3*(c*x^2+b*x+a)^(3/
2)/c-7/64*f*h^2/c^4*b^3*(c*x^2+b*x+a)^(3/2)+21/512*f*h^2/c^5*b^5*(c*x^2+b*x
+a)^(1/2)-21/1024*f*h^2/c^(11/2)*b^6*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))+1/16*f*h^2*a^3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-5/
24/c^2*b*(c*x^2+b*x+a)^(3/2)*f*g^2+3/16/c^2*b*a*(c*x^2+b*x+a)^(1/2)*x*e*h^2
+3/16/c^3*b^2*a*(c*x^2+b*x+a)^(1/2)*f*g*h+3/8/c^(5/2)*b*a^2*ln((c*x+1/2*b)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g*h+5/16/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*e*g*h
+3/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g*h-1/2/c*
b*(c*x^2+b*x+a)^(1/2)*x*d*g*h-7/32*f*h^2/c^3*b^2*a*(c*x^2+b*x+a)^(1/2)*x-7/
20/c^2*b*x*(c*x^2+b*x+a)^(3/2)*f*g*h+1/2*d*g^2/c^(1/2)*ln((c*x+1/2*b)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))*a+1/4*d*g^2/c*(c*x^2+b*x+a)^(1/2)*b+5/64/c^3*b^3*(c
*x^2+b*x+a)^(1/2)*d*h^2+5/64/c^3*b^3*(c*x^2+b*x+a)^(1/2)*f*g^2-5/128/c^(7/2
)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2-5/128/c^(7/2)*b^4*l
n((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2-1/8*a^2/c^(3/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2-1/8*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))*f*g^2+2/3*(c*x^2+b*x+a)^(3/2)/c*d*g*h-1/8/c^2*b^2*
(c*x^2+b*x+a)^(1/2)*e*g^2+1/16/c^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))*e*g^2+1/4*x*(c*x^2+b*x+a)^(3/2)/c*d*h^2+1/4*x*(c*x^2+b*x+a)^(3/
2)/c*f*g^2-5/24/c^2*b*(c*x^2+b*x+a)^(3/2)*d*h^2+3/8/c^2*b*a*(c*x^2+b*x+a)^(
1/2)*x*f*g*h+3/16/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a
```

```
*f*g^2-5/64/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g*h+1
/2*x*(c*x^2+b*x+a)^(3/2)/c*e*g*h-1/4/c*b*(c*x^2+b*x+a)^(1/2)*x*e*g^2-1/4/c^
2*b^2*(c*x^2+b*x+a)^(1/2)*d*g*h-1/4/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*a*e*g^2+1/8/c^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))*d*g*h
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [**B**] time = 7.91, size = 1881, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*g^2*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (e*h^2*x^2*(a + b*x + c*x^2
)^(3/2))/(5*c) + (f*h^2*x^3*(a + b*x + c*x^2)^(3/2))/(6*c) - (a*d*h^2*((x/2
 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x +
 c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (a*f*g^2*((x/2 + b/(4*c
))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(
1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) + (d*g^2*log((b/2 + c*x)/c^(1/2) +
 (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*g^2*log((b + 2*c*
x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (2*
a*e*h^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b
*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^
(1/2))/(24*c^2)))/(5*c) - (5*b*d*h^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x
 + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2
+ 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (5*b*f*g^2*((log((b
+ 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))
 + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))
/(8*c) + (e*g^2*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2)
)/(24*c^2) + (d*h^2*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (f*g^2*x*(a + b*x +
c*x^2)^(3/2))/(4*c) + (a*f*h^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x
 + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2
+ 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)
^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c
*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))
/(2*c) + (7*b*e*h^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(
1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*
(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*
c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2)
 + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (
3*b*f*h^2*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2)
)*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a +
 b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) +
 (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (
a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (2*a*
((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16
*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(
```

```
24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(4*c) + (2*f*g*h*x^
2*(a + b*x + c*x^2)^(3/2))/(5*c) - (a*e*g*h*((x/2 + b/(4*c))*(a + b*x + c*x
^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/
4))/(2*c^(3/2))))/(2*c) + (d*g*h*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x
^2)^(1/2))*(b^3 - 4*a*b*c))/(8*c^(5/2)) - (4*a*f*g*h*((log((b + 2*c*x)/c^(1
/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a +
 c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (5*b
*e*g*h*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*
c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(
1/2))/(24*c^2)))/(4*c) + (d*g*h*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*
x + c*x^2)^(1/2))/(12*c^2) + (e*g*h*x*(a + b*x + c*x^2)^(3/2))/(2*c) + (7*b
*f*g*h*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2
 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x +
 c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)))))/(4*c)))/(5*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (g+hx)^2 \sqrt{a+bx+cx^2} \left(d+ex+fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**2*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

3.188 $\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=322

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-8c^2(aeh+afg+2bdh+2beg)+2bc(6afh+5b(eh+fg))-7b^3fh+32c^3dg\right)}{128c^4}+\frac{(a+b\ldots}{}$$

```
[Out] 1/5*f*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c/h+1/240*(35*b^2*f*h^2-16*c^2*(3*f*g^2
-5*h*(d*h+e*g))-2*c*h*(16*a*f*h+25*b*(e*h+f*g))-6*c*h*(7*b*f*h-10*c*e*h+6*c
*f*g)*x)*(c*x^2+b*x+a)^(3/2)/c^3/h-1/256*(-4*a*c+b^2)*(32*c^3*d*g-7*b^3*f*h
-8*c^2*(a*e*h+a*f*g+2*b*d*h+2*b*e*g)+2*b*c*(6*a*f*h+5*b*(e*h+f*g)))*arctanh
(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)+1/128*(32*c^3*d*g-7*b^3
*f*h-8*c^2*(a*e*h+a*f*g+2*b*d*h+2*b*e*g)+2*b*c*(6*a*f*h+5*b*(e*h+f*g)))*(2*
c*x+b)*(c*x^2+b*x+a)^(1/2)/c^4
```

Rubi [A] time = 0.50, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1653, 779, 612, 621, 206}

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(-2ch(16afh+25b(eh+fg))+35b^2fh^2-6chx(7bfh-10ceh+6cfg)+c^2\left(-\left(48fg^2-80h(dh\ldots\right.\right.\right.}{240c^3h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((32*c^3*d*g - 7*b^3*f*h - 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a*e*h) + 2*b*
c*(6*a*f*h + 5*b*(f*g + e*h)))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^4)
 + (f*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(5*c*h) + ((35*b^2*f*h^2 - c^2*(
48*f*g^2 - 80*h*(e*g + d*h)) - 2*c*h*(16*a*f*h + 25*b*(f*g + e*h)) - 6*c*h*
(6*c*f*g - 10*c*e*h + 7*b*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(240*c^3*h) - ((
b^2 - 4*a*c)*(32*c^3*d*g - 7*b^3*f*h - 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a
*e*h) + 2*b*c*(6*a*f*h + 5*b*(f*g + e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(256*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
```

```
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch} + \frac{\int (g+hx)\left(-\frac{1}{2}h(3bfg-10cdh+\right.}{}$$

$$= \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch} + \frac{\left(35b^2fh^2-c^2\left(48fg^2-80h(eg\right.\right.}{}$$

$$= \frac{\left(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh\right.}{128c^4}$$

$$= \frac{\left(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh\right.}{128c^4}$$

$$= \frac{\left(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh\right.}{128c^4}$$

Mathematica [A] time = 0.48, size = 258, normalized size = 0.80

$$\frac{\frac{(a+x(b+cx))^{3/2}\left(-2ch(16afh+b(25eh+25fg+21fhx))+35b^2fh^2+c^2(20h(4dh+4eg+3ehx)-12fg(4g+3hx))\right)}{48c^2} - \frac{5h\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4a\right.\right.}{}}{5ch}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^2*(a + x*(b + c*x))^(3/2) + ((a + x*(b + c*x))^(3/2)*(35*b^2*f
*h^2 + c^2*(-12*f*g*(4*g + 3*h*x) + 20*h*(4*e*g + 4*d*h + 3*e*h*x)) - 2*c*h
*(16*a*f*h + b*(25*f*g + 25*e*h + 21*f*h*x))))/(48*c^2) - (5*h*(-32*c^3*d*g
 + 7*b^3*f*h + 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a*e*h) - 2*b*c*(6*a*f*h +
 5*b*(f*g + e*h)))*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*
a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(256*c^(7/2))
)/(5*c*h)
```

fricas [A] time = 1.16, size = 1009, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(2*(16*(b^2*c^3 - 4*a*c^4)*d - 8*(b^3*c^2 - 4*a*b*c^3)*e + (5*
b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*g - (16*(b^3*c^2 - 4*a*b*c^3)*d - 2*(
5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*e + (7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2
)*f)*h)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*f*h*x^4 + 48*(10*c^5*f*g + (10*c^5*
e + b*c^4*f)*h)*x^3 + 8*(10*(8*c^5*e + b*c^4*f)*g + (80*c^5*d + 10*b*c^4*e
- (7*b^2*c^3 - 16*a*c^4)*f)*h)*x^2 + 10*(48*b*c^4*d - 8*(3*b^2*c^3 - 8*a*c^
4)*e + (15*b^3*c^2 - 52*a*b*c^3)*f)*g - (80*(3*b^2*c^3 - 8*a*c^4)*d - 10*(1
5*b^3*c^2 - 52*a*b*c^3)*e + (105*b^4*c - 460*a*b^2*c^2 + 256*a^2*c^3)*f)*h
+ 2*(10*(48*c^5*d + 8*b*c^4*e - (5*b^2*c^3 - 12*a*c^4)*f)*g + (80*b*c^4*d -
 10*(5*b^2*c^3 - 12*a*c^4)*e + (35*b^3*c^2 - 116*a*b*c^3)*f)*h)*x)*sqrt(c*x
^2 + b*x + a))/c^5, 1/3840*(15*(2*(16*(b^2*c^3 - 4*a*c^4)*d - 8*(b^3*c^2 -
4*a*b*c^3)*e + (5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*g - (16*(b^3*c^2 -
4*a*b*c^3)*d - 2*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*e + (7*b^5 - 40*a*b^
3*c + 48*a^2*b*c^2)*f)*h)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(384*c^5*f*h*x^4 + 48*(10*c^5*f*
g + (10*c^5*e + b*c^4*f)*h)*x^3 + 8*(10*(8*c^5*e + b*c^4*f)*g + (80*c^5*d +
 10*b*c^4*e - (7*b^2*c^3 - 16*a*c^4)*f)*h)*x^2 + 10*(48*b*c^4*d - 8*(3*b^2*
c^3 - 8*a*c^4)*e + (15*b^3*c^2 - 52*a*b*c^3)*f)*g - (80*(3*b^2*c^3 - 8*a*c^
4)*d - 10*(15*b^3*c^2 - 52*a*b*c^3)*e + (105*b^4*c - 460*a*b^2*c^2 + 256*a^
2*c^3)*f)*h + 2*(10*(48*c^5*d + 8*b*c^4*e - (5*b^2*c^3 - 12*a*c^4)*f)*g + (
80*b*c^4*d - 10*(5*b^2*c^3 - 12*a*c^4)*e + (35*b^3*c^2 - 116*a*b*c^3)*f)*h)
*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

giac [A] time = 0.24, size = 495, normalized size = 1.54

$$\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(8\,fhx+\frac{10\,c^4fg+bc^3fh+10\,c^4he}{c^4}\right)x+\frac{10\,bc^3fg+80\,c^4dh-7\,b^2c^2fh+16\,ac^3fh+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*f*h*x + (10*c^4*f*g + b*c^3*f*h +
10*c^4*h*e)/c^4)*x + (10*b*c^3*f*g + 80*c^4*d*h - 7*b^2*c^2*f*h + 16*a*c^3*
f*h + 80*c^4*g*e + 10*b*c^3*h*e)/c^4)*x + (480*c^4*d*g - 50*b^2*c^2*f*g + 1
20*a*c^3*f*g + 80*b*c^3*d*h + 35*b^3*c*f*h - 116*a*b*c^2*f*h + 80*b*c^3*g*e
 - 50*b^2*c^2*h*e + 120*a*c^3*h*e)/c^4)*x + (480*b*c^3*d*g + 150*b^3*c*f*g
- 520*a*b*c^2*f*g - 240*b^2*c^2*d*h + 640*a*c^3*d*h - 105*b^4*f*h + 460*a*b
^2*c*f*h - 256*a^2*c^2*f*h - 240*b^2*c^2*g*e + 640*a*c^3*g*e + 150*b^3*c*h*
e - 520*a*b*c^2*h*e)/c^4) + 1/256*(32*b^2*c^3*d*g - 128*a*c^4*d*g + 10*b^4*
c*f*g - 48*a*b^2*c^2*f*g + 32*a^2*c^3*f*g - 16*b^3*c^2*d*h + 64*a*b*c^3*d*h
 - 7*b^5*f*h + 40*a*b^3*c*f*h - 48*a^2*b*c^2*f*h - 16*b^3*c^2*g*e + 64*a*b*
c^3*g*e + 10*b^4*c*h*e - 48*a*b^2*c^2*h*e + 32*a^2*c^3*h*e)*log(abs(-2*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

maple [B] time = 0.01, size = 1117, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/3*(c*x^2+b*x+a)^(3/2)/c*d*h+1/3*(c*x^2+b*x+a)^(3/2)/c*e*g+1/2*d*g*(c*x^2+
b*x+a)^(1/2)*x+3/16*h*f/c^2*b*a*(c*x^2+b*x+a)^(1/2)*x-7/40*h*f/c^2*b*x*(c*x
^2+b*x+a)^(3/2)+5/32/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*e*h+3/16*h*f/c^(5/2)*b*a
^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/32*h*f/c^3*b^2*a*(c*x^2+b*
x+a)^(1/2)+1/4*x*(c*x^2+b*x+a)^(3/2)/c*e*h+1/4*x*(c*x^2+b*x+a)^(3/2)/c*f*g-
```

```
5/32*h*f/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-7/64*h*f
/c^3*b^3*(c*x^2+b*x+a)^(1/2)*x+5/32/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*f*g+3/16/
c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*h+3/16/c^(5/2)*
b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f*g-1/8*a/c*(c*x^2+b*x+a)
^(1/2)*x*e*h-1/8*a/c*(c*x^2+b*x+a)^(1/2)*x*f*g-1/16*a/c^2*(c*x^2+b*x+a)^(1/
2)*b*e*h-1/4/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*h-5/
24/c^2*b*(c*x^2+b*x+a)^(3/2)*e*h-5/24/c^2*b*(c*x^2+b*x+a)^(3/2)*f*g+5/64/c^
3*b^3*(c*x^2+b*x+a)^(1/2)*e*h+1/2*d*g/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*a-1/8*d*g/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
)*b^2-1/8/c^2*b^2*(c*x^2+b*x+a)^(1/2)*d*h-1/8/c^2*b^2*(c*x^2+b*x+a)^(1/2)*e
*g+1/16/c^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h+1/16/c^
(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g+1/4*d*g/c*(c*x^2+
b*x+a)^(1/2)*b+1/5*h*f*x^2*(c*x^2+b*x+a)^(3/2)/c+7/48*h*f/c^3*b^2*(c*x^2+b*
x+a)^(3/2)-7/128*h*f/c^4*b^4*(c*x^2+b*x+a)^(1/2)+7/256*h*f/c^(9/2)*b^5*ln((
c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-2/15*h*f*a/c^2*(c*x^2+b*x+a)^(3/2)+
5/64/c^3*b^3*(c*x^2+b*x+a)^(1/2)*f*g-5/128/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))*e*h-5/128/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*f*g-1/8*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))*e*h-1/8*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g-1/1
6*a/c^2*(c*x^2+b*x+a)^(1/2)*b*f*g-1/4/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*a*e*g-1/4/c*b*(c*x^2+b*x+a)^(1/2)*x*d*h-1/4/c*b*(c*x^2+b*x
+a)^(1/2)*x*e*g
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 5.62, size = 877, normalized size = 2.72

$$d\,g\,\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a}-\frac{2\,a\,f\,h\,\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{5\,c}-\frac{5\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*g*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (2*a*f*h*((log((b + 2*c*x)/c^
(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a
 + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (5
*b*e*h*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*
c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(
1/2))/(24*c^2)))/(8*c) - (5*b*f*g*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x +
c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2
*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (d*h*(8*c*(a + c*x^2) -
 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2) + (e*g*(8*c*(a + c*x^2)
```

```
 - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2) + (e*h*x*(a + b*x + c
*x^2)^(3/2))/(4*c) + (f*g*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (7*b*f*h*((5*b
*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(1
6*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/
(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*
(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2
))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) + (f*h*x^2*(a + b*x + c*x^2)
^(3/2))/(5*c) - (a*e*h*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2
 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*
c) - (a*f*g*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(
1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) + (d*g*l
og((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)
) + (d*h*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*
c))/(16*c^(5/2)) + (e*g*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2)
)*(b^3 - 4*a*b*c))/(16*c^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (g + hx) \sqrt{a + bx + cx^2} \left(d + ex + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

3.189 $\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=175

$$-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+2be)+5b^2f+16c^2d\right)}{128c^{7/2}}+\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acf+5b^2f\right.}{64c^3}$$

```
[Out] 1/24*(-5*b*f+8*c*e)*(c*x^2+b*x+a)^(3/2)/c^2+1/4*f*x*(c*x^2+b*x+a)^(3/2)/c-1
/128*(-4*a*c+b^2)*(16*c^2*d+5*b^2*f-4*c*(a*f+2*b*e))*arctanh(1/2*(2*c*x+b)/
c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/64*(-4*a*c*f+5*b^2*f-8*b*c*e+16*c^2*
d)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^3
```

Rubi [A] time = 0.17, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acf+5b^2f-8bce+16c^2d\right)}{64c^3}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+2be)+\right.}{128c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
)/(64*c^3) + ((8*c*e - 5*b*f)*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (f*x*(a +
 b*x + c*x^2)^(3/2))/(4*c) - ((b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*
e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7
/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
```

```
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx = \frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}+\frac{\int\left(4cd-af+\frac{1}{2}(8ce-5bf)x\right)\sqrt{a+bx+cx^2}\,dx}{4c}$$

$$= \frac{(8ce-5bf)\left(a+bx+cx^2\right)^{3/2}}{24c^2}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}+\frac{\left(16c^2d-8bce+5b\right.}{}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{(8ce-5bf)\left(a+\right.}{24c^2}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{(8ce-5bf)\left(a+\right.}{24c^2}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{(8ce-5bf)\left(a+\right.}{24c^2}$$

Mathematica [A] time = 0.30, size = 173, normalized size = 0.99

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(4bc\left(2c\left(6d+2ex+fx^2\right)-13af\right)+8c^2\left(a(8e+3fx)+2cx\left(6d+4ex+3fx^2\right)\right)+15b^3f-2\right.}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^3*f - 2*b^2*c*(12*e + 5*f*x) + 4*b*c
*(-13*a*f + 2*c*(6*d + 2*e*x + f*x^2)) + 8*c^2*(a*(8*e + 3*f*x) + 2*c*x*(6*
d + 4*e*x + 3*f*x^2))) - 3*(b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*e +
 a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(384*c^(7/2)
)
```

fricas [A] time = 0.80, size = 465, normalized size = 2.66

$$\left[\frac{3\left(16\left(b^2c^2-4\,ac^3\right)d-8\left(b^3c-4\,abc^2\right)e+\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)f\right)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c^2)*e + (5*b^4 - 24
*a*b^2*c + 16*a^2*c^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*x^3 + 48*b*c^3*d
 + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15*b^3*c - 52*a
*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f)*x)*sqrt(c*x
^2 + b*x + a))/c^4, 1/384*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c
^2)*e + (5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*f*x^3
 + 48*b*c^3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15
```

```
*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f
)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

giac [A] time = 0.24, size = 212, normalized size = 1.21

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,fx+\frac{bc^2f+8\,c^3e}{c^3}\right)x+\frac{48\,c^3d-5\,b^2cf+12\,ac^2f+8\,bc^2e}{c^3}\right)x+\frac{48\,bc^2d+15\,b^3f-5}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*x + (b*c^2*f + 8*c^3*e)/c^3)*x + (48
*c^3*d - 5*b^2*c*f + 12*a*c^2*f + 8*b*c^2*e)/c^3)*x + (48*b*c^2*d + 15*b^3*
f - 52*a*b*c*f - 24*b^2*c*e + 64*a*c^2*e)/c^3) + 1/128*(16*b^2*c^2*d - 64*a
*c^3*d + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2*f - 8*b^3*c*e + 32*a*b*c^2*e)*
log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

maple [B] time = 0.01, size = 453, normalized size = 2.59

$$-\frac{a^2 f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}+\frac{3a\,b^2 f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{16c^{\frac{5}{2}}}-\frac{abe\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{4c^{\frac{3}{2}}}+\frac{ad\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/4*f*x*(c*x^2+b*x+a)^(3/2)/c-5/24*f/c^2*b*(c*x^2+b*x+a)^(3/2)+5/32*f/c^2*b
^2*(c*x^2+b*x+a)^(1/2)*x+5/64*f/c^3*b^3*(c*x^2+b*x+a)^(1/2)+3/16*f/c^(5/2)*
b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-5/128*f/c^(7/2)*b^4*ln((c
*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/8*f*a/c*(c*x^2+b*x+a)^(1/2)*x-1/16
*f*a/c^2*(c*x^2+b*x+a)^(1/2)*b-1/8*f*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))+1/3*e*(c*x^2+b*x+a)^(3/2)/c-1/4*e/c*b*(c*x^2+b*x+a)^(1/2)
*x-1/8*e/c^2*b^2*(c*x^2+b*x+a)^(1/2)-1/4*e/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*a+1/16*e/c^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))+1/2*d*(c*x^2+b*x+a)^(1/2)*x+1/4*d/c*(c*x^2+b*x+a)^(1/2)*b+1/2*d
/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*d/c^(3/2)*ln((c*
x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 4.24, size = 320, normalized size = 1.83

$$d\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a}-\frac{a\,f\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}+\frac{d\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (a*f*((x/2 + b/(4*c))*(a + b*x
+ c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c -
 b^2/4))/(2*c^(3/2))))/(4*c) + (d*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^
2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*log((b + 2*c*x)/c^(1/2) + 2*(a +
b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (5*b*f*((log((b + 2*c*x
)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*
c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c)
+ (e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)
+ (f*x*(a + b*x + c*x^2)^(3/2))/(4*c)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

3.190 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{g+hx}\,dx$

Optimal. Leaf size=321

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(2cg-bh)(bfg-2cdh)-\left(-4ch(bg-ah)-b^2h^2+8c^2g^2\right)(bfh-2ceh+2cfg)\right)}{16c^{5/2}h^4}$$

```
[Out] 1/3*f*(c*x^2+b*x+a)^(3/2)/c/h+1/16*(4*c*h*(-b*h+2*c*g)*(b*f*g-2*c*d*h)-(b*f
*h-2*c*e*h+2*c*f*g)*(8*c^2*g^2-b^2*h^2-4*c*h*(-a*h+b*g)))*arctanh(1/2*(2*c*
x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/h^4+(d*h^2-e*g*h+f*g^2)*arctanh(1
/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2)
)*(a*h^2-b*g*h+c*g^2)^(1/2)/h^4-1/8*(4*c*h*(b*f*g-2*c*d*h)-(-b*h+4*c*g)*(b*
f*h-2*c*e*h+2*c*f*g)+2*c*h*(b*f*h-2*c*e*h+2*c*f*g)*x)*(c*x^2+b*x+a)^(1/2)/c
^2/h^3
```

Rubi [A] time = 0.78, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 814, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(2cg-bh)(bfg-2cdh)-\left(-4ch(bg-ah)-b^2h^2+8c^2g^2\right)(bfh-2ceh+2cfg)\right)}{16c^{5/2}h^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] -((4*c*h*(b*f*g - 2*c*d*h) - (4*c*g - b*h)*(2*c*f*g - 2*c*e*h + b*f*h) + 2*
c*h*(2*c*f*g - 2*c*e*h + b*f*h)*x)*Sqrt[a + b*x + c*x^2])/(8*c^2*h^3) + (f*
(a + b*x + c*x^2)^(3/2))/(3*c*h) + ((4*c*h*(2*c*g - b*h)*(b*f*g - 2*c*d*h)
- (2*c*f*g - 2*c*e*h + b*f*h)*(8*c^2*g^2 - b^2*h^2 - 4*c*h*(b*g - a*h)))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*h^4) + (S
qrt[c*g^2 - b*g*h + a*h^2]*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(b*g - 2*a*h + (
2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/h^4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
```

```
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{g+hx}\,dx = \frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch} + \frac{\int \frac{\left(-\frac{3}{2}h(bfg-2cdh)-\frac{3}{2}h(2cfg-2ceh+bfh)x\right)\sqrt{a+bx+cx^2}}{g+hx}\,dx}{3ch^2}$$

$$= -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+}{8c^2h^3}$$

$$= -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+}{8c^2h^3}$$

$$= -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+}{8c^2h^3}$$

$$= -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+}{8c^2h^3}$$

Mathematica [A] time = 0.79, size = 331, normalized size = 1.03

$$2\sqrt{c}\left(h\sqrt{a+x(b+cx)}\left(2ch(4afh+b(3eh-3fg+fhx))-3b^2fh^2+4c^2\left(3h(2dh-2eg+ehx)+f\left(6g^2-3ghx+\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] (-3*(-(b^3*f*h^3) + 2*b*c*h^2*(-(b*f*g) + b*e*h + 2*a*f*h) + 16*c^3*g*(f*g^
2 + h*(-(e*g) + d*h)) - 8*c^2*h*(b*f*g^2 + b*h*(-(e*g) + d*h) + a*h*(-(f*g)
 + e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c
]*(h*Sqrt[a + x*(b + c*x)]*(-3*b^2*f*h^2 + 2*c*h*(4*a*f*h + b*(-3*f*g + 3*e
*h + f*h*x)) + 4*c^2*(3*h*(-2*e*g + 2*d*h + e*h*x) + f*(6*g^2 - 3*g*h*x + 2
*h^2*x^2))) - 24*c^2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*(f*g^2 + h*(-(e*g) + d*
h))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) +
a*h)]*Sqrt[a + x*(b + c*x)])]))/(48*c^(5/2)*h^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 2549, normalized size = 7.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x)
```

```
[Out] 1/3*f*(c*x^2+b*x+a)^(3/2)/c/h+1/2/h*e*(c*x^2+b*x+a)^(1/2)*x-1/h^2*((x+g/h)^
2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e*g+1/h^3*((x+g/h)
^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*f*g^2+1/h*((x+g/h
)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*d+1/h^3*ln((1/2*
(b*h-2*c*g)/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-
b*g*h+c*g^2)/h^2)^(1/2))*c^(1/2)*g^2*e-1/h^4*ln((1/2*(b*h-2*c*g)/h+(x+g/h)*
c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
))*c^(1/2)*g^3*f-1/h/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g
^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*
c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*a*d-1/8/h*
f/c^2*b^2*(c*x^2+b*x+a)^(1/2)+1/16/h*f/c^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))+1/4/h*e/c*(c*x^2+b*x+a)^(1/2)*b+1/2/h*e/c^(1/2)*ln((c*x
+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8/h*e/c^(3/2)*ln((c*x+1/2*b)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))*b^2-1/2/h^2*f*g*(c*x^2+b*x+a)^(1/2)*x-1/2/h^2*ln((1
/2*(b*h-2*c*g)/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h
^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)*b*e*g+1/2/h^3*ln((1/2*(b*h-2*c*g)/h+(x+
g/h)*c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)
^(1/2))/c^(1/2)*b*f*g^2+1/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-
b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((
x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*a
*e*g-1/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b
```

```
*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c
*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*a*f*g^2+1/h^2/((a*h^
2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/
h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*g*d-1/h^3/((a*h^2-b*g*h+c*g^2)/h^2)
^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+
c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^
2)^(1/2))/(x+g/h))*b*g^2*e+1/2/h*ln((1/2*(b*h-2*c*g)/h+(x+g/h)*c)/c^(1/2)+(
(x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)*b
*d-1/h^2*ln((1/2*(b*h-2*c*g)/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/
h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))*c^(1/2)*g*d+1/h^4/((a*h^2-b*g*h+c
*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h
+c*g^2)/h^2)^(1/2))/(x+g/h))*b*g^3*f-1/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*
ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
))/(x+g/h))*c*g^2*d+1/h^4/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*
h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/
h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^3
*e-1/h^5/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h
-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g
)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^4*f-1/4/h*f/c*b*(c
*x^2+b*x+a)^(1/2)*x-1/4/h*f/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))*a-1/4/h^2*f*g/c*(c*x^2+b*x+a)^(1/2)*b-1/2/h^2*f*g/c^(1/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/8/h^2*f*g/c^(3/2)*ln((c*x+1/2*b)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))*b^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re details)Is b*h-2*c*g zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

```
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}\left(d + ex + fx^2\right)}{g + hx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x), x)
```

3.191 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx$

Optimal. Leaf size=459

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(-afh-beh+2bfg)+b^2fh^2-8c^2\left(3fg^2-h(2eg-dh)\right)\right)}{8c^{3/2}h^4}-\frac{\sqrt{a+bx+cx^2}\left(2ch\right.}{}$$

```
[Out] -(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)-1/8
*(b^2*f*h^2+4*c*h*(-a*f*h-b*e*h+2*b*f*g)-8*c^2*(3*f*g^2-h*(-d*h+2*e*g)))*ar
ctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/h^4-1/2*(2*c*g*(3*
f*g^2-h*(-d*h+2*e*g))+h*(2*a*h*(-e*h+2*f*g)-b*(d*h^2-3*e*g*h+5*f*g^2)))*arc
tanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)
^(1/2))/h^4/(a*h^2-b*g*h+c*g^2)^(1/2)-1/4*(b*f*h^2*(-a*h+b*g)+4*c^2*g*(3*f*
g^2-h*(-d*h+2*e*g))+c*h*(4*a*h*(-e*h+2*f*g)-b*(4*d*h^2-8*e*g*h+13*f*g^2))+2
*c*h^2*(2*c*e*g+b*f*g-3*c*f*g^2/h-2*c*d*h-a*f*h)*x)*(c*x^2+b*x+a)^(1/2)/c/h
^3/(a*h^2-b*g*h+c*g^2)
```

Rubi [A] time = 1.10, antiderivative size = 453, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 814, 843, 621, 206, 724}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(-afh-beh+2bfg)+b^2fh^2-8c^2\left(3fg^2-h(2eg-dh)\right)\right)}{8c^{3/2}h^4}-\frac{\sqrt{a+bx+cx^2}\left(2chx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -((b*f*h*(b*g - a*h) - 4*c^2*g*(2*e*g - (3*f*g^2)/h - d*h) + 4*a*c*h*(2*f*g
 - e*h) - b*c*(13*f*g^2 - 8*e*g*h + 4*d*h^2) + 2*c*h*(2*c*e*g + b*f*g - (3*
c*f*g^2)/h - 2*c*d*h - a*f*h)*x)*Sqrt[a + b*x + c*x^2])/(4*c*h^2*(c*g^2 - b
*g*h + a*h^2)) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(h*(c*g^
2 - b*g*h + a*h^2)*(g + h*x)) - ((b^2*f*h^2 + 4*c*h*(2*b*f*g - b*e*h - a*f*
h) - 8*c^2*(3*f*g^2 - h*(2*e*g - d*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/(8*c^(3/2)*h^4) - ((2*c*(3*f*g^3 - g*h*(2*e*g - d*h))
- h*(5*b*f*g^2 - b*h*(3*e*g - d*h) - 2*a*h*(2*f*g - e*h)))*ArcTanh[(b*g - 2
*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2
])])/(2*h^4*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} - \int \frac{\left(\frac{1}{2}\left(-2cdg+3beg+2afg-\frac{3bfg^2}{h}-bdh-2aeh \ldots\right.\right.}{cg^2-\ldots}$$

$$= -\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc\left(13fg^2-\ldots\right.\right.}{4ch^2\left(cg^2-\ldots\right.}$$

$$= -\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc\left(13fg^2-\ldots\right.\right.}{4ch^2\left(cg^2-\ldots\right.}$$

$$= -\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc\left(13fg^2-\ldots\right.\right.}{4ch^2\left(cg^2-\ldots\right.}$$

$$= -\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc\left(13fg^2-\ldots\right.\right.}{4ch^2\left(cg^2-\ldots\right.}$$

Mathematica [A] time = 1.56, size = 486, normalized size = 1.06

$$-\frac{\frac{\left(h(ah-bg)+cg^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(4ch(afh+beh-2bfg)-b^2fh^2+8c^2\left(h(dh-2eg)+3fg^2\right)\right)}{\sqrt{c}}+2h\sqrt{a+x(b+cx)}\left(ch(2ah(2eh-4fg+fhx)+4bh(dh-2eg)+bfg(1\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] ((f*(a + x*(b + c*x))^(3/2))/(g + h*x) - ((3*c*f*g^2 + f*h*(-(b*g) + a*h) +
 2*c*h*(-(e*g) + d*h))*(a + x*(b + c*x))^(3/2))/((c*g^2 + h*(-(b*g) + a*h))
*(g + h*x)) - (2*h*Sqrt[a + x*(b + c*x)]*(b*f*h^2*(-(b*g) + a*h) + c*h*(4*b
*h*(-2*e*g + d*h) + b*f*g*(13*g - 2*h*x) + 2*a*h*(-4*f*g + 2*e*h + f*h*x))
+ c^2*(6*f*g^2*(-2*g + h*x) + 4*h*(e*g*(2*g - h*x) + d*h*(-g + h*x)))) + ((
c*g^2 + h*(-(b*g) + a*h))*(-(b^2*f*h^2) + 4*c*h*(-2*b*f*g + b*e*h + a*f*h)
+ 8*c^2*(3*f*g^2 + h*(-2*e*g + d*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + x*(b + c*x)])]])/Sqrt[c] + 4*c*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*(2*c*(3*f*g
^3 + g*h*(-2*e*g + d*h)) - h*(5*b*f*g^2 + b*h*(-3*e*g + d*h) + 2*a*h*(-2*f*
g + e*h)))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(
b*g) + a*h)]*Sqrt[a + x*(b + c*x)])]])/(4*h^3*(-(c*g^2) + h*(b*g - a*h))))/(
2*c*h)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 6218, normalized size = 13.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re details)Is b*h-2*c*g zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

```
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}{\left(g+hx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**2,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

3.192 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

Optimal. Leaf size=448

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2\left(8a^2fh^2-4abh(6fg-eh)+b^2\left(15fg^2-h(dh+3eg)\right)\right)-4ch\left(bg^2(10fg-\right.\right.}{8h^4\left(ah^2-bgh+cg^2\right)^{3/2}}$$

```
[Out] -1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
^2+1/8*(8*c^2*g^3*(-e*h+3*f*g)-4*c*h*(b*g^2*(-3*e*h+10*f*g)-a*h*(d*h^2-3*e*
g*h+9*f*g^2))+h^2*(8*a^2*f*h^2-4*a*b*h*(-e*h+6*f*g)+b^2*(15*f*g^2-h*(d*h+3*
e*g))))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c
*x^2+b*x+a)^(1/2))/h^4/(a*h^2-b*g*h+c*g^2)^(3/2)-1/2*(-b*f*h-2*c*e*h+6*c*f*
g)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/h^4/c^(1/2)+1/4*(4*c*
g^2*(-e*h+3*f*g)/h+4*a*h*(-e*h+3*f*g)-b*(-d*h^2-3*e*g*h+11*f*g^2)-2*h*(c*e*
g+2*b*f*g-3*c*f*g^2/h-c*d*h-2*a*f*h)*x)*(c*x^2+b*x+a)^(1/2)/h^2/(a*h^2-b*g*
h+c*g^2)/(h*x+g)
```

Rubi [A] time = 0.87, antiderivative size = 446, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 812, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2\left(8a^2fh^2-4abh(6fg-eh)+b^2\left(15fg^2-h(dh+3eg)\right)\right)-4ch\left(bg^2(10fg-\right.\right.}{8h^4\left(ah^2-bgh+cg^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] -((11*b*f*g^2 - b*h*(3*e*g + d*h) - (4*c*g^2*(3*f*g - e*h))/h - 4*a*h*(3*f*
g - e*h) + 2*h*(c*e*g + 2*b*f*g - (3*c*f*g^2)/h - c*d*h - 2*a*f*h)*x)*Sqrt[
a + b*x + c*x^2])/(4*h^2*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) - ((f*g^2 - h*(
e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(2*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)
^2) - ((6*c*f*g - 2*c*e*h - b*f*h)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(2*Sqrt[c]*h^4) + ((8*c^2*g^3*(3*f*g - e*h) - 4*c*h*(b*g^2*
(10*f*g - 3*e*h) - a*h*(9*f*g^2 - 3*e*g*h + d*h^2)) + h^2*(8*a^2*f*h^2 - 4*
a*b*h*(6*f*g - e*h) + b^2*(15*f*g^2 - h*(3*e*g + d*h))))*ArcTanh[(b*g - 2*a
*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])
])/(8*h^4*(c*g^2 - b*g*h + a*h^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} - \frac{\int \frac{\left(\frac{1}{2}\left(-4cdg+3beg+4afg-\frac{3bfg^2}{h}+bdh-4aeh \cdots\right.\right.}{\cdots}}{2\left(cg^2 \cdots\right.}$$

$$= -\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg+2bfg \cdots\right.\right.}{4h^2\left(cg^2-bgh+ah^2\right)(g+hx) \cdots}$$

$$= -\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg+2bfg \cdots\right.\right.}{4h^2\left(cg^2-bgh+ah^2\right)(g+hx) \cdots}$$

$$= -\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg+2bfg \cdots\right.\right.}{4h^2\left(cg^2-bgh+ah^2\right)(g+hx) \cdots}$$

$$= -\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg+2bfg \cdots\right.\right.}{4h^2\left(cg^2-bgh+ah^2\right)(g+hx) \cdots}$$

Mathematica [A] time = 3.58, size = 645, normalized size = 1.44

$$-\frac{\frac{2c\sqrt{a+x(b+cx)}\left(h^2\left(-4a^2fh^2-4abh(eh-4fg)+b^2\left(dh^2+3egh-11fg^2\right)\right)+ch\left(b\left(h(dh(hx-g)+eg(3hx-7g))+fg^2(23g-7hx)\right)-2ah(h(dh-3eg+2ehx)+fg(9g-4hx))\right)-2c^2\left(gh\left(dh^2x+eg(h\right.\right.\right.}{h^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((f*(a + x*(b + c*x))^(3/2))/(g + h*x)^2 - ((3*c*f*g^2 + 2*f*h*(-(b*g) + a*
h) + c*h*(-(e*g) + d*h))*(a + x*(b + c*x))^(3/2))/(2*(c*g^2 + h*(-(b*g) + a
*h))*(g + h*x)^2) - ((-2*c*(6*c*f*g^3 - 2*c*g*h*(e*g + d*h) - 4*a*h^2*(-2*f
*g + e*h) + b*h*(-7*f*g^2 + h*(3*e*g + d*h)))*(a + x*(b + c*x))^(3/2))/(g +
 h*x) + (2*c*Sqrt[a + x*(b + c*x)]*(h^2*(-4*a^2*f*h^2 - 4*a*b*h*(-4*f*g + e
*h) + b^2*(-11*f*g^2 + 3*e*g*h + d*h^2)) - 2*c^2*(3*f*g^3*(2*g - h*x) + g*h
*(d*h^2*x + e*g*(-2*g + h*x))) + c*h*(-2*a*h*(f*g*(9*g - 4*h*x) + h*(-3*e*g
 + d*h + 2*e*h*x)) + b*(f*g^2*(23*g - 7*h*x) + h*(d*h*(-g + h*x) + e*g*(-7*
g + 3*h*x))))))/h^2 + (4*Sqrt[c]*(6*c*f*g - 2*c*e*h - b*f*h)*(c*g^2 + h*(-(
b*g) + a*h))^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + c*S
qrt[c*g^2 + h*(-(b*g) + a*h)]*(8*c^2*g^3*(3*f*g - e*h) + 4*c*h*(b*g^2*(-10*
f*g + 3*e*h) + a*h*(9*f*g^2 - 3*e*g*h + d*h^2)) + h^2*(8*a^2*f*h^2 + 4*a*b*
h*(-6*f*g + e*h) + b^2*(15*f*g^2 - h*(3*e*g + d*h))))*ArcTanh[(-(b*g) + 2*a
*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x
)])])/h^3)/(8*(c*g^2 + h*(-(b*g) + a*h))^2))/(c*h)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.81Unable to divide, perha
ps due to rounding error%%%{1,[6,0,0,7,0,0,0,0]%%%}+%%%{%%{[-6,0]:[1,0,%%%{
-1,[1]%%%}]%%},[5,0,0,6,1,0,0,0]%%%}+%%%{3,[4,1,0,6,1,0,0,0]%%%}+%%%{-3,[4,
0,0,7,0,0,1,0]%%%}+%%%{%%%{12,[1]%%%},[4,0,0,5,2,0,0,0]%%%}+%%%{%%{[-12,0]:
[1,0,%%%{-1,[1]%%%}]%%},[3,1,0,5,2,0,0,0]%%%}+%%%{%%{[12,0]:[1,0,%%%{-1,[1]
%%%}]%%},[3,0,0,6,1,0,1,0]%%%}+%%%{%%{[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[3,0,0,4,3,0,0,0]%%%}+%%%{3,[2,2,0,5,2,0,0,0]%%%}+%%%{-6,[2,1,0,6,1,0
,1,0]%%%}+%%%{%%%{12,[1]%%%},[2,1,0,4,3,0,0,0]%%%}+%%%{3,[2,0,0,7,0,0,2,0]%
%%}+%%%{%%%{-12,[1]%%%},[2,0,0,5,2,0,1,0]%%%}+%%%{%%{[-6,0]:[1,0,%%%{-1,[1]
%%%}]%%},[1,2,0,4,3,0,0,0]%%%}+%%%{%%{[12,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,0
,5,2,0,1,0]%%%}+%%%{%%{[-6,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,0,6,1,0,2,0]%%%}
+%%%{1,[0,3,0,4,3,0,0,0]%%%}+%%%{-3,[0,2,0,5,2,0,1,0]%%%}+%%%{3,[0,1,0,6,1,
0,2,0]%%%}+%%%{-1,[0,0,0,7,0,0,3,0]%%%} / %%%{%%{poly1[%%%{1,[1]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[6,0,0,3,0,0,0,0]%%%}+%%%{%%%{-6,[2]%%%},[5,0,0,2,1,0,
```

```
0,0]%%%}+%%%{%%{[%%%{3,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,1,0,2,1,0,0,0]
%%%}+%%%{%%{poly1[%%%{-3,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0,3,0,0,1,
0]%%%}+%%%{%%{poly1[%%%{12,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0,1,2,0,
0,0]%%%}+%%%{%%%{-12,[2]%%%},[3,1,0,1,2,0,0,0]%%%}+%%%{%%%{12,[2]%%%},[3,0,
0,2,1,0,1,0]%%%}+%%%{%%%{-8,[3]%%%},[3,0,0,0,3,0,0,0]%%%}+%%%{%%{[%%%{3,[1]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,0,1,2,0,0,0]%%%}+%%%{%%{[%%%{-6,[1]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,1,0,2,1,0,1,0]%%%}+%%%{%%{[%%%{12,[2]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[2,1,0,0,3,0,0,0]%%%}+%%%{%%{poly1[%%%{3,[1]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,0,3,0,0,2,0]%%%}+%%%{%%{poly1[%%%{-12,[2]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,0,1,2,0,1,0]%%%}+%%%{%%%{-6,[2]%%%},[1,2
,0,0,3,0,0,0]%%%}+%%%{%%%{12,[2]%%%},[1,1,0,1,2,0,1,0]%%%}+%%%{%%%{-6,[2]%%
%},[1,0,0,2,1,0,2,0]%%%}+%%%{%%{[%%%{1,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
0,3,0,0,3,0,0,0]%%%}+%%%{%%{[%%%{-3,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2
,0,1,2,0,1,0]%%%}+%%%{%%{[%%%{3,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,1,0,2
,1,0,2,0]%%%}+%%%{%%{poly1[%%%{-1,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,0
,3,0,0,3,0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.02, size = 12139, normalized size = 27.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x)
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="maxima"
)
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                                   +c*g^2 zero or nonze
ro?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**3,x)
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

3.193 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$

Optimal. Leaf size=603

$$-\frac{\sqrt{a+bx+cx^2}\left(hx\left(h^2\left(8a^2fh^2-2abh(10fg-eh)+b^2\left(11fg^2-h(dh+eg)\right)\right)+2cgh\left(2ah(6fg-eh)-b\left(12\right.\right.\right.\right.}{}$$

```
[Out] -1/3*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
^3-1/16*(16*c^3*f*g^5-8*c^2*g*h*(a*d*h^3-5*a*f*g^2*h+5*b*f*g^3)-b*h^3*(8*a^
2*f*h^2-2*a*b*h*(e*h+6*f*g)+b^2*(d*h^2+e*g*h+5*f*g^2))+2*c*h^2*(4*a^2*h^2*(
-e*h+4*f*g)-2*a*b*h*(-d*h^2-e*g*h+15*f*g^2)+b^2*(d*g*h^2+15*f*g^3)))*arctan
h(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1
/2))/h^4/(a*h^2-b*g*h+c*g^2)^(5/2)+f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b
*x+a)^(1/2))*c^(1/2)/h^4-1/8*(8*c^2*f*g^5-2*c*g*h*(-2*a*d*h^3-6*a*f*g^2*h+b
*d*g*h^2+7*b*f*g^3)+h^2*(4*a^2*e*h^3+b^2*g*(d*h^2+e*g*h+5*f*g^2)-2*a*b*h*(d
*h^2+2*e*g*h+3*f*g^2))+h*(4*c^2*(-d*g^2*h^2+3*f*g^4)+h^2*(8*a^2*f*h^2-2*a*b
*h*(-e*h+10*f*g)+b^2*(11*f*g^2-h*(d*h+e*g)))+2*c*g*h*(2*a*h*(-e*h+6*f*g)-b*
(12*f*g^2-h*(2*d*h+e*g))))*x)*(c*x^2+b*x+a)^(1/2)/h^3/(a*h^2-b*g*h+c*g^2)^2
/(h*x+g)^2
```

Rubi [A] time = 1.45, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 810, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx+cx^2}\left(hx\left(8a^2fh^3-2b\left(ah^2(10fg-eh)-cgh(2dh+eg)+12cfg^3\right)+4acgh(6fg-eh)+b^2h\left(11fg^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -(((8*c^2*f*g^5)/h + 4*a^2*e*h^4 + 4*a*c*g*h*(3*f*g^2 + d*h^2) + b^2*g*h*(5
*f*g^2 + h*(e*g + d*h)) - 2*b*(a*h^2*(3*f*g^2 + 2*e*g*h + d*h^2) + c*(7*f*g
^4 + d*g^2*h^2)) + h*(8*a^2*f*h^3 + 4*a*c*g*h*(6*f*g - e*h) + c^2*((12*f*g^
4)/h - 4*d*g^2*h) + b^2*h*(11*f*g^2 - h*(e*g + d*h)) - 2*b*(12*c*f*g^3 - c*
g*h*(e*g + 2*d*h) + a*h^2*(10*f*g - e*h)))*x)*Sqrt[a + b*x + c*x^2])/(8*h^2
*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b*x
 + c*x^2)^(3/2))/(3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) + (Sqrt[c]*f*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h^4 - ((16*c^3*f*g^5 -
 8*c^2*g*h*(5*b*f*g^3 - 5*a*f*g^2*h + a*d*h^3) - b*h^3*(8*a^2*f*h^2 - 2*a*b
*h*(6*f*g + e*h) + b^2*(5*f*g^2 + e*g*h + d*h^2)) + 2*c*h^2*(4*a^2*h^2*(4*f
*g - e*h) - 2*a*b*h*(15*f*g^2 - e*g*h - d*h^2) + b^2*(15*f*g^3 + d*g*h^2)))
*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqr
t[a + b*x + c*x^2]])])/(16*h^4*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3} - \frac{\int \frac{\left(-\frac{3}{2}\left(2cdg-beg-2afg+\frac{bfg^2}{h}-bdh+2a\right.\right.}{(g+h}}{3\left(cg^2-b\right.}$$

$$= -\frac{\left(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh\left(3fg^2+dh^2\right)+b^2gh\left(5fg^2+h(eg+dh)\right)-\right.}{}$$

$$= -\frac{\left(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh\left(3fg^2+dh^2\right)+b^2gh\left(5fg^2+h(eg+dh)\right)-\right.}{}$$

$$= -\frac{\left(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh\left(3fg^2+dh^2\right)+b^2gh\left(5fg^2+h(eg+dh)\right)-\right.}{}$$

$$= -\frac{\left(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh\left(3fg^2+dh^2\right)+b^2gh\left(5fg^2+h(eg+dh)\right)-\right.}{}$$

Mathematica [A] time = 1.93, size = 439, normalized size = 0.73

$$\frac{\frac{\left(\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)}{8\left(h(ah-bg)+cg^2\right)^{3/2}}+\frac{\sqrt{a+x(b+cx)}\,(-2ah+b(g-hx)+2cgx)}{4(g+hx)^2\left(h(ah-bg)+cg^2\right)}\right)\left(2ah^2(eh-2fg)-bh\left(h(dh+eg)-3fg^2\right)+c\left(2dgh^2-2fg^3\right)\right)}{2\left(h(ah-bg)+cg^2\right)} - \frac{h(a+x(b+cx}{3(g+hx)}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] (-1/3*(h*(f*g^2 + h*(-(e*g) + d*h))*(a + x*(b + c*x))^(3/2))/((c*g^2 + h*(-
(b*g) + a*h))*(g + h*x)^3) + ((2*a*h^2*(-2*f*g + e*h) + c*(-2*f*g^3 + 2*d*g
*h^2) - b*h*(-3*f*g^2 + h*(e*g + d*h)))*((Sqrt[a + x*(b + c*x)]*(-2*a*h + 2
*c*g*x + b*(g - h*x)))/(4*(c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) + ((b^2 -
 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g
) + a*h)]*Sqrt[a + x*(b + c*x)])])/(8*(c*g^2 + h*(-(b*g) + a*h))^(3/2))))/(
2*(c*g^2 + h*(-(b*g) + a*h))) + (f*(-((h*Sqrt[a + x*(b + c*x)])/(g + h*x))
+ Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + ((2*c*g
- b*h)*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g)
 + a*h)]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)])))/h^2)
/h^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 42.01Unable to divide, perh
aps due to rounding error%%%{%%{[-1,0]:[1,0,%%%{-1,[1]%%%}]%%},[8,0,0,0,0,0
,8,0]%%%}+%%%{%%%{8,[1]%%%},[7,0,0,0,0,0,7,1]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1
,[1]%%%}]%%},[6,0,1,0,0,0,7,1]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[6,
0,0,0,1,0,8,0]%%%}+%%%{%%{[%%%{-24,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,0,
0,0,0,0,6,2]%%%}+%%%{%%%{24,[1]%%%},[5,0,1,0,0,0,6,2]%%%}+%%%{%%%{-24,[1]%%
%},[5,0,0,0,1,0,7,1]%%%}+%%%{%%%{32,[2]%%%},[5,0,0,0,0,0,5,3]%%%}+%%%{%%{[-
6,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,2,0,0,0,6,2]%%%}+%%%{%%{[12,0]:[1,0,%%%{-
1,[1]%%%}]%%},[4,0,1,0,1,0,7,1]%%%}+%%%{%%{[%%%{-48,[1]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[4,0,1,0,0,0,5,3]%%%}+%%%{%%{[-6,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,
0,0,0,2,0,8,0]%%%}+%%%{%%{[%%%{48,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0
,0,1,0,6,2]%%%}+%%%{%%{[%%%{-16,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0,0
,0,0,4,4]%%%}+%%%{%%%{24,[1]%%%},[3,0,2,0,0,0,5,3]%%%}+%%%{%%%{-48,[1]%%%},
[3,0,1,0,1,0,6,2]%%%}+%%%{%%%{32,[2]%%%},[3,0,1,0,0,0,4,4]%%%}+%%%{%%%{24,[
1]%%%},[3,0,0,0,2,0,7,1]%%%}+%%%{%%%{-32,[2]%%%},[3,0,0,0,1,0,5,3]%%%}+%%%{
%%{[-4,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,3,0,0,0,5,3]%%%}+%%%{%%{[12,0]:[1,0,
%%%{-1,[1]%%%}]%%},[2,0,2,0,1,0,6,2]%%%}+%%%{%%{[%%%{-24,[1]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[2,0,2,0,0,0,4,4]%%%}+%%%{%%{[-12,0]:[1,0,%%%{-1,[1]%%%}]%
%},[2,0,1,0,2,0,7,1]%%%}+%%%{%%{[%%%{48,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[2,0,1,0,1,0,5,3]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,0,0,3,0,8,0
]%%%}+%%%{%%{[%%%{-24,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,0,0,2,0,6,2]%
%%}+%%%{%%%{8,[1]%%%},[1,0,3,0,0,0,4,4]%%%}+%%%{%%%{-24,[1]%%%},[1,0,2,0,1,
0,5,3]%%%}+%%%{%%%{24,[1]%%%},[1,0,1,0,2,0,6,2]%%%}+%%%{%%%{-8,[1]%%%},[1,0
,0,0,3,0,7,1]%%%}+%%%{%%{[-1,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,4,0,0,0,4,4]%%
%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,3,0,1,0,5,3]%%%}+%%%{%%{[-6,0]
:[1,0,%%%{-1,[1]%%%}]%%},[0,0,2,0,2,0,6,2]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]
%%%}]%%},[0,0,1,0,3,0,7,1]%%%}+%%%{%%{[-1,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,0
,0,4,0,8,0]%%%} / %%%{%%%{1,[2]%%%},[8,0,0,0,0,0,4,0]%%%}+%%%{%%{poly1[%%%{
-8,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,0,0,0,0,0,3,1]%%%}+%%%{%%%{4,[2]%%
%},[6,0,1,0,0,0,3,1]%%%}+%%%{%%%{-4,[2]%%%},[6,0,0,0,1,0,4,0]%%%}+%%%{%%%{2
4,[3]%%%},[6,0,0,0,0,0,2,2]%%%}+%%%{%%{[%%%{-24,[2]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[5,0,1,0,0,0,2,2]%%%}+%%%{%%{poly1[%%%{24,[2]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[5,0,0,0,1,0,3,1]%%%}+%%%{%%{poly1[%%%{-32,[3]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[5,0,0,0,0,0,1,3]%%%}+%%%{%%%{6,[2]%%%},[4,0,2,0,0,0,2,2]%%%}+
%%%{%%%{-12,[2]%%%},[4,0,1,0,1,0,3,1]%%%}+%%%{%%%{48,[3]%%%},[4,0,1,0,0,0,1
,3]%%%}+%%%{%%%{6,[2]%%%},[4,0,0,0,2,0,4,0]%%%}+%%%{%%%{-48,[3]%%%},[4,0,0,
0,1,0,2,2]%%%}+%%%{%%%{16,[4]%%%},[4,0,0,0,0,0,0,4]%%%}+%%%{%%{poly1[%%%{-2
4,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,2,0,0,0,1,3]%%%}+%%%{%%{[%%%{48,[
2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,1,0,1,0,2,2]%%%}+%%%{%%{[%%%{-32,[3]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,1,0,0,0,0,4]%%%}+%%%{%%{poly1[%%%{-24,
[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,0,0,2,0,3,1]%%%}+%%%{%%{poly1[%%%{3
2,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,0,0,1,0,1,3]%%%}+%%%{%%%{4,[2]%%%
},[2,0,3,0,0,0,1,3]%%%}+%%%{%%%{-12,[2]%%%},[2,0,2,0,1,0,2,2]%%%}+%%%{%%%{2
4,[3]%%%},[2,0,2,0,0,0,0,4]%%%}+%%%{%%%{12,[2]%%%},[2,0,1,0,2,0,3,1]%%%}+%%
%{%%%{-48,[3]%%%},[2,0,1,0,1,0,1,3]%%%}+%%%{%%%{-4,[2]%%%},[2,0,0,0,3,0,4,0
]%%%}+%%%{%%%{24,[3]%%%},[2,0,0,0,2,0,2,2]%%%}+%%%{%%{[%%%{-8,[2]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[1,0,3,0,0,0,0,4]%%%}+%%%{%%{poly1[%%%{24,[2]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[1,0,2,0,1,0,1,3]%%%}+%%%{%%{[%%%{-24,[2]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[1,0,1,0,2,0,2,2]%%%}+%%%{%%{poly1[%%%{8,[2]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[1,0,0,0,3,0,3,1]%%%}+%%%{%%%{1,[2]%%%},[0,0,4,0,0,0
,0,4]%%%}+%%%{%%%{-4,[2]%%%},[0,0,3,0,1,0,1,3]%%%}+%%%{%%%{6,[2]%%%},[0,0,2
,0,2,0,2,2]%%%}+%%%{%%%{-4,[2]%%%},[0,0,1,0,3,0,3,1]%%%}+%%%{%%%{1,[2]%%%},
[0,0,0,0,4,0,4,0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.02, size = 19321, normalized size = 32.04

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                            +c*g^2 zero or nonze
ro?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
```

```
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**4,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

3.194 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^5} dx$

Optimal. Leaf size=497

$$\frac{\sqrt{a+bx+cx^2}\,(-2ah+x(2cg-bh)+bg)\left(16a^2fh^2-4c\left(a\left(dh^2-5egh+fg^2\right)+2bg(2dh+eg)\right)-8abh(eh+2fg\right.}{64(g+hx)^2\left(ah^2-bgh+cg^2\right)^3}$$

```
[Out] -1/4*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
^4+1/24*(2*c*g*(3*f*g^2+h*(-5*d*h+e*g))+h*(8*a*h*(-e*h+2*f*g)-b*(-5*d*h^2-3
*e*g*h+11*f*g^2)))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^3-1/
128*(-4*a*c+b^2)*(16*c^2*d*g^2+16*a^2*f*h^2-8*a*b*h*(e*h+2*f*g)+b^2*(5*d*h^
2+3*e*g*h+5*f*g^2)-4*c*(2*b*g*(2*d*h+e*g)+a*(d*h^2-5*e*g*h+f*g^2)))*arctanh
(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/
2))/(a*h^2-b*g*h+c*g^2)^(7/2)+1/64*(16*c^2*d*g^2+16*a^2*f*h^2-8*a*b*h*(e*h+
2*f*g)+b^2*(5*d*h^2+3*e*g*h+5*f*g^2)-4*c*(2*b*g*(2*d*h+e*g)+a*(d*h^2-5*e*g*
h+f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^
2)^3/(h*x+g)^2
```

Rubi [A] time = 0.86, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1650, 806, 720, 724, 206}

$$\frac{\sqrt{a+bx+cx^2}\,(-2ah+x(2cg-bh)+bg)\left(16a^2fh^2-4c\left(-ah(5eg-dh)+afg^2+2bg(2dh+eg)\right)-8abh(eh+2f\right.}{64(g+hx)^2\left(ah^2-bgh+cg^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] ((16*c^2*d*g^2 + 16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*
(5*e*g - d*h) + 2*b*g*(e*g + 2*d*h)) + b^2*(5*f*g^2 + h*(3*e*g + 5*d*h)))*(
b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(64*(c*g^2 - b*g*h +
a*h^2)^3*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(
4*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) + ((6*c*f*g^3 + 2*c*g*h*(e*g - 5*d
*h) + 8*a*h^2*(2*f*g - e*h) - b*h*(11*f*g^2 - h*(3*e*g + 5*d*h)))*(a + b*x
+ c*x^2)^(3/2))/(24*h*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^3) - ((b^2 - 4*a*
c)*(16*c^2*d*g^2 + 16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*
h*(5*e*g - d*h) + 2*b*g*(e*g + 2*d*h)) + b^2*(5*f*g^2 + h*(3*e*g + 5*d*h)))
*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqr
t[a + b*x + c*x^2]])])/(128*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
```

```
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4} - \frac{\int \frac{\left(\frac{1}{2}\left(-8cdg+3beg+8afg-\frac{3bfg^2}{h}+5bdh \cdots\right.\right.}{\cdots}}{4\left(\cdots\right.}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4} + \frac{\left(6cfg^3+2cgh(eg-5dh)+8a \cdots\right.}{2 \cdots}$$

$$= \frac{\left(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c\left(afg^2-ah(5eg-dh)+2bg \cdots\right.\right.}{64\left(cg^2-b \cdots\right.}$$

$$= \frac{\left(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c\left(afg^2-ah(5eg-dh)+2bg \cdots\right.\right.}{64\left(cg^2-b \cdots\right.}$$

$$= \frac{\left(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c\left(afg^2-ah(5eg-dh)+2bg \cdots\right.\right.}{64\left(cg^2-b \cdots\right.}$$

Mathematica [A] time = 3.90, size = 447, normalized size = 0.90

$$\frac{\frac{\frac{3}{2}ch\left(\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)}{8\left(h(ah-bg)+cg^2\right)^{3/2}}+\frac{\sqrt{a+x(b+cx)}\,(-2ah+b(g-hx)+2cgx)}{4(g+hx)^2\left(h(ah-bg)+cg^2\right)}\right)\left(16a^2fh^2-4c\left(ah(dh-5eg)+afg^2+2bg(2dh+eg)\right)-8abh(eh+2fg)+b^2\left(h(\right.\right.}{24\left(h(ah-bg)+cg^2\right)^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (-((f*(a + x*(b + c*x))^(3/2))/(g + h*x)^4) + ((3*c*f*g^2 + 4*f*h*(-(b*g) +
 a*h) + c*h*(e*g - d*h))*(a + x*(b + c*x))^(3/2))/(4*(c*g^2 + h*(-(b*g) + a
*h))*(g + h*x)^4) + ((c*(6*c*f*g^3 + 2*c*g*h*(e*g - 5*d*h) - 8*a*h^2*(-2*f*
g + e*h) + b*h*(-11*f*g^2 + h*(3*e*g + 5*d*h)))*(a + x*(b + c*x))^(3/2))/(g
 + h*x)^3 + (3*c*h*(16*c^2*d*g^2 + 16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) - 4
*c*(a*f*g^2 + a*h*(-5*e*g + d*h) + 2*b*g*(e*g + 2*d*h)) + b^2*(5*f*g^2 + h*
(3*e*g + 5*d*h)))*((Sqrt[a + x*(b + c*x)]*(-2*a*h + 2*c*g*x + b*(g - h*x)))
/(4*(c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g
) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(
b + c*x)])])/(8*(c*g^2 + h*(-(b*g) + a*h))^(3/2))))/2)/(24*(c*g^2 + h*(-(b*
g) + a*h))^2))/(c*h)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 29161, normalized size = 58.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                            +c*g^2 zero or nonze
ro?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
```

```
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{\left(g+hx\right)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**5,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

3.195 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

Optimal. Leaf size=824

$$\frac{\left(4c^2\left(3fg^2+h(2eg-27dh)\right)g^2-5h^2\left(\left(3fg^2+3ehg+7dh^2\right)b^2-2ah(6fg+5eh)b+16a^2fh^2\right)-2ch\left(bg\left(16fg^2\right.\right.\right.}{240h\left(cg^2-bhg+ah^2\right)^3(g+hx)^3}$$

```
[Out] -1/5*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
^5+1/40*(2*c*g*(3*f*g^2+h*(-7*d*h+2*e*g))+h*(10*a*h*(-e*h+2*f*g)-b*(-7*d*h^
2-3*e*g*h+13*f*g^2)))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^4
+1/240*(4*c^2*g^2*(3*f*g^2+h*(-27*d*h+2*e*g))-5*h^2*(16*a^2*f*h^2-2*a*b*h*(
5*e*h+6*f*g)+b^2*(7*d*h^2+3*e*g*h+3*f*g^2))-2*c*h*(b*g*(-54*d*h^2-21*e*g*h+
16*f*g^2)-2*a*h*(8*d*h^2-33*e*g*h+18*f*g^2)))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-
b*g*h+c*g^2)^3/(h*x+g)^3-1/256*(-4*a*c+b^2)*(32*c^3*d*g^3-8*c^2*g*(2*b*g*(3
*d*h+e*g)+a*(3*d*h^2-6*e*g*h+f*g^2))-b*h*(16*a^2*f*h^2-2*a*b*h*(5*e*h+6*f*g
)+b^2*(7*d*h^2+3*e*g*h+3*f*g^2))+2*c*(4*a^2*h^2*(-e*h+6*f*g)-6*a*b*h*(-d*h^
2+3*e*g*h+3*f*g^2)+b^2*g*(15*d*h^2+6*e*g*h+5*f*g^2)))*arctanh(1/2*(b*g-2*a*
h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g
*h+c*g^2)^(9/2)+1/128*(32*c^3*d*g^3-8*c^2*g*(2*b*g*(3*d*h+e*g)+a*(3*d*h^2-6
*e*g*h+f*g^2))-b*h*(16*a^2*f*h^2-2*a*b*h*(5*e*h+6*f*g)+b^2*(7*d*h^2+3*e*g*h
+3*f*g^2))+2*c*(4*a^2*h^2*(-e*h+6*f*g)-6*a*b*h*(-d*h^2+3*e*g*h+3*f*g^2)+b^2
*g*(15*d*h^2+6*e*g*h+5*f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1
/2)/(a*h^2-b*g*h+c*g^2)^4/(h*x+g)^2
```

Rubi [A] time = 2.33, antiderivative size = 826, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1650, 834, 806, 720, 724, 206}

$$\frac{\left(4\left(3fg^4+h(2eg-27dh)g^2\right)c^2-2h\left(bg\left(16fg^2-21ehg-54dh^2\right)-2ah\left(18fg^2-33ehg+8dh^2\right)\right)c-5h^2\left(\left(3fg^2\right.\right.\right.}{240h\left(cg^2-bhg+ah^2\right)^3(g+hx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] ((32*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - 3*a*h*(2*e*g - d*h) + 2*b*g*(e*g + 3*d*
h)) + 2*c*(4*a^2*h^2*(6*f*g - e*h) - 6*a*b*h*(3*f*g^2 + h*(3*e*g - d*h)) +
b^2*(5*f*g^3 + 3*g*h*(2*e*g + 5*d*h))) - b*h*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g
 + 5*e*h) + b^2*(3*f*g^2 + h*(3*e*g + 7*d*h))))*(b*g - 2*a*h + (2*c*g - b*h
)*x)*Sqrt[a + b*x + c*x^2])/(128*(c*g^2 - b*g*h + a*h^2)^4*(g + h*x)^2) - (
(f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(5*h*(c*g^2 - b*g*h + a*h^
2)*(g + h*x)^5) + ((6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g -
 e*h) - b*h*(13*f*g^2 - h*(3*e*g + 7*d*h)))*(a + b*x + c*x^2)^(3/2))/(40*h*
(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^4) + ((4*c^2*(3*f*g^4 + g^2*h*(2*e*g -
27*d*h)) - 5*h^2*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g + 5*e*h) + b^2*(3*f*g^2 + 3
*e*g*h + 7*d*h^2)) - 2*c*h*(b*g*(16*f*g^2 - 21*e*g*h - 54*d*h^2) - 2*a*h*(1
8*f*g^2 - 33*e*g*h + 8*d*h^2)))*(a + b*x + c*x^2)^(3/2))/(240*h*(c*g^2 - b*
g*h + a*h^2)^3*(g + h*x)^3) - ((b^2 - 4*a*c)*(32*c^3*d*g^3 - 8*c^2*g*(a*f*g
^2 - 3*a*h*(2*e*g - d*h) + 2*b*g*(e*g + 3*d*h)) + 2*c*(4*a^2*h^2*(6*f*g - e
*h) - 6*a*b*h*(3*f*g^2 + h*(3*e*g - d*h)) + b^2*(5*f*g^3 + 3*g*h*(2*e*g + 5
*d*h))) - b*h*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g + 5*e*h) + b^2*(3*f*g^2 + h*(3
*e*g + 7*d*h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*
g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(256*(c*g^2 - b*g*h + a*h^2)^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} - \int \frac{\left(\frac{1}{2}\left(-10cdg+3beg+10afg-\frac{3bfg^2}{h}+7bdh-\right.\right.}{5\left(c\right.}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} + \frac{\left(6cfg^3+2cgh(2eg-7dh)+10a\right.}{40}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} + \frac{\left(6cfg^3+2cgh(2eg-7dh)+10a\right.}{40}$$

$$= \frac{\left(32c^3dg^3-8c^2g\left(afg^2-3ah(2eg-dh)+2bg(eg+3dh)\right)+2c\left(4a^2h^2(6fg\right.\right.}{}$$

$$= \frac{\left(32c^3dg^3-8c^2g\left(afg^2-3ah(2eg-dh)+2bg(eg+3dh)\right)+2c\left(4a^2h^2(6fg\right.\right.}{}$$

$$= \frac{\left(32c^3dg^3-8c^2g\left(afg^2-3ah(2eg-dh)+2bg(eg+3dh)\right)+2c\left(4a^2h^2(6fg\right.\right.}{}$$

Mathematica [A] time = 6.33, size = 1128, normalized size = 1.37

$$\frac{\sqrt{a+x(b+cx)}\left(-\frac{\left(\frac{1}{2}h(3bfg+4cdh-10afh)-\frac{1}{2}g(6cfg+4ceh-7bfh)\right)\left(cx^2+bx+a\right)^{3/2}}{5\left(cg^2-bhg+ah^2\right)(g+hx)^5} - \frac{-\frac{\left(2cg\left(3cfg^2-5fh(bg-ah)+2ch(eg-dh)\right)-ch\left(3bfg^2-bh(3eg+7dh)+10h\right.\right.}{4\left(cg^2-bhg+ah^2\right)(g+hx)^4}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -1/2*(f*(a + b*x + c*x^2)*Sqrt[a + x*(b + c*x)])/(c*h*(g + h*x)^5) + (Sqrt[
a + x*(b + c*x)]*(-1/5*(((h*(3*b*f*g + 4*c*d*h - 10*a*f*h))/2 - (g*(6*c*f*g
 + 4*c*e*h - 7*b*f*h))/2)*(a + b*x + c*x^2)^(3/2))/((c*g^2 - b*g*h + a*h^2)
*(g + h*x)^5) - (-1/4*((2*c*g*(3*c*f*g^2 - 5*f*h*(b*g - a*h) + 2*c*h*(e*g -
 d*h)) - c*h*(3*b*f*g^2 - b*h*(3*e*g + 7*d*h) + 10*h*(c*d*g - a*f*g + a*e*h
)))*(a + b*x + c*x^2)^(3/2))/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) - (((c^2
*g*(6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g - e*h) - b*h*(13*
f*g^2 - h*(3*e*g + 7*d*h))) - (c*h*(5*b^2*h*(3*f*g^2 + h*(3*e*g + 7*d*h)) +
 2*b*(3*c*f*g^3 - c*g*h*(18*e*g + 47*d*h) - 5*a*h^2*(6*f*g + 5*e*h)) + 16*h
*(5*c^2*d*g^2 + 5*a^2*f*h^2 - a*c*(2*f*g^2 - h*(7*e*g - 2*d*h)))))/2)*(a +
b*x + c*x^2)^(3/2))/(3*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) - ((-2*(a*c^2*h
*(6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g - e*h) - b*h*(13*f*
g^2 - h*(3*e*g + 7*d*h))) + (c^2*g*(5*b^2*h*(3*f*g^2 + h*(3*e*g + 7*d*h)) +
```

```
 2*b*(3*c*f*g^3 - c*g*h*(18*e*g + 47*d*h) - 5*a*h^2*(6*f*g + 5*e*h)) + 16*h
*(5*c^2*d*g^2 + 5*a^2*f*h^2 - a*c*(2*f*g^2 - h*(7*e*g - 2*d*h)))))/2) + b*(
c^2*g*(6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g - e*h) - b*h*(
13*f*g^2 - h*(3*e*g + 7*d*h))) + (c*h*(5*b^2*h*(3*f*g^2 + h*(3*e*g + 7*d*h)
) + 2*b*(3*c*f*g^3 - c*g*h*(18*e*g + 47*d*h) - 5*a*h^2*(6*f*g + 5*e*h)) + 1
6*h*(5*c^2*d*g^2 + 5*a^2*f*h^2 - a*c*(2*f*g^2 - h*(7*e*g - 2*d*h)))))/2))*(
((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*
h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[c*g^2
 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))))/(2*(c*g^2 - b*g*h + a*h^
2)))/(4*(c*g^2 - b*g*h + a*h^2)))/(5*(c*g^2 - b*g*h + a*h^2))))/(2*c*h*Sqrt
[a + b*x + c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 40336, normalized size = 48.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                                   +c*g^2 zero or nonze
ro?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}{\left(g+hx\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**6,x)
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

3.196 $\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=1169

$$\frac{f\left(cx^2+bx+a\right)^{5/2}(g+hx)^4}{9ch}-\frac{(10cfg-18ceh+13bfh)\left(cx^2+bx+a\right)^{5/2}(g+hx)^3}{144c^2h}+\frac{\left(-12\left(5fg^2-3h(3eg+8a\right.\right.}{}$$

```
[Out] 1/12288*(1536*c^5*d*g^3-143*b^5*f*h^3+22*b^3*c*h^2*(20*a*f*h+9*b*(e*h+3*f*g
))-48*b*c^2*h*(5*a^2*f*h^2+9*a*b*h*(e*h+3*f*g)+6*b^2*(d*h^2+3*e*g*h+3*f*g^2
))-256*c^4*g*(3*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+32*c^3*(3*a^2*h^2*
(e*h+3*f*g)+14*b^2*g*(f*g^2+3*h*(d*h+e*g))+12*a*b*h*(3*f*g^2+h*(d*h+3*e*g))
))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^6+1/2016*(143*b^2*f*h^2-2*c*h*(64*a*f*h+
99*b*e*h+24*b*f*g)-12*c^2*(5*f*g^2-3*h*(8*d*h+3*e*g)))*(h*x+g)^2*(c*x^2+b*x
+a)^(5/2)/c^3/h-1/144*(13*b*f*h-18*c*e*h+10*c*f*g)*(h*x+g)^3*(c*x^2+b*x+a)^
(5/2)/c^2/h+1/9*f*(h*x+g)^4*(c*x^2+b*x+a)^(5/2)/c/h+1/80640*(3003*b^4*f*h^4
-192*c^4*g^2*(5*f*g^2-3*h*(64*d*h+3*e*g))-198*b^2*c*h^3*(38*a*f*h+21*b*(e*h
+3*f*g))+8*c^2*h^2*(256*a^2*f*h^2+837*a*b*h*(e*h+3*f*g)+b^2*(1553*f*g^2+756
*h*(d*h+3*e*g)))-16*c^3*h*(32*a*h*(17*f*g^2+9*h*(d*h+3*e*g))+b*g*(13*f*g^2+
9*h*(196*d*h+141*e*g)))-10*c*h*(429*b^3*f*h^3-22*b*c*h^2*(34*a*f*h+27*b*e*h
+29*b*f*g)+16*c^3*g*(5*f*g^2-9*h*(12*d*h+e*g))+8*c^2*h*(a*h*(63*e*h+61*f*g)
+3*b*(f*g^2+6*h*(6*d*h+7*e*g)))))*x)*(c*x^2+b*x+a)^(5/2)/c^5/h+1/65536*(-4*a
*c+b^2)^2*(1536*c^5*d*g^3-143*b^5*f*h^3+22*b^3*c*h^2*(20*a*f*h+9*b*(e*h+3*f
*g))-48*b*c^2*h*(5*a^2*f*h^2+9*a*b*h*(e*h+3*f*g)+6*b^2*(d*h^2+3*e*g*h+3*f*g
^2))-256*c^4*g*(3*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+32*c^3*(3*a^2*h^
2*(e*h+3*f*g)+14*b^2*g*(f*g^2+3*h*(d*h+e*g))+12*a*b*h*(3*f*g^2+h*(d*h+3*e*g
))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(15/2)-1/32768*(-
4*a*c+b^2)*(1536*c^5*d*g^3-143*b^5*f*h^3+22*b^3*c*h^2*(20*a*f*h+9*b*(e*h+3*
f*g))-48*b*c^2*h*(5*a^2*f*h^2+9*a*b*h*(e*h+3*f*g)+6*b^2*(d*h^2+3*e*g*h+3*f*
g^2))-256*c^4*g*(3*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+32*c^3*(3*a^2*h
^2*(e*h+3*f*g)+14*b^2*g*(f*g^2+3*h*(d*h+e*g))+12*a*b*h*(3*f*g^2+h*(d*h+3*e*
g))))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^7
```

Rubi [A] time = 3.70, antiderivative size = 1166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1653, 832, 779, 612, 621, 206}

$$\frac{f\left(cx^2+bx+a\right)^{5/2}(g+hx)^4}{9ch}-\frac{(10cfg-18ceh+13bfh)\left(cx^2+bx+a\right)^{5/2}(g+hx)^3}{144c^2h}+\frac{\left(-12\left(5fg^2-3h(3eg+8a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(1536*c^5*d*g^3 - 143*b^5*f*h^3 - 256*c^4*g*(a*f*g^2 + 3*a*
h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h)) + 22*b^3*c*h^2*(20*a*f*h + 9*b*(3*f*g
+ e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) + 6*b^2*(3*f*g^2
+ 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*b^2*g*(f*g^2 + 3
*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b + 2*c*x)*Sqrt[a
 + b*x + c*x^2])/(32768*c^7) + ((1536*c^5*d*g^3 - 143*b^5*f*h^3 - 256*c^4*g
*(a*f*g^2 + 3*a*h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h)) + 22*b^3*c*h^2*(20*a*f
*h + 9*b*(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) +
 6*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*
b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b
 + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(12288*c^6) + ((143*b^2*f*h^2 - 2*c*h*(2
4*b*f*g + 99*b*e*h + 64*a*f*h) - 12*c^2*(5*f*g^2 - 3*h*(3*e*g + 8*d*h)))*(g
 + h*x)^2*(a + b*x + c*x^2)^(5/2))/(2016*c^3*h) - ((10*c*f*g - 18*c*e*h + 1
3*b*f*h)*(g + h*x)^3*(a + b*x + c*x^2)^(5/2))/(144*c^2*h) + (f*(g + h*x)^4*
```

```
(a + b*x + c*x^2)^(5/2))/(9*c*h) + ((3003*b^4*f*h^4 - 192*c^4*(5*f*g^4 - 3*
g^2*h*(3*e*g + 64*d*h)) - 198*b^2*c*h^3*(38*a*f*h + 21*b*(3*f*g + e*h)) + 8
*c^2*h^2*(256*a^2*f*h^2 + 837*a*b*h*(3*f*g + e*h) + b^2*(1553*f*g^2 + 756*h
*(3*e*g + d*h))) - 16*c^3*h*(32*a*h*(17*f*g^2 + 9*h*(3*e*g + d*h)) + b*g*(1
3*f*g^2 + 9*h*(141*e*g + 196*d*h))) - 10*c*h*(429*b^3*f*h^3 - 22*b*c*h^2*(2
9*b*f*g + 27*b*e*h + 34*a*f*h) + 16*c^3*(5*f*g^3 - 9*g*h*(e*g + 12*d*h)) +
8*c^2*h*(a*h*(61*f*g + 63*e*h) + 3*b*(f*g^2 + 6*h*(7*e*g + 6*d*h))))*x)*(a
+ b*x + c*x^2)^(5/2))/(80640*c^5*h) + ((b^2 - 4*a*c)^2*(1536*c^5*d*g^3 - 14
3*b^5*f*h^3 - 256*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h))
 + 22*b^3*c*h^2*(20*a*f*h + 9*b*(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 +
9*a*b*h*(3*f*g + e*h) + 6*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*
h^2*(3*f*g + e*h) + 14*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2
+ h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(65536*c^(15/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
```

```
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \frac{f(g+hx)^4 \left(a+bx+cx^2\right)^{5/2}}{9ch} + \frac{\int (g+hx)^3 \left(-\frac{1}{2}h(5bfg-18c \cdots\right.}{\cdots}$$

$$= -\frac{(10cfg-18ceh+13bfh)(g+hx)^3 \left(a+bx+cx^2\right)^{5/2}}{144c^2h} + \frac{f(g \cdots}{\cdots}$$

$$= \frac{\left(143b^2fh^2-2ch(24bfg+99beh+64afh)-12c^2\left(5fg^2-3 \cdots\right.\right.}{2016c^3h \cdots}$$

$$= \frac{\left(143b^2fh^2-2ch(24bfg+99beh+64afh)-12c^2\left(5fg^2-3 \cdots\right.\right.}{2016c^3h \cdots}$$

$$= \frac{\left(1536c^5dg^3-143b^5fh^3-256c^4g\left(afg^2+3ah(eg+dh)+3 \cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(b^2-4ac\right)\left(1536c^5dg^3-143b^5fh^3-256c^4g\left(afg^2+3ah(\cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(b^2-4ac\right)\left(1536c^5dg^3-143b^5fh^3-256c^4g\left(afg^2+3ah(\cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(b^2-4ac\right)\left(1536c^5dg^3-143b^5fh^3-256c^4g\left(afg^2+3ah(\cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 2.71, size = 721, normalized size = 0.62

$$\frac{(a+x(b+cx))^{5/2}\left(4c^2h^2\left(512a^2fh^2+2abh(837eh+2511fg+935fhx)+b^2(27h(56dh+168eg+55ehx)+fg(3106g+1595hx))\right)-66b^2ch^3(114afh+b(63eh+18 \cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (((143*b^2*f*h^2 - 2*c*h*(24*b*f*g + 99*b*e*h + 64*a*f*h) - 12*c^2*(5*f*g^2
 - 3*h*(3*e*g + 8*d*h)))*(g + h*x)^2*(a + x*(b + c*x))^(5/2))/(224*c^2) - (
(13*b*f*h + 2*c*(5*f*g - 9*e*h))*(g + h*x)^3*(a + x*(b + c*x))^(5/2))/(16*c
) + f*(g + h*x)^4*(a + x*(b + c*x))^(5/2) + ((a + x*(b + c*x))^(5/2)*(3003*
b^4*f*h^4 - 66*b^2*c*h^3*(114*a*f*h + b*(189*f*g + 63*e*h + 65*f*h*x)) - 32
*c^4*(5*f*g^3*(6*g + 5*h*x) - 9*g*h*(e*g*(6*g + 5*h*x) + 4*d*h*(32*g + 15*h
*x))) + 4*c^2*h^2*(512*a^2*f*h^2 + 2*a*b*h*(2511*f*g + 837*e*h + 935*f*h*x)
 + b^2*(f*g*(3106*g + 1595*h*x) + 27*h*(168*e*g + 56*d*h + 55*e*h*x))) - 16
*c^3*h*(a*h*(f*g*(544*g + 305*h*x) + 9*h*(96*e*g + 32*d*h + 35*e*h*x)) + b*
(f*g^2*(13*g + 15*h*x) + 9*h*(4*d*h*(49*g + 15*h*x) + e*g*(141*g + 70*h*x))
))))/(8960*c^4) + (3*h*(1536*c^5*d*g^3 - 143*b^5*f*h^3 - 256*c^4*g*(a*f*g^2
 + 3*a*h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h)) + 22*b^3*c*h^2*(20*a*f*h + 9*b*
(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) + 6*b^2*(3
*f*g^2 + 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*b^2*g*(f*
```

```
g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(2*Sqrt[c]*
(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2))
+ 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])
)/(65536*c^(13/2)))/(9*c*h)
```

fricas [B] time = 10.43, size = 4751, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas"
)
```

```
[Out] [-1/41287680*(315*(64*(24*(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d - 12*(b^5*
c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*e + (7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b
^2*c^5 - 64*a^3*c^6)*f)*g^3 - 96*(24*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)
*d - 2*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*e + 3*(3*b
^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*f)*g^2*h + 6*(32*(7*
b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*d - 48*(3*b^7*c^2 -
28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*e + 3*(33*b^8*c - 336*a*b^6*c
^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*f)*g*h^2 - (96*(3*b
^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*d - 6*(33*b^8*c - 33
6*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*e + (143*b
^9 - 1584*a*b^7*c + 6048*a^2*b^5*c^2 - 8960*a^3*b^3*c^3 + 3840*a^4*b*c^4)*f
)*h^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(c) - 4*a*c) - 4*(1146880*c^9*f*h^3*x^8 + 71680*(54*c^9*f*g*h^
2 + (18*c^9*e + 19*b*c^8*f)*h^3)*x^7 + 5120*(864*c^9*f*g^2*h + 54*(16*c^9*e
 + 17*b*c^8*f)*g*h^2 + (288*c^9*d + 306*b*c^8*e + (3*b^2*c^7 + 320*a*c^8)*f
)*h^3)*x^6 + 1280*(1344*c^9*f*g^3 + 288*(14*c^9*e + 15*b*c^8*f)*g^2*h + 18*
(224*c^9*d + 240*b*c^8*e + 3*(b^2*c^7 + 84*a*c^8)*f)*g*h^2 + (1440*b*c^8*d
+ 18*(b^2*c^7 + 84*a*c^8)*e - (13*b^3*c^6 - 60*a*b*c^7)*f)*h^3)*x^5 + 128*(
1344*(12*c^9*e + 13*b*c^8*f)*g^3 + 288*(168*c^9*d + 182*b*c^8*e + 3*(b^2*c^
7 + 64*a*c^8)*f)*g^2*h + 18*(2912*b*c^8*d + 48*(b^2*c^7 + 64*a*c^8)*e - 3*(
11*b^3*c^6 - 52*a*b*c^7)*f)*g*h^2 + (288*(b^2*c^7 + 64*a*c^8)*d - 18*(11*b^
3*c^6 - 52*a*b*c^7)*e + (143*b^4*c^5 - 804*a*b^2*c^6 + 768*a^2*c^7)*f)*h^3)
*x^4 - 1344*(120*(3*b^3*c^6 - 20*a*b*c^7)*d - 12*(15*b^4*c^5 - 100*a*b^2*c^
6 + 128*a^2*c^7)*e + (105*b^5*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*f)*g^3
+ 288*(168*(15*b^4*c^5 - 100*a*b^2*c^6 + 128*a^2*c^7)*d - 14*(105*b^5*c^4 -
 760*a*b^3*c^5 + 1296*a^2*b*c^6)*e + 3*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488
*a^2*b^2*c^5 - 2048*a^3*c^6)*f)*g^2*h - 18*(224*(105*b^5*c^4 - 760*a*b^3*c^
5 + 1296*a^2*b*c^6)*d - 48*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2*b^2*c^5
 - 2048*a^3*c^6)*e + 3*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2*b^3*c^4
- 58816*a^3*b*c^5)*f)*g*h^2 + (288*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2
*b^2*c^5 - 2048*a^3*c^6)*d - 18*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2
*b^3*c^4 - 58816*a^3*b*c^5)*e + (45045*b^8*c - 438900*a*b^6*c^2 + 1383984*a
^2*b^4*c^3 - 1467072*a^3*b^2*c^4 + 262144*a^4*c^5)*f)*h^3 + 16*(1344*(120*c
^9*d + 132*b*c^8*e + (3*b^2*c^7 + 140*a*c^8)*f)*g^3 + 288*(1848*b*c^8*d + 1
4*(3*b^2*c^7 + 140*a*c^8)*e - 3*(9*b^3*c^6 - 44*a*b*c^7)*f)*g^2*h + 18*(224
*(3*b^2*c^7 + 140*a*c^8)*d - 48*(9*b^3*c^6 - 44*a*b*c^7)*e + 3*(99*b^4*c^5
- 568*a*b^2*c^6 + 560*a^2*c^7)*f)*g*h^2 - (288*(9*b^3*c^6 - 44*a*b*c^7)*d -
 18*(99*b^4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*e + (1287*b^5*c^4 - 8536*a*b
^3*c^5 + 12912*a^2*b*c^6)*f)*h^3)*x^3 + 8*(1344*(360*b*c^8*d + 12*(b^2*c^7
+ 32*a*c^8)*e - (7*b^3*c^6 - 36*a*b*c^7)*f)*g^3 + 288*(168*(b^2*c^7 + 32*a*
c^8)*d - 14*(7*b^3*c^6 - 36*a*b*c^7)*e + 3*(21*b^4*c^5 - 124*a*b^2*c^6 + 12
8*a^2*c^7)*f)*g^2*h - 18*(224*(7*b^3*c^6 - 36*a*b*c^7)*d - 48*(21*b^4*c^5 -
 124*a*b^2*c^6 + 128*a^2*c^7)*e + 3*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2416*a^
2*b*c^6)*f)*g*h^2 + (288*(21*b^4*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*d - 18*
(231*b^5*c^4 - 1560*a*b^3*c^5 + 2416*a^2*b*c^6)*e + (3003*b^6*c^3 - 22968*a
*b^4*c^4 + 47280*a^2*b^2*c^5 - 16384*a^3*c^6)*f)*h^3)*x^2 + 2*(1344*(120*(b
^2*c^7 + 20*a*c^8)*d - 12*(5*b^3*c^6 - 28*a*b*c^7)*e + (35*b^4*c^5 - 216*a*
```

```
b^2*c^6 + 240*a^2*c^7)*f)*g^3 - 288*(168*(5*b^3*c^6 - 28*a*b*c^7)*d - 14*(3
5*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*e + 3*(105*b^5*c^4 - 728*a*b^3*c^5
 + 1168*a^2*b*c^6)*f)*g^2*h + 18*(224*(35*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2
*c^7)*d - 48*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*e + 3*(1155*b^6
*c^3 - 8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*f)*g*h^2 - (288*(
105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*d - 18*(1155*b^6*c^3 - 8988*a
*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*e + (15015*b^7*c^2 - 130284*a*
b^5*c^3 + 338832*a^2*b^3*c^4 - 236864*a^3*b*c^5)*f)*h^3)*x)*sqrt(c*x^2 + b*
x + a))/c^8, -1/20643840*(315*(64*(24*(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*
d - 12*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*e + (7*b^6*c^3 - 60*a*b^4*c^4
 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*f)*g^3 - 96*(24*(b^5*c^4 - 8*a*b^3*c^5 + 1
6*a^2*b*c^6)*d - 2*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6
)*e + 3*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*f)*g^2*h
 + 6*(32*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*d - 48*(
3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*e + 3*(33*b^8*c -
 336*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*f)*g*h^
2 - (96*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*d - 6*(3
3*b^8*c - 336*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5
)*e + (143*b^9 - 1584*a*b^7*c + 6048*a^2*b^5*c^2 - 8960*a^3*b^3*c^3 + 3840*
a^4*b*c^4)*f)*h^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sq
rt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1146880*c^9*f*h^3*x^8 + 71680*(54*c^9*
f*g*h^2 + (18*c^9*e + 19*b*c^8*f)*h^3)*x^7 + 5120*(864*c^9*f*g^2*h + 54*(16
*c^9*e + 17*b*c^8*f)*g*h^2 + (288*c^9*d + 306*b*c^8*e + (3*b^2*c^7 + 320*a*
c^8)*f)*h^3)*x^6 + 1280*(1344*c^9*f*g^3 + 288*(14*c^9*e + 15*b*c^8*f)*g^2*h
 + 18*(224*c^9*d + 240*b*c^8*e + 3*(b^2*c^7 + 84*a*c^8)*f)*g*h^2 + (1440*b*
c^8*d + 18*(b^2*c^7 + 84*a*c^8)*e - (13*b^3*c^6 - 60*a*b*c^7)*f)*h^3)*x^5 +
 128*(1344*(12*c^9*e + 13*b*c^8*f)*g^3 + 288*(168*c^9*d + 182*b*c^8*e + 3*(
b^2*c^7 + 64*a*c^8)*f)*g^2*h + 18*(2912*b*c^8*d + 48*(b^2*c^7 + 64*a*c^8)*e
 - 3*(11*b^3*c^6 - 52*a*b*c^7)*f)*g*h^2 + (288*(b^2*c^7 + 64*a*c^8)*d - 18*
(11*b^3*c^6 - 52*a*b*c^7)*e + (143*b^4*c^5 - 804*a*b^2*c^6 + 768*a^2*c^7)*f
)*h^3)*x^4 - 1344*(120*(3*b^3*c^6 - 20*a*b*c^7)*d - 12*(15*b^4*c^5 - 100*a*
b^2*c^6 + 128*a^2*c^7)*e + (105*b^5*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*f
)*g^3 + 288*(168*(15*b^4*c^5 - 100*a*b^2*c^6 + 128*a^2*c^7)*d - 14*(105*b^5
*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*e + 3*(315*b^6*c^3 - 2520*a*b^4*c^4
 + 5488*a^2*b^2*c^5 - 2048*a^3*c^6)*f)*g^2*h - 18*(224*(105*b^5*c^4 - 760*a*
b^3*c^5 + 1296*a^2*b*c^6)*d - 48*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2*b
^2*c^5 - 2048*a^3*c^6)*e + 3*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2*b^
3*c^4 - 58816*a^3*b*c^5)*f)*g*h^2 + (288*(315*b^6*c^3 - 2520*a*b^4*c^4 + 54
88*a^2*b^2*c^5 - 2048*a^3*c^6)*d - 18*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 816
48*a^2*b^3*c^4 - 58816*a^3*b*c^5)*e + (45045*b^8*c - 438900*a*b^6*c^2 + 138
3984*a^2*b^4*c^3 - 1467072*a^3*b^2*c^4 + 262144*a^4*c^5)*f)*h^3 + 16*(1344*
(120*c^9*d + 132*b*c^8*e + (3*b^2*c^7 + 140*a*c^8)*f)*g^3 + 288*(1848*b*c^8
*d + 14*(3*b^2*c^7 + 140*a*c^8)*e - 3*(9*b^3*c^6 - 44*a*b*c^7)*f)*g^2*h + 1
8*(224*(3*b^2*c^7 + 140*a*c^8)*d - 48*(9*b^3*c^6 - 44*a*b*c^7)*e + 3*(99*b^
4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*f)*g*h^2 - (288*(9*b^3*c^6 - 44*a*b*c^
7)*d - 18*(99*b^4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*e + (1287*b^5*c^4 - 85
36*a*b^3*c^5 + 12912*a^2*b*c^6)*f)*h^3)*x^3 + 8*(1344*(360*b*c^8*d + 12*(b^
2*c^7 + 32*a*c^8)*e - (7*b^3*c^6 - 36*a*b*c^7)*f)*g^3 + 288*(168*(b^2*c^7 +
 32*a*c^8)*d - 14*(7*b^3*c^6 - 36*a*b*c^7)*e + 3*(21*b^4*c^5 - 124*a*b^2*c^
6 + 128*a^2*c^7)*f)*g^2*h - 18*(224*(7*b^3*c^6 - 36*a*b*c^7)*d - 48*(21*b^4
*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*e + 3*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2
416*a^2*b*c^6)*f)*g*h^2 + (288*(21*b^4*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*d
 - 18*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2416*a^2*b*c^6)*e + (3003*b^6*c^3 - 2
2968*a*b^4*c^4 + 47280*a^2*b^2*c^5 - 16384*a^3*c^6)*f)*h^3)*x^2 + 2*(1344*(
120*(b^2*c^7 + 20*a*c^8)*d - 12*(5*b^3*c^6 - 28*a*b*c^7)*e + (35*b^4*c^5 -
216*a*b^2*c^6 + 240*a^2*c^7)*f)*g^3 - 288*(168*(5*b^3*c^6 - 28*a*b*c^7)*d -
 14*(35*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*e + 3*(105*b^5*c^4 - 728*a*b
^3*c^5 + 1168*a^2*b*c^6)*f)*g^2*h + 18*(224*(35*b^4*c^5 - 216*a*b^2*c^6 + 2
40*a^2*c^7)*d - 48*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*e + 3*(11
```

```
55*b^6*c^3 - 8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*f)*g*h^2 -
(288*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*d - 18*(1155*b^6*c^3 -
8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*e + (15015*b^7*c^2 - 130
284*a*b^5*c^3 + 338832*a^2*b^3*c^4 - 236864*a^3*b*c^5)*f)*h^3)*x)*sqrt(c*x^
2 + b*x + a))/c^8]
```

giac [B] time = 0.44, size = 2977, normalized size = 2.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/10321920*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*(16*c*f*h^3*x + (54
*c^9*f*g*h^2 + 19*b*c^8*f*h^3 + 18*c^9*h^3*e)/c^8)*x + (864*c^9*f*g^2*h + 9
18*b*c^8*f*g*h^2 + 288*c^9*d*h^3 + 3*b^2*c^7*f*h^3 + 320*a*c^8*f*h^3 + 864*
c^9*g*h^2*e + 306*b*c^8*h^3*e)/c^8)*x + (1344*c^9*f*g^3 + 4320*b*c^8*f*g^2*
h + 4032*c^9*d*g*h^2 + 54*b^2*c^7*f*g*h^2 + 4536*a*c^8*f*g*h^2 + 1440*b*c^8
*d*h^3 - 13*b^3*c^6*f*h^3 + 60*a*b*c^7*f*h^3 + 4032*c^9*g^2*h*e + 4320*b*c^
8*g*h^2*e + 18*b^2*c^7*h^3*e + 1512*a*c^8*h^3*e)/c^8)*x + (17472*b*c^8*f*g^
3 + 48384*c^9*d*g^2*h + 864*b^2*c^7*f*g^2*h + 55296*a*c^8*f*g^2*h + 52416*b
*c^8*d*g*h^2 - 594*b^3*c^6*f*g*h^2 + 2808*a*b*c^7*f*g*h^2 + 288*b^2*c^7*d*h
^3 + 18432*a*c^8*d*h^3 + 143*b^4*c^5*f*h^3 - 804*a*b^2*c^6*f*h^3 + 768*a^2*
c^7*f*h^3 + 16128*c^9*g^3*e + 52416*b*c^8*g^2*h*e + 864*b^2*c^7*g*h^2*e + 5
5296*a*c^8*g*h^2*e - 198*b^3*c^6*h^3*e + 936*a*b*c^7*h^3*e)/c^8)*x + (16128
0*c^9*d*g^3 + 4032*b^2*c^7*f*g^3 + 188160*a*c^8*f*g^3 + 532224*b*c^8*d*g^2*
h - 7776*b^3*c^6*f*g^2*h + 38016*a*b*c^7*f*g^2*h + 12096*b^2*c^7*d*g*h^2 +
564480*a*c^8*d*g*h^2 + 5346*b^4*c^5*f*g*h^2 - 30672*a*b^2*c^6*f*g*h^2 + 302
40*a^2*c^7*f*g*h^2 - 2592*b^3*c^6*d*h^3 + 12672*a*b*c^7*d*h^3 - 1287*b^5*c^
4*f*h^3 + 8536*a*b^3*c^5*f*h^3 - 12912*a^2*b*c^6*f*h^3 + 177408*b*c^8*g^3*e
 + 12096*b^2*c^7*g^2*h*e + 564480*a*c^8*g^2*h*e - 7776*b^3*c^6*g*h^2*e + 38
016*a*b*c^7*g*h^2*e + 1782*b^4*c^5*h^3*e - 10224*a*b^2*c^6*h^3*e + 10080*a^
2*c^7*h^3*e)/c^8)*x + (483840*b*c^8*d*g^3 - 9408*b^3*c^6*f*g^3 + 48384*a*b*
c^7*f*g^3 + 48384*b^2*c^7*d*g^2*h + 1548288*a*c^8*d*g^2*h + 18144*b^4*c^5*f
*g^2*h - 107136*a*b^2*c^6*f*g^2*h + 110592*a^2*c^7*f*g^2*h - 28224*b^3*c^6*
d*g*h^2 + 145152*a*b*c^7*d*g*h^2 - 12474*b^5*c^4*f*g*h^2 + 84240*a*b^3*c^5*
f*g*h^2 - 130464*a^2*b*c^6*f*g*h^2 + 6048*b^4*c^5*d*h^3 - 35712*a*b^2*c^6*d
*h^3 + 36864*a^2*c^7*d*h^3 + 3003*b^6*c^3*f*h^3 - 22968*a*b^4*c^4*f*h^3 + 4
7280*a^2*b^2*c^5*f*h^3 - 16384*a^3*c^6*f*h^3 + 16128*b^2*c^7*g^3*e + 516096
*a*c^8*g^3*e - 28224*b^3*c^6*g^2*h*e + 145152*a*b*c^7*g^2*h*e + 18144*b^4*c
^5*g*h^2*e - 107136*a*b^2*c^6*g*h^2*e + 110592*a^2*c^7*g*h^2*e - 4158*b^5*c
^4*h^3*e + 28080*a*b^3*c^5*h^3*e - 43488*a^2*b*c^6*h^3*e)/c^8)*x + (161280*
b^2*c^7*d*g^3 + 3225600*a*c^8*d*g^3 + 47040*b^4*c^5*f*g^3 - 290304*a*b^2*c^
6*f*g^3 + 322560*a^2*c^7*f*g^3 - 241920*b^3*c^6*d*g^2*h + 1354752*a*b*c^7*d
*g^2*h - 90720*b^5*c^4*f*g^2*h + 628992*a*b^3*c^5*f*g^2*h - 1009152*a^2*b*c
^6*f*g^2*h + 141120*b^4*c^5*d*g*h^2 - 870912*a*b^2*c^6*d*g*h^2 + 967680*a^2
*c^7*d*g*h^2 + 62370*b^6*c^3*f*g*h^2 - 485352*a*b^4*c^4*f*g*h^2 + 1020384*a
^2*b^2*c^5*f*g*h^2 - 362880*a^3*c^6*f*g*h^2 - 30240*b^5*c^4*d*h^3 + 209664*
a*b^3*c^5*d*h^3 - 336384*a^2*b*c^6*d*h^3 - 15015*b^7*c^2*f*h^3 + 130284*a*b
^5*c^3*f*h^3 - 338832*a^2*b^3*c^4*f*h^3 + 236864*a^3*b*c^5*f*h^3 - 80640*b^
3*c^6*g^3*e + 451584*a*b*c^7*g^3*e + 141120*b^4*c^5*g^2*h*e - 870912*a*b^2*
c^6*g^2*h*e + 967680*a^2*c^7*g^2*h*e - 90720*b^5*c^4*g*h^2*e + 628992*a*b^3
*c^5*g*h^2*e - 1009152*a^2*b*c^6*g*h^2*e + 20790*b^6*c^3*h^3*e - 161784*a*b
^4*c^4*h^3*e + 340128*a^2*b^2*c^5*h^3*e - 120960*a^3*c^6*h^3*e)/c^8)*x - (4
83840*b^3*c^6*d*g^3 - 3225600*a*b*c^7*d*g^3 + 141120*b^5*c^4*f*g^3 - 102144
0*a*b^3*c^5*f*g^3 + 1741824*a^2*b*c^6*f*g^3 - 725760*b^4*c^5*d*g^2*h + 4838
400*a*b^2*c^6*d*g^2*h - 6193152*a^2*c^7*d*g^2*h - 272160*b^6*c^3*f*g^2*h +
2177280*a*b^4*c^4*f*g^2*h - 4741632*a^2*b^2*c^5*f*g^2*h + 1769472*a^3*c^6*f
*g^2*h + 423360*b^5*c^4*d*g*h^2 - 3064320*a*b^3*c^5*d*g*h^2 + 5225472*a^2*b
*c^6*d*g*h^2 + 187110*b^7*c^2*f*g*h^2 - 1655640*a*b^5*c^3*f*g*h^2 + 4408992
```

```
*a^2*b^3*c^4*f*g*h^2 - 3176064*a^3*b*c^5*f*g*h^2 - 90720*b^6*c^3*d*h^3 + 72
5760*a*b^4*c^4*d*h^3 - 1580544*a^2*b^2*c^5*d*h^3 + 589824*a^3*c^6*d*h^3 - 4
5045*b^8*c*f*h^3 + 438900*a*b^6*c^2*f*h^3 - 1383984*a^2*b^4*c^3*f*h^3 + 146
7072*a^3*b^2*c^4*f*h^3 - 262144*a^4*c^5*f*h^3 - 241920*b^4*c^5*g^3*e + 1612
800*a*b^2*c^6*g^3*e - 2064384*a^2*c^7*g^3*e + 423360*b^5*c^4*g^2*h*e - 3064
320*a*b^3*c^5*g^2*h*e + 5225472*a^2*b*c^6*g^2*h*e - 272160*b^6*c^3*g*h^2*e
+ 2177280*a*b^4*c^4*g*h^2*e - 4741632*a^2*b^2*c^5*g*h^2*e + 1769472*a^3*c^6
*g*h^2*e + 62370*b^7*c^2*h^3*e - 551880*a*b^5*c^3*h^3*e + 1469664*a^2*b^3*c
^4*h^3*e - 1058688*a^3*b*c^5*h^3*e)/c^8) - 1/65536*(1536*b^4*c^5*d*g^3 - 12
288*a*b^2*c^6*d*g^3 + 24576*a^2*c^7*d*g^3 + 448*b^6*c^3*f*g^3 - 3840*a*b^4*
c^4*f*g^3 + 9216*a^2*b^2*c^5*f*g^3 - 4096*a^3*c^6*f*g^3 - 2304*b^5*c^4*d*g^
2*h + 18432*a*b^3*c^5*d*g^2*h - 36864*a^2*b*c^6*d*g^2*h - 864*b^7*c^2*f*g^2
*h + 8064*a*b^5*c^3*f*g^2*h - 23040*a^2*b^3*c^4*f*g^2*h + 18432*a^3*b*c^5*f
*g^2*h + 1344*b^6*c^3*d*g*h^2 - 11520*a*b^4*c^4*d*g*h^2 + 27648*a^2*b^2*c^5
*d*g*h^2 - 12288*a^3*c^6*d*g*h^2 + 594*b^8*c*f*g*h^2 - 6048*a*b^6*c^2*f*g*h
^2 + 20160*a^2*b^4*c^3*f*g*h^2 - 23040*a^3*b^2*c^4*f*g*h^2 + 4608*a^4*c^5*f
*g*h^2 - 288*b^7*c^2*d*h^3 + 2688*a*b^5*c^3*d*h^3 - 7680*a^2*b^3*c^4*d*h^3
+ 6144*a^3*b*c^5*d*h^3 - 143*b^9*f*h^3 + 1584*a*b^7*c*f*h^3 - 6048*a^2*b^5*
c^2*f*h^3 + 8960*a^3*b^3*c^3*f*h^3 - 3840*a^4*b*c^4*f*h^3 - 768*b^5*c^4*g^3
*e + 6144*a*b^3*c^5*g^3*e - 12288*a^2*b*c^6*g^3*e + 1344*b^6*c^3*g^2*h*e -
11520*a*b^4*c^4*g^2*h*e + 27648*a^2*b^2*c^5*g^2*h*e - 12288*a^3*c^6*g^2*h*e
 - 864*b^7*c^2*g*h^2*e + 8064*a*b^5*c^3*g*h^2*e - 23040*a^2*b^3*c^4*g*h^2*e
 + 18432*a^3*b*c^5*g*h^2*e + 198*b^8*c*h^3*e - 2016*a*b^6*c^2*h^3*e + 6720*
a^2*b^4*c^3*h^3*e - 7680*a^3*b^2*c^4*h^3*e + 1536*a^4*c^5*h^3*e)*log(abs(-2
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(15/2)
```

maple [B] time = 0.03, size = 5881, normalized size = 5.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (g + h\,x)^3\,\left(c\,x^2 + b\,x + a\right)^{3/2}\,\left(f\,x^2 + e\,x + d\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (g + hx)^3 \left(a + bx + cx^2\right)^{\frac{3}{2}} \left(d + ex + fx^2\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((g + h*x)**3*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

3.197 $\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=753

$$\frac{\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(dh^2+2egh+fg^2\right)\right)-72b^2ch(3a\right.}{32768c^{13/2}}$$

```
[Out] 1/6144*(768*c^4*d*g^2+99*b^4*f*h^2-72*b^2*c*h*(3*a*f*h+2*b*e*h+4*b*f*g)-128
*c^3*(3*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+16*c^2*(3*a^2*f*h^2+12*a*b
*h*(e*h+2*f*g)+14*b^2*(d*h^2+2*e*g*h+f*g^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)
/c^5-1/112*(11*b*f*h-16*c*e*h+10*c*f*g)*(h*x+g)^2*(c*x^2+b*x+a)^(5/2)/c^2/h
+1/8*f*(h*x+g)^3*(c*x^2+b*x+a)^(5/2)/c/h-1/13440*(693*b^3*f*h^3+96*c^3*g*(5
*f*g^2-8*h*(7*d*h+e*g))-36*b*c*h^2*(31*a*f*h+28*b*(e*h+2*f*g))+8*c^2*h*(96*
a*h*(e*h+2*f*g)+b*(31*f*g^2+196*h*(d*h+2*e*g)))-10*c*h*(99*b^2*f*h^2-8*c^2*
(5*f*g^2-4*h*(7*d*h+2*e*g))-12*c*h*(7*a*f*h+2*b*(6*e*h+f*g)))*x)*(c*x^2+b*x
+a)^(5/2)/c^4/h+1/32768*(-4*a*c+b^2)^2*(768*c^4*d*g^2+99*b^4*f*h^2-72*b^2*c
*h*(3*a*f*h+2*b*e*h+4*b*f*g)-128*c^3*(3*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*
g^2))+16*c^2*(3*a^2*f*h^2+12*a*b*h*(e*h+2*f*g)+14*b^2*(d*h^2+2*e*g*h+f*g^2)
))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13/2)-1/16384*(-4*
a*c+b^2)*(768*c^4*d*g^2+99*b^4*f*h^2-72*b^2*c*h*(3*a*f*h+2*b*e*h+4*b*f*g)-1
28*c^3*(3*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+16*c^2*(3*a^2*f*h^2+12*a
*b*h*(e*h+2*f*g)+14*b^2*(d*h^2+2*e*g*h+f*g^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(1/
2)/c^6
```

Rubi [A] time = 2.10, antiderivative size = 749, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1653, 832, 779, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(h(dh+2eg)+fg^2\right)\right)-72b^2ch(3afh+2b\right.}{6144c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(768*c^4*d*g^2 + 99*b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e
*h + 3*a*f*h) - 128*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 3*b*g*(e*g + 2*d*h))
 + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2*f*g + e*h) + 14*b^2*(f*g^2 + h*(2*e*g
+ d*h))))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(16384*c^6) + ((768*c^4*d*g^2
+ 99*b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^
2 + a*h*(2*e*g + d*h) + 3*b*g*(e*g + 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b
*h*(2*f*g + e*h) + 14*b^2*(f*g^2 + h*(2*e*g + d*h))))*(b + 2*c*x)*(a + b*x
+ c*x^2)^(3/2))/(6144*c^5) - ((10*c*f*g - 16*c*e*h + 11*b*f*h)*(g + h*x)^2*
(a + b*x + c*x^2)^(5/2))/(112*c^2*h) + (f*(g + h*x)^3*(a + b*x + c*x^2)^(5/
2))/(8*c*h) - ((693*b^3*f*h^3 + 96*c^3*(5*f*g^3 - 8*g*h*(e*g + 7*d*h)) - 36
*b*c*h^2*(31*a*f*h + 28*b*(2*f*g + e*h)) + 8*c^2*h*(31*b*f*g^2 + 196*b*h*(2
*e*g + d*h) + 96*a*h*(2*f*g + e*h)) - 10*c*h*(99*b^2*f*h^2 - 8*c^2*(5*f*g^2
 - 4*h*(2*e*g + 7*d*h)) - 12*c*h*(7*a*f*h + 2*b*(f*g + 6*e*h)))*x)*(a + b*x
 + c*x^2)^(5/2))/(13440*c^4*h) + ((b^2 - 4*a*c)^2*(768*c^4*d*g^2 + 99*b^4*f
*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^2 + a*h*(2
*e*g + d*h) + 3*b*g*(e*g + 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2*f*g
+ e*h) + 14*b^2*(f*g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(32768*c^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \frac{f(g+hx)^3 \left(a+bx+cx^2\right)^{5/2}}{8ch} + \frac{\int (g+hx)^2 \left(-\frac{1}{2}h(5bfg-16c\right.}{}$$

$$= -\frac{(10cfg-16ceh+11bfh)(g+hx)^2 \left(a+bx+cx^2\right)^{5/2}}{112c^2h} + \frac{f(g}{}$$

$$= -\frac{(10cfg-16ceh+11bfh)(g+hx)^2 \left(a+bx+cx^2\right)^{5/2}}{112c^2h} + \frac{f(g}{}$$

$$= \frac{\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+\right.}{}$$

Mathematica [A] time = 1.60, size = 468, normalized size = 0.62

$$\frac{h\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(h(dh+2eg)+fg^2\right.\right.\right.}{12288c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (-1/14*((11*b*f*h + 2*c*(5*f*g - 8*e*h))*(g + h*x)^2*(a + x*(b + c*x))^(5/2
))/c + f*(g + h*x)^3*(a + x*(b + c*x))^(5/2) - ((a + x*(b + c*x))^(5/2)*(69
3*b^3*f*h^3 + 8*c^2*h*(b*f*g*(31*g + 30*h*x) + 4*b*h*(98*e*g + 49*d*h + 45*
e*h*x) + 3*a*h*(64*f*g + 32*e*h + 35*f*h*x)) - 18*b*c*h^2*(62*a*f*h + b*(11
2*f*g + 56*e*h + 55*f*h*x)) + 16*c^3*(5*f*g^2*(6*g + 5*h*x) - 4*h*(2*e*g*(6
*g + 5*h*x) + 7*d*h*(12*g + 5*h*x)))))/(1680*c^3) + (h*(768*c^4*d*g^2 + 99*
b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^2 + a
*h*(2*e*g + d*h) + 3*b*g*(e*g + 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2
*f*g + e*h) + 14*b^2*(f*g^2 + h*(2*e*g + d*h))))*(2*Sqrt[c]*(b + 2*c*x)*Sqr
t[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*
c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(12288*c^(11/
2)))/(8*c*h)
```

fricas [B] time = 4.35, size = 3145, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas"
)
```

```
[Out] [1/6881280*(105*(32*(24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d - 12*(b^5*c^
3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e + (7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2
*c^4 - 64*a^3*c^5)*f)*g^2 - 32*(24*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d
 - 2*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e + 3*(3*b^7
*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*f)*g*h + (32*(7*b^6*c^2
```

```
- 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 48*(3*b^7*c - 28*a*b^5*c
^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*e + 3*(33*b^8 - 336*a*b^6*c + 1120*a^2*
b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f)*h^2)*sqrt(c)*log(-8*c^2*x^2 -
8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2
15040*c^8*f*h^2*x^7 + 15360*(32*c^8*f*g*h + (16*c^8*e + 17*b*c^7*f)*h^2)*x^
6 + 1280*(224*c^8*f*g^2 + 32*(14*c^8*e + 15*b*c^7*f)*g*h + (224*c^8*d + 240
*b*c^7*e + 3*(b^2*c^6 + 84*a*c^7)*f)*h^2)*x^5 + 128*(224*(12*c^8*e + 13*b*c
^7*f)*g^2 + 32*(168*c^8*d + 182*b*c^7*e + 3*(b^2*c^6 + 64*a*c^7)*f)*g*h + (
2912*b*c^7*d + 48*(b^2*c^6 + 64*a*c^7)*e - 3*(11*b^3*c^5 - 52*a*b*c^6)*f)*h
^2)*x^4 + 16*(224*(120*c^8*d + 132*b*c^7*e + (3*b^2*c^6 + 140*a*c^7)*f)*g^2
 + 32*(1848*b*c^7*d + 14*(3*b^2*c^6 + 140*a*c^7)*e - 3*(9*b^3*c^5 - 44*a*b*
c^6)*f)*g*h + (224*(3*b^2*c^6 + 140*a*c^7)*d - 48*(9*b^3*c^5 - 44*a*b*c^6)*
e + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f)*h^2)*x^3 - 224*(120*(3*
b^3*c^5 - 20*a*b*c^6)*d - 12*(15*b^4*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*e +
 (105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*f)*g^2 + 32*(168*(15*b^4*c^
4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d - 14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296
*a^2*b*c^5)*e + 3*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a
^3*c^5)*f)*g*h - (224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 48
*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e + 3*(34
65*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f)*h^2 +
8*(224*(360*b*c^7*d + 12*(b^2*c^6 + 32*a*c^7)*e - (7*b^3*c^5 - 36*a*b*c^6)*
f)*g^2 + 32*(168*(b^2*c^6 + 32*a*c^7)*d - 14*(7*b^3*c^5 - 36*a*b*c^6)*e + 3
*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*f)*g*h - (224*(7*b^3*c^5 - 36*a
*b*c^6)*d - 48*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*e + 3*(231*b^5*c^
3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f)*h^2)*x^2 + 2*(224*(120*(b^2*c^6 + 2
0*a*c^7)*d - 12*(5*b^3*c^5 - 28*a*b*c^6)*e + (35*b^4*c^4 - 216*a*b^2*c^5 +
240*a^2*c^6)*f)*g^2 - 32*(168*(5*b^3*c^5 - 28*a*b*c^6)*d - 14*(35*b^4*c^4 -
 216*a*b^2*c^5 + 240*a^2*c^6)*e + 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2
*b*c^5)*f)*g*h + (224*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*d - 48*(10
5*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e + 3*(1155*b^6*c^2 - 8988*a*b^
4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))
/c^7, -1/3440640*(105*(32*(24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d - 12*(
b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e + (7*b^6*c^2 - 60*a*b^4*c^3 + 144*a
^2*b^2*c^4 - 64*a^3*c^5)*f)*g^2 - 32*(24*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*
c^5)*d - 2*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e + 3*
(3*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*f)*g*h + (32*(7*b^
6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 48*(3*b^7*c - 28*a
*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*e + 3*(33*b^8 - 336*a*b^6*c + 112
0*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f)*h^2)*sqrt(-c)*arctan(1/2
*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2
15040*c^8*f*h^2*x^7 + 15360*(32*c^8*f*g*h + (16*c^8*e + 17*b*c^7*f)*h^2)*x^
6 + 1280*(224*c^8*f*g^2 + 32*(14*c^8*e + 15*b*c^7*f)*g*h + (224*c^8*d + 240
*b*c^7*e + 3*(b^2*c^6 + 84*a*c^7)*f)*h^2)*x^5 + 128*(224*(12*c^8*e + 13*b*c
^7*f)*g^2 + 32*(168*c^8*d + 182*b*c^7*e + 3*(b^2*c^6 + 64*a*c^7)*f)*g*h + (
2912*b*c^7*d + 48*(b^2*c^6 + 64*a*c^7)*e - 3*(11*b^3*c^5 - 52*a*b*c^6)*f)*h
^2)*x^4 + 16*(224*(120*c^8*d + 132*b*c^7*e + (3*b^2*c^6 + 140*a*c^7)*f)*g^2
 + 32*(1848*b*c^7*d + 14*(3*b^2*c^6 + 140*a*c^7)*e - 3*(9*b^3*c^5 - 44*a*b*
c^6)*f)*g*h + (224*(3*b^2*c^6 + 140*a*c^7)*d - 48*(9*b^3*c^5 - 44*a*b*c^6)*
e + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f)*h^2)*x^3 - 224*(120*(3*
b^3*c^5 - 20*a*b*c^6)*d - 12*(15*b^4*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*e +
 (105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*f)*g^2 + 32*(168*(15*b^4*c^
4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d - 14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296
*a^2*b*c^5)*e + 3*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a
^3*c^5)*f)*g*h - (224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 48
*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e + 3*(34
65*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f)*h^2 +
8*(224*(360*b*c^7*d + 12*(b^2*c^6 + 32*a*c^7)*e - (7*b^3*c^5 - 36*a*b*c^6)*
f)*g^2 + 32*(168*(b^2*c^6 + 32*a*c^7)*d - 14*(7*b^3*c^5 - 36*a*b*c^6)*e + 3
*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*f)*g*h - (224*(7*b^3*c^5 - 36*a
```

```
*b*c^6)*d - 48*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*e + 3*(231*b^5*c^
3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f)*h^2)*x^2 + 2*(224*(120*(b^2*c^6 + 2
0*a*c^7)*d - 12*(5*b^3*c^5 - 28*a*b*c^6)*e + (35*b^4*c^4 - 216*a*b^2*c^5 +
240*a^2*c^6)*f)*g^2 - 32*(168*(5*b^3*c^5 - 28*a*b*c^6)*d - 14*(35*b^4*c^4 -
 216*a*b^2*c^5 + 240*a^2*c^6)*e + 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2
*b*c^5)*f)*g*h + (224*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*d - 48*(10
5*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e + 3*(1155*b^6*c^2 - 8988*a*b^
4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))
/c^7]
```

giac [B] time = 0.37, size = 1852, normalized size = 2.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/1720320*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*(14*c*f*h^2*x + (32*c^8
*f*g*h + 17*b*c^7*f*h^2 + 16*c^8*h^2*e)/c^7)*x + (224*c^8*f*g^2 + 480*b*c^7
*f*g*h + 224*c^8*d*h^2 + 3*b^2*c^6*f*h^2 + 252*a*c^7*f*h^2 + 448*c^8*g*h*e
+ 240*b*c^7*h^2*e)/c^7)*x + (2912*b*c^7*f*g^2 + 5376*c^8*d*g*h + 96*b^2*c^6
*f*g*h + 6144*a*c^7*f*g*h + 2912*b*c^7*d*h^2 - 33*b^3*c^5*f*h^2 + 156*a*b*c
^6*f*h^2 + 2688*c^8*g^2*e + 5824*b*c^7*g*h*e + 48*b^2*c^6*h^2*e + 3072*a*c^
7*h^2*e)/c^7)*x + (26880*c^8*d*g^2 + 672*b^2*c^6*f*g^2 + 31360*a*c^7*f*g^2
+ 59136*b*c^7*d*g*h - 864*b^3*c^5*f*g*h + 4224*a*b*c^6*f*g*h + 672*b^2*c^6*
d*h^2 + 31360*a*c^7*d*h^2 + 297*b^4*c^4*f*h^2 - 1704*a*b^2*c^5*f*h^2 + 1680
*a^2*c^6*f*h^2 + 29568*b*c^7*g^2*e + 1344*b^2*c^6*g*h*e + 62720*a*c^7*g*h*e
 - 432*b^3*c^5*h^2*e + 2112*a*b*c^6*h^2*e)/c^7)*x + (80640*b*c^7*d*g^2 - 15
68*b^3*c^5*f*g^2 + 8064*a*b*c^6*f*g^2 + 5376*b^2*c^6*d*g*h + 172032*a*c^7*d
*g*h + 2016*b^4*c^4*f*g*h - 11904*a*b^2*c^5*f*g*h + 12288*a^2*c^6*f*g*h - 1
568*b^3*c^5*d*h^2 + 8064*a*b*c^6*d*h^2 - 693*b^5*c^3*f*h^2 + 4680*a*b^3*c^4
*f*h^2 - 7248*a^2*b*c^5*f*h^2 + 2688*b^2*c^6*g^2*e + 86016*a*c^7*g^2*e - 31
36*b^3*c^5*g*h*e + 16128*a*b*c^6*g*h*e + 1008*b^4*c^4*h^2*e - 5952*a*b^2*c^
5*h^2*e + 6144*a^2*c^6*h^2*e)/c^7)*x + (26880*b^2*c^6*d*g^2 + 537600*a*c^7*
d*g^2 + 7840*b^4*c^4*f*g^2 - 48384*a*b^2*c^5*f*g^2 + 53760*a^2*c^6*f*g^2 -
26880*b^3*c^5*d*g*h + 150528*a*b*c^6*d*g*h - 10080*b^5*c^3*f*g*h + 69888*a*
b^3*c^4*f*g*h - 112128*a^2*b*c^5*f*g*h + 7840*b^4*c^4*d*h^2 - 48384*a*b^2*c
^5*d*h^2 + 53760*a^2*c^6*d*h^2 + 3465*b^6*c^2*f*h^2 - 26964*a*b^4*c^3*f*h^2
 + 56688*a^2*b^2*c^4*f*h^2 - 20160*a^3*c^5*f*h^2 - 13440*b^3*c^5*g^2*e + 75
264*a*b*c^6*g^2*e + 15680*b^4*c^4*g*h*e - 96768*a*b^2*c^5*g*h*e + 107520*a^
2*c^6*g*h*e - 5040*b^5*c^3*h^2*e + 34944*a*b^3*c^4*h^2*e - 56064*a^2*b*c^5*
h^2*e)/c^7)*x - (80640*b^3*c^5*d*g^2 - 537600*a*b*c^6*d*g^2 + 23520*b^5*c^3
*f*g^2 - 170240*a*b^3*c^4*f*g^2 + 290304*a^2*b*c^5*f*g^2 - 80640*b^4*c^4*d*
g*h + 537600*a*b^2*c^5*d*g*h - 688128*a^2*c^6*d*g*h - 30240*b^6*c^2*f*g*h +
 241920*a*b^4*c^3*f*g*h - 526848*a^2*b^2*c^4*f*g*h + 196608*a^3*c^5*f*g*h +
 23520*b^5*c^3*d*h^2 - 170240*a*b^3*c^4*d*h^2 + 290304*a^2*b*c^5*d*h^2 + 10
395*b^7*c*f*h^2 - 91980*a*b^5*c^2*f*h^2 + 244944*a^2*b^3*c^3*f*h^2 - 176448
*a^3*b*c^4*f*h^2 - 40320*b^4*c^4*g^2*e + 268800*a*b^2*c^5*g^2*e - 344064*a^
2*c^6*g^2*e + 47040*b^5*c^3*g*h*e - 340480*a*b^3*c^4*g*h*e + 580608*a^2*b*c
^5*g*h*e - 15120*b^6*c^2*h^2*e + 120960*a*b^4*c^3*h^2*e - 263424*a^2*b^2*c^
4*h^2*e + 98304*a^3*c^5*h^2*e)/c^7) - 1/32768*(768*b^4*c^4*d*g^2 - 6144*a*b
^2*c^5*d*g^2 + 12288*a^2*c^6*d*g^2 + 224*b^6*c^2*f*g^2 - 1920*a*b^4*c^3*f*g
^2 + 4608*a^2*b^2*c^4*f*g^2 - 2048*a^3*c^5*f*g^2 - 768*b^5*c^3*d*g*h + 6144
*a*b^3*c^4*d*g*h - 12288*a^2*b*c^5*d*g*h - 288*b^7*c*f*g*h + 2688*a*b^5*c^2
*f*g*h - 7680*a^2*b^3*c^3*f*g*h + 6144*a^3*b*c^4*f*g*h + 224*b^6*c^2*d*h^2
- 1920*a*b^4*c^3*d*h^2 + 4608*a^2*b^2*c^4*d*h^2 - 2048*a^3*c^5*d*h^2 + 99*b
^8*f*h^2 - 1008*a*b^6*c*f*h^2 + 3360*a^2*b^4*c^2*f*h^2 - 3840*a^3*b^2*c^3*f
*h^2 + 768*a^4*c^4*f*h^2 - 384*b^5*c^3*g^2*e + 3072*a*b^3*c^4*g^2*e - 6144*
a^2*b*c^5*g^2*e + 448*b^6*c^2*g*h*e - 3840*a*b^4*c^3*g*h*e + 9216*a^2*b^2*c
^4*g*h*e - 4096*a^3*c^5*g*h*e - 144*b^7*c*h^2*e + 1344*a*b^5*c^2*h^2*e - 38
```

```
40*a^2*b^3*c^3*h^2*e + 3072*a^3*b*c^4*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2)
```

maple [B] time = 0.02, size = 3769, normalized size = 5.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/5*(c*x^2+b*x+a)^(5/2)/c*e*g^2+1/4*d*g^2*(c*x^2+b*x+a)^(3/2)*x+3/16/c^(5/2
)*b*a^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g*h-15/128/c^(7/2)*b^
4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g*h-1/12*a/c*(c*x^2+b*x+a
)^(3/2)*x*e*g*h-1/24*a/c^2*(c*x^2+b*x+a)^(3/2)*b*e*g*h-1/8*a^2/c*(c*x^2+b*x
+a)^(1/2)*x*e*g*h-7/30/c^2*b*(c*x^2+b*x+a)^(5/2)*e*g*h+3/20/c^3*b^2*(c*x^2+
b*x+a)^(5/2)*f*g*h-3/64/c^4*b^4*(c*x^2+b*x+a)^(3/2)*f*g*h+9/512/c^5*b^6*(c*
x^2+b*x+a)^(1/2)*f*g*h-15/128/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))*a^2*e*h^2+21/512/c^(9/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))*a*e*h^2+3/32/c^(5/2)*b*a^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*e*h^2-3/28/c^2*b*x*(c*x^2+b*x+a)^(5/2)*e*h^2-3/64/c^3*b^3*(c*x^2+b*x+a
)^(3/2)*x*e*h^2-9/256*f*h^2/c^4*b^3*a*(c*x^2+b*x+a)^(3/2)-57/1024*f*h^2/c^4
*b^3*a^2*(c*x^2+b*x+a)^(1/2)-11/112*f*h^2/c^2*b*x^2*(c*x^2+b*x+a)^(5/2)+93/
1120*f*h^2/c^3*b*a*(c*x^2+b*x+a)^(5/2)+105/1024*f*h^2/c^(9/2)*b^4*ln((c*x+1
/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-63/2048*f*h^2/c^(11/2)*b^6*ln((c*x+1
/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+3/128/c^3*b^4*(c*x^2+b*x+a)^(1/2)*e*g^
2-3/256/c^(7/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g^2+1/8*d
*g^2/c*(c*x^2+b*x+a)^(3/2)*b+3/8*d*g^2*(c*x^2+b*x+a)^(1/2)*x*a-3/64*d*g^2/c
^2*(c*x^2+b*x+a)^(1/2)*b^3+3/8*d*g^2/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))*a^2+7/96/c^2*b^2*(c*x^2+b*x+a)^(3/2)*x*d*h^2+7/96/c^2*b^2*(c*
x^2+b*x+a)^(3/2)*x*f*g^2+7/96/c^3*b^3*(c*x^2+b*x+a)^(3/2)*e*g*h-7/256/c^3*b
^4*(c*x^2+b*x+a)^(1/2)*x*d*h^2-9/128*f*h^2/c^3*b^2*a*(c*x^2+b*x+a)^(3/2)*x-
57/512*f*h^2/c^3*b^2*a^2*(c*x^2+b*x+a)^(1/2)*x-1/16/c^2*b^2*(c*x^2+b*x+a)^(
3/2)*e*g^2+2/5*(c*x^2+b*x+a)^(5/2)/c*d*g*h-3/32/c^3*b^3*(c*x^2+b*x+a)^(1/2)
*x*a*e*h^2-15/64/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^
2*f*g*h+21/256/c^(9/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f*
g*h+1/16/c^2*b*a*(c*x^2+b*x+a)^(3/2)*x*e*h^2+1/16/c^3*b^2*a*(c*x^2+b*x+a)^(
3/2)*f*g*h+3/32/c^3*b^2*a^2*(c*x^2+b*x+a)^(1/2)*f*g*h+3/32/c^2*b*a^2*(c*x^2
+b*x+a)^(1/2)*x*e*h^2+9/32/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))*a^2*e*g*h+1/8/c^3*b^3*(c*x^2+b*x+a)^(1/2)*a*e*g*h-1/4/c*b*(c*x^2+b*
x+a)^(3/2)*x*d*g*h-3/16/c*b*(c*x^2+b*x+a)^(1/2)*x*a*e*g^2+3/32/c^2*b^3*(c*x
^2+b*x+a)^(1/2)*x*d*g*h-3/16/c^2*b^2*(c*x^2+b*x+a)^(1/2)*a*d*g*h-3/8/c^(3/2
)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*d*g*h+3/16/c^(5/2)*b^3*
ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*g*h+9/256/c^4*b^5*(c*x^2+b*
x+a)^(1/2)*x*f*g*h-3/14/c^2*b*x*(c*x^2+b*x+a)^(5/2)*f*g*h-3/32/c^3*b^3*(c*x
^2+b*x+a)^(3/2)*x*f*g*h-3/32/c^4*b^4*(c*x^2+b*x+a)^(1/2)*a*f*g*h-3/32/c^2*b
^2*(c*x^2+b*x+a)^(1/2)*a*e*g^2+3/64/c^3*b^4*(c*x^2+b*x+a)^(1/2)*d*g*h-3/16/
c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*e*g^2+3/32/c^(5/2
)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g^2-3/128/c^(7/2)*b^5
*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*g*h-3/32*d*g^2/c*(c*x^2+b*x+
a)^(1/2)*x*b^2+2/7*x^2*(c*x^2+b*x+a)^(5/2)/c*f*g*h-4/35*a/c^2*(c*x^2+b*x+a)
^(5/2)*f*g*h-15/128*f*h^2/c^(7/2)*b^2*a^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))+1/128*f*h^2*a^2/c^3*(c*x^2+b*x+a)^(3/2)*b+3/128*f*h^2*a^3/c^2*(c
*x^2+b*x+a)^(1/2)*x+3/256*f*h^2*a^3/c^3*(c*x^2+b*x+a)^(1/2)*b-1/16*f*h^2*a/
c^2*x*(c*x^2+b*x+a)^(5/2)+1/64*f*h^2*a^2/c^2*(c*x^2+b*x+a)^(3/2)*x+33/448*f
*h^2/c^3*b^2*x*(c*x^2+b*x+a)^(5/2)+33/1024*f*h^2/c^4*b^4*(c*x^2+b*x+a)^(3/2
)*x-99/8192*f*h^2/c^5*b^6*(c*x^2+b*x+a)^(1/2)*x-1/8/c*b*(c*x^2+b*x+a)^(3/2)
*x*e*g^2-1/8/c^2*b^2*(c*x^2+b*x+a)^(3/2)*d*g*h+3/64/c^2*b^3*(c*x^2+b*x+a)^(
1/2)*x*e*g^2+9/512/c^4*b^5*(c*x^2+b*x+a)^(1/2)*x*e*h^2-3/64/c^4*b^4*(c*x^2+
b*x+a)^(1/2)*a*e*h^2-9/1024/c^(11/2)*b^7*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))*f*g*h+1/32/c^3*b^2*a*(c*x^2+b*x+a)^(3/2)*e*h^2+3/64/c^3*b^2*a^2*(
```

```
c*x^2+b*x+a)^(1/2)*e*h^2-15/256/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))*a*d*h^2-15/256/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))*a*f*g^2+7/512/c^(9/2)*b^6*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))*e*g*h-1/24*a/c*(c*x^2+b*x+a)^(3/2)*x*d*h^2-1/24*a/c*(c*x^2+b*x+a)^(3/2)
*x*f*g^2-1/48*a/c^2*(c*x^2+b*x+a)^(3/2)*b*d*h^2-1/48*a/c^2*(c*x^2+b*x+a)^(3
/2)*b*f*g^2-1/16*a^2/c*(c*x^2+b*x+a)^(1/2)*x*d*h^2-1/16*a^2/c*(c*x^2+b*x+a)
^(1/2)*x*f*g^2-1/32*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b*d*h^2-1/32*a^2/c^2*(c*x^2
+b*x+a)^(1/2)*b*f*g^2-1/8*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))*e*g*h+1/3*x*(c*x^2+b*x+a)^(5/2)/c*e*g*h+3/16*d*g^2/c*(c*x^2+b*x+a)^(
1/2)*b*a-3/16*d*g^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2
*a-1/16*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b*e*g*h+7/48/c^2*b^2*(c*x^2+b*x+a)^(3/2
)*x*e*g*h+1/8/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*a*d*h^2+1/8/c^2*b^2*(c*x^2+b*x+
a)^(1/2)*x*a*f*g^2-7/128/c^3*b^4*(c*x^2+b*x+a)^(1/2)*x*e*g*h+153/2048*f*h^2
/c^4*b^4*(c*x^2+b*x+a)^(1/2)*x*a-7/256/c^3*b^4*(c*x^2+b*x+a)^(1/2)*x*f*g^2+
1/16/c^3*b^3*(c*x^2+b*x+a)^(1/2)*a*d*h^2+1/16/c^3*b^3*(c*x^2+b*x+a)^(1/2)*a
*f*g^2-7/256/c^4*b^5*(c*x^2+b*x+a)^(1/2)*e*g*h+9/64/c^(5/2)*b^2*ln((c*x+1/2
*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*d*h^2+9/64/c^(5/2)*b^2*ln((c*x+1/2*b)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*f*g^2+153/4096*f*h^2/c^5*b^5*(c*x^2+b*x+a)
^(1/2)*a+33/2048*f*h^2/c^5*b^5*(c*x^2+b*x+a)^(3/2)-99/16384*f*h^2/c^6*b^7*(
c*x^2+b*x+a)^(1/2)+3/128*d*g^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))*b^4-9/2048/c^(11/2)*b^7*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*e*h^2+1/7*x^2*(c*x^2+b*x+a)^(5/2)/c*e*h^2-2/35*a/c^2*(c*x^2+b*x+a)^(5/2)*e
*h^2+3/40/c^3*b^2*(c*x^2+b*x+a)^(5/2)*e*h^2-3/128/c^4*b^4*(c*x^2+b*x+a)^(3/
2)*e*h^2+9/1024/c^5*b^6*(c*x^2+b*x+a)^(1/2)*e*h^2+1/6*x*(c*x^2+b*x+a)^(5/2)
/c*d*h^2+1/6*x*(c*x^2+b*x+a)^(5/2)/c*f*g^2-7/60/c^2*b*(c*x^2+b*x+a)^(5/2)*d
*h^2-7/60/c^2*b*(c*x^2+b*x+a)^(5/2)*f*g^2+7/192/c^3*b^3*(c*x^2+b*x+a)^(3/2)
*d*h^2+7/192/c^3*b^3*(c*x^2+b*x+a)^(3/2)*f*g^2-7/512/c^4*b^5*(c*x^2+b*x+a)^
(1/2)*d*h^2-7/512/c^4*b^5*(c*x^2+b*x+a)^(1/2)*f*g^2+7/1024/c^(9/2)*b^6*ln((
c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2+7/1024/c^(9/2)*b^6*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2-1/16*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2-1/16*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*f*g^2+3/128*f*h^2*a^4/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))+99/32768*f*h^2/c^(13/2)*b^8*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))+1/8*f*h^2*x^3*(c*x^2+b*x+a)^(5/2)/c-33/640*f*h^2/c^4*b^3*(c*x
^2+b*x+a)^(5/2)-3/16/c^3*b^3*(c*x^2+b*x+a)^(1/2)*x*a*f*g*h+1/8/c^2*b*a*(c*x
^2+b*x+a)^(3/2)*x*f*g*h+3/16/c^2*b*a^2*(c*x^2+b*x+a)^(1/2)*x*f*g*h+1/4/c^2*
b^2*(c*x^2+b*x+a)^(1/2)*x*a*e*g*h-3/8/c*b*(c*x^2+b*x+a)^(1/2)*x*a*d*g*h
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g+h\,x\right)^2\,{\left(c\,x^2+b\,x+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g+hx\right)^2\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((g + h*x)**2*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

3.198 $\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=418

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-8c^2(aeh+afg+3bdh+3beg)+2bc(6afh+7b(eh+fg))-9b^3fh+48c^3dg\right)}{384c^4}+$$

```
[Out] 1/384*(48*c^3*d*g-9*b^3*f*h-8*c^2*(a*e*h+a*f*g+3*b*d*h+3*b*e*g)+2*b*c*(6*a*
f*h+7*b*(e*h+f*g)))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^4+1/7*f*(h*x+g)^2*(c*x^
2+b*x+a)^(5/2)/c/h+1/840*(63*b^2*f*h^2-24*c^2*(5*f*g^2-7*h*(d*h+e*g))-2*c*h
*(24*a*f*h+49*b*(e*h+f*g))-10*c*h*(9*b*f*h-14*c*e*h+10*c*f*g)*x)*(c*x^2+b*x
+a)^(5/2)/c^3/h+1/2048*(-4*a*c+b^2)^2*(48*c^3*d*g-9*b^3*f*h-8*c^2*(a*e*h+a*
f*g+3*b*d*h+3*b*e*g)+2*b*c*(6*a*f*h+7*b*(e*h+f*g)))*arctanh(1/2*(2*c*x+b)/c
^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)-1/1024*(-4*a*c+b^2)*(48*c^3*d*g-9*b^3*
f*h-8*c^2*(a*e*h+a*f*g+3*b*d*h+3*b*e*g)+2*b*c*(6*a*f*h+7*b*(e*h+f*g)))*(2*c
*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

Rubi [A] time = 0.65, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1653, 779, 612, 621, 206}

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(-2ch(24afh+49b(eh+fg))+63b^2fh^2-10chx(9bfh-14ceh+10cfg)-24c^2\left(5fg^2-7h(\right.\right.}{840c^3h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b*e*g + a*f*g + 3*b*d*h
 + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*(b + 2*c*x)*Sqrt[a + b*x + c*
x^2])/(1024*c^5) + ((48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b*e*g + a*f*g + 3*b*
d*h + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*(b + 2*c*x)*(a + b*x + c*
x^2)^(3/2))/(384*c^4) + (f*(g + h*x)^2*(a + b*x + c*x^2)^(5/2))/(7*c*h) + (
(63*b^2*f*h^2 - 24*c^2*(5*f*g^2 - 7*h*(e*g + d*h)) - 2*c*h*(24*a*f*h + 49*b
*(f*g + e*h)) - 10*c*h*(10*c*f*g - 14*c*e*h + 9*b*f*h)*x)*(a + b*x + c*x^2)
^(5/2))/(840*c^3*h) + ((b^2 - 4*a*c)^2*(48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b
*e*g + a*f*g + 3*b*d*h + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx=\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}+\frac{\int (g+hx)\left(-\frac{1}{2}h(5bfg-14cdh+\right.}{}$$

$$=\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}+\frac{\left(63b^2fh^2-24c^2\left(5fg^2-7h(eg\right.\right.}{}$$

$$=\frac{\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+\right.}{384c^4}$$

$$=-\frac{\left(b^2-4ac\right)\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+\right.}{1024c^5}$$

$$=-\frac{\left(b^2-4ac\right)\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+\right.}{1024c^5}$$

$$=-\frac{\left(b^2-4ac\right)\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+\right.}{1024c^5}$$

Mathematica [A] time = 0.78, size = 285, normalized size = 0.68

$$\frac{(a+x(b+cx))^{5/2}\left(-2ch(24afh+b(49eh+49fg+45fhx))+63b^2fh^2-4c^2(5fg(6g+5hx)-7h(6dh+6eg+5ehx))\right)}{120c^2}-\frac{7h\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^2*(a + x*(b + c*x))^(5/2) + ((a + x*(b + c*x))^(5/2)*(63*b^2*f
*h^2 - 4*c^2*(5*f*g*(6*g + 5*h*x) - 7*h*(6*e*g + 6*d*h + 5*e*h*x)) - 2*c*h*
(24*a*f*h + b*(49*f*g + 49*e*h + 45*f*h*x))))/(120*c^2) - (7*h*(-48*c^3*d*g
 + 9*b^3*f*h + 8*c^2*(3*b*e*g + a*f*g + 3*b*d*h + a*e*h) - 2*b*c*(6*a*f*h +
 7*b*(f*g + e*h)))*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8
```

```
*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + x*(b + c*x)])])]))/(6144*c^(9/2)))/(7*c*h)
```

fricas [B] time = 1.03, size = 1833, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/430080*(105*(2*(24*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 12*(b^5*c^2
- 8*a*b^3*c^3 + 16*a^2*b*c^4)*e + (7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3
 - 64*a^3*c^4)*f)*g - (24*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(7*b
^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*e + 3*(3*b^7 - 28*a*b^5
*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*f)*h)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x
- b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(15360*c^7
*f*h*x^6 + 1280*(14*c^7*f*g + (14*c^7*e + 15*b*c^6*f)*h)*x^5 + 128*(14*(12*
c^7*e + 13*b*c^6*f)*g + (168*c^7*d + 182*b*c^6*e + 3*(b^2*c^5 + 64*a*c^6)*f
)*h)*x^4 + 16*(14*(120*c^7*d + 132*b*c^6*e + (3*b^2*c^5 + 140*a*c^6)*f)*g +
 (1848*b*c^6*d + 14*(3*b^2*c^5 + 140*a*c^6)*e - 3*(9*b^3*c^4 - 44*a*b*c^5)*
f)*h)*x^3 + 8*(14*(360*b*c^6*d + 12*(b^2*c^5 + 32*a*c^6)*e - (7*b^3*c^4 - 3
6*a*b*c^5)*f)*g + (168*(b^2*c^5 + 32*a*c^6)*d - 14*(7*b^3*c^4 - 36*a*b*c^5)
*e + 3*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*f)*h)*x^2 - 14*(120*(3*b^
3*c^4 - 20*a*b*c^5)*d - 12*(15*b^4*c^3 - 100*a*b^2*c^4 + 128*a^2*c^5)*e + (
105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*c^4)*f)*g + (168*(15*b^4*c^3 - 100
*a*b^2*c^4 + 128*a^2*c^5)*d - 14*(105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*
c^4)*e + 3*(315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4)*f
)*h + 2*(14*(120*(b^2*c^5 + 20*a*c^6)*d - 12*(5*b^3*c^4 - 28*a*b*c^5)*e + (
35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*f)*g - (168*(5*b^3*c^4 - 28*a*b*c
^5)*d - 14*(35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*e + 3*(105*b^5*c^2 -
728*a*b^3*c^3 + 1168*a^2*b*c^4)*f)*h)*x)*sqrt(c*x^2 + b*x + a))/c^6, -1/215
040*(105*(2*(24*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 12*(b^5*c^2 - 8*a*
b^3*c^3 + 16*a^2*b*c^4)*e + (7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*
a^3*c^4)*f)*g - (24*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(7*b^6*c -
 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*e + 3*(3*b^7 - 28*a*b^5*c + 8
0*a^2*b^3*c^2 - 64*a^3*b*c^3)*f)*h)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(15360*c^7*f*h*x^6 + 1
280*(14*c^7*f*g + (14*c^7*e + 15*b*c^6*f)*h)*x^5 + 128*(14*(12*c^7*e + 13*b
*c^6*f)*g + (168*c^7*d + 182*b*c^6*e + 3*(b^2*c^5 + 64*a*c^6)*f)*h)*x^4 + 1
6*(14*(120*c^7*d + 132*b*c^6*e + (3*b^2*c^5 + 140*a*c^6)*f)*g + (1848*b*c^6
*d + 14*(3*b^2*c^5 + 140*a*c^6)*e - 3*(9*b^3*c^4 - 44*a*b*c^5)*f)*h)*x^3 +
8*(14*(360*b*c^6*d + 12*(b^2*c^5 + 32*a*c^6)*e - (7*b^3*c^4 - 36*a*b*c^5)*f
)*g + (168*(b^2*c^5 + 32*a*c^6)*d - 14*(7*b^3*c^4 - 36*a*b*c^5)*e + 3*(21*b
^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*f)*h)*x^2 - 14*(120*(3*b^3*c^4 - 20*a
*b*c^5)*d - 12*(15*b^4*c^3 - 100*a*b^2*c^4 + 128*a^2*c^5)*e + (105*b^5*c^2
- 760*a*b^3*c^3 + 1296*a^2*b*c^4)*f)*g + (168*(15*b^4*c^3 - 100*a*b^2*c^4 +
 128*a^2*c^5)*d - 14*(105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*c^4)*e + 3*(
315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4)*f)*h + 2*(14*
(120*(b^2*c^5 + 20*a*c^6)*d - 12*(5*b^3*c^4 - 28*a*b*c^5)*e + (35*b^4*c^3 -
 216*a*b^2*c^4 + 240*a^2*c^5)*f)*g - (168*(5*b^3*c^4 - 28*a*b*c^5)*d - 14*(
35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*e + 3*(105*b^5*c^2 - 728*a*b^3*c^
3 + 1168*a^2*b*c^4)*f)*h)*x)*sqrt(c*x^2 + b*x + a))/c^6]
```

giac [B] time = 0.29, size = 955, normalized size = 2.28

$$\frac{1}{107520}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\left(12\,cfhx+\frac{14\,c^7fg+15\,bc^6fh+14\,c^7he}{c^6}\right)x+\frac{182\,bc^6fg+168\,c^7dh+3\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*c*f*h*x + (14*c^7*f*g +
15*b*c^6*f*h + 14*c^7*h*e)/c^6)*x + (182*b*c^6*f*g + 168*c^7*d*h + 3*b^2*c^
5*f*h + 192*a*c^6*f*h + 168*c^7*g*e + 182*b*c^6*h*e)/c^6)*x + (1680*c^7*d*g
 + 42*b^2*c^5*f*g + 1960*a*c^6*f*g + 1848*b*c^6*d*h - 27*b^3*c^4*f*h + 132*
a*b*c^5*f*h + 1848*b*c^6*g*e + 42*b^2*c^5*h*e + 1960*a*c^6*h*e)/c^6)*x + (5
040*b*c^6*d*g - 98*b^3*c^4*f*g + 504*a*b*c^5*f*g + 168*b^2*c^5*d*h + 5376*a
*c^6*d*h + 63*b^4*c^3*f*h - 372*a*b^2*c^4*f*h + 384*a^2*c^5*f*h + 168*b^2*c
^5*g*e + 5376*a*c^6*g*e - 98*b^3*c^4*h*e + 504*a*b*c^5*h*e)/c^6)*x + (1680*
b^2*c^5*d*g + 33600*a*c^6*d*g + 490*b^4*c^3*f*g - 3024*a*b^2*c^4*f*g + 3360
*a^2*c^5*f*g - 840*b^3*c^4*d*h + 4704*a*b*c^5*d*h - 315*b^5*c^2*f*h + 2184*
a*b^3*c^3*f*h - 3504*a^2*b*c^4*f*h - 840*b^3*c^4*g*e + 4704*a*b*c^5*g*e + 4
90*b^4*c^3*h*e - 3024*a*b^2*c^4*h*e + 3360*a^2*c^5*h*e)/c^6)*x - (5040*b^3*
c^4*d*g - 33600*a*b*c^5*d*g + 1470*b^5*c^2*f*g - 10640*a*b^3*c^3*f*g + 1814
4*a^2*b*c^4*f*g - 2520*b^4*c^3*d*h + 16800*a*b^2*c^4*d*h - 21504*a^2*c^5*d*
h - 945*b^6*c*f*h + 7560*a*b^4*c^2*f*h - 16464*a^2*b^2*c^3*f*h + 6144*a^3*c
^4*f*h - 2520*b^4*c^3*g*e + 16800*a*b^2*c^4*g*e - 21504*a^2*c^5*g*e + 1470*
b^5*c^2*h*e - 10640*a*b^3*c^3*h*e + 18144*a^2*b*c^4*h*e)/c^6) - 1/2048*(48*
b^4*c^3*d*g - 384*a*b^2*c^4*d*g + 768*a^2*c^5*d*g + 14*b^6*c*f*g - 120*a*b^
4*c^2*f*g + 288*a^2*b^2*c^3*f*g - 128*a^3*c^4*f*g - 24*b^5*c^2*d*h + 192*a*
b^3*c^3*d*h - 384*a^2*b*c^4*d*h - 9*b^7*f*h + 84*a*b^5*c*f*h - 240*a^2*b^3*
c^2*f*h + 192*a^3*b*c^3*f*h - 24*b^5*c^2*g*e + 192*a*b^3*c^3*g*e - 384*a^2*
b*c^4*g*e + 14*b^6*c*h*e - 120*a*b^4*c^2*h*e + 288*a^2*b^2*c^3*h*e - 128*a^
3*c^4*h*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(
11/2)
```

maple [B] time = 0.01, size = 2026, normalized size = 4.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/4*d*g*(c*x^2+b*x+a)^(3/2)*x+1/5*(c*x^2+b*x+a)^(5/2)/c*d*h+1/5*(c*x^2+b*x+
a)^(5/2)/c*e*g+9/1024*h*f/c^5*b^6*(c*x^2+b*x+a)^(1/2)-3/32*h*f/c^3*b^3*(c*x
^2+b*x+a)^(1/2)*x*a+3/32*h*f/c^2*b*a^2*(c*x^2+b*x+a)^(1/2)*x+1/16*h*f/c^2*b
*a*(c*x^2+b*x+a)^(3/2)*x-3/16/c*b*(c*x^2+b*x+a)^(1/2)*x*a*d*h-3/16/c*b*(c*x
^2+b*x+a)^(1/2)*x*a*e*g+1/8/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*a*f*g+1/8/c^2*b^2
*(c*x^2+b*x+a)^(1/2)*x*a*e*h-3/128*h*f/c^4*b^4*(c*x^2+b*x+a)^(3/2)-15/128*h
*f/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-15/256/c^(7/
2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f*g-1/24*a/c*(c*x^2+b*
x+a)^(3/2)*x*e*h-1/48*a/c^2*(c*x^2+b*x+a)^(3/2)*b*f*g-1/48*a/c^2*(c*x^2+b*x
+a)^(3/2)*b*e*h+1/6*x*(c*x^2+b*x+a)^(5/2)/c*e*h+1/6*x*(c*x^2+b*x+a)^(5/2)/c
*f*g-1/16/c^2*b^2*(c*x^2+b*x+a)^(3/2)*d*h-1/16/c^2*b^2*(c*x^2+b*x+a)^(3/2)*
e*g+3/128/c^3*b^4*(c*x^2+b*x+a)^(1/2)*d*h+3/128/c^3*b^4*(c*x^2+b*x+a)^(1/2)
*e*g-3/256/c^(7/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h-7/25
6/c^3*b^4*(c*x^2+b*x+a)^(1/2)*x*e*h-7/256/c^3*b^4*(c*x^2+b*x+a)^(1/2)*x*f*g
+1/16/c^3*b^3*(c*x^2+b*x+a)^(1/2)*a*e*h+1/16/c^3*b^3*(c*x^2+b*x+a)^(1/2)*a*
f*g+9/64/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*e*h+9/
64/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*f*g-15/256/c
^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*h-1/16*a^2/c*(c*
x^2+b*x+a)^(1/2)*x*e*h-1/16*a^2/c*(c*x^2+b*x+a)^(1/2)*x*f*g-1/32*a^2/c^2*(c
*x^2+b*x+a)^(1/2)*b*e*h-1/32*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b*f*g+7/96/c^2*b^2
*(c*x^2+b*x+a)^(3/2)*x*e*h+7/96/c^2*b^2*(c*x^2+b*x+a)^(3/2)*x*f*g-1/8/c*b*(
c*x^2+b*x+a)^(3/2)*x*d*h-1/8/c*b*(c*x^2+b*x+a)^(3/2)*x*e*g+3/64/c^2*b^3*(c*
x^2+b*x+a)^(1/2)*x*d*h+3/64/c^2*b^3*(c*x^2+b*x+a)^(1/2)*x*e*g-3/32/c^2*b^2*
(c*x^2+b*x+a)^(1/2)*a*d*h-3/32/c^2*b^2*(c*x^2+b*x+a)^(1/2)*a*e*g-3/16/c^(3/
2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*d*h-3/16/c^(3/2)*b*ln(
(c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*e*g+3/32/c^(5/2)*b^3*ln((c*x+1
```

```
/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*h+3/32/c^(5/2)*b^3*ln((c*x+1/2*b)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g-3/32*d*g/c*(c*x^2+b*x+a)^(1/2)*x*b^2+3/16*
d*g/c*(c*x^2+b*x+a)^(1/2)*b*a-3/16*d*g/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))*b^2*a-1/24*a/c*(c*x^2+b*x+a)^(3/2)*x*f*g+1/32*h*f/c^3*b^2*a
*(c*x^2+b*x+a)^(3/2)+3/64*h*f/c^3*b^2*a^2*(c*x^2+b*x+a)^(1/2)-3/28*h*f/c^2*
b*x*(c*x^2+b*x+a)^(5/2)+21/512*h*f/c^(9/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))*a+3/32*h*f/c^(5/2)*b*a^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))-3/64*h*f/c^3*b^3*(c*x^2+b*x+a)^(3/2)*x+9/512*h*f/c^4*b^5*(c*x^2+b
*x+a)^(1/2)*x-3/64*h*f/c^4*b^4*(c*x^2+b*x+a)^(1/2)*a-3/256/c^(7/2)*b^5*ln((
c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g+1/8*d*g/c*(c*x^2+b*x+a)^(3/2)*b
+3/8*d*g*(c*x^2+b*x+a)^(1/2)*x*a-3/64*d*g/c^2*(c*x^2+b*x+a)^(1/2)*b^3+3/8*d
*g/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/128*d*g/c^(5/2
)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4-7/60/c^2*b*(c*x^2+b*x+a)^
(5/2)*e*h-7/60/c^2*b*(c*x^2+b*x+a)^(5/2)*f*g+7/192/c^3*b^3*(c*x^2+b*x+a)^(3
/2)*e*h+7/192/c^3*b^3*(c*x^2+b*x+a)^(3/2)*f*g-7/512/c^4*b^5*(c*x^2+b*x+a)^(
1/2)*e*h-7/512/c^4*b^5*(c*x^2+b*x+a)^(1/2)*f*g+7/1024/c^(9/2)*b^6*ln((c*x+1
/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h+7/1024/c^(9/2)*b^6*ln((c*x+1/2*b)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*f*g-1/16*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*e*h-1/16*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))*f*g-9/2048*h*f/c^(11/2)*b^7*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))+1/7*h*f*x^2*(c*x^2+b*x+a)^(5/2)/c-2/35*h*f*a/c^2*(c*x^2+b*x+a)^(5/2)+3/
40*h*f/c^3*b^2*(c*x^2+b*x+a)^(5/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g+h\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g + hx\right) \left(a + bx + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((g + h*x)*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

3.199 $\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=236

$$\frac{\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+3be)+7b^2f+24c^2d\right)}{1024c^{9/2}}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4c(af\right.}{512c^4}$$

```
[Out] 1/192*(-4*a*c*f+7*b^2*f-12*b*c*e+24*c^2*d)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^
3+1/60*(-7*b*f+12*c*e)*(c*x^2+b*x+a)^(5/2)/c^2+1/6*f*x*(c*x^2+b*x+a)^(5/2)/
c+1/1024*(-4*a*c+b^2)^2*(24*c^2*d+7*b^2*f-4*c*(a*f+3*b*e))*arctanh(1/2*(2*c
*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)-1/512*(-4*a*c+b^2)*(24*c^2*d+7*b
^2*f-4*c*(a*f+3*b*e))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^4
```

Rubi [A] time = 0.24, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acf+7b^2f-12bce+24c^2d\right)}{192c^3}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4c(af+3be)\right.}{512c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*(b + 2*c*x)*Sqrt[a
 + b*x + c*x^2])/(512*c^4) + ((24*c^2*d - 12*b*c*e + 7*b^2*f - 4*a*c*f)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3) + ((12*c*e - 7*b*f)*(a + b*x +
c*x^2)^(5/2))/(60*c^2) + (f*x*(a + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*
c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}+\frac{\int \left(6cd-af+\frac{1}{2}(12ce-7bf)x\right)\left(a+bx+cx^2\right.}{6c}$$

$$=\frac{(12ce-7bf)\left(a+bx+cx^2\right)^{5/2}}{60c^2}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}+\frac{\left(2c(6cd-a\right.}{}$$

$$=\frac{\left(24c^2d-12bce+7b^2f-4acf\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3}+\frac{(12ce-}{}$$

$$=-\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}$$

$$=-\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}$$

$$=-\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}$$

Mathematica [A] time = 0.69, size = 392, normalized size = 1.66

$$\frac{\frac{360d\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{3/2}}-60be\left(\frac{3\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2c\right.}{c^{5/2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (1920*d*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 3072*e*(a + x*(b + c*x))^(5/2
) + 2560*f*x*(a + x*(b + c*x))^(5/2) + (360*(b^2 - 4*a*c)*d*(-2*Sqrt[c]*(b
+ 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2) - 60*b*e*((16*(b + 2*c*x)*(a + x*(b +
c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c
*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
))/c^(5/2)) + (f*(-1792*b*(a + x*(b + c*x))^(5/2) + 5*(7*b^2 - 4*a*c)*((16*
(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b +
2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])]))/c^(5/2))))/c)/(15360*c)
```

fricas [A] time = 0.51, size = 839, normalized size = 3.56

$$\left[-\frac{15\left(24\left(b^4c^2-8ab^2c^3+16a^2c^4\right)d-12\left(b^5c-8ab^3c^2+16a^2bc^3\right)e+\left(7b^6-60ab^4c+144a^2b^2c^2-64a^3c^3\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*
c^3)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f
)*x^4 + 16*(120*c^6*d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(3
60*b*c^5*d + 12*(b^2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 -
120*(3*b^3*c^3 - 20*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c
^4)*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 +
20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 +
 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/15360*(15*(24*(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e +
(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*f)*sqrt(-c)*arctan(1/2*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(12
80*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f)*x^4 + 16*(120*c^6*d + 132*b*c^5*
e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(360*b*c^5*d + 12*(b^2*c^4 + 32*a*c^
5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d + 1
2*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4)*e - (105*b^5*c - 760*a*b^3*c^2
 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 + 20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a
*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b
*x + a))/c^5]
```

giac [A] time = 0.26, size = 417, normalized size = 1.77

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,cfx+\frac{13\,bc^5f+12\,c^6e}{c^5}\right)x+\frac{120\,c^6d+3\,b^2c^4f+140\,ac^5f+132\,bc^5e}{c^5}\right)x+\frac{360\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*f*x + (13*b*c^5*f + 12*c^6*e
)/c^5)*x + (120*c^6*d + 3*b^2*c^4*f + 140*a*c^5*f + 132*b*c^5*e)/c^5)*x + (
360*b*c^5*d - 7*b^3*c^3*f + 36*a*b*c^4*f + 12*b^2*c^4*e + 384*a*c^5*e)/c^5)
*x + (120*b^2*c^4*d + 2400*a*c^5*d + 35*b^4*c^2*f - 216*a*b^2*c^3*f + 240*a
^2*c^4*f - 60*b^3*c^3*e + 336*a*b*c^4*e)/c^5)*x - (360*b^3*c^3*d - 2400*a*b
*c^4*d + 105*b^5*c*f - 760*a*b^3*c^2*f + 1296*a^2*b*c^3*f - 180*b^4*c^2*e +
 1200*a*b^2*c^3*e - 1536*a^2*c^4*e)/c^5) - 1/1024*(24*b^4*c^2*d - 192*a*b^2
*c^3*d + 384*a^2*c^4*d + 7*b^6*f - 60*a*b^4*c*f + 144*a^2*b^2*c^2*f - 64*a^
3*c^3*f - 12*b^5*c*e + 96*a*b^3*c^2*e - 192*a^2*b*c^3*e)*log(abs(-2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

maple [B] time = 0.01, size = 862, normalized size = 3.65

$$-\frac{a^3f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{16c^{\frac{3}{2}}}+\frac{9a^2b^2f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{64c^{\frac{5}{2}}}-\frac{3a^2be\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{16c^{\frac{3}{2}}}+\frac{3a^2d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/8*f/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*a-3/16*e/c*b*(c*x^2+b*x+a)^(1/2)*x*a+1/
6*f*x*(c*x^2+b*x+a)^(5/2)/c+1/8*d/c*(c*x^2+b*x+a)^(3/2)*b+3/8*d*(c*x^2+b*x+
a)^(1/2)*x*a-3/64*d/c^2*(c*x^2+b*x+a)^(1/2)*b^3+3/8*d/c^(1/2)*ln((c*x+1/2*b
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/128*d/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))*b^4-1/16*e/c^2*b^2*(c*x^2+b*x+a)^(3/2)+3/128*e/c^3*b^4*
```

```
(c*x^2+b*x+a)^(1/2)-3/256*e/c^(7/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))-7/60*f/c^2*b*(c*x^2+b*x+a)^(5/2)+7/192*f/c^3*b^3*(c*x^2+b*x+a)^(3/
2)-7/512*f/c^4*b^5*(c*x^2+b*x+a)^(1/2)+7/1024*f/c^(9/2)*b^6*ln((c*x+1/2*b)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/16*f*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))+3/16*d/c*(c*x^2+b*x+a)^(1/2)*b*a-3/16*d/c^(3/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a+1/4*d*(c*x^2+b*x+a)^(3/2)*x+1/5*e*(
c*x^2+b*x+a)^(5/2)/c-1/24*f*a/c*(c*x^2+b*x+a)^(3/2)*x-1/8*e/c*b*(c*x^2+b*x+
a)^(3/2)*x-1/32*f*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b-1/48*f*a/c^2*(c*x^2+b*x+a)^
(3/2)*b-1/16*f*a^2/c*(c*x^2+b*x+a)^(1/2)*x-3/32*d/c*(c*x^2+b*x+a)^(1/2)*x*b
^2+3/64*e/c^2*b^3*(c*x^2+b*x+a)^(1/2)*x-3/32*e/c^2*b^2*(c*x^2+b*x+a)^(1/2)*
a-3/16*e/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/32*e/c
^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+7/96*f/c^2*b^2*(c*
x^2+b*x+a)^(3/2)*x-7/256*f/c^3*b^4*(c*x^2+b*x+a)^(1/2)*x+1/16*f/c^3*b^3*(c*
x^2+b*x+a)^(1/2)*a+9/64*f/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))*a^2-15/256*f/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

3.200 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx$

Optimal. Leaf size=660

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(2cg-bh)\left(8ch(bg-2ah)(bfg-2cdh)-g\left(-4ach-3b^2h+8bcg\right)(bfh-2ceh+2c\right.\right.}{}$$

```
[Out] -1/48*(8*c*h*(b*f*g-2*c*d*h)-(-3*b*h+8*c*g)*(b*f*h-2*c*e*h+2*c*f*g)+6*c*h*(
b*f*h-2*c*e*h+2*c*f*g)*x)*(c*x^2+b*x+a)^(3/2)/c^2/h^3+1/5*f*(c*x^2+b*x+a)^(
5/2)/c/h-1/256*(4*c*h*(-b*h+2*c*g)*(8*c*h*(-2*a*h+b*g)*(b*f*g-2*c*d*h)-g*(-
4*a*c*h-3*b^2*h+8*b*c*g)*(b*f*h-2*c*e*h+2*c*f*g))-2*(4*c^2*g^2-1/2*b^2*h^2-
2*c*h*(-a*h+b*g))*(8*c*h*(-b*h+2*c*g)*(b*f*g-2*c*d*h)-(b*f*h-2*c*e*h+2*c*f*
g)*(16*c^2*g^2-3*b^2*h^2-4*c*h*(-3*a*h+2*b*g))))*arctanh(1/2*(2*c*x+b)/c^(1
/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)/h^6+(a*h^2-b*g*h+c*g^2)^(3/2)*(f*g^2-h*(-d
*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(
c*x^2+b*x+a)^(1/2))/h^6+1/128*(3*b^4*f*h^4+6*b^2*c*h^3*(-2*a*f*h-b*e*h+b*f*
g)+128*c^4*g^2*(f*g^2-h*(-d*h+e*g))-32*c^3*h*(-4*a*h+5*b*g)*(f*g^2-h*(-d*h+
e*g))-8*b*c^2*h^2*(3*a*h*(-e*h+f*g)-2*b*(d*h^2-e*g*h+f*g^2))+2*c*h*(8*c*h*(
-b*h+2*c*g)*(b*f*g-2*c*d*h)-(b*f*h-2*c*e*h+2*c*f*g)*(16*c^2*g^2-3*b^2*h^2-4
*c*h*(-3*a*h+2*b*g)))*x)*(c*x^2+b*x+a)^(1/2)/c^3/h^5
```

Rubi [A] time = 1.83, antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1653, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2bg-3ah)-3b^2h^2+16c^2g^2\right)(bfh-2ceh+2cfg)\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((3*b^4*f*h^4 + 6*b^2*c*h^3*(b*f*g - b*e*h - 2*a*f*h) - 32*c^3*h*(5*b*g - 4
*a*h)*(f*g^2 - h*(e*g - d*h)) + 128*c^4*(f*g^4 - g^2*h*(e*g - d*h)) - 8*b*c
^2*h^2*(3*a*h*(f*g - e*h) - 2*b*(f*g^2 - e*g*h + d*h^2)) + 2*c*h*(8*c*h*(2*
c*g - b*h)*(b*f*g - 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*
b^2*h^2 - 4*c*h*(2*b*g - 3*a*h)))*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*h^5) -
 ((8*c*h*(b*f*g - 2*c*d*h) - (8*c*g - 3*b*h)*(2*c*f*g - 2*c*e*h + b*f*h) +
6*c*h*(2*c*f*g - 2*c*e*h + b*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(48*c^2*h^3)
+ (f*(a + b*x + c*x^2)^(5/2))/(5*c*h) - ((4*c*h*(2*c*g - b*h)*(8*c*h*(b*g -
 2*a*h)*(b*f*g - 2*c*d*h) - g*(8*b*c*g - 3*b^2*h - 4*a*c*h)*(2*c*f*g - 2*c*
e*h + b*f*h)) - 2*(4*c^2*g^2 - (b^2*h^2)/2 - 2*c*h*(b*g - a*h))*(8*c*h*(2*c
*g - b*h)*(b*f*g - 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*b
^2*h^2 - 4*c*h*(2*b*g - 3*a*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(256*c^(7/2)*h^6) + ((c*g^2 - b*g*h + a*h^2)^(3/2)*(f*g^2 -
h*(e*g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*
h + a*h^2]*Sqrt[a + b*x + c*x^2])])/h^6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \frac{f(a+bx+cx^2)^{5/2}}{5ch} + \frac{\int \frac{\left(-\frac{5}{2}h(bfg-2cdh)-\frac{5}{2}h(2cfg-2ceh+bfh)x\right)(a+bx+cx^2)^{3/2}}{g+hx}dx}{5ch^2}$$

$$= -\frac{(8ch(bfg-2cdh)-(8cg-3bh)(2cfg-2ceh+bfh)+6ch(2cfg-2ce}{48c^2h^3}$$

$$= \frac{\left(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h(eg-\right.\right.}{}$$

$$= \frac{\left(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h(eg-\right.\right.}{}$$

$$= \frac{\left(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h(eg-\right.\right.}{}$$

$$= \frac{\left(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h(eg-\right.\right.}{}$$

Mathematica [A] time = 2.21, size = 635, normalized size = 0.96

$$-\frac{(a+x(b+cx))^{3/2}\left(3b^2fh^2+6bch(f(g+hx)-eh)-4c^2(h(4dh-4eg+3ehx)+fg(4g-3hx))\right)}{48c^2h^3}+\frac{-\tanh^{-1}\left(\frac{}{2\sqrt{c}\ldots}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] (f*(a + x*(b + c*x))^(5/2))/(5*c*h) - ((a + x*(b + c*x))^(3/2)*(3*b^2*f*h^2
 + 6*b*c*h*(-(e*h) + f*(g + h*x)) - 4*c^2*(f*g*(4*g - 3*h*x) + h*(-4*e*g +
4*d*h + 3*e*h*x))))/(48*c^2*h^3) + (Sqrt[c]*h*Sqrt[a + x*(b + c*x)]*(3*b^4*
f*h^4 + 64*c^4*g*(f*g^2 + h*(-(e*g) + d*h))*(2*g - h*x) + 6*b^2*c*h^3*(-(b*
e*h) - 2*a*f*h + b*f*(g + h*x)) + 4*b*c^2*h^2*(6*a*e*h^2 - 6*a*f*h*(g + h*x
) + b*f*g*(4*g + 3*h*x) + b*h*(-4*e*g + 4*d*h - 3*e*h*x)) - 16*c^3*h*(2*b*(
f*g^2 + h*(-(e*g) + d*h))*(5*g - h*x) + a*h*(f*g*(-8*g + 3*h*x) + h*(8*e*g
- 8*d*h - 3*e*h*x)))) - (2*c*h*(2*c*g - b*h)*(8*c*h*(b*g - 2*a*h)*(b*f*g -
2*c*d*h) - g*(8*b*c*g - 3*b^2*h - 4*a*c*h)*(2*c*f*g - 2*c*e*h + b*f*h)) + (
-4*c^2*g^2 + (b^2*h^2)/2 + 2*c*h*(b*g - a*h))*(8*c*h*(2*c*g - b*h)*(b*f*g -
 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*b^2*h^2 + 4*c*h*(-2
*b*g + 3*a*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 1
28*c^(7/2)*(c*g^2 + h*(-(b*g) + a*h))^(3/2)*(f*g^2 + h*(-(e*g) + d*h))*ArcT
anh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sq
rt[a + x*(b + c*x)])])/(128*c^(7/2)*h^6)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 6715, normalized size = 10.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re details)Is b*h-2*c*g zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{g+h\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}{g + hx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x), x)
```

3.201 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx$

Optimal. Leaf size=754

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2h^2\left(a^2fh^2-2abh(2fg-eh)+b^2\left(dh^2-2egh+3fg^2\right)\right)+8b^2ch^3(-3afh-beh+2bf}{128c^{5/}}$$

[Out] -1/24*(3*b*f*h^2*(-a*h+b*g)+8*c^2*g*(5*f*g^2-h*(-3*d*h+4*e*g))+c*h*(8*a*h*(-e*h+2*f*g)-b*(43*f*g^2-8*h*(-3*d*h+4*e*g)))+6*c*h^2*(4*c*e*g+b*f*g-5*c*f*g^2/h-4*c*d*h-a*f*h)*x)*(c*x^2+b*x+a)^(3/2)/c/h^3/(a*h^2-b*g*h+c*g^2)-(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)+1/128*(3*b^4*f*h^4+8*b^2*c*h^3*(-3*a*f*h-b*e*h+2*b*f*g)+128*c^4*g^2*(5*f*g^2-h*(-3*d*h+4*e*g))+48*c^2*h^2*(a^2*f*h^2-2*a*b*h*(-e*h+2*f*g)+b^2*(d*h^2-2*e*g*h+3*f*g^2))+192*c^3*h*(a*h*(d*h^2-2*e*g*h+3*f*g^2)-b*g*(2*d*h^2-3*e*g*h+4*f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/h^6-1/2*(2*c*g*(5*f*g^2-h*(-3*d*h+4*e*g))+h*(2*a*h*(-e*h+2*f*g)-b*(3*d*h^2-5*e*g*h+7*f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*h^2-b*g*h+c*g^2)^(1/2)/h^6-1/64*(3*b^3*f*h^3+4*b*c*h^2*(-3*a*f*h-2*b*e*h+4*b*f*g)+64*c^3*g*(5*f*g^2-h*(-3*d*h+4*e*g))+16*c^2*h*(4*a*h*(-e*h+2*f*g)-b*(9*d*h^2-14*e*g*h+19*f*g^2))+2*c*h*(3*b^2*f*h^2+4*c*h*(-3*a*f*h-2*b*e*h+4*b*f*g)-16*c^2*(5*f*g^2-h*(-3*d*h+4*e*g)))*x)*(c*x^2+b*x+a)^(1/2)/c^2/h^5

Rubi [A] time = 2.50, antiderivative size = 750, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1650, 814, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2h^2\left(a^2fh^2-2abh(2fg-eh)+b^2\left(dh^2-2egh+3fg^2\right)\right)+8b^2ch^3(-3afh-beh+2bf}{128c^{5/}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]

[Out] -((3*b^3*f*h^3 + 4*b*c*h^2*(4*b*f*g - 2*b*e*h - 3*a*f*h) + 64*c^3*(5*f*g^3 - g*h*(4*e*g - 3*d*h)) - 16*c^2*h*(19*b*f*g^2 - b*h*(14*e*g - 9*d*h) - 4*a*h*(2*f*g - e*h)) + 2*c*h*(3*b^2*f*h^2 + 4*c*h*(4*b*f*g - 2*b*e*h - 3*a*f*h) - 16*c^2*(5*f*g^2 - h*(4*e*g - 3*d*h)))*x)*Sqrt[a + b*x + c*x^2])/(64*c^2*h^5) - ((3*b*f*h*(b*g - a*h) + (8*c^2*(5*f*g^3 - g*h*(4*e*g - 3*d*h)))/h - c*(43*b*f*g^2 - 8*b*h*(4*e*g - 3*d*h) - 8*a*h*(2*f*g - e*h)) + 6*c*h*(4*c*e*g + b*f*g - (5*c*f*g^2)/h - 4*c*d*h - a*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(24*c*h^2*(c*g^2 - b*g*h + a*h^2)) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) + ((3*b^4*f*h^4 + 8*b^2*c*h^3*(2*b*f*g - b*e*h - 3*a*f*h) + 128*c^4*(5*f*g^4 - g^2*h*(4*e*g - 3*d*h)) + 48*c^2*h^2*(a^2*f*h^2 - 2*a*b*h*(2*f*g - e*h) + b^2*(3*f*g^2 - 2*e*g*h + d*h^2)) + 192*c^3*h*(a*h*(3*f*g^2 - 2*e*g*h + d*h^2) - b*g*(4*f*g^2 - 3*e*g*h + 2*d*h^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(5/2)*h^6) - (Sqrt[c*g^2 - b*g*h + a*h^2]*(2*c*(5*f*g^3 - g*h*(4*e*g - 3*d*h)) - h*(7*b*f*g^2 - b*h*(5*e*g - 3*d*h) - 2*a*h*(2*f*g - e*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*h^6)

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} - \frac{\int \frac{\left(\frac{1}{2}\left(-2cdg+5beg+2afg-\frac{5bfg^2}{h}-3bdh-\right.\right.}{}}{cg}$$

$$= -\frac{\left(3bfh(bg-ah)+\frac{8c^2\left(5fg^3-gh(4eg-3dh)\right)}{h}-c\left(43bfg^2-8bh(4eg-3dh)-\right.\right.}{24ch^2\left(cg\right.}$$

$$= -\frac{\left(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3\left(5fg^3-gh(4eg-3dh)\right)\right.}{}$$

$$= -\frac{\left(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3\left(5fg^3-gh(4eg-3dh)\right)\right.}{}$$

$$= -\frac{\left(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3\left(5fg^3-gh(4eg-3dh)\right)\right.}{}$$

$$= -\frac{\left(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3\left(5fg^3-gh(4eg-3dh)\right)\right.}{}$$

Mathematica [A] time = 4.16, size = 756, normalized size = 1.00

$$-\frac{\frac{-2ch\sqrt{a+x(b+cx)}\left(h(ah-bg)+cg^2\right)\left(-4c^2h(ah(8eh-16fg+3fhx)+2b(h(9dh-14eg+ehx)+fg(19g-2hx)))+bch^2(b(-4eh+8fg+3fhx)-6afh)+\frac{3}{2}b^3fh^3+16c^3(2g-hx)\left(h(3dh-4eg)+5fg^2\right)\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -1/4*(-((f*(a + x*(b + c*x))^(5/2))/(g + h*x)) + ((5*c*f*g^2 + f*h*(-(b*g)
+ a*h) + 4*c*h*(-(e*g) + d*h))*(a + x*(b + c*x))^(5/2))/((c*g^2 + h*(-(b*g)
 + a*h))*(g + h*x)) + (((a + x*(b + c*x))^(3/2)*(3*b*f*h^2*(-(b*g) + a*h) +
 c*h*(8*b*h*(-4*e*g + 3*d*h) + b*f*g*(43*g - 6*h*x) + 2*a*h*(-8*f*g + 4*e*h
 + 3*f*h*x)) + c^2*(10*f*g^2*(-4*g + 3*h*x) + 8*h*(e*g*(4*g - 3*h*x) + 3*d*
h*(-g + h*x)))))/(6*h^2) + (-2*c*h*(c*g^2 + h*(-(b*g) + a*h))*Sqrt[a + x*(b
 + c*x)]*((3*b^3*f*h^3)/2 + 16*c^3*(5*f*g^2 + h*(-4*e*g + 3*d*h))*(2*g - h*
x) + b*c*h^2*(-6*a*f*h + b*(8*f*g - 4*e*h + 3*f*h*x)) - 4*c^2*h*(a*h*(-16*f
*g + 8*e*h + 3*f*h*x) + 2*b*(f*g*(19*g - 2*h*x) + h*(-14*e*g + 9*d*h + e*h*
x)))) + Sqrt[c]*(c*g^2 + h*(-(b*g) + a*h))*(2*c*h*(2*c*g - b*h)*(3*b^2*f*g*
h + 4*a*c*h*(5*f*g - 4*e*h) - 8*b*c*(5*f*g^2 + h*(-4*e*g + 3*d*h))) + ((8*c
^2*g^2 - b^2*h^2 + 4*c*h*(-(b*g) + a*h))*(-3*b^2*f*h^2 + 4*c*h*(-4*b*f*g +
2*b*e*h + 3*a*f*h) + 16*c^2*(5*f*g^2 + h*(-4*e*g + 3*d*h))))/2)*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 32*c^3*(c*g^2 + h*(-(b*g) + a
*h))^(3/2)*(2*c*(5*f*g^3 + g*h*(-4*e*g + 3*d*h)) + h*(-7*b*f*g^2 + b*h*(5*e
*g - 3*d*h) - 2*a*h*(-2*f*g + e*h)))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*
h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])]])/(16*c^2*h^5
))/(-(c*g^2) + h*(b*g - a*h)))/(c*h)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 14734, normalized size = 19.54

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re details)Is b*h-2*c*g zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**2,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

3.202 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

Optimal. Leaf size=824

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{2h\left(cg^2-bhg+ah^2\right)(g+hx)^2}-\frac{\left(4cg\left(-\frac{10fg^2}{h}+6eg-3dh\right)-4ah(7fg-3eh)+b\left(31fg^2-3h(5eg-dh)\right)\right.}{12h^2\left(cg^2-bhg+ah\right.}$$

```
[Out] -1/12*(4*c*g*(6*e*g-10*f*g^2/h-3*d*h)-4*a*h*(-3*e*h+7*f*g)+b*(31*f*g^2-3*h*
(-d*h+5*e*g))+2*h*(3*c*e*g+2*b*f*g-5*c*f*g^2/h-3*c*d*h-2*a*f*h)*x)*(c*x^2+b
*x+a)^(3/2)/h^2/(a*h^2-b*g*h+c*g^2)/(h*x+g)-1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2
+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^2-1/16*(b^3*f*h^3+6*b*c*h^2*(-2
*a*f*h-b*e*h+3*b*f*g)+16*c^3*g*(10*f*g^2-3*h*(-d*h+2*e*g))+24*c^2*h*(a*h*(-
e*h+3*f*g)-b*(d*h^2-3*e*g*h+6*f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2
+b*x+a)^(1/2))/c^(3/2)/h^6+1/8*(8*c^2*g^2*(10*f*g^2-3*h*(-d*h+2*e*g))+4*c*h
*(a*h*(3*d*h^2-9*e*g*h+19*f*g^2)-b*g*(6*d*h^2-15*e*g*h+28*f*g^2))+h^2*(8*a^
2*f*h^2-4*a*b*h*(-3*e*h+10*f*g)+b^2*(35*f*g^2-3*h*(-d*h+5*e*g))))*arctanh(1
/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2)
)/h^6/(a*h^2-b*g*h+c*g^2)^(1/2)-1/8*(b^2*f*h^3*(-a*h+b*g)-8*c^3*g^2*(10*f*g
^2-3*h*(-d*h+2*e*g))-2*c^2*h*(2*a*h*(3*d*h^2-9*e*g*h+19*f*g^2)-3*b*g*(5*d*h
^2-12*e*g*h+22*f*g^2))-c*h^2*(8*a^2*f*h^2-18*a*b*h*(-e*h+3*f*g)+b^2*(53*f*g
^2-6*h*(-d*h+4*e*g)))+2*c*h*(b*f*h^2*(-a*h+b*g)+2*c^2*g*(10*f*g^2-3*h*(-d*h
+2*e*g))+c*h*(2*a*h*(-3*e*h+7*f*g)-3*b*(d*h^2-3*e*g*h+6*f*g^2)))*x)*(c*x^2+
b*x+a)^(1/2)/c/h^5/(a*h^2-b*g*h+c*g^2)
```

Rubi [A] time = 2.14, antiderivative size = 819, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1650, 812, 814, 843, 621, 206, 724}

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{2h\left(cg^2-bhg+ah^2\right)(g+hx)^2}-\frac{\left(31bfg^2+4c\left(-\frac{10fg^2}{h}+6eg-3dh\right)g-3bh(5eg-dh)-4ah(7fg-3eh)+\right.}{12h^2\left(cg^2-bhg+ah^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] -((b^2*f*h^2*(b*g - a*h) + 8*c^3*g^2*(6*e*g - (10*f*g^2)/h - 3*d*h) - 2*c^2
*(2*a*h*(19*f*g^2 - 9*e*g*h + 3*d*h^2) - 3*b*g*(22*f*g^2 - 12*e*g*h + 5*d*h
^2)) - c*h*(8*a^2*f*h^2 - 18*a*b*h*(3*f*g - e*h) + b^2*(53*f*g^2 - 6*h*(4*e
*g - d*h))) + 2*c*(b*f*h^2*(b*g - a*h) + 2*c^2*(10*f*g^3 - 3*g*h*(2*e*g - d
*h)) + c*h*(2*a*h*(7*f*g - 3*e*h) - 3*b*(6*f*g^2 - 3*e*g*h + d*h^2)))*x)*Sq
rt[a + b*x + c*x^2])/(8*c*h^4*(c*g^2 - b*g*h + a*h^2)) - ((31*b*f*g^2 + 4*c
*g*(6*e*g - (10*f*g^2)/h - 3*d*h) - 3*b*h*(5*e*g - d*h) - 4*a*h*(7*f*g - 3*
e*h) + 2*h*(3*c*e*g + 2*b*f*g - (5*c*f*g^2)/h - 3*c*d*h - 2*a*f*h)*x)*(a +
b*x + c*x^2)^(3/2))/(12*h^2*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) - ((f*g^2 -
h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(2*h*(c*g^2 - b*g*h + a*h^2)*(g + h
*x)^2) - ((b^3*f*h^3 + 6*b*c*h^2*(3*b*f*g - b*e*h - 2*a*f*h) + 16*c^3*(10*f
*g^3 - 3*g*h*(2*e*g - d*h)) - 24*c^2*h*(6*b*f*g^2 - b*h*(3*e*g - d*h) - a*h
*(3*f*g - e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1
6*c^(3/2)*h^6) + ((8*c^2*(10*f*g^4 - 3*g^2*h*(2*e*g - d*h)) - 4*c*h*(28*b*f
*g^3 - 3*b*g*h*(5*e*g - 2*d*h) - a*h*(19*f*g^2 - 9*e*g*h + 3*d*h^2)) + h^2*
(8*a^2*f*h^2 - 4*a*b*h*(10*f*g - 3*e*h) + b^2*(35*f*g^2 - 3*h*(5*e*g - d*h)
)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*
Sqrt[a + b*x + c*x^2]])])/(8*h^6*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
```

```
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} - \int \frac{\left(\frac{1}{2}\left(-4cdg+5beg+4afg-\frac{5bfg^2}{h}-bdh-4 \ldots\right.\right.}{2\left(\ldots\right.}$$

$$= -\frac{\left(31bfg^2+4cg\left(6eg-\frac{10fg^2}{h}-3dh\right)-3bh(5eg-dh)-4ah(7fg-3eh) \ldots\right.}{12h^2\left(cg^2-bgh+a \ldots\right.}$$

$$= -\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2\left(2ah\left(19fg^2-9eg \ldots\right.\right.\right.}{\ldots}$$

$$= -\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2\left(2ah\left(19fg^2-9eg \ldots\right.\right.\right.}{\ldots}$$

$$= -\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2\left(2ah\left(19fg^2-9eg \ldots\right.\right.\right.}{\ldots}$$

$$= -\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2\left(2ah\left(19fg^2-9eg \ldots\right.\right.\right.}{\ldots}$$

Mathematica [B] time = 6.27, size = 4162, normalized size = 5.05

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] (f*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(3*c*h*(g + h*x)^2) - ((a + x
*(b + c*x))^(3/2)*(-1/2*(((h*(5*b*f*g - 6*c*d*h - 4*a*f*h))/2 - (g*(10*c*f*
g - 6*c*e*h + b*f*h))/2)*(a + b*x + c*x^2)^(5/2))/((c*g^2 - b*g*h + a*h^2)*
(g + h*x)^2) - (((-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)
) + (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2
)*(a + b*x + c*x^2)^(5/2))/((-(c*g^2) + b*g*h - a*h^2)*(g + h*x)) + (((4*c*
(4*c*g - (3*b*h)/2)*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d
*h)) + (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h))
)/2) + 4*c*h*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h))
- (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2
 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/
2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*
h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2) - 1
2*c^2*h*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*
h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2)*x)*(a +
 b*x + c*x^2)^(3/2))/(12*c*h^2) - (((2*c*h*(-4*c*(2*a*c*g*h + b*g*(-4*c*g +
 (3*b*h)/2))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) +
(3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) +
```

```
4*c*h*(b*g - 2*a*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g -
 d*h)) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e
*h)))/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*
e*h)))/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h))
- (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))
/2)) - (2*c*g - (b*h)/2)*(-4*c*(-8*c^2*g^2 + (3*b^2*h^2)/2 - c*h*(-4*b*g +
6*a*h))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*
h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h
*(2*c*g - b*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)
) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))
/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h))
)/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*
c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2))
+ c*h*(-4*c*(-8*c^2*g^2 + (3*b^2*h^2)/2 - c*h*(-4*b*g + 6*a*h))*(-3*c*g*(5*
c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*(5*b*f*g^2 - b*h*
(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*(2*c*g - b*h)*(-3*
a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c^2*g*(5*b*f
*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (3*b*c*h*(5*b*
f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (5*b*(3*c*g*(
5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*(5*b*f*g^2 - b*
h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2))*x)*Sqrt[a + b*x + c
*x^2])/(2*c*h^2) - (((2*c*h*(2*c*g - b*h)*(-4*c*(2*a*c*g*h + b*g*(-4*c*g +
(3*b*h)/2))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (
3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4
*c*h*(b*g - 2*a*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g -
d*h)) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*
h)))/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e
*h)))/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) -
 (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/
2)) + (-4*c^2*g^2 + (b^2*h^2)/2 - c*h*(-2*b*g + 2*a*h))*(-4*c*(-8*c^2*g^2 +
 (3*b^2*h^2)/2 - c*h*(-4*b*g + 6*a*h))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*
h) - 3*c*h*(e*g - d*h)) + (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*
g - a*f*g + a*e*h)))/2) + 4*c*h*(2*c*g - b*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*
(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h)
+ 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h)
 + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g -
a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*
d*g - a*f*g + a*e*h)))/2))/2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(Sqrt[c]*h) - (4*Sqrt[c*g^2 - b*g*h + a*h^2]*(-(g*(2*c*h*(2*c*
g - b*h)*(-4*c*(2*a*c*g*h + b*g*(-4*c*g + (3*b*h)/2))*(-3*c*g*(5*c*f*g^2 -
2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d
*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*(b*g - 2*a*h)*(-3*a*c*h*(5*c
*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c^2*g*(5*b*f*g^2 - b*h
*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (3*b*c*h*(5*b*f*g^2 - b*
h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (5*b*(3*c*g*(5*c*f*g^2
- 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*(5*b*f*g^2 - b*h*(5*e*g -
 d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2)) + (-4*c^2*g^2 + (b^2*h^2)/2 -
 c*h*(-2*b*g + 2*a*h))*(-4*c*(-8*c^2*g^2 + (3*b^2*h^2)/2 - c*h*(-4*b*g + 6*
a*h))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*
(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*(
2*c*g - b*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h))
- (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2
 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/
2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*
h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2))))
+ h*(2*c*h*(b*g - 2*a*h)*(-4*c*(2*a*c*g*h + b*g*(-4*c*g + (3*b*h)/2))*(-3*c
*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*(5*b*f*g^2
- b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*(b*g - 2*a*h
)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c^2*g*
```

```
(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (3*b*c*h
*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (5*b*(3
*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*(5*b*f*g^
2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2)) + (2*a*c*g*h
+ b*g*(-2*c*g + (b*h)/2))*(-4*c*(-8*c^2*g^2 + (3*b^2*h^2)/2 - c*h*(-4*b*g +
 6*a*h))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c
*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*
h*(2*c*g - b*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h
)) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h))
)/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)
))/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3
*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2))
))*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2
]*Sqrt[a + b*x + c*x^2])]])/(h*(4*c*g^2 - 4*b*g*h + 4*a*h^2)))/(4*c*h^2))/(8
*c*h^2))/(-(c*g^2) + b*g*h - a*h^2))/(2*(c*g^2 - b*g*h + a*h^2)))))/(3*c*h*(
a + b*x + c*x^2)^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.47Unable to divide, perha
ps due to rounding error%%%{1,[6,0,0,0,9,0,0,0]%%%}+%%%{%%{[-6,0]:[1,0,%%%{
-1,[1]%%%}]%%},[5,0,0,0,8,1,0,0]%%%}+%%%{-3,[4,1,0,0,9,0,0,0]%%%}+%%%{3,[4,
0,1,0,8,1,0,0]%%%}+%%%{%%%{12,[1]%%%},[4,0,0,0,7,2,0,0]%%%}+%%%{%%{[12,0]:[
1,0,%%%{-1,[1]%%%}]%%},[3,1,0,0,8,1,0,0]%%%}+%%%{%%{[-12,0]:[1,0,%%%{-1,[1]
%%%}]%%},[3,0,1,0,7,2,0,0]%%%}+%%%{%%{[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[3,0,0,0,6,3,0,0]%%%}+%%%{3,[2,2,0,0,9,0,0,0]%%%}+%%%{-6,[2,1,1,0,8,1
,0,0]%%%}+%%%{%%%{-12,[1]%%%},[2,1,0,0,7,2,0,0]%%%}+%%%{3,[2,0,2,0,7,2,0,0]
%%%}+%%%{%%%{12,[1]%%%},[2,0,1,0,6,3,0,0]%%%}+%%%{%%{[-6,0]:[1,0,%%%{-1,[1]
%%%}]%%},[1,2,0,0,8,1,0,0]%%%}+%%%{%%{[12,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,1
,0,7,2,0,0]%%%}+%%%{%%{[-6,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,2,0,6,3,0,0]%%%}
+%%%{-1,[0,3,0,0,9,0,0,0]%%%}+%%%{3,[0,2,1,0,8,1,0,0]%%%}+%%%{-3,[0,1,2,0,7
,2,0,0]%%%}+%%%{1,[0,0,3,0,6,3,0,0]%%%} / %%%{%%{poly1[%%%{1,[1]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[6,0,0,0,3,0,0,0]%%%}+%%%{%%%{-6,[2]%%%},[5,0,0,0,2,1,
0,0]%%%}+%%%{%%{[%%%{-3,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,1,0,0,3,0,0,0
]%%%}+%%%{%%{poly1[%%%{3,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,1,0,2,1,0,
0]%%%}+%%%{%%{poly1[%%%{12,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0,0,1,2,
0,0]%%%}+%%%{%%%{12,[2]%%%},[3,1,0,0,2,1,0,0]%%%}+%%%{%%%{-12,[2]%%%},[3,0,
1,0,1,2,0,0]%%%}+%%%{%%%{-8,[3]%%%},[3,0,0,0,0,3,0,0]%%%}+%%%{%%{[%%%{3,[1]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,0,0,3,0,0,0]%%%}+%%%{%%{[%%%{-6,[1]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,1,1,0,2,1,0,0]%%%}+%%%{%%{[%%%{-12,[2]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[2,1,0,0,1,2,0,0]%%%}+%%%{%%{poly1[%%%{3,[1]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,2,0,1,2,0,0]%%%}+%%%{%%{poly1[%%%{12,[2]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,1,0,0,3,0,0]%%%}+%%%{%%%{-6,[2]%%%},[1,2
,0,0,2,1,0,0]%%%}+%%%{%%%{12,[2]%%%},[1,1,1,0,1,2,0,0]%%%}+%%%{%%%{-6,[2]%%
```

```
%},[1,0,2,0,0,3,0,0]%%%}+%%%{%%{[%%%{-1,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[0,3,0,0,3,0,0,0]%%%}+%%%{%%{[%%%{3,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2
,1,0,2,1,0,0]%%%}+%%%{%%{[%%%{-3,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,1,2,
0,1,2,0,0]%%%}+%%%{%%{poly1[%%%{1,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,3
,0,0,3,0,0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.02, size = 26596, normalized size = 32.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                                +c*g^2 zero or nonze
ro?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**3,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

3.203 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$

Optimal. Leaf size=833

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{3h\left(cg^2-bhg+ah^2\right)(g+hx)^3}-\frac{\left(2cg\left(-\frac{10fg^2}{h}+4eg-dh\right)-6ah(3fg-eh)+b\left(17fg^2-h(5eg+dh)\right)+2h\right.}{12h^2\left(cg^2-bhg+ah^2\right)}$$

```
[Out] -1/12*(2*c*g*(4*e*g-10*f*g^2/h-d*h)-6*a*h*(-e*h+3*f*g)+b*(17*f*g^2-h*(d*h+5
*e*g))+2*h*(2*c*e*g+3*b*f*g-5*c*f*g^2/h-2*c*d*h-3*a*f*h)*x)*(c*x^2+b*x+a)^(
3/2)/h^2/(a*h^2-b*g*h+c*g^2)/(h*x+g)^2-1/3*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+
a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^3-1/16*(16*c^3*g^3*(10*f*g^2-h*(-d*h
+4*e*g))-b*h^3*(24*a^2*f*h^2-6*a*b*h*(-e*h+10*f*g)+b^2*(-d*h^2-5*e*g*h+35*f
*g^2))+6*c*h^2*(4*a^2*h^2*(-e*h+4*f*g)+b^2*g*(d*h^2-10*e*g*h+35*f*g^2)-2*a*
b*h*(d*h^2-7*e*g*h+25*f*g^2))-24*c^2*g*h*(b*g*(d*h^2-5*e*g*h+14*f*g^2)-a*h*
(d*h^2-4*e*g*h+11*f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*
g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/(a*h^2-b*g*h+c*g^2)^(3/2)+1/8*(3*
b^2*f*h^2-12*c*h*(-a*f*h-b*e*h+4*b*f*g)+8*c^2*(10*f*g^2-h*(-d*h+4*e*g)))*ar
ctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/c^(1/2)-1/8*(8*c^2*g^2
*(10*f*g^2-h*(-d*h+4*e*g))-2*c*h*(3*b*g*(d*h^2-6*e*g*h+18*f*g^2)-2*a*h*(2*d
*h^2-8*e*g*h+23*f*g^2))+h^2*(12*a^2*f*h^2-6*a*b*h*(-e*h+7*f*g)+b^2*(29*f*g^
2-h*(d*h+5*e*g)))+2*h*(3*b*f*h^2*(-a*h+b*g)+2*c^2*g*(10*f*g^2-h*(-d*h+4*e*g
))+c*h*(6*a*h*(-e*h+3*f*g)-b*(d*h^2-7*e*g*h+22*f*g^2)))*x)*(c*x^2+b*x+a)^(1
/2)/h^5/(a*h^2-b*g*h+c*g^2)/(h*x+g)
```

Rubi [A] time = 2.26, antiderivative size = 829, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 812, 843, 621, 206, 724}

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{3h\left(cg^2-bhg+ah^2\right)(g+hx)^3}-\frac{\left(17bfg^2+2c\left(-\frac{10fg^2}{h}+4eg-dh\right)g-bh(5eg+dh)-6ah(3fg-eh)+2h\right.}{12h^2\left(cg^2-bhg+ah^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -((12*a^2*f*h^3 - 8*c^2*g^2*(4*e*g - (10*f*g^2)/h - d*h) - 6*a*b*h^2*(7*f*g
 - e*h) + 4*a*c*h*(23*f*g^2 - 2*h*(4*e*g - d*h)) - 6*b*c*g*(18*f*g^2 - h*(6
*e*g - d*h)) + b^2*h*(29*f*g^2 - h*(5*e*g + d*h)) + 2*(3*b*f*h^2*(b*g - a*h
) + 2*c^2*(10*f*g^3 - g*h*(4*e*g - d*h)) - c*h*(22*b*f*g^2 - b*h*(7*e*g - d
*h) - 6*a*h*(3*f*g - e*h)))*x)*Sqrt[a + b*x + c*x^2])/(8*h^4*(c*g^2 - b*g*h
 + a*h^2)*(g + h*x)) - ((17*b*f*g^2 + 2*c*g*(4*e*g - (10*f*g^2)/h - d*h) -
b*h*(5*e*g + d*h) - 6*a*h*(3*f*g - e*h) + 2*h*(2*c*e*g + 3*b*f*g - (5*c*f*g
^2)/h - 2*c*d*h - 3*a*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(12*h^2*(c*g^2 - b*g
*h + a*h^2)*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2)
)/(3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) + ((3*b^2*f*h^2 - 12*c*h*(4*b*f
*g - b*e*h - a*f*h) + 8*c^2*(10*f*g^2 - h*(4*e*g - d*h)))*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*h^6) - ((16*c^3*(10*f*g^5
 - g^3*h*(4*e*g - d*h)) - b*h^3*(24*a^2*f*h^2 - 6*a*b*h*(10*f*g - e*h) + b^
2*(35*f*g^2 - 5*e*g*h - d*h^2)) + 6*c*h^2*(4*a^2*h^2*(4*f*g - e*h) + b^2*g*
(35*f*g^2 - 10*e*g*h + d*h^2) - 2*a*b*h*(25*f*g^2 - 7*e*g*h + d*h^2)) - 24*
c^2*g*h*(b*g*(14*f*g^2 - 5*e*g*h + d*h^2) - a*h*(11*f*g^2 - 4*e*g*h + d*h^2
)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*
Sqrt[a + b*x + c*x^2])])/(16*h^6*(c*g^2 - b*g*h + a*h^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3} - \frac{\int \frac{\left(\frac{1}{2}\left(-6cdg+5beg+6afg-\frac{5bfg^2}{h}+bdh-6\right.\right.}{}}{3\left(\right.}$$

$$= -\frac{\left(17bfg^2+2cg\left(4eg-\frac{10fg^2}{h}-dh\right)-bh(5eg+dh)-6ah(3fg-eh)+2\right.}{12h^2\left(cg^2-bgh+ah^2\right.}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach\left(23fg^2\right.\right.}{}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach\left(23fg^2\right.\right.}{}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach\left(23fg^2\right.\right.}{}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach\left(23fg^2\right.\right.}{}$$

Mathematica [B] time = 6.49, size = 7806, normalized size = 9.37

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [B] time = 26.17, size = 7319, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*c*f*x/h^4 - (16*c^2*f*g*h^10 - 5*b*c*f*h^11 -
4*c^2*h^11*e)/(c*h^15)) - 1/8*(160*c^3*f*g^5 - 336*b*c^2*f*g^4*h + 16*c^3*d
*g^3*h^2 + 210*b^2*c*f*g^3*h^2 + 264*a*c^2*f*g^3*h^2 - 24*b*c^2*d*g^2*h^3 -
 35*b^3*f*g^2*h^3 - 300*a*b*c*f*g^2*h^3 + 6*b^2*c*d*g*h^4 + 24*a*c^2*d*g*h^
4 + 60*a*b^2*f*g*h^4 + 96*a^2*c*f*g*h^4 + b^3*d*h^5 - 12*a*b*c*d*h^5 - 24*a
^2*b*f*h^5 - 64*c^3*g^4*h*e + 120*b*c^2*g^3*h^2*e - 60*b^2*c*g^2*h^3*e - 96
```

*a*c^2*g^2*h^3*e + 5*b^3*g*h^4*e + 84*a*b*c*g*h^4*e - 6*a*b^2*h^5*e - 24*a^2*c*h^5*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c*g^2*h^6 - b*g*h^7 + a*h^8)*sqrt(-c*g^2 + b*g*h - a*h^2)) - 1/24*(480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^(7/2)*f*g^5*h^2 - 912*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^(5/2)*f*g^4*h^3 + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^(7/2)*d*g^3*h^4 + 522*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c^(3/2)*f*g^3*h^4 + 552*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^(5/2)*f*g^3*h^4 - 216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^(5/2)*d*g^2*h^5 - 87*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*sqrt(c)*f*g^2*h^5 - 540*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^(3/2)*f*g^2*h^5 + 78*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c^(3/2)*d*g*h^6 + 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^(5/2)*d*g*h^6 + 108*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*sqrt(c)*f*g*h^6 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c^(3/2)*f*g*h^6 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*sqrt(c)*d*h^7 - 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^(3/2)*d*h^7 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*sqrt(c)*f*h^7 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^(7/2)*g^4*h^3*e + 504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^(5/2)*g^3*h^4*e - 252*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c^(3/2)*g^2*h^5*e - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^(5/2)*g^2*h^5*e + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*sqrt(c)*g*h^6*e + 228*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^(3/2)*g*h^6*e - 30*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*sqrt(c)*h^7*e - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c^(3/2)*h^7*e + 1680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^4*f*g^6*h - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^3*f*g^5*h^2 + 432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^4*d*g^4*h^3 + 1362*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^2*f*g^4*h^3 + 1464*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^3*f*g^4*h^3 - 504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^3*d*g^3*h^4 - 147*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c*f*g^3*h^4 - 876*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^2*f*g^3*h^4 + 54*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^2*d*g^2*h^5 + 216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^3*d*g^2*h^5 - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c*f*g^2*h^5 - 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^2*f*g^2*h^5 + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c*d*g*h^6 + 84*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^2*d*g*h^6 + 216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*c*f*g*h^6 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c*d*h^7 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^2*d*h^7 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*c*f*h^7 - 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^4*g^5*h^2*e + 1464*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^3*g^4*h^3*e - 540*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^2*g^3*h^4*e - 672*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^3*g^3*h^4*e + 21*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c*g^2*h^5*e + 180*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^2*g^2*h^5*e + 90*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c*g*h^6*e + 168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^2*g*h^6*e - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*c*h^7*e + 1504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^(9/2)*f*g^7 - 1072*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^(7/2)*f*g^6*h + 352*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^(9/2)*d*g^5*h^2 - 1308*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^(5/2)*f*g^5*h^2 - 656*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^(7/2)*f*g^5*h^2 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^(7/2)*d*g^4*h^3 + 1042*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c^(3/2)*f*g^4*h^3 + 4056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^(5/2)*f*g^4*h^3 - 420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^(5/2)*d*g^3*h^4 - 272*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^(7/2)*d*g^3*h^4 - 136*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*sqrt(c)*f*g^3*h^4 - 2712*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c^(3/2)*f*g^3*h^4 - 2208*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^(5/2)*f*g^3*h^4 + 106*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c^(3/2)*d*g^2*h^5 + 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^(5/2)*d*g^2*h^5 + 328*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*sqrt(c)*f*g^2*h^5 + 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^(3/2)*f*g^2*h^5 +

8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*sqrt(c)*d*g*h^6 - 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c^(3/2)*d*g*h^6 - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^(5/2)*d*g*h^6 - 240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*sqrt(c)*f*g*h^6 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*c^(3/2)*f*g*h^6 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*sqrt(c)*d*h^7 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*sqrt(c)*f*h^7 - 832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^(9/2)*g^6*h*e + 400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^(7/2)*g^5*h^2*e + 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^(5/2)*g^4*h^3*e + 512*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^(7/2)*g^4*h^3*e - 478*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c^(3/2)*g^3*h^4*e - 2232*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^(5/2)*g^3*h^4*e + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*sqrt(c)*g^2*h^5*e + 1092*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c^(3/2)*g^2*h^5*e + 1104*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^(5/2)*g^2*h^5*e - 88*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*sqrt(c)*g*h^6*e - 576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^(3/2)*g*h^6*e + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*sqrt(c)*h^7*e + 2256*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^4*f*g^7 - 3420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c^3*f*g^6*h - 2832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^4*f*g^6*h + 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^4*d*g^5*h^2 + 1218*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^2*f*g^5*h^2 + 5976*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*c^3*f*g^5*h^2 - 516*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c^3*d*g^4*h^3 - 624*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^4*d*g^4*h^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c*f*g^4*h^3 - 1944*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^2*f*g^4*h^3 - 2208*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*c^3*f*g^4*h^3 - 6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^2*d*g^3*h^4 + 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*c^3*d*g^3*h^4 - 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*c*f*g^3*h^4 + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^2*f*g^3*h^4 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c*d*g^2*h^5 + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^2*d*g^2*h^5 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*c^3*d*g^2*h^5 + 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*c*f*g^2*h^5 + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^2*f*g^2*h^5 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*c*d*g*h^6 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^2*d*g*h^6 - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b*c*f*g*h^6 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^2*d*h^7 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*c*f*h^7 - 1248*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^4*g^6*h*e + 1656*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c^3*g^5*h^2*e + 1536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^4*g^5*h^2*e - 414*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^2*g^4*h^3*e - 2760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*c^3*g^4*h^3*e - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c*g^3*h^4*e + 420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^2*g^3*h^4*e + 912*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*c^3*g^3*h^4*e + 168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*c*g^2*h^5*e + 432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^2*g^2*h^5*e - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*c*g*h^6*e - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^2*g*h^6*e + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b*c*h^7*e + 1128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^(7/2)*f*g^7 - 1776*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^(5/2)*f*g^6*h - 2832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(7/2)*f*g^6*h + 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^(7/2)*d*g^5*h^2 + 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^4*c^(3/2)*f*g^5*h^2 + 5580*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*c^(5/2)*f*g^5*h^2 + 1776*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c^(7/2)*f*g^5*h^2 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^(5/2)*d*g^4*h^3 - 624*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(7/2)*d*g^4*h^3 - 57*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*sqrt(c)*f*g^4*h^3 - 2514*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^3*c^(3/2)*f*g^4*h^3 - 5688*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^(5/2)*f*g^4*h^3 + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^4*c^(3/2)*d*g^3*h^4 + 852*(sqrt(c)*x - sqrt(c*x^2 + b*x

```
+ a))*a*b^2*c^(5/2)*d*g^3*h^4 + 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2
*c^(7/2)*d*g^3*h^4 + 198*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*sqrt(c)*
f*g^3*h^4 + 3078*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*c^(3/2)*f*g^3*
h^4 + 1848*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*c^(5/2)*f*g^3*h^4 + 3*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*sqrt(c)*d*g^2*h^5 - 90*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a*b^3*c^(3/2)*d*g^2*h^5 - 864*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*a^2*b*c^(5/2)*d*g^2*h^5 - 249*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a^2*b^3*sqrt(c)*f*g^2*h^5 - 1476*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^
3*b*c^(3/2)*f*g^2*h^5 - 6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*sqrt(c)
*d*g*h^6 + 90*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*c^(3/2)*d*g*h^6 +
 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*c^(5/2)*d*g*h^6 + 132*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*sqrt(c)*f*g*h^6 + 192*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a^4*c^(3/2)*f*g*h^6 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a^2*b^3*sqrt(c)*d*h^7 - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c^
(3/2)*d*h^7 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*sqrt(c)*f*h^7 -
624*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^(7/2)*g^6*h*e + 876*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*b^3*c^(5/2)*g^5*h^2*e + 1536*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a*b*c^(7/2)*g^5*h^2*e - 282*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*b^4*c^(3/2)*g^4*h^3*e - 2664*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^
2*c^(5/2)*g^4*h^3*e - 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c^(7/2)*g
^4*h^3*e + 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*sqrt(c)*g^3*h^4*e + 8
94*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^3*c^(3/2)*g^3*h^4*e + 2640*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^(5/2)*g^3*h^4*e - 48*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*a*b^4*sqrt(c)*g^2*h^5*e - 936*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*a^2*b^2*c^(3/2)*g^2*h^5*e - 816*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^3*c^(5/2)*g^2*h^5*e + 51*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3
*sqrt(c)*g*h^6*e + 300*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c^(3/2)*g*
h^6*e - 18*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*sqrt(c)*h^7*e + 24*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*c^(3/2)*h^7*e + 188*b^3*c^3*f*g^7 -
272*b^4*c^2*f*g^6*h - 708*a*b^2*c^3*f*g^6*h + 44*b^3*c^3*d*g^5*h^2 + 87*b^5
*c*f*g^5*h^2 + 1214*a*b^3*c^2*f*g^5*h^2 + 888*a^2*b*c^3*f*g^5*h^2 - 44*b^4*
c^2*d*g^4*h^3 - 156*a*b^2*c^3*d*g^4*h^3 - 426*a*b^4*c*f*g^4*h^3 - 2010*a^2*
b^2*c^2*f*g^4*h^3 - 376*a^3*c^3*f*g^4*h^3 + 3*b^5*c*d*g^3*h^4 + 182*a*b^3*c
^2*d*g^3*h^4 + 192*a^2*b*c^3*d*g^3*h^4 + 807*a^2*b^3*c*f*g^3*h^4 + 1468*a^3
*b*c^2*f*g^3*h^4 - 6*a*b^4*c*d*g^2*h^5 - 294*a^2*b^2*c^2*d*g^2*h^5 - 88*a^3
*c^3*d*g^2*h^5 - 732*a^3*b^2*c*f*g^2*h^5 - 400*a^4*c^2*f*g^2*h^5 + 3*a^2*b^
3*c*d*g*h^6 + 220*a^3*b*c^2*d*g*h^6 + 312*a^4*b*c*f*g*h^6 - 64*a^4*c^2*d*h^
7 - 48*a^5*c*f*h^7 - 104*b^3*c^3*g^6*h*e + 134*b^4*c^2*g^5*h^2*e + 384*a*b^
2*c^3*g^5*h^2*e - 33*b^5*c*g^4*h^3*e - 578*a*b^3*c^2*g^4*h^3*e - 480*a^2*b*
c^3*g^4*h^3*e + 144*a*b^4*c*g^3*h^4*e + 936*a^2*b^2*c^2*g^3*h^4*e + 208*a^3
*c^3*g^3*h^4*e - 237*a^2*b^3*c*g^2*h^5*e - 676*a^3*b*c^2*g^2*h^5*e + 174*a^
3*b^2*c*g*h^6*e + 184*a^4*c^2*g*h^6*e - 48*a^4*b*c*h^7*e)/((c^(3/2)*g^2*h^6
 - b*sqrt(c)*g*h^7 + a*sqrt(c)*h^8)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*g + b*g - a*h)^3) - 1/8*(
80*c^2*f*g^2 - 48*b*c*f*g*h + 8*c^2*d*h^2 + 3*b^2*f*h^2 + 12*a*c*f*h^2 - 32
*c^2*g*h*e + 12*b*c*h^2*e)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sq
rt(c) + b))/(sqrt(c)*h^6)
```

maple [B] time = 0.03, size = 40092, normalized size = 48.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                             +c*g^2 zero or nonze
ro?
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{\left(g+hx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**4,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

3.204 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx$

Optimal. Leaf size=1097

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}-\frac{\left(16c^2(5fg-eh)g^4-4ch\left(bg\left(31fg^2-5ehg+3dh^2\right)-ah\left(25fg^2-5eh\right.\right.\right.}{}$$

```
[Out] -1/96*(16*c^2*g^4*(-e*h+5*f*g)-h^2*(16*a^2*h^2*(-2*e*h+f*g)-b^2*g*(3*d*h^2+
5*e*g*h+35*f*g^2)+4*a*b*h*(3*d*h^2+7*e*g*h+7*f*g^2))-4*c*g*h*(b*g*(3*d*h^2-
5*e*g*h+31*f*g^2)-a*h*(9*d*h^2-5*e*g*h+25*f*g^2))+3*h*(8*c^2*g^2*(5*f*g^2-h
*(d*h+e*g))+h^2*(16*a^2*f*h^2-8*a*b*h*(-e*h+6*f*g)+b^2*(-3*d*h^2-5*e*g*h+29
*f*g^2))-4*c*h*(2*b*g*(-d*h^2-2*e*g*h+9*f*g^2)-a*h*(d*h^2-5*e*g*h+17*f*g^2)
))*x)*(c*x^2+b*x+a)^(3/2)/h^3/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^3-1/4*(f*g^2-h*
(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^4+1/128*(128*
c^4*g^5*(-e*h+5*f*g)-64*c^3*g^3*h*(b*g*(-5*e*h+28*f*g)-5*a*h*(-e*h+5*f*g))+
8*c*h^3*(24*a^3*f*h^3-12*a^2*b*h^2*(-e*h+10*f*g)-5*b^3*g^2*(-e*h+14*f*g)+3*
a*b^2*h*(-d*h^2-5*e*g*h+55*f*g^2))-48*c^2*h^2*(10*a*b*g^2*h*(-e*h+6*f*g)-5*
b^2*g^3*(-e*h+7*f*g)-a^2*h^2*(d*h^2-5*e*g*h+25*f*g^2))+b^2*h^4*(48*a^2*f*h^
2-8*a*b*h*(e*h+10*f*g)+b^2*(3*d*h^2+5*e*g*h+35*f*g^2)))*arctanh(1/2*(b*g-2*
a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/(a*h
^2-b*g*h+c*g^2)^(5/2)-1/2*(-3*b*f*h-2*c*e*h+10*c*f*g)*arctanh(1/2*(2*c*x+b)
/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/h^6+1/64*(64*c^3*g^4*(-e*h+5*f*g)-16*
c^2*g^2*h*(b*g*(-7*e*h+41*f*g)-8*a*h*(-e*h+5*f*g))+4*c*h^2*(2*b^2*g^2*(-5*e
*h+46*f*g)+16*a^2*h^2*(-e*h+5*f*g)-a*b*h*(-3*d*h^2-25*e*g*h+173*f*g^2))-b*h
^3*(48*a^2*f*h^2-8*a*b*h*(e*h+10*f*g)+b^2*(3*d*h^2+5*e*g*h+35*f*g^2))+2*c*h
*(16*c^2*g^3*(-e*h+5*f*g)-4*c*h*(6*b*g^2*(-e*h+6*f*g)-a*h*(35*f*g^2-h*(-3*d
*h+7*e*g)))+h^2*(48*a^2*f*h^2-8*a*b*h*(-e*h+14*f*g)+b^2*(61*f*g^2-h*(3*d*h+
5*e*g))))*x)*(c*x^2+b*x+a)^(1/2)/h^5/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)
```

Rubi [A] time = 3.12, antiderivative size = 1096, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 810, 812, 843, 621, 206, 724}

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}-\frac{\left(\frac{16c^2(5fg-eh)g^4}{h}-4c\left(bg\left(31fg^2-5ehg+3dh^2\right)-ah\left(25fg^2-5ehg+9a\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (((64*c^3*g^4*(5*f*g - e*h))/h - 16*c^2*g^2*(b*g*(41*f*g - 7*e*h) - 8*a*h*(
5*f*g - e*h)) + 4*c*h*(2*b^2*g^2*(46*f*g - 5*e*h) + 16*a^2*h^2*(5*f*g - e*h
) - a*b*h*(173*f*g^2 - 25*e*g*h - 3*d*h^2)) - b*h^2*(48*a^2*f*h^2 - 8*a*b*h
*(10*f*g + e*h) + b^2*(35*f*g^2 + 5*e*g*h + 3*d*h^2)) + 2*c*(16*c^2*g^3*(5*
f*g - e*h) - 4*c*h*(6*b*g^2*(6*f*g - e*h) - a*h*(35*f*g^2 - h*(7*e*g - 3*d*
h))) + h^2*(48*a^2*f*h^2 - 8*a*b*h*(14*f*g - e*h) + b^2*(61*f*g^2 - h*(5*e*
g + 3*d*h))))*x)*Sqrt[a + b*x + c*x^2])/(64*h^4*(c*g^2 - b*g*h + a*h^2)^2*(
g + h*x)) - (((16*c^2*g^4*(5*f*g - e*h))/h - h*(16*a^2*h^2*(f*g - 2*e*h) -
b^2*g*(35*f*g^2 + 5*e*g*h + 3*d*h^2) + 4*a*b*h*(7*f*g^2 + 7*e*g*h + 3*d*h^2
)) - 4*c*g*(b*g*(31*f*g^2 - 5*e*g*h + 3*d*h^2) - a*h*(25*f*g^2 - 5*e*g*h +
9*d*h^2)) + 3*h*((40*c^2*f*g^4)/h + 16*a^2*f*h^3 - 8*c^2*g^2*(e*g + d*h) -
8*a*b*h^2*(6*f*g - e*h) + 4*a*c*h*(17*f*g^2 - h*(5*e*g - d*h)) - 8*b*c*g*(9
*f*g^2 - h*(2*e*g + d*h)) + b^2*h*(29*f*g^2 - h*(5*e*g + 3*d*h)))*x)*(a + b
*x + c*x^2)^(3/2))/(96*h^2*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^3) - ((f*g^2
 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(4*h*(c*g^2 - b*g*h + a*h^2)*(g
```

```
+ h*x)^4) - (Sqrt[c]*(10*c*f*g - 2*c*e*h - 3*b*f*h)*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*h^6) + ((128*c^4*g^5*(5*f*g - e*h) - 64
*c^3*g^3*h*(b*g*(28*f*g - 5*e*h) - 5*a*h*(5*f*g - e*h)) + 8*c*h^3*(24*a^3*f
*h^3 - 12*a^2*b*h^2*(10*f*g - e*h) - 5*b^3*g^2*(14*f*g - e*h) + 3*a*b^2*h*(
55*f*g^2 - 5*e*g*h - d*h^2)) - 48*c^2*h^2*(10*a*b*g^2*h*(6*f*g - e*h) - 5*b
^2*g^3*(7*f*g - e*h) - a^2*h^2*(25*f*g^2 - 5*e*g*h + d*h^2)) + b^2*h^4*(48*
a^2*f*h^2 - 8*a*b*h*(10*f*g + e*h) + b^2*(35*f*g^2 + 5*e*g*h + 3*d*h^2)))*A
rcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[
a + b*x + c*x^2])])/(128*h^6*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
```

```
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4} - \int \frac{\left(\frac{1}{2}\left(-8cdg+5beg+8afg-\frac{5bfg^2}{h}+3b\ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(\frac{16c^2g^4(5fg-eh)}{h} - h\left(16a^2h^2(fg-2eh) - b^2g\left(35fg^2+5egh+3dh^2\right)\ldots\right.\right.}{\ldots}$$

$$= \frac{\left(\frac{64c^3g^4(5fg-eh)}{h} - 16c^2g^2(bg(41fg-7eh)-8ah(5fg-eh)) + 4ch\left(2b^2g\ldots\right.\right.}{\ldots}$$

$$= \frac{\left(\frac{64c^3g^4(5fg-eh)}{h} - 16c^2g^2(bg(41fg-7eh)-8ah(5fg-eh)) + 4ch\left(2b^2g\ldots\right.\right.}{\ldots}$$

$$= \frac{\left(\frac{64c^3g^4(5fg-eh)}{h} - 16c^2g^2(bg(41fg-7eh)-8ah(5fg-eh)) + 4ch\left(2b^2g\ldots\right.\right.}{\ldots}$$

$$= \frac{\left(\frac{64c^3g^4(5fg-eh)}{h} - 16c^2g^2(bg(41fg-7eh)-8ah(5fg-eh)) + 4ch\left(2b^2g\ldots\right.\right.}{\ldots}$$

Mathematica [B] time = 6.62, size = 46895, normalized size = 42.75

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.06, size = 57957, normalized size = 52.83

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                                    +c*g^2 zero or nonze
ro?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**5,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

3.205 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

Optimal. Leaf size=1226

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}-\frac{\left(16c^2fg^5-2ch\left(13bfg^3-10afhg^2+3bdh^2g-6adh^3\right)g-h^2\left(-g\left(7fg\right.\right.\right.}{}$$

```
[Out] -1/48*(16*c^2*f*g^5-2*c*g*h*(-6*a*d*h^3-10*a*f*g^2*h+3*b*d*g*h^2+13*b*f*g^3
)-h^2*(4*a^2*h^2*(-3*e*h+2*f*g)-b^2*g*(7*f*g^2+3*h*(d*h+e*g))+2*a*b*h*(f*g^
2+3*h*(d*h+2*e*g)))+h*(4*c^2*(-3*d*g^2*h^2+7*f*g^4)+2*c*g*h*(2*a*h*(-3*e*h+
14*f*g)-b*(-6*d*h^2-3*e*g*h+28*f*g^2))+h^2*(16*a^2*f*h^2-2*a*b*h*(-3*e*h+22
*f*g)+b^2*(25*f*g^2-3*h*(d*h+e*g))))*x)*(c*x^2+b*x+a)^(3/2)/h^3/(a*h^2-b*g*
h+c*g^2)^2/(h*x+g)^4-1/5*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-
b*g*h+c*g^2)/(h*x+g)^5+c^(3/2)*f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a
)^(1/2))/h^6-1/256*(256*c^5*f*g^7-896*c^4*f*g^5*h*(-a*h+b*g)+32*c^3*g*h^2*(
35*b^2*f*g^4-70*a*b*f*g^3*h+a^2*h^2*(-3*d*h^2+35*f*g^2))-16*c^2*h^3*(35*b^3
*f*g^4-6*a^3*h^3*(-e*h+6*f*g)+3*a^2*b*h^2*(-d*h^2-e*g*h+35*f*g^2)-3*a*b^2*g
*h*(d*h^2+35*f*g^2))+b^3*h^5*(16*a^2*f*h^2-2*a*b*h*(3*e*h+10*f*g)+b^2*(7*f*
g^2+3*h*(d*h+e*g)))-2*b*c*h^4*(96*a^3*f*h^3-24*a^2*b*h^2*(e*h+8*f*g)-b^3*(-
3*d*g*h^2+35*f*g^3)+4*a*b^2*h*(35*f*g^2+3*h*(d*h+e*g))))*arctanh(1/2*(b*g-2
*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/(a*
h^2-b*g*h+c*g^2)^(7/2)-1/128*(128*c^4*f*g^7-32*c^3*f*g^5*h*(-10*a*h+11*b*g)
+8*c^2*g*h^2*(38*b^2*f*g^4+2*a^2*h^2*(3*d*h^2+13*f*g^2)-a*b*g*h*(3*d*h^2+65
*f*g^2))-2*c*h^3*(8*a^3*h^3*(-3*e*h+2*f*g)-2*a*b^2*g^2*h*(3*e*h+34*f*g)+4*a
^2*b*h^2*(3*d*h^2+6*e*g*h+5*f*g^2)+b^3*(-3*d*g^2*h^2+35*f*g^4))-b*h^4*(-2*a
*h+b*g)*(16*a^2*f*h^2-2*a*b*h*(3*e*h+10*f*g)+b^2*(7*f*g^2+3*h*(d*h+e*g)))+h
*(128*c*f*(c*g^2-h*(-a*h+b*g))^3+(-b*h+2*c*g)*(32*c^3*f*g^5-8*c^2*g*h*(3*a*
d*h^3-11*a*f*g^2*h+10*b*f*g^3)+2*c*h^2*(4*a^2*h^2*(-3*e*h+10*f*g)-6*a*b*h*(
-d*h^2-e*g*h+11*f*g^2)+b^2*(3*d*g*h^2+29*f*g^3))-b*h^3*(16*a^2*f*h^2-2*a*b*
h*(3*e*h+10*f*g)+b^2*(7*f*g^2+3*h*(d*h+e*g)))))*x)*(c*x^2+b*x+a)^(1/2)/h^5/
(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^2
```

Rubi [A] time = 4.00, antiderivative size = 1223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1650, 810, 843, 621, 206, 724}

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}-\frac{\left(16c^2fg^5-2ch\left(13bfg^3-10afhg^2+3bdh^2g-6adh^3\right)g-h^2\left(-g\left(7fg\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -(((128*c^4*f*g^7)/h - 32*c^3*f*g^5*(11*b*g - 10*a*h) + 8*c^2*g*h*(38*b^2*f
*g^4 + 2*a^2*h^2*(13*f*g^2 + 3*d*h^2) - a*b*g*h*(65*f*g^2 + 3*d*h^2)) - b*h
^3*(b*g - 2*a*h)*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^2 +
3*h*(e*g + d*h))) - 2*c*h^2*(8*a^3*h^3*(2*f*g - 3*e*h) - 2*a*b^2*g^2*h*(34*
f*g + 3*e*h) + b^3*(35*f*g^4 - 3*d*g^2*h^2) + 4*a^2*b*h^2*(5*f*g^2 + 3*h*(2
*e*g + d*h))) + (128*c*f*(c*g^2 - h*(b*g - a*h))^3 + (2*c*g - b*h)*(32*c^3*
f*g^5 - 8*c^2*g*h*(10*b*f*g^3 - 11*a*f*g^2*h + 3*a*d*h^3) + 2*c*h^2*(4*a^2*
h^2*(10*f*g - 3*e*h) - 6*a*b*h*(11*f*g^2 - e*g*h - d*h^2) + b^2*(29*f*g^3 +
 3*d*g*h^2)) - b*h^3*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^
2 + 3*h*(e*g + d*h)))))*x)*Sqrt[a + b*x + c*x^2])/(128*h^4*(c*g^2 - b*g*h +
 a*h^2)^3*(g + h*x)^2) - ((16*c^2*f*g^5 - 2*c*g*h*(13*b*f*g^3 - 10*a*f*g^2*
h + 3*b*d*g*h^2 - 6*a*d*h^3) - h^2*(4*a^2*h^2*(2*f*g - 3*e*h) - b^2*g*(7*f*
```

```
g^2 + 3*h*(e*g + d*h)) + 2*a*b*h*(f*g^2 + 3*h*(2*e*g + d*h))) + h^2*(16*a^2
*f*h^3 + 4*a*c*g*h*(14*f*g - 3*e*h) + c^2*((28*f*g^4)/h - 12*d*g^2*h) + b^2
*h*(25*f*g^2 - 3*h*(e*g + d*h)) - b*(56*c*f*g^3 - 6*c*g*h*(e*g + 2*d*h) + 2
*a*h^2*(22*f*g - 3*e*h)))*x)*(a + b*x + c*x^2)^(3/2))/(48*h^3*(c*g^2 - b*g*
h + a*h^2)^2*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2
))/(5*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) + (c^(3/2)*f*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h^6 - ((256*c^5*f*g^7 - 896*c^4*f*g^
5*h*(b*g - a*h) + 32*c^3*g*h^2*(35*b^2*f*g^4 - 70*a*b*f*g^3*h + a^2*h^2*(35
*f*g^2 - 3*d*h^2)) - 16*c^2*h^3*(35*b^3*f*g^4 - 6*a^3*h^3*(6*f*g - e*h) + 3
*a^2*b*h^2*(35*f*g^2 - e*g*h - d*h^2) - 3*a*b^2*g*h*(35*f*g^2 + d*h^2)) + b
^3*h^5*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^2 + 3*h*(e*g +
 d*h))) - 2*b*c*h^4*(96*a^3*f*h^3 - 24*a^2*b*h^2*(8*f*g + e*h) - b^3*(35*f*
g^3 - 3*d*g*h^2) + 4*a*b^2*h*(35*f*g^2 + 3*h*(e*g + d*h))))*ArcTanh[(b*g -
2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^
2])])/(256*h^6*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} - \int \frac{\left(-\frac{5}{2}\left(2cdg-beg-2afg+\frac{bfg^2}{h}-bdh+\cdots\right.\right.}{5\left(cg^2\cdots\right.}$$

$$= -\frac{\left(16c^2fg^5-2cgh\left(13bfg^3-10afg^2h+3bdgh^2-6adh^3\right)-h^2\left(4a^2h\cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(\frac{128c^4fg^7}{h}-32c^3fg^5(11bg-10ah)+8c^2gh\left(38b^2fg^4+2a^2h^2\left(13fg\cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{\left(\frac{128c^4fg^7}{h}-32c^3fg^5(11bg-10ah)+8c^2gh\left(38b^2fg^4+2a^2h^2\left(13fg\cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{\left(\frac{128c^4fg^7}{h}-32c^3fg^5(11bg-10ah)+8c^2gh\left(38b^2fg^4+2a^2h^2\left(13fg\cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{\left(\frac{128c^4fg^7}{h}-32c^3fg^5(11bg-10ah)+8c^2gh\left(38b^2fg^4+2a^2h^2\left(13fg\cdots\right.\right.\right.}{\cdots}$$

Mathematica [A] time = 6.30, size = 1111, normalized size = 0.91

$$f(a+x(b+cx))^{3/2}\left(\frac{-\frac{(bh-2cg)\left(cx^2+bx+a\right)^{3/2}}{2\left(cg^2-bhg+ah^2\right)(g+hx)^2}-\frac{\frac{\left(\frac{1}{2}h\left(hb^2+2cgb-8ach\right)-cg(2cg-bh)\right)\left(cx^2+bx+a\right)^{3/2}}{\left(-cg^2+bhg-ah^2\right)(g+hx)}+\frac{\left(h\left(4c^2g^2-b^2h^2-4ch(bg-2ah)\right)xc^2-\left(2cg-\frac{bh}{2}\right)\left(4c^2g^2-b^2h^2-\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -(((a + x*(b + c*x))^(3/2)*(((g*h*(2*f*g - e*h) - h*(f*g^2 - d*h^2))*(a + b
*x + c*x^2)^(5/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) - ((-2*(a*h^2*(2
*f*g - e*h) + c*g*(f*g^2 - d*h^2)) + b*(g*h*(2*f*g - e*h) + h*(f*g^2 - d*h^
2)))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 -
 b*g*h + a*h^2)*(g + h*x)^4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b
*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b
^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*
h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2
 - 4*b*g*h + 4*a*h^2))))/(16*(c*g^2 - b*g*h + a*h^2))))/(2*(c*g^2 - b*g*h +
 a*h^2))))/(h^2*(a + b*x + c*x^2)^(3/2))) + (f*(a + x*(b + c*x))^(3/2)*(-1/
3*(a + b*x + c*x^2)^(3/2)/(h*(g + h*x)^3) + (-1/2*((-2*c*g + b*h)*(a + b*x
+ c*x^2)^(3/2))/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) - (((-(c*g*(2*c*g - b
*h)) + (h*(2*b*c*g + b^2*h - 8*a*c*h))/2)*(a + b*x + c*x^2)^(3/2))/((-(c*g^
2) + b*g*h - a*h^2)*(g + h*x)) + (((-(c*(2*c*g - (b*h)/2)*(4*c^2*g^2 - b^2*
h^2 - 4*c*h*(b*g - 2*a*h))) + (c*h*(-10*b^2*c*g*h + 8*a*c^2*g*h - b^3*h^2 +
 4*b*c*(2*c*g^2 + 3*a*h^2)))/2 + c^2*h*(4*c^2*g^2 - b^2*h^2 - 4*c*h*(b*g -
2*a*h))*x)*Sqrt[a + b*x + c*x^2])/(2*c*h^2) - ((-16*c^(5/2)*(c*g^2 - h*(b*g
 - a*h))^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h - (4*S
qrt[c*g^2 - b*g*h + a*h^2]*(-(c*h*(c*g^2 - b*g*h + a*h^2)*(8*b*c^2*g^2 - 6*
b^2*c*g*h - 8*a*c^2*g*h - b^3*h^2 + 12*a*b*c*h^2)) + 16*c^3*g*(c*g^2 - h*(b
*g - a*h))^2)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*
g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(h*(4*c*g^2 - 4*b*g*h + 4*a*h^2)))/(4
*c*h^2))/(-(c*g^2) + b*g*h - a*h^2))/(2*(c*g^2 - b*g*h + a*h^2)))/(2*h)))/(
h^2*(a + b*x + c*x^2)^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.07, size = 76693, normalized size = 62.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                               +c*g^2 zero or nonze
ro?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{\left(g+hx\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**6,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

3.206 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^7}\,dx$

Optimal. Leaf size=657

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-2ah+x(2cg-bh)+bg)\left(24a^2fh^2-4c\left(a\left(dh^2-7egh+fg^2\right)+3bg(2dh+eg)\right)-12abh(eh+\right.}{192(g+hx)^4\left(ah^2-bgh+cg^2\right)^3}$$

```
[Out] 1/192*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h*(e*h+2*f*g)+b^2*(7*d*h^2+5*e*g*h+
7*f*g^2)-4*c*(3*b*g*(2*d*h+e*g)+a*(d*h^2-7*e*g*h+f*g^2)))*(b*g-2*a*h+(-b*h+
2*c*g)*x)*(c*x^2+b*x+a)^(3/2)/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^4-1/6*(f*g^2-h*
(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^6+1/60*(2*c*g
*(5*f*g^2+h*(-7*d*h+e*g))+h*(12*a*h*(-e*h+2*f*g)-b*(-7*d*h^2-5*e*g*h+17*f*g
^2)))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^5+1/1024*(-4*a*c+
b^2)^2*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h*(e*h+2*f*g)+b^2*(7*d*h^2+5*e*g*h
+7*f*g^2)-4*c*(3*b*g*(2*d*h+e*g)+a*(d*h^2-7*e*g*h+f*g^2)))*arctanh(1/2*(b*g
-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^
2-b*g*h+c*g^2)^(9/2)-1/512*(-4*a*c+b^2)*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h
*(e*h+2*f*g)+b^2*(7*d*h^2+5*e*g*h+7*f*g^2)-4*c*(3*b*g*(2*d*h+e*g)+a*(d*h^2-
7*e*g*h+f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*
h+c*g^2)^4/(h*x+g)^2
```

Rubi [A] time = 1.22, antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1650, 806, 720, 724, 206}

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-2ah+x(2cg-bh)+bg)\left(24a^2fh^2-4c\left(-ah(7eg-dh)+afg^2+3bg(2dh+eg)\right)-12abh(eh+\right.}{192(g+hx)^4\left(ah^2-bgh+cg^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] -((b^2 - 4*a*c)*(24*c^2*d*g^2 + 24*a^2*f*h^2 - 12*a*b*h*(2*f*g + e*h) - 4*c
*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*(e*g + 2*d*h)) + b^2*(7*f*g^2 + h*(5*
e*g + 7*d*h)))*(b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(512*
(c*g^2 - b*g*h + a*h^2)^4*(g + h*x)^2) + ((24*c^2*d*g^2 + 24*a^2*f*h^2 - 12
*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*(e*g + 2*d*
h)) + b^2*(7*f*g^2 + h*(5*e*g + 7*d*h)))*(b*g - 2*a*h + (2*c*g - b*h)*x)*(a
 + b*x + c*x^2)^(3/2))/(192*(c*g^2 - b*g*h + a*h^2)^3*(g + h*x)^4) - ((f*g^
2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(6*h*(c*g^2 - b*g*h + a*h^2)*(g
 + h*x)^6) + ((2*c*(5*f*g^3 + g*h*(e*g - 7*d*h)) - h*(17*b*f*g^2 - b*h*(5*e
*g + 7*d*h) - 12*a*h*(2*f*g - e*h)))*(a + b*x + c*x^2)^(5/2))/(60*h*(c*g^2
- b*g*h + a*h^2)^2*(g + h*x)^5) + ((b^2 - 4*a*c)^2*(24*c^2*d*g^2 + 24*a^2*f
*h^2 - 12*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*(e
*g + 2*d*h)) + b^2*(7*f*g^2 + h*(5*e*g + 7*d*h)))*ArcTanh[(b*g - 2*a*h + (2
*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(102
4*(c*g^2 - b*g*h + a*h^2)^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^7}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{6h\left(cg^2-bgh+ah^2\right)(g+hx)^6} - \frac{\int \frac{\left(\frac{1}{2}\left(-12cdg+5beg+12afg-\frac{5bfg^2}{h}+7bd\right.\right.}{}}{6}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{6h\left(cg^2-bgh+ah^2\right)(g+hx)^6} + \frac{\left(2c\left(5fg^3+gh(eg-7dh)\right)-h\right.}{6}$$

$$= \frac{\left(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c\left(afg^2-ah(7eg-dh)+3b\right.\right.}{192\left(cg^2-\right.}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c\left(afg^2-ah(7e\right.\right.}{512\,(}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c\left(afg^2-ah(7e\right.\right.}{512\,(}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c\left(afg^2-ah(7e\right.\right.}{512\,(}$$

Mathematica [A] time = 6.24, size = 766, normalized size = 1.17

$$(a+x(b+cx))^{3/2}\left(-\frac{\frac{\left(a+bx+cx^2\right)^{5/2}\left(\frac{1}{2}ch\left(12h(aeh-afg+cdg)-bh(7dh+5eg)+5bfg^2\right)-cg\left(-6fh(bg-ah)+ch(eg-dh)+5cfg^2\right)\right)}{5(g+hx)^5\left(ah^2-bgh+cg^2\right)} - \left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ah+x(2cg-bh)+bg)}{8(g+hx)^4\left(ah^2-bgh+cg^2\right)} - \frac{3(\ldots}{}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] -((f*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(c*h*(g + h*x)^6)) + ((a +
x*(b + c*x))^(3/2)*(-1/6*(((h*(5*b*f*g + 2*c*d*h - 12*a*f*h))/2 - (g*(-7*b*
f*h + 2*c*(5*f*g + e*h)))/2)*(a + b*x + c*x^2)^(5/2))/((c*g^2 - b*g*h + a*h
^2)*(g + h*x)^6) - (((-(c*g*(5*c*f*g^2 - 6*f*h*(b*g - a*h) + c*h*(e*g - d*h
))) + (c*h*(5*b*f*g^2 - b*h*(5*e*g + 7*d*h) + 12*h*(c*d*g - a*f*g + a*e*h))
)/2)*(a + b*x + c*x^2)^(5/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) - ((-
```

```
2*(-(a*c*h*(5*c*f*g^2 - 6*f*h*(b*g - a*h) + c*h*(e*g - d*h))) - (c^2*g*(5*b
*f*g^2 - b*h*(5*e*g + 7*d*h) + 12*h*(c*d*g - a*f*g + a*e*h)))/2) + b*(-(c*g
*(5*c*f*g^2 - 6*f*h*(b*g - a*h) + c*h*(e*g - d*h))) - (c*h*(5*b*f*g^2 - b*h
*(5*e*g + 7*d*h) + 12*h*(c*d*g - a*f*g + a*e*h)))/2))*(((b*g - 2*a*h + (2*c
*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^
4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^
2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g
) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x +
c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))))/(
16*(c*g^2 - b*g*h + a*h^2))))/(2*(c*g^2 - b*g*h + a*h^2)))/(6*(c*g^2 - b*g*
h + a*h^2))))/(c*h*(a + b*x + c*x^2)^(3/2))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**B**] time = 0.13, size = 100754, normalized size = 153.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x)
```

```
[Out] result too large to display
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                                +c*g^2 zero or nonze
ro?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^7} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{7}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**7,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**7, x)
```

3.207 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx$

Optimal. Leaf size=1062

$$\frac{\left(4c^2\left(5fg^2+h(2eg-51dh)\right)g^2-7h^2\left(\left(5fg^2+5ehg+9dh^2\right)b^2-2ah(10fg+7eh)b+24a^2fh^2\right)-2ch\left(3bg\left(8\right.\right.\right.}{840h\left(cg^2-bhg+ah^2\right)^3(g+hx)^5}$$

```
[Out] 1/384*(48*c^3*d*g^3-8*c^2*g*(3*b*g*(3*d*h+e*g)+a*(3*d*h^2-8*e*g*h+f*g^2))-b
*h*(24*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))+2*c*
(4*a^2*h^2*(-e*h+8*f*g)-2*a*b*h*(-3*d*h^2+13*e*g*h+13*f*g^2)+b^2*g*(21*d*h^
2+10*e*g*h+7*f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(3/2)/(a*h^2
-b*g*h+c*g^2)^4/(h*x+g)^4-1/7*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a
*h^2-b*g*h+c*g^2)/(h*x+g)^7+1/84*(2*c*g*(5*f*g^2+h*(-9*d*h+2*e*g))+h*(14*a*
h*(-e*h+2*f*g)-b*(-9*d*h^2-5*e*g*h+19*f*g^2)))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2
-b*g*h+c*g^2)^2/(h*x+g)^6+1/840*(4*c^2*g^2*(5*f*g^2+h*(-51*d*h+2*e*g))-7*h^
2*(24*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))-2*c*h
*(3*b*g*(-34*d*h^2-15*e*g*h+8*f*g^2)-2*a*h*(12*d*h^2-61*e*g*h+26*f*g^2)))*(
c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^5+1/2048*(-4*a*c+b^2)^2*
(48*c^3*d*g^3-8*c^2*g*(3*b*g*(3*d*h+e*g)+a*(3*d*h^2-8*e*g*h+f*g^2))-b*h*(24
*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))+2*c*(4*a^2
*h^2*(-e*h+8*f*g)-2*a*b*h*(-3*d*h^2+13*e*g*h+13*f*g^2)+b^2*g*(21*d*h^2+10*e
*g*h+7*f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^
(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(11/2)-1/1024*(-4*a*c+b^2)*(
48*c^3*d*g^3-8*c^2*g*(3*b*g*(3*d*h+e*g)+a*(3*d*h^2-8*e*g*h+f*g^2))-b*h*(24*
a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))+2*c*(4*a^2*
h^2*(-e*h+8*f*g)-2*a*b*h*(-3*d*h^2+13*e*g*h+13*f*g^2)+b^2*g*(21*d*h^2+10*e*
g*h+7*f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+
c*g^2)^5/(h*x+g)^2
```

Rubi [A] time = 3.00, antiderivative size = 1062, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1650, 834, 806, 720, 724, 206}

$$\frac{\left(4\left(5fg^4+h(2eg-51dh)g^2\right)c^2-2h\left(3bg\left(8fg^2-15ehg-34dh^2\right)-2ah\left(26fg^2-61ehg+12dh^2\right)\right)c-7h^2\left(\left(5\right.\right.\right.}{840h\left(cg^2-bhg+ah^2\right)^3(g+hx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -((b^2 - 4*a*c)*(48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - a*h*(8*e*g - 3*d*h) + 3*
b*g*(e*g + 3*d*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) + b^2*(5*
f*g^2 + h*(5*e*g + 9*d*h))) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h*(13*f*
g^2 + h*(13*e*g - 3*d*h)) + b^2*(7*f*g^3 + g*h*(10*e*g + 21*d*h))))*(b*g -
2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(1024*(c*g^2 - b*g*h + a*h^
2)^5*(g + h*x)^2) + ((48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - a*h*(8*e*g - 3*d*h)
 + 3*b*g*(e*g + 3*d*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) + b^
2*(5*f*g^2 + h*(5*e*g + 9*d*h))) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h*(
13*f*g^2 + h*(13*e*g - 3*d*h)) + b^2*(7*f*g^3 + g*h*(10*e*g + 21*d*h))))*(b
*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(384*(c*g^2 - b*g*h
+ a*h^2)^4*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))
/(7*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^7) + ((2*c*(5*f*g^3 + g*h*(2*e*g -
9*d*h)) - h*(19*b*f*g^2 - b*h*(5*e*g + 9*d*h) - 14*a*h*(2*f*g - e*h)))*(a +
 b*x + c*x^2)^(5/2))/(84*h*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^6) + ((4*c^2
*(5*f*g^4 + g^2*h*(2*e*g - 51*d*h)) - 7*h^2*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g
```

```
 + 7*e*h) + b^2*(5*f*g^2 + 5*e*g*h + 9*d*h^2)) - 2*c*h*(3*b*g*(8*f*g^2 - 15
*e*g*h - 34*d*h^2) - 2*a*h*(26*f*g^2 - 61*e*g*h + 12*d*h^2)))*(a + b*x + c*
x^2)^(5/2))/(840*h*(c*g^2 - b*g*h + a*h^2)^3*(g + h*x)^5) + ((b^2 - 4*a*c)^
2*(48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - a*h*(8*e*g - 3*d*h) + 3*b*g*(e*g + 3*d
*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) + b^2*(5*f*g^2 + h*(5*e
*g + 9*d*h))) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h*(13*f*g^2 + h*(13*e*
g - 3*d*h)) + b^2*(7*f*g^3 + g*h*(10*e*g + 21*d*h)))))*ArcTanh[(b*g - 2*a*h
+ (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/
(2048*(c*g^2 - b*g*h + a*h^2)^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
```

```
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{7h\left(cg^2-bgh+ah^2\right)(g+hx)^7} - \int \frac{\left(\frac{1}{2}\left(-14cdg+5beg+14afg-\frac{5bfg^2}{h}+\right.\right.}{}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{7h\left(cg^2-bgh+ah^2\right)(g+hx)^7} + \frac{\left(2c\left(5fg^3+gh(2eg-9dh)\right)\right.}{}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{7h\left(cg^2-bgh+ah^2\right)(g+hx)^7} + \frac{\left(2c\left(5fg^3+gh(2eg-9dh)\right)\right.}{}$$

$$= \frac{\left(48c^3dg^3-8c^2g\left(afg^2-ah(8eg-3dh)+3bg(eg+3dh)\right)-bh\left(24a^2\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(48c^3dg^3-8c^2g\left(afg^2-ah(8eg-3dh)+3bg(eg+3dh)\right)\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(48c^3dg^3-8c^2g\left(afg^2-ah(8eg-3dh)+3bg(eg+3dh)\right)\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(48c^3dg^3-8c^2g\left(afg^2-ah(8eg-3dh)+3bg(eg+3dh)\right)\right.}{}$$

Mathematica [A] time = 6.42, size = 1221, normalized size = 1.15

$$(a+x(b+cx))^{3/2}\left(-\frac{\left(\frac{1}{2}h(5bfg+4cdh-14afh)-\frac{1}{2}g(10cfg+4ceh-9bfh)\right)\left(cx^2+bx+a\right)^{5/2}}{7\left(cg^2-bhg+ah^2\right)(g+hx)^7}-\frac{-\frac{\left(2cg\left(5cfg^2-7fh(bg-ah)+2ch(eg-dh)\right)-ch\left(5bfg^2-bh(5eg+9dh)\right.\right.}{6\left(cg^2-bhg+ah^2\right)(g+hx)^6}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -1/2*(f*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(c*h*(g + h*x)^7) + ((a
 + x*(b + c*x))^(3/2)*(-1/7*(((h*(5*b*f*g + 4*c*d*h - 14*a*f*h))/2 - (g*(10*
c*f*g + 4*c*e*h - 9*b*f*h))/2)*(a + b*x + c*x^2)^(5/2))/((c*g^2 - b*g*h + a
*h^2)*(g + h*x)^7) - (-1/6*((2*c*g*(5*c*f*g^2 - 7*f*h*(b*g - a*h) + 2*c*h*(
e*g - d*h)) - c*h*(5*b*f*g^2 - b*h*(5*e*g + 9*d*h) + 14*h*(c*d*g - a*f*g +
a*e*h)))*(a + b*x + c*x^2)^(5/2))/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^6) - (
((c^2*g*(2*c*(5*f*g^3 + g*h*(2*e*g - 9*d*h)) - h*(19*b*f*g^2 - b*h*(5*e*g +
 9*d*h) - 14*a*h*(2*f*g - e*h))) - (c*h*(7*b^2*h*(5*f*g^2 + h*(5*e*g + 9*d*
h)) + 2*b*(5*c*f*g^3 - c*g*h*(40*e*g + 93*d*h) - 7*a*h^2*(10*f*g + 7*e*h))
+ 24*h*(7*c^2*d*g^2 + 7*a^2*f*h^2 - a*c*(2*f*g^2 - h*(9*e*g - 2*d*h)))))/2)
*(a + b*x + c*x^2)^(5/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) - ((-2*(a
*c^2*h*(2*c*(5*f*g^3 + g*h*(2*e*g - 9*d*h)) - h*(19*b*f*g^2 - b*h*(5*e*g +
9*d*h) - 14*a*h*(2*f*g - e*h))) + (c^2*g*(7*b^2*h*(5*f*g^2 + h*(5*e*g + 9*d
*h)) + 2*b*(5*c*f*g^3 - c*g*h*(40*e*g + 93*d*h) - 7*a*h^2*(10*f*g + 7*e*h))
 + 24*h*(7*c^2*d*g^2 + 7*a^2*f*h^2 - a*c*(2*f*g^2 - h*(9*e*g - 2*d*h)))))/2
) + b*(c^2*g*(2*c*(5*f*g^3 + g*h*(2*e*g - 9*d*h)) - h*(19*b*f*g^2 - b*h*(5*
e*g + 9*d*h) - 14*a*h*(2*f*g - e*h))) + (c*h*(7*b^2*h*(5*f*g^2 + h*(5*e*g +
 9*d*h)) + 2*b*(5*c*f*g^3 - c*g*h*(40*e*g + 93*d*h) - 7*a*h^2*(10*f*g + 7*e
*h)) + 24*h*(7*c^2*d*g^2 + 7*a^2*f*h^2 - a*c*(2*f*g^2 - h*(9*e*g - 2*d*h)))
))/2))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2
 - b*g*h + a*h^2)*(g + h*x)^4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g -
 b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + (
(b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*
g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g
^2 - 4*b*g*h + 4*a*h^2))))/(16*(c*g^2 - b*g*h + a*h^2))))/(2*(c*g^2 - b*g*h
 + a*h^2)))/(6*(c*g^2 - b*g*h + a*h^2)))/(7*(c*g^2 - b*g*h + a*h^2))))/(2*c
*h*(a + b*x + c*x^2)^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.18, size = 126612, normalized size = 119.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                                +c*g^2 zero or nonze
ro?
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**8,x)
```

```
[Out] Timed out
```

3.208 $\int (1+2x)^3\sqrt{2-x+3x^2}\,\left(1+3x+4x^2\right)\,dx$

Optimal. Leaf size=143

$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4+\frac{67}{378}\left(3x^2-x+2\right)^{3/2}(2x+1)^3+\frac{17}{105}\left(3x^2-x+2\right)^{3/2}(2x+1)^2-\frac{(26982x+75295)\left(3x^2\right.}{68040}$$

```
[Out] 17/105*(1+2*x)^2*(3*x^2-x+2)^(3/2)+67/378*(1+2*x)^3*(3*x^2-x+2)^(3/2)+2/21*
(1+2*x)^4*(3*x^2-x+2)^(3/2)-1/68040*(75295+26982*x)*(3*x^2-x+2)^(3/2)+12403
9/93312*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+5393/15552*(1-6*x)*(3*x^2-x+
2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4+\frac{67}{378}\left(3x^2-x+2\right)^{3/2}(2x+1)^3+\frac{17}{105}\left(3x^2-x+2\right)^{3/2}(2x+1)^2-\frac{(26982x+75295)\left(3x^2\right.}{68040}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^3*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (5393*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/15552 + (17*(1 + 2*x)^2*(2 - x + 3*x^2
)^(3/2))/105 + (67*(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2))/378 + (2*(1 + 2*x)^4*
(2 - x + 3*x^2)^(3/2))/21 - ((75295 + 26982*x)*(2 - x + 3*x^2)^(3/2))/68040
 + (124039*ArcSinh[(1 - 6*x)/Sqrt[23]])/(31104*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
```

```
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)dx = \frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2}+\frac{1}{84}\int (1+2x)^3(-32+268x)\sqrt{2$$

$$= \frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2}+$$

$$= \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+$$

$$= \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+$$

$$= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552}+\frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+$$

$$= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552}+\frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+$$

$$= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552}+\frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+$$

Mathematica [A] time = 0.05, size = 70, normalized size = 0.49

$$\frac{6\sqrt{3x^2-x+2}\left(2488320x^6+6462720x^5+7491456x^4+5497776x^3+3280872x^2+1493894x-543069\right)-4}{3265920}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^3*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(-543069 + 1493894*x + 3280872*x^2 + 5497776*x^3 + 7
491456*x^4 + 6462720*x^5 + 2488320*x^6) - 4341365*Sqrt[3]*ArcSinh[(-1 + 6*x
)/Sqrt[23]])/3265920
```

fricas [A] time = 0.75, size = 83, normalized size = 0.58

$$\frac{1}{544320}\left(2488320\,x^6+6462720\,x^5+7491456\,x^4+5497776\,x^3+3280872\,x^2+1493894\,x-543069\right)\sqrt{3\,x^2-x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/544320*(2488320*x^6 + 6462720*x^5 + 7491456*x^4 + 5497776*x^3 + 3280872*x
^2 + 1493894*x - 543069)*sqrt(3*x^2 - x + 2) + 124039/186624*sqrt(3)*log(4*
sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

giac [A] time = 0.26, size = 78, normalized size = 0.55

$$\frac{1}{544320}\left(2\left(12\left(6\left(8\left(30\left(72\,x+187\right)x+6503\right)x+38179\right)x+136703\right)x+746947\right)x-543069\right)\sqrt{3\,x^2-x+2}+\frac{124}{933}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/544320*(2*(12*(6*(8*(30*(72*x + 187)*x + 6503)*x + 38179)*x + 136703)*x +
 746947)*x - 543069)*sqrt(3*x^2 - x + 2) + 124039/93312*sqrt(3)*log(-2*sqrt
(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

maple [A] time = 0.02, size = 115, normalized size = 0.80

$$\frac{32\left(3x^2-x+2\right)^{\frac{3}{2}}x^4}{21}+\frac{844\left(3x^2-x+2\right)^{\frac{3}{2}}x^3}{189}+\frac{1594\left(3x^2-x+2\right)^{\frac{3}{2}}x^2}{315}+\frac{7849\left(3x^2-x+2\right)^{\frac{3}{2}}x}{3780}-\frac{124039\sqrt{3}\ \operatorname{arcsi}}{9331}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x)
```

```
[Out] 32/21*x^4*(3*x^2-x+2)^(3/2)+844/189*x^3*(3*x^2-x+2)^(3/2)+7849/3780*x*(3*x^
2-x+2)^(3/2)+1594/315*x^2*(3*x^2-x+2)^(3/2)-124039/93312*3^(1/2)*arcsinh(6/
23*23^(1/2)*(x-1/6))-5393/15552*(6*x-1)*(3*x^2-x+2)^(1/2)-45739/68040*(3*x^
2-x+2)^(3/2)
```

maxima [A] time = 0.95, size = 126, normalized size = 0.88

$$\frac{32}{21}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x^4+\frac{844}{189}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x^3+\frac{1594}{315}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x^2+\frac{7849}{3780}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x-\frac{45739}{68040}\left(3\,x^2-x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/21*(3*x^2 - x + 2)^(3/2)*x^4 + 844/189*(3*x^2 - x + 2)^(3/2)*x^3 + 1594/
315*(3*x^2 - x + 2)^(3/2)*x^2 + 7849/3780*(3*x^2 - x + 2)^(3/2)*x - 45739/6
8040*(3*x^2 - x + 2)^(3/2) - 5393/2592*sqrt(3*x^2 - x + 2)*x - 124039/93312
*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 5393/15552*sqrt(3*x^2 - x + 2)
```

mupad [B] time = 5.54, size = 170, normalized size = 1.19

$$\frac{1594\,x^2\left(3\,x^2-x+2\right)^{3/2}}{315}+\frac{844\,x^3\left(3\,x^2-x+2\right)^{3/2}}{189}+\frac{32\,x^4\left(3\,x^2-x+2\right)^{3/2}}{21}-\frac{137057\,\sqrt{3}\ \ln\left(\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,(}{}\right.}{136080}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] (1594*x^2*(3*x^2 - x + 2)^(3/2))/315 + (844*x^3*(3*x^2 - x + 2)^(3/2))/189
+ (32*x^4*(3*x^2 - x + 2)^(3/2))/21 - (137057*3^(1/2)*log((3*x^2 - x + 2)^(
1/2) + (3^(1/2)*(3*x - 1/2))/3))/136080 - (5959*(x/2 - 1/12)*(3*x^2 - x + 2
)^(1/2))/1890 - (45739*(3*x^2 - x + 2)^(1/2)*(72*x^2 - 6*x + 45))/1632960 +
 (7849*x*(3*x^2 - x + 2)^(3/2))/3780 - (1051997*3^(1/2)*log(2*(3*x^2 - x +
2)^(1/2) + (3^(1/2)*(6*x - 1))/3))/3265920
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)^3 \sqrt{3x^2-x+2} \left(4x^2+3x+1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)**3*sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1), x)
```

3.209 $\int (1+2x)^2\sqrt{2-x+3x^2}\,\left(1+3x+4x^2\right)\,dx$

Optimal. Leaf size=118

$$\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3+\frac{1}{5}\left(3x^2-x+2\right)^{3/2}(2x+1)^2+\frac{1}{810}(306x+25)\left(3x^2-x+2\right)^{3/2}+\frac{235(1-6x)\sqrt{3x^2-x+}}{1296}$$

[Out] 1/5*(1+2*x)^2*(3*x^2-x+2)^(3/2)+1/9*(1+2*x)^3*(3*x^2-x+2)^(3/2)+1/810*(25+306*x)*(3*x^2-x+2)^(3/2)+5405/7776*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+235/1296*(1-6*x)*(3*x^2-x+2)^(1/2)

Rubi [A] time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3+\frac{1}{5}\left(3x^2-x+2\right)^{3/2}(2x+1)^2+\frac{1}{810}(306x+25)\left(3x^2-x+2\right)^{3/2}+\frac{235(1-6x)\sqrt{3x^2-x+}}{1296}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)^2*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]

[Out] (235*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/1296 + ((1 + 2*x)^2*(2 - x + 3*x^2)^(3/2))/5 + ((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2))/9 + ((25 + 306*x)*(2 - x + 3*x^2)^(3/2))/810 + (5405*ArcSinh[(1 - 6*x)/Sqrt[23]])/(2592*Sqrt[3])

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) - 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
```

```
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)dx &= \frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{1}{72}\int (1+2x)^2(-12+216x)\sqrt{2-\\ &= \frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{\int (1\\ &= \frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{1}{810\\ &= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1\\ &= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1\\ &= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1\end{aligned}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.55

$$\frac{6\sqrt{3x^2-x+2}\left(17280x^5+35712x^4+33552x^3+22344x^2+14638x+5607\right)-27025\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{38880}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^2*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(5607 + 14638*x + 22344*x^2 + 33552*x^3 + 35712*x^4
+ 17280*x^5) - 27025*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/38880
```

fricas [A] time = 0.95, size = 78, normalized size = 0.66

$$\frac{1}{6480}\left(17280\,x^5+35712\,x^4+33552\,x^3+22344\,x^2+14638\,x+5607\right)\sqrt{3\,x^2-x+2}+\frac{5405}{15552}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6480*(17280*x^5 + 35712*x^4 + 33552*x^3 + 22344*x^2 + 14638*x + 5607)*sqr
t(3*x^2 - x + 2) + 5405/15552*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*
x - 1) - 72*x^2 + 24*x - 25)
```

giac [A] time = 0.30, size = 73, normalized size = 0.62

$$\frac{1}{6480}\,(2\,(12\,(6\,(8\,(15\,x+31)x+233)x+931)x+7319)x+5607)\sqrt{3\,x^2-x+2}+\frac{5405}{7776}\,\sqrt{3}\,\log\left(-2\,\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6480*(2*(12*(6*(8*(15*x + 31)*x + 233)*x + 931)*x + 7319)*x + 5607)*sqrt(
3*x^2 - x + 2) + 5405/7776*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 -
 x + 2)) + 1)
```

maple [A] time = 0.01, size = 98, normalized size = 0.83

$$\frac{8\left(3x^2-x+2\right)^{\frac{3}{2}}x^3}{9}+\frac{32\left(3x^2-x+2\right)^{\frac{3}{2}}x^2}{15}+\frac{83\left(3x^2-x+2\right)^{\frac{3}{2}}x}{45}-\frac{5405\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{7776}+\frac{277\left(3x^2-x+\right.}{810}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x)
```

```
[Out] 8/9*(3*x^2-x+2)^(3/2)*x^3+32/15*(3*x^2-x+2)^(3/2)*x^2+83/45*(3*x^2-x+2)^(3/
2)*x+277/810*(3*x^2-x+2)^(3/2)-235/1296*(6*x-1)*(3*x^2-x+2)^(1/2)-5405/7776
*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

maxima [A] time = 0.96, size = 109, normalized size = 0.92

$$\frac{8}{9}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x^3+\frac{32}{15}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x^2+\frac{83}{45}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x+\frac{277}{810}\left(3\,x^2-x+2\right)^{\frac{3}{2}}-\frac{235}{216}\,\sqrt{3\,x^2-x+2}\,x-\frac{5405}{7776}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*(3*x^2 - x + 2)^(3/2)*x^3 + 32/15*(3*x^2 - x + 2)^(3/2)*x^2 + 83/45*(3*
x^2 - x + 2)^(3/2)*x + 277/810*(3*x^2 - x + 2)^(3/2) - 235/216*sqrt(3*x^2 -
 x + 2)*x - 5405/7776*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 235/1296*s
qrt(3*x^2 - x + 2)
```

mupad [B] time = 5.15, size = 153, normalized size = 1.30

$$\frac{32\,x^2\left(3\,x^2-x+2\right)^{3/2}}{15}+\frac{8\,x^3\left(3\,x^2-x+2\right)^{3/2}}{9}-\frac{2783\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\left(3\,x-\frac{1}{2}\right)}{3}\right)}{3240}-\frac{121\left(\frac{x}{2}-\frac{1}{12}\right)\sqrt{3\,x^2-x}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] (32*x^2*(3*x^2 - x + 2)^(3/2))/15 + (8*x^3*(3*x^2 - x + 2)^(3/2))/9 - (2783
*3^(1/2)*log((3*x^2 - x + 2)^(1/2) + (3^(1/2)*(3*x - 1/2))/3))/3240 - (121*
(x/2 - 1/12)*(3*x^2 - x + 2)^(1/2))/45 + (277*(3*x^2 - x + 2)^(1/2)*(72*x^2
 - 6*x + 45))/19440 + (83*x*(3*x^2 - x + 2)^(3/2))/45 + (6371*3^(1/2)*log(2
*(3*x^2 - x + 2)^(1/2) + (3^(1/2)*(6*x - 1))/3))/38880
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)^2 \sqrt{3x^2-x+2}\left(4x^2+3x+1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)**2*sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1), x)
```

3.210 $\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)dx$

Optimal. Leaf size=93

$$\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2+\frac{(738x+745)\left(3x^2-x+2\right)^{3/2}}{1620}+\frac{19(1-6x)\sqrt{3x^2-x+2}}{2592}+\frac{437\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{5184\sqrt{3}}$$

```
[Out] 2/15*(1+2*x)^2*(3*x^2-x+2)^(3/2)+1/1620*(745+738*x)*(3*x^2-x+2)^(3/2)+437/1
5552*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+19/2592*(1-6*x)*(3*x^2-x+2)^(1/
2)
```

Rubi [A] time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1653, 779, 612, 619, 215}

$$\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2+\frac{(738x+745)\left(3x^2-x+2\right)^{3/2}}{1620}+\frac{19(1-6x)\sqrt{3x^2-x+2}}{2592}+\frac{437\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{5184\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (19*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/2592 + (2*(1 + 2*x)^2*(2 - x + 3*x^2)^(3
/2))/15 + ((745 + 738*x)*(2 - x + 3*x^2)^(3/2))/1620 + (437*ArcSinh[(1 - 6*
x)/Sqrt[23]])/(5184*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
```

```
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)dx = \frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{60}\int (1+2x)(8+164x)\sqrt{2-x+\ldots}$$

$$= \frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{(745+738x)\left(2-x+3x^2\right)^{3/2}}{1620}-\ldots$$

$$= \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592}+\frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{(745\ldots}{\ldots}$$

$$= \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592}+\frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{(745\ldots}{\ldots}$$

$$= \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592}+\frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{(745\ldots}{\ldots}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.65

$$\frac{6\sqrt{3x^2-x+2}\left(20736x^4+31536x^3+24072x^2+17374x+15471\right)-2185\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{77760}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(15471 + 17374*x + 24072*x^2 + 31536*x^3 + 20736*x^4
) - 2185*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/77760
```

fricas [A] time = 0.89, size = 73, normalized size = 0.78

$$\frac{1}{12960}\left(20736\,x^4+31536\,x^3+24072\,x^2+17374\,x+15471\right)\sqrt{3\,x^2-x+2}+\frac{437}{31104}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12960*(20736*x^4 + 31536*x^3 + 24072*x^2 + 17374*x + 15471)*sqrt(3*x^2 -
x + 2) + 437/31104*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72
*x^2 + 24*x - 25)
```

giac [A] time = 0.26, size = 68, normalized size = 0.73

$$\frac{1}{12960}\left(2\left(12\left(18\left(48\,x+73\right)x+1003\right)x+8687\right)x+15471\right)\sqrt{3\,x^2-x+2}+\frac{437}{15552}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

[Out] 1/12960*(2*(12*(18*(48*x + 73)*x + 1003)*x + 8687)*x + 15471)*sqrt(3*x^2 - x + 2) + 437/15552*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)

maple [A] time = 0.01, size = 81, normalized size = 0.87

$$\frac{8\left(3x^2-x+2\right)^{\frac{3}{2}}x^2}{15}+\frac{89\left(3x^2-x+2\right)^{\frac{3}{2}}x}{90}-\frac{437\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{15552}+\frac{961\left(3x^2-x+2\right)^{\frac{3}{2}}}{1620}-\frac{19\left(6x-1\right)\sqrt{3x^2-}}{2592}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x+1)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x)

[Out] 8/15*(3*x^2-x+2)^(3/2)*x^2+89/90*(3*x^2-x+2)^(3/2)*x+961/1620*(3*x^2-x+2)^(3/2)-19/2592*(6*x-1)*(3*x^2-x+2)^(1/2)-437/15552*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))

maxima [A] time = 0.96, size = 92, normalized size = 0.99

$$\frac{8}{15}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x^2+\frac{89}{90}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x+\frac{961}{1620}\left(3\,x^2-x+2\right)^{\frac{3}{2}}-\frac{19}{432}\sqrt{3\,x^2-x+2}\,x-\frac{437}{15552}\sqrt{3}\ \operatorname{arsinh}\left(\frac{1}{23}\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")

[Out] 8/15*(3*x^2 - x + 2)^(3/2)*x^2 + 89/90*(3*x^2 - x + 2)^(3/2)*x + 961/1620*(3*x^2 - x + 2)^(3/2) - 19/432*sqrt(3*x^2 - x + 2)*x - 437/15552*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 19/2592*sqrt(3*x^2 - x + 2)

mupad [B] time = 4.88, size = 136, normalized size = 1.46

$$\frac{8\,x^2\left(3\,x^2-x+2\right)^{3/2}}{15}-\frac{253\sqrt{3}\ \ln\left(\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\left(3\,x-\frac{1}{2}\right)}{3}\right)}{810}-\frac{44\left(\frac{x}{2}-\frac{1}{12}\right)\sqrt{3\,x^2-x+2}}{45}+\frac{961\sqrt{3\,x^2-x+2}\ (}{388}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 1)*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)

[Out] (8*x^2*(3*x^2 - x + 2)^(3/2))/15 - (253*3^(1/2)*log((3*x^2 - x + 2)^(1/2) + (3^(1/2)*(3*x - 1/2))/3))/810 - (44*(x/2 - 1/12)*(3*x^2 - x + 2)^(1/2))/45 + (961*(3*x^2 - x + 2)^(1/2)*(72*x^2 - 6*x + 45))/38880 + (89*x*(3*x^2 - x + 2)^(3/2))/90 + (22103*3^(1/2)*log(2*(3*x^2 - x + 2)^(1/2) + (3^(1/2)*(6*x - 1))/3))/77760

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x+1\right)\sqrt{3x^2-x+2}\left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)

[Out] Integral((2*x + 1)*sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1), x)

3.211 $\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{1+2x} \, dx$

Optimal. Leaf size=101

$$\frac{2}{9}\left(3x^2-x+2\right)^{3/2}+\frac{1}{72}(30x+13)\sqrt{3x^2-x+2}-\frac{1}{8}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)-\frac{43\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}}$$

```
[Out] 2/9*(3*x^2-x+2)^(3/2)-43/432*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1/8*arc
tanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+1/72*(13+30*x)*(3*x^
2-x+2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{2}{9}\left(3x^2-x+2\right)^{3/2}+\frac{1}{72}(30x+13)\sqrt{3x^2-x+2}-\frac{1}{8}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)-\frac{43\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] ((13 + 30*x)*Sqrt[2 - x + 3*x^2])/72 + (2*(2 - x + 3*x^2)^(3/2))/9 - (43*Ar
cSinh[(1 - 6*x)/Sqrt[23]])/(144*Sqrt[3]) - (Sqrt[13]*ArcTanh[(9 - 8*x)/(2*S
qrt[13]*Sqrt[2 - x + 3*x^2])])/8
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
```

```
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-x+3x^2}\,\left(1+3x+4x^2\right)}{1+2x}\,dx &= \frac{2}{9}\left(2-x+3x^2\right)^{3/2}+\frac{1}{36}\int \frac{(48+60x)\sqrt{2-x+3x^2}}{1+2x}\,dx\\ &= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2}+\frac{2}{9}\left(2-x+3x^2\right)^{3/2}-\frac{\int \frac{-3324-1032x}{(1+2x)\sqrt{2-x+3x^2}}\,dx}{1728}\\ &= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2}+\frac{2}{9}\left(2-x+3x^2\right)^{3/2}+\frac{43}{144}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx\\ &= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2}+\frac{2}{9}\left(2-x+3x^2\right)^{3/2}-\frac{13}{4}\,\mathrm{Subst}\left(\int \frac{1}{52-x^2}\,dx\right.\\ &= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2}+\frac{2}{9}\left(2-x+3x^2\right)^{3/2}-\frac{43\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}}-\frac{1}{8}\sqrt{1}\end{aligned}$$

Mathematica [A] time = 0.05, size = 86, normalized size = 0.85

$$\frac{1}{432}\left(6\sqrt{3x^2-x+2}\,\left(48x^2+14x+45\right)-54\sqrt{13}\,\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\,\sqrt{3x^2-x+2}}\right)+43\sqrt{3}\,\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(45 + 14*x + 48*x^2) + 43*Sqrt[3]*ArcSinh[(-1 + 6*x)
/Sqrt[23]] - 54*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])
])/432
```

fricas [A] time = 0.91, size = 115, normalized size = 1.14

$$\frac{1}{72}\left(48\,x^2+14\,x+45\right)\sqrt{3\,x^2-x+2}+\frac{43}{864}\,\sqrt{3}\,\log\left(-4\,\sqrt{3}\,\sqrt{3\,x^2-x+2}\,(6\,x-1)-72\,x^2+24\,x-25\right)+\frac{1}{16}\,\sqrt{1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="fricas")
```

```
[Out] 1/72*(48*x^2 + 14*x + 45)*sqrt(3*x^2 - x + 2) + 43/864*sqrt(3)*log(-4*sqrt(
3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 1/16*sqrt(13)*log(
-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2
+ 4*x + 1))
```

giac [A] time = 0.39, size = 126, normalized size = 1.25

$$\frac{1}{72}\,(2\,(24\,x+7)x+45)\sqrt{3\,x^2-x+2}-\frac{43}{432}\,\sqrt{3}\,\log\left(-6\,\sqrt{3}\,x+\sqrt{3}+6\,\sqrt{3\,x^2-x+2}\right)+\frac{1}{8}\,\sqrt{13}\,\log\left(-\frac{\left|-4\,\sqrt{3}\,x\right.}{2\left(2\,\sqrt{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="giac")
```

```
[Out] 1/72*(2*(24*x + 7)*x + 45)*sqrt(3*x^2 - x + 2) - 43/432*sqrt(3)*log(-6*sqrt
(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 1/8*sqrt(13)*log(-1/2*abs(-4*sqr
t(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqr
t(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

maple [A] time = 0.01, size = 95, normalized size = 0.94

$$\frac{43\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{432}-\frac{\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{8}+\frac{2\left(3x^2-x+2\right)^{\frac{3}{2}}}{9}+\frac{5\,(6x-1)\,\sqrt{3x^2-x+2}}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(2*x+1),x)
```

```
[Out] 2/9*(3*x^2-x+2)^(3/2)+5/72*(6*x-1)*(3*x^2-x+2)^(1/2)+43/432*3^(1/2)*arcsinh
(6/23*23^(1/2)*(x-1/6))+1/8*(12*(x+1/2)^2-16*x+5)^(1/2)-1/8*13^(1/2)*arctan
h(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

maxima [A] time = 0.96, size = 96, normalized size = 0.95

$$\frac{2}{9}\left(3\,x^2-x+2\right)^{\frac{3}{2}}+\frac{5}{12}\,\sqrt{3\,x^2-x+2}\,x+\frac{43}{432}\,\sqrt{3}\ \operatorname{arsinh}\left(\frac{6}{23}\,\sqrt{23}\,x-\frac{1}{23}\,\sqrt{23}\right)+\frac{1}{8}\,\sqrt{13}\ \operatorname{arsinh}\left(\frac{8\,\sqrt{23}\,x}{23\,|2\,x+1|}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="maxima")
```

```
[Out] 2/9*(3*x^2 - x + 2)^(3/2) + 5/12*sqrt(3*x^2 - x + 2)*x + 43/432*sqrt(3)*arc
sinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1/8*sqrt(13)*arcsinh(8/23*sqrt(23)*
x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 13/72*sqrt(3*x^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3\,x^2-x+2}\,\left(4\,x^2+3\,x+1\right)}{2\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
[Out] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3x^2-x+2}\left(4x^2+3x+1\right)}{2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x),x)
[Out] Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

3.212 $\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx$

Optimal. Leaf size=108

$$-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}-\frac{1}{156}(67-96x)\sqrt{3x^2-x+2}+\frac{17\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8\sqrt{13}}-\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

```
[Out] -1/13*(3*x^2-x+2)^(3/2)/(1+2*x)-11/18*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2
)+17/104*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1/156*(6
7-96*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}-\frac{1}{156}(67-96x)\sqrt{3x^2-x+2}+\frac{17\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8\sqrt{13}}-\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] -((67 - 96*x)*Sqrt[2 - x + 3*x^2])/156 - (2 - x + 3*x^2)^(3/2)/(13*(1 + 2*x
)) - (11*ArcSinh[(1 - 6*x)/Sqrt[23]])/(6*Sqrt[3]) + (17*ArcTanh[(9 - 8*x)/(
2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(8*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
```

```
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx = -\frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)} - \frac{1}{13}\int \frac{\left(-\frac{15}{2}-32x\right)\sqrt{2-x+3x^2}}{1+2x}\,dx$$

$$= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2} - \frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)} + \frac{1}{624}\int \frac{-182+228}{(1+2x)\sqrt{2-x}}$$

$$= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2} - \frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)} + \frac{11}{6}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx$$

$$= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2} - \frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)} + \frac{17}{4}\text{Subst}\left(\int \frac{1}{52-x^2}\,d\right.$$

$$= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2} - \frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)} - \frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}} + \frac{17\,\mathrm{t}}{}$$

Mathematica [A] time = 0.09, size = 92, normalized size = 0.85

$$\frac{\sqrt{3x^2-x+2}\left(12x^2-2x-7\right)}{24x+12} + \frac{17\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8\sqrt{13}} + \frac{11\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] (Sqrt[2 - x + 3*x^2]*(-7 - 2*x + 12*x^2))/(12 + 24*x) + (11*ArcSinh[(-1 + 6
*x)/Sqrt[23]])/(6*Sqrt[3]) + (17*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x +
 3*x^2])])/(8*Sqrt[13])
```

fricas [A] time = 0.91, size = 133, normalized size = 1.23

$$\frac{572\sqrt{3}\,(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)+153\sqrt{13}\,(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+\cdots}}{\cdots}\right)\cdots}{1872\,(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/1872*(572*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
- 72*x^2 + 24*x - 25) + 153*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2 -
 x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 156*(12*x^2
 - 2*x - 7)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

giac [B] time = 0.72, size = 380, normalized size = 3.52

$$\frac{17}{104}\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)-\frac{11}{18}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\cdots}\right.}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2x}\cdots}\right.}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 17/104*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqr
t(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 11/18*sqrt(3)*log(1/2*abs(-2*sqrt(
3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(sqr
t(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))*sgn(1
/(2*x + 1)) - 1/8*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1/(2*x + 1))
+ 1/12*(67*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^3
*sgn(1/(2*x + 1)) - 57*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) +
sqrt(13)/(2*x + 1))^2*sgn(1/(2*x + 1)) + 129*(sqrt(-8/(2*x + 1) + 13/(2*x +
 1)^2 + 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 27*sqrt(13)*sgn(1/(2*x
+ 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^2 -
3)^2
```

maple [A] time = 0.01, size = 123, normalized size = 1.14

$$\frac{11\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{18}+\frac{17\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{104}+\frac{(6x-1)\sqrt{3x^2-x+2}}{12}-\frac{17\sqrt{-16x+12\left(\cdots\right.}}{104}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(2*x+1)^2,x)
```

```
[Out] 1/12*(6*x-1)*(3*x^2-x+2)^(1/2)+11/18*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
-17/104*(-16*x+12*(x+1/2)^2+5)^(1/2)+17/104*13^(1/2)*arctanh(2/13*(-4*x+9/2
)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))-1/26/(x+1/2)*(3*(x+1/2)^2-4*x+5/4)
^(3/2)+1/52*(6*x-1)*(3*(x+1/2)^2-4*x+5/4)^(1/2)
```

maxima [A] time = 0.97, size = 103, normalized size = 0.95

$$\frac{1}{2}\sqrt{3x^2-x+2}\,x+\frac{11}{18}\sqrt{3}\ \operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{17}{104}\sqrt{13}\ \operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right)-\frac{1}{3}\sqrt{3x\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 - x + 2)*x + 11/18*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sq
rt(23)) - 17/104*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(
23)/abs(2*x + 1)) - 1/3*sqrt(3*x^2 - x + 2) - 1/4*sqrt(3*x^2 - x + 2)/(2*x
+ 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3\,x^2-x+2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
```

```
[Out] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3x^{2}-x+2}\,\left(4x^{2}+3x+1\right)}{\left(2x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x)**2,x)
```

```
[Out] Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

3.213 $\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx$

Optimal. Leaf size=115

$$-\frac{\left(3x^2-x+2\right)^{3/2}}{26(2x+1)^2}+\frac{11(10x+7)\sqrt{3x^2-x+2}}{104(2x+1)}-\frac{803\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{208\sqrt{13}}+\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}}$$

```
[Out] -1/26*(3*x^2-x+2)^(3/2)/(1+2*x)^2+11/24*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1
/2)-803/2704*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+11/1
04*(7+10*x)*(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 812, 843, 619, 215, 724, 206}

$$-\frac{\left(3x^2-x+2\right)^{3/2}}{26(2x+1)^2}+\frac{11(10x+7)\sqrt{3x^2-x+2}}{104(2x+1)}-\frac{803\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{208\sqrt{13}}+\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] (11*(7 + 10*x)*Sqrt[2 - x + 3*x^2])/(104*(1 + 2*x)) - (2 - x + 3*x^2)^(3/2)
/(26*(1 + 2*x)^2) + (11*ArcSinh[(1 - 6*x)/Sqrt[23]])/(8*Sqrt[3]) - (803*Arc
Tanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(208*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx = -\frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2}-\frac{1}{26}\int \frac{\left(-\frac{33}{2}-55x\right)\sqrt{2-x+3x^2}}{(1+2x)^2}\,dx$$

$$= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)}-\frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2}+\frac{1}{208}\int \frac{517-572x}{(1+2x)\sqrt{2-x+3x^2}}$$

$$= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)}-\frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2}-\frac{11}{8}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx+\frac{8}{2}$$

$$= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)}-\frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2}-\frac{803}{104}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,$$

$$= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)}-\frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2}+\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}}-\frac{803\tan}{}$$

Mathematica [A] time = 0.09, size = 93, normalized size = 0.81

$$\frac{\frac{78\sqrt{3x^2-x+2}\left(208x^2+268x+69\right)}{(2x+1)^2}-2409\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)-3718\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{8112}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((78*Sqrt[2 - x + 3*x^2]*(69 + 268*x + 208*x^2))/(1 + 2*x)^2 - 3718*Sqrt[3]
*ArcSinh[(-1 + 6*x)/Sqrt[23]] - 2409*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]
*Sqrt[2 - x + 3*x^2])])/8112
```

fricas [A] time = 0.85, size = 149, normalized size = 1.30

$$\frac{3718\sqrt{3}\left(4x^2+4x+1\right)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+2409\sqrt{13}\left(4x^2+4x+1\right)\log}{16224\left(4x^2+4x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/16224*(3718*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(
6*x - 1) - 72*x^2 + 24*x - 25) + 2409*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*sq
rt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x
+ 1)) + 156*(208*x^2 + 268*x + 69)*sqrt(3*x^2 - x + 2))/(4*x^2 + 4*x + 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{-389344,[6]%%%}+%%%{%%{[1168032,0]:[1,0,-3]%%},[5]%%%}+%%%{-584016,[
4]%%%}+%%%{%%{[-4672128,0]:[1,0,-3]%%},[3]%%%}+%%%{1460040,[2]%%%}+%%%{%%{[
7300200,0]:[1,0,-3]%%},[1]%%%}+%%%{6083500,[0]%%%} / %%%{%%{[24,0]:[1,0,-3]
%%},[6]%%%}+%%%{-216,[5]%%%}+%%%{%%{[36,0]:[1,0,-3]%%},[4]%%%}+%%%{864,[3]%
%%}+%%%{%%{[-90,0]:[1,0,-3]%%},[2]%%%}+%%%{-1350,[1]%%%}+%%%{%%{[-375,0]:[1
,0,-3]%%},[0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.01, size = 125, normalized size = 1.09

$$-\frac{11\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{24}-\frac{803\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{2704}+\frac{803\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}{2704}+\frac{11\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(2*x+1)^3,x)
```

```
[Out] 803/2704*(-16*x+12*(x+1/2)^2+5)^(1/2)-11/24*3^(1/2)*arcsinh(6/23*23^(1/2)*(
x-1/6))-803/2704*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2
)^2+5)^(1/2))+11/338/(x+1/2)*(-4*x+3*(x+1/2)^2+5/4)^(3/2)-11/676*(6*x-1)*(-
4*x+3*(x+1/2)^2+5/4)^(1/2)-1/104/(x+1/2)^2*(-4*x+3*(x+1/2)^2+5/4)^(3/2)
```

maxima [A] time = 0.99, size = 114, normalized size = 0.99

$$-\frac{11}{24}\sqrt{3}\ \operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{803}{2704}\sqrt{13}\ \operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right)+\frac{55}{104}\sqrt{3x^2-x+2}-\frac{}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="maxima")
```

```
[Out] -11/24*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 803/2704*sqrt(13)
*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 55/10
```

```
4*sqrt(3*x^2 - x + 2) - 1/26*(3*x^2 - x + 2)^(3/2)/(4*x^2 + 4*x + 1) + 11/5
2*sqrt(3*x^2 - x + 2)/(2*x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3\,x^2-x+2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
[Out] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3x^2-x+2}\,\left(4x^2+3x+1\right)}{\left(2x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x)**3,x)
[Out] Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

3.214 $\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx$

Optimal. Leaf size=158

$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4+\frac{913}{486}x^2\left(3x^2-x+2\right)^{5/2}-\frac{11(283-5850x)\left(3x^2-x+2\right)^{5/2}}{58320}+\frac{54593(1-6x)\left(3x^2-\right.}{559872}$$

```
[Out] 54593/559872*(1-6*x)*(3*x^2-x+2)^(3/2)-11/58320*(283-5850*x)*(3*x^2-x+2)^(5
/2)+913/486*x^2*(3*x^2-x+2)^(5/2)+77/81*x^3*(3*x^2-x+2)^(5/2)+2/27*(1+2*x)^
4*(3*x^2-x+2)^(5/2)+28879697/26873856*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2
)+1255639/4478976*(1-6*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1653, 12, 779, 612, 619, 215}

$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4+\frac{77}{81}x^3\left(3x^2-x+2\right)^{5/2}+\frac{913}{486}x^2\left(3x^2-x+2\right)^{5/2}-\frac{11(283-5850x)\left(3x^2-x+2\right)^{5/2}}{58320}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (1255639*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/4478976 + (54593*(1 - 6*x)*(2 - x +
 3*x^2)^(3/2))/559872 - (11*(283 - 5850*x)*(2 - x + 3*x^2)^(5/2))/58320 + (
913*x^2*(2 - x + 3*x^2)^(5/2))/486 + (77*x^3*(2 - x + 3*x^2)^(5/2))/81 + (2
*(1 + 2*x)^4*(2 - x + 3*x^2)^(5/2))/27 + (28879697*ArcSinh[(1 - 6*x)/Sqrt[2
3]])/(8957952*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
```

```
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx = \frac{2}{27}(1+2x)^4 \left(2-x+3x^2\right)^{5/2} + \frac{1}{108}\int 308x(1+2x)^3 \left(2-x+3x\right.$$

$$= \frac{2}{27}(1+2x)^4 \left(2-x+3x^2\right)^{5/2} + \frac{77}{27}\int x(1+2x)^3 \left(2-x+3x^2\right)^{3/2}$$

$$= \frac{77}{81}x^3 \left(2-x+3x^2\right)^{5/2} + \frac{2}{27}(1+2x)^4 \left(2-x+3x^2\right)^{5/2} + \frac{77}{648}\int x$$

$$= \frac{913}{486}x^2 \left(2-x+3x^2\right)^{5/2} + \frac{77}{81}x^3 \left(2-x+3x^2\right)^{5/2} + \frac{2}{27}(1+2x)^4 \left(\right.$$

$$= -\frac{11(283-5850x)\left(2-x+3x^2\right)^{5/2}}{58320} + \frac{913}{486}x^2 \left(2-x+3x^2\right)^{5/2} +$$

$$= \frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872} - \frac{11(283-5850x)\left(2-x+3x^2\right)}{58320}$$

$$= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872}$$

$$= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872}$$

$$= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872}$$

Mathematica [**A**] time = 0.05, size = 80, normalized size = 0.51

$$\frac{6\sqrt{3x^2-x+2}\left(238878720x^8+510105600x^7+635765760x^6+711210240x^5+649452672x^4+421626672x^3+\right.}{134369280}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(12499587 + 84014278*x + 201289704*x^2 + 421626672*x
^3 + 649452672*x^4 + 711210240*x^5 + 635765760*x^6 + 510105600*x^7 + 238878
720*x^8) - 144398485*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/134369280
```

fricas [A] time = 0.91, size = 93, normalized size = 0.59

$$\frac{1}{22394880}\left(238878720\,x^8+510105600\,x^7+635765760\,x^6+711210240\,x^5+649452672\,x^4+421626672\,x^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/22394880*(238878720*x^8 + 510105600*x^7 + 635765760*x^6 + 711210240*x^5 +
 649452672*x^4 + 421626672*x^3 + 201289704*x^2 + 84014278*x + 12499587)*sqr
t(3*x^2 - x + 2) + 28879697/53747712*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x +
 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

giac [A] time = 0.26, size = 88, normalized size = 0.56

$$\frac{1}{22394880}\,(2\,(12\,(6\,(8\,(30\,(36\,(2\,(96\,x+205)x+511)x+20579)x+563761)x+2927963)x+8387071)x+420$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/22394880*(2*(12*(6*(8*(30*(36*(2*(96*x + 205)*x + 511)*x + 20579)*x + 563
761)*x + 2927963)*x + 8387071)*x + 42007139)*x + 12499587)*sqrt(3*x^2 - x +
 2) + 28879697/26873856*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x
+ 2)) + 1)
```

maple [A] time = 0.02, size = 134, normalized size = 0.85

$$\frac{32\left(3x^2-x+2\right)^{\frac{5}{2}}x^4}{27}+\frac{269\left(3x^2-x+2\right)^{\frac{5}{2}}x^3}{81}+\frac{1777\left(3x^2-x+2\right)^{\frac{5}{2}}x^2}{486}+\frac{1099\left(3x^2-x+2\right)^{\frac{5}{2}}x}{648}-\frac{28879697\sqrt{3}\,a}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x)
```

```
[Out] 32/27*x^4*(3*x^2-x+2)^(5/2)+269/81*x^3*(3*x^2-x+2)^(5/2)+1777/486*x^2*(3*x^
2-x+2)^(5/2)+1099/648*x*(3*x^2-x+2)^(5/2)-54593/559872*(6*x-1)*(3*x^2-x+2)^
(3/2)-28879697/26873856*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-1255639/4478
976*(6*x-1)*(3*x^2-x+2)^(1/2)+1207/58320*(3*x^2-x+2)^(5/2)
```

maxima [A] time = 0.98, size = 155, normalized size = 0.98

$$\frac{32}{27}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^4+\frac{269}{81}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^3+\frac{1777}{486}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^2+\frac{1099}{648}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x+\frac{1207}{58320}\left(3\,x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 32/27*(3*x^2 - x + 2)^(5/2)*x^4 + 269/81*(3*x^2 - x + 2)^(5/2)*x^3 + 1777/4
86*(3*x^2 - x + 2)^(5/2)*x^2 + 1099/648*(3*x^2 - x + 2)^(5/2)*x + 1207/5832
0*(3*x^2 - x + 2)^(5/2) - 54593/93312*(3*x^2 - x + 2)^(3/2)*x + 54593/55987
2*(3*x^2 - x + 2)^(3/2) - 1255639/746496*sqrt(3*x^2 - x + 2)*x - 28879697/2
6873856*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 1255639/4478976*sqrt(3*x
^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(2\,x+1\right)}^3\,{\left(3\,x^2-x+2\right)}^{3/2}\,\left(4\,x^2+3\,x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
[Out] int((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)^3 \left(3x^2-x+2\right)^{\frac{3}{2}} \left(4x^2+3x+1\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
[Out] Integral((2*x + 1)**3*(3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1), x)
```

3.215 $\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx$

Optimal. Leaf size=141

$$\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3+\frac{8}{63}\left(3x^2-x+2\right)^{5/2}(2x+1)^2+\frac{13(50x+29)\left(3x^2-x+2\right)^{5/2}}{2520}+\frac{91(1-6x)\left(3x^2-x\right.}{3456}$$

```
[Out] 91/3456*(1-6*x)*(3*x^2-x+2)^(3/2)+8/63*(1+2*x)^2*(3*x^2-x+2)^(5/2)+1/12*(1+
2*x)^3*(3*x^2-x+2)^(5/2)+13/2520*(29+50*x)*(3*x^2-x+2)^(5/2)+48139/165888*a
rcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2093/27648*(1-6*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3+\frac{8}{63}\left(3x^2-x+2\right)^{5/2}(2x+1)^2+\frac{13(50x+29)\left(3x^2-x+2\right)^{5/2}}{2520}+\frac{91(1-6x)\left(3x^2-x\right.}{3456}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (2093*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/27648 + (91*(1 - 6*x)*(2 - x + 3*x^2)^
(3/2))/3456 + (8*(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2))/63 + ((1 + 2*x)^3*(2 -
x + 3*x^2)^(5/2))/12 + (13*(29 + 50*x)*(2 - x + 3*x^2)^(5/2))/2520 + (48139
*ArcSinh[(1 - 6*x)/Sqrt[23]])/(55296*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
```

```
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^2\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)dx = \frac{1}{12}(1+2x)^3\left(2-x+3x^2\right)^{5/2}+\frac{1}{96}\int (1+2x)^2(20+256x)\left(2-\right.$$

$$= \frac{8}{63}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{12}(1+2x)^3\left(2-x+3x^2\right)^{5/2}+$$

$$= \frac{8}{63}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{12}(1+2x)^3\left(2-x+3x^2\right)^{5/2}+$$

$$= \frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456}+\frac{8}{63}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{12}$$

$$= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648}+\frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456}+\frac{8}{63}($$

$$= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648}+\frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456}+\frac{8}{63}($$

$$= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648}+\frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456}+\frac{8}{63}($$

Mathematica [A] time = 0.05, size = 75, normalized size = 0.53

$$\frac{6\sqrt{3x^2-x+2}\left(5806080x^7+9262080x^6+10656000x^5+12173952x^4+10119792x^3+5694024x^2+2735918x\right.}{5806080}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(1517367 + 2735918*x + 5694024*x^2 + 10119792*x^3 +
12173952*x^4 + 10656000*x^5 + 9262080*x^6 + 5806080*x^7) - 1684865*Sqrt[3]*
ArcSinh[(-1 + 6*x)/Sqrt[23]])/5806080
```

fricas [A] time = 0.91, size = 88, normalized size = 0.62

$$\frac{1}{967680}\left(5806080\,x^7+9262080\,x^6+10656000\,x^5+12173952\,x^4+10119792\,x^3+5694024\,x^2+2735918\,x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/967680*(5806080*x^7 + 9262080*x^6 + 10656000*x^5 + 12173952*x^4 + 1011979
2*x^3 + 5694024*x^2 + 2735918*x + 1517367)*sqrt(3*x^2 - x + 2) + 48139/3317
76*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25
)
```

giac [A] time = 0.27, size = 83, normalized size = 0.59

$$\frac{1}{967680}\,(2\,(12\,(2\,(8\,(30\,(12\,(42\,x+67)x+925)x+31703)x+210829)x+237251)x+1367959)x+1517367)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/967680*(2*(12*(2*(8*(30*(12*(42*x + 67)*x + 925)*x + 31703)*x + 210829)*x
 + 237251)*x + 1367959)*x + 1517367)*sqrt(3*x^2 - x + 2) + 48139/165888*sqr
t(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

maple [A] time = 0.01, size = 117, normalized size = 0.83

$$\frac{2\left(3x^2-x+2\right)^{\frac{5}{2}}x^3}{3}+\frac{95\left(3x^2-x+2\right)^{\frac{5}{2}}x^2}{63}+\frac{319\left(3x^2-x+2\right)^{\frac{5}{2}}x}{252}-\frac{48139\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{165888}-\frac{91\,(6x-1)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x)
```

```
[Out] 2/3*(3*x^2-x+2)^(5/2)*x^3+95/63*(3*x^2-x+2)^(5/2)*x^2+319/252*(3*x^2-x+2)^(
5/2)*x-91/3456*(6*x-1)*(3*x^2-x+2)^(3/2)-48139/165888*3^(1/2)*arcsinh(6/23*
23^(1/2)*(x-1/6))-2093/27648*(6*x-1)*(3*x^2-x+2)^(1/2)+907/2520*(3*x^2-x+2)
^(5/2)
```

maxima [A] time = 0.93, size = 138, normalized size = 0.98

$$\frac{2}{3}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^3+\frac{95}{63}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^2+\frac{319}{252}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x+\frac{907}{2520}\left(3\,x^2-x+2\right)^{\frac{5}{2}}-\frac{91}{576}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 2/3*(3*x^2 - x + 2)^(5/2)*x^3 + 95/63*(3*x^2 - x + 2)^(5/2)*x^2 + 319/252*(
3*x^2 - x + 2)^(5/2)*x + 907/2520*(3*x^2 - x + 2)^(5/2) - 91/576*(3*x^2 - x
 + 2)^(3/2)*x + 91/3456*(3*x^2 - x + 2)^(3/2) - 2093/4608*sqrt(3*x^2 - x +
2)*x - 48139/165888*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 2093/27648*s
qrt(3*x^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2\,x+1)^2\,\left(3\,x^2-x+2\right)^{3/2}\,\left(4\,x^2+3\,x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] int((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)^2 \left(3x^2-x+2\right)^{\frac{3}{2}} \left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
```

```
[Out] Integral((2*x + 1)**2*(3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1), x)
```

3.216 $\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx$

Optimal. Leaf size=116

$$\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}+\frac{1}{378}(102x+109)\left(3x^2-x+2\right)^{5/2}-\frac{71(1-6x)\left(3x^2-x+2\right)^{3/2}}{2592}-\frac{1633(1-6x)\sqrt{3x^2}}{20736}$$

```
[Out] -71/2592*(1-6*x)*(3*x^2-x+2)^(3/2)+2/21*(1+2*x)^2*(3*x^2-x+2)^(5/2)+1/378*(
109+102*x)*(3*x^2-x+2)^(5/2)-37559/124416*arcsinh(1/23*(1-6*x)*23^(1/2))*3^
(1/2)-1633/20736*(1-6*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1653, 779, 612, 619, 215}

$$\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}+\frac{1}{378}(102x+109)\left(3x^2-x+2\right)^{5/2}-\frac{71(1-6x)\left(3x^2-x+2\right)^{3/2}}{2592}-\frac{1633(1-6x)\sqrt{3x^2}}{20736}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (-1633*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/20736 - (71*(1 - 6*x)*(2 - x + 3*x^2)
^(3/2))/2592 + (2*(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2))/21 + ((109 + 102*x)*(2
 - x + 3*x^2)^(5/2))/378 - (37559*ArcSinh[(1 - 6*x)/Sqrt[23]])/(41472*Sqrt[
3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
```

```
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)dx = \frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{84}\int (1+2x)(40+204x)\left(2-x\right.$$

$$= \frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{378}(109+102x)\left(2-x+3x^2\right)^{5/2}$$

$$= -\frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592}+\frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{37}$$

$$= -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736}-\frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592}+\frac{2}{21}($$

$$= -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736}-\frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592}+\frac{2}{21}($$

$$= -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736}-\frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592}+\frac{2}{21}($$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.60

$$\frac{6\sqrt{3x^2-x+2}\left(497664x^6+518400x^5+653184x^4+744336x^3+531384x^2+275410x+203337\right)+262913\sqrt{3}}{870912}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(203337 + 275410*x + 531384*x^2 + 744336*x^3 + 65318
4*x^4 + 518400*x^5 + 497664*x^6) + 262913*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[2
3]])/870912
```

fricas [A] time = 0.93, size = 83, normalized size = 0.72

$$\frac{1}{145152}\left(497664\,x^6+518400\,x^5+653184\,x^4+744336\,x^3+531384\,x^2+275410\,x+203337\right)\sqrt{3\,x^2-x+2}+\frac{37}{248}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/145152*(497664*x^6 + 518400*x^5 + 653184*x^4 + 744336*x^3 + 531384*x^2 +
275410*x + 203337)*sqrt(3*x^2 - x + 2) + 37559/248832*sqrt(3)*log(-4*sqrt(3
)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

giac [A] time = 0.21, size = 78, normalized size = 0.67

$$\frac{1}{145152}\left(2\left(12\left(18\left(24\left(2\left(24\,x+25\right)x+63\right)x+1723\right)x+22141\right)x+137705\right)x+203337\right)\sqrt{3\,x^2-x+2}-\frac{37559}{124416}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/145152*(2*(12*(18*(24*(2*(24*x + 25)*x + 63)*x + 1723)*x + 22141)*x + 137
705)*x + 203337)*sqrt(3*x^2 - x + 2) - 37559/124416*sqrt(3)*log(-2*sqrt(3)*
(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

maple [A] time = 0.00, size = 100, normalized size = 0.86

$$\frac{8\left(3x^2-x+2\right)^{\frac{5}{2}}x^2}{21}+\frac{41\left(3x^2-x+2\right)^{\frac{5}{2}}x}{63}+\frac{37559\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{124416}+\frac{145\left(3x^2-x+2\right)^{\frac{5}{2}}}{378}+\frac{71\left(6x-1\right)\left(3\right.}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x)
```

```
[Out] 8/21*(3*x^2-x+2)^(5/2)*x^2+41/63*(3*x^2-x+2)^(5/2)*x+145/378*(3*x^2-x+2)^(5
/2)+71/2592*(6*x-1)*(3*x^2-x+2)^(3/2)+1633/20736*(6*x-1)*(3*x^2-x+2)^(1/2)+
37559/124416*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

maxima [A] time = 0.97, size = 121, normalized size = 1.04

$$\frac{8}{21}\left(3x^2-x+2\right)^{\frac{5}{2}}x^2+\frac{41}{63}\left(3x^2-x+2\right)^{\frac{5}{2}}x+\frac{145}{378}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{71}{432}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{71}{2592}\left(3x^2-x+2\right)^{\frac{3}{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 8/21*(3*x^2 - x + 2)^(5/2)*x^2 + 41/63*(3*x^2 - x + 2)^(5/2)*x + 145/378*(3
*x^2 - x + 2)^(5/2) + 71/432*(3*x^2 - x + 2)^(3/2)*x - 71/2592*(3*x^2 - x +
 2)^(3/2) + 1633/3456*sqrt(3*x^2 - x + 2)*x + 37559/124416*sqrt(3)*arcsinh(
1/23*sqrt(23)*(6*x - 1)) - 1633/20736*sqrt(3*x^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x+1)\left(3x^2-x+2\right)^{3/2}\left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] int((2*x + 1)*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)\left(3x^2-x+2\right)^{\frac{3}{2}}\left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
```

```
[Out] Integral((2*x + 1)*(3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1), x)
```

3.217 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x} \, dx$

Optimal. Leaf size=124

$$\frac{2}{15}\left(3x^2-x+2\right)^{5/2}+\frac{1}{144}(30x+7)\left(3x^2-x+2\right)^{3/2}+\frac{(402x+869)\sqrt{3x^2-x+2}}{1152}-\frac{13}{32}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x}}\right.$$

```
[Out] 1/144*(7+30*x)*(3*x^2-x+2)^(3/2)+2/15*(3*x^2-x+2)^(5/2)+2203/6912*arcsinh(1
/23*(1-6*x)*23^(1/2))*3^(1/2)-13/32*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+
2)^(1/2))*13^(1/2)+1/1152*(869+402*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{2}{15}\left(3x^2-x+2\right)^{5/2}+\frac{1}{144}(30x+7)\left(3x^2-x+2\right)^{3/2}+\frac{(402x+869)\sqrt{3x^2-x+2}}{1152}-\frac{13}{32}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] ((869 + 402*x)*Sqrt[2 - x + 3*x^2])/1152 + ((7 + 30*x)*(2 - x + 3*x^2)^(3/2
))/144 + (2*(2 - x + 3*x^2)^(5/2))/15 + (2203*ArcSinh[(1 - 6*x)/Sqrt[23]])/
(2304*Sqrt[3]) - (13*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*
x^2])])/32
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
```

```
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{1+2x}\,dx = \frac{2}{15}\left(2-x+3x^2\right)^{5/2}+\frac{1}{60}\int \frac{(80+100x)\left(2-x+3x^2\right)^{3/2}}{1+2x}\,dx$$

$$= \frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2}+\frac{2}{15}\left(2-x+3x^2\right)^{5/2}-\frac{\int \frac{(-13380-8040x)}{1+2x}}{576}$$

$$= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152}+\frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2}+\frac{2}{15}\left(2-\right.$$

$$= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152}+\frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2}+\frac{2}{15}\left(2-\right.$$

$$= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152}+\frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2}+\frac{2}{15}\left(2-\right.$$

$$= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152}+\frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2}+\frac{2}{15}\left(2-\right.$$

Mathematica [A] time = 0.06, size = 96, normalized size = 0.77

$$\frac{-14040\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)+6\sqrt{3x^2-x+2}\left(6912x^4-1008x^3+9624x^2+1058x+7977\right)-11015\sqrt{}}{34560}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(7977 + 1058*x + 9624*x^2 - 1008*x^3 + 6912*x^4) - 1
1015*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]] - 14040*Sqrt[13]*ArcTanh[(9 - 8*x
)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/34560
```

fricas [A] time = 0.91, size = 125, normalized size = 1.01

$$\frac{1}{5760}\left(6912\,x^4-1008\,x^3+9624\,x^2+1058\,x+7977\right)\sqrt{3\,x^2-x+2}+\frac{2203}{13824}\,\sqrt{3}\,\log\left(4\,\sqrt{3}\,\sqrt{3\,x^2-x+2}\,(6\,x-1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="fricas")
```

```
[Out] 1/5760*(6912*x^4 - 1008*x^3 + 9624*x^2 + 1058*x + 7977)*sqrt(3*x^2 - x + 2)
 + 2203/13824*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2
+ 24*x - 25) + 13/64*sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9
) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1))
```

giac [A] time = 0.28, size = 136, normalized size = 1.10

$$\frac{1}{5760}\,(2\,(12\,(6\,(48\,x-7)x+401)x+529)x+7977)\sqrt{3\,x^2-x+2}+\frac{2203}{6912}\,\sqrt{3}\,\log\left(-6\,\sqrt{3}\,x+\sqrt{3}\,+6\,\sqrt{3\,x^2-x+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="giac")
```

```
[Out] 1/5760*(2*(12*(6*(48*x - 7)*x + 401)*x + 529)*x + 7977)*sqrt(3*x^2 - x + 2)
 + 2203/6912*sqrt(3)*log(-6*sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) +
13/32*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(
3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

maple [A] time = 0.01, size = 151, normalized size = 1.22

$$-\frac{2203\sqrt{3}\,\ \mathrm{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{6912}-\frac{13\sqrt{13}\,\ \mathrm{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{32}+\frac{2\left(3x^2-x+2\right)^{\frac{5}{2}}}{15}+\frac{5\,(6x-1)\left(3x^2-x+\right.}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(2*x+1),x)
```

```
[Out] 2/15*(3*x^2-x+2)^(5/2)+5/144*(6*x-1)*(3*x^2-x+2)^(3/2)+115/1152*(6*x-1)*(3*
x^2-x+2)^(1/2)-2203/6912*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+1/12*(-4*x+
3*(x+1/2)^2+5/4)^(3/2)-1/24*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(1/2)+13/32*(-16
*x+12*(x+1/2)^2+5)^(1/2)-13/32*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-
16*x+12*(x+1/2)^2+5)^(1/2))
```

maxima [A] time = 0.99, size = 125, normalized size = 1.01

$$\frac{2}{15}\left(3\,x^2-x+2\right)^{\frac{5}{2}}+\frac{5}{24}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x+\frac{7}{144}\left(3\,x^2-x+2\right)^{\frac{3}{2}}+\frac{67}{192}\,\sqrt{3\,x^2-x+2}\,x-\frac{2203}{6912}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\,\sqrt{23}\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="maxima")
```

```
[Out] 2/15*(3*x^2 - x + 2)^(5/2) + 5/24*(3*x^2 - x + 2)^(3/2)*x + 7/144*(3*x^2 -
x + 2)^(3/2) + 67/192*sqrt(3*x^2 - x + 2)*x - 2203/6912*sqrt(3)*arcsinh(6/2
3*sqrt(23)*x - 1/23*sqrt(23)) + 13/32*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(
2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 869/1152*sqrt(3*x^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2-x+2\right)^{3/2}\,\left(4\,x^2+3\,x+1\right)}{2\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
```

```
[Out] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^{2} - x + 2\right)^{\frac{3}{2}} \left(4x^{2} + 3x + 1\right)}{2x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x),x)
```

```
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

3.218 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2} dx$

Optimal. Leaf size=131

$$-\frac{(3x^2-x+2)^{5/2}}{13(2x+1)}-\frac{1}{104}(23-38x)\left(3x^2-x+2\right)^{3/2}-\frac{1}{192}(349-294x)\sqrt{3x^2-x+2}+\frac{25}{32}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-}}\right.$$

```
[Out] -1/104*(23-38*x)*(3*x^2-x+2)^(3/2)-1/13*(3*x^2-x+2)^(5/2)/(1+2*x)-2327/1152
*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+25/32*arctanh(1/26*(9-8*x)*13^(1/2)
/(3*x^2-x+2)^(1/2))*13^(1/2)-1/192*(349-294*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{(3x^2-x+2)^{5/2}}{13(2x+1)}-\frac{1}{104}(23-38x)\left(3x^2-x+2\right)^{3/2}-\frac{1}{192}(349-294x)\sqrt{3x^2-x+2}+\frac{25}{32}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] -((349 - 294*x)*Sqrt[2 - x + 3*x^2])/192 - ((23 - 38*x)*(2 - x + 3*x^2)^(3/
2))/104 - (2 - x + 3*x^2)^(5/2)/(13*(1 + 2*x)) - (2327*ArcSinh[(1 - 6*x)/Sq
rt[23]])/(384*Sqrt[3]) + (25*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2
 - x + 3*x^2])])/32
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
```

```
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx = -\frac{\left(2-x+3x^2\right)^{5/2}}{13(1+2x)} - \frac{1}{13}\int \frac{\left(-\frac{13}{2}-38x\right)\left(2-x+3x^2\right)^{3/2}}{1+2x}\,dx$$

$$= -\frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \frac{\left(2-x+3x^2\right)^{5/2}}{13(1+2x)} + \frac{\int \frac{(-78+7644x)\sqrt{2-}}{1+2x}}{1248}$$

$$= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \big($$

$$= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \big($$

$$= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \big($$

$$= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \big($$

Mathematica [A] time = 0.09, size = 103, normalized size = 0.79

$$\frac{900\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)+\frac{6\sqrt{3x^2-x+2}\left(288x^4-96x^3+564x^2-332x-493\right)}{2x+1}+2327\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{1152}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] ((6*Sqrt[2 - x + 3*x^2]*(-493 - 332*x + 564*x^2 - 96*x^3 + 288*x^4))/(1 + 2
*x) + 2327*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]] + 900*Sqrt[13]*ArcTanh[(9 -
 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/1152
```

fricas [A] time = 0.88, size = 143, normalized size = 1.09

$$\frac{2327\sqrt{3}\,(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)+900\sqrt{13}\,(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}\,(\ldots}{4\ldots}\right.}{2304\,(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2304*(2327*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
 - 72*x^2 + 24*x - 25) + 900*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2
- x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 12*(288*x^
4 - 96*x^3 + 564*x^2 - 332*x - 493)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

giac [B] time = 0.84, size = 570, normalized size = 4.35

$$\frac{25}{32}\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)-\frac{2327}{1152}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{8}{2x\ldots}}\right.}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2x+\ldots}}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 25/32*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt
(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 2327/1152*sqrt(3)*log(1/2*abs(-2*sq
rt(3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(
sqrt(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))*sg
n(1/(2*x + 1)) - 13/32*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1/(2*x +
 1)) + 1/192*(5929*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x
 + 1))^7*sgn(1/(2*x + 1)) - 7272*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)
^2 + 3) + sqrt(13)/(2*x + 1))^6*sgn(1/(2*x + 1)) + 25101*(sqrt(-8/(2*x + 1)
 + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^5*sgn(1/(2*x + 1)) - 48*sqrt(1
3)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^4*sgn(1/(
2*x + 1)) + 112359*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x
 + 1))^3*sgn(1/(2*x + 1)) - 69336*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1
)^2 + 3) + sqrt(13)/(2*x + 1))^2*sgn(1/(2*x + 1)) + 71955*(sqrt(-8/(2*x + 1
) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 24624*sqrt
(13)*sgn(1/(2*x + 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)
/(2*x + 1))^2 - 3)^4
```

maple [A] time = 0.01, size = 179, normalized size = 1.37

$$\frac{2327\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{1152}+\frac{25\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{32}+\frac{(6x-1)\left(3x^2-x+2\right)^{\frac{3}{2}}}{24}+\frac{23\,(6x-1)\sqrt{3\ldots}}{192\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(2*x+1)^2,x)
```

```
[Out] 1/24*(6*x-1)*(3*x^2-x+2)^(3/2)+23/192*(6*x-1)*(3*x^2-x+2)^(1/2)+2327/1152*3
^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-25/156*(-4*x+3*(x+1/2)^2+5/4)^(3/2)+1
3/96*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(1/2)-25/32*(-16*x+12*(x+1/2)^2+5)^(1/2
)+25/32*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1
/2))-1/26/(x+1/2)*(-4*x+3*(x+1/2)^2+5/4)^(5/2)+1/52*(6*x-1)*(-4*x+3*(x+1/2)
^2+5/4)^(3/2)
```

maxima [A] time = 0.98, size = 132, normalized size = 1.01

$$\frac{1}{4}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x-\frac{1}{8}\left(3\,x^2-x+2\right)^{\frac{3}{2}}+\frac{49}{32}\,\sqrt{3\,x^2-x+2}\,x+\frac{2327}{1152}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\,\sqrt{23}\,x-\frac{1}{23}\,\sqrt{23}\right)-\frac{25}{32}\,\sqrt{13}\,\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(3*x^2 - x + 2)^(3/2)*x - 1/8*(3*x^2 - x + 2)^(3/2) + 49/32*sqrt(3*x^2
- x + 2)*x + 2327/1152*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 2
5/32*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x
+ 1)) - 349/192*sqrt(3*x^2 - x + 2) - 1/4*(3*x^2 - x + 2)^(3/2)/(2*x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2-x+2\right)^{3/2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
```

```
[Out] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2-x+2\right)^{\frac{3}{2}}\left(4x^2+3x+1\right)}{\left(2x+1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)
```

```
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

3.219 $\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}+\frac{(122x+151)\left(3x^2-x+2\right)^{3/2}}{312(2x+1)}+\frac{1}{624}(1858-771x)\sqrt{3x^2-x+2}-\frac{1153\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{64\sqrt{13}}+$$

```
[Out] 1/312*(151+122*x)*(3*x^2-x+2)^(3/2)/(1+2*x)-1/26*(3*x^2-x+2)^(5/2)/(1+2*x)^
2+1519/576*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1153/832*arctanh(1/26*(9-
8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+1/624*(1858-771*x)*(3*x^2-x+2)^(1
/2)
```

Rubi [A] time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1650, 812, 814, 843, 619, 215, 724, 206}

$$-\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}+\frac{(122x+151)\left(3x^2-x+2\right)^{3/2}}{312(2x+1)}+\frac{1}{624}(1858-771x)\sqrt{3x^2-x+2}-\frac{1153\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{64\sqrt{13}}+$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((1858 - 771*x)*Sqrt[2 - x + 3*x^2])/624 + ((151 + 122*x)*(2 - x + 3*x^2)^(
3/2))/(312*(1 + 2*x)) - (2 - x + 3*x^2)^(5/2)/(26*(1 + 2*x)^2) + (1519*ArcS
inh[(1 - 6*x)/Sqrt[23]])/(192*Sqrt[3]) - (1153*ArcTanh[(9 - 8*x)/(2*Sqrt[13
]*Sqrt[2 - x + 3*x^2])])/(64*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
```

```
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx = -\frac{\left(2-x+3x^2\right)^{5/2}}{26(1+2x)^2} - \frac{1}{26}\int \frac{\left(-\frac{31}{2}-61x\right)\left(2-x+3x^2\right)^{3/2}}{(1+2x)^2}\,dx$$

$$= \frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)} - \frac{\left(2-x+3x^2\right)^{5/2}}{26(1+2x)^2} + \frac{1}{208}\int \frac{(639-1028x}{1+}$$

$$= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)} - \frac{\left(2-\right.}{26(}$$

$$= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)} - \frac{\left(2-\right.}{26(}$$

$$= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)} - \frac{\left(2-\right.}{26(}$$

$$= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)} - \frac{\left(2-\right.}{26(}$$

Mathematica [A] time = 0.11, size = 103, normalized size = 0.75

$$\frac{-10377\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)+\frac{156\sqrt{3x^2-x+2}\left(96x^4-68x^3+390x^2+627x+182\right)}{(2x+1)^2}-19747\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{7488}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((156*Sqrt[2 - x + 3*x^2]*(182 + 627*x + 390*x^2 - 68*x^3 + 96*x^4))/(1 + 2
*x)^2 - 19747*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]] - 10377*Sqrt[13]*ArcTanh
[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/7488
```

fricas [A] time = 0.89, size = 159, normalized size = 1.15

$$\frac{19747\sqrt{3}\left(4x^2+4x+1\right)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+10377\sqrt{13}\left(4x^2+4x+1\right)\log}{14976\left(4x^2+4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/14976*(19747*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*
(6*x - 1) - 72*x^2 + 24*x - 25) + 10377*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*
sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*
x + 1)) + 312*(96*x^4 - 68*x^3 + 390*x^2 + 627*x + 182)*sqrt(3*x^2 - x + 2)
)/(4*x^2 + 4*x + 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{-18688512,[6]%%%}+%%%{%%{[56065536,0]:[1,0,-3]%%},[5]%%%}+%%%{-28032
768,[4]%%%}+%%%{%%{[-224262144,0]:[1,0,-3]%%},[3]%%%}+%%%{70081920,[2]%%%}+
%%%{%%{[350409600,0]:[1,0,-3]%%},[1]%%%}+%%%{292008000,[0]%%%} / %%%{%%{[24
,0]:[1,0,-3]%%},[6]%%%}+%%%{-216,[5]%%%}+%%%{%%{[36,0]:[1,0,-3]%%},[4]%%%}+
%%%{864,[3]%%%}+%%%{%%{[-90,0]:[1,0,-3]%%},[2]%%%}+%%%{-1350,[1]%%%}+%%%{%%
{[-375,0]:[1,0,-3]%%},[0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.01, size = 162, normalized size = 1.17

$$-\frac{1519\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{576}-\frac{1153\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^{2}+5}}\right)}{832}+\frac{1153\left(-4x+3\left(x+\frac{1}{2}\right)^{2}+\frac{5}{4}\right)^{\frac{3}{2}}}{4056}-\frac{25}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(2*x+1)^3,x)
```

```
[Out] 1153/4056*(-4*x+3*(x+1/2)^2+5/4)^(3/2)-257/1248*(6*x-1)*(-4*x+3*(x+1/2)^2+5
/4)^(1/2)-1519/576*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+1153/832*(-16*x+1
2*(x+1/2)^2+5)^(1/2)-1153/832*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-1
6*x+12*(x+1/2)^2+5)^(1/2))+15/338/(x+1/2)*(-4*x+3*(x+1/2)^2+5/4)^(5/2)-15/6
76*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(3/2)-1/104/(x+1/2)^2*(-4*x+3*(x+1/2)^2+5
/4)^(5/2)
```

maxima [A] time = 0.98, size = 143, normalized size = 1.04

$$\frac{61}{312}\left(3\,x^{2}-x+2\right)^{\frac{3}{2}}-\frac{\left(3\,x^{2}-x+2\right)^{\frac{5}{2}}}{26\left(4\,x^{2}+4\,x+1\right)}-\frac{257}{208}\sqrt{3\,x^{2}-x+2}\,x-\frac{1519}{576}\sqrt{3}\ \operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{1153}{832}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="maxima")
```

```
[Out] 61/312*(3*x^2 - x + 2)^(3/2) - 1/26*(3*x^2 - x + 2)^(5/2)/(4*x^2 + 4*x + 1)
 - 257/208*sqrt(3*x^2 - x + 2)*x - 1519/576*sqrt(3)*arcsinh(6/23*sqrt(23)*x
 - 1/23*sqrt(23)) + 1153/832*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1)
- 9/23*sqrt(23)/abs(2*x + 1)) + 929/312*sqrt(3*x^2 - x + 2) + 15/52*(3*x^2
- x + 2)^(3/2)/(2*x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2-x+2\right)^{3/2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

```
[Out] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^{2}-x+2\right)^{\frac{3}{2}}\left(4x^{2}+3x+1\right)}{\left(2x+1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

3.220 $\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx$

Optimal. Leaf size=189

$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4+\frac{29}{330}\left(3x^2-x+2\right)^{7/2}(2x+1)^3+\frac{133\left(3x^2-x+2\right)^{7/2}(2x+1)^2}{1485}-\frac{(26353-21350x)}{4989}$$

```
[Out] 117047/1492992*(1-6*x)*(3*x^2-x+2)^(3/2)+5089/155520*(1-6*x)*(3*x^2-x+2)^(5
/2)-1/498960*(26353-21350*x)*(3*x^2-x+2)^(7/2)+133/1485*(1+2*x)^2*(3*x^2-x+
2)^(7/2)+29/330*(1+2*x)^3*(3*x^2-x+2)^(7/2)+2/33*(1+2*x)^4*(3*x^2-x+2)^(7/2
)+61917863/71663616*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2692081/11943936
*(1-6*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4+\frac{29}{330}\left(3x^2-x+2\right)^{7/2}(2x+1)^3+\frac{133\left(3x^2-x+2\right)^{7/2}(2x+1)^2}{1485}-\frac{(26353-21350x)}{4989}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (2692081*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/11943936 + (117047*(1 - 6*x)*(2 - x
 + 3*x^2)^(3/2))/1492992 + (5089*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/155520 -
((26353 - 21350*x)*(2 - x + 3*x^2)^(7/2))/498960 + (133*(1 + 2*x)^2*(2 - x
+ 3*x^2)^(7/2))/1485 + (29*(1 + 2*x)^3*(2 - x + 3*x^2)^(7/2))/330 + (2*(1 +
 2*x)^4*(2 - x + 3*x^2)^(7/2))/33 + (61917863*ArcSinh[(1 - 6*x)/Sqrt[23]])/
(23887872*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^3\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)dx = \frac{2}{33}(1+2x)^4\left(2-x+3x^2\right)^{7/2}+\frac{1}{132}\int (1+2x)^3(32+348x)\left(2-\right.$$

$$= \frac{29}{330}(1+2x)^3\left(2-x+3x^2\right)^{7/2}+\frac{2}{33}(1+2x)^4\left(2-x+3x^2\right)^{7/2}+$$

$$= \frac{133(1+2x)^2\left(2-x+3x^2\right)^{7/2}}{1485}+\frac{29}{330}(1+2x)^3\left(2-x+3x^2\right)^{7/2}+$$

$$= -\frac{(26353-21350x)\left(2-x+3x^2\right)^{7/2}}{498960}+\frac{133(1+2x)^2\left(2-x+3x\right.}{1485}$$

$$= \frac{5089(1-6x)\left(2-x+3x^2\right)^{5/2}}{155520}-\frac{(26353-21350x)\left(2-x+3x^2\right)}{498960}$$

$$= \frac{117047(1-6x)\left(2-x+3x^2\right)^{3/2}}{1492992}+\frac{5089(1-6x)\left(2-x+3x^2\right)^{5/2}}{155520}$$

$$= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936}+\frac{117047(1-6x)\left(2-x+3x^2\right)^{3}}{1492992}$$

$$= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936}+\frac{117047(1-6x)\left(2-x+3x^2\right)^{3}}{1492992}$$

$$= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936}+\frac{117047(1-6x)\left(2-x+3x^2\right)^{3}}{1492992}$$

Mathematica [A] time = 0.06, size = 90, normalized size = 0.48

$$6\sqrt{3x^2-x+2}\left(120394874880x^{10}+207681159168x^9+308846297088x^8+419978151936x^7+415908006912x^6\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(9173509857 + 26646633218*x + 72088585464*x^2 + 1612
69204752*x^3 + 263636134272*x^4 + 347247744768*x^5 + 415908006912*x^6 + 419
978151936*x^7 + 308846297088*x^8 + 207681159168*x^9 + 120394874880*x^10) -
23838377255*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/27590492160
```

fricas [A] time = 0.89, size = 103, normalized size = 0.54

$$\frac{1}{4598415360}\left(120394874880\,x^{10}+207681159168\,x^{9}+308846297088\,x^{8}+419978151936\,x^{7}+41590800691\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/4598415360*(120394874880*x^10 + 207681159168*x^9 + 308846297088*x^8 + 419
978151936*x^7 + 415908006912*x^6 + 347247744768*x^5 + 263636134272*x^4 + 16
1269204752*x^3 + 72088585464*x^2 + 26646633218*x + 9173509857)*sqrt(3*x^2 -
 x + 2) + 61917863/143327232*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
 - 1) - 72*x^2 + 24*x - 25)
```

giac [A] time = 0.20, size = 98, normalized size = 0.52

$$\frac{1}{4598415360}\left(2\left(12\left(6\left(8\left(6\left(36\left(14\left(48\left(18\left(40\,x+69\right)x+1847\right)x+120557\right)x+1671441\right)x+50238389\right)x+228\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/4598415360*(2*(12*(6*(8*(6*(36*(14*(48*(18*(40*x + 69)*x + 1847)*x + 1205
57)*x + 1671441)*x + 50238389)*x + 228850811)*x + 1119925033)*x + 300369106
1)*x + 13323316609)*x + 9173509857)*sqrt(3*x^2 - x + 2) + 61917863/71663616
*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

maple [A] time = 0.02, size = 153, normalized size = 0.81

$$\frac{32\left(3x^{2}-x+2\right)^{\frac{7}{2}}x^{4}}{33}+\frac{436\left(3x^{2}-x+2\right)^{\frac{7}{2}}x^{3}}{165}+\frac{4258\left(3x^{2}-x+2\right)^{\frac{7}{2}}x^{2}}{1485}+\frac{10073\left(3x^{2}-x+2\right)^{\frac{7}{2}}x}{7128}-\frac{61917863\sqrt{3}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x)
```

```
[Out] 32/33*x^4*(3*x^2-x+2)^(7/2)+436/165*x^3*(3*x^2-x+2)^(7/2)+4258/1485*x^2*(3*
x^2-x+2)^(7/2)+10073/7128*x*(3*x^2-x+2)^(7/2)-5089/155520*(6*x-1)*(3*x^2-x+
2)^(5/2)-117047/1492992*(6*x-1)*(3*x^2-x+2)^(3/2)-61917863/71663616*3^(1/2)
*arcsinh(6/23*23^(1/2)*(x-1/6))-2692081/11943936*(6*x-1)*(3*x^2-x+2)^(1/2)+
92423/498960*(3*x^2-x+2)^(7/2)
```

maxima [A] time = 0.98, size = 184, normalized size = 0.97

$$\frac{32}{33}\left(3\,x^{2}-x+2\right)^{\frac{7}{2}}x^{4}+\frac{436}{165}\left(3\,x^{2}-x+2\right)^{\frac{7}{2}}x^{3}+\frac{4258}{1485}\left(3\,x^{2}-x+2\right)^{\frac{7}{2}}x^{2}+\frac{10073}{7128}\left(3\,x^{2}-x+2\right)^{\frac{7}{2}}x+\frac{92423}{498960}\left(3\,x^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 32/33*(3*x^2 - x + 2)^(7/2)*x^4 + 436/165*(3*x^2 - x + 2)^(7/2)*x^3 + 4258/
1485*(3*x^2 - x + 2)^(7/2)*x^2 + 10073/7128*(3*x^2 - x + 2)^(7/2)*x + 92423
/498960*(3*x^2 - x + 2)^(7/2) - 5089/25920*(3*x^2 - x + 2)^(5/2)*x + 5089/1
55520*(3*x^2 - x + 2)^(5/2) - 117047/248832*(3*x^2 - x + 2)^(3/2)*x + 11704
7/1492992*(3*x^2 - x + 2)^(3/2) - 2692081/1990656*sqrt(3*x^2 - x + 2)*x - 6
1917863/71663616*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 2692081/1194393
6*sqrt(3*x^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2\,x+1)^3\,\left(3\,x^2-x+2\right)^{5/2}\,\left(4\,x^2+3\,x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] int((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)^3 \left(3x^2-x+2\right)^{\frac{5}{2}} \left(4x^2+3x+1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

```
[Out] Integral((2*x + 1)**3*(3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1), x)
```

3.221 $\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx$

Optimal. Leaf size=164

$$\frac{1}{15}(2x+1)^3 \left(3x^2-x+2\right)^{7/2}+\frac{37}{405}(2x+1)^2 \left(3x^2-x+2\right)^{7/2}+\frac{(3430x+2731)\left(3x^2-x+2\right)^{7/2}}{17010}-\frac{293(1-6x)\left(3x^2\right.}{58320}$$

```
[Out] -6739/559872*(1-6*x)*(3*x^2-x+2)^(3/2)-293/58320*(1-6*x)*(3*x^2-x+2)^(5/2)+
37/405*(1+2*x)^2*(3*x^2-x+2)^(7/2)+1/15*(1+2*x)^3*(3*x^2-x+2)^(7/2)+1/17010
*(2731+3430*x)*(3*x^2-x+2)^(7/2)-3564931/26873856*arcsinh(1/23*(1-6*x)*23^(
1/2))*3^(1/2)-154997/4478976*(1-6*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{1}{15}(2x+1)^3 \left(3x^2-x+2\right)^{7/2}+\frac{37}{405}(2x+1)^2 \left(3x^2-x+2\right)^{7/2}+\frac{(3430x+2731)\left(3x^2-x+2\right)^{7/2}}{17010}-\frac{293(1-6x)\left(3x^2\right.}{58320}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (-154997*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/4478976 - (6739*(1 - 6*x)*(2 - x +
3*x^2)^(3/2))/559872 - (293*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/58320 + (37*(1
 + 2*x)^2*(2 - x + 3*x^2)^(7/2))/405 + ((1 + 2*x)^3*(2 - x + 3*x^2)^(7/2))/
15 + ((2731 + 3430*x)*(2 - x + 3*x^2)^(7/2))/17010 - (3564931*ArcSinh[(1 -
6*x)/Sqrt[23]])/(8957952*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^2\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)dx = \frac{1}{15}(1+2x)^3\left(2-x+3x^2\right)^{7/2}+\frac{1}{120}\int (1+2x)^2(52+296x)\left(2-\right.$$

$$=\frac{37}{405}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{15}(1+2x)^3\left(2-x+3x^2\right)^{7/2}+$$

$$=\frac{37}{405}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{15}(1+2x)^3\left(2-x+3x^2\right)^{7/2}+$$

$$=-\frac{293(1-6x)\left(2-x+3x^2\right)^{5/2}}{58320}+\frac{37}{405}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+$$

$$=-\frac{6739(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872}-\frac{293(1-6x)\left(2-x+3x^2\right)^{5/2}}{58320}+$$

$$=-\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976}-\frac{6739(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872}$$

$$=-\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976}-\frac{6739(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872}$$

$$=-\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976}-\frac{6739(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872}$$

Mathematica [A] time = 0.06, size = 85, normalized size = 0.52

$$\frac{6\sqrt{3x^2-x+2}\left(2257403904x^9+2675441664x^8+4427716608x^7+5671627776x^6+4996802304x^5+41715797\right.}{940584960}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(387182961 + 692659234*x + 1693765752*x^2 + 30961049
76*x^3 + 4171579776*x^4 + 4996802304*x^5 + 5671627776*x^6 + 4427716608*x^7
+ 2675441664*x^8 + 2257403904*x^9) + 124772585*Sqrt[3]*ArcSinh[(-1 + 6*x)/S
qrt[23]])/940584960
```

fricas [A] time = 0.95, size = 98, normalized size = 0.60

$$\frac{1}{156764160}\left(2257403904\,x^9 + 2675441664\,x^8 + 4427716608\,x^7 + 5671627776\,x^6 + 4996802304\,x^5 + 417157\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/156764160*(2257403904*x^9 + 2675441664*x^8 + 4427716608*x^7 + 5671627776*
x^6 + 4996802304*x^5 + 4171579776*x^4 + 3096104976*x^3 + 1693765752*x^2 + 6
92659234*x + 387182961)*sqrt(3*x^2 - x + 2) + 3564931/53747712*sqrt(3)*log(
-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

giac [A] time = 0.21, size = 93, normalized size = 0.57

$$\frac{1}{156764160}\left(2\left(12\left(6\left(8\left(6\left(36\left(14\left(24\left(27\,x + 32\right)x + 1271\right)x + 22793\right)x + 722917\right)x + 3621163\right)x + 21500729\right)x\right.\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/156764160*(2*(12*(6*(8*(6*(36*(14*(24*(27*x + 32)*x + 1271)*x + 22793)*x
+ 722917)*x + 3621163)*x + 21500729)*x + 70573573)*x + 346329617)*x + 38718
2961)*sqrt(3*x^2 - x + 2) - 3564931/26873856*sqrt(3)*log(-2*sqrt(3)*(sqrt(3
)*x - sqrt(3*x^2 - x + 2)) + 1)
```

maple [A] time = 0.01, size = 136, normalized size = 0.83

$$\frac{8\left(3x^2 - x + 2\right)^{\frac{7}{2}} x^3}{15} + \frac{472\left(3x^2 - x + 2\right)^{\frac{7}{2}} x^2}{405} + \frac{235\left(3x^2 - x + 2\right)^{\frac{7}{2}} x}{243} + \frac{3564931\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{26873856} + \frac{293\,(6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x)
```

```
[Out] 8/15*(3*x^2-x+2)^(7/2)*x^3+472/405*(3*x^2-x+2)^(7/2)*x^2+235/243*(3*x^2-x+2
)^(7/2)*x+293/58320*(6*x-1)*(3*x^2-x+2)^(5/2)+6739/559872*(6*x-1)*(3*x^2-x+
2)^(3/2)+3564931/26873856*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+154997/447
8976*(6*x-1)*(3*x^2-x+2)^(1/2)+5419/17010*(3*x^2-x+2)^(7/2)
```

maxima [A] time = 0.97, size = 167, normalized size = 1.02

$$\frac{8}{15}\left(3\,x^2 - x + 2\right)^{\frac{7}{2}}x^3 + \frac{472}{405}\left(3\,x^2 - x + 2\right)^{\frac{7}{2}}x^2 + \frac{235}{243}\left(3\,x^2 - x + 2\right)^{\frac{7}{2}}x + \frac{5419}{17010}\left(3\,x^2 - x + 2\right)^{\frac{7}{2}} + \frac{293}{9720}\left(3\,x^2 - x + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 8/15*(3*x^2 - x + 2)^(7/2)*x^3 + 472/405*(3*x^2 - x + 2)^(7/2)*x^2 + 235/24
3*(3*x^2 - x + 2)^(7/2)*x + 5419/17010*(3*x^2 - x + 2)^(7/2) + 293/9720*(3*
x^2 - x + 2)^(5/2)*x - 293/58320*(3*x^2 - x + 2)^(5/2) + 6739/93312*(3*x^2
- x + 2)^(3/2)*x - 6739/559872*(3*x^2 - x + 2)^(3/2) + 154997/746496*sqrt(3
*x^2 - x + 2)*x + 3564931/26873856*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1))
 - 154997/4478976*sqrt(3*x^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2\,x+1)^2\,{\left(3\,x^2-x+2\right)}^{5/2}\,\left(4\,x^2+3\,x+1\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
[Out] int((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)^2 \left(3x^2-x+2\right)^{\frac{5}{2}} \left(4x^2+3x+1\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
[Out] Integral((2*x + 1)**2*(3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1), x)
```

3.222 $\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx$

Optimal. Leaf size=139

$$\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}+\frac{1}{648}(122x+137)\left(3x^2-x+2\right)^{7/2}-\frac{445(1-6x)\left(3x^2-x+2\right)^{5/2}}{15552}-\frac{51175(1-6x)\left(3x\right.}{74649}$$

```
[Out] -51175/746496*(1-6*x)*(3*x^2-x+2)^(3/2)-445/15552*(1-6*x)*(3*x^2-x+2)^(5/2)
+2/27*(1+2*x)^2*(3*x^2-x+2)^(7/2)+1/648*(137+122*x)*(3*x^2-x+2)^(7/2)-27071
575/35831808*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1177025/5971968*(1-6*x)
*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1653, 779, 612, 619, 215}

$$\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}+\frac{1}{648}(122x+137)\left(3x^2-x+2\right)^{7/2}-\frac{445(1-6x)\left(3x^2-x+2\right)^{5/2}}{15552}-\frac{51175(1-6x)\left(3x\right.}{74649}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (-1177025*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/5971968 - (51175*(1 - 6*x)*(2 - x
+ 3*x^2)^(3/2))/746496 - (445*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/15552 + (2*(
1 + 2*x)^2*(2 - x + 3*x^2)^(7/2))/27 + ((137 + 122*x)*(2 - x + 3*x^2)^(7/2)
)/648 - (27071575*ArcSinh[(1 - 6*x)/Sqrt[23]])/(11943936*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
```

```
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)dx = \frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2} + \frac{1}{108}\int (1+2x)(72+244x)\left(2-x\right.$$

$$= \frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2} + \frac{1}{648}(137+122x)\left(2-x+3x^2\right)^{7/2}$$

$$= -\frac{445(1-6x)\left(2-x+3x^2\right)^{5/2}}{15552} + \frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2} + \cdots$$

$$= -\frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496} - \frac{445(1-6x)\left(2-x+3x^2\right)^{5/2}}{15552} + \cdots$$

$$= -\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968} - \frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496} \cdots$$

$$= -\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968} - \frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496} \cdots$$

$$= -\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968} - \frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496} \cdots$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.58

$$\frac{6\sqrt{3x^2-x+2}\left(47775744x^8+30357504x^7+79377408x^6+80034048x^5+66969216x^4+58946544x^3+410310\cdots\right.}{35831808}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(10960335 + 19860062*x + 41031048*x^2 + 58946544*x^3
 + 66969216*x^4 + 80034048*x^5 + 79377408*x^6 + 30357504*x^7 + 47775744*x^8
) + 27071575*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/35831808
```

fricas [A] time = 0.90, size = 93, normalized size = 0.67

$$\frac{1}{5971968}\left(47775744\,x^8+30357504\,x^7+79377408\,x^6+80034048\,x^5+66969216\,x^4+58946544\,x^3+41031048\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/5971968*(47775744*x^8 + 30357504*x^7 + 79377408*x^6 + 80034048*x^5 + 6696
9216*x^4 + 58946544*x^3 + 41031048*x^2 + 19860062*x + 10960335)*sqrt(3*x^2
- x + 2) + 27071575/71663616*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*
x - 1) - 72*x^2 + 24*x - 25)
```

giac [A] time = 0.19, size = 88, normalized size = 0.63

$$\frac{1}{5971968}\left(2\left(12\left(6\left(8\left(6\left(36\left(2\left(96x+61\right)x+319\right)x+11579\right)x+58133\right)x+409351\right)x+1709627\right)x+9930031\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/5971968*(2*(12*(6*(8*(6*(36*(2*(96*x + 61)*x + 319)*x + 11579)*x + 58133)
*x + 409351)*x + 1709627)*x + 9930031)*x + 10960335)*sqrt(3*x^2 - x + 2) -
27071575/35831808*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
+ 1)
```

maple [A] time = 0.01, size = 119, normalized size = 0.86

$$\frac{8\left(3x^{2}-x+2\right)^{\frac{7}{2}}x^{2}}{27}+\frac{157\left(3x^{2}-x+2\right)^{\frac{7}{2}}x}{324}+\frac{27071575\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{35831808}+\frac{185\left(3x^{2}-x+2\right)^{\frac{7}{2}}}{648}+\frac{445\left(6x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x)
```

```
[Out] 8/27*(3*x^2-x+2)^(7/2)*x^2+157/324*(3*x^2-x+2)^(7/2)*x+185/648*(3*x^2-x+2)^
(7/2)+445/15552*(6*x-1)*(3*x^2-x+2)^(5/2)+51175/746496*(6*x-1)*(3*x^2-x+2)^
(3/2)+1177025/5971968*(6*x-1)*(3*x^2-x+2)^(1/2)+27071575/35831808*3^(1/2)*a
rcsinh(6/23*23^(1/2)*(x-1/6))
```

maxima [A] time = 0.97, size = 150, normalized size = 1.08

$$\frac{8}{27}\left(3x^{2}-x+2\right)^{\frac{7}{2}}x^{2}+\frac{157}{324}\left(3x^{2}-x+2\right)^{\frac{7}{2}}x+\frac{185}{648}\left(3x^{2}-x+2\right)^{\frac{7}{2}}+\frac{445}{2592}\left(3x^{2}-x+2\right)^{\frac{5}{2}}x-\frac{445}{15552}\left(3x^{2}-x+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 8/27*(3*x^2 - x + 2)^(7/2)*x^2 + 157/324*(3*x^2 - x + 2)^(7/2)*x + 185/648*
(3*x^2 - x + 2)^(7/2) + 445/2592*(3*x^2 - x + 2)^(5/2)*x - 445/15552*(3*x^2
 - x + 2)^(5/2) + 51175/124416*(3*x^2 - x + 2)^(3/2)*x - 51175/746496*(3*x^
2 - x + 2)^(3/2) + 1177025/995328*sqrt(3*x^2 - x + 2)*x + 27071575/35831808
*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 1177025/5971968*sqrt(3*x^2 - x
+ 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x+1)\,\left(3x^2-x+2\right)^{5/2}\,\left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] int((2*x + 1)*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)\left(3x^{2}-x+2\right)^{\frac{5}{2}}\left(4x^{2}+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

```
[Out] Integral((2*x + 1)*(3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1), x)
```

3.223 $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx$

Optimal. Leaf size=147

$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}+\frac{(150x+29)\left(3x^2-x+2\right)^{5/2}}{1080}+\frac{(2154x+2449)\left(3x^2-x+2\right)^{3/2}}{10368}+\frac{(221999-17850x)\sqrt{3x^2-x}}{82944}$$

```
[Out] 1/10368*(2449+2154*x)*(3*x^2-x+2)^(3/2)+1/1080*(29+150*x)*(3*x^2-x+2)^(5/2)
+2/21*(3*x^2-x+2)^(7/2)+944521/497664*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2
)-169/128*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+1/82944
*(221999-17850*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}+\frac{(150x+29)\left(3x^2-x+2\right)^{5/2}}{1080}+\frac{(2154x+2449)\left(3x^2-x+2\right)^{3/2}}{10368}+\frac{(221999-17850x)\sqrt{3x^2-x}}{82944}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] ((221999 - 17850*x)*Sqrt[2 - x + 3*x^2])/82944 + ((2449 + 2154*x)*(2 - x +
3*x^2)^(3/2))/10368 + ((29 + 150*x)*(2 - x + 3*x^2)^(5/2))/1080 + (2*(2 - x
 + 3*x^2)^(7/2))/21 + (944521*ArcSinh[(1 - 6*x)/Sqrt[23]])/(165888*Sqrt[3])
 - (169*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/128
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
```

```
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{1+2x}\,dx &= \frac{2}{21}\left(2-x+3x^2\right)^{7/2}+\frac{1}{84}\int \frac{(112+140x)\left(2-x+3x^2\right)^{5/2}}{1+2x}\,dx\\
&= \frac{(29+150x)\left(2-x+3x^2\right)^{5/2}}{1080}+\frac{2}{21}\left(2-x+3x^2\right)^{7/2}-\frac{\int \frac{(-29708-20104x)(\cdots}{1+2x}}{1209\cdots}\\
&= \frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368}+\frac{(29+150x)\left(2-x+3x^2\right)^{5/2}}{1080}+\frac{2}{21}\cdots\\
&= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944}+\frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368}-\cdots\\
&= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944}+\frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368}-\cdots\\
&= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944}+\frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368}-\cdots\\
&= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944}+\frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368}-\cdots
\end{aligned}$$

Mathematica [A] time = 0.08, size = 106, normalized size = 0.72

$$\frac{-22997520\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)+6\sqrt{3x^2-x+2}\left(7464960x^6-3836160x^5+15700608x^4-3646512x^3\right.}{17418240}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(11665053 - 2120998*x + 12466776*x^2 - 3646512*x^3 +
 15700608*x^4 - 3836160*x^5 + 7464960*x^6) - 33058235*Sqrt[3]*ArcSinh[(-1 +
 6*x)/Sqrt[23]] - 22997520*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 -
x + 3*x^2])])/17418240
```

fricas [A] time = 0.88, size = 135, normalized size = 0.92

$$\frac{1}{2903040}\left(7464960\,x^6-3836160\,x^5+15700608\,x^4-3646512\,x^3+12466776\,x^2-2120998\,x+11665053\right)\sqrt{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="fricas")
```

```
[Out] 1/2903040*(7464960*x^6 - 3836160*x^5 + 15700608*x^4 - 3646512*x^3 + 1246677
6*x^2 - 2120998*x + 11665053)*sqrt(3*x^2 - x + 2) + 944521/995328*sqrt(3)*l
og(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 169/256*
sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x +
 185)/(4*x^2 + 4*x + 1))
```

giac [A] time = 0.31, size = 146, normalized size = 0.99

$$\frac{1}{2903040}\left(2\left(12\left(18\left(8\left(30\left(72\,x-37\right)x+4543\right)x-8441\right)x+519449\right)x-1060499\right)x+11665053\right)\sqrt{3\,x^2-x+2}+\frac{9}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="giac")
```

```
[Out] 1/2903040*(2*(12*(18*(8*(30*(72*x - 37)*x + 4543)*x - 8441)*x + 519449)*x -
 1060499)*x + 11665053)*sqrt(3*x^2 - x + 2) + 944521/497664*sqrt(3)*log(-6*
sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 169/128*sqrt(13)*log(-1/2*ab
s(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)
*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

maple [A] time = 0.01, size = 207, normalized size = 1.41

$$-\frac{944521\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{497664}-\frac{169\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{128}+\frac{2\left(3x^2-x+2\right)^{\frac{7}{2}}}{21}+\frac{5\left(6x-1\right)\left(3x^2-\right.}{216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(2*x+1),x)
```

```
[Out] 2/21*(3*x^2-x+2)^(7/2)+5/216*(6*x-1)*(3*x^2-x+2)^(5/2)+575/10368*(6*x-1)*(3
*x^2-x+2)^(3/2)+13225/82944*(6*x-1)*(3*x^2-x+2)^(1/2)-944521/497664*3^(1/2)
*arcsinh(6/23*23^(1/2)*(x-1/6))+1/20*(-4*x+3*(x+1/2)^2+5/4)^(5/2)-1/48*(6*x
-1)*(-4*x+3*(x+1/2)^2+5/4)^(3/2)-25/128*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(1/2
)+13/48*(-4*x+3*(x+1/2)^2+5/4)^(3/2)+169/128*(-16*x+12*(x+1/2)^2+5)^(1/2)-1
```

```
69/128*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/
2))
```

maxima [A] time = 0.97, size = 154, normalized size = 1.05

$$\frac{2}{21}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{5}{36}\left(3x^2-x+2\right)^{\frac{5}{2}}x+\frac{29}{1080}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{359}{1728}\left(3x^2-x+2\right)^{\frac{3}{2}}x+\frac{2449}{10368}\left(3x^2-x+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="maxima")
```

```
[Out] 2/21*(3*x^2 - x + 2)^(7/2) + 5/36*(3*x^2 - x + 2)^(5/2)*x + 29/1080*(3*x^2
- x + 2)^(5/2) + 359/1728*(3*x^2 - x + 2)^(3/2)*x + 2449/10368*(3*x^2 - x +
 2)^(3/2) - 2975/13824*sqrt(3*x^2 - x + 2)*x - 944521/497664*sqrt(3)*arcsin
h(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 169/128*sqrt(13)*arcsinh(8/23*sqrt(23)
*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 221999/82944*sqrt(3*x^2 - x
 + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3x^2-x+2\right)^{5/2}\left(4x^2+3x+1\right)}{2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
```

```
[Out] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2-x+2\right)^{\frac{5}{2}}\left(4x^2+3x+1\right)}{2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x),x)
```

```
[Out] Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

3.224 $\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx$

Optimal. Leaf size=154

$$-\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}-\frac{11(37-60x)\left(3x^2-x+2\right)^{5/2}}{2340}-\frac{11}{864}(67-78x)\left(3x^2-x+2\right)^{3/2}-\frac{11(4727-3090x)\sqrt{3x^2-x+2}}{6912}$$

```
[Out] -11/864*(67-78*x)*(3*x^2-x+2)^(3/2)-11/2340*(37-60*x)*(3*x^2-x+2)^(5/2)-1/1
3*(3*x^2-x+2)^(7/2)/(1+2*x)-315623/41472*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(
1/2)+429/128*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-11/6
912*(4727-3090*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}-\frac{11(37-60x)\left(3x^2-x+2\right)^{5/2}}{2340}-\frac{11}{864}(67-78x)\left(3x^2-x+2\right)^{3/2}-\frac{11(4727-3090x)\sqrt{3x^2-x+2}}{6912}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] (-11*(4727 - 3090*x)*Sqrt[2 - x + 3*x^2])/6912 - (11*(67 - 78*x)*(2 - x + 3
*x^2)^(3/2))/864 - (11*(37 - 60*x)*(2 - x + 3*x^2)^(5/2))/2340 - (2 - x + 3
*x^2)^(7/2)/(13*(1 + 2*x)) - (315623*ArcSinh[(1 - 6*x)/Sqrt[23]])/(13824*Sq
rt[3]) + (429*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])
/128
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
```

```
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx = -\frac{\left(2-x+3x^2\right)^{7/2}}{13(1+2x)} - \frac{1}{13}\int \frac{\left(-\frac{11}{2}-44x\right)\left(2-x+3x^2\right)^{5/2}}{1+2x}\,dx$$

$$= -\frac{11(37-60x)\left(2-x+3x^2\right)^{5/2}}{2340} - \frac{\left(2-x+3x^2\right)^{7/2}}{13(1+2x)} + \frac{\int \frac{(-286+14872x)(2-}{1+2x}}{1872}$$

$$= -\frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{11(37-60x)\left(2-x+3x^2\right)^{5/2}}{2340} - \frac{(2}{}$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} -$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} -$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} -$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} -$$

Mathematica [A] time = 0.12, size = 113, normalized size = 0.73

$$\frac{694980\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)+\frac{6\sqrt{3x^2-x+2}\left(103680x^6-65664x^5+251424x^4-115680x^3+310660x^2-322972x-364257\right)}{2x+1}+15781}{207360}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] ((6*Sqrt[2 - x + 3*x^2]*(-364257 - 322972*x + 310660*x^2 - 115680*x^3 + 251
424*x^4 - 65664*x^5 + 103680*x^6))/(1 + 2*x) + 1578115*Sqrt[3]*ArcSinh[(-1
+ 6*x)/Sqrt[23]] + 694980*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x
 + 3*x^2])])/207360
```

fricas [A] time = 0.96, size = 153, normalized size = 0.99

$$\frac{1578115\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+694980\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/414720*(1578115*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
 - 1) - 72*x^2 + 24*x - 25) + 694980*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqr
t(3*x^2 - x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 12
*(103680*x^6 - 65664*x^5 + 251424*x^4 - 115680*x^3 + 310660*x^2 - 322972*x
- 364257)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

giac [B] time = 1.10, size = 760, normalized size = 4.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 429/128*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sq
rt(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 315623/41472*sqrt(3)*log(1/2*abs(
-2*sqrt(3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x +
1))/(sqrt(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)
))*sgn(1/(2*x + 1)) - 169/128*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1
/(2*x + 1)) + 1/34560*(5154065*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + s
qrt(13)/(2*x + 1))^11*sgn(1/(2*x + 1)) - 7837020*sqrt(13)*(sqrt(-8/(2*x + 1
) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^10*sgn(1/(2*x + 1)) + 3946881
5*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^9*sgn(1/(2
*x + 1)) - 14445540*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqr
t(13)/(2*x + 1))^8*sgn(1/(2*x + 1)) + 460893402*(sqrt(-8/(2*x + 1) + 13/(2*
x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^7*sgn(1/(2*x + 1)) - 343084680*sqrt(13)
*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^6*sgn(1/(2*
x + 1)) + 944150094*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*
x + 1))^5*sgn(1/(2*x + 1)) - 22871160*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x
 + 1)^2 + 3) + sqrt(13)/(2*x + 1))^4*sgn(1/(2*x + 1)) + 1397032245*(sqrt(-8
/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^3*sgn(1/(2*x + 1)) -
 683367516*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*
x + 1))^2*sgn(1/(2*x + 1)) + 392684355*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2
+ 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 197538588*sqrt(13)*sgn(1/(2*x
 + 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^2 -
 3)^6
```

maple [A] time = 0.01, size = 235, normalized size = 1.53

$$\frac{315623\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{41472}+\frac{429\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^{2}+5}}\right)}{128}+\frac{(6x-1)\left(3x^{2}-x+2\right)^{\frac{5}{2}}}{36}+\frac{115\,(6x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(2*x+1)^2,x)
```

```
[Out] 1/36*(6*x-1)*(3*x^2-x+2)^(5/2)+115/1728*(6*x-1)*(3*x^2-x+2)^(3/2)+2645/1382
4*(6*x-1)*(3*x^2-x+2)^(1/2)+315623/41472*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1
/6))-33/260*(-4*x+3*(x+1/2)^2+5/4)^(5/2)+19/192*(6*x-1)*(-4*x+3*(x+1/2)^2+5
/4)^(3/2)+965/1536*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(1/2)-11/16*(-4*x+3*(x+1/
2)^2+5/4)^(3/2)-429/128*(-16*x+12*(x+1/2)^2+5)^(1/2)+429/128*13^(1/2)*arcta
nh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))-1/26/(x+1/2)*(-4*
x+3*(x+1/2)^2+5/4)^(7/2)+1/52*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(5/2)
```

maxima [A] time = 1.00, size = 161, normalized size = 1.05

$$\frac{1}{6}\left(3\,x^{2}-x+2\right)^{\frac{5}{2}}x-\frac{7}{90}\left(3\,x^{2}-x+2\right)^{\frac{5}{2}}+\frac{143}{144}\left(3\,x^{2}-x+2\right)^{\frac{3}{2}}x-\frac{737}{864}\left(3\,x^{2}-x+2\right)^{\frac{3}{2}}-\frac{\left(3\,x^{2}-x+2\right)^{\frac{5}{2}}}{4\,(2\,x+1)}+\frac{5665}{1152}\,\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*x^2 - x + 2)^(5/2)*x - 7/90*(3*x^2 - x + 2)^(5/2) + 143/144*(3*x^2 -
 x + 2)^(3/2)*x - 737/864*(3*x^2 - x + 2)^(3/2) - 1/4*(3*x^2 - x + 2)^(5/2)
/(2*x + 1) + 5665/1152*sqrt(3*x^2 - x + 2)*x + 315623/41472*sqrt(3)*arcsinh
(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 429/128*sqrt(13)*arcsinh(8/23*sqrt(23)*
x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) - 51997/6912*sqrt(3*x^2 - x +
2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2-x+2\right)^{5/2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
```

```
[Out] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^{2}-x+2\right)^{\frac{5}{2}}\left(4x^{2}+3x+1\right)}{\left(2x+1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)
```

```
[Out] Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

3.225 $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx$

Optimal. Leaf size=161

$$-\frac{(3x^2-x+2)^{7/2}}{26(2x+1)^2}+\frac{(134x+257)(3x^2-x+2)^{5/2}}{520(2x+1)}+\frac{1}{832}(1227-838x)(3x^2-x+2)^{3/2}+\frac{(21317-10470x)\sqrt{3x^2-x}}{1536}$$

```
[Out] 1/832*(1227-838*x)*(3*x^2-x+2)^(3/2)+1/520*(257+134*x)*(3*x^2-x+2)^(5/2)/(1
+2*x)-1/26*(3*x^2-x+2)^(7/2)/(1+2*x)^2+118423/9216*arcsinh(1/23*(1-6*x)*23^
(1/2))*3^(1/2)-1631/256*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13
^(1/2)+1/1536*(21317-10470*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1650, 812, 814, 843, 619, 215, 724, 206}

$$-\frac{(3x^2-x+2)^{7/2}}{26(2x+1)^2}+\frac{(134x+257)(3x^2-x+2)^{5/2}}{520(2x+1)}+\frac{1}{832}(1227-838x)(3x^2-x+2)^{3/2}+\frac{(21317-10470x)\sqrt{3x^2-x}}{1536}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((21317 - 10470*x)*Sqrt[2 - x + 3*x^2])/1536 + ((1227 - 838*x)*(2 - x + 3*x
^2)^(3/2))/832 + ((257 + 134*x)*(2 - x + 3*x^2)^(5/2))/(520*(1 + 2*x)) - (2
 - x + 3*x^2)^(7/2)/(26*(1 + 2*x)^2) + (118423*ArcSinh[(1 - 6*x)/Sqrt[23]])
/(3072*Sqrt[3]) - (1631*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x +
 3*x^2])])/256
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
```

```
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = -\frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2} - \frac{1}{26}\int \frac{\left(-\frac{29}{2}-67x\right)(2-x+3x^2)^{5/2}}{(1+2x)^2}\,dx$$

$$= \frac{(257+134x)(2-x+3x^2)^{5/2}}{520(1+2x)} - \frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2} + \frac{1}{208}\int \frac{(793-1676x}{1}$$

$$= \frac{1}{832}(1227-838x)(2-x+3x^2)^{3/2} + \frac{(257+134x)(2-x+3x^2)^{5/2}}{520(1+2x)} - \frac{(2}{2}$$

$$= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)(2-x+3x^2)^{3/2} + $$

$$= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)(2-x+3x^2)^{3/2} + $$

$$= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)(2-x+3x^2)^{3/2} + $$

$$= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)(2-x+3x^2)^{3/2} + $$

Mathematica [A] time = 0.11, size = 113, normalized size = 0.70

$$\frac{-293580\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) + \frac{6\sqrt{3x^2-x+2}\left(27648x^6-22464x^5+83616x^4-76200x^3+256564x^2+464446x+142057\right)}{(2x+1)^2} - 592115}{46080}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((6*Sqrt[2 - x + 3*x^2]*(142057 + 464446*x + 256564*x^2 - 76200*x^3 + 83616
*x^4 - 22464*x^5 + 27648*x^6))/(1 + 2*x)^2 - 592115*Sqrt[3]*ArcSinh[(-1 + 6
*x)/Sqrt[23]] - 293580*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x +
3*x^2])])/46080
```

fricas [A] time = 0.94, size = 169, normalized size = 1.05

$$592115\sqrt{3}\left(4x^2+4x+1\right)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+293580\sqrt{13}\left(4x^2+4x+1\right)\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/92160*(592115*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)
*(6*x - 1) - 72*x^2 + 24*x - 25) + 293580*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(
4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 +
4*x + 1)) + 12*(27648*x^6 - 22464*x^5 + 83616*x^4 - 76200*x^3 + 256564*x^2
+ 464446*x + 142057)*sqrt(3*x^2 - x + 2))/(4*x^2 + 4*x + 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{-299016192,[6]%%%}+%%%{%%{[897048576,0]:[1,0,-3]%%},[5]%%%}+%%%{-448
524288,[4]%%%}+%%%{%%{[-3588194304,0]:[1,0,-3]%%},[3]%%%}+%%%{1121310720,[2
]%%%}+%%%{%%{[5606553600,0]:[1,0,-3]%%},[1]%%%}+%%%{4672128000,[0]%%%} / %%
%{%%{[24,0]:[1,0,-3]%%},[6]%%%}+%%%{-216,[5]%%%}+%%%{%%{[36,0]:[1,0,-3]%%},
[4]%%%}+%%%{864,[3]%%%}+%%%{%%{[-90,0]:[1,0,-3]%%},[2]%%%}+%%%{-1350,[1]%%%
}+%%%{%%{[-375,0]:[1,0,-3]%%},[0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.02, size = 199, normalized size = 1.24

$$-\frac{118423\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{9216}-\frac{1631\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^{2}+5}}\right)}{256}+\frac{1631\left(-4x+3\left(x+\frac{1}{2}\right)^{2}+\frac{5}{4}\right)^{\frac{5}{2}}}{6760}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(2*x+1)^3,x)
```

```
[Out] 1631/6760*(-4*x+3*(x+1/2)^2+5/4)^(5/2)+1631/1248*(-4*x+3*(x+1/2)^2+5/4)^(3/
2)+1631/256*(-16*x+12*(x+1/2)^2+5)^(1/2)-1/104/(x+1/2)^2*(-4*x+3*(x+1/2)^2+
5/4)^(7/2)-1745/1536*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(1/2)+19/338/(x+1/2)*(-
4*x+3*(x+1/2)^2+5/4)^(7/2)-19/676*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(5/2)-419/
2496*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(3/2)-1631/256*13^(1/2)*arctanh(2/13*(-
4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))-118423/9216*3^(1/2)*arcsinh
(6/23*23^(1/2)*(x-1/6))
```

maxima [A] time = 0.99, size = 172, normalized size = 1.07

$$\frac{67}{520}\left(3x^{2}-x+2\right)^{\frac{5}{2}}-\frac{\left(3x^{2}-x+2\right)^{\frac{7}{2}}}{26\left(4x^{2}+4x+1\right)}-\frac{419}{416}\left(3x^{2}-x+2\right)^{\frac{3}{2}}x+\frac{1227}{832}\left(3x^{2}-x+2\right)^{\frac{3}{2}}+\frac{19\left(3x^{2}-x+2\right)^{\frac{5}{2}}}{52\left(2x+1\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="maxima")
```

```
[Out] 67/520*(3*x^2 - x + 2)^(5/2) - 1/26*(3*x^2 - x + 2)^(7/2)/(4*x^2 + 4*x + 1)
 - 419/416*(3*x^2 - x + 2)^(3/2)*x + 1227/832*(3*x^2 - x + 2)^(3/2) + 19/52
*(3*x^2 - x + 2)^(5/2)/(2*x + 1) - 1745/256*sqrt(3*x^2 - x + 2)*x - 118423/
9216*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1631/256*sqrt(13)*a
rcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 21317/1
536*sqrt(3*x^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3x^{2}-x+2\right)^{5/2}\left(4x^{2}+3x+1\right)}{\left(2x+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

```
[Out] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2-x+2\right)^{\frac{5}{2}}\left(4x^2+3x+1\right)}{(2x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
```

```
[Out] Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

3.226 $\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=693

$$\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(96c^3\left(a^2h^2(eh+3fg)+2abh\left(h(dh+3eg)+3fg^2\right)+b^2g\left(3h(dh+eg)+fg^2\right)\right)-80bc$$

```
[Out] 1/256*(256*c^5*d*g^3-63*b^5*f*h^3+70*b^3*c*h^2*(4*a*f*h+b*e*h+3*b*f*g)-80*b
*c^2*h*(3*a^2*f*h^2+3*a*b*h*(e*h+3*f*g)+b^2*(d*h^2+3*e*g*h+3*f*g^2))-128*c^
4*g*(b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+96*c^3*(a^2*h^2*(e*h+3*f*g)+b
^2*g*(f*g^2+3*h*(d*h+e*g))+2*a*b*h*(3*f*g^2+h*(d*h+3*e*g))))*arctanh(1/2*(2
*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/240*(63*b^2*f*h^2-2*c*h*(32
*a*f*h+35*b*e*h+24*b*f*g)-c^2*(12*f*g^2-20*h*(4*d*h+3*e*g)))*(h*x+g)^2*(c*x
^2+b*x+a)^(1/2)/c^3/h-1/40*(9*b*f*h+2*c*(-5*e*h+f*g))*(h*x+g)^3*(c*x^2+b*x+
a)^(1/2)/c^2/h+1/5*f*(h*x+g)^4*(c*x^2+b*x+a)^(1/2)/c/h+1/1920*(945*b^4*f*h^
4-64*c^4*g^2*(3*f*g^2-5*h*(16*d*h+3*e*g))-210*b^2*c*h^3*(14*a*f*h+5*b*(e*h+
3*f*g))+8*c^2*h^2*(128*a^2*f*h^2+275*a*b*h*(e*h+3*f*g)+3*b^2*(129*f*g^2+50*
h*(d*h+3*e*g)))-16*c^3*h*(16*a*h*(13*f*g^2+5*h*(d*h+3*e*g))+b*g*(39*f*g^2+5
*h*(54*d*h+47*e*g)))-2*c*h*(315*b^3*f*h^3-14*b*c*h^2*(46*a*f*h+25*b*e*h+39*
b*f*g)+16*c^3*g*(3*f*g^2-5*h*(10*d*h+3*e*g))+8*c^2*h*(a*h*(45*e*h+71*f*g)+b
*(50*d*h^2+80*e*g*h+21*f*g^2)))*x)*(c*x^2+b*x+a)^(1/2)/c^5/h
```

Rubi [A] time = 2.10, antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1653, 832, 779, 621, 206}

$$\sqrt{a+bx+cx^2}\left(8c^2h^2\left(128a^2fh^2+275abh(eh+3fg)+3b^2\left(50h(dh+3eg)+129fg^2\right)\right)-2chx\left(8c^2h\left(ah(45eh\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((63*b^2*f*h^2 - 2*c*h*(24*b*f*g + 35*b*e*h + 32*a*f*h) - c^2*(12*f*g^2 - 2
0*h*(3*e*g + 4*d*h)))*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(240*c^3*h) - ((9*
b*f*h + 2*c*(f*g - 5*e*h))*(g + h*x)^3*Sqrt[a + b*x + c*x^2])/(40*c^2*h) +
(f*(g + h*x)^4*Sqrt[a + b*x + c*x^2])/(5*c*h) + ((945*b^4*f*h^4 - 64*c^4*(3
*f*g^4 - 5*g^2*h*(3*e*g + 16*d*h)) - 210*b^2*c*h^3*(14*a*f*h + 5*b*(3*f*g +
 e*h)) + 8*c^2*h^2*(128*a^2*f*h^2 + 275*a*b*h*(3*f*g + e*h) + 3*b^2*(129*f*
g^2 + 50*h*(3*e*g + d*h))) - 16*c^3*h*(16*a*h*(13*f*g^2 + 5*h*(3*e*g + d*h)
) + b*g*(39*f*g^2 + 5*h*(47*e*g + 54*d*h))) - 2*c*h*(315*b^3*f*h^3 - 14*b*c
*h^2*(39*b*f*g + 25*b*e*h + 46*a*f*h) + 16*c^3*(3*f*g^3 - 5*g*h*(3*e*g + 10
*d*h)) + 8*c^2*h*(21*b*f*g^2 + 10*b*h*(8*e*g + 5*d*h) + a*h*(71*f*g + 45*e*
h)))*x)*Sqrt[a + b*x + c*x^2])/(1920*c^5*h) + ((256*c^5*d*g^3 - 63*b^5*f*h^
3 + 70*b^3*c*h^2*(3*b*f*g + b*e*h + 4*a*f*h) - 128*c^4*g*(a*f*g^2 + 3*a*h*(
e*g + d*h) + b*g*(e*g + 3*d*h)) - 80*b*c^2*h*(3*a^2*f*h^2 + 3*a*b*h*(3*f*g
+ e*h) + b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 96*c^3*(a^2*h^2*(3*f*g + e*h) +
 b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 2*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch} + \frac{\int \frac{(g+hx)^3\left(-\frac{1}{2}h(bfg-10cdh+8afh)-\frac{1}{2}h(2cfg-10ceh+9bfh)x\right)}{\sqrt{a+bx+cx^2}}}{5ch^2}$$

$$= -\frac{(9bfh+2c(fg-5eh))(g+hx)^3\sqrt{a+bx+cx^2}}{40c^2h} + \frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}$$

$$= \frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g}{240c^3h}$$

$$= \frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g}{240c^3h}$$

$$= \frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g}{240c^3h}$$

$$= \frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g}{240c^3h}$$

Mathematica [A] time = 1.25, size = 588, normalized size = 0.85

$$\frac{\sqrt{a+x(b+cx)}\left(4c^2h\left(256a^2fh^2+2abh(275eh+825fg+161fhx)+b^2\left(25h(12dh+36eg+7ehx)+3f\left(300g\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(945*b^4*f*h^3 - 210*b^2*c*h^2*(5*b*e*h + 14*a*f*h +
 3*b*f*(5*g + h*x)) + 32*c^4*(10*d*h*(18*g^2 + 9*g*h*x + 2*h^2*x^2) + 15*e*
(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3) + 3*f*x*(10*g^3 + 20*g^2*h*x +
15*g*h^2*x^2 + 4*h^3*x^3)) + 4*c^2*h*(256*a^2*f*h^2 + 2*a*b*h*(825*f*g + 27
5*e*h + 161*f*h*x) + b^2*(25*h*(36*e*g + 12*d*h + 7*e*h*x) + 3*f*(300*g^2 +
 175*g*h*x + 42*h^2*x^2))) - 16*c^3*(a*h*(5*h*(48*e*g + 16*d*h + 9*e*h*x) +
 f*(240*g^2 + 135*g*h*x + 32*h^2*x^2)) + b*(3*f*(30*g^3 + 50*g^2*h*x + 35*g
*h^2*x^2 + 9*h^3*x^3) + 5*h*(2*d*h*(27*g + 5*h*x) + e*(54*g^2 + 30*g*h*x +
7*h^2*x^2))))))/(1920*c^5) + ((256*c^5*d*g^3 - 63*b^5*f*h^3 + 70*b^3*c*h^2*
(3*b*f*g + b*e*h + 4*a*f*h) - 128*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + b*g*
(e*g + 3*d*h)) - 80*b*c^2*h*(3*a^2*f*h^2 + 3*a*b*h*(3*f*g + e*h) + b^2*(3*f
*g^2 + 3*e*g*h + d*h^2)) + 96*c^3*(a^2*h^2*(3*f*g + e*h) + b^2*g*(f*g^2 + 3
*h*(e*g + d*h)) + 2*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(256*c^(11/2))
```

fricas [A] time = 1.73, size = 1435, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/7680*(15*(32*(8*c^5*d - 4*b*c^4*e + (3*b^2*c^3 - 4*a*c^4)*f)*g^3 - 48*(
8*b*c^4*d - 2*(3*b^2*c^3 - 4*a*c^4)*e + (5*b^3*c^2 - 12*a*b*c^3)*f)*g^2*h +
 6*(16*(3*b^2*c^3 - 4*a*c^4)*d - 8*(5*b^3*c^2 - 12*a*b*c^3)*e + (35*b^4*c -
 120*a*b^2*c^2 + 48*a^2*c^3)*f)*g*h^2 - (16*(5*b^3*c^2 - 12*a*b*c^3)*d - 2*
```

```
(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*e + (63*b^5 - 280*a*b^3*c + 240*a^2
*b*c^2)*f)*h^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x
 + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*f*h^3*x^4 + 480*(4*c^5*e -
3*b*c^4*f)*g^3 + 240*(24*c^5*d - 18*b*c^4*e + (15*b^2*c^3 - 16*a*c^4)*f)*g^
2*h - 30*(144*b*c^4*d - 8*(15*b^2*c^3 - 16*a*c^4)*e + 5*(21*b^3*c^2 - 44*a*
b*c^3)*f)*g*h^2 + (80*(15*b^2*c^3 - 16*a*c^4)*d - 50*(21*b^3*c^2 - 44*a*b*c
^3)*e + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*f)*h^3 + 48*(30*c^5*f*g
*h^2 + (10*c^5*e - 9*b*c^4*f)*h^3)*x^3 + 8*(240*c^5*f*g^2*h + 30*(8*c^5*e -
 7*b*c^4*f)*g*h^2 + (80*c^5*d - 70*b*c^4*e + (63*b^2*c^3 - 64*a*c^4)*f)*h^3
)*x^2 + 2*(480*c^5*f*g^3 + 240*(6*c^5*e - 5*b*c^4*f)*g^2*h + 30*(48*c^5*d -
 40*b*c^4*e + (35*b^2*c^3 - 36*a*c^4)*f)*g*h^2 - (400*b*c^4*d - 10*(35*b^2*
c^3 - 36*a*c^4)*e + 7*(45*b^3*c^2 - 92*a*b*c^3)*f)*h^3)*x)*sqrt(c*x^2 + b*x
 + a))/c^6, -1/3840*(15*(32*(8*c^5*d - 4*b*c^4*e + (3*b^2*c^3 - 4*a*c^4)*f)
*g^3 - 48*(8*b*c^4*d - 2*(3*b^2*c^3 - 4*a*c^4)*e + (5*b^3*c^2 - 12*a*b*c^3)
*f)*g^2*h + 6*(16*(3*b^2*c^3 - 4*a*c^4)*d - 8*(5*b^3*c^2 - 12*a*b*c^3)*e +
(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f)*g*h^2 - (16*(5*b^3*c^2 - 12*a*b*
c^3)*d - 2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*e + (63*b^5 - 280*a*b^3*
c + 240*a^2*b*c^2)*f)*h^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(384*c^5*f*h^3*x^4 + 480*(4*c^5
*e - 3*b*c^4*f)*g^3 + 240*(24*c^5*d - 18*b*c^4*e + (15*b^2*c^3 - 16*a*c^4)*
f)*g^2*h - 30*(144*b*c^4*d - 8*(15*b^2*c^3 - 16*a*c^4)*e + 5*(21*b^3*c^2 -
44*a*b*c^3)*f)*g*h^2 + (80*(15*b^2*c^3 - 16*a*c^4)*d - 50*(21*b^3*c^2 - 44*
a*b*c^3)*e + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*f)*h^3 + 48*(30*c^
5*f*g*h^2 + (10*c^5*e - 9*b*c^4*f)*h^3)*x^3 + 8*(240*c^5*f*g^2*h + 30*(8*c^
5*e - 7*b*c^4*f)*g*h^2 + (80*c^5*d - 70*b*c^4*e + (63*b^2*c^3 - 64*a*c^4)*f
)*h^3)*x^2 + 2*(480*c^5*f*g^3 + 240*(6*c^5*e - 5*b*c^4*f)*g^2*h + 30*(48*c^
5*d - 40*b*c^4*e + (35*b^2*c^3 - 36*a*c^4)*f)*g*h^2 - (400*b*c^4*d - 10*(35
*b^2*c^3 - 36*a*c^4)*e + 7*(45*b^3*c^2 - 92*a*b*c^3)*f)*h^3)*x)*sqrt(c*x^2
+ b*x + a))/c^6]
```

giac [A] time = 0.32, size = 822, normalized size = 1.19

$$\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(\frac{8fh^3x}{c}+\frac{30c^4fgh^2-9bc^3fh^3+10c^4h^3e}{c^5}\right)x+\frac{240c^4fg^2h-210bc^3fgh^2+80c^4dh\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*f*h^3*x/c + (30*c^4*f*g*h^2 - 9*b*
c^3*f*h^3 + 10*c^4*h^3*e)/c^5)*x + (240*c^4*f*g^2*h - 210*b*c^3*f*g*h^2 + 8
0*c^4*d*h^3 + 63*b^2*c^2*f*h^3 - 64*a*c^3*f*h^3 + 240*c^4*g*h^2*e - 70*b*c^
3*h^3*e)/c^5)*x + (480*c^4*f*g^3 - 1200*b*c^3*f*g^2*h + 1440*c^4*d*g*h^2 +
1050*b^2*c^2*f*g*h^2 - 1080*a*c^3*f*g*h^2 - 400*b*c^3*d*h^3 - 315*b^3*c*f*h
^3 + 644*a*b*c^2*f*h^3 + 1440*c^4*g^2*h*e - 1200*b*c^3*g*h^2*e + 350*b^2*c^
2*h^3*e - 360*a*c^3*h^3*e)/c^5)*x - (1440*b*c^3*f*g^3 - 5760*c^4*d*g^2*h -
3600*b^2*c^2*f*g^2*h + 3840*a*c^3*f*g^2*h + 4320*b*c^3*d*g*h^2 + 3150*b^3*c
*f*g*h^2 - 6600*a*b*c^2*f*g*h^2 - 1200*b^2*c^2*d*h^3 + 1280*a*c^3*d*h^3 - 9
45*b^4*f*h^3 + 2940*a*b^2*c*f*h^3 - 1024*a^2*c^2*f*h^3 - 1920*c^4*g^3*e + 4
320*b*c^3*g^2*h*e - 3600*b^2*c^2*g*h^2*e + 3840*a*c^3*g*h^2*e + 1050*b^3*c*
h^3*e - 2200*a*b*c^2*h^3*e)/c^5) - 1/256*(256*c^5*d*g^3 + 96*b^2*c^3*f*g^3
- 128*a*c^4*f*g^3 - 384*b*c^4*d*g^2*h - 240*b^3*c^2*f*g^2*h + 576*a*b*c^3*f
*g^2*h + 288*b^2*c^3*d*g*h^2 - 384*a*c^4*d*g*h^2 + 210*b^4*c*f*g*h^2 - 720*
a*b^2*c^2*f*g*h^2 + 288*a^2*c^3*f*g*h^2 - 80*b^3*c^2*d*h^3 + 192*a*b*c^3*d*
h^3 - 63*b^5*f*h^3 + 280*a*b^3*c*f*h^3 - 240*a^2*b*c^2*f*h^3 - 128*b*c^4*g^
3*e + 288*b^2*c^3*g^2*h*e - 384*a*c^4*g^2*h*e - 240*b^3*c^2*g*h^2*e + 576*a
*b*c^3*g*h^2*e + 70*b^4*c*h^3*e - 240*a*b^2*c^2*h^3*e + 96*a^2*c^3*h^3*e)*l
og(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

maple [B] time = 0.02, size = 1869, normalized size = 2.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] g^3*d*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/c*(c*x^2+b*x+a)
^(1/2)*g^3*e-3/4/c^2*b*(c*x^2+b*x+a)^(1/2)*g^3*f+3/8/c^(5/2)*b^2*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^3*f-1/2*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))*g^3*f+161/240*h^3*f/c^3*b*a*x*(c*x^2+b*x+a)^(1/2)-7/
8/c^2*b*x^2*(c*x^2+b*x+a)^(1/2)*g*h^2*f+35/32/c^3*b^2*x*(c*x^2+b*x+a)^(1/2)
*g*h^2*f-45/16/c^(7/2)*b^2*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*
h^2*f+55/16/c^3*b*a*(c*x^2+b*x+a)^(1/2)*g*h^2*f-9/8*a/c^2*x*(c*x^2+b*x+a)^(
1/2)*g*h^2*f-5/4/c^2*b*x*(c*x^2+b*x+a)^(1/2)*g*h^2*e-5/4/c^2*b*x*(c*x^2+b*x
+a)^(1/2)*g^2*h*f+9/4/c^(5/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))*g*h^2*e+9/4/c^(5/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^2*
h*f+3/4*x^3/c*(c*x^2+b*x+a)^(1/2)*g*h^2*f-7/24/c^2*b*x^2*(c*x^2+b*x+a)^(1/2
)*h^3*e-21/64*h^3*f/c^4*b^3*x*(c*x^2+b*x+a)^(1/2)+35/32*h^3*f/c^(9/2)*b^3*a
*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-49/32*h^3*f/c^4*b^2*a*(c*x^2+b
*x+a)^(1/2)-15/16*h^3*f/c^(7/2)*b*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))-9/40*h^3*f/c^2*b*x^3*(c*x^2+b*x+a)^(1/2)+21/80*h^3*f/c^3*b^2*x^2*(c*
x^2+b*x+a)^(1/2)+15/8/c^3*b^2*(c*x^2+b*x+a)^(1/2)*g^2*h*f-15/16/c^(7/2)*b^3
*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*e-15/16/c^(7/2)*b^3*ln((
c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^2*h*f+3/4/c^(5/2)*b*a*ln((c*x+1/2
*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*d-2*a/c^2*(c*x^2+b*x+a)^(1/2)*g*h^2*e-
2*a/c^2*(c*x^2+b*x+a)^(1/2)*g^2*h*f+3/2*x/c*(c*x^2+b*x+a)^(1/2)*g*h^2*d+3/2
*x/c*(c*x^2+b*x+a)^(1/2)*g^2*h*e-9/4/c^2*b*(c*x^2+b*x+a)^(1/2)*g*h^2*d-9/4/
c^2*b*(c*x^2+b*x+a)^(1/2)*g^2*h*e+9/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))*g*h^2*d+9/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))*g^2*h*e-3/2*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))*g*h^2*d-3/2*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^2*h*
e-3/2*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^2*h*d+1/5*h^3
*f*x^4/c*(c*x^2+b*x+a)^(1/2)+63/128*h^3*f/c^5*b^4*(c*x^2+b*x+a)^(1/2)-63/25
6*h^3*f/c^(11/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+35/96/c^3*
b^2*x*(c*x^2+b*x+a)^(1/2)*h^3*e+8/15*h^3*f*a^2/c^3*(c*x^2+b*x+a)^(1/2)+1/4*
x^3/c*(c*x^2+b*x+a)^(1/2)*h^3*e-35/64/c^4*b^3*(c*x^2+b*x+a)^(1/2)*h^3*e+35/
128/c^(9/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*e+3/8*a^2/c
^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*e+1/3*x^2/c*(c*x^2+b
*x+a)^(1/2)*h^3*d+5/8/c^3*b^2*(c*x^2+b*x+a)^(1/2)*h^3*d-5/16/c^(7/2)*b^3*ln
((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*d-2/3*a/c^2*(c*x^2+b*x+a)^(1/
2)*h^3*d+1/2*x/c*(c*x^2+b*x+a)^(1/2)*g^3*f+3/c*(c*x^2+b*x+a)^(1/2)*g^2*h*d-
1/2*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^3*e-105/64/c^4*
b^3*(c*x^2+b*x+a)^(1/2)*g*h^2*f+105/128/c^(9/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+
(c*x^2+b*x+a)^(1/2))*g*h^2*f-15/16/c^(7/2)*b^2*a*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*h^3*e+55/48/c^3*b*a*(c*x^2+b*x+a)^(1/2)*h^3*e-3/8*a/c^2*x
*(c*x^2+b*x+a)^(1/2)*h^3*e+9/8*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))*g*h^2*f+x^2/c*(c*x^2+b*x+a)^(1/2)*g*h^2*e+x^2/c*(c*x^2+b*x+a)^(
1/2)*g^2*h*f-5/12/c^2*b*x*(c*x^2+b*x+a)^(1/2)*h^3*d+15/8/c^3*b^2*(c*x^2+b*x
+a)^(1/2)*g*h^2*e-4/15*h^3*f*a/c^2*x^2*(c*x^2+b*x+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g+h\,x\right)^3\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g+hx\right)^3\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**3*(d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

3.227 $\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=420

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(dh^2+2egh+fg^2\right)\right)-40b^2ch(3afh+beh+2bfg\right.}{128c^{9/2}}$$

```
[Out] 1/128*(128*c^4*d*g^2+35*b^4*f*h^2-40*b^2*c*h*(3*a*f*h+b*e*h+2*b*f*g)-64*c^3
*(b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+48*c^2*(a^2*f*h^2+2*a*b*h*(e*h+2
*f*g)+b^2*(d*h^2+2*e*g*h+f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+
a)^(1/2))/c^(9/2)-1/24*(7*b*f*h-8*c*e*h+2*c*f*g)*(h*x+g)^2*(c*x^2+b*x+a)^(1
/2)/c^2/h+1/4*f*(h*x+g)^3*(c*x^2+b*x+a)^(1/2)/c/h-1/192*(105*b^3*f*h^3+32*c
^3*g*(f*g^2-4*h*(3*d*h+e*g))-20*b*c*h^2*(11*a*f*h+6*b*(e*h+2*f*g))+8*c^2*h*
(16*a*h*(e*h+2*f*g)+b*(11*f*g^2+18*h*(d*h+2*e*g)))-2*c*h*(35*b^2*f*h^2-4*c*
h*(9*a*f*h+10*b*e*h+6*b*f*g)-8*c^2*(f*g^2-2*h*(3*d*h+2*e*g)))*x)*(c*x^2+b*x
+a)^(1/2)/c^4/h
```

Rubi [A] time = 1.01, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1653, 832, 779, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(h(dh+2eg)+fg^2\right)\right)-40b^2ch(3afh+beh+2bfg\right.}{128c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] -((2*c*f*g - 8*c*e*h + 7*b*f*h)*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(24*c^2*
h) + (f*(g + h*x)^3*Sqrt[a + b*x + c*x^2])/(4*c*h) - ((105*b^3*f*h^3 + 32*c
^3*(f*g^3 - 4*g*h*(e*g + 3*d*h)) - 20*b*c*h^2*(11*a*f*h + 6*b*(2*f*g + e*h)
) + 8*c^2*h*(11*b*f*g^2 + 18*b*h*(2*e*g + d*h) + 16*a*h*(2*f*g + e*h)) - 2*
c*h*(35*b^2*f*h^2 - 4*c*h*(6*b*f*g + 10*b*e*h + 9*a*f*h) - 8*c^2*(f*g^2 - 2
*h*(2*e*g + 3*d*h)))*x)*Sqrt[a + b*x + c*x^2])/(192*c^4*h) + ((128*c^4*d*g^
2 + 35*b^4*f*h^2 - 40*b^2*c*h*(2*b*f*g + b*e*h + 3*a*f*h) - 64*c^3*(a*f*g^2
 + a*h*(2*e*g + d*h) + b*g*(e*g + 2*d*h)) + 48*c^2*(a^2*f*h^2 + 2*a*b*h*(2*
f*g + e*h) + b^2*(f*g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + b*x + c*x^2])])/(128*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} + \frac{\int \frac{(g+hx)^2\left(-\frac{1}{2}h(bfg-8cdh+6afh)-\frac{1}{2}h(2cfg-8ceh+7bfh)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{4ch^2}$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} +$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} -$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} -$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} -$$

Mathematica [A] time = 0.65, size = 343, normalized size = 0.82

$$3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(h(dh+2eg)+fg^2\right)\right)-40b^2ch(3afh+beh+2bfg$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^3*f*h^2 + 10*b*c*h*(22*a*f*h + b*(
24*f*g + 12*e*h + 7*f*h*x)) + 16*c^3*(6*d*h*(4*g + h*x) + 4*e*(3*g^2 + 3*g*
h*x + h^2*x^2) + f*x*(6*g^2 + 8*g*h*x + 3*h^2*x^2)) - 8*c^2*(2*b*h*(18*e*g
```

```
+ 9*d*h + 5*e*h*x) + a*h*(32*f*g + 16*e*h + 9*f*h*x) + b*f*(18*g^2 + 20*g*h
*x + 7*h^2*x^2))) + 3*(128*c^4*d*g^2 + 35*b^4*f*h^2 - 40*b^2*c*h*(2*b*f*g +
 b*e*h + 3*a*f*h) - 64*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + b*g*(e*g + 2*d*h)
) + 48*c^2*(a^2*f*h^2 + 2*a*b*h*(2*f*g + e*h) + b^2*(f*g^2 + h*(2*e*g + d*h
))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(384*c^(9/2))
```

fricas [A] time = 1.33, size = 861, normalized size = 2.05

$$\left[\frac{3\left(16\left(8c^4d-4bc^3e+\left(3b^2c^2-4ac^3\right)f\right)g^2-16\left(8bc^3d-2\left(3b^2c^2-4ac^3\right)e+\left(5b^3c-12abc^2\right)f\right)gh+\left(16\left(3\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/768*(3*(16*(8*c^4*d - 4*b*c^3*e + (3*b^2*c^2 - 4*a*c^3)*f)*g^2 - 16*(8*b
*c^3*d - 2*(3*b^2*c^2 - 4*a*c^3)*e + (5*b^3*c - 12*a*b*c^2)*f)*g*h + (16*(3
*b^2*c^2 - 4*a*c^3)*d - 8*(5*b^3*c - 12*a*b*c^2)*e + (35*b^4 - 120*a*b^2*c
+ 48*a^2*c^2)*f)*h^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*h^2*x^3 + 48*(4*c^4*
e - 3*b*c^3*f)*g^2 + 16*(24*c^4*d - 18*b*c^3*e + (15*b^2*c^2 - 16*a*c^3)*f)
*g*h - (144*b*c^3*d - 8*(15*b^2*c^2 - 16*a*c^3)*e + 5*(21*b^3*c - 44*a*b*c^
2)*f)*h^2 + 8*(16*c^4*f*g*h + (8*c^4*e - 7*b*c^3*f)*h^2)*x^2 + 2*(48*c^4*f*
g^2 + 16*(6*c^4*e - 5*b*c^3*f)*g*h + (48*c^4*d - 40*b*c^3*e + (35*b^2*c^2 -
 36*a*c^3)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/384*(3*(16*(8*c^4*d -
4*b*c^3*e + (3*b^2*c^2 - 4*a*c^3)*f)*g^2 - 16*(8*b*c^3*d - 2*(3*b^2*c^2 - 4
*a*c^3)*e + (5*b^3*c - 12*a*b*c^2)*f)*g*h + (16*(3*b^2*c^2 - 4*a*c^3)*d - 8
*(5*b^3*c - 12*a*b*c^2)*e + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f)*h^2)*sqr
t(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*
x + a*c)) - 2*(48*c^4*f*h^2*x^3 + 48*(4*c^4*e - 3*b*c^3*f)*g^2 + 16*(24*c^4
*d - 18*b*c^3*e + (15*b^2*c^2 - 16*a*c^3)*f)*g*h - (144*b*c^3*d - 8*(15*b^2
*c^2 - 16*a*c^3)*e + 5*(21*b^3*c - 44*a*b*c^2)*f)*h^2 + 8*(16*c^4*f*g*h + (
8*c^4*e - 7*b*c^3*f)*h^2)*x^2 + 2*(48*c^4*f*g^2 + 16*(6*c^4*e - 5*b*c^3*f)*
g*h + (48*c^4*d - 40*b*c^3*e + (35*b^2*c^2 - 36*a*c^3)*f)*h^2)*x)*sqrt(c*x^
2 + b*x + a))/c^5]
```

giac [A] time = 0.30, size = 457, normalized size = 1.09

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6fh^2x}{c}+\frac{16c^3fgh-7bc^2fh^2+8c^3h^2e}{c^4}\right)x+\frac{48c^3fg^2-80bc^2fgh+48c^3dh^2+35b}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*h^2*x/c + (16*c^3*f*g*h - 7*b*c^2*f*
h^2 + 8*c^3*h^2*e)/c^4)*x + (48*c^3*f*g^2 - 80*b*c^2*f*g*h + 48*c^3*d*h^2 +
 35*b^2*c*f*h^2 - 36*a*c^2*f*h^2 + 96*c^3*g*h*e - 40*b*c^2*h^2*e)/c^4)*x -
(144*b*c^2*f*g^2 - 384*c^3*d*g*h - 240*b^2*c*f*g*h + 256*a*c^2*f*g*h + 144*
b*c^2*d*h^2 + 105*b^3*f*h^2 - 220*a*b*c*f*h^2 - 192*c^3*g^2*e + 288*b*c^2*g
*h*e - 120*b^2*c*h^2*e + 128*a*c^2*h^2*e)/c^4) - 1/128*(128*c^4*d*g^2 + 48*
b^2*c^2*f*g^2 - 64*a*c^3*f*g^2 - 128*b*c^3*d*g*h - 80*b^3*c*f*g*h + 192*a*b
*c^2*f*g*h + 48*b^2*c^2*d*h^2 - 64*a*c^3*d*h^2 + 35*b^4*f*h^2 - 120*a*b^2*c
*f*h^2 + 48*a^2*c^2*f*h^2 - 64*b*c^3*g^2*e + 96*b^2*c^2*g*h*e - 128*a*c^3*g
*h*e - 40*b^3*c*h^2*e + 96*a*b*c^2*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

maple [B] time = 0.01, size = 1069, normalized size = 2.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/c*(c*x^2+b*x+a)^(1/2)*g^2*e+g^2*d*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))/c^(1/2)-5/6/c^2*b*x*(c*x^2+b*x+a)^(1/2)*g*h*f+3/2/c^(5/2)*b*a*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h*f+5/4/c^3*b^2*(c*x^2+b*x+a)^(1/2)*g
*h*f-4/3*a/c^2*(c*x^2+b*x+a)^(1/2)*g*h*f+x/c*(c*x^2+b*x+a)^(1/2)*e*g*h+3/4/
c^(5/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^2*e+2/3*x^2/c*(c*
x^2+b*x+a)^(1/2)*g*h*f-5/12/c^2*b*x*(c*x^2+b*x+a)^(1/2)*h^2*e+2/c*(c*x^2+b*
x+a)^(1/2)*g*h*d-1/2*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
g^2*e+3/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2+1/3
*x^2/c*(c*x^2+b*x+a)^(1/2)*h^2*e-5/8/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*g*h*f+35/96*h^2*f/c^3*b^2*x*(c*x^2+b*x+a)^(1/2)-15/16*h^2
*f/c^(7/2)*b^2*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+55/48*h^2*f/c^
3*b*a*(c*x^2+b*x+a)^(1/2)-3/8*h^2*f*a/c^2*x*(c*x^2+b*x+a)^(1/2)-7/24*h^2*f/
c^2*b*x^2*(c*x^2+b*x+a)^(1/2)-3/2/c^2*b*(c*x^2+b*x+a)^(1/2)*e*g*h+3/4/c^(5/
2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g*h-a/c^(3/2)*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g*h-b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+
(c*x^2+b*x+a)^(1/2))*g*h*d-2/3*a/c^2*(c*x^2+b*x+a)^(1/2)*h^2*e+1/2*x/c*(c*x
^2+b*x+a)^(1/2)*d*h^2+1/4*h^2*f*x^3/c*(c*x^2+b*x+a)^(1/2)-35/64*h^2*f/c^4*b
^3*(c*x^2+b*x+a)^(1/2)+35/128*h^2*f/c^(9/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))+3/8*h^2*f*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))+1/2*x/c*(c*x^2+b*x+a)^(1/2)*f*g^2-3/4/c^2*b*(c*x^2+b*x+a)^(1/2)*d*h
^2-1/2*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2-1/2*a/c^
(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2-3/4/c^2*b*(c*x^2+b*
x+a)^(1/2)*f*g^2+3/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
)*f*g^2+5/8/c^3*b^2*(c*x^2+b*x+a)^(1/2)*h^2*e-5/16/c^(7/2)*b^3*ln((c*x+1/2*
b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^2*e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g+h\,x\right)^2\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g + hx\right)^{2} \left(d + ex + fx^{2}\right)}{\sqrt{a + bx + cx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**2*(d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

3.228 $\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=223

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(aeh+afg+bdh+beg)+6bc(2afh+beh+bfg)-5b^3fh+16c^3dg\right)}{16c^{7/2}}+\sqrt{a+bx+c}$$

```
[Out] 1/16*(16*c^3*d*g-5*b^3*f*h-8*c^2*(a*e*h+a*f*g+b*d*h+b*e*g)+6*b*c*(2*a*f*h+b
*e*h+b*f*g))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/3
*f*(h*x+g)^2*(c*x^2+b*x+a)^(1/2)/c/h+1/24*(15*b^2*f*h^2-8*c^2*(f*g^2-3*h*(d
*h+e*g))-2*c*h*(8*a*f*h+9*b*(e*h+f*g))-2*c*h*(5*b*f*h-6*c*e*h+2*c*f*g)*x)*(
c*x^2+b*x+a)^(1/2)/c^3/h
```

Rubi [A] time = 0.30, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1653, 779, 621, 206}

$$\frac{\sqrt{a+bx+cx^2}\left(-2ch(8afh+9b(eh+fg))+15b^2fh^2-2chx(5bfh-6ceh+2cfg)-8c^2\left(fg^2-3h(dh+eg)\right)\right)}{24c^3h}+\text{t}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (f*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c*h) + ((15*b^2*f*h^2 - 8*c^2*(f*g
^2 - 3*h*(e*g + d*h)) - 2*c*h*(8*a*f*h + 9*b*(f*g + e*h)) - 2*c*h*(2*c*f*g
- 6*c*e*h + 5*b*f*h)*x)*Sqrt[a + b*x + c*x^2])/(24*c^3*h) + ((16*c^3*d*g -
5*b^3*f*h - 8*c^2*(b*e*g + a*f*g + b*d*h + a*e*h) + 6*b*c*(b*f*g + b*e*h +
2*a*f*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(7/
2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
```

```
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\int \frac{(g+hx)\left(-\frac{1}{2}h(bfg-6cdh+4afh)-\frac{1}{2}h(2cfg-6ceh+5bfh)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{3ch^2}$$

$$= \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\left(15b^2fh^2-8c^2\left(fg^2-3h(eg+dh)\right)-2ch(8afh\right.}{}$$

$$= \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\left(15b^2fh^2-8c^2\left(fg^2-3h(eg+dh)\right)-2ch(8afh\right.}{}$$

$$= \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\left(15b^2fh^2-8c^2\left(fg^2-3h(eg+dh)\right)-2ch(8afh\right.}{}$$

Mathematica [A] time = 0.23, size = 215, normalized size = 0.96

$$\frac{-\frac{3h\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(8c^2(aeh+afg+bdh+beg)-6bc(2afh+beh+bfg)+5b^3fh-16c^3dg\right)}{16c^{5/2}} + \frac{\sqrt{a+x(b+cx)}\left(-2ch(8afh+b(9eh+9fg+5fhx))+1\right.}{8c^2}}{3ch}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (f*(g + h*x)^2*Sqrt[a + x*(b + c*x)] + (Sqrt[a + x*(b + c*x)]*(15*b^2*f*h^2
 - 4*c^2*(f*g*(2*g + h*x) - 3*h*(2*e*g + 2*d*h + e*h*x)) - 2*c*h*(8*a*f*h +
 b*(9*f*g + 9*e*h + 5*f*h*x))))/(8*c^2) - (3*h*(-16*c^3*d*g + 5*b^3*f*h + 8
*c^2*(b*e*g + a*f*g + b*d*h + a*e*h) - 6*b*c*(b*f*g + b*e*h + 2*a*f*h))*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(16*c^(5/2)))/(3*c*h)
```

fricas [A] time = 1.11, size = 461, normalized size = 2.07

$$\left[\frac{3\left(2\left(8c^3d-4bc^2e+\left(3b^2c-4ac^2\right)f\right)g-\left(8bc^2d-2\left(3b^2c-4ac^2\right)e+\left(5b^3-12abc\right)f\right)h\right)\sqrt{c}\log\left(-8c^2x^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(2*(8*c^3*d - 4*b*c^2*e + (3*b^2*c - 4*a*c^2)*f)*g - (8*b*c^2*d -
2*(3*b^2*c - 4*a*c^2)*e + (5*b^3 - 12*a*b*c)*f)*h)*sqrt(c)*log(-8*c^2*x^2 -
 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(
8*c^3*f*h*x^2 + 6*(4*c^3*e - 3*b*c^2*f)*g + (24*c^3*d - 18*b*c^2*e + (15*b^
2*c - 16*a*c^2)*f)*h + 2*(6*c^3*f*g + (6*c^3*e - 5*b*c^2*f)*h)*x)*sqrt(c*x^
2 + b*x + a))/c^4, -1/48*(3*(2*(8*c^3*d - 4*b*c^2*e + (3*b^2*c - 4*a*c^2)*f
)*g - (8*b*c^2*d - 2*(3*b^2*c - 4*a*c^2)*e + (5*b^3 - 12*a*b*c)*f)*h)*sqrt(
-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x
```

```
+ a*c)) - 2*(8*c^3*f*h*x^2 + 6*(4*c^3*e - 3*b*c^2*f)*g + (24*c^3*d - 18*b*c
^2*e + (15*b^2*c - 16*a*c^2)*f)*h + 2*(6*c^3*f*g + (6*c^3*e - 5*b*c^2*f)*h)
*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

giac [A] time = 0.27, size = 210, normalized size = 0.94

$$\frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(\frac{4\,fhx}{c}+\frac{6\,c^2fg-5\,bcfh+6\,c^2he}{c^3}\right)x-\frac{18\,bcfg-24\,c^2dh-15\,b^2fh+16\,acfh-24\,c^2ge+1}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*f*h*x/c + (6*c^2*f*g - 5*b*c*f*h + 6*c^2*h
*e)/c^3)*x - (18*b*c*f*g - 24*c^2*d*h - 15*b^2*f*h + 16*a*c*f*h - 24*c^2*g*
e + 18*b*c*h*e)/c^3) - 1/16*(16*c^3*d*g + 6*b^2*c*f*g - 8*a*c^2*f*g - 8*b*c
^2*d*h - 5*b^3*f*h + 12*a*b*c*f*h - 8*b*c^2*g*e + 6*b^2*c*h*e - 8*a*c^2*h*e
)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

maple [B] time = 0.01, size = 505, normalized size = 2.26

$$\frac{\sqrt{c\,x^2+bx+a}\;fh\,x^2}{3c}+\frac{3abfh\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{4c^{\frac{5}{2}}}-\frac{aeh\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}-\frac{afg\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2}\right.}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/3*h*f*x^2/c*(c*x^2+b*x+a)^(1/2)-5/12*h*f/c^2*b*x*(c*x^2+b*x+a)^(1/2)+5/8*
h*f/c^3*b^2*(c*x^2+b*x+a)^(1/2)-5/16*h*f/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))+3/4*h*f/c^(5/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))-2/3*h*f*a/c^2*(c*x^2+b*x+a)^(1/2)+1/2*x/c*(c*x^2+b*x+a)^(1/2)*e*
h+1/2*x/c*(c*x^2+b*x+a)^(1/2)*f*g-3/4/c^2*b*(c*x^2+b*x+a)^(1/2)*e*h-3/4/c^2
*b*(c*x^2+b*x+a)^(1/2)*f*g+3/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))*e*h+3/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*f*g-1/2*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h-1/2*a/c^
(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g+1/c*(c*x^2+b*x+a)^(1/
2)*d*h+1/c*(c*x^2+b*x+a)^(1/2)*e*g-1/2*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*d*h-1/2*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*e*g+d*g*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g+h\,x\right)\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g + hx)(d + ex + fx^2)}{\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((g + h*x)*(d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

3.229 $\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=116

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}\,(4ce-3bf)}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

```
[Out] 1/8*(8*c^2*d+3*b^2*f-4*c*(a*f+b*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*
x+a)^(1/2))/c^(5/2)+1/4*(-3*b*f+4*c*e)*(c*x^2+b*x+a)^(1/2)/c^2+1/2*f*x*(c*x
^2+b*x+a)^(1/2)/c
```

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}\,(4ce-3bf)}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((4*c*e - 3*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (f*x*Sqrt[a + b*x + c*x^2
])/(2*c) + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{2cd-af+\frac{1}{2}(4ce-3bf)x}{\sqrt{a+bx+cx^2}}\,dx}{2c}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3bf)\right)\int \frac{}{\sqrt{}}}{4c^2}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3bf)\right)\text{Sub}}{2c^2}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(8c^2d+3b^2f-4c(be+af)\right)\tanh^{-1}}{8c^{5/2}}$$

Mathematica [A] time = 0.15, size = 96, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}} + \frac{\sqrt{a+x(b+cx)}\,(-3bf+4ce+2cfx)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((4*c*e - 3*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) + ((8*c^2*d + 3*b
^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)
])])/(8*c^(5/2))
```

fricas [A] time = 0.86, size = 227, normalized size = 1.96

$$\left[-\frac{\left(8\,c^2d-4\,bce+\left(3\,b^2-4\,ac\right)f\right)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{c}-4\,ac\right)-4\,\left(2\,c\right.}{16\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*
b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*c
^2*f*x + 4*c^2*e - 3*b*c*f)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((8*c^2*d - 4*
b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*f*x + 4*c^2*e - 3*b*c*f)
*sqrt(c*x^2 + b*x + a))/c^3]
```

giac [A] time = 0.25, size = 98, normalized size = 0.84

$$\frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2\,fx}{c}-\frac{3\,bf-4\,ce}{c^2}\right)-\frac{\left(8\,c^2d+3\,b^2f-4\,acf-4\,bce\right)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right.\right.}{8\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*f*x/c - (3*b*f - 4*c*e)/c^2) - 1/8*(8*c^2*d +
3*b^2*f - 4*a*c*f - 4*b*c*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*sqrt(c) - b))/c^(5/2)
```

maple [A] time = 0.01, size = 185, normalized size = 1.59

$$-\frac{af\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}+\frac{3b^2f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{5}{2}}}-\frac{be\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}+\frac{d\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*f*x*(c*x^2+b*x+a)^(1/2)/c-3/4*f/c^2*b*(c*x^2+b*x+a)^(1/2)+3/8*f/c^(5/2)
*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*f*a/c^(3/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+e/c*(c*x^2+b*x+a)^(1/2)-1/2*e*b/c^(3/2)*l
n((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+d*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))/c^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2+e\,x+d}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

3.230 $\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=179

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bfh-2ceh+2cfg)}{2c^{3/2}h^2}+\frac{\left(fg^2-h(eg-dh)\right)\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{h^2\sqrt{ah^2-bgh+cg^2}}+\frac{f\sqrt{a+b\ldots}}{c\ldots}$$

```
[Out] -1/2*(b*f*h-2*c*e*h+2*c*f*g)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/c^(3/2)/h^2+(f*g^2-h*(-d*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x
)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^2/(a*h^2-b*g*h+c*g^2)^(1
/2)+f*(c*x^2+b*x+a)^(1/2)/c/h
```

Rubi [A] time = 0.29, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1653, 843, 621, 206, 724}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bfh-2ceh+2cfg)}{2c^{3/2}h^2}+\frac{\left(fg^2-h(eg-dh)\right)\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{h^2\sqrt{ah^2-bgh+cg^2}}+\frac{f\sqrt{a+b\ldots}}{c\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (f*Sqrt[a + b*x + c*x^2])/(c*h) - ((2*c*f*g - 2*c*e*h + b*f*h)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*h^2) + ((f*g^2 - h*(
e*g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
 a*h^2]*Sqrt[a + b*x + c*x^2])])/(h^2*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \frac{f\sqrt{a+bx+cx^2}}{ch} + \frac{\int \frac{-\frac{1}{2}h(bfg-2cdh)-\frac{1}{2}h(2cfg-2ceh+bfh)x}{(g+hx)\sqrt{a+bx+cx^2}}\,dx}{ch^2}$$

$$= \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2ch^2} + \frac{\left(fg^2-egh+dh^2\right)\int \frac{}{(g}}{h^2}$$

$$= \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{ch^2} - \frac{\left(2\left(f\right.\right.}{}$$

$$= \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}h^2} + \frac{\left(fg^2-h(eg-\right.}{}$$

Mathematica [A] time = 0.28, size = 172, normalized size = 0.96

$$-\frac{\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)(bfh-2ceh+2cfg)}{c^{3/2}} + \frac{2\left(h(dh-eg)+fg^2\right)\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)}{\sqrt{h(ah-bg)+cg^2}} - \frac{2fh\sqrt{a+x(b+cx)}}{c}}{2h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/2*((-2*f*h*Sqrt[a + x*(b + c*x)])/c + ((2*c*f*g - 2*c*e*h + b*f*h)*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + (2*(f*g^2 + h*
(-(e*g) + d*h))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 +
h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*g^2 + h*(-(b*g) + a*h)])/
h^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 599, normalized size = 3.35

$$-\frac{d\ln\left(\frac{\frac{(hb-2cg)\left(x+\frac{g}{h}\right)}{h}+\frac{2a\,h^2-2bgh+2c\,g^2}{h^2}+2\sqrt{\frac{a\,h^2-bgh+c\,g^2}{h^2}}\,\sqrt{\left(x+\frac{g}{h}\right)^2c+\frac{(hb-2cg)\left(x+\frac{g}{h}\right)}{h}+\frac{a\,h^2-bgh+c\,g^2}{h^2}}}{x+\frac{g}{h}}\right)}{\sqrt{\frac{a\,h^2-bgh+c\,g^2}{h^2}}\,h}+\frac{eg\ln\left(\frac{\frac{(hb-2cg)\left(x+\frac{g}{h}\right)}{h}+\frac{2a\,h^2-2bgh+2c\,g^2}{h^2}\cdots}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] f*(c*x^2+b*x+a)^(1/2)/c/h-1/2/h*f*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))+1/h*e*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/h^
2*f*g*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/h/((a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2))/(x+g/h))*d+1/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((
(b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)
^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(
x+g/h))*e*g-1/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h
+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(
b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*f*g^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `assum
e?` for more details)Is (b/h-(2*c*g)/h^2)^2    -(4*c       *((-(b*g)/h)
    +(c*g^2)/h^2+a))     /h^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)\,\sqrt{c\,x^2+b\,x+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^{2}}{\left(g + hx\right) \sqrt{a + bx + cx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)*sqrt(a + b*x + c*x**2)), x)
```

3.231 $\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=241

$$-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}-\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h\left(2ah(2fg-eh)-b\left(-dh^2-egh+3fg^2\right)\right.\right.}{2h^2\left(ah^2-bgh+cg^2\right)^{3/2}}$$

```
[Out] -1/2*(2*c*(-d*g*h^2+f*g^3)+h*(2*a*h*(-e*h+2*f*g)-b*(-d*h^2-e*g*h+3*f*g^2)))
*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*
x+a)^(1/2))/h^2/(a*h^2-b*g*h+c*g^2)^(3/2)+f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(
c*x^2+b*x+a)^(1/2))/h^2/c^(1/2)-(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2)/h/
(a*h^2-b*g*h+c*g^2)/(h*x+g)
```

Rubi [A] time = 0.37, antiderivative size = 239, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1650, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}-\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(2c\left(fg^3-dgh^2\right)-h\left(-2ah(2fg-eh)-bh\right.\right.}{2h^2\left(ah^2-bgh+cg^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(((f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(h*(c*g^2 - b*g*h + a*h^2
)*(g + h*x))) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
(Sqrt[c]*h^2) - ((2*c*(f*g^3 - d*g*h^2) - h*(3*b*f*g^2 - b*h*(e*g + d*h) -
2*a*h*(2*f*g - e*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2
 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*h^2*(c*g^2 - b*g*h + a*h^2)^(
3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} - \frac{\int \frac{\frac{1}{2}\left(-2cdg+beg+2afg-\frac{bfg^2}{h}+bdh-2aeh\right)+f\left(bg-\cdots\right.}{(g+hx)\sqrt{a+bx+cx^2}}}{cg^2-bgh+ah^2}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} + \frac{f\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{h^2} - \frac{\left(2c\left(fg^3-dgh^2\right)\cdots\right.}{\cdots}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} + \frac{(2f)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{h^2} \cdots$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} + \frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,h^2} - \frac{\left(2c\left(fg^3-\cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 0.35, size = 227, normalized size = 0.94

$$\frac{-\frac{h\sqrt{a+x(b+cx)}\left(h(dh-eg)+fg^2\right)}{(g+hx)\left(h(ah-bg)+cg^2\right)} + \frac{\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)\left(h\left(-2ah(eh-2fg)+bh(dh+eg)-3bfg^2\right)+2c\left(fg^3-dgh^2\right)\right)}{2\left(h(ah-bg)+cg^2\right)^{3/2}} + \frac{f\tanh^{-1}\left(\frac{\cdots}{2\sqrt{c}\cdots}\right)}{\sqrt{c}}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-((h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(-(b*g)
 + a*h))*(g + h*x))) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*
x)])])/Sqrt[c] + ((2*c*(f*g^3 - d*g*h^2) + h*(-3*b*f*g^2 + b*h*(e*g + d*h)
- 2*a*h*(-2*f*g + e*h)))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt
[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(2*(c*g^2 + h*(-(b*g) +
 a*h))^(3/2)))/h^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 0.71Error: Bad Argument Type
```

maple [B] time = 0.02, size = 1671, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] f/h^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/(a*h^2-b*g*h+c*
g^2)/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1
/2)*d+1/h/(a*h^2-b*g*h+c*g^2)/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a
*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e*g-1/h^2/(a*h^2-b*g*h+c*g^2)/(x+g/h)*((x+g/h)
^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*f*g^2+1/2/(a*h^2-
b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a
*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2
*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*d-1/2/h/(a*h^2-b
*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*
h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*
c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*e*g+1/2/h^2/(a*h^
2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*
(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h
-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*f*g^2-1/h/(a*h
^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2
*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*
h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g*d+1/h^2/(a*
h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+
2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b
*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^2*e-1/h^3/
(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)
/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c
+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^3*f-1/h
^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h
+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g
/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*e+2/h^3/((a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b
*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^
2)/h^2)^(1/2))/(x+g/h))*f*g
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `assum
e?` for more details)Is (b/h-(2*c*g)/h^2)^2    -(4*c       *((-(b*g)/h)
    +(c*g^2)/h^2+a))     /h^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)^2\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\left(g+hx\right)^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)**2*sqrt(a + b*x + c*x**2)), x)
```

3.232 $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=336

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(8a^2fh^2-4c\left(a\left(dh^2-3egh+fg^2\right)+bg(2dh+eg)\right)-4abh(eh+2fg)+b^2\left(3dh\right.\right.}{8\left(ah^2-bgh+cg^2\right)^{5/2}}$$

```
[Out] 1/8*(8*c^2*d*g^2+8*a^2*f*h^2-4*a*b*h*(e*h+2*f*g)+b^2*(3*d*h^2+e*g*h+3*f*g^2
)-4*c*(b*g*(2*d*h+e*g)+a*(d*h^2-3*e*g*h+f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b
*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*
g^2)^(5/2)-1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2)/h/(a*h^2-b*g*h+c*g^
2)/(h*x+g)^2+1/4*(2*c*g*(f*g^2+h*(-3*d*h+e*g))+h*(4*a*h*(-e*h+2*f*g)-b*(-3*
d*h^2-e*g*h+5*f*g^2)))*(c*x^2+b*x+a)^(1/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)
```

Rubi [A] time = 0.66, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1650, 806, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(8a^2fh^2-4c\left(-ah(3eg-dh)+afg^2+bg(2dh+eg)\right)-4abh(eh+2fg)+b^2\left(h(3\right.\right.}{8\left(ah^2-bgh+cg^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(2*h*(c*g^2 - b*g*h + a*h^
2)*(g + h*x)^2) + ((2*c*(f*g^3 + g*h*(e*g - 3*d*h)) - h*(5*b*f*g^2 - b*h*(e
*g + 3*d*h) - 4*a*h*(2*f*g - e*h)))*Sqrt[a + b*x + c*x^2])/(4*h*(c*g^2 - b*
g*h + a*h^2)^2*(g + h*x)) + ((8*c^2*d*g^2 + 8*a^2*f*h^2 - 4*a*b*h*(2*f*g +
e*h) - 4*c*(a*f*g^2 - a*h*(3*e*g - d*h) + b*g*(e*g + 2*d*h)) + b^2*(3*f*g^2
 + h*(e*g + 3*d*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2
- b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} - \frac{\int \frac{\frac{1}{2}\left(-4cdg+beg+4afg-\frac{bfg^2}{h}+3bdh-4aeh\right)-\left(ceg-\right.}{(g+hx)^2\sqrt{a+bx+cx^2}}}{2\left(cg^2-bgh+ah^2\right.}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{\left(2c\left(fg^3+gh(eg-3dh)\right)-h\left(5bfg^2-\right.\right.}{4h\left(cg^2-\right.}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{\left(2c\left(fg^3+gh(eg-3dh)\right)-h\left(5bfg^2-\right.\right.}{4h\left(cg^2-\right.}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{\left(2c\left(fg^3+gh(eg-3dh)\right)-h\left(5bfg^2-\right.\right.}{4h\left(cg^2-\right.}$$

Mathematica [A] time = 1.10, size = 367, normalized size = 1.09

$$\frac{-\frac{ch\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)\left(8a^2fh^2-4c\left(ah(dh-3eg)+afg^2+bg(2dh+eg)\right)-4abh(eh+2fg)+b^2\left(h(3dh+eg)+3fg^2\right)+8c^2dg^2\right)}{8\left(h(ah-bg)+cg^2\right)^{5/2}}+\frac{\sqrt{a+x(b+}}{}}{ch}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-((f*Sqrt[a + x*(b + c*x)])/(g + h*x)^2) + ((c*f*g^2 + 2*f*h*(-(b*g) + a*h
) + c*h*(e*g - d*h))*Sqrt[a + x*(b + c*x)])/(2*(c*g^2 + h*(-(b*g) + a*h))*(
g + h*x)^2) + (c*(2*c*(f*g^3 + g*h*(e*g - 3*d*h)) + h*(-5*b*f*g^2 + b*h*(e*
g + 3*d*h) - 4*a*h*(-2*f*g + e*h)))*Sqrt[a + x*(b + c*x)])/(4*(c*g^2 + h*(-
(b*g) + a*h))^2*(g + h*x)) - (c*h*(8*c^2*d*g^2 + 8*a^2*f*h^2 - 4*a*b*h*(2*f
*g + e*h) - 4*c*(a*f*g^2 + a*h*(-3*e*g + d*h) + b*g*(e*g + 2*d*h)) + b^2*(3
*f*g^2 + h*(e*g + 3*d*h)))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sq
rt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(8*(c*g^2 + h*(-(b*g)
 + a*h))^(5/2)))/(c*h)
```

fricas [B] time = 174.73, size = 2034, normalized size = 6.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*g^4 - (8*b*c*d + 8*a*b*f -
(b^2 + 12*a*c)*e)*g^3*h - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*g^2*h^2 +
 ((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*g^2*h^2 - (8*b*c*d + 8*a*b*f - (b
^2 + 12*a*c)*e)*g*h^3 - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*h^4)*x^2 +
2*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*g^3*h - (8*b*c*d + 8*a*b*f - (b^
2 + 12*a*c)*e)*g^2*h^2 - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*g*h^3)*x)*
sqrt(c*g^2 - b*g*h + a*h^2)*log((8*a*b*g*h - 8*a^2*h^2 - (b^2 + 4*a*c)*g^2
- (8*c^2*g^2 - 8*b*c*g*h + (b^2 + 4*a*c)*h^2)*x^2 - 4*sqrt(c*g^2 - b*g*h +
a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x) - 2*(4*b*c*g^2
 + 4*a*b*h^2 - (3*b^2 + 4*a*c)*g*h)*x)/(h^2*x^2 + 2*g*h*x + g^2)) - 4*(2*a^
2*d*h^5 - (4*c^2*e - 3*b*c*f)*g^5 + (8*c^2*d + 5*b*c*e - 3*(b^2 + 2*a*c)*f)
*g^4*h - (13*b*c*d - 9*a*b*f + (b^2 + 2*a*c)*e)*g^3*h^2 - (a*b*e + 6*a^2*f
- 5*(b^2 + 2*a*c)*d)*g^2*h^3 - (7*a*b*d - 2*a^2*e)*g*h^4 - (2*c^2*f*g^5 + (
2*c^2*e - 7*b*c*f)*g^4*h - (6*c^2*d + b*c*e - 5*(b^2 + 2*a*c)*f)*g^3*h^2 +
(9*b*c*d - 13*a*b*f - (b^2 + 2*a*c)*e)*g^2*h^3 + (5*a*b*e + 8*a^2*f - 3*(b^
2 + 2*a*c)*d)*g*h^4 + (3*a*b*d - 4*a^2*e)*h^5)*x)*sqrt(c*x^2 + b*x + a))/(c
^3*g^8 - 3*b*c^2*g^7*h - 3*a^2*b*g^3*h^5 + a^3*g^2*h^6 + 3*(b^2*c + a*c^2)*
g^6*h^2 - (b^3 + 6*a*b*c)*g^5*h^3 + 3*(a*b^2 + a^2*c)*g^4*h^4 + (c^3*g^6*h^
2 - 3*b*c^2*g^5*h^3 - 3*a^2*b*g*h^7 + a^3*h^8 + 3*(b^2*c + a*c^2)*g^4*h^4 -
 (b^3 + 6*a*b*c)*g^3*h^5 + 3*(a*b^2 + a^2*c)*g^2*h^6)*x^2 + 2*(c^3*g^7*h -
3*b*c^2*g^6*h^2 - 3*a^2*b*g^2*h^6 + a^3*g*h^7 + 3*(b^2*c + a*c^2)*g^5*h^3 -
 (b^3 + 6*a*b*c)*g^4*h^4 + 3*(a*b^2 + a^2*c)*g^3*h^5)*x), 1/8*(((8*c^2*d -
4*b*c*e + (3*b^2 - 4*a*c)*f)*g^4 - (8*b*c*d + 8*a*b*f - (b^2 + 12*a*c)*e)*g
^3*h - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*g^2*h^2 + ((8*c^2*d - 4*b*c*
e + (3*b^2 - 4*a*c)*f)*g^2*h^2 - (8*b*c*d + 8*a*b*f - (b^2 + 12*a*c)*e)*g*h
^3 - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*h^4)*x^2 + 2*((8*c^2*d - 4*b*c
*e + (3*b^2 - 4*a*c)*f)*g^3*h - (8*b*c*d + 8*a*b*f - (b^2 + 12*a*c)*e)*g^2*
h^2 - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*g*h^3)*x)*sqrt(-c*g^2 + b*g*h
 - a*h^2)*arctan(-1/2*sqrt(-c*g^2 + b*g*h - a*h^2)*sqrt(c*x^2 + b*x + a)*(b
*g - 2*a*h + (2*c*g - b*h)*x)/(a*c*g^2 - a*b*g*h + a^2*h^2 + (c^2*g^2 - b*c
*g*h + a*c*h^2)*x^2 + (b*c*g^2 - b^2*g*h + a*b*h^2)*x)) - 2*(2*a^2*d*h^5 -
(4*c^2*e - 3*b*c*f)*g^5 + (8*c^2*d + 5*b*c*e - 3*(b^2 + 2*a*c)*f)*g^4*h - (
13*b*c*d - 9*a*b*f + (b^2 + 2*a*c)*e)*g^3*h^2 - (a*b*e + 6*a^2*f - 5*(b^2 +
 2*a*c)*d)*g^2*h^3 - (7*a*b*d - 2*a^2*e)*g*h^4 - (2*c^2*f*g^5 + (2*c^2*e -
7*b*c*f)*g^4*h - (6*c^2*d + b*c*e - 5*(b^2 + 2*a*c)*f)*g^3*h^2 + (9*b*c*d -
 13*a*b*f - (b^2 + 2*a*c)*e)*g^2*h^3 + (5*a*b*e + 8*a^2*f - 3*(b^2 + 2*a*c)
*d)*g*h^4 + (3*a*b*d - 4*a^2*e)*h^5)*x)*sqrt(c*x^2 + b*x + a))/(c^3*g^8 - 3
*b*c^2*g^7*h - 3*a^2*b*g^3*h^5 + a^3*g^2*h^6 + 3*(b^2*c + a*c^2)*g^6*h^2 -
(b^3 + 6*a*b*c)*g^5*h^3 + 3*(a*b^2 + a^2*c)*g^4*h^4 + (c^3*g^6*h^2 - 3*b*c^
2*g^5*h^3 - 3*a^2*b*g*h^7 + a^3*h^8 + 3*(b^2*c + a*c^2)*g^4*h^4 - (b^3 + 6*
a*b*c)*g^3*h^5 + 3*(a*b^2 + a^2*c)*g^2*h^6)*x^2 + 2*(c^3*g^7*h - 3*b*c^2*g^
6*h^2 - 3*a^2*b*g^2*h^6 + a^3*g*h^7 + 3*(b^2*c + a*c^2)*g^5*h^3 - (b^3 + 6*
a*b*c)*g^4*h^4 + 3*(a*b^2 + a^2*c)*g^3*h^5)*x)]
```

giac [B] time = 0.54, size = 2307, normalized size = 6.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(8*c^2*d*g^2 + 3*b^2*f*g^2 - 4*a*c*f*g^2 - 8*b*c*d*g*h - 8*a*b*f*g*h +
3*b^2*d*h^2 - 4*a*c*d*h^2 + 8*a^2*f*h^2 - 4*b*c*g^2*e + b^2*g*h*e + 12*a*c*
g*h*e - 4*a*b*h^2*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(
c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c^2*g^4 - 2*b*c*g^3*h + b^2*g^2*h^2 +
 2*a*c*g^2*h^2 - 2*a*b*g*h^3 + a^2*h^4)*sqrt(-c*g^2 + b*g*h - a*h^2)) + 1/4
*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*f*g^4*h - 16*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*b*c*f*g^3*h^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^3*c^2*d*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^2*h^3 +
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 + 8*(sqrt(c)*x - sqr
```

```
t(c*x^2 + b*x + a))^3*b*c*d*g*h^4 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*a*b*f*g*h^4 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*h^5 + 4*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*h^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*b*c*g^2*h^3*e - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*g*h^4*e
- 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*g*h^4*e + 4*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a*b*h^5*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c
^(5/2)*f*g^5 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*f*g^4*h -
 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*g^3*h^2 - (sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*f*g^3*h^2 + 28*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*a*c^(3/2)*f*g^3*h^2 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*b*c^(3/2)*d*g^2*h^3 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c
)*f*g^2*h^3 - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*g*h^4 +
 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*g*h^4 - 16*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*f*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*c^(5/2)*g^4*h*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*
c^(3/2)*g^3*h^2*e + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*g^2
*h^3*e - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*g^2*h^3*e - 4*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*g*h^4*e + 8*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*h^5*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*b*c^2*f*g^5 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*f*g^4*h -
 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*f*g^4*h - 24*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*b*c^2*d*g^3*h^2 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
b^3*f*g^3*h^2 + 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*f*g^3*h^2 + 20
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*g^2*h^3 + 40*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a*c^2*d*g^2*h^3 - 11*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a*b^2*f*g^2*h^3 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*f*g^2*h^3 -
 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*g*h^4 - 28*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a*b*c*d*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*
b*f*g*h^4 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d*h^5 + 4*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a^2*c*d*h^5 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*b*c^2*g^4*h*e - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*g^3*h^2*e
+ (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*g^2*h^3*e - 16*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*a*b*c*g^2*h^3*e + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
*b^2*g*h^4*e + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*g*h^4*e - 4*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*h^5*e + 2*b^2*c^(3/2)*f*g^5 - 5*b^3*
sqrt(c)*f*g^4*h - 4*a*b*c^(3/2)*f*g^4*h - 6*b^2*c^(3/2)*d*g^3*h^2 + 21*a*b^
2*sqrt(c)*f*g^3*h^2 + 4*a^2*c^(3/2)*f*g^3*h^2 + 3*b^3*sqrt(c)*d*g^2*h^3 + 2
0*a*b*c^(3/2)*d*g^2*h^3 - 32*a^2*b*sqrt(c)*f*g^2*h^3 - 11*a*b^2*sqrt(c)*d*g
*h^4 - 12*a^2*c^(3/2)*d*g*h^4 + 16*a^3*sqrt(c)*f*g*h^4 + 8*a^2*b*sqrt(c)*d*
h^5 + 2*b^2*c^(3/2)*g^4*h*e + b^3*sqrt(c)*g^3*h^2*e - 8*a*b*c^(3/2)*g^3*h^2
*e - 5*a*b^2*sqrt(c)*g^2*h^3*e + 4*a^2*c^(3/2)*g^2*h^3*e + 12*a^2*b*sqrt(c)
*g*h^4*e - 8*a^3*sqrt(c)*h^5*e)/((c^2*g^4*h^2 - 2*b*c*g^3*h^3 + b^2*g^2*h^4
 + 2*a*c*g^2*h^4 - 2*a*b*g*h^5 + a^2*h^6)*((sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*g + b*g - a*h)^2)
```

maple [B] time = 0.02, size = 3615, normalized size = 10.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -f/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b
*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*
(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))-3/2/h/(a*h^2-b*g*h+c*g^2
)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*
h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+
g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*c*g^2*e+3/2/h^2/(a*h^2-b*
g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a
*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2
```

```
*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*c*g^3*f-3/2/h^2/
(a*h^2-b*g*h+c*g^2)^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2)*c*g^3*f-3/8/h/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^
2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2
-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*
g^2)/h^2)^(1/2))/(x+g/h))*b^2*f*g^2+3/2/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*c*g*d-3/2/h/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-
b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2
+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^
2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c^2*g^2*d+3/2/h^2/(a*h^2-b*g*h+c*g^2)^2
/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c
*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h
)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c^2*g^3*e-3/2/h^3/(a*h^2-b*g*h
+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^
2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*
g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c^2*g^4*f-1/2/h/(a*h^
2-b*g*h+c*g^2)/(x+g/h)^2*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*
g^2)/h^2)^(1/2)*d-1/h/(a*h^2-b*g*h+c*g^2)/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*
(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e+2/h^2/(a*h^2-b*g*h+c*g^2)/(x+g/h
)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*f*g+1/2
/h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g
/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^
2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*e+1/2/
h^2/(a*h^2-b*g*h+c*g^2)/(x+g/h)^2*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2
-b*g*h+c*g^2)/h^2)^(1/2)*e*g-1/2/h^3/(a*h^2-b*g*h+c*g^2)/(x+g/h)^2*((x+g/h)
^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*f*g^2+3/4*h/(a*h^
2-b*g*h+c*g^2)^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*
g^2)/h^2)^(1/2)*b*d-3/4/(a*h^2-b*g*h+c*g^2)^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c
*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*e*g-3/2/(a*h^2-b*g*h+c*g^2)^
2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
*c*g*d-3/8*h/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h
-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/
2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/
h))*b^2*d+3/8/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*
h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1
/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g
/h))*b^2*e*g+1/2/h*c/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln
(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^
2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))
/(x+g/h))*d-1/h^2/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((
b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^
(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x
+g/h))*b*f*g-3/2/h^2/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln
(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^
2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))
/(x+g/h))*c*g*e+5/2/h^3/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2))/(x+g/h))*c*g^2*f+3/4/h/(a*h^2-b*g*h+c*g^2)^2/(x+g/h)*((x+g/h)^2*c+(b*h-
2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*f*g^2+3/2/h/(a*h^2-b*g*h+
c*g^2)^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2
)^(1/2)*c*g^2*e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                            +c*g^2    positive,
negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\left(g+hx\right)^3\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)**3*sqrt(a + b*x + c*x**2)), x)
```

3.233 $\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=504

$$\frac{h\sqrt{a+bx+cx^2}\left(8c^2\left(32a^2fh^2+39abh(eh+3fg)+b^2\left(9h(dh+3eg)+20fg^2\right)\right)+2chx\left(-8c^2(9aeh+11afg+3b\right.\right.}{}$$

```
[Out] -1/16*(35*b^3*f*h^3-30*b*c*h^2*(2*a*f*h+b*e*h+3*b*f*g)-16*c^3*g*(f*g^2+3*h*
(d*h+e*g))+24*c^2*h*(a*h*(e*h+3*f*g)+b*(d*h^2+3*e*g*h+3*f*g^2)))*arctanh(1/
2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)+2*(c*(2*a*e-b*(d+a*f/c))-(
-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)*(h*x+g)^3/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/
2)+1/3*(-16*a*c*f+7*b^2*f-6*b*c*e+12*c^2*d)*h*(h*x+g)^2*(c*x^2+b*x+a)^(1/2)
/c^2/(-4*a*c+b^2)+1/24*h*(192*c^4*d*g^2+105*b^4*f*h^2-10*b^2*c*h*(46*a*f*h+
9*b*(e*h+3*f*g))-16*c^3*(3*b*g*(3*d*h+2*e*g)+4*a*(3*d*h^2+9*e*g*h+7*f*g^2))
+8*c^2*(32*a^2*f*h^2+39*a*b*h*(e*h+3*f*g)+b^2*(20*f*g^2+9*h*(d*h+3*e*g)))+2
*c*h*(48*c^3*d*g-35*b^3*f*h-8*c^2*(9*a*e*h+11*a*f*g+3*b*d*h+3*b*e*g)+2*b*c*
(58*a*f*h+15*b*e*h+17*b*f*g))*x)*(c*x^2+b*x+a)^(1/2)/c^4/(-4*a*c+b^2)
```

Rubi [A] time = 1.18, antiderivative size = 502, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1644, 832, 779, 621, 206}

$$\frac{h\sqrt{a+bx+cx^2}\left(8c\left(32a^2fh^2+39abh(eh+3fg)+b^2\left(9h(dh+3eg)+20fg^2\right)\right)+2hx\left(-8c^2(9aeh+11afg+3bd\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x)^3)/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + ((12*c^2*d - 6*b*c*e +
 7*b^2*f - 16*a*c*f)*h*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c^2*(b^2 - 4*a
*c)) + (h*(192*c^3*d*g^2 + (105*b^4*f*h^2)/c - 10*b^2*h*(46*a*f*h + 9*b*(3*
f*g + e*h)) - 16*c^2*(3*b*g*(2*e*g + 3*d*h) + 4*a*(7*f*g^2 + 9*e*g*h + 3*d*
h^2)) + 8*c*(32*a^2*f*h^2 + 39*a*b*h*(3*f*g + e*h) + b^2*(20*f*g^2 + 9*h*(3
*e*g + d*h))) + 2*h*(48*c^3*d*g - 35*b^3*f*h - 8*c^2*(3*b*e*g + 11*a*f*g +
3*b*d*h + 9*a*e*h) + 2*b*c*(17*b*f*g + 15*b*e*h + 58*a*f*h))*x)*Sqrt[a + b*
x + c*x^2])/(24*c^3*(b^2 - 4*a*c)) - ((35*b^3*f*h^3 - 30*b*c*h^2*(3*b*f*g +
 b*e*h + 2*a*f*h) - 16*c^3*(f*g^3 + 3*g*h*(e*g + d*h)) + 24*c^2*h*(3*b*f*g^
2 + b*h*(3*e*g + d*h) + a*h*(3*f*g + e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(16*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{(g+hx)}{}}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(12c^2d+\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(12c^2d+\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(12c^2d+\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(12c^2d+\right.}{}$$

Mathematica [A] time = 1.59, size = 715, normalized size = 1.42

$$\frac{3\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(24c^2h\left(ah(eh+3fg)+bh(dh+3eg)+3bfg^2\right)-30bch^2(2af}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[c]*(105*b^5*f*h^3*x + 5*b^4*h^2*(21*a*f*h + c*x*(-54*f*g - 18*e*h
+ 7*f*h*x)) - 2*b^3*c*h*(5*a*h*(27*f*g + 9*e*h + 53*f*h*x) + c*x*(3*h*(-36*
e*g - 12*d*h + 5*e*h*x) + f*(-108*g^2 + 45*g*h*x + 7*h^2*x^2))) - 16*c^2*(-
16*a^3*f*h^3 + 6*c^3*d*g^3*x + a*c^2*(6*d*h*(-3*g^2 - 3*g*h*x + h^2*x^2) -
3*e*(2*g^3 + 6*g^2*h*x - 6*g*h^2*x^2 - h^3*x^3) + f*x*(-6*g^3 + 18*g^2*h*x
+ 9*g*h^2*x^2 + 2*h^3*x^3)) + a^2*c*h*(f*(36*g^2 + 27*g*h*x - 8*h^2*x^2) +
3*h*(4*d*h + 3*e*(4*g + h*x)))) - 8*b*c^2*(-6*c^2*g^2*(-(d*g) + e*g*x + 3*d
*h*x) - a^2*h^2*(117*f*g + 39*e*h + 61*f*h*x) + a*c*(f*(6*g^3 + 90*g^2*h*x
- 45*g*h^2*x^2 - 7*h^3*x^3) + 3*h*(2*d*h*(3*g + 5*h*x) + e*(6*g^2 + 30*g*h*
x - 5*h^2*x^2)))) + 4*b^2*c*(-115*a^2*f*h^3 + a*c*h*(3*h*(18*e*g + 6*d*h +
31*e*h*x) + f*(54*g^2 + 279*g*h*x - 43*h^2*x^2)) + c^2*x*(f*(-12*g^3 + 18*g
^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^3) + 3*h*(2*d*h*(-6*g + h*x) + e*(-12*g^2 +
6*g*h*x + h^2*x^2))))) + 3*(b^2 - 4*a*c)*(35*b^3*f*h^3 - 30*b*c*h^2*(3*b*f*
g + b*e*h + 2*a*f*h) - 16*c^3*g*(f*g^2 + 3*h*(e*g + d*h)) + 24*c^2*h*(3*b*f
*g^2 + b*h*(3*e*g + d*h) + a*h*(3*f*g + e*h)))*Sqrt[a + x*(b + c*x)]*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(48*c^(9/2)*(-b^2 + 4*a*c
)*Sqrt[a + x*(b + c*x)])
```

fricas [B] time = 28.67, size = 2937, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/96*(3*(16*(a*b^2*c^3 - 4*a^2*c^4)*f*g^3 + 24*(2*(a*b^2*c^3 - 4*a^2*c^4)*
e - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*f)*g^2*h + 6*(8*(a*b^2*c^3 - 4*a^2*c^4)*d -
 12*(a*b^3*c^2 - 4*a^2*b*c^3)*e + 3*(5*a*b^4*c - 24*a^2*b^2*c^2 + 16*a^3*c^
3)*f)*g*h^2 - (24*(a*b^3*c^2 - 4*a^2*b*c^3)*d - 6*(5*a*b^4*c - 24*a^2*b^2*c
^2 + 16*a^3*c^3)*e + 5*(7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*f)*h^3 + (16
*(b^2*c^4 - 4*a*c^5)*f*g^3 + 24*(2*(b^2*c^4 - 4*a*c^5)*e - 3*(b^3*c^3 - 4*a
*b*c^4)*f)*g^2*h + 6*(8*(b^2*c^4 - 4*a*c^5)*d - 12*(b^3*c^3 - 4*a*b*c^4)*e
+ 3*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g*h^2 - (24*(b^3*c^3 - 4*a*b
*c^4)*d - 6*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + 5*(7*b^5*c - 40*a*b
^3*c^2 + 48*a^2*b*c^3)*f)*h^3)*x^2 + (16*(b^3*c^3 - 4*a*b*c^4)*f*g^3 + 24*(
2*(b^3*c^3 - 4*a*b*c^4)*e - 3*(b^4*c^2 - 4*a*b^2*c^3)*f)*g^2*h + 6*(8*(b^3*
c^3 - 4*a*b*c^4)*d - 12*(b^4*c^2 - 4*a*b^2*c^3)*e + 3*(5*b^5*c - 24*a*b^3*c
^2 + 16*a^2*b*c^3)*f)*g*h^2 - (24*(b^4*c^2 - 4*a*b^2*c^3)*d - 6*(5*b^5*c -
24*a*b^3*c^2 + 16*a^2*b*c^3)*e + 5*(7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*f)
*h^3)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(
2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*(b^2*c^4 - 4*a*c^5)*f*h^3*x^4 - 48*(b*c^
5*d - 2*a*c^5*e + a*b*c^4*f)*g^3 + 72*(4*a*c^5*d - 2*a*b*c^4*e + (3*a*b^2*c
^3 - 8*a^2*c^4)*f)*g^2*h - 18*(8*a*b*c^4*d - 4*(3*a*b^2*c^3 - 8*a^2*c^4)*e
+ (15*a*b^3*c^2 - 52*a^2*b*c^3)*f)*g*h^2 + (24*(3*a*b^2*c^3 - 8*a^2*c^4)*d
- 6*(15*a*b^3*c^2 - 52*a^2*b*c^3)*e + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256*
a^3*c^3)*f)*h^3 + 2*(18*(b^2*c^4 - 4*a*c^5)*f*g*h^2 + (6*(b^2*c^4 - 4*a*c^5
)*e - 7*(b^3*c^3 - 4*a*b*c^4)*f)*h^3)*x^3 + (72*(b^2*c^4 - 4*a*c^5)*f*g^2*h
 + 18*(4*(b^2*c^4 - 4*a*c^5)*e - 5*(b^3*c^3 - 4*a*b*c^4)*f)*g*h^2 + (24*(b^
2*c^4 - 4*a*c^5)*d - 30*(b^3*c^3 - 4*a*b*c^4)*e + (35*b^4*c^2 - 172*a*b^2*c
```

```
^3 + 128*a^2*c^4)*f)*h^3)*x^2 - (48*(2*c^6*d - b*c^5*e + (b^2*c^4 - 2*a*c^5
)*f)*g^3 - 72*(2*b*c^5*d - 2*(b^2*c^4 - 2*a*c^5)*e + (3*b^3*c^3 - 10*a*b*c^
4)*f)*g^2*h + 18*(8*(b^2*c^4 - 2*a*c^5)*d - 4*(3*b^3*c^3 - 10*a*b*c^4)*e +
(15*b^4*c^2 - 62*a*b^2*c^3 + 24*a^2*c^4)*f)*g*h^2 - (24*(3*b^3*c^3 - 10*a*b
*c^4)*d - 6*(15*b^4*c^2 - 62*a*b^2*c^3 + 24*a^2*c^4)*e + (105*b^5*c - 530*a
*b^3*c^2 + 488*a^2*b*c^3)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^5 - 4*
a^2*c^6 + (b^2*c^6 - 4*a*c^7)*x^2 + (b^3*c^5 - 4*a*b*c^6)*x), -1/48*(3*(16*
(a*b^2*c^3 - 4*a^2*c^4)*f*g^3 + 24*(2*(a*b^2*c^3 - 4*a^2*c^4)*e - 3*(a*b^3*
c^2 - 4*a^2*b*c^3)*f)*g^2*h + 6*(8*(a*b^2*c^3 - 4*a^2*c^4)*d - 12*(a*b^3*c^
2 - 4*a^2*b*c^3)*e + 3*(5*a*b^4*c - 24*a^2*b^2*c^2 + 16*a^3*c^3)*f)*g*h^2 -
 (24*(a*b^3*c^2 - 4*a^2*b*c^3)*d - 6*(5*a*b^4*c - 24*a^2*b^2*c^2 + 16*a^3*c
^3)*e + 5*(7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*f)*h^3 + (16*(b^2*c^4 - 4
*a*c^5)*f*g^3 + 24*(2*(b^2*c^4 - 4*a*c^5)*e - 3*(b^3*c^3 - 4*a*b*c^4)*f)*g^
2*h + 6*(8*(b^2*c^4 - 4*a*c^5)*d - 12*(b^3*c^3 - 4*a*b*c^4)*e + 3*(5*b^4*c^
2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g*h^2 - (24*(b^3*c^3 - 4*a*b*c^4)*d - 6*(
5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + 5*(7*b^5*c - 40*a*b^3*c^2 + 48*a
^2*b*c^3)*f)*h^3)*x^2 + (16*(b^3*c^3 - 4*a*b*c^4)*f*g^3 + 24*(2*(b^3*c^3 -
4*a*b*c^4)*e - 3*(b^4*c^2 - 4*a*b^2*c^3)*f)*g^2*h + 6*(8*(b^3*c^3 - 4*a*b*c
^4)*d - 12*(b^4*c^2 - 4*a*b^2*c^3)*e + 3*(5*b^5*c - 24*a*b^3*c^2 + 16*a^2*b
*c^3)*f)*g*h^2 - (24*(b^4*c^2 - 4*a*b^2*c^3)*d - 6*(5*b^5*c - 24*a*b^3*c^2
+ 16*a^2*b*c^3)*e + 5*(7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*f)*h^3)*x)*sqrt
(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x
 + a*c)) - 2*(8*(b^2*c^4 - 4*a*c^5)*f*h^3*x^4 - 48*(b*c^5*d - 2*a*c^5*e + a
*b*c^4*f)*g^3 + 72*(4*a*c^5*d - 2*a*b*c^4*e + (3*a*b^2*c^3 - 8*a^2*c^4)*f)*
g^2*h - 18*(8*a*b*c^4*d - 4*(3*a*b^2*c^3 - 8*a^2*c^4)*e + (15*a*b^3*c^2 - 5
2*a^2*b*c^3)*f)*g*h^2 + (24*(3*a*b^2*c^3 - 8*a^2*c^4)*d - 6*(15*a*b^3*c^2 -
 52*a^2*b*c^3)*e + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*f)*h^3 + 2
*(18*(b^2*c^4 - 4*a*c^5)*f*g*h^2 + (6*(b^2*c^4 - 4*a*c^5)*e - 7*(b^3*c^3 -
4*a*b*c^4)*f)*h^3)*x^3 + (72*(b^2*c^4 - 4*a*c^5)*f*g^2*h + 18*(4*(b^2*c^4 -
 4*a*c^5)*e - 5*(b^3*c^3 - 4*a*b*c^4)*f)*g*h^2 + (24*(b^2*c^4 - 4*a*c^5)*d
- 30*(b^3*c^3 - 4*a*b*c^4)*e + (35*b^4*c^2 - 172*a*b^2*c^3 + 128*a^2*c^4)*f
)*h^3)*x^2 - (48*(2*c^6*d - b*c^5*e + (b^2*c^4 - 2*a*c^5)*f)*g^3 - 72*(2*b*
c^5*d - 2*(b^2*c^4 - 2*a*c^5)*e + (3*b^3*c^3 - 10*a*b*c^4)*f)*g^2*h + 18*(8
*(b^2*c^4 - 2*a*c^5)*d - 4*(3*b^3*c^3 - 10*a*b*c^4)*e + (15*b^4*c^2 - 62*a*
b^2*c^3 + 24*a^2*c^4)*f)*g*h^2 - (24*(3*b^3*c^3 - 10*a*b*c^4)*d - 6*(15*b^4
*c^2 - 62*a*b^2*c^3 + 24*a^2*c^4)*e + (105*b^5*c - 530*a*b^3*c^2 + 488*a^2*
b*c^3)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^5 - 4*a^2*c^6 + (b^2*c^6
- 4*a*c^7)*x^2 + (b^3*c^5 - 4*a*b*c^6)*x)]
```

giac [B] time = 0.35, size = 1054, normalized size = 2.09

$$\left(\left(2\left(\frac{4\left(b^2c^3fh^3-4\,ac^4fh^3\right)x}{b^2c^4-4\,ac^5}+\frac{18\,b^2c^3fgh^2-72\,ac^4fgh^2-7\,b^3c^2fh^3+28\,abc^3fh^3+6\,b^2c^3h^3e-24\,ac^4h^3e}{b^2c^4-4\,ac^5}\right)x+\frac{72\,b^2c^3fg^2h-288\,ac^4fg^2h-90\,b^3c^2\ldots}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*(((2*(4*(b^2*c^3*f*h^3 - 4*a*c^4*f*h^3)*x/(b^2*c^4 - 4*a*c^5) + (18*b^
2*c^3*f*g*h^2 - 72*a*c^4*f*g*h^2 - 7*b^3*c^2*f*h^3 + 28*a*b*c^3*f*h^3 + 6*b
^2*c^3*h^3*e - 24*a*c^4*h^3*e)/(b^2*c^4 - 4*a*c^5))*x + (72*b^2*c^3*f*g^2*h
 - 288*a*c^4*f*g^2*h - 90*b^3*c^2*f*g*h^2 + 360*a*b*c^3*f*g*h^2 + 24*b^2*c^
3*d*h^3 - 96*a*c^4*d*h^3 + 35*b^4*c*f*h^3 - 172*a*b^2*c^2*f*h^3 + 128*a^2*c
^3*f*h^3 + 72*b^2*c^3*g*h^2*e - 288*a*c^4*g*h^2*e - 30*b^3*c^2*h^3*e + 120*
a*b*c^3*h^3*e)/(b^2*c^4 - 4*a*c^5))*x - (96*c^5*d*g^3 + 48*b^2*c^3*f*g^3 -
96*a*c^4*f*g^3 - 144*b*c^4*d*g^2*h - 216*b^3*c^2*f*g^2*h + 720*a*b*c^3*f*g^
2*h + 144*b^2*c^3*d*g*h^2 - 288*a*c^4*d*g*h^2 + 270*b^4*c*f*g*h^2 - 1116*a*
b^2*c^2*f*g*h^2 + 432*a^2*c^3*f*g*h^2 - 72*b^3*c^2*d*h^3 + 240*a*b*c^3*d*h^
3 - 105*b^5*f*h^3 + 530*a*b^3*c*f*h^3 - 488*a^2*b*c^2*f*h^3 - 48*b*c^4*g^3*
```

```
e + 144*b^2*c^3*g^2*h*e - 288*a*c^4*g^2*h*e - 216*b^3*c^2*g*h^2*e + 720*a*b
*c^3*g*h^2*e + 90*b^4*c*h^3*e - 372*a*b^2*c^2*h^3*e + 144*a^2*c^3*h^3*e)/(b
^2*c^4 - 4*a*c^5))*x - (48*b*c^4*d*g^3 + 48*a*b*c^3*f*g^3 - 288*a*c^4*d*g^2
*h - 216*a*b^2*c^2*f*g^2*h + 576*a^2*c^3*f*g^2*h + 144*a*b*c^3*d*g*h^2 + 27
0*a*b^3*c*f*g*h^2 - 936*a^2*b*c^2*f*g*h^2 - 72*a*b^2*c^2*d*h^3 + 192*a^2*c^
3*d*h^3 - 105*a*b^4*f*h^3 + 460*a^2*b^2*c*f*h^3 - 256*a^3*c^2*f*h^3 - 96*a*
c^4*g^3*e + 144*a*b*c^3*g^2*h*e - 216*a*b^2*c^2*g*h^2*e + 576*a^2*c^3*g*h^2
*e + 90*a*b^3*c*h^3*e - 312*a^2*b*c^2*h^3*e)/(b^2*c^4 - 4*a*c^5))/sqrt(c*x^
2 + b*x + a) - 1/16*(16*c^3*f*g^3 - 72*b*c^2*f*g^2*h + 48*c^3*d*g*h^2 + 90*
b^2*c*f*g*h^2 - 72*a*c^2*f*g*h^2 - 24*b*c^2*d*h^3 - 35*b^3*f*h^3 + 60*a*b*c
*f*h^3 + 48*c^3*g^2*h*e - 72*b*c^2*g*h^2*e + 30*b^2*c*h^3*e - 24*a*c^2*h^3*
e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

maple [B] time = 0.02, size = 2780, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 1/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^3*f-1/c/(c*x^2+b*x+
a)^(1/2)*g^3*e-3/4/c^3*b^2/(c*x^2+b*x+a)^(1/2)*h^3*d-3/2/c^(5/2)*b*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*d+2*a/c^2/(c*x^2+b*x+a)^(1/2)*h^3*d
-35/32*h^3*f/c^5*b^4/(c*x^2+b*x+a)^(1/2)-35/16*h^3*f/c^(9/2)*b^3*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-8/3*h^3*f*a^2/c^3/(c*x^2+b*x+a)^(1/2)+1/2
*x^3/c/(c*x^2+b*x+a)^(1/2)*h^3*e+15/16/c^4*b^3/(c*x^2+b*x+a)^(1/2)*h^3*e+15
/8/c^(7/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*e-3/2*a/c^(5
/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*e+1/3*h^3*f*x^4/c/(c*x^
2+b*x+a)^(1/2)+2*g^3*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+3/c^(3/2)*
ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*d+3/c^(3/2)*ln((c*x+1/2*b
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^2*h*e-3/c/(c*x^2+b*x+a)^(1/2)*g^2*h*d+x^2/
c/(c*x^2+b*x+a)^(1/2)*h^3*d+45/16/c^4*b^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g
*h^2*f-39/4/c^3*b*a/(c*x^2+b*x+a)^(1/2)*g*h^2*f-13/4/c^3*b^3*a/(4*a*c-b^2)/
(c*x^2+b*x+a)^(1/2)*h^3*e-9/4/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g*h^2
*e-9/4/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g^2*h*f+2*a/c^2*b^2/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*h^3*d+9/2/c^2*b*x/(c*x^2+b*x+a)^(1/2)*g*h^2*e+12*a/
c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^2*h*f+3/2/c^2*b^3/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)*g*h^2*d+1/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^3*f-3/2/
c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*h^3*d-3*b^2/c/(4*a*c-b^2)/(c*x^2+
b*x+a)^(1/2)*g^2*h*d+115/24*h^3*f/c^4*b^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
-15/4*h^3*f/c^3*b*a*x/(c*x^2+b*x+a)^(1/2)-8/3*h^3*f*a^2/c^3*b^2/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)-35/16*h^3*f/c^4*b^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+
9/2/c^2*b*x/(c*x^2+b*x+a)^(1/2)*g^2*h*f-15/4/c^2*b*x^2/(c*x^2+b*x+a)^(1/2)*
g*h^2*f-45/8/c^3*b^2*x/(c*x^2+b*x+a)^(1/2)*g*h^2*f+15/8/c^3*b^4/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)*x*h^3*e-9/4/c^3*b^2/(c*x^2+b*x+a)^(1/2)*g*h^2*e-9/4/c^
3*b^2/(c*x^2+b*x+a)^(1/2)*g^2*h*f-3/4/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/
2)*h^3*d-9/2/c^(5/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*e+
3/2*x^3/c/(c*x^2+b*x+a)^(1/2)*g*h^2*f-5/4/c^2*b*x^2/(c*x^2+b*x+a)^(1/2)*h^3
*e-15/8/c^3*b^2*x/(c*x^2+b*x+a)^(1/2)*h^3*e+45/16/c^4*b^3/(c*x^2+b*x+a)^(1/
2)*g*h^2*f+15/16/c^4*b^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*h^3*e+45/8/c^(7/2)
*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*f-13/4/c^3*b*a/(c*x^
2+b*x+a)^(1/2)*h^3*e+3/2*a/c^2*x/(c*x^2+b*x+a)^(1/2)*h^3*e-9/2*a/c^(5/2)*ln
((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*f-9/2/c^(5/2)*b*ln((c*x+1/2
*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^2*h*f+6*a/c^2/(c*x^2+b*x+a)^(1/2)*g*h^2*
e+6*a/c^2/(c*x^2+b*x+a)^(1/2)*g^2*h*f+3*x^2/c/(c*x^2+b*x+a)^(1/2)*g*h^2*e+9
/2*a/c^2*x/(c*x^2+b*x+a)^(1/2)*g*h^2*f+3/2/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2)*g^2*h*e-6*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^2*h*d+45/8/c^3*b^4/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g*h^2*f-13/2/c^2*b^2*a/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)*x*h^3*e-39/4/c^3*b^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g*h^2*
f+115/12*h^3*f/c^3*b^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-16/3*h^3*f*a^2/c
```

```
^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+6*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2)*g*h^2*e+6*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g^2*h*f+4*a/c*b
/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*h^3*d+3/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2)*x*g*h^2*d+3/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^2*h*e-9/2/c^2*b
^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g*h^2*e-9/2/c^2*b^3/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)*x*g^2*h*f+3*x^2/c/(c*x^2+b*x+a)^(1/2)*g^2*h*f+3/2/c^2*b*x/(c*
x^2+b*x+a)^(1/2)*h^3*d-3*x/c/(c*x^2+b*x+a)^(1/2)*g*h^2*d-3*x/c/(c*x^2+b*x+a
)^(1/2)*g^2*h*e+3/2/c^2*b/(c*x^2+b*x+a)^(1/2)*g*h^2*d+3/2/c^2*b/(c*x^2+b*x+
a)^(1/2)*g^2*h*e+1/2/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g^3*f-2*b/(4*a
*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^3*e-b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g
^3*e-7/12*h^3*f/c^2*b*x^3/(c*x^2+b*x+a)^(1/2)+35/24*h^3*f/c^3*b^2*x^2/(c*x^
2+b*x+a)^(1/2)+35/16*h^3*f/c^4*b^3*x/(c*x^2+b*x+a)^(1/2)-35/32*h^3*f/c^5*b^
6/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+115/24*h^3*f/c^4*b^2*a/(c*x^2+b*x+a)^(1/2
)+15/4*h^3*f/c^(7/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/3*h^
3*f*a/c^2*x^2/(c*x^2+b*x+a)^(1/2)+12*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*
x*g*h^2*e-39/2/c^2*b^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g*h^2*f-x/c/(c*x
^2+b*x+a)^(1/2)*g^3*f+1/2/c^2*b/(c*x^2+b*x+a)^(1/2)*g^3*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g+h\,x)^3\,(f\,x^2+e\,x+d)}{(c\,x^2+b\,x+a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**3*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

3.234 $\int \frac{(g+hx)^2(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=289

$$\frac{2(g+hx)^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-12ch(afh+beh+2\ldots\right.}{8c^{7/\ldots}}$$

```
[Out] 1/8*(15*b^2*f*h^2-12*c*h*(a*f*h+b*e*h+2*b*f*g)+8*c^2*(f*g^2+h*(d*h+2*e*g)))
*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+2*(c*(2*a*e-b*(
d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)*(h*x+g)^2/c/(-4*a*c+b^2)/(c*x^2
+b*x+a)^(1/2)+1/4*h*(32*c^3*d*g-15*b^3*f*h-8*c^2*(4*a*e*h+8*a*f*g+b*d*h+2*b
*e*g)+4*b*c*(13*a*f*h+3*b*e*h+6*b*f*g)+2*c*(-12*a*c*f+5*b^2*f-4*b*c*e+8*c^2
*d)*h*x)*(c*x^2+b*x+a)^(1/2)/c^3/(-4*a*c+b^2)
```

Rubi [A] time = 0.39, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1644, 779, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-12ch(afh+beh+2bfg)+15b^2fh^2+8c^2\left(h(dh+2eg)+fg^2\right)\right)}{8c^{7/2}}+\frac{2(g+hx)^2\left(c\left(2ae-b\ldots\right.\right.}{c\left(\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x)^2)/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (h*(32*c^2*d*g - (15*b
^3*f*h)/c - 8*c*(2*b*e*g + 8*a*f*g + b*d*h + 4*a*e*h) + 4*b*(6*b*f*g + 3*b*
e*h + 13*a*f*h) + 2*(8*c^2*d - 4*b*c*e + 5*b^2*f - 12*a*c*f)*h*x)*Sqrt[a +
b*x + c*x^2])/(4*c^2*(b^2 - 4*a*c)) + ((15*b^2*f*h^2 - 12*c*h*(2*b*f*g + b*
e*h + a*f*h) + 8*c^2*(f*g^2 + h*(2*e*g + d*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{(g+hx)}{}}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{h\left(32c^2dg\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{h\left(32c^2dg\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{h\left(32c^2dg\right.}{}$$

Mathematica [A] time = 0.83, size = 412, normalized size = 1.43

$$\frac{2\sqrt{c}\left(4bc\left(-13a^2fh^2+ac\left(2h(dh+2eg+5ehx)+f\left(2g^2+20ghx-5h^2x^2\right)\right)+2c^2g(d(g-2hx)-egx)\right)+8c^2\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[c]*(15*b^4*f*h^2*x + b^3*h*(15*a*f*h + c*x*(-24*f*g - 12*e*h + 5*f*
h*x)) + 4*b*c*(-13*a^2*f*h^2 + 2*c^2*g*(-(e*g*x) + d*(g - 2*h*x)) + a*c*(2*
h*(2*e*g + d*h + 5*e*h*x) + f*(2*g^2 + 20*g*h*x - 5*h^2*x^2))) - 2*b^2*c*(a
*h*(12*f*g + 6*e*h + 31*f*h*x) + c*x*(2*h*(-4*e*g - 2*d*h + e*h*x) + f*(-4*
g^2 + 4*g*h*x + h^2*x^2))) + 8*c^2*(2*c^2*d*g^2*x + a^2*h*(8*f*g + 4*e*h +
3*f*h*x) + a*c*(-2*d*h*(2*g + h*x) - 2*e*(g^2 + 2*g*h*x - h^2*x^2) + f*x*(-
2*g^2 + 4*g*h*x + h^2*x^2)))) - (b^2 - 4*a*c)*(15*b^2*f*h^2 - 12*c*h*(2*b*f
*g + b*e*h + a*f*h) + 8*c^2*(f*g^2 + h*(2*e*g + d*h)))*Sqrt[a + x*(b + c*x)
]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(8*c^(7/2)*(-b^2
+ 4*a*c)*Sqrt[a + x*(b + c*x)])
```

fricas [B] time = 20.32, size = 1769, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/16*((8*(a*b^2*c^2 - 4*a^2*c^3)*f*g^2 + 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*e -
 3*(a*b^3*c - 4*a^2*b*c^2)*f)*g*h + (8*(a*b^2*c^2 - 4*a^2*c^3)*d - 12*(a*b^
3*c - 4*a^2*b*c^2)*e + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f)*h^2 + (8*
(b^2*c^3 - 4*a*c^4)*f*g^2 + 8*(2*(b^2*c^3 - 4*a*c^4)*e - 3*(b^3*c^2 - 4*a*b
*c^3)*f)*g*h + (8*(b^2*c^3 - 4*a*c^4)*d - 12*(b^3*c^2 - 4*a*b*c^3)*e + 3*(5
*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*h^2)*x^2 + (8*(b^3*c^2 - 4*a*b*c^3)*
f*g^2 + 8*(2*(b^3*c^2 - 4*a*b*c^3)*e - 3*(b^4*c - 4*a*b^2*c^2)*f)*g*h + (8*
(b^3*c^2 - 4*a*b*c^3)*d - 12*(b^4*c - 4*a*b^2*c^2)*e + 3*(5*b^5 - 24*a*b^3*
c + 16*a^2*b*c^2)*f)*h^2)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*
f*h^2*x^3 - 8*(b*c^4*d - 2*a*c^4*e + a*b*c^3*f)*g^2 + 8*(4*a*c^4*d - 2*a*b*
c^3*e + (3*a*b^2*c^2 - 8*a^2*c^3)*f)*g*h - (8*a*b*c^3*d - 4*(3*a*b^2*c^2 -
8*a^2*c^3)*e + (15*a*b^3*c - 52*a^2*b*c^2)*f)*h^2 + (8*(b^2*c^3 - 4*a*c^4)*
f*g*h + (4*(b^2*c^3 - 4*a*c^4)*e - 5*(b^3*c^2 - 4*a*b*c^3)*f)*h^2)*x^2 - (8
*(2*c^5*d - b*c^4*e + (b^2*c^3 - 2*a*c^4)*f)*g^2 - 8*(2*b*c^4*d - 2*(b^2*c^
3 - 2*a*c^4)*e + (3*b^3*c^2 - 10*a*b*c^3)*f)*g*h + (8*(b^2*c^3 - 2*a*c^4)*d
 - 4*(3*b^3*c^2 - 10*a*b*c^3)*e + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*f)
*h^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6
)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x), -1/8*((8*(a*b^2*c^2 - 4*a^2*c^3)*f*g^2 +
8*(2*(a*b^2*c^2 - 4*a^2*c^3)*e - 3*(a*b^3*c - 4*a^2*b*c^2)*f)*g*h + (8*(a*b
^2*c^2 - 4*a^2*c^3)*d - 12*(a*b^3*c - 4*a^2*b*c^2)*e + 3*(5*a*b^4 - 24*a^2*
b^2*c + 16*a^3*c^2)*f)*h^2 + (8*(b^2*c^3 - 4*a*c^4)*f*g^2 + 8*(2*(b^2*c^3 -
 4*a*c^4)*e - 3*(b^3*c^2 - 4*a*b*c^3)*f)*g*h + (8*(b^2*c^3 - 4*a*c^4)*d - 1
2*(b^3*c^2 - 4*a*b*c^3)*e + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*h^2)
*x^2 + (8*(b^3*c^2 - 4*a*b*c^3)*f*g^2 + 8*(2*(b^3*c^2 - 4*a*b*c^3)*e - 3*(b
^4*c - 4*a*b^2*c^2)*f)*g*h + (8*(b^3*c^2 - 4*a*b*c^3)*d - 12*(b^4*c - 4*a*b
^2*c^2)*e + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f)*h^2)*x)*sqrt(-c)*arcta
n(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) -
 2*(2*(b^2*c^3 - 4*a*c^4)*f*h^2*x^3 - 8*(b*c^4*d - 2*a*c^4*e + a*b*c^3*f)*g
^2 + 8*(4*a*c^4*d - 2*a*b*c^3*e + (3*a*b^2*c^2 - 8*a^2*c^3)*f)*g*h - (8*a*b
*c^3*d - 4*(3*a*b^2*c^2 - 8*a^2*c^3)*e + (15*a*b^3*c - 52*a^2*b*c^2)*f)*h^2
 + (8*(b^2*c^3 - 4*a*c^4)*f*g*h + (4*(b^2*c^3 - 4*a*c^4)*e - 5*(b^3*c^2 - 4
*a*b*c^3)*f)*h^2)*x^2 - (8*(2*c^5*d - b*c^4*e + (b^2*c^3 - 2*a*c^4)*f)*g^2
- 8*(2*b*c^4*d - 2*(b^2*c^3 - 2*a*c^4)*e + (3*b^3*c^2 - 10*a*b*c^3)*f)*g*h
+ (8*(b^2*c^3 - 2*a*c^4)*d - 4*(3*b^3*c^2 - 10*a*b*c^3)*e + (15*b^4*c - 62*
a*b^2*c^2 + 24*a^2*c^3)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^
2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x)]
```

giac [B] time = 0.32, size = 580, normalized size = 2.01

$$\left(\left(\frac{2\left(b^2c^2fh^2-4\,ac^3fh^2\right)x}{b^2c^3-4\,ac^4}+\frac{8\,b^2c^2fgh-32\,ac^3fgh-5\,b^3cfh^2+20\,abc^2fh^2+4\,b^2c^2h^2e-16\,ac^3h^2e}{b^2c^3-4\,ac^4}\right)x-\frac{16\,c^4dg^2+8\,b^2c^2fg^2-16\,ac^3fg^2-16\,bc^3dgh-24\,b\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(((2*(b^2*c^2*f*h^2 - 4*a*c^3*f*h^2)*x/(b^2*c^3 - 4*a*c^4) + (8*b^2*c^2
*f*g*h - 32*a*c^3*f*g*h - 5*b^3*c*f*h^2 + 20*a*b*c^2*f*h^2 + 4*b^2*c^2*h^2*
e - 16*a*c^3*h^2*e)/(b^2*c^3 - 4*a*c^4))*x - (16*c^4*d*g^2 + 8*b^2*c^2*f*g^
2 - 16*a*c^3*f*g^2 - 16*b*c^3*d*g*h - 24*b^3*c*f*g*h + 80*a*b*c^2*f*g*h + 8
*b^2*c^2*d*h^2 - 16*a*c^3*d*h^2 + 15*b^4*f*h^2 - 62*a*b^2*c*f*h^2 + 24*a^2*
c^2*f*h^2 - 8*b*c^3*g^2*e + 16*b^2*c^2*g*h*e - 32*a*c^3*g*h*e - 12*b^3*c*h^
2*e + 40*a*b*c^2*h^2*e)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d*g^2 + 8*a*b*c^2
*f*g^2 - 32*a*c^3*d*g*h - 24*a*b^2*c*f*g*h + 64*a^2*c^2*f*g*h + 8*a*b*c^2*d
*h^2 + 15*a*b^3*f*h^2 - 52*a^2*b*c*f*h^2 - 16*a*c^3*g^2*e + 16*a*b*c^2*g*h*
e - 12*a*b^2*c*h^2*e + 32*a^2*c^2*h^2*e)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^2 +
```

```
b*x + a) - 1/8*(8*c^2*f*g^2 - 24*b*c*f*g*h + 8*c^2*d*h^2 + 15*b^2*f*h^2 - 1
2*a*c*f*h^2 + 16*c^2*g*h*e - 12*b*c*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

maple [B] time = 0.01, size = 1557, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -13/2*h^2*f/c^2*b^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+1/c^(3/2)*ln((c*x+1
/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2+1/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))*f*g^2-1/c/(c*x^2+b*x+a)^(1/2)*g^2*e+4*a/c*b/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)*x*h^2*e+4*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g*
h*f+2/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e*g*h-3/c^2*b^3/(4*a*c-b^2)/(
c*x^2+b*x+a)^(1/2)*x*g*h*f+8*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g*h*f+
2*g^2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+15/16*h^2*f/c^4*b^3/(c*x^
2+b*x+a)^(1/2)+1/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*f*g^2-5/4*h^2*f/c^
2*b*x^2/(c*x^2+b*x+a)^(1/2)-15/8*h^2*f/c^3*b^2*x/(c*x^2+b*x+a)^(1/2)+15/16*
h^2*f/c^4*b^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-13/4*h^2*f/c^3*b*a/(c*x^2+b*x
+a)^(1/2)+3/2*h^2*f*a/c^2*x/(c*x^2+b*x+a)^(1/2)+2*x^2/c/(c*x^2+b*x+a)^(1/2)
*g*h*f+3/2/c^2*b*x/(c*x^2+b*x+a)^(1/2)*h^2*e-3/2/c^3*b^2/(c*x^2+b*x+a)^(1/2
)*g*h*f-3/4/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*h^2*e-3/c^(5/2)*b*ln((c
*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h*f+4*a/c^2/(c*x^2+b*x+a)^(1/2)*g*
h*f-2*x/c/(c*x^2+b*x+a)^(1/2)*e*g*h+1/c^2*b/(c*x^2+b*x+a)^(1/2)*e*g*h+1/2/c
^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*h^2+1/2/c^2*b^3/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)*f*g^2-2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^2*e-b^2/c/(4*a*
c-b^2)/(c*x^2+b*x+a)^(1/2)*g^2*e+1/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*
e*g*h-3/2/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*h^2*e-3/2/c^3*b^4/(4*a*
c-b^2)/(c*x^2+b*x+a)^(1/2)*g*h*f+2*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2
)*h^2*e+1/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*h^2+3/c^2*b*x/(c*x^2+b*
x+a)^(1/2)*g*h*f-4*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g*h*d-2*b^2/c/(4*a*c
-b^2)/(c*x^2+b*x+a)^(1/2)*g*h*d+15/8*h^2*f/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2)*x-13/4*h^2*f/c^3*b^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+15/8*h^2*f/c
^(7/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/2*h^2*f*a/c^(5/2)*
ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-x/c/(c*x^2+b*x+a)^(1/2)*d*h^2-x
/c/(c*x^2+b*x+a)^(1/2)*f*g^2+1/2/c^2*b/(c*x^2+b*x+a)^(1/2)*d*h^2+1/2/c^2*b/
(c*x^2+b*x+a)^(1/2)*f*g^2+2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*e*g*h-2/c/(c*x^2+b*x+a)^(1/2)*g*h*d+1/2*h^2*f*x^3/c/(c*x^2+b*x+a)^(1/2
)+x^2/c/(c*x^2+b*x+a)^(1/2)*h^2*e-3/4/c^3*b^2/(c*x^2+b*x+a)^(1/2)*h^2*e-3/2
/c^(5/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^2*e+2*a/c^2/(c*x^2
+b*x+a)^(1/2)*h^2*e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g+h\,x\right)}^2\,\left(f\,x^2+e\,x+d\right)}{{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g+hx\right)^2\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**2*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

3.235 $\int \frac{(g+hx)(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=186

$$\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{h\sqrt{a+bx+cx^2}\left(-8acf+3b^2f-2bce+4c\right.}{c^2\left(b^2-4ac\right)}$$

```
[Out] -1/2*(3*b*f*h-2*c*(e*h+f*g))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/c^(5/2)+2*(c*(2*a*e-b*(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)*(h*
x+g)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)+(4*c^2*d+3*b^2*f-2*c*(4*a*f+b*e))*h
*(c*x^2+b*x+a)^(1/2)/c^2/(-4*a*c+b^2)
```

Rubi [A] time = 0.23, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1644, 640, 621, 206}

$$\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{h\sqrt{a+bx+cx^2}\left(-8acf+3b^2f-2bce+4c\right.}{c^2\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x))/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + ((4*c^2*d - 2*b*c*e + 3*
b^2*f - 8*a*c*f)*h*Sqrt[a + b*x + c*x^2])/(c^2*(b^2 - 4*a*c)) - ((3*b*f*h -
 2*c*(f*g + e*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
2*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
```

```
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - 2\int \frac{-\frac{b^2fg+2b(cd+a}{}}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(4c^2d+3b^2f\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(4c^2d+3b^2f\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(4c^2d+3b^2f\right.}{}$$

Mathematica [A] time = 0.73, size = 205, normalized size = 1.10

$$\frac{\frac{2\sqrt{c}\left(4c\left(2a^2fh-ac(dh+e(g+hx)+fx(g-hx))+c^2dgx\right)+b^2(cx(2eh+2fg-fhx)-3afh)+2bc(aeh+af(g+5hx)+cd(g-hx)-cegx)-3b^3fhx\right)}{\sqrt{a+x(b+cx)}}+\left(b^2-4ac\right)\text{lo}}{2c^{5/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(-3*b^3*f*h*x + 2*b*c*(a*e*h - c*e*g*x + c*d*(g - h*x) + a*f*(g
 + 5*h*x)) + b^2*(-3*a*f*h + c*x*(2*f*g + 2*e*h - f*h*x)) + 4*c*(2*a^2*f*h
+ c^2*d*g*x - a*c*(d*h + f*x*(g - h*x) + e*(g + h*x)))))/Sqrt[a + x*(b + c*
x)] + (b^2 - 4*a*c)*(3*b*f*h - 2*c*(f*g + e*h))*Log[b + 2*c*x + 2*Sqrt[c]*S
qrt[a + x*(b + c*x)]])/(2*c^(5/2)*(-b^2 + 4*a*c))
```

fricas [B] time = 14.45, size = 905, normalized size = 4.87

$$\left[-\frac{\left(2\left(ab^2c-4a^2c^2\right)fg+\left(2\left(b^2c^2-4ac^3\right)fg+\left(2\left(b^2c^2-4ac^3\right)e-3\left(b^3c-4abc^2\right)f\right)h\right)x^2+\left(2\left(ab^2c-4a^2c^2\right)e-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*(a*b^2*c - 4*a^2*c^2)*f*g + (2*(b^2*c^2 - 4*a*c^3)*f*g + (2*(b^2*
c^2 - 4*a*c^3)*e - 3*(b^3*c - 4*a*b*c^2)*f)*h)*x^2 + (2*(a*b^2*c - 4*a^2*c^
2)*e - 3*(a*b^3 - 4*a^2*b*c)*f)*h + (2*(b^3*c - 4*a*b*c^2)*f*g + (2*(b^3*c
- 4*a*b*c^2)*e - 3*(b^4 - 4*a*b^2*c)*f)*h)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*
c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*((b^2*
```

```
c^2 - 4*a*c^3)*f*h*x^2 - 2*(b*c^3*d - 2*a*c^3*e + a*b*c^2*f)*g + (4*a*c^3*d
 - 2*a*b*c^2*e + (3*a*b^2*c - 8*a^2*c^2)*f)*h - (2*(2*c^4*d - b*c^3*e + (b^
2*c^2 - 2*a*c^3)*f)*g - (2*b*c^3*d - 2*(b^2*c^2 - 2*a*c^3)*e + (3*b^3*c - 1
0*a*b*c^2)*f)*h)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^
4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c^4)*x), -1/2*((2*(a*b^2*c - 4*a^2*c^2)
*f*g + (2*(b^2*c^2 - 4*a*c^3)*f*g + (2*(b^2*c^2 - 4*a*c^3)*e - 3*(b^3*c - 4
*a*b*c^2)*f)*h)*x^2 + (2*(a*b^2*c - 4*a^2*c^2)*e - 3*(a*b^3 - 4*a^2*b*c)*f)
*h + (2*(b^3*c - 4*a*b*c^2)*f*g + (2*(b^3*c - 4*a*b*c^2)*e - 3*(b^4 - 4*a*b
^2*c)*f)*h)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-
c)/(c^2*x^2 + b*c*x + a*c)) - 2*((b^2*c^2 - 4*a*c^3)*f*h*x^2 - 2*(b*c^3*d -
 2*a*c^3*e + a*b*c^2*f)*g + (4*a*c^3*d - 2*a*b*c^2*e + (3*a*b^2*c - 8*a^2*c
^2)*f)*h - (2*(2*c^4*d - b*c^3*e + (b^2*c^2 - 2*a*c^3)*f)*g - (2*b*c^3*d -
2*(b^2*c^2 - 2*a*c^3)*e + (3*b^3*c - 10*a*b*c^2)*f)*h)*x)*sqrt(c*x^2 + b*x
+ a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c
^4)*x)]
```

giac [A] time = 0.28, size = 271, normalized size = 1.46

$$\frac{\left(\frac{(b^2cfh-4\,ac^2fh)x}{b^2c^2-4\,ac^3} - \frac{4\,c^3dg+2\,b^2cfg-4\,ac^2fg-2\,bc^2dh-3\,b^3fh+10\,abcfh-2\,bc^2ge+2\,b^2che-4\,ac^2he}{b^2c^2-4\,ac^3}\right)x - \frac{2\,bc^2dg+2\,abcfg-4\,ac^2dh-3\,ab^2fh+8\,a}{b^2c^2-4\,ac^3}}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] (((b^2*c*f*h - 4*a*c^2*f*h)*x/(b^2*c^2 - 4*a*c^3) - (4*c^3*d*g + 2*b^2*c*f*
g - 4*a*c^2*f*g - 2*b*c^2*d*h - 3*b^3*f*h + 10*a*b*c*f*h - 2*b*c^2*g*e + 2*
b^2*c*h*e - 4*a*c^2*h*e)/(b^2*c^2 - 4*a*c^3))*x - (2*b*c^2*d*g + 2*a*b*c*f*
g - 4*a*c^2*d*h - 3*a*b^2*f*h + 8*a^2*c*f*h - 4*a*c^2*g*e + 2*a*b*c*h*e)/(b
^2*c^2 - 4*a*c^3))/sqrt(c*x^2 + b*x + a) - 1/2*(2*c*f*g - 3*b*f*h + 2*c*h*e
)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2)
```

maple [B] time = 0.01, size = 735, normalized size = 3.95

$$\frac{4abfhx}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c} - \frac{3b^3fhx}{2\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c^2} + \frac{b^2ehx}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c} + \frac{b^2fgx}{\left(4ac-b^2\right)\sqrt{c\,x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 1/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g-1/c/(c*x^2+b*x+a)
^(1/2)*d*h-1/c/(c*x^2+b*x+a)^(1/2)*e*g+1/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*e*h-3/2*f*h/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+2*f
*h*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+4*f*h*a/c*b/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)*x+1/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e*h+1/c*b^2/(4*a*
c-b^2)/(c*x^2+b*x+a)^(1/2)*x*f*g-x/c/(c*x^2+b*x+a)^(1/2)*e*h+2*f*h*a/c^2/(c
*x^2+b*x+a)^(1/2)-3/2*f*h/c^(5/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))+f*h*x^2/c/(c*x^2+b*x+a)^(1/2)-3/4*f*h/c^3*b^2/(c*x^2+b*x+a)^(1/2)+2*d*
g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/2/c^2*b/(c*x^2+b*x+a)^(1/2)*f
*g-2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e*g-2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2)*x*d*h+1/2/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*f*g+1/2/c^2*b^3/(4*
a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e*h+3/2*f*h/c^2*b*x/(c*x^2+b*x+a)^(1/2)-3/4*f*
h/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)*d*h-b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e*g-x/c/(c*x^2+b*x+a)^(1/2)*
f*g+1/2/c^2*b/(c*x^2+b*x+a)^(1/2)*e*h
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g+hx)\left(fx^2+ex+d\right)}{\left(cx^2+bx+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

3.236 $\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

```
[Out] f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)+2*(c*(2*a*e-b*
(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(
1/2)
```

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1660, 12, 621, 206}

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(
c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{\left(b^2-4ac\right)f}{2c\sqrt{a+bx+cx^2}}dx}{b^2-4ac}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\int \frac{1}{\sqrt{a+bx+cx^2}}dx}{c}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{(2f)\,\text{Subst}\left(\int \frac{1}{4c-x^2}dx,x,\frac{b}{\sqrt{a+}}\right.}{c}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.30, size = 113, normalized size = 1.02

$$\frac{\frac{2\sqrt{c}\left(abf-2ac(e+fx)+b^2fx+bc(d-ex)+2c^2dx\right)}{\sqrt{a+x(b+cx)}}-f\left(b^2-4ac\right)\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{c^{3/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(a*b*f + 2*c^2*d*x + b^2*f*x + b*c*(d - e*x) - 2*a*c*(e + f*x))
)/Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*f*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)]])/(c^(3/2)*(-b^2 + 4*a*c))
```

fricas [B] time = 1.37, size = 429, normalized size = 3.86

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)fx^2+\left(b^3-4\,abc\right)fx+\left(ab^2-4\,a^2c\right)f\right)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{}\right.}{2\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^2+\left(b^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)
*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x +
b)*sqrt(c) - 4*a*c) - 4*(b*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e
 + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 +
(b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -(((b^2*c - 4*a*c^2)*f*
x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(b*c^2*d
- 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c
*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^
2 - 4*a*b*c^3)*x)]
```

giac [A] time = 0.27, size = 122, normalized size = 1.10

$$-\frac{2\left(\frac{\left(2\,c^2d+b^2f-2\,acf-bce\right)x}{b^2c-4\,ac^2}+\frac{bcd+abf-2\,ace}{b^2c-4\,ac^2}\right)}{\sqrt{cx^2+bx+a}}-\frac{f\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^2*d + b^2*f - 2*a*c*f - b*c*e)*x/(b^2*c - 4*a*c^2) + (b*c*d + a*b*
f - 2*a*c*e)/(b^2*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - f*log(abs(-2*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2)
```

maple [B] time = 0.01, size = 249, normalized size = 2.24

$$\frac{b^2 f x}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c}-\frac{2bex}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+\frac{b^3 f}{2\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c^2}-\frac{b^2 e}{\left(4ac-b^2\right)\sqrt{c\,x^2+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -f*x/c/(c*x^2+b*x+a)^(1/2)+1/2*f/c^2*b/(c*x^2+b*x+a)^(1/2)+f/c*b^2/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*x+1/2*f/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+f/c
^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-e/c/(c*x^2+b*x+a)^(1/2)-
2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)+2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 4.54, size = 143, normalized size = 1.29

$$\frac{f\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}}-\frac{e\,\left(4\,a+2\,b\,x\right)}{\left(4\,a\,c-b^2\right)\,\sqrt{c\,x^2+b\,x+a}}+\frac{d\,\left(\frac{b}{2}+c\,x\right)}{\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}+\frac{f\,\left(\frac{a\,b}{2}-x\,\left(a\,c-\right.\right.}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] (f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) - (e*(4*a +
2*b*x))/((4*a*c - b^2)*(a + b*x + c*x^2)^(1/2)) + (d*(b/2 + c*x))/((a*c - b
^2/4)*(a + b*x + c*x^2)^(1/2)) + (f*((a*b)/2 - x*(a*c - b^2/2)))/(c*(a*c -
b^2/4)*(a + b*x + c*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

3.237 $\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=225

$$\frac{2\left(-x\left(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg\right)-b(aeh+afg+cdg)+2a(afh-cdh+ceg)+b^2d\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ah^2-bgh+cg^2\right)}$$

```
[Out] (f*g^2-h*(-d*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*
g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(3/2)+2*(b^2*d*h-b*(a*e
*h+a*f*g+c*d*g)+2*a*(a*f*h-c*d*h+c*e*g)-(2*c^2*d*g+b*f*(-a*h+b*g)-c*(-2*a*e
*h+2*a*f*g+b*d*h+b*e*g))*x)/(-4*a*c+b^2)/(a*h^2-b*g*h+c*g^2)/(c*x^2+b*x+a)^
(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 12, 724, 206}

$$\frac{2\left(-x\left(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg\right)-b(aeh+afg+cdg)+2a(afh-cdh+ceg)+b^2d\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ah^2-bgh+cg^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^2*d*h - b*(c*d*g + a*f*g + a*e*h) + 2*a*(c*e*g - c*d*h + a*f*h) - (2*
c^2*d*g + b*f*(b*g - a*h) - c*(b*e*g + 2*a*f*g + b*d*h - 2*a*e*h))*x))/((b^
2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)*Sqrt[a + b*x + c*x^2]) + ((f*g^2 - h*(e*
g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a
*h^2]*Sqrt[a + b*x + c*x^2])])/(c*g^2 - b*g*h + a*h^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
```

```
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-\left(2c^2dg+bf(bg-ah\right.\right.}{\left(b^2-4ac\right)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx+cx}}$$

$$=\frac{2\left(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-\left(2c^2dg+bf(bg-ah\right.\right.}{\left(b^2-4ac\right)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx+cx}}$$

$$=\frac{2\left(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-\left(2c^2dg+bf(bg-ah\right.\right.}{\left(b^2-4ac\right)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx+cx}}$$

$$=\frac{2\left(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-\left(2c^2dg+bf(bg-ah\right.\right.}{\left(b^2-4ac\right)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx+cx}}$$

Mathematica [A] time = 0.49, size = 271, normalized size = 1.20

$$\frac{\frac{-b^2\left(afh^2+2cdh^2+cfg(g-2hx)\right)-2bch(-aeh+af(g+hx)+c(-dg+dhx+egx))+4c^2(ah(dh-eg+ehx)+afg(g-hx)+cdghx)+b^3fgh}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(h(bg-ah)-cg^2\right)}-\frac{ch\left(h(dh-eg)+fg^2\right)\text{ta}}{(h}}{ch}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-(f/Sqrt[a + x*(b + c*x)]) + (b^3*f*g*h - b^2*(2*c*d*h^2 + a*f*h^2 + c*f*g
*(g - 2*h*x)) - 2*b*c*h*(-(a*e*h) + a*f*(g + h*x) + c*(-(d*g) + e*g*x + d*h
*x)) + 4*c^2*(c*d*g*h*x + a*f*g*(g - h*x) + a*h*(-(e*g) + d*h + e*h*x)))/((
b^2 - 4*a*c)*(-(c*g^2) + h*(b*g - a*h))*Sqrt[a + x*(b + c*x)]) - (c*h*(f*g^
2 + h*(-(e*g) + d*h))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*
g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))
^(3/2))/(c*h)
```

fricas [B] time = 34.40, size = 1905, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((a*b^2 - 4*a^2*c)*f*g^2 - (a*b^2 - 4*a^2*c)*e*g*h + (a*b^2 - 4*a^2*c
)*d*h^2 + ((b^2*c - 4*a*c^2)*f*g^2 - (b^2*c - 4*a*c^2)*e*g*h + (b^2*c - 4*a
*c^2)*d*h^2)*x^2 + ((b^3 - 4*a*b*c)*f*g^2 - (b^3 - 4*a*b*c)*e*g*h + (b^3 -
4*a*b*c)*d*h^2)*x)*sqrt(c*g^2 - b*g*h + a*h^2)*log((8*a*b*g*h - 8*a^2*h^2 -
 (b^2 + 4*a*c)*g^2 - (8*c^2*g^2 - 8*b*c*g*h + (b^2 + 4*a*c)*h^2)*x^2 - 4*sq
rt(c*g^2 - b*g*h + a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h
)*x) - 2*(4*b*c*g^2 + 4*a*b*h^2 - (3*b^2 + 4*a*c)*g*h)*x)/(h^2*x^2 + 2*g*h*
x + g^2)) - 4*((b*c^2*d - 2*a*c^2*e + a*b*c*f)*g^3 + (3*a*b*c*e - 2*(b^2*c
```

```
- a*c^2)*d - (a*b^2 + 2*a^2*c)*f)*g^2*h + (3*a^2*b*f + (b^3 - a*b*c)*d - (a
*b^2 + 2*a^2*c)*e)*g*h^2 + (a^2*b*e - 2*a^3*f - (a*b^2 - 2*a^2*c)*d)*h^3 +
((2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*g^3 - (3*b*c^2*d - (b^2*c + 2*a*
c^2)*e + (b^3 - a*b*c)*f)*g^2*h - (3*a*b*c*e - (b^2*c + 2*a*c^2)*d - 2*(a*b
^2 - a^2*c)*f)*g*h^2 - (a*b*c*d - 2*a^2*c*e + a^2*b*f)*h^3)*x)*sqrt(c*x^2 +
 b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*g^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*g^3*h +
 (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*g^2*h^2 - 2*(a^2*b^3 - 4*a^3*b*c)*g*h^3
+ (a^3*b^2 - 4*a^4*c)*h^4 + ((b^2*c^3 - 4*a*c^4)*g^4 - 2*(b^3*c^2 - 4*a*b*c
^3)*g^3*h + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*g^2*h^2 - 2*(a*b^3*c - 4*a^2*
b*c^2)*g*h^3 + (a^2*b^2*c - 4*a^3*c^2)*h^4)*x^2 + ((b^3*c^2 - 4*a*b*c^3)*g^
4 - 2*(b^4*c - 4*a*b^2*c^2)*g^3*h + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*g^2*h^2
 - 2*(a*b^4 - 4*a^2*b^2*c)*g*h^3 + (a^2*b^3 - 4*a^3*b*c)*h^4)*x), (((a*b^2
- 4*a^2*c)*f*g^2 - (a*b^2 - 4*a^2*c)*e*g*h + (a*b^2 - 4*a^2*c)*d*h^2 + ((b^
2*c - 4*a*c^2)*f*g^2 - (b^2*c - 4*a*c^2)*e*g*h + (b^2*c - 4*a*c^2)*d*h^2)*x
^2 + ((b^3 - 4*a*b*c)*f*g^2 - (b^3 - 4*a*b*c)*e*g*h + (b^3 - 4*a*b*c)*d*h^2
)*x)*sqrt(-c*g^2 + b*g*h - a*h^2)*arctan(-1/2*sqrt(-c*g^2 + b*g*h - a*h^2)*
sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x)/(a*c*g^2 - a*b*g*h +
a^2*h^2 + (c^2*g^2 - b*c*g*h + a*c*h^2)*x^2 + (b*c*g^2 - b^2*g*h + a*b*h^2)
*x)) - 2*((b*c^2*d - 2*a*c^2*e + a*b*c*f)*g^3 + (3*a*b*c*e - 2*(b^2*c - a*c
^2)*d - (a*b^2 + 2*a^2*c)*f)*g^2*h + (3*a^2*b*f + (b^3 - a*b*c)*d - (a*b^2
+ 2*a^2*c)*e)*g*h^2 + (a^2*b*e - 2*a^3*f - (a*b^2 - 2*a^2*c)*d)*h^3 + ((2*c
^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*g^3 - (3*b*c^2*d - (b^2*c + 2*a*c^2)*
e + (b^3 - a*b*c)*f)*g^2*h - (3*a*b*c*e - (b^2*c + 2*a*c^2)*d - 2*(a*b^2 -
a^2*c)*f)*g*h^2 - (a*b*c*d - 2*a^2*c*e + a^2*b*f)*h^3)*x)*sqrt(c*x^2 + b*x
+ a))/((a*b^2*c^2 - 4*a^2*c^3)*g^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*g^3*h + (a*b
^4 - 2*a^2*b^2*c - 8*a^3*c^2)*g^2*h^2 - 2*(a^2*b^3 - 4*a^3*b*c)*g*h^3 + (a^
3*b^2 - 4*a^4*c)*h^4 + ((b^2*c^3 - 4*a*c^4)*g^4 - 2*(b^3*c^2 - 4*a*b*c^3)*g
^3*h + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*g^2*h^2 - 2*(a*b^3*c - 4*a^2*b*c^2
)*g*h^3 + (a^2*b^2*c - 4*a^3*c^2)*h^4)*x^2 + ((b^3*c^2 - 4*a*b*c^3)*g^4 - 2
*(b^4*c - 4*a*b^2*c^2)*g^3*h + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*g^2*h^2 - 2*
(a*b^4 - 4*a^2*b^2*c)*g*h^3 + (a^2*b^3 - 4*a^3*b*c)*h^4)*x)]
```

giac [B] time = 0.29, size = 719, normalized size = 3.20

$$-\frac{2\left(\frac{(2\,c^3dg^3+b^2cfg^3-2\,ac^2fg^3-3\,bc^2dg^2h-b^3fg^2h+abcfg^2h+b^2cdgh^2+2\,ac^2dgh^2+2\,ab^2fgh^2-2\,a^2cfgh^2-abcdh^3-a^2bfh^3-bc^2g^3e+b^2cg^2he+2\,ac^2g^2h}{b^2c^2g^4-4\,ac^3g^4-2\,b^3cg^3h+8\,abc^2g^3h+b^4g^2h^2-2\,ab^2cg^2h^2-8\,a^2c^2g^2h^2-2\,ab^3gh^3+8\,a^2bcgh^3+a^2b^2h^4-4\,a^3ch^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d*g^3 + b^2*c*f*g^3 - 2*a*c^2*f*g^3 - 3*b*c^2*d*g^2*h - b^3*f*g^
2*h + a*b*c*f*g^2*h + b^2*c*d*g*h^2 + 2*a*c^2*d*g*h^2 + 2*a*b^2*f*g*h^2 - 2
*a^2*c*f*g*h^2 - a*b*c*d*h^3 - a^2*b*f*h^3 - b*c^2*g^3*e + b^2*c*g^2*h*e +
2*a*c^2*g^2*h*e - 3*a*b*c*g*h^2*e + 2*a^2*c*h^3*e)*x/(b^2*c^2*g^4 - 4*a*c^3
*g^4 - 2*b^3*c*g^3*h + 8*a*b*c^2*g^3*h + b^4*g^2*h^2 - 2*a*b^2*c*g^2*h^2 -
8*a^2*c^2*g^2*h^2 - 2*a*b^3*g*h^3 + 8*a^2*b*c*g*h^3 + a^2*b^2*h^4 - 4*a^3*c
*h^4) + (b*c^2*d*g^3 + a*b*c*f*g^3 - 2*b^2*c*d*g^2*h + 2*a*c^2*d*g^2*h - a*
b^2*f*g^2*h - 2*a^2*c*f*g^2*h + b^3*d*g*h^2 - a*b*c*d*g*h^2 + 3*a^2*b*f*g*h
^2 - a*b^2*d*h^3 + 2*a^2*c*d*h^3 - 2*a^3*f*h^3 - 2*a*c^2*g^3*e + 3*a*b*c*g^
2*h*e - a*b^2*g*h^2*e - 2*a^2*c*g*h^2*e + a^2*b*h^3*e)/(b^2*c^2*g^4 - 4*a*c
^3*g^4 - 2*b^3*c*g^3*h + 8*a*b*c^2*g^3*h + b^4*g^2*h^2 - 2*a*b^2*c*g^2*h^2
- 8*a^2*c^2*g^2*h^2 - 2*a*b^3*g*h^3 + 8*a^2*b*c*g*h^3 + a^2*b^2*h^4 - 4*a^3
*c*h^4))/sqrt(c*x^2 + b*x + a) + 2*(f*g^2 + d*h^2 - g*h*e)*arctan(-((sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/(
(c*g^2 - b*g*h + a*h^2)*sqrt(-c*g^2 + b*g*h - a*h^2))
```

maple [B] time = 0.01, size = 2079, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -4/h/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*c^2*g^2*e+4/h^2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^
2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*c^2*
g^3*f-2*h/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/
h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b*c*d+2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/
((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b*c*e*g
-2/h/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*c*g^2*e+2/h^2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)
/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*c*g^3*
f-1/h*f/c/(c*x^2+b*x+a)^(1/2)+h/(a*h^2-b*g*h+c*g^2)/((x+g/h)^2*c+(b*h-2*c*g
)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*d-1/(a*h^2-b*g*h+c*g^2)/((x+g/h)
^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e*g-h/(a*h^2-b*g*
h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)*b^2*d+1/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g
)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b^2*e*g-1/h/(a*h^2-b*g*h+c*g^2)/
((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*
g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)
/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*f*g^2-2/h*f*b/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)*x-1/h*f*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+4/h*e/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*x*c-2/h^2*f*g/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b-2/h
/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-
b*g*h+c*g^2)/h^2)^(1/2)*x*b*c*f*g^2+2/h*e/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b
+1/h/(a*h^2-b*g*h+c*g^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*
g^2)/h^2)^(1/2)*f*g^2-h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2))/(x+g/h))*d+1/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b
*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(
1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+
g/h))*e*g+2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h
)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*c*g*d-4/h^2*f*g/(4*a*c-b^2)/(c*x^2+b*x
+a)^(1/2)*x*c+4/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x
+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*c^2*g*d-1/h/(a*h^2-b*g*h+c*g^2)/(4
*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
*b^2*f*g^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `assum
e?` for more details)Is (b/h-(2*c*g)/h^2)^2    -(4*c       *((-(b*g)/h)
    +(c*g^2)/h^2+a))     /h^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2}{(g + hx)\left(a + bx + cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.238 $\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=421

$$-\frac{2\left(cx\left(2a^2fh^2-c\left(2a\left(dh^2-2egh+fg^2\right)+bg(2dh+eg)\right)-abh(eh+2fg)+b^2\left(dh^2+fg^2\right)+2c^2dg^2\right)+b\left(a\cdots\right.\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx\cdots}}$$

```
[Out] 1/2*(2*c*g*(f*g^2-h*(-3*d*h+2*e*g))-h*(2*a*h*(-e*h+2*f*g)-b*(-3*d*h^2+e*g*h
+f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/
(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(5/2)-2*(b^3*d*h^2-b^2*h*(a*e*h+2*
c*d*g)-2*a*c*(c*g*(-2*d*h+e*g)+a*h*(-e*h+2*f*g))+b*(c^2*d*g^2+a^2*f*h^2+a*c
*(-3*d*h^2+2*e*g*h+f*g^2))+c*(2*c^2*d*g^2+2*a^2*f*h^2-a*b*h*(e*h+2*f*g)+b^2
*(d*h^2+f*g^2)-c*(b*g*(2*d*h+e*g)+2*a*(d*h^2-2*e*g*h+f*g^2)))*x)/(-4*a*c+b^
2)/(a*h^2-b*g*h+c*g^2)^2/(c*x^2+b*x+a)^(1/2)-h*(f*g^2-h*(-d*h+e*g))*(c*x^2+
b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)
```

Rubi [A] time = 0.80, antiderivative size = 418, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 806, 724, 206}

$$-\frac{2\left(cx\left(2a^2fh^2-c\left(2a\left(dh^2-2egh+fg^2\right)+bg(2dh+eg)\right)-abh(eh+2fg)+b^2\left(dh^2+fg^2\right)+2c^2dg^2\right)+b\left(a\cdots\right.\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b^3*d*h^2 - b^2*h*(2*c*d*g + a*e*h) - 2*a*c*(c*g*(e*g - 2*d*h) + a*h*(
2*f*g - e*h)) + b*(c^2*d*g^2 + a^2*f*h^2 + a*c*(f*g^2 + 2*e*g*h - 3*d*h^2))
 + c*(2*c^2*d*g^2 + 2*a^2*f*h^2 - a*b*h*(2*f*g + e*h) + b^2*(f*g^2 + d*h^2)
 - c*(b*g*(e*g + 2*d*h) + 2*a*(f*g^2 - 2*e*g*h + d*h^2)))*x))/((b^2 - 4*a*c
)*(c*g^2 - b*g*h + a*h^2)^2*Sqrt[a + b*x + c*x^2]) - (h*(f*g^2 - h*(e*g - d
*h))*Sqrt[a + b*x + c*x^2])/((c*g^2 - b*g*h + a*h^2)^2*(g + h*x)) + ((2*c*(
f*g^3 - g*h*(2*e*g - 3*d*h)) + h*(b*f*g^2 + b*h*(e*g - 3*d*h) - 2*a*h*(2*f*
g - e*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
```

```
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b\left(c^2dg^2+\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b\left(c^2dg^2+\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b\left(c^2dg^2+\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b\left(c^2dg^2+\cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 2.46, size = 487, normalized size = 1.16

$$\frac{ch\left(\frac{\left(4ac-b^2\right)\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)\left(h\left(2ah(eh-2fg)+bh(eg-3dh)+bfg^2\right)+2c\left(gh(3dh-2eg)+fg^3\right)\right)}{\left(h(ah-bg)+cg^2\right)^{5/2}}-\frac{2h\sqrt{a+x(b+cx)}\left(4a^2fh^2-2c\left(2ah(2dh-3eg)+4afg^2+bg(2dh+eg\cdots\right.\right.}{(g+hx)\left(h(ah-bg)+\cdots\right.}\cdots\right)}{b^2-4ac}\cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-(f/((g + h*x)*Sqrt[a + x*(b + c*x)])) + (b^3*f*g*h - b^2*(4*c*d*h^2 + a*f
*h^2 + c*f*g*(g - 4*h*x)) - 4*b*c*h*(a*h*(-e + f*x) + c*(-(d*g) + e*g*x + d
*h*x)) + 4*c*(-(a^2*f*h^2) + 2*c^2*d*g*h*x + a*c*(f*g*(g - 2*h*x) + 2*h*(-(
e*g) + d*h + e*h*x))))/((b^2 - 4*a*c)*(-(c*g^2) + h*(b*g - a*h))*(g + h*x)*
Sqrt[a + x*(b + c*x)]) + (c*h*((-2*h*(4*c^2*d*g^2 + 4*a^2*f*h^2 - 2*a*b*h*(
2*f*g + e*h) - 2*c*(4*a*f*g^2 + 2*a*h*(-3*e*g + 2*d*h) + b*g*(e*g + 2*d*h))
 + b^2*(3*f*g^2 + h*(-(e*g) + 3*d*h)))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(
-(b*g) + a*h))^2*(g + h*x)) + ((-b^2 + 4*a*c)*(2*c*(f*g^3 + g*h*(-2*e*g + 3
```

```
*d*h)) + h*(b*f*g^2 + b*h*(e*g - 3*d*h) + 2*a*h*(-2*f*g + e*h)))*ArcTanh[(-
(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a +
 x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(5/2)))/(b^2 - 4*a*c))/(2*c*h)
```

fricas [B] time = 107.70, size = 5098, normalized size = 12.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*((2*(a*b^2*c - 4*a^2*c^2)*f*g^4 - (4*(a*b^2*c - 4*a^2*c^2)*e - (a*b^3
- 4*a^2*b*c)*f)*g^3*h + (6*(a*b^2*c - 4*a^2*c^2)*d + (a*b^3 - 4*a^2*b*c)*e
- 4*(a^2*b^2 - 4*a^3*c)*f)*g^2*h^2 - (3*(a*b^3 - 4*a^2*b*c)*d - 2*(a^2*b^2
- 4*a^3*c)*e)*g*h^3 + (2*(b^2*c^2 - 4*a*c^3)*f*g^3*h - (4*(b^2*c^2 - 4*a*c^
3)*e - (b^3*c - 4*a*b*c^2)*f)*g^2*h^2 + (6*(b^2*c^2 - 4*a*c^3)*d + (b^3*c -
 4*a*b*c^2)*e - 4*(a*b^2*c - 4*a^2*c^2)*f)*g*h^3 - (3*(b^3*c - 4*a*b*c^2)*d
 - 2*(a*b^2*c - 4*a^2*c^2)*e)*h^4)*x^3 + (2*(b^2*c^2 - 4*a*c^3)*f*g^4 - (4*
(b^2*c^2 - 4*a*c^3)*e - 3*(b^3*c - 4*a*b*c^2)*f)*g^3*h + (6*(b^2*c^2 - 4*a*
c^3)*d - 3*(b^3*c - 4*a*b*c^2)*e + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*f)*g^2*h^
2 + (3*(b^3*c - 4*a*b*c^2)*d + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*e - 4*(a*b^3 -
 4*a^2*b*c)*f)*g*h^3 - (3*(b^4 - 4*a*b^2*c)*d - 2*(a*b^3 - 4*a^2*b*c)*e)*h^
4)*x^2 + (2*(b^3*c - 4*a*b*c^2)*f*g^4 - (4*(b^3*c - 4*a*b*c^2)*e - (b^4 - 2
*a*b^2*c - 8*a^2*c^2)*f)*g^3*h + (6*(b^3*c - 4*a*b*c^2)*d + (b^4 - 8*a*b^2*
c + 16*a^2*c^2)*e - 3*(a*b^3 - 4*a^2*b*c)*f)*g^2*h^2 - (3*(b^4 - 6*a*b^2*c
+ 8*a^2*c^2)*d - 3*(a*b^3 - 4*a^2*b*c)*e + 4*(a^2*b^2 - 4*a^3*c)*f)*g*h^3 -
 (3*(a*b^3 - 4*a^2*b*c)*d - 2*(a^2*b^2 - 4*a^3*c)*e)*h^4)*x)*sqrt(c*g^2 - b
*g*h + a*h^2)*log((8*a*b*g*h - 8*a^2*h^2 - (b^2 + 4*a*c)*g^2 - (8*c^2*g^2 -
 8*b*c*g*h + (b^2 + 4*a*c)*h^2)*x^2 - 4*sqrt(c*g^2 - b*g*h + a*h^2)*sqrt(c*
x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x) - 2*(4*b*c*g^2 + 4*a*b*h^2 -
 (3*b^2 + 4*a*c)*g*h)*x)/(h^2*x^2 + 2*g*h*x + g^2)) - 4*((a^2*b^2 - 4*a^3*c
)*d*h^5 + 2*(b*c^3*d - 2*a*c^3*e + a*b*c^2*f)*g^5 + (8*a*b*c^2*e - 2*(3*b^2
*c^2 - 4*a*c^3)*d - (a*b^2*c + 12*a^2*c^2)*f)*g^4*h + (6*(b^3*c - 2*a*b*c^2
)*d - (7*a*b^2*c - 4*a^2*c^2)*e - (a*b^3 - 16*a^2*b*c)*f)*g^3*h^2 - ((2*b^4
 - 3*a*b^2*c - 4*a^2*c^2)*d - (3*a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 + 12*a^3*c
)*f)*g^2*h^3 + (2*a^3*b*f + (a*b^3 - 2*a^2*b*c)*d - (3*a^2*b^2 - 8*a^3*c)*e
)*g*h^4 + ((4*c^4*d - 2*b*c^3*e + (3*b^2*c^2 - 8*a*c^3)*f)*g^4*h - (8*b*c^3
*d - (b^2*c^2 + 12*a*c^3)*e + (3*b^3*c - 4*a*b*c^2)*f)*g^3*h^2 + ((7*b^2*c^
2 - 4*a*c^3)*d + (b^3*c - 16*a*b*c^2)*e + (7*a*b^2*c - 4*a^2*c^2)*f)*g^2*h^
3 - (8*a^2*b*c*f + (3*b^3*c - 4*a*b*c^2)*d - (a*b^2*c + 12*a^2*c^2)*e)*g*h^
4 - (2*a^2*b*c*e - 4*a^3*c*f - (3*a*b^2*c - 8*a^2*c^2)*d)*h^5)*x^2 + (2*(2*
c^4*d - b*c^3*e + (b^2*c^2 - 2*a*c^3)*f)*g^5 - (6*b*c^3*d - 2*(b^2*c^2 + 2*
a*c^3)*e + (b^3*c + 2*a*b*c^2)*f)*g^4*h + (8*a*c^3*d - b^3*c*e - (b^4 - 8*a
*b^2*c + 8*a^2*c^2)*f)*g^3*h^2 + (a*b^3*f + (5*b^3*c - 16*a*b*c^2)*d + (b^4
 - 8*a*b^2*c + 8*a^2*c^2)*e)*g^2*h^3 - ((3*b^4 - 8*a*b^2*c - 4*a^2*c^2)*d -
 (a*b^3 + 2*a^2*b*c)*e + 2*(a^2*b^2 + 2*a^3*c)*f)*g*h^4 + (2*a^3*b*f + (3*a
*b^3 - 10*a^2*b*c)*d - 2*(a^2*b^2 - 2*a^3*c)*e)*h^5)*x)*sqrt(c*x^2 + b*x +
a))/((a*b^2*c^3 - 4*a^2*c^4)*g^7 - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*g^6*h + 3*(a
*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*g^5*h^2 - (a*b^5 + 2*a^2*b^3*c - 24*a^3
*b*c^2)*g^4*h^3 + 3*(a^2*b^4 - 3*a^3*b^2*c - 4*a^4*c^2)*g^3*h^4 - 3*(a^3*b^
3 - 4*a^4*b*c)*g^2*h^5 + (a^4*b^2 - 4*a^5*c)*g*h^6 + ((b^2*c^4 - 4*a*c^5)*g
^6*h - 3*(b^3*c^3 - 4*a*b*c^4)*g^5*h^2 + 3*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c
^4)*g^4*h^3 - (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*g^3*h^4 + 3*(a*b^4*c - 3
*a^2*b^2*c^2 - 4*a^3*c^3)*g^2*h^5 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*g*h^6 + (a^
3*b^2*c - 4*a^4*c^2)*h^7)*x^3 + ((b^2*c^4 - 4*a*c^5)*g^7 - 2*(b^3*c^3 - 4*a
*b*c^4)*g^6*h + 3*(a*b^2*c^3 - 4*a^2*c^4)*g^5*h^2 + (2*b^5*c - 11*a*b^3*c^2
 + 12*a^2*b*c^3)*g^4*h^3 - (b^6 - a*b^4*c - 15*a^2*b^2*c^2 + 12*a^3*c^3)*g^
3*h^4 + 3*(a*b^5 - 4*a^2*b^3*c)*g^2*h^5 - (3*a^2*b^4 - 13*a^3*b^2*c + 4*a^4
*c^2)*g*h^6 + (a^3*b^3 - 4*a^4*b*c)*h^7)*x^2 + ((b^3*c^3 - 4*a*b*c^4)*g^7 -
```

```
(3*b^4*c^2 - 13*a*b^2*c^3 + 4*a^2*c^4)*g^6*h + 3*(b^5*c - 4*a*b^3*c^2)*g^5
*h^2 - (b^6 - a*b^4*c - 15*a^2*b^2*c^2 + 12*a^3*c^3)*g^4*h^3 + (2*a*b^5 - 1
1*a^2*b^3*c + 12*a^3*b*c^2)*g^3*h^4 + 3*(a^3*b^2*c - 4*a^4*c^2)*g^2*h^5 - 2
*(a^3*b^3 - 4*a^4*b*c)*g*h^6 + (a^4*b^2 - 4*a^5*c)*h^7)*x), 1/2*((2*(a*b^2*
c - 4*a^2*c^2)*f*g^4 - (4*(a*b^2*c - 4*a^2*c^2)*e - (a*b^3 - 4*a^2*b*c)*f)*
g^3*h + (6*(a*b^2*c - 4*a^2*c^2)*d + (a*b^3 - 4*a^2*b*c)*e - 4*(a^2*b^2 - 4
*a^3*c)*f)*g^2*h^2 - (3*(a*b^3 - 4*a^2*b*c)*d - 2*(a^2*b^2 - 4*a^3*c)*e)*g*
h^3 + (2*(b^2*c^2 - 4*a*c^3)*f*g^3*h - (4*(b^2*c^2 - 4*a*c^3)*e - (b^3*c -
4*a*b*c^2)*f)*g^2*h^2 + (6*(b^2*c^2 - 4*a*c^3)*d + (b^3*c - 4*a*b*c^2)*e -
4*(a*b^2*c - 4*a^2*c^2)*f)*g*h^3 - (3*(b^3*c - 4*a*b*c^2)*d - 2*(a*b^2*c -
4*a^2*c^2)*e)*h^4)*x^3 + (2*(b^2*c^2 - 4*a*c^3)*f*g^4 - (4*(b^2*c^2 - 4*a*c
^3)*e - 3*(b^3*c - 4*a*b*c^2)*f)*g^3*h + (6*(b^2*c^2 - 4*a*c^3)*d - 3*(b^3*
c - 4*a*b*c^2)*e + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*f)*g^2*h^2 + (3*(b^3*c -
4*a*b*c^2)*d + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*e - 4*(a*b^3 - 4*a^2*b*c)*f)*g
*h^3 - (3*(b^4 - 4*a*b^2*c)*d - 2*(a*b^3 - 4*a^2*b*c)*e)*h^4)*x^2 + (2*(b^3
*c - 4*a*b*c^2)*f*g^4 - (4*(b^3*c - 4*a*b*c^2)*e - (b^4 - 2*a*b^2*c - 8*a^2
*c^2)*f)*g^3*h + (6*(b^3*c - 4*a*b*c^2)*d + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*
e - 3*(a*b^3 - 4*a^2*b*c)*f)*g^2*h^2 - (3*(b^4 - 6*a*b^2*c + 8*a^2*c^2)*d -
 3*(a*b^3 - 4*a^2*b*c)*e + 4*(a^2*b^2 - 4*a^3*c)*f)*g*h^3 - (3*(a*b^3 - 4*a
^2*b*c)*d - 2*(a^2*b^2 - 4*a^3*c)*e)*h^4)*x)*sqrt(-c*g^2 + b*g*h - a*h^2)*a
rctan(-1/2*sqrt(-c*g^2 + b*g*h - a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h
+ (2*c*g - b*h)*x)/(a*c*g^2 - a*b*g*h + a^2*h^2 + (c^2*g^2 - b*c*g*h + a*c*
h^2)*x^2 + (b*c*g^2 - b^2*g*h + a*b*h^2)*x)) - 2*((a^2*b^2 - 4*a^3*c)*d*h^5
 + 2*(b*c^3*d - 2*a*c^3*e + a*b*c^2*f)*g^5 + (8*a*b*c^2*e - 2*(3*b^2*c^2 -
4*a*c^3)*d - (a*b^2*c + 12*a^2*c^2)*f)*g^4*h + (6*(b^3*c - 2*a*b*c^2)*d - (
7*a*b^2*c - 4*a^2*c^2)*e - (a*b^3 - 16*a^2*b*c)*f)*g^3*h^2 - ((2*b^4 - 3*a*
b^2*c - 4*a^2*c^2)*d - (3*a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 + 12*a^3*c)*f)*g^
2*h^3 + (2*a^3*b*f + (a*b^3 - 2*a^2*b*c)*d - (3*a^2*b^2 - 8*a^3*c)*e)*g*h^4
 + ((4*c^4*d - 2*b*c^3*e + (3*b^2*c^2 - 8*a*c^3)*f)*g^4*h - (8*b*c^3*d - (b
^2*c^2 + 12*a*c^3)*e + (3*b^3*c - 4*a*b*c^2)*f)*g^3*h^2 + ((7*b^2*c^2 - 4*a
*c^3)*d + (b^3*c - 16*a*b*c^2)*e + (7*a*b^2*c - 4*a^2*c^2)*f)*g^2*h^3 - (8*
a^2*b*c*f + (3*b^3*c - 4*a*b*c^2)*d - (a*b^2*c + 12*a^2*c^2)*e)*g*h^4 - (2*
a^2*b*c*e - 4*a^3*c*f - (3*a*b^2*c - 8*a^2*c^2)*d)*h^5)*x^2 + (2*(2*c^4*d -
 b*c^3*e + (b^2*c^2 - 2*a*c^3)*f)*g^5 - (6*b*c^3*d - 2*(b^2*c^2 + 2*a*c^3)*
e + (b^3*c + 2*a*b*c^2)*f)*g^4*h + (8*a*c^3*d - b^3*c*e - (b^4 - 8*a*b^2*c
+ 8*a^2*c^2)*f)*g^3*h^2 + (a*b^3*f + (5*b^3*c - 16*a*b*c^2)*d + (b^4 - 8*a*
b^2*c + 8*a^2*c^2)*e)*g^2*h^3 - ((3*b^4 - 8*a*b^2*c - 4*a^2*c^2)*d - (a*b^3
 + 2*a^2*b*c)*e + 2*(a^2*b^2 + 2*a^3*c)*f)*g*h^4 + (2*a^3*b*f + (3*a*b^3 -
10*a^2*b*c)*d - 2*(a^2*b^2 - 2*a^3*c)*e)*h^5)*x)*sqrt(c*x^2 + b*x + a))/((a
*b^2*c^3 - 4*a^2*c^4)*g^7 - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*g^6*h + 3*(a*b^4*c
- 3*a^2*b^2*c^2 - 4*a^3*c^3)*g^5*h^2 - (a*b^5 + 2*a^2*b^3*c - 24*a^3*b*c^2)
*g^4*h^3 + 3*(a^2*b^4 - 3*a^3*b^2*c - 4*a^4*c^2)*g^3*h^4 - 3*(a^3*b^3 - 4*a
^4*b*c)*g^2*h^5 + (a^4*b^2 - 4*a^5*c)*g*h^6 + ((b^2*c^4 - 4*a*c^5)*g^6*h -
3*(b^3*c^3 - 4*a*b*c^4)*g^5*h^2 + 3*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*g^4
*h^3 - (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*g^3*h^4 + 3*(a*b^4*c - 3*a^2*b^
2*c^2 - 4*a^3*c^3)*g^2*h^5 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*g*h^6 + (a^3*b^2*c
 - 4*a^4*c^2)*h^7)*x^3 + ((b^2*c^4 - 4*a*c^5)*g^7 - 2*(b^3*c^3 - 4*a*b*c^4)
*g^6*h + 3*(a*b^2*c^3 - 4*a^2*c^4)*g^5*h^2 + (2*b^5*c - 11*a*b^3*c^2 + 12*a
^2*b*c^3)*g^4*h^3 - (b^6 - a*b^4*c - 15*a^2*b^2*c^2 + 12*a^3*c^3)*g^3*h^4 +
 3*(a*b^5 - 4*a^2*b^3*c)*g^2*h^5 - (3*a^2*b^4 - 13*a^3*b^2*c + 4*a^4*c^2)*g
*h^6 + (a^3*b^3 - 4*a^4*b*c)*h^7)*x^2 + ((b^3*c^3 - 4*a*b*c^4)*g^7 - (3*b^4
*c^2 - 13*a*b^2*c^3 + 4*a^2*c^4)*g^6*h + 3*(b^5*c - 4*a*b^3*c^2)*g^5*h^2 -
(b^6 - a*b^4*c - 15*a^2*b^2*c^2 + 12*a^3*c^3)*g^4*h^3 + (2*a*b^5 - 11*a^2*b
^3*c + 12*a^3*b*c^2)*g^3*h^4 + 3*(a^3*b^2*c - 4*a^4*c^2)*g^2*h^5 - 2*(a^3*b
^3 - 4*a^4*b*c)*g*h^6 + (a^4*b^2 - 4*a^5*c)*h^7)*x)]
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 4930, normalized size = 11.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -1/(a*h^2-b*g*h+c*g^2)/(x+g/h)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*
g*h+c*g^2)/h^2)^(1/2)*d-1/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/
2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^
2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(
1/2))/(x+g/h))*e+3/2/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*
ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
))/(x+g/h))*b*f*g^2+3*h/(a*h^2-b*g*h+c*g^2)^2/((x+g/h)^2*c+(b*h-2*c*g)*(x+g
/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*c*g*d+3/h/(a*h^2-b*g*h+c*g^2)^2/((x+g/
h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*c*g^3*f+3/2*h^2
/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^
2-b*g*h+c*g^2)/h^2)^(1/2)*b^3*d+3/2*h^2/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*d+1/h/(a*h^2-b*g*h+c*g^2)/(x+g/h)/((x+g/h)
^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e*g-2/h/(a*h^2-b*
g*h+c*g^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
)*f*g-1/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b^2*e+2/h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+
c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a
*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*
h+c*g^2)/h^2)^(1/2))/(x+g/h))*f*g-1/h^2/(a*h^2-b*g*h+c*g^2)/(x+g/h)/((x+g/h
)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*f*g^2+3/2*h/(a*h
^2-b*g*h+c*g^2)^2/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^
2)^(1/2)*b*e*g+3/2/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*
g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b^3*f*g^2+1/(a*h^2-b*g*h+c*g^2)
/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e-12/h/(
a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-
b*g*h+c*g^2)/h^2)^(1/2)*x*b*c^2*g^3*f-3*h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)
/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b^2*c*
e*g+4/h/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b*c*f*g-12*h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-
b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b*
c^2*g*d-3/2/(a*h^2-b*g*h+c*g^2)^2/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2
-b*g*h+c*g^2)/h^2)^(1/2)*b*f*g^2-3/(a*h^2-b*g*h+c*g^2)^2/((x+g/h)^2*c+(b*h-
2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*c*g^2*e+3/(a*h^2-b*g*h+c*g^
2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g
*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x
+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^2*e-8*c^2/(a*h^2-b*g*h
+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*x*d-4*c/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)
*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*d+2*f/h^2*(2*c*x+b)/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)-3/2*h^2/(a*h^2-b*g*h+c*g^2)^2/((x+g/h)^2*c+(b*h-2*c*g)
*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*d+12/h^2/(a*h^2-b*g*h+c*g^2)^2/
(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2)*x*c^3*g^4*f-6*h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*
g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b^2*c*g*d+6/h^2/(a*h^2-b*g*h+c*
g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h
^2)^(1/2)*b*c^2*g^4*f-6/h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b
```

```
*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b^2*c*g^3*f-16/h^2*c^2/(
a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*
g*h+c*g^2)/h^2)^(1/2)*x*f*g^2+6/h*c/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h
)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*e*g-8/h^2*c/(a
*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2)*b*f*g^2+12/h*c^2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g
/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*e*g-6/h/(a*h
^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2)*b*c^2*g^3*e+3*h^2/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((
x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b^2*c*d+1
2/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h
^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b*c^2*g^2*e+3/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2
)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b^2*c
*f*g^2-3/2*h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+
g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b^3*e*g-3/2*h/(a*h^2-b*g*h+c*g^2)^2/(
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g
^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/
h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*e*g+12/(a*h^2-b*g*h+c*g^2)^2/(
4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
)*x*c^3*g^2*d+6/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*
(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*c^2*g^2*d+6/(a*h^2-b*g*h+c*g^2)^
2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(
1/2)*b^2*c*g^2*e-3*h/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*
ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
))/(x+g/h))*c*g*d-3/h/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2))/(x+g/h))*c*g^3*f-2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*
c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b*c*e+2/h/(a*h^2-b*g*h+c*g^
2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^
(1/2)*b^2*f*g-12/h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*
g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*c^3*g^3*e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `assum
e?` for more details)Is (b/h-(2*c*g)/h^2)^2    -(4*c       *((-(b*g)/h)
    +(c*g^2)/h^2+a))     /h^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^2\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

3.239 $\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=713

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2\left(8a^2fh^2+4abh(2fg-3eh)-\left(b^2\left(3h(eg-5dh)+fg^2\right)\right)\right)-4ch\left(ah\left(3dh^2-9\right.\right.\right.}{8\left(ah^2-bgh+cg^2\right)^{7/2}}$$

```
[Out] 1/8*(8*c^2*g^2*(6*d*h^2-3*e*g*h+f*g^2)+h^2*(8*a^2*f*h^2+4*a*b*h*(-3*e*h+2*f
*g)-b^2*(f*g^2+3*h*(-5*d*h+e*g)))-4*c*h*(a*h*(3*d*h^2-9*e*g*h+11*f*g^2)-b*g
*(2*f*g^2+3*h*(-4*d*h+e*g))))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2
-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(7/2)+2*(b^4*d
*h^3-b^3*h^2*(a*e*h+3*c*d*g)+b^2*h*(3*c^2*d*g^2+a^2*f*h^2+a*c*h*(-4*d*h+3*e
*g))-b*c*(c^2*d*g^3+3*a^2*h^2*(-e*h+f*g)+a*c*g*(-9*d*h^2+3*e*g*h+f*g^2))-2*
a*c*(a^2*f*h^3-c^2*g^2*(-3*d*h+e*g)-a*c*h*(d*h^2-3*e*g*h+3*f*g^2))-c*(2*c^3
*d*g^3-b*(a^2*f-a*b*e+b^2*d)*h^3-c^2*g*(b*g*(3*d*h+e*g)+2*a*(3*d*h^2-3*e*g*
h+f*g^2))+c*(2*a^2*h^2*(-e*h+3*f*g)-3*a*b*h*(-d*h^2+e*g*h+f*g^2)+b^2*(3*d*g
*h^2+f*g^3)))*x)/(-4*a*c+b^2)/(a*h^2-b*g*h+c*g^2)^3/(c*x^2+b*x+a)^(1/2)-1/2
*h*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^2
-1/4*h*(2*c*g*(3*f*g^2-h*(-7*d*h+5*e*g))-h*(4*a*h*(-e*h+2*f*g)-b*(-7*d*h^2+
3*e*g*h+f*g^2)))*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)
```

Rubi [A] time = 2.67, antiderivative size = 707, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1646, 1650, 806, 724, 206}

$$\frac{2\left(-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg^3\right)\right)-bh^3\left(a^2f-abe+b^2d\right)-c^2g\left(-6ah(e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^4*d*h^3 - b^3*h^2*(3*c*d*g + a*e*h) + b^2*h*(3*c^2*d*g^2 + a^2*f*h^2
+ a*c*h*(3*e*g - 4*d*h)) - b*c*(c^2*d*g^3 + 3*a^2*h^2*(f*g - e*h) + a*c*g*(
f*g^2 + 3*e*g*h - 9*d*h^2)) - 2*a*c*(a^2*f*h^3 - c^2*g^2*(e*g - 3*d*h) - a*
c*h*(3*f*g^2 - 3*e*g*h + d*h^2)) - c*(2*c^3*d*g^3 - b*(b^2*d - a*b*e + a^2*
f)*h^3 - c^2*g*(2*a*f*g^2 - 6*a*h*(e*g - d*h) + b*g*(e*g + 3*d*h)) + c*(2*a
^2*h^2*(3*f*g - e*h) + b^2*(f*g^3 + 3*d*g*h^2) - 3*a*b*h*(f*g^2 + h*(e*g -
d*h))))*x))/((b^2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)^3*Sqrt[a + b*x + c*x^2])
 - (h*(f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(2*(c*g^2 - b*g*h + a*
h^2)^2*(g + h*x)^2) - (h*(6*c*f*g^3 - 2*c*g*h*(5*e*g - 7*d*h) - 4*a*h^2*(2*
f*g - e*h) + b*h*(f*g^2 + h*(3*e*g - 7*d*h)))*Sqrt[a + b*x + c*x^2])/(4*(c*
g^2 - b*g*h + a*h^2)^3*(g + h*x)) + ((8*c^2*g^2*(f*g^2 - 3*e*g*h + 6*d*h^2)
 + 4*c*h*(2*b*f*g^3 + 3*b*g*h*(e*g - 4*d*h) - a*h*(11*f*g^2 - 9*e*g*h + 3*d
*h^2)) + h^2*(8*a^2*f*h^2 + 4*a*b*h*(2*f*g - 3*e*h) - b^2*(f*g^2 + 3*h*(e*g
 - 5*d*h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h
+ a*h^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h\left(3c^2dg^2+a^2fh^2+ach(3eg-4dh)\right)-bc\left(c\right.\right.}{}$$

$$= \frac{2\left(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h\left(3c^2dg^2+a^2fh^2+ach(3eg-4dh)\right)-bc\left(c\right.\right.}{}$$

$$= \frac{2\left(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h\left(3c^2dg^2+a^2fh^2+ach(3eg-4dh)\right)-bc\left(c\right.\right.}{}$$

$$= \frac{2\left(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h\left(3c^2dg^2+a^2fh^2+ach(3eg-4dh)\right)-bc\left(c\right.\right.}{}$$

$$= \frac{2\left(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h\left(3c^2dg^2+a^2fh^2+ach(3eg-4dh)\right)-bc\left(c\right.\right.}{}$$

Mathematica [A] time = 5.29, size = 762, normalized size = 1.07

$$\frac{3ch\left(-\frac{4h\sqrt{a+x(b+cx)}\left(8a^2fh^2-4c\left(ah(3dh-5eg)+3afg^2+bg(2dh+eg)\right)-4abh(eh+2fg)+b^2\left(h(5dh-eg)+5fg^2\right)+8c^2dg^2\right)}{(g+hx)^2}+\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)\left(h^2\left(-8a^2\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-(f/((g + h*x)^2*Sqrt[a + x*(b + c*x)])) + (b^3*f*g*h - b^2*(6*c*d*h^2 + a
*f*h^2 + c*f*g*(g - 6*h*x)) + 2*b*c*h*(3*a*e*h + a*f*(g - 3*h*x) - 3*c*(-(d
*g) + e*g*x + d*h*x)) + 4*c*(-2*a^2*f*h^2 + 3*c^2*d*g*h*x + a*c*(f*g*(g - 3
*h*x) + 3*h*(-(e*g) + d*h + e*h*x))))/((b^2 - 4*a*c)*(-(c*g^2) + h*(b*g - a
*h))*(g + h*x)^2*Sqrt[a + x*(b + c*x)]) + (3*c*h*((-4*h*(8*c^2*d*g^2 + 8*a^
2*f*h^2 - 4*a*b*h*(2*f*g + e*h) - 4*c*(3*a*f*g^2 + b*g*(e*g + 2*d*h) + a*h*
(-5*e*g + 3*d*h)) + b^2*(5*f*g^2 + h*(-(e*g) + 5*d*h)))*Sqrt[a + x*(b + c*x
)])/(g + h*x)^2 - (2*h*(16*c^3*d*g^3 - 8*c^2*g*(5*a*f*g^2 + b*g*(e*g + 3*d*
h) + a*h*(-11*e*g + 13*d*h)) + b*h*(-8*a^2*f*h^2 + 4*a*b*h*(-2*f*g + 3*e*h)
 + b^2*(f*g^2 + 3*h*(e*g - 5*d*h))) + 2*c*(8*a^2*h^2*(5*f*g - 2*e*h) + 2*a*
b*h*(-7*f*g^2 + h*(-9*e*g + 13*d*h)) + b^2*(7*f*g^3 + g*h*(-5*e*g + 19*d*h)
)))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)) + ((b^2 -
 4*a*c)*(-8*c^2*g^2*(f*g^2 - 3*e*g*h + 6*d*h^2) - 4*c*h*(2*b*f*g^3 + 3*b*g*
h*(e*g - 4*d*h) + a*h*(-11*f*g^2 + 9*e*g*h - 3*d*h^2)) + h^2*(-8*a^2*f*h^2
+ 4*a*b*h*(-2*f*g + 3*e*h) + b^2*(f*g^2 + 3*h*(e*g - 5*d*h))))*ArcTanh[(-(b
*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x
*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2)))/(8*(b^2 - 4*a*c)*(c*g^2 +
 h*(-(b*g) + a*h))^2))/(3*c*h)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [B] time = 0.88, size = 5637, normalized size = 7.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^7*d*g^9 + b^2*c^5*f*g^9 - 2*a*c^6*f*g^9 - 9*b*c^6*d*g^8*h - 3*b^3*
c^4*f*g^8*h + 3*a*b*c^5*f*g^8*h + 18*b^2*c^5*d*g^7*h^2 + 3*b^4*c^3*f*g^7*h^
2 + 6*a*b^2*c^4*f*g^7*h^2 - 21*b^3*c^4*d*g^6*h^3 - b^5*c^2*f*g^6*h^3 - 13*a
*b^3*c^3*f*g^6*h^3 - 16*a^2*b*c^4*f*g^6*h^3 + 15*b^4*c^3*d*g^5*h^4 + 6*a*b^
2*c^4*d*g^5*h^4 - 12*a^2*c^5*d*g^5*h^4 + 6*a*b^4*c^2*f*g^5*h^4 + 36*a^2*b^2
*c^3*f*g^5*h^4 + 12*a^3*c^4*f*g^5*h^4 - 6*b^5*c^2*d*g^4*h^5 - 15*a*b^3*c^3*
d*g^4*h^5 + 30*a^2*b*c^4*d*g^4*h^5 - 21*a^2*b^3*c^2*f*g^4*h^5 - 42*a^3*b*c^
3*f*g^4*h^5 + b^6*c*d*g^3*h^6 + 12*a*b^4*c^2*d*g^3*h^6 - 18*a^2*b^2*c^3*d*g
^3*h^6 - 16*a^3*c^4*d*g^3*h^6 + a^2*b^4*c*f*g^3*h^6 + 34*a^3*b^2*c^2*f*g^3*
h^6 + 16*a^4*c^3*f*g^3*h^6 - 3*a*b^5*c*d*g^2*h^7 - 3*a^2*b^3*c^2*d*g^2*h^7
+ 24*a^3*b*c^3*d*g^2*h^7 - 3*a^3*b^3*c*f*g^2*h^7 - 24*a^4*b*c^2*f*g^2*h^7 +
 3*a^2*b^4*c*d*g*h^8 - 6*a^3*b^2*c^2*d*g*h^8 - 6*a^4*c^3*d*g*h^8 + 3*a^4*b^
2*c*f*g*h^8 + 6*a^5*c^2*f*g*h^8 - a^3*b^3*c*d*h^9 + 3*a^4*b*c^2*d*h^9 - a^5
*b*c*f*h^9 - b*c^6*g^9*e + 3*b^2*c^5*g^8*h*e + 6*a*c^6*g^8*h*e - 3*b^3*c^4*
g^7*h^2*e - 24*a*b*c^5*g^7*h^2*e + b^4*c^3*g^6*h^3*e + 34*a*b^2*c^4*g^6*h^3
*e + 16*a^2*c^5*g^6*h^3*e - 21*a*b^3*c^3*g^5*h^4*e - 42*a^2*b*c^4*g^5*h^4*e
 + 6*a*b^4*c^2*g^4*h^5*e + 36*a^2*b^2*c^3*g^4*h^5*e + 12*a^3*c^4*g^4*h^5*e
- a*b^5*c*g^3*h^6*e - 13*a^2*b^3*c^2*g^3*h^6*e - 16*a^3*b*c^3*g^3*h^6*e + 3
*a^2*b^4*c*g^2*h^7*e + 6*a^3*b^2*c^2*g^2*h^7*e - 3*a^3*b^3*c*g*h^8*e + 3*a^
4*b*c^2*g*h^8*e + a^4*b^2*c*h^9*e - 2*a^5*c^2*h^9*e)*x/(b^2*c^6*g^12 - 4*a*
c^7*g^12 - 6*b^3*c^5*g^11*h + 24*a*b*c^6*g^11*h + 15*b^4*c^4*g^10*h^2 - 54*
a*b^2*c^5*g^10*h^2 - 24*a^2*c^6*g^10*h^2 - 20*b^5*c^3*g^9*h^3 + 50*a*b^3*c^
4*g^9*h^3 + 120*a^2*b*c^5*g^9*h^3 + 15*b^6*c^2*g^8*h^4 - 225*a^2*b^2*c^4*g^
8*h^4 - 60*a^3*c^5*g^8*h^4 - 6*b^7*c*g^7*h^5 - 36*a*b^5*c^2*g^7*h^5 + 180*a
^2*b^3*c^3*g^7*h^5 + 240*a^3*b*c^4*g^7*h^5 + b^8*g^6*h^6 + 26*a*b^6*c*g^6*h
^6 - 30*a^2*b^4*c^2*g^6*h^6 - 340*a^3*b^2*c^3*g^6*h^6 - 80*a^4*c^4*g^6*h^6
- 6*a*b^7*g^5*h^7 - 36*a^2*b^5*c*g^5*h^7 + 180*a^3*b^3*c^2*g^5*h^7 + 240*a^
4*b*c^3*g^5*h^7 + 15*a^2*b^6*g^4*h^8 - 225*a^4*b^2*c^2*g^4*h^8 - 60*a^5*c^3
*g^4*h^8 - 20*a^3*b^5*g^3*h^9 + 50*a^4*b^3*c*g^3*h^9 + 120*a^5*b*c^2*g^3*h^
9 + 15*a^4*b^4*g^2*h^10 - 54*a^5*b^2*c*g^2*h^10 - 24*a^6*c^2*g^2*h^10 - 6*a
^5*b^3*g*h^11 + 24*a^6*b*c*g*h^11 + a^6*b^2*h^12 - 4*a^7*c*h^12) + (b*c^6*d
*g^9 + a*b*c^5*f*g^9 - 6*b^2*c^5*d*g^8*h + 6*a*c^6*d*g^8*h - 3*a*b^2*c^4*f*
g^8*h - 6*a^2*c^5*f*g^8*h + 15*b^3*c^4*d*g^7*h^2 - 24*a*b*c^5*d*g^7*h^2 + 3
*a*b^3*c^3*f*g^7*h^2 + 24*a^2*b*c^4*f*g^7*h^2 - 20*b^4*c^3*d*g^6*h^3 + 34*a
*b^2*c^4*d*g^6*h^3 + 16*a^2*c^5*d*g^6*h^3 - a*b^4*c^2*f*g^6*h^3 - 34*a^2*b^
2*c^3*f*g^6*h^3 - 16*a^3*c^4*f*g^6*h^3 + 15*b^5*c^2*d*g^5*h^4 - 15*a*b^3*c^
3*d*g^5*h^4 - 54*a^2*b*c^4*d*g^5*h^4 + 21*a^2*b^3*c^2*f*g^5*h^4 + 42*a^3*b*
c^3*f*g^5*h^4 - 6*b^6*c*d*g^4*h^5 - 9*a*b^4*c^2*d*g^4*h^5 + 66*a^2*b^2*c^3*
d*g^4*h^5 + 12*a^3*c^4*d*g^4*h^5 - 6*a^2*b^4*c*f*g^4*h^5 - 36*a^3*b^2*c^2*f
*g^4*h^5 - 12*a^4*c^3*f*g^4*h^5 + b^7*d*g^3*h^6 + 11*a*b^5*c*d*g^3*h^6 - 31
*a^2*b^3*c^2*d*g^3*h^6 - 32*a^3*b*c^3*d*g^3*h^6 + a^2*b^5*f*g^3*h^6 + 13*a^
3*b^3*c*f*g^3*h^6 + 16*a^4*b*c^2*f*g^3*h^6 - 3*a*b^6*d*g^2*h^7 + 30*a^3*b^2
*c^2*d*g^2*h^7 - 3*a^3*b^4*f*g^2*h^7 - 6*a^4*b^2*c*f*g^2*h^7 + 3*a^2*b^5*d*
g*h^8 - 9*a^3*b^3*c*d*g*h^8 - 3*a^4*b*c^2*d*g*h^8 + 3*a^4*b^3*f*g*h^8 - 3*a
^5*b*c*f*g*h^8 - a^3*b^4*d*h^9 + 4*a^4*b^2*c*d*h^9 - 2*a^5*c^2*d*h^9 - a^5*
b^2*f*h^9 + 2*a^6*c*f*h^9 - 2*a*c^6*g^9*e + 9*a*b*c^5*g^8*h*e - 18*a*b^2*c^
```

4*g^7*h^2*e + 21*a*b^3*c^3*g^6*h^3*e - 15*a*b^4*c^2*g^5*h^4*e - 6*a^2*b^2*c
^3*g^5*h^4*e + 12*a^3*c^4*g^5*h^4*e + 6*a*b^5*c*g^4*h^5*e + 15*a^2*b^3*c^2*
g^4*h^5*e - 30*a^3*b*c^3*g^4*h^5*e - a*b^6*g^3*h^6*e - 12*a^2*b^4*c*g^3*h^6
*e + 18*a^3*b^2*c^2*g^3*h^6*e + 16*a^4*c^3*g^3*h^6*e + 3*a^2*b^5*g^2*h^7*e
+ 3*a^3*b^3*c*g^2*h^7*e - 24*a^4*b*c^2*g^2*h^7*e - 3*a^3*b^4*g*h^8*e + 6*a^
4*b^2*c*g*h^8*e + 6*a^5*c^2*g*h^8*e + a^4*b^3*h^9*e - 3*a^5*b*c*h^9*e)/(b^2
*c^6*g^12 - 4*a*c^7*g^12 - 6*b^3*c^5*g^11*h + 24*a*b*c^6*g^11*h + 15*b^4*c^
4*g^10*h^2 - 54*a*b^2*c^5*g^10*h^2 - 24*a^2*c^6*g^10*h^2 - 20*b^5*c^3*g^9*h
^3 + 50*a*b^3*c^4*g^9*h^3 + 120*a^2*b*c^5*g^9*h^3 + 15*b^6*c^2*g^8*h^4 - 22
5*a^2*b^2*c^4*g^8*h^4 - 60*a^3*c^5*g^8*h^4 - 6*b^7*c*g^7*h^5 - 36*a*b^5*c^2
*g^7*h^5 + 180*a^2*b^3*c^3*g^7*h^5 + 240*a^3*b*c^4*g^7*h^5 + b^8*g^6*h^6 +
26*a*b^6*c*g^6*h^6 - 30*a^2*b^4*c^2*g^6*h^6 - 340*a^3*b^2*c^3*g^6*h^6 - 80*
a^4*c^4*g^6*h^6 - 6*a*b^7*g^5*h^7 - 36*a^2*b^5*c*g^5*h^7 + 180*a^3*b^3*c^2*
g^5*h^7 + 240*a^4*b*c^3*g^5*h^7 + 15*a^2*b^6*g^4*h^8 - 225*a^4*b^2*c^2*g^4*
h^8 - 60*a^5*c^3*g^4*h^8 - 20*a^3*b^5*g^3*h^9 + 50*a^4*b^3*c*g^3*h^9 + 120*
a^5*b*c^2*g^3*h^9 + 15*a^4*b^4*g^2*h^10 - 54*a^5*b^2*c*g^2*h^10 - 24*a^6*c^
2*g^2*h^10 - 6*a^5*b^3*g*h^11 + 24*a^6*b*c*g*h^11 + a^6*b^2*h^12 - 4*a^7*c*
h^12))/sqrt(c*x^2 + b*x + a) + 1/4*(8*c^2*f*g^4 + 8*b*c*f*g^3*h + 48*c^2*d*
g^2*h^2 - b^2*f*g^2*h^2 - 44*a*c*f*g^2*h^2 - 48*b*c*d*g*h^3 + 8*a*b*f*g*h^3
 + 15*b^2*d*h^4 - 12*a*c*d*h^4 + 8*a^2*f*h^4 - 24*c^2*g^3*h*e + 12*b*c*g^2*
h^2*e - 3*b^2*g*h^3*e + 36*a*c*g*h^3*e - 12*a*b*h^4*e)*arctan(-((sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c^3
*g^6 - 3*b*c^2*g^5*h + 3*b^2*c*g^4*h^2 + 3*a*c^2*g^4*h^2 - b^3*g^3*h^3 - 6*
a*b*c*g^3*h^3 + 3*a*b^2*g^2*h^4 + 3*a^2*c*g^2*h^4 - 3*a^2*b*g*h^5 + a^3*h^6
)*sqrt(-c*g^2 + b*g*h - a*h^2)) - 1/4*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^3*c^2*f*g^4*h + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d*g^2*h^3 -
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^2*h^3 - 20*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*b*c*d*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*f*g*h^4 + 7*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*h^5 - 4*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*a*c*d*h^5 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*g^3*h^
2*e + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*g^2*h^3*e - 3*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*b^2*g*h^4*e + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*a*c*g*h^4*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*h^5*e + 2
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*f*g^5 - 8*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*b*c^(3/2)*f*g^4*h + 56*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^2*c^(5/2)*d*g^3*h^2 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt
(c)*f*g^3*h^2 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*f*g^3*h^
2 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*d*g^2*h^3 + 13*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*g*h^4 - 28*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*a*c^(3/2)*d*g*h^4 + 16*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^2*a^2*sqrt(c)*f*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sq
rt(c)*d*h^5 - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*g^4*h*e + 28
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*g^3*h^2*e - 9*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*g^2*h^3*e + 36*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*a*c^(3/2)*g^2*h^3*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a*b*sqrt(c)*g*h^4*e - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c
)*h^5*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*f*g^5 - 4*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*b^2*c*f*g^4*h - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a*c^2*f*g^4*h + 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*d*g^3*h^
2 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*f*g^3*h^2 - 28*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a*b*c*f*g^3*h^2 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*b^2*c*d*g^2*h^3 - 88*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*g^2*h^3
+ 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*f*g^2*h^3 + 44*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^2*c*f*g^2*h^3 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*b^3*d*g*h^4 + 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*g*h^4 - 8*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*f*g*h^4 - 9*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*a*b^2*d*h^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d*h
^5 - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*g^4*h*e + 24*(sqrt(c)*x -

```
sqrt(c*x^2 + b*x + a))*b^2*c*g^3*h^2*e + 64*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a*c^2*g^3*h^2*e - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*g^2*h^3*e
 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*g^2*h^3*e + (sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*a*b^2*g*h^4*e - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a^2*c*g*h^4*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*h^5*e + 6*b^2*
c^(3/2)*f*g^5 + b^3*sqrt(c)*f*g^4*h - 20*a*b*c^(3/2)*f*g^4*h + 14*b^2*c^(3/
2)*d*g^3*h^2 - 9*a*b^2*sqrt(c)*f*g^3*h^2 + 12*a^2*c^(3/2)*f*g^3*h^2 - 7*b^3
*sqrt(c)*d*g^2*h^3 - 44*a*b*c^(3/2)*d*g^2*h^3 + 24*a^2*b*sqrt(c)*f*g^2*h^3
+ 23*a*b^2*sqrt(c)*d*g*h^4 + 28*a^2*c^(3/2)*d*g*h^4 - 16*a^3*sqrt(c)*f*g*h^
4 - 16*a^2*b*sqrt(c)*d*h^5 - 10*b^2*c^(3/2)*g^4*h*e + 3*b^3*sqrt(c)*g^3*h^2
*e + 32*a*b*c^(3/2)*g^3*h^2*e - 7*a*b^2*sqrt(c)*g^2*h^3*e - 20*a^2*c^(3/2)*
g^2*h^3*e - 4*a^2*b*sqrt(c)*g*h^4*e + 8*a^3*sqrt(c)*h^5*e)/((c^3*g^6 - 3*b*
c^2*g^5*h + 3*b^2*c*g^4*h^2 + 3*a*c^2*g^4*h^2 - b^3*g^3*h^3 - 6*a*b*c*g^3*h
^3 + 3*a*b^2*g^2*h^4 + 3*a^2*c*g^2*h^4 - 3*a^2*b*g*h^5 + a^3*h^6)*((sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqr
t(c)*g + b*g - a*h)^2)
```

maple [B] time = 0.02, size = 9126, normalized size = 12.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                              +c*g^2    positive,
negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

3.240 $\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=120

$$\frac{2}{15}\sqrt{3x^2-x+2}\,(2x+1)^4+\frac{19}{60}\sqrt{3x^2-x+2}\,(2x+1)^3+\frac{44}{135}\sqrt{3x^2-x+2}\,(2x+1)^2-\frac{(6298x+24897)\sqrt{3x^2-x+2}}{3240}+$$

```
[Out] 9211/3888*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+44/135*(1+2*x)^2*(3*x^2-x+
2)^(1/2)+19/60*(1+2*x)^3*(3*x^2-x+2)^(1/2)+2/15*(1+2*x)^4*(3*x^2-x+2)^(1/2)
-1/3240*(24897+6298*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1653, 832, 779, 619, 215}

$$\frac{2}{15}\sqrt{3x^2-x+2}\,(2x+1)^4+\frac{19}{60}\sqrt{3x^2-x+2}\,(2x+1)^3+\frac{44}{135}\sqrt{3x^2-x+2}\,(2x+1)^2-\frac{(6298x+24897)\sqrt{3x^2-x+2}}{3240}+$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (44*(1 + 2*x)^2*Sqrt[2 - x + 3*x^2])/135 + (19*(1 + 2*x)^3*Sqrt[2 - x + 3*x
^2])/60 + (2*(1 + 2*x)^4*Sqrt[2 - x + 3*x^2])/15 - ((24897 + 6298*x)*Sqrt[2
 - x + 3*x^2])/3240 + (9211*ArcSinh[(1 - 6*x)/Sqrt[23]])/(1296*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2}+\frac{1}{60}\int \frac{(1+2x)^3(-64+228x)}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2}+\frac{1}{720}\int \frac{(1+2x)^2(-}{\sqrt{2-}}$$

$$= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2}+\frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2-}$$

$$= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2}+\frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2-}$$

$$= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2}+\frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2-}$$

$$= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2}+\frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2-}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.50

$$\frac{6\sqrt{3x^2-x+2}\left(6912x^4+22032x^3+26904x^2+7538x-22383\right)-46055\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{19440}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(-22383 + 7538*x + 26904*x^2 + 22032*x^3 + 6912*x^4)
 - 46055*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/19440
```

fricas [A] time = 0.84, size = 73, normalized size = 0.61

$$\frac{1}{3240}\left(6912\,x^4+22032\,x^3+26904\,x^2+7538\,x-22383\right)\sqrt{3\,x^2-x+2}+\frac{9211}{7776}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}\,(6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3240*(6912*x^4 + 22032*x^3 + 26904*x^2 + 7538*x - 22383)*sqrt(3*x^2 - x +
 2) + 9211/7776*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^
2 + 24*x - 25)
```

giac [A] time = 0.24, size = 68, normalized size = 0.57

$$\frac{1}{3240}\left(2\left(12\left(18\left(16x+51\right)x+1121\right)x+3769\right)x-22383\right)\sqrt{3x^2-x+2}+\frac{9211}{3888}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3240*(2*(12*(18*(16*x + 51)*x + 1121)*x + 3769)*x - 22383)*sqrt(3*x^2 - x
 + 2) + 9211/3888*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
+ 1)
```

maple [A] time = 0.02, size = 96, normalized size = 0.80

$$\frac{32\sqrt{3x^2-x+2}\,x^4}{15}+\frac{34\sqrt{3x^2-x+2}\,x^3}{5}+\frac{1121\sqrt{3x^2-x+2}\,x^2}{135}+\frac{3769\sqrt{3x^2-x+2}\,x}{1620}-\frac{9211\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,(x}{23}\right.}{3888}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x)
```

```
[Out] 32/15*x^4*(3*x^2-x+2)^(1/2)+34/5*x^3*(3*x^2-x+2)^(1/2)+1121/135*x^2*(3*x^2-
x+2)^(1/2)+3769/1620*x*(3*x^2-x+2)^(1/2)-829/120*(3*x^2-x+2)^(1/2)-9211/388
8*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

maxima [A] time = 0.95, size = 97, normalized size = 0.81

$$\frac{32}{15}\sqrt{3x^2-x+2}\,x^4+\frac{34}{5}\sqrt{3x^2-x+2}\,x^3+\frac{1121}{135}\sqrt{3x^2-x+2}\,x^2+\frac{3769}{1620}\sqrt{3x^2-x+2}\,x-\frac{9211}{3888}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/15*sqrt(3*x^2 - x + 2)*x^4 + 34/5*sqrt(3*x^2 - x + 2)*x^3 + 1121/135*sqr
t(3*x^2 - x + 2)*x^2 + 3769/1620*sqrt(3*x^2 - x + 2)*x - 9211/3888*sqrt(3)*
arcsinh(1/23*sqrt(23)*(6*x - 1)) - 829/120*sqrt(3*x^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
```

```
[Out] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/sqrt(3*x**2 - x + 2), x)
```

3.241 $\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=95

$$\frac{1}{6}\sqrt{3x^2-x+2}\,(2x+1)^3+\frac{11}{27}\sqrt{3x^2-x+2}\,(2x+1)^2-\frac{143}{324}(3-2x)\sqrt{3x^2-x+2}+\frac{4147\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{648\sqrt{3}}$$

```
[Out] 4147/1944*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-143/324*(3-2*x)*(3*x^2-x+2
)^(1/2)+11/27*(1+2*x)^2*(3*x^2-x+2)^(1/2)+1/6*(1+2*x)^3*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1653, 832, 779, 619, 215}

$$\frac{1}{6}\sqrt{3x^2-x+2}\,(2x+1)^3+\frac{11}{27}\sqrt{3x^2-x+2}\,(2x+1)^2-\frac{143}{324}(3-2x)\sqrt{3x^2-x+2}+\frac{4147\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{648\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (-143*(3 - 2*x)*Sqrt[2 - x + 3*x^2])/324 + (11*(1 + 2*x)^2*Sqrt[2 - x + 3*x
^2])/27 + ((1 + 2*x)^3*Sqrt[2 - x + 3*x^2])/6 + (4147*ArcSinh[(1 - 6*x)/Sqr
t[23]])/(648*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx &= \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2}+\frac{1}{48}\int \frac{(1+2x)^2(-44+176x)}{\sqrt{2-x+3x^2}}\,dx\\
&= \frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2}+\frac{1}{432}\int \frac{(1+2x)(-1716}{\sqrt{2-x+3}}\\
&= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2}+\frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2-x+}\\
&= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2}+\frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2-x+}\\
&= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2}+\frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2-x+}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.58

$$\frac{6\sqrt{3x^2-x+2}\left(432x^3+1176x^2+1138x-243\right)-4147\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{1944}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(-243 + 1138*x + 1176*x^2 + 432*x^3) - 4147*Sqrt[3]*
ArcSinh[(-1 + 6*x)/Sqrt[23]])/1944
```

fricas [A] time = 0.76, size = 68, normalized size = 0.72

$$\frac{1}{324}\left(432\,x^3+1176\,x^2+1138\,x-243\right)\sqrt{3\,x^2-x+2}+\frac{4147}{3888}\,\sqrt{3}\,\log\left(4\,\sqrt{3}\,\sqrt{3\,x^2-x+2}\,(6\,x-1)-72\,x^2+24\,x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/324*(432*x^3 + 1176*x^2 + 1138*x - 243)*sqrt(3*x^2 - x + 2) + 4147/3888*s
qrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

giac [A] time = 0.22, size = 63, normalized size = 0.66

$$\frac{1}{324}\,(2\,(12\,(18\,x+49)x+569)x-243)\sqrt{3\,x^2-x+2}+\frac{4147}{1944}\,\sqrt{3}\,\log\left(-2\,\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/324*(2*(12*(18*x + 49)*x + 569)*x - 243)*sqrt(3*x^2 - x + 2) + 4147/1944*
sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

maple [A] time = 0.01, size = 79, normalized size = 0.83

$$\frac{4\sqrt{3x^2-x+2}\,x^3}{3}+\frac{98\sqrt{3x^2-x+2}\,x^2}{27}+\frac{569\sqrt{3x^2-x+2}\,x}{162}-\frac{4147\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{1944}-\frac{3\sqrt{3x^2-x+2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x)
```

```
[Out] 4/3*(3*x^2-x+2)^(1/2)*x^3+98/27*(3*x^2-x+2)^(1/2)*x^2+569/162*(3*x^2-x+2)^(
1/2)*x-3/4*(3*x^2-x+2)^(1/2)-4147/1944*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6
))
```

maxima [A] time = 0.97, size = 80, normalized size = 0.84

$$\frac{4}{3}\sqrt{3x^2-x+2}\,x^3+\frac{98}{27}\sqrt{3x^2-x+2}\,x^2+\frac{569}{162}\sqrt{3x^2-x+2}\,x-\frac{4147}{1944}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(6x-1)\right)-\frac{3}{4}\sqrt{3x^2-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3*x^2 - x + 2)*x^3 + 98/27*sqrt(3*x^2 - x + 2)*x^2 + 569/162*sqrt(
3*x^2 - x + 2)*x - 4147/1944*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 3/4
*sqrt(3*x^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+1)^2\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
```

```
[Out] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^2\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/sqrt(3*x**2 - x + 2), x)
```

3.242 $\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=70

$$\frac{2}{9}\sqrt{3x^2-x+2}\,(2x+1)^2+\frac{1}{54}(62x+69)\sqrt{3x^2-x+2}+\frac{251\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}$$

```
[Out] 251/324*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2/9*(1+2*x)^2*(3*x^2-x+2)^(1
/2)+1/54*(69+62*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1653, 779, 619, 215}

$$\frac{2}{9}\sqrt{3x^2-x+2}\,(2x+1)^2+\frac{1}{54}(62x+69)\sqrt{3x^2-x+2}+\frac{251\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (2*(1 + 2*x)^2*Sqrt[2 - x + 3*x^2])/9 + ((69 + 62*x)*Sqrt[2 - x + 3*x^2])/5
4 + (251*ArcSinh[(1 - 6*x)/Sqrt[23]])/(108*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{36}\int \frac{(1+2x)(-24+124x)}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{54}(69+62x)\sqrt{2-x+3x^2} - \frac{251}{108}\int \frac{1}{\sqrt{2-x+3x}}$$

$$= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{54}(69+62x)\sqrt{2-x+3x^2} - \frac{251\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\right.}{108\sqrt{6}}$$

$$= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{54}(69+62x)\sqrt{2-x+3x^2} + \frac{251\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.71

$$\frac{1}{324}\left(6\sqrt{3x^2-x+2}\left(48x^2+110x+81\right)-251\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(81 + 110*x + 48*x^2) - 251*Sqrt[3]*ArcSinh[(-1 + 6*
x)/Sqrt[23]])/324
```

fricas [A] time = 0.76, size = 63, normalized size = 0.90

$$\frac{1}{54}\left(48\,x^2+110\,x+81\right)\sqrt{3\,x^2-x+2}+\frac{251}{648}\,\sqrt{3}\,\log\left(4\,\sqrt{3}\,\sqrt{3\,x^2-x+2}\,(6\,x-1)-72\,x^2+24\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(48*x^2 + 110*x + 81)*sqrt(3*x^2 - x + 2) + 251/648*sqrt(3)*log(4*sqrt
(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

giac [A] time = 0.26, size = 58, normalized size = 0.83

$$\frac{1}{54}\,(2\,(24\,x+55)x+81)\sqrt{3\,x^2-x+2}\,+\,\frac{251}{324}\,\sqrt{3}\,\log\left(-2\,\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/54*(2*(24*x + 55)*x + 81)*sqrt(3*x^2 - x + 2) + 251/324*sqrt(3)*log(-2*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

maple [A] time = 0.01, size = 62, normalized size = 0.89

$$\frac{8\sqrt{3x^2-x+2}\,x^2}{9}+\frac{55\sqrt{3x^2-x+2}\,x}{27}-\frac{251\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{324}+\frac{3\sqrt{3x^2-x+2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x)
```

```
[Out] 8/9*(3*x^2-x+2)^(1/2)*x^2+55/27*(3*x^2-x+2)^(1/2)*x+3/2*(3*x^2-x+2)^(1/2)-2
51/324*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

maxima [A] time = 0.96, size = 63, normalized size = 0.90

$$\frac{8}{9}\sqrt{3x^2-x+2}\,x^2+\frac{55}{27}\sqrt{3x^2-x+2}\,x-\frac{251}{324}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(6x-1)\right)+\frac{3}{2}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*sqrt(3*x^2 - x + 2)*x^2 + 55/27*sqrt(3*x^2 - x + 2)*x - 251/324*sqrt(3)
*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 3/2*sqrt(3*x^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
```

```
[Out] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)*(4*x**2 + 3*x + 1)/sqrt(3*x**2 - x + 2), x)
```

3.243 $\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}} \, dx$

Optimal. Leaf size=78

$$\frac{2}{3}\sqrt{3x^2-x+2}-\frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}-\frac{5\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

```
[Out] -5/18*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1/26*arctanh(1/26*(9-8*x)*13^(
1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+2/3*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1653, 843, 619, 215, 724, 206}

$$\frac{2}{3}\sqrt{3x^2-x+2}-\frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}-\frac{5\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] (2*Sqrt[2 - x + 3*x^2])/3 - (5*ArcSinh[(1 - 6*x)/Sqrt[23]])/(6*Sqrt[3]) - A
rcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])]/(2*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx &= \frac{2}{3}\sqrt{2-x+3x^2}+\frac{1}{12}\int \frac{16+20x}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= \frac{2}{3}\sqrt{2-x+3x^2}+\frac{1}{2}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx+\frac{5}{6}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx\\
&= \frac{2}{3}\sqrt{2-x+3x^2}+\frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{6\sqrt{69}}-\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{}{\sqrt{}}\right.\\
&= \frac{2}{3}\sqrt{2-x+3x^2}-\frac{5\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2\sqrt{13}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 1.00

$$\frac{2}{3}\sqrt{3x^2-x+2}-\frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}+\frac{5\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] (2*Sqrt[2 - x + 3*x^2])/3 + (5*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(6*Sqrt[3]) -
ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])]/(2*Sqrt[13])
```

fricas [A] time = 0.77, size = 105, normalized size = 1.35

$$\frac{5}{36}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+\frac{1}{52}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+22}{4x^2+4x+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 5/36*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x -
 25) + 1/52*sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x
^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 2/3*sqrt(3*x^2 - x + 2)
```

giac [A] time = 0.57, size = 116, normalized size = 1.49

$$-\frac{5}{18}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3x^2-x+2}\right)+\frac{1}{26}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -5/18*sqrt(3)*log(-6*sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 1/26*sq
rt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 -
x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))) + 2/3*s
qrt(3*x^2 - x + 2)
```

maple [A] time = 0.01, size = 60, normalized size = 0.77

$$\frac{5\sqrt{3}\ \mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{18}-\frac{\sqrt{13}\ \mathrm{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^{2}+5}}\right)}{26}+\frac{2\sqrt{3x^{2}-x+2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)/(3*x^2-x+2)^(1/2),x)
```

```
[Out] 2/3*(3*x^2-x+2)^(1/2)+5/18*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-1/26*13^(
1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))
```

maxima [A] time = 0.97, size = 67, normalized size = 0.86

$$\frac{5}{18}\sqrt{3}\ \mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)+\frac{1}{26}\sqrt{13}\ \mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|}-\frac{9\sqrt{23}}{23|2x+1|}\right)+\frac{2}{3}\sqrt{3x^{2}-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/18*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1/26*sqrt(13)*arcsi
nh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 2/3*sqrt(3*
x^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4x^{2}+3x+1}{(2x+1)\sqrt{3x^{2}-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(1/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^{2}+3x+1}{(2x+1)\sqrt{3x^{2}-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*sqrt(3*x**2 - x + 2)), x)
```

3.244 $\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=83

$$-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}+\frac{9\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{26\sqrt{13}}-\frac{\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+9/338*arctanh(1/26*(9-8*x)*13^(
1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1/13*(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 843, 619, 215, 724, 206}

$$-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}+\frac{9\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{26\sqrt{13}}-\frac{\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] -Sqrt[2 - x + 3*x^2]/(13*(1 + 2*x)) - ArcSinh[(1 - 6*x)/Sqrt[23]]/Sqrt[3] +
 (9*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(26*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx = -\frac{\sqrt{2-x+3x^2}}{13(1+2x)} - \frac{1}{13}\int \frac{-\frac{17}{2}-26x}{(1+2x)\sqrt{2-x+3x^2}}\,dx$$

$$= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)} - \frac{9}{26}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx + \int \frac{1}{\sqrt{2-x+3x^2}}\,dx$$

$$= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)} + \frac{9}{13}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx, x, \frac{9-8x}{\sqrt{2-x+3x^2}}\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1\cdots}}\right.}{\cdots}$$

$$= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)} - \frac{\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}} + \frac{9\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{26\sqrt{13}}$$

Mathematica [A] time = 0.05, size = 82, normalized size = 0.99

$$-\frac{\sqrt{3x^2-x+2}}{13(2x+1)} + \frac{9\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{26\sqrt{13}} + \frac{\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] -1/13*Sqrt[2 - x + 3*x^2]/(1 + 2*x) + ArcSinh[(-1 + 6*x)/Sqrt[23]]/Sqrt[3]
+ (9*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(26*Sqrt[13])
```

fricas [A] time = 0.87, size = 123, normalized size = 1.48

$$\frac{338\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+27\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}\cdots}{4\cdots}\right.}{2028(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2028*(338*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
- 72*x^2 + 24*x - 25) + 27*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2 -
x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) - 156*sqrt(3*x
^2 - x + 2))/(2*x + 1)
```

giac [A] time = 0.28, size = 48, normalized size = 0.58

$$\frac{1}{26}\sqrt{3}\,\text{sgn}\left(\frac{1}{2x+1}\right) - \frac{\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}}{26\,\text{sgn}\left(\frac{1}{2x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/26*sqrt(3)*sgn(1/(2*x + 1)) - 1/26*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3
)/sgn(1/(2*x + 1))
```

maple [A] time = 0.01, size = 67, normalized size = 0.81

$$\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3}+\frac{9\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^{2}+5}}\right)}{338}-\frac{\sqrt{-4x+3\left(x+\frac{1}{2}\right)^{2}+\frac{5}{4}}}{26\left(x+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^2/(3*x^2-x+2)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+9/338*13^(1/2)*arctanh(2/13*(-4*
x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))-1/26/(x+1/2)*(-4*x+3*(x+1/2)^
2+5/4)^(1/2)
```

maxima [A] time = 0.98, size = 74, normalized size = 0.89

$$\frac{1}{3}\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{9}{338}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)-\frac{\sqrt{3\,x^{2}-x+2}}{13\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 9/338*sqrt(13)*arcsi
nh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) - 1/13*sqrt(3
*x^2 - x + 2)/(2*x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^2\,\sqrt{3\,x^2-x+2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^{2}+3x+1}{\left(2x+1\right)^{2}\sqrt{3x^{2}-x+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*sqrt(3*x**2 - x + 2)), x)
```

3.245 $\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=89

$$\frac{7\sqrt{3x^2-x+2}}{169(2x+1)}-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}-\frac{581\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{676\sqrt{13}}$$

```
[Out] -581/8788*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1/26*(3
*x^2-x+2)^(1/2)/(1+2*x)^2+7/169*(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1650, 806, 724, 206}

$$\frac{7\sqrt{3x^2-x+2}}{169(2x+1)}-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}-\frac{581\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{676\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] -Sqrt[2 - x + 3*x^2]/(26*(1 + 2*x)^2) + (7*Sqrt[2 - x + 3*x^2])/(169*(1 + 2
*x)) - (581*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(676*Sqrt[
13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx &= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}-\frac{1}{26}\int \frac{-\frac{35}{2}-49x}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx\\ &= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}+\frac{7\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{581}{676}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\ &= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}+\frac{7\sqrt{2-x+3x^2}}{169(1+2x)}-\frac{581}{338}\operatorname{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)\\ &= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}+\frac{7\sqrt{2-x+3x^2}}{169(1+2x)}-\frac{581\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{676\sqrt{13}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.78

$$\frac{\frac{26(28x+1)\sqrt{3x^2-x+2}}{(2x+1)^2}-581\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8788}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] ((26*(1 + 28*x)*Sqrt[2 - x + 3*x^2])/(1 + 2*x)^2 - 581*Sqrt[13]*ArcTanh[(9
- 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/8788
```

fricas [A] time = 0.68, size = 96, normalized size = 1.08

$$\frac{581\sqrt{13}\left(4x^2+4x+1\right)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+52\sqrt{3x^2-x+2}(28x+1)}{17576\left(4x^2+4x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/17576*(581*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2
)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 52*sqrt(3*x^2 - x
 + 2)*(28*x + 1))/(4*x^2 + 4*x + 1)
```

giac [B] time = 0.33, size = 204, normalized size = 2.29

$$\frac{581}{8788}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{190\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^3-53\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right.}{338\left(2\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 581/8788*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sq
rt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2
))) + 1/338*(190*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 - 53*sqrt(3)*(sqrt(3)*
x - sqrt(3*x^2 - x + 2))^2 - 489*sqrt(3)*x + 289*sqrt(3) + 489*sqrt(3*x^2 -
```

```
 x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x - sq
rt(3*x^2 - x + 2)) - 5)^2
```

maple [A] time = 0.04, size = 74, normalized size = 0.83

$$-\frac{581\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{8788}+\frac{7\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{338\left(x+\frac{1}{2}\right)}-\frac{\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{104\left(x+\frac{1}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^3/(3*x^2-x+2)^(1/2),x)
```

```
[Out] -581/8788*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^
(1/2))+7/338/(x+1/2)*(-4*x+3*(x+1/2)^2+5/4)^(1/2)-1/104/(x+1/2)^2*(-4*x+3*(
x+1/2)^2+5/4)^(1/2)
```

maxima [A] time = 0.98, size = 82, normalized size = 0.92

$$\frac{581}{8788}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)-\frac{\sqrt{3\,x^2-x+2}}{26\left(4\,x^2+4\,x+1\right)}+\frac{7\sqrt{3\,x^2-x+2}}{169\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 581/8788*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(
2*x + 1)) - 1/26*sqrt(3*x^2 - x + 2)/(4*x^2 + 4*x + 1) + 7/169*sqrt(3*x^2 -
 x + 2)/(2*x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{{(2\,x+1)}^3\,\sqrt{3\,x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^3\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*sqrt(3*x**2 - x + 2)), x)
```

3.246 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{32}{27}\sqrt{3x^2-x+2}\,x^2+\frac{412}{81}\sqrt{3x^2-x+2}\,x+\frac{746}{81}\sqrt{3x^2-x+2}+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}+\frac{353\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{81\sqrt{3}}$$

```
[Out] 353/243*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2/1863*(12839-3871*x)/(3*x^2
-x+2)^(1/2)+746/81*(3*x^2-x+2)^(1/2)+412/81*x*(3*x^2-x+2)^(1/2)+32/27*x^2*(
3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{32}{27}\sqrt{3x^2-x+2}\,x^2+\frac{412}{81}\sqrt{3x^2-x+2}\,x+\frac{746}{81}\sqrt{3x^2-x+2}+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}+\frac{353\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{81\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(12839 - 3871*x))/(1863*Sqrt[2 - x + 3*x^2]) + (746*Sqrt[2 - x + 3*x^2])
/81 + (412*x*Sqrt[2 - x + 3*x^2])/81 + (32*x^2*Sqrt[2 - x + 3*x^2])/27 + (3
53*ArcSinh[(1 - 6*x)/Sqrt[23]])/(81*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{2}{23}\int \frac{\frac{1127}{81}+\frac{7682x}{27}+\frac{2852x^2}{9}+\frac{368x^3}{3}}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{32}{27}x^2\sqrt{2-x+3x^2} + \frac{2}{207}\int \frac{\frac{1127}{9}+2070x+\frac{9476x^2}{3}}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x+3x^2} + \frac{1}{621}\int \frac{-55\ldots}{\sqrt{\ldots}}$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{746}{81}\sqrt{2-x+3x^2} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x\ldots}$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{746}{81}\sqrt{2-x+3x^2} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x\ldots}$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{746}{81}\sqrt{2-x+3x^2} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x\ldots}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.67

$$\frac{6\left(3312x^4+13110x^3+23207x^2-2974x+29997\right)-8119\sqrt{9x^2-3x+6}\,\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{5589\sqrt{3x^2-x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (6*(29997 - 2974*x + 23207*x^2 + 13110*x^3 + 3312*x^4) - 8119*Sqrt[6 - 3*x
+ 9*x^2]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(5589*Sqrt[2 - x + 3*x^2])
```

fricas [A] time = 0.71, size = 97, normalized size = 0.94

$$\frac{8119\sqrt{3}\left(3x^2-x+2\right)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)+12\left(3312x^4+13110x^3+2320\ldots\right.}{11178\left(3x^2-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/11178*(8119*sqrt(3)*(3*x^2 - x + 2)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*
x - 1) - 72*x^2 + 24*x - 25) + 12*(3312*x^4 + 13110*x^3 + 23207*x^2 - 2974*
x + 29997)*sqrt(3*x^2 - x + 2))/(3*x^2 - x + 2)
```

giac [A] time = 0.21, size = 67, normalized size = 0.65

$$\frac{353}{243}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)+\frac{2\left(\left(23\left(6\left(24x+95\right)x+1009\right)x-2974\right)x+29997\right)}{1863\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 353/243*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/1
863*((23*(6*(24*x + 95)*x + 1009)*x - 2974)*x + 29997)/sqrt(3*x^2 - x + 2)
```

maple [A] time = 0.01, size = 115, normalized size = 1.12

$$\frac{32x^4}{9\sqrt{3x^2-x+2}}+\frac{380x^3}{27\sqrt{3x^2-x+2}}+\frac{2018x^2}{81\sqrt{3x^2-x+2}}+\frac{353x}{81\sqrt{3x^2-x+2}}-\frac{353\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{243}-\frac{521\,(6x}{414\sqrt{3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x)
```

```
[Out] 32/9*x^4/(3*x^2-x+2)^(1/2)+380/27*x^3/(3*x^2-x+2)^(1/2)+2018/81*x^2/(3*x^2-
x+2)^(1/2)+353/81*x/(3*x^2-x+2)^(1/2)-521/414*(6*x-1)/(3*x^2-x+2)^(1/2)-353
/243*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+557/18/(3*x^2-x+2)^(1/2)
```

maxima [A] time = 0.96, size = 97, normalized size = 0.94

$$\frac{32\,x^4}{9\,\sqrt{3\,x^2-x+2}}+\frac{380\,x^3}{27\,\sqrt{3\,x^2-x+2}}+\frac{2018\,x^2}{81\,\sqrt{3\,x^2-x+2}}-\frac{353}{243}\sqrt{3}\ \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(6\,x-1)\right)-\frac{5948\,x}{1863\,\sqrt{3\,x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 32/9*x^4/sqrt(3*x^2 - x + 2) + 380/27*x^3/sqrt(3*x^2 - x + 2) + 2018/81*x^2
/sqrt(3*x^2 - x + 2) - 353/243*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 5
948/1863*x/sqrt(3*x^2 - x + 2) + 2222/69/sqrt(3*x^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+1)^3\,\left(4\,x^2+3\,x+1\right)}{\left(3\,x^2-x+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
```

```
[Out] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

3.247 $\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}+\frac{8}{9}x\sqrt{3x^2-x+2}+\frac{112}{27}\sqrt{3x^2-x+2}-\frac{64\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

```
[Out] -64/27*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2/621*(1249-2273*x)/(3*x^2-x+
2)^(1/2)+112/27*(3*x^2-x+2)^(1/2)+8/9*x*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}+\frac{8}{9}x\sqrt{3x^2-x+2}+\frac{112}{27}\sqrt{3x^2-x+2}-\frac{64\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(1249 - 2273*x))/(621*Sqrt[2 - x + 3*x^2]) + (112*Sqrt[2 - x + 3*x^2])/2
7 + (8*x*Sqrt[2 - x + 3*x^2])/9 - (64*ArcSinh[(1 - 6*x)/Sqrt[23]])/(9*Sqrt[
3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
```

```
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx &= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{\frac{2116}{27}+\frac{1150x}{9}+\frac{184x^2}{3}}{\sqrt{2-x+3x^2}}\,dx\\
&= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{8}{9}x\sqrt{2-x+3x^2}+\frac{1}{69}\int \frac{\frac{3128}{9}+\frac{2576x}{3}}{\sqrt{2-x+3x^2}}\,dx\\
&= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{112}{27}\sqrt{2-x+3x^2}+\frac{8}{9}x\sqrt{2-x+3x^2}+\frac{64}{9}\int \frac{1}{\sqrt{2-x+3x}}\\
&= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{112}{27}\sqrt{2-x+3x^2}+\frac{8}{9}x\sqrt{2-x+3x^2}+\frac{64\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\right.}{9\sqrt{6}}\\
&= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{112}{27}\sqrt{2-x+3x^2}+\frac{8}{9}x\sqrt{2-x+3x^2}-\frac{64\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.74

$$\frac{2\left(828x^3+3588x^2+736\sqrt{9x^2-3x+6}\,\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)-3009x+3825\right)}{621\sqrt{3x^2-x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(3825 - 3009*x + 3588*x^2 + 828*x^3 + 736*Sqrt[6 - 3*x + 9*x^2]*ArcSinh[
(-1 + 6*x)/Sqrt[23]]))/(621*Sqrt[2 - x + 3*x^2])
```

fricas [A] time = 0.60, size = 92, normalized size = 1.12

$$\frac{2\left(368\sqrt{3}\left(3x^2-x+2\right)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)+3\left(276x^3+1196x^2-1003x+\right.\right.}{621\left(3x^2-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/621*(368*sqrt(3)*(3*x^2 - x + 2)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
- 1) - 72*x^2 + 24*x - 25) + 3*(276*x^3 + 1196*x^2 - 1003*x + 1275)*sqrt(3*
x^2 - x + 2))/(3*x^2 - x + 2)
```

giac [A] time = 0.27, size = 62, normalized size = 0.76

$$-\frac{64}{27}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)+\frac{2\left((92\,(3x+13)x-1003)x+1275\right)}{207\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -64/27*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/20
7*((92*(3*x + 13)*x - 1003)*x + 1275)/sqrt(3*x^2 - x + 2)
```

maple [A] time = 0.01, size = 98, normalized size = 1.20

$$\frac{8x^3}{3\sqrt{3x^2-x+2}}+\frac{104x^2}{9\sqrt{3x^2-x+2}}-\frac{64x}{9\sqrt{3x^2-x+2}}+\frac{64\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{27}+\frac{107}{9\sqrt{3x^2-x+2}}-\frac{89\,(6x-}{207\sqrt{3x^2-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x)
```

```
[Out] 8/3/(3*x^2-x+2)^(1/2)*x^3+104/9/(3*x^2-x+2)^(1/2)*x^2-64/9/(3*x^2-x+2)^(1/2
)*x+107/9/(3*x^2-x+2)^(1/2)-89/207*(6*x-1)/(3*x^2-x+2)^(1/2)+64/27*3^(1/2)*
arcsinh(6/23*23^(1/2)*(x-1/6))
```

maxima [A] time = 0.97, size = 80, normalized size = 0.98

$$\frac{8\,x^3}{3\,\sqrt{3\,x^2-x+2}}+\frac{104\,x^2}{9\,\sqrt{3\,x^2-x+2}}+\frac{64}{27}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(6\,x-1)\right)-\frac{2006\,x}{207\,\sqrt{3\,x^2-x+2}}+\frac{850}{69\,\sqrt{3\,x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^3/sqrt(3*x^2 - x + 2) + 104/9*x^2/sqrt(3*x^2 - x + 2) + 64/27*sqrt(3)
*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 2006/207*x/sqrt(3*x^2 - x + 2) + 850/69
/sqrt(3*x^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+1)^2\,\left(4\,x^2+3\,x+1\right)}{\left(3\,x^2-x+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
```

```
[Out] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^2\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

3.248 $\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}+\frac{8}{9}\sqrt{3x^2-x+2}-\frac{14\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}$$

```
[Out] -14/9*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-2/207*(73+367*x)/(3*x^2-x+2)^(
1/2)+8/9*(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1660, 640, 619, 215}

$$-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}+\frac{8}{9}\sqrt{3x^2-x+2}-\frac{14\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (-2*(73 + 367*x))/(207*Sqrt[2 - x + 3*x^2]) + (8*Sqrt[2 - x + 3*x^2])/9 - (
14*ArcSinh[(1 - 6*x)/Sqrt[23]])/(3*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx = -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{\frac{437}{9}+\frac{92x}{3}}{\sqrt{2-x+3x^2}}\,dx$$
$$= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}+\frac{14}{3}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx$$
$$= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}+\frac{14\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{3\sqrt{69}}$$
$$= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}-\frac{14\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}$$

Mathematica [A] time = 0.12, size = 50, normalized size = 0.79

$$\frac{2\left(92x^2-153x+37\right)}{69\sqrt{3x^2-x+2}}+\frac{14\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(37 - 153*x + 92*x^2))/(69*Sqrt[2 - x + 3*x^2]) + (14*ArcSinh[(-1 + 6*x)
/Sqrt[23]])/(3*Sqrt[3])
```

fricas [A] time = 0.65, size = 87, normalized size = 1.38

$$\frac{161\sqrt{3}\left(3x^2-x+2\right)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)+6\left(92x^2-153x+37\right)\sqrt{3x^2-}}{207\left(3x^2-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/207*(161*sqrt(3)*(3*x^2 - x + 2)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
- 1) - 72*x^2 + 24*x - 25) + 6*(92*x^2 - 153*x + 37)*sqrt(3*x^2 - x + 2))/(
3*x^2 - x + 2)
```

giac [A] time = 0.27, size = 57, normalized size = 0.90

$$-\frac{14}{9}\sqrt{3}\,\log\left(-2\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3x^2-x+2}\right)+1\right)+\frac{2\,((92x-153)x+37)}{69\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -14/9*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/69*
((92*x - 153)*x + 37)/sqrt(3*x^2 - x + 2)
```

maple [A] time = 0.01, size = 81, normalized size = 1.29

$$\frac{8x^2}{3\sqrt{3x^2-x+2}}-\frac{14x}{3\sqrt{3x^2-x+2}}+\frac{14\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{9}+\frac{10}{9\sqrt{3x^2-x+2}}+\frac{\frac{16x}{69}-\frac{8}{207}}{\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x)
```

```
[Out] 8/3/(3*x^2-x+2)^(1/2)*x^2-14/3/(3*x^2-x+2)^(1/2)*x+10/9/(3*x^2-x+2)^(1/2)+8
/207*(6*x-1)/(3*x^2-x+2)^(1/2)+14/9*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

maxima [A] time = 0.94, size = 63, normalized size = 1.00

$$\frac{8\,x^2}{3\,\sqrt{3\,x^2-x+2}}+\frac{14}{9}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(6\,x-1)\right)-\frac{102\,x}{23\,\sqrt{3\,x^2-x+2}}+\frac{74}{69\,\sqrt{3\,x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^2/sqrt(3*x^2 - x + 2) + 14/9*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1))
 - 102/23*x/sqrt(3*x^2 - x + 2) + 74/69/sqrt(3*x^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(2\,x+1)\,\left(4\,x^2+3\,x+1\right)}{{\left(3\,x^2-x+2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
```

```
[Out] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

3.249 $\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}}-\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{13\sqrt{13}}$$

```
[Out] -2/169*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-2/299*(101
-77*x)/(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 12, 724, 206}

$$-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}}-\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{13\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (-2*(101 - 77*x))/(299*Sqrt[2 - x + 3*x^2]) - (2*ArcTanh[(9 - 8*x)/(2*Sqrt[
13]*Sqrt[2 - x + 3*x^2])])/(13*Sqrt[13])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx = -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}} + \frac{2}{23}\int \frac{23}{13(1+2x)\sqrt{2-x+3x^2}}\,dx$$
$$= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}} + \frac{2}{13}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx$$
$$= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}} - \frac{4}{13}\,\text{Subst}\left(\int \frac{1}{52-x^2}\,dx, x, \frac{9-8x}{\sqrt{2-x+3x^2}}\right)$$
$$= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}} - \frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{13\sqrt{13}}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 1.18

$$-\frac{2\left(23\sqrt{13}\sqrt{3x^2-x+2}\,\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)-1001x+1313\right)}{3887\sqrt{3x^2-x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (-2*(1313 - 1001*x + 23*Sqrt[13]*Sqrt[2 - x + 3*x^2]*ArcTanh[(9 - 8*x)/(2*S
qrt[13]*Sqrt[2 - x + 3*x^2])]))/(3887*Sqrt[2 - x + 3*x^2])
```

fricas [A] time = 0.61, size = 96, normalized size = 1.55

$$\frac{23\sqrt{13}\left(3x^2-x+2\right)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}\,(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+26\sqrt{3x^2-x+2}\,(77x-101)}{3887\left(3x^2-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3887*(23*sqrt(13)*(3*x^2 - x + 2)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8
*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 26*sqrt(3*x^2 - x + 2
)*(77*x - 101))/(3*x^2 - x + 2)
```

giac [A] time = 0.58, size = 91, normalized size = 1.47

$$\frac{2}{169}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{2(77x-101)}{299\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/169*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(
3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
 + 2/299*(77*x - 101)/sqrt(3*x^2 - x + 2)
```

maple [B] time = 0.01, size = 102, normalized size = 1.65

$$-\frac{2\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{169}-\frac{2}{3\sqrt{3x^2-x+2}}+\frac{\frac{10x}{23}-\frac{5}{69}}{\sqrt{3x^2-x+2}}+\frac{1}{13\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}+\frac{\frac{24x}{299}}{\sqrt{-4x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)/(3*x^2-x+2)^(3/2),x)
```

```
[Out] -2/3/(3*x^2-x+2)^(1/2)+5/69*(6*x-1)/(3*x^2-x+2)^(1/2)+1/13/(-4*x+3*(x+1/2)^
2+5/4)^(1/2)+4/299*(6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^(1/2)-2/169*13^(1/2)*arct
anh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))
```

maxima [A] time = 0.97, size = 64, normalized size = 1.03

$$\frac{2}{169}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{154\,x}{299\,\sqrt{3\,x^2-x+2}}-\frac{202}{299\,\sqrt{3\,x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/169*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x
 + 1)) + 154/299*x/sqrt(3*x^2 - x + 2) - 202/299/sqrt(3*x^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{4\,x^2+3\,x+1}{(2\,x+1)\,\left(3\,x^2-x+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(3/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 - x + 2)**(3/2)), x)
```

3.250 $\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}} dx$

Optimal. Leaf size=87

$$-\frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{169(2x+1)}+\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

```
[Out] 2/2197*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-2/3887*(19
7-837*x)/(3*x^2-x+2)^(1/2)-4/169*(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A] time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 806, 724, 206}

$$-\frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{169(2x+1)}+\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (-2*(197 - 837*x))/(3887*Sqrt[2 - x + 3*x^2]) - (4*Sqrt[2 - x + 3*x^2])/(16
9*(1 + 2*x)) + (2*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(169
*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
```

```
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx = -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{\frac{184}{169}-\frac{230x}{169}}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx$$

$$= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}-\frac{2}{169}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx$$

$$= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{4}{169}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9 \ldots}{\sqrt{2 \ldots}}\right.$$

$$= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{169\sqrt{13}}$$

Mathematica [A] time = 0.04, size = 74, normalized size = 0.85

$$\frac{2\left(1536x^2+489x-289\right)}{3887(2x+1)\sqrt{3x^2-x+2}}+\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*(-289 + 489*x + 1536*x^2))/(3887*(1 + 2*x)*Sqrt[2 - x + 3*x^2]) + (2*Arc
Tanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(169*Sqrt[13])
```

fricas [A] time = 0.69, size = 106, normalized size = 1.22

$$\frac{23\sqrt{13}\left(6x^3+x^2+3x+2\right)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}\,(8x-9)-220x^2+196x-185}{4x^2+4x+1}\right)+26\left(1536x^2+489x-289\right)\sqrt{3x^2-x \ldots}}{50531\left(6x^3+x^2+3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/50531*(23*sqrt(13)*(6*x^3 + x^2 + 3*x + 2)*log((4*sqrt(13)*sqrt(3*x^2 - x
 + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 26*(1536*x^2
+ 489*x - 289)*sqrt(3*x^2 - x + 2))/(6*x^3 + x^2 + 3*x + 2)
```

giac [B] time = 0.30, size = 168, normalized size = 1.93

$$-\frac{2}{50531}\sqrt{13}\left(256\sqrt{13}\sqrt{3}+23\log\left(\sqrt{13}\sqrt{3}-4\right)\right)\text{sgn}\left(\frac{1}{2x+1}\right)-\frac{2\left(\frac{\frac{1047}{\text{sgn}\left(\frac{1}{2x+1}\right)}+\frac{299}{(2x+1)\text{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}-\frac{768}{\text{sgn}\left(\frac{1}{2x+1}\right)}\right)}{3887\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}}+\frac{2 \ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -2/50531*sqrt(13)*(256*sqrt(13)*sqrt(3) + 23*log(sqrt(13)*sqrt(3) - 4))*sgn
(1/(2*x + 1)) - 2/3887*((1047/sgn(1/(2*x + 1)) + 299/((2*x + 1)*sgn(1/(2*x
+ 1))))/(2*x + 1) - 768/sgn(1/(2*x + 1)))/sqrt(-8/(2*x + 1) + 13/(2*x + 1)^
2 + 3) + 2/2197*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 +
 3) + sqrt(13)/(2*x + 1)) - 4)/sgn(1/(2*x + 1))
```

maple [A] time = 0.01, size = 109, normalized size = 1.25

$$\frac{2\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{2197}+\frac{\frac{12x}{23}-\frac{2}{23}}{\sqrt{3x^2-x+2}}-\frac{1}{169\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}-\frac{82\,(6x-1)}{3887\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^2/(3*x^2-x+2)^(3/2),x)
```

```
[Out] 2/23*(6*x-1)/(3*x^2-x+2)^(1/2)-1/169/(-4*x+3*(x+1/2)^2+5/4)^(1/2)-82/3887*(
6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^(1/2)+2/2197*13^(1/2)*arctanh(2/13*(-4*x+9/2)
*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))-1/26/(x+1/2)/(-4*x+3*(x+1/2)^2+5/4)
^(1/2)
```

maxima [A] time = 0.96, size = 96, normalized size = 1.10

$$-\frac{2}{2197}\sqrt{13}\ \operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{1536\,x}{3887\sqrt{3\,x^2-x+2}}-\frac{279}{3887\sqrt{3\,x^2-x+2}}-\frac{1}{13\left(2\sqrt{3\,x^2-x+2}\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/2197*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2
*x + 1)) + 1536/3887*x/sqrt(3*x^2 - x + 2) - 279/3887/sqrt(3*x^2 - x + 2) -
 1/13/(2*sqrt(3*x^2 - x + 2)*x + sqrt(3*x^2 - x + 2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^2\,{\left(3\,x^2-x+2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^2\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 - x + 2)**(3/2)), x)
```

3.251 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=112

$$\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{2197(2x+1)}-\frac{2\sqrt{3x^2-x+2}}{169(2x+1)^2}-\frac{487\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

```
[Out] -487/28561*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+2/5053
1*(2363+3693*x)/(3*x^2-x+2)^(1/2)-2/169*(3*x^2-x+2)^(1/2)/(1+2*x)^2-4/2197*
(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A] time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1646, 1650, 806, 724, 206}

$$\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{2197(2x+1)}-\frac{2\sqrt{3x^2-x+2}}{169(2x+1)^2}-\frac{487\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*(2363 + 3693*x))/(50531*Sqrt[2 - x + 3*x^2]) - (2*Sqrt[2 - x + 3*x^2])/(
169*(1 + 2*x)^2) - (4*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)) - (487*ArcTanh[
(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2197*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
```

```
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}} + \frac{2}{23}\int \frac{\frac{8349}{2197}+\frac{20838x}{2197}+\frac{23828x^2}{2197}}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}} - \frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2} - \frac{1}{299}\int \frac{-\frac{11615}{169}-\frac{22034x}{169}}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}} - \frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2} - \frac{4\sqrt{2-x+3x^2}}{2197(1+2x)} + \frac{487\int \frac{1}{(1+2x)\sqrt{2-x+3x}}}{2197}$$

$$= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}} - \frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2} - \frac{4\sqrt{2-x+3x^2}}{2197(1+2x)} - \frac{974\,\text{Subst}\left(\int \frac{1}{52-x^2}\right.}{219}$$

$$= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}} - \frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2} - \frac{4\sqrt{2-x+3x^2}}{2197(1+2x)} - \frac{487\tanh^{-1}\left(\frac{9}{2\sqrt{13}\sqrt{}}\right.}{2197\sqrt{13}}$$

Mathematica [A] time = 0.06, size = 79, normalized size = 0.71

$$\frac{2\left(14496x^3+23281x^2+13306x+1673\right)}{50531(2x+1)^2\sqrt{3x^2-x+2}} - \frac{487\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*(1673 + 13306*x + 23281*x^2 + 14496*x^3))/(50531*(1 + 2*x)^2*Sqrt[2 - x
+ 3*x^2]) - (487*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2197
*Sqrt[13])
```

fricas [A] time = 0.64, size = 126, normalized size = 1.12

$$\frac{11201\sqrt{13}\left(12x^4+8x^3+7x^2+7x+2\right)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+52\left(14496x^3+23281x^2+\right.}{1313806\left(12x^4+8x^3+7x^2+7x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1313806*(11201*sqrt(13)*(12*x^4 + 8*x^3 + 7*x^2 + 7*x + 2)*log(-(4*sqrt(1
3)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)
) + 52*(14496*x^3 + 23281*x^2 + 13306*x + 1673)*sqrt(3*x^2 - x + 2))/(12*x^
4 + 8*x^3 + 7*x^2 + 7*x + 2)
```

giac [B] time = 0.31, size = 223, normalized size = 1.99

$$\frac{487}{28561}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}+\frac{2\left(62\left(\sqrt{3}x-\sqrt{3x^2\right.\right.}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 487/28561*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*s
qrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x +
2))) + 2/50531*(3693*x + 2363)/sqrt(3*x^2 - x + 2) + 2/2197*(62*(sqrt(3)*x
- sqrt(3*x^2 - x + 2))^3 - 37*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 +
 263*sqrt(3)*x - 71*sqrt(3) - 263*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt
(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

maple [A] time = 0.01, size = 111, normalized size = 0.99

$$-\frac{487\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{28561}+\frac{487}{4394\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}+\frac{\frac{7248x}{50531}-\frac{1208}{50531}}{\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}+\frac{}{338\left(x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^3/(3*x^2-x+2)^(3/2),x)
```

```
[Out] 487/4394/(-4*x+3*(x+1/2)^2+5/4)^(1/2)+1208/50531*(6*x-1)/(-4*x+3*(x+1/2)^2+
5/4)^(1/2)-487/28561*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x
+1/2)^2+5)^(1/2))+3/338/(x+1/2)/(-4*x+3*(x+1/2)^2+5/4)^(1/2)-1/104/(x+1/2)^
2/(-4*x+3*(x+1/2)^2+5/4)^(1/2)
```

maxima [A] time = 0.97, size = 145, normalized size = 1.29

$$\frac{487}{28561}\sqrt{13}\ \operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|}-\frac{9\sqrt{23}}{23|2x+1|}\right)+\frac{7248x}{50531\sqrt{3x^2-x+2}}+\frac{8785}{101062\sqrt{3x^2-x+2}}-\frac{}{26\left(4\sqrt{3x^2-x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 487/28561*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs
(2*x + 1)) + 7248/50531*x/sqrt(3*x^2 - x + 2) + 8785/101062/sqrt(3*x^2 - x
+ 2) - 1/26/(4*sqrt(3*x^2 - x + 2)*x^2 + 4*sqrt(3*x^2 - x + 2)*x + sqrt(3*x
^2 - x + 2)) + 3/169/(2*sqrt(3*x^2 - x + 2)*x + sqrt(3*x^2 - x + 2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4x^2+3x+1}{(2x+1)^3\left(3x^2-x+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)),x)
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^3\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(3/2),x)
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*(3*x**2 - x + 2)**(3/2)), x)
```

3.252 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx$

Optimal. Leaf size=86

$$\frac{2(12839-3871x)}{5589(3x^2-x+2)^{3/2}}+\frac{32}{27}\sqrt{3x^2-x+2}-\frac{28(42240x+35809)}{128547\sqrt{3x^2-x+2}}-\frac{296\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{27\sqrt{3}}$$

```
[Out] 2/5589*(12839-3871*x)/(3*x^2-x+2)^(3/2)-296/81*arcsinh(1/23*(1-6*x)*23^(1/2
))*3^(1/2)-28/128547*(35809+42240*x)/(3*x^2-x+2)^(1/2)+32/27*(3*x^2-x+2)^(1
/2)
```

Rubi [A] time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 640, 619, 215}

$$\frac{2(12839-3871x)}{5589(3x^2-x+2)^{3/2}}+\frac{32}{27}\sqrt{3x^2-x+2}-\frac{28(42240x+35809)}{128547\sqrt{3x^2-x+2}}-\frac{296\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{27\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(12839 - 3871*x))/(5589*(2 - x + 3*x^2)^(3/2)) - (28*(35809 + 42240*x))/
(128547*Sqrt[2 - x + 3*x^2]) + (32*Sqrt[2 - x + 3*x^2])/27 - (296*ArcSinh[(
1 - 6*x)/Sqrt[23]])/(27*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}dx = \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{4361}{81}+\frac{7682x}{9}+\frac{2852x^2}{3}+368x^3}{\left(2-x+3x^2\right)^{3/2}}dx$$

$$= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}}+\frac{4\int \frac{\frac{37030}{9}+\frac{4232x}{3}}{\sqrt{2-x+3x^2}}dx}{1587}$$

$$= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}}+\frac{32}{27}\sqrt{2-x+3x^2}+\frac{296}{27}\int \frac{}{\sqrt{2}}$$

$$= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}}+\frac{32}{27}\sqrt{2-x+3x^2}+\frac{296\,\text{Subst}}{}$$

$$= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}}+\frac{32}{27}\sqrt{2-x+3x^2}-\frac{296\sinh^{-1}}{27\sqrt{3}}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 0.83

$$\frac{2\left(228528x^4-743712x^3+25890x^2+78292\sqrt{3}\left(3x^2-x+2\right)^{3/2}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)-358377x-134217\right)}{42849\left(3x^2-x+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(-134217 - 358377*x + 25890*x^2 - 743712*x^3 + 228528*x^4 + 78292*Sqrt[3
]*(2 - x + 3*x^2)^(3/2)*ArcSinh[(-1 + 6*x)/Sqrt[23]]))/(42849*(2 - x + 3*x^
2)^(3/2))
```

fricas [A] time = 0.59, size = 117, normalized size = 1.36

$$\frac{2\left(39146\sqrt{3}\left(9x^4-6x^3+13x^2-4x+4\right)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)+3\left(76176x^4\right.\right.}{42849\left(9x^4-6x^3+13x^2-4x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/42849*(39146*sqrt(3)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-4*sqrt(3)*sq
rt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 3*(76176*x^4 - 247904*x
^3 + 8630*x^2 - 119459*x - 44739)*sqrt(3*x^2 - x + 2))/(9*x^4 - 6*x^3 + 13*
x^2 - 4*x + 4)
```

giac [A] time = 0.21, size = 67, normalized size = 0.78

$$-\frac{296}{81}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)+\frac{2\left(\left(2\left(8\left(4761x-15494\right)x+4315\right)x-119459\right)x-44739\right)}{14283\left(3x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -296/81*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/1
4283*((2*(8*(4761*x - 15494)*x + 4315)*x - 119459)*x - 44739)/(3*x^2 - x +
2)^(3/2)
```

maple [B] time = 0.01, size = 163, normalized size = 1.90

$$\frac{32x^4}{3\left(3x^2-x+2\right)^{\frac{3}{2}}}-\frac{296x^3}{27\left(3x^2-x+2\right)^{\frac{3}{2}}}+\frac{8x^2}{27\left(3x^2-x+2\right)^{\frac{3}{2}}}-\frac{296x}{27\sqrt{3x^2-x+2}}-\frac{461x}{81\left(3x^2-x+2\right)^{\frac{3}{2}}}+\frac{296\sqrt{3}\,\operatorname{arcsi}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x)
```

```
[Out] 13763/33534*(6*x-1)/(3*x^2-x+2)^(3/2)-296/27/(3*x^2-x+2)^(1/2)*x+65264/1285
47*(6*x-1)/(3*x^2-x+2)^(1/2)+296/81*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-
148/81/(3*x^2-x+2)^(1/2)-1727/1458/(3*x^2-x+2)^(3/2)+32/3*x^4/(3*x^2-x+2)^(
3/2)-296/27*x^3/(3*x^2-x+2)^(3/2)+8/27*x^2/(3*x^2-x+2)^(3/2)-461/81*x/(3*x^
2-x+2)^(3/2)
```

maxima [B] time = 0.96, size = 202, normalized size = 2.35

$$\frac{32\,x^4}{3\left(3\,x^2-x+2\right)^{\frac{3}{2}}}+\frac{296}{42849}\,x\left(\frac{426\,x}{\sqrt{3\,x^2-x+2}}-\frac{4761\,x^2}{\left(3\,x^2-x+2\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{3\,x^2-x+2}}+\frac{805\,x}{\left(3\,x^2-x+2\right)^{\frac{3}{2}}}-\frac{2162}{\left(3\,x^2-x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/3*x^4/(3*x^2 - x + 2)^(3/2) + 296/42849*x*(426*x/sqrt(3*x^2 - x + 2) - 4
761*x^2/(3*x^2 - x + 2)^(3/2) - 71/sqrt(3*x^2 - x + 2) + 805*x/(3*x^2 - x +
 2)^(3/2) - 2162/(3*x^2 - x + 2)^(3/2)) + 296/81*sqrt(3)*arcsinh(1/23*sqrt(
23)*(6*x - 1)) - 42032/42849*sqrt(3*x^2 - x + 2) - 47072/42849*x/sqrt(3*x^2
 - x + 2) + 52/9*x^2/(3*x^2 - x + 2)^(3/2) - 23104/14283/sqrt(3*x^2 - x + 2
) - 7742/1863*x/(3*x^2 - x + 2)^(3/2) + 1666/1863/(3*x^2 - x + 2)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+1)^3\,\left(4\,x^2+3\,x+1\right)}{\left(3\,x^2-x+2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
```

```
[Out] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

3.253 $\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=68

$$\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{3x^2-x+2}}-\frac{16\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

```
[Out] 2/1863*(1249-2273*x)/(3*x^2-x+2)^(3/2)-16/27*arcsinh(1/23*(1-6*x)*23^(1/2))
*3^(1/2)-8/42849*(23257-1473*x)/(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 12, 619, 215}

$$\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{3x^2-x+2}}-\frac{16\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(1249 - 2273*x))/(1863*(2 - x + 3*x^2)^(3/2)) - (8*(23257 - 1473*x))/(42
849*Sqrt[2 - x + 3*x^2]) - (16*ArcSinh[(1 - 6*x)/Sqrt[23]])/(9*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)^2 \left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}} \, dx = \frac{2(1249-2273x)}{1863 \left(2-x+3x^2\right)^{3/2}} + \frac{2}{69} \int \frac{\frac{1802}{27}+\frac{1150x}{3}+184x^2}{\left(2-x+3x^2\right)^{3/2}} \, dx$$

$$= \frac{2(1249-2273x)}{1863 \left(2-x+3x^2\right)^{3/2}} - \frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}} + \frac{4\int \frac{2116}{3\sqrt{2-x+3x^2}} \, dx}{1587}$$

$$= \frac{2(1249-2273x)}{1863 \left(2-x+3x^2\right)^{3/2}} - \frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}} + \frac{16}{9} \int \frac{1}{\sqrt{2-x+3x^2}} \, dx$$

$$= \frac{2(1249-2273x)}{1863 \left(2-x+3x^2\right)^{3/2}} - \frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}} + \frac{16 \,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}} \, dx, x, -1\right.}{9\sqrt{69}}$$

$$= \frac{2(1249-2273x)}{1863 \left(2-x+3x^2\right)^{3/2}} - \frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}} - \frac{16 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

Mathematica [A] time = 0.13, size = 66, normalized size = 0.97

$$\frac{2 \left(5892x^3 - 94992x^2 + 4232\sqrt{3} \left(3x^2-x+2\right)^{3/2} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right) + 17511x - 52443\right)}{14283 \left(3x^2-x+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(-52443 + 17511*x - 94992*x^2 + 5892*x^3 + 4232*Sqrt[3]*(2 - x + 3*x^2)^
(3/2)*ArcSinh[(-1 + 6*x)/Sqrt[23]]))/(14283*(2 - x + 3*x^2)^(3/2))
```

fricas [B] time = 0.86, size = 112, normalized size = 1.65

$$\frac{2 \left(2116 \sqrt{3} \left(9 x^4 - 6 x^3 + 13 x^2 - 4 x + 4\right) \log\left(-4 \sqrt{3} \sqrt{3 x^2 - x + 2} \, (6 x - 1) - 72 x^2 + 24 x - 25\right) + 3 \left(1964 x^3\right.}{14283 \left(9 x^4 - 6 x^3 + 13 x^2 - 4 x + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/14283*(2116*sqrt(3)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-4*sqrt(3)*sqr
t(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 3*(1964*x^3 - 31664*x^2
+ 5837*x - 17481)*sqrt(3*x^2 - x + 2))/(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)
```

giac [A] time = 0.24, size = 62, normalized size = 0.91

$$-\frac{16}{27} \sqrt{3} \log\left(-2 \sqrt{3} \left(\sqrt{3} x - \sqrt{3 x^2 - x + 2}\right) + 1\right) + \frac{2 \left(\left(4 \left(491 x - 7916\right) x + 5837\right) x - 17481\right)}{4761 \left(3 x^2 - x + 2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -16/27*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/47
61*((4*(491*x - 7916)*x + 5837)*x - 17481)/(3*x^2 - x + 2)^(3/2)
```

maple [B] time = 0.01, size = 146, normalized size = 2.15

$$-\frac{16x^3}{9\left(3x^2-x+2\right)^{\frac{3}{2}}}-\frac{92x^2}{9\left(3x^2-x+2\right)^{\frac{3}{2}}}-\frac{67x}{27\left(3x^2-x+2\right)^{\frac{3}{2}}}-\frac{16x}{9\sqrt{3x^2-x+2}}+\frac{16\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{27}-\frac{2}{486\left(3x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x)
```

```
[Out] -16/9/(3*x^2-x+2)^(3/2)*x^3-92/9/(3*x^2-x+2)^(3/2)*x^2-67/27/(3*x^2-x+2)^(3
/2)*x-2653/486/(3*x^2-x+2)^(3/2)+4585/11178*(6*x-1)/(3*x^2-x+2)^(3/2)+18892
/42849*(6*x-1)/(3*x^2-x+2)^(1/2)-16/9/(3*x^2-x+2)^(1/2)*x-8/27/(3*x^2-x+2)^
(1/2)+16/27*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

maxima [B] time = 0.97, size = 185, normalized size = 2.72

$$\frac{16}{14283}x\left(\frac{426\,x}{\sqrt{3\,x^2-x+2}}-\frac{4761\,x^2}{\left(3\,x^2-x+2\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{3\,x^2-x+2}}+\frac{805\,x}{\left(3\,x^2-x+2\right)^{\frac{3}{2}}}-\frac{2162}{\left(3\,x^2-x+2\right)^{\frac{3}{2}}}\right)+\frac{16}{27}\sqrt{3}\ \operatorname{arsinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 16/14283*x*(426*x/sqrt(3*x^2 - x + 2) - 4761*x^2/(3*x^2 - x + 2)^(3/2) - 71
/sqrt(3*x^2 - x + 2) + 805*x/(3*x^2 - x + 2)^(3/2) - 2162/(3*x^2 - x + 2)^(
3/2)) + 16/27*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 2272/14283*sqrt(3*
x^2 - x + 2) + 28184/14283*x/sqrt(3*x^2 - x + 2) - 28/3*x^2/(3*x^2 - x + 2)
^(3/2) - 2956/4761/sqrt(3*x^2 - x + 2) - 106/621*x/(3*x^2 - x + 2)^(3/2) -
3394/621/(3*x^2 - x + 2)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+1)^2\,\left(4\,x^2+3\,x+1\right)}{\left(3\,x^2-x+2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
```

```
[Out] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^2\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

3.254 $\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}} dx$

Optimal. Leaf size=47

$$-\frac{4(3889-4290x)}{14283\sqrt{3x^2-x+2}}-\frac{2(367x+73)}{621\left(3x^2-x+2\right)^{3/2}}$$

```
[Out] -2/621*(73+367*x)/(3*x^2-x+2)^(3/2)-4/14283*(3889-4290*x)/(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1660, 636}

$$-\frac{4(3889-4290x)}{14283\sqrt{3x^2-x+2}}-\frac{2(367x+73)}{621\left(3x^2-x+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (-2*(73 + 367*x))/(621*(2 - x + 3*x^2)^(3/2)) - (4*(3889 - 4290*x))/(14283*
Sqrt[2 - x + 3*x^2])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx &= -\frac{2(73+367x)}{621\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{577}{9}+92x}{\left(2-x+3x^2\right)^{3/2}}\,dx\\ &= -\frac{2(73+367x)}{621\left(2-x+3x^2\right)^{3/2}}-\frac{4(3889-4290x)}{14283\sqrt{2-x+3x^2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 33, normalized size = 0.70

$$\frac{2\left(2860x^3-3546x^2+1833x-1915\right)}{1587\left(3x^2-x+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(-1915 + 1833*x - 3546*x^2 + 2860*x^3))/(1587*(2 - x + 3*x^2)^(3/2))
```

fricas [A] time = 0.80, size = 51, normalized size = 1.09

$$\frac{2\left(2860\,x^3-3546\,x^2+1833\,x-1915\right)\sqrt{3\,x^2-x+2}}{1587\left(9\,x^4-6\,x^3+13\,x^2-4\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/1587*(2860*x^3 - 3546*x^2 + 1833*x - 1915)*sqrt(3*x^2 - x + 2)/(9*x^4 - 6
*x^3 + 13*x^2 - 4*x + 4)
```

giac [A] time = 0.20, size = 28, normalized size = 0.60

$$\frac{2\,((2\,(1430\,x-1773)x+1833)x-1915)}{1587\left(3\,x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/1587*((2*(1430*x - 1773)*x + 1833)*x - 1915)/(3*x^2 - x + 2)^(3/2)
```

maple [A] time = 0.00, size = 30, normalized size = 0.64

$$\frac{\frac{5720}{1587}x^3-\frac{2364}{529}x^2+\frac{1222}{529}x-\frac{3830}{1587}}{\left(3x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x)
```

```
[Out] 2/1587/(3*x^2-x+2)^(3/2)*(2860*x^3-3546*x^2+1833*x-1915)
```

maxima [A] time = 0.44, size = 76, normalized size = 1.62

$$\frac{5720\,x}{4761\,\sqrt{3\,x^2-x+2}}-\frac{8\,x^2}{3\left(3\,x^2-x+2\right)^{\frac{3}{2}}}-\frac{2860}{14283\,\sqrt{3\,x^2-x+2}}-\frac{182\,x}{621\left(3\,x^2-x+2\right)^{\frac{3}{2}}}-\frac{1250}{621\left(3\,x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5720/4761*x/sqrt(3*x^2 - x + 2) - 8/3*x^2/(3*x^2 - x + 2)^(3/2) - 2860/1428
3/sqrt(3*x^2 - x + 2) - 182/621*x/(3*x^2 - x + 2)^(3/2) - 1250/621/(3*x^2 -
 x + 2)^(3/2)
```

mupad [B] time = 4.20, size = 49, normalized size = 1.04

$$-\frac{442\,x-5720\,x\,\left(3\,x^2-x+2\right)+15556\,x^2+11490}{\sqrt{3\,x^2-x+2}\,\left(14283\,x^2-4761\,x+9522\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
```

```
[Out] -(442*x - 5720*x*(3*x^2 - x + 2) + 15556*x^2 + 11490)/((3*x^2 - x + 2)^(1/2
)*(14283*x^2 - 4761*x + 9522))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

3.255 $\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{4(691-13668x)}{268203\sqrt{3x^2-x+2}}-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}-\frac{8\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

```
[Out] -2/897*(101-77*x)/(3*x^2-x+2)^(3/2)-8/2197*arctanh(1/26*(9-8*x)*13^(1/2)/(3
*x^2-x+2)^(1/2))*13^(1/2)-4/268203*(691-13668*x)/(3*x^2-x+2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1646, 822, 12, 724, 206}

$$-\frac{4(691-13668x)}{268203\sqrt{3x^2-x+2}}-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}-\frac{8\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (-2*(101 - 77*x))/(897*(2 - x + 3*x^2)^(3/2)) - (4*(691 - 13668*x))/(268203
*Sqrt[2 - x + 3*x^2]) - (8*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2
]])])/(169*Sqrt[13])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{223}{13}+\frac{308x}{13}}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx$$

$$= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}}-\frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}}+\frac{4\int \frac{3174}{13(1+2x)\sqrt{2-x+3x^2}}\,dx}{20631}$$

$$= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}}-\frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}}+\frac{8}{169}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}$$

$$= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}}-\frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}}-\frac{16}{169}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,\right.$$

$$= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}}-\frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}}-\frac{8\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{169\sqrt{13}}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.85

$$\frac{2\left(82008x^3-31482x^2+79077x-32963\right)}{268203\left(3x^2-x+2\right)^{3/2}}-\frac{8\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (2*(-32963 + 79077*x - 31482*x^2 + 82008*x^3))/(268203*(2 - x + 3*x^2)^(3/2
)) - (8*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(169*Sqrt[13])
```

fricas [A] time = 0.68, size = 126, normalized size = 1.48

$$\frac{2\left(3174\sqrt{13}\left(9x^4-6x^3+13x^2-4x+4\right)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+13\left(82008x^3-31482\right.\right.}{3486639\left(9x^4-6x^3+13x^2-4x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3486639*(3174*sqrt(13)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-(4*sqrt(13
)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1))
```

```
+ 13*(82008*x^3 - 31482*x^2 + 79077*x - 32963)*sqrt(3*x^2 - x + 2))/(9*x^4
- 6*x^3 + 13*x^2 - 4*x + 4)
```

giac [A] time = 0.43, size = 101, normalized size = 1.19

$$\frac{8}{2197}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{2\left(3\left(6\left(4556x-1749\right)x+26359\right)x-32963\right)}{268203\left(3x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 8/2197*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt
(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))
) + 2/268203*(3*(6*(4556*x - 1749)*x + 26359)*x - 32963)/(3*x^2 - x + 2)^(3
/2)
```

maple [B] time = 0.01, size = 158, normalized size = 1.86

$$-\frac{8\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{2197}-\frac{2}{9\left(3x^2-x+2\right)^{\frac{3}{2}}}+\frac{\frac{10x}{69}-\frac{5}{207}}{\left(3x^2-x+2\right)^{\frac{3}{2}}}+\frac{\frac{80x}{529}-\frac{40}{1587}}{\sqrt{3x^2-x+2}}+\frac{1}{39\left(-4x+3\left(x+\frac{1}{2}\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)/(3*x^2-x+2)^(5/2),x)
```

```
[Out] -2/9/(3*x^2-x+2)^(3/2)+5/207*(6*x-1)/(3*x^2-x+2)^(3/2)+40/1587*(6*x-1)/(3*x
^2-x+2)^(1/2)+1/39/(-4*x+3*(x+1/2)^2+5/4)^(3/2)+4/897*(6*x-1)/(-4*x+3*(x+1/
2)^2+5/4)^(3/2)+784/89401*(6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^(1/2)+4/169/(-4*x+
3*(x+1/2)^2+5/4)^(1/2)-8/2197*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-1
6*x+12*(x+1/2)^2+5)^(1/2))
```

maxima [A] time = 0.96, size = 93, normalized size = 1.09

$$\frac{8}{2197}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|}-\frac{9\sqrt{23}}{23|2x+1|}\right)+\frac{18224x}{89401\sqrt{3x^2-x+2}}-\frac{2764}{268203\sqrt{3x^2-x+2}}+\frac{154x}{897\left(3x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/2197*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*
x + 1)) + 18224/89401*x/sqrt(3*x^2 - x + 2) - 2764/268203/sqrt(3*x^2 - x +
2) + 154/897*x/(3*x^2 - x + 2)^(3/2) - 202/897/(3*x^2 - x + 2)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(5/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)\left(3x^2 - x + 2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(5/2),x)
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 - x + 2)**(5/2)), x)
```

3.256 $\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{24(841-6633x)}{1162213\sqrt{3x^2-x+2}}-\frac{16\sqrt{3x^2-x+2}}{2197(2x+1)}-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}-\frac{56\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

```
[Out] -2/11661*(197-837*x)/(3*x^2-x+2)^(3/2)-56/28561*arctanh(1/26*(9-8*x)*13^(1/
2)/(3*x^2-x+2)^(1/2))*13^(1/2)-24/1162213*(841-6633*x)/(3*x^2-x+2)^(1/2)-16
/2197*(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A] time = 0.15, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 806, 724, 206}

$$-\frac{24(841-6633x)}{1162213\sqrt{3x^2-x+2}}-\frac{16\sqrt{3x^2-x+2}}{2197(2x+1)}-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}-\frac{56\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (-2*(197 - 837*x))/(11661*(2 - x + 3*x^2)^(3/2)) - (24*(841 - 6633*x))/(116
2213*Sqrt[2 - x + 3*x^2]) - (16*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)) - (56
*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2197*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
```

```
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{2226}{169}+\frac{462x}{13}+\frac{6696x^2}{169}}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}} - \frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}} + \frac{4\int \frac{\frac{50784}{2197}+\frac{19044x}{2197}}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx}{1587}$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}} - \frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}} - \frac{16\sqrt{2-x+3x^2}}{2197(1+2x)} + \frac{56 \int}{}$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}} - \frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}} - \frac{16\sqrt{2-x+3x^2}}{2197(1+2x)} - \frac{112}{}$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}} - \frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}} - \frac{16\sqrt{2-x+3x^2}}{2197(1+2x)} - \frac{56\,\mathrm{t}}{}$$

Mathematica [A] time = 0.06, size = 111, normalized size = 1.01

$$\frac{26\left(1318464x^4+133308x^3+1021566x^2+569989x-170239\right)-88872\sqrt{13}\sqrt{3x^2-x+2}\left(6x^3+x^2+3x+2\right.}{45326307(2x+1)\left(3x^2-x+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (26*(-170239 + 569989*x + 1021566*x^2 + 133308*x^3 + 1318464*x^4) - 88872*S
qrt[13]*Sqrt[2 - x + 3*x^2]*(2 + 3*x + x^2 + 6*x^3)*ArcTanh[(9 - 8*x)/(2*Sq
rt[13]*Sqrt[2 - x + 3*x^2])])/(45326307*(1 + 2*x)*(2 - x + 3*x^2)^(3/2))
```

fricas [A] time = 0.74, size = 141, normalized size = 1.28

$$\frac{2\left(22218\sqrt{13}\left(18x^5-3x^4+20x^3+5x^2+4x+4\right)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+13\left(1318464\right.\right.}{45326307\left(18x^5-3x^4+20x^3+5x^2+4x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45326307*(22218*sqrt(13)*(18*x^5 - 3*x^4 + 20*x^3 + 5*x^2 + 4*x + 4)*log(
-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2
+ 4*x + 1)) + 13*(1318464*x^4 + 133308*x^3 + 1021566*x^2 + 569989*x - 17023
9)*sqrt(3*x^2 - x + 2))/(18*x^5 - 3*x^4 + 20*x^3 + 5*x^2 + 4*x + 4)
```

giac [B] time = 0.36, size = 233, normalized size = 2.12

$$-\frac{56}{15108769}\sqrt{13}\left(872\sqrt{13}\sqrt{3}-529\log\left(\sqrt{13}\sqrt{3}-4\right)\right)\mathrm{sgn}\left(\frac{1}{2x+1}\right)-\frac{56\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}\right.\right.}{28561\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -56/15108769*sqrt(13)*(872*sqrt(13)*sqrt(3) - 529*log(sqrt(13)*sqrt(3) - 4)
)*sgn(1/(2*x + 1)) - 56/28561*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13
/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)) - 4)/sgn(1/(2*x + 1)) + 8/3486639*(
((13*(77756/sgn(1/(2*x + 1)) + 20631/((2*x + 1)*sgn(1/(2*x + 1))))/(2*x + 1
) - 1399650/sgn(1/(2*x + 1)))/(2*x + 1) + 625905/sgn(1/(2*x + 1)))/(2*x + 1
) - 164808/sgn(1/(2*x + 1)))/((8/(2*x + 1) - 13/(2*x + 1)^2 - 3)*sqrt(-8/(2
*x + 1) + 13/(2*x + 1)^2 + 3))
```

maple [A] time = 0.01, size = 165, normalized size = 1.50

$$-\frac{56\sqrt{13}\ \mathrm{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{28561}+\frac{\frac{4x}{23}-\frac{2}{69}}{\left(3x^2-x+2\right)^{\frac{3}{2}}}+\frac{\frac{96x}{529}-\frac{16}{529}}{\sqrt{3x^2-x+2}}+\frac{7}{507\left(-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}\right)^{\frac{3}{2}}}-\frac{}{11661\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^2/(3*x^2-x+2)^(5/2),x)
```

```
[Out] 2/69*(6*x-1)/(3*x^2-x+2)^(3/2)+16/529*(6*x-1)/(3*x^2-x+2)^(1/2)+7/507/(-4*x
+3*(x+1/2)^2+5/4)^(3/2)-128/11661*(6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^(3/2)-1073
6/1162213*(6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^(1/2)+28/2197/(-4*x+3*(x+1/2)^2+5/
4)^(1/2)-56/28561*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/
2)^2+5)^(1/2))-1/26/(x+1/2)/(-4*x+3*(x+1/2)^2+5/4)^(3/2)
```

maxima [A] time = 0.97, size = 125, normalized size = 1.14

$$\frac{56}{28561}\sqrt{13}\ \mathrm{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right)+\frac{146496\,x}{1162213\sqrt{3x^2-x+2}}-\frac{9604}{1162213\sqrt{3x^2-x+2}}+\frac{420\,x}{3887\left(3x^2-x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 56/28561*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(
2*x + 1)) + 146496/1162213*x/sqrt(3*x^2 - x + 2) - 9604/1162213/sqrt(3*x^2
- x + 2) + 420/3887*x/(3*x^2 - x + 2)^(3/2) - 1/13/(2*(3*x^2 - x + 2)^(3/2)
*x + (3*x^2 - x + 2)^(3/2)) - 49/11661/(3*x^2 - x + 2)^(3/2)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^2\,{\left(3\,x^2-x+2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)),x)
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^{2}+3x+1}{\left(2x+1\right)^{2}\left(3x^{2}-x+2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(5/2),x)
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 - x + 2)**(5/2)), x)
```

3.257 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}-\frac{144\sqrt{3x^2-x+2}}{28561(2x+1)}-\frac{8\sqrt{3x^2-x+2}}{2197(2x+1)^2}+\frac{12(103526x+25771)}{15108769\sqrt{3x^2-x+2}}-\frac{2084\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{28561\sqrt{13}}$$

```
[Out] 2/151593*(2363+3693*x)/(3*x^2-x+2)^(3/2)-2084/371293*arctanh(1/26*(9-8*x)*1
3^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+12/15108769*(25771+103526*x)/(3*x^2-x+2
)^(1/2)-8/2197*(3*x^2-x+2)^(1/2)/(1+2*x)^2-144/28561*(3*x^2-x+2)^(1/2)/(1+2
*x)
```

Rubi [A] time = 0.21, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1646, 1650, 806, 724, 206}

$$\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}-\frac{144\sqrt{3x^2-x+2}}{28561(2x+1)}-\frac{8\sqrt{3x^2-x+2}}{2197(2x+1)^2}+\frac{12(103526x+25771)}{15108769\sqrt{3x^2-x+2}}-\frac{2084\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{28561\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (2*(2363 + 3693*x))/(151593*(2 - x + 3*x^2)^(3/2)) + (12*(25771 + 103526*x)
)/(15108769*Sqrt[2 - x + 3*x^2]) - (8*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)^
2) - (144*Sqrt[2 - x + 3*x^2])/(28561*(1 + 2*x)) - (2084*ArcTanh[(9 - 8*x)/
(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(28561*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
```

```
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{32433}{2197}+\frac{106830x}{2197}+\frac{160116x^2}{2197}+\frac{59088x^3}{2197}}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}}+\frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}}+\frac{4\int \frac{\frac{1434648}{28561}+\frac{3345396x}{28561}+\frac{309\ldots}{28\ldots}}{(1+2x)^3\sqrt{2-x+3x^2}}}{1587}$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}}+\frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}}-\frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2}-\frac{2\int \ldots}{\ldots}$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}}+\frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}}-\frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2}-\frac{144\ldots}{28\ldots}$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}}+\frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}}-\frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2}-\frac{144\ldots}{28\ldots}$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}}+\frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}}-\frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2}-\frac{144\ldots}{28\ldots}$$

Mathematica [A] time = 0.07, size = 89, normalized size = 0.66

$$\frac{2\left(20304864x^5+20074356x^4+19381992x^3+21890266x^2+10777477x+847141\right)}{45326307(2x+1)^2\left(3x^2-x+2\right)^{3/2}}-\frac{2084\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2\ldots}}\right)}{28561\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (2*(847141 + 10777477*x + 21890266*x^2 + 19381992*x^3 + 20074356*x^4 + 2030
4864*x^5))/(45326307*(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)) - (2084*ArcTanh[(9
- 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(28561*Sqrt[13])
```

fricas [A] time = 0.83, size = 156, normalized size = 1.16

$$\frac{2\left(826827\sqrt{13}\left(36x^6+12x^5+37x^4+30x^3+13x^2+12x+4\right)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+13\right.}{589241991\left(36x^6+12x^5+37x^4+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/589241991*(826827*sqrt(13)*(36*x^6 + 12*x^5 + 37*x^4 + 30*x^3 + 13*x^2 +
12*x + 4)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x
+ 185)/(4*x^2 + 4*x + 1)) + 13*(20304864*x^5 + 20074356*x^4 + 19381992*x^3
+ 21890266*x^2 + 10777477*x + 847141)*sqrt(3*x^2 - x + 2))/(36*x^6 + 12*x^5
 + 37*x^4 + 30*x^3 + 13*x^2 + 12*x + 4)
```

giac [B] time = 0.35, size = 233, normalized size = 1.73

$$\frac{2084}{371293}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{2(3(6(310578x-26213)x+1455755)x+16}{45326307\left(3x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 2084/371293*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4
*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x
+ 2))) + 2/45326307*(3*(6*(310578*x - 26213)*x + 1455755)*x + 1634293)/(3*x
^2 - x + 2)^(3/2) - 8/28561*(66*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 + 21*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 - 1015*sqrt(3)*x + 431*sqrt(3) +
1015*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3
)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

maple [A] time = 0.01, size = 148, normalized size = 1.10

$$-\frac{2084\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{371293}+\frac{521}{13182\left(-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{\frac{1772x}{50531}-\frac{886}{151593}}{\left(-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{\frac{11280}{15108}}{\sqrt{-4x+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^3/(3*x^2-x+2)^(5/2),x)
```

```
[Out] 521/13182/(-4*x+3*(x+1/2)^2+5/4)^(3/2)+886/151593*(6*x-1)/(-4*x+3*(x+1/2)^2
+5/4)^(3/2)+188008/15108769*(6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^(1/2)+1042/28561
/(-4*x+3*(x+1/2)^2+5/4)^(1/2)-2084/371293*13^(1/2)*arctanh(2/13*(-4*x+9/2)*
13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))-1/338/(x+1/2)/(-4*x+3*(x+1/2)^2+5/4)
^(3/2)-1/104/(x+1/2)^2/(-4*x+3*(x+1/2)^2+5/4)^(3/2)
```

maxima [A] time = 0.99, size = 174, normalized size = 1.29

$$\frac{2084}{371293}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|}-\frac{9\sqrt{23}}{23|2x+1|}\right)+\frac{1128048x}{15108769\sqrt{3x^2-x+2}}+\frac{363210}{15108769\sqrt{3x^2-x+2}}+\frac{17}{50531(3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2084/371293*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/a
bs(2*x + 1)) + 1128048/15108769*x/sqrt(3*x^2 - x + 2) + 363210/15108769/sqr
t(3*x^2 - x + 2) + 1772/50531*x/(3*x^2 - x + 2)^(3/2) - 1/26/(4*(3*x^2 - x
+ 2)^(3/2)*x^2 + 4*(3*x^2 - x + 2)^(3/2)*x + (3*x^2 - x + 2)^(3/2)) - 1/169
/(2*(3*x^2 - x + 2)^(3/2)*x + (3*x^2 - x + 2)^(3/2)) + 10211/303186/(3*x^2
- x + 2)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^3\,{\left(3\,x^2-x+2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^3\left(3x^2-x+2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*(3*x**2 - x + 2)**(5/2)), x)
```

3.258 $\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=208

$$\frac{(b+2cx)\left(-3b^2fh^2+6bceh^2+4c^2\left(fg^2-h(2dh+eg)\right)\right)}{3ch^2(2cg-bh)^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}+\frac{2\left(dh^2-egh+fg^2\right)}{3h^3(g+hx)(2cg-bh)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}-$$

```
[Out] -f/c/h^3/(-g*(-b*h+c*g)+b*h^2*x+c*h^2*x^2)^(1/2)+1/3*(6*b*c*e*h^2-3*b^2*f*h
^2+4*c^2*(f*g^2-h*(2*d*h+e*g)))*(2*c*x+b)/c/h^2/(-b*h+2*c*g)^3/(-g*(-b*h+c*
g)+b*h^2*x+c*h^2*x^2)^(1/2)+2/3*(d*h^2-e*g*h+f*g^2)/h^3/(-b*h+2*c*g)/(h*x+g
)/(-g*(-b*h+c*g)+b*h^2*x+c*h^2*x^2)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.064, Rules used = {1638, 792, 613}

$$\frac{(b+2cx)\left(-3b^2fh^2+6bceh^2+4c^2\left(fg^2-h(2dh+eg)\right)\right)}{3ch^2(2cg-bh)^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}+\frac{2\left(dh^2-egh+fg^2\right)}{3h^3(g+hx)(2cg-bh)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*(-(c*g^2) + b*g*h + b*h^2*x + c*h^2*x^2)^(
3/2)),x]
```

```
[Out] -(f/(c*h^3*Sqrt[-(g*(c*g - b*h)) + b*h^2*x + c*h^2*x^2])) + ((6*b*c*e*h^2 -
 3*b^2*f*h^2 + 4*c^2*(f*g^2 - h*(e*g + 2*d*h)))*(b + 2*c*x))/(3*c*h^2*(2*c*
g - b*h)^3*Sqrt[-(g*(c*g - b*h)) + b*h^2*x + c*h^2*x^2]) + (2*(f*g^2 - e*g*
h + d*h^2))/(3*h^3*(2*c*g - b*h)*(g + h*x)*Sqrt[-(g*(c*g - b*h)) + b*h^2*x
+ c*h^2*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 1638

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d - b
*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = -\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} - \frac{\int \frac{\frac{1}{2}h^3(bfg-2cdh)+\frac{1}{2}h^3(2cfg}{(g+hx)\left(-cg^2+bgh+bh^2x\right.}}{ch^4}$$

$$= -\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} + \frac{2\left(fg^2\right.}{3h^3(2cg-bh)(g+hx)}$$

$$= -\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} + \frac{\left(6bceh^2-3b^2fh^2+4c\right.}{3ch^2(2cg-bh)^3\sqrt{}}$$

Mathematica [A] time = 0.51, size = 219, normalized size = 1.05

$$\frac{2b^2h^2\left(f\left(8g^2+12ghx+3h^2x^2\right)-h(dh+2eg+3ehx)\right)-4bch\left(h\left(e\left(g^2+2ghx+3h^2x^2\right)-2dh(2g+hx)\right)+2f\right.}{3h^3(g+hx)(bh-2cg)^3\sqrt{(g}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*(-(c*g^2) + b*g*h + b*h^2*x + c*h^2*
x^2)^(3/2)),x]
```

```
[Out] (2*b^2*h^2*(-(h*(2*e*g + d*h + 3*e*h*x)) + f*(8*g^2 + 12*g*h*x + 3*h^2*x^2)
) + 8*c^2*(f*g^2*(2*g^2 + 2*g*h*x - h^2*x^2) + h*(e*g*(g^2 + g*h*x + h^2*x^
2) + d*h*(-g^2 + 2*g*h*x + 2*h^2*x^2))) - 4*b*c*h*(2*f*g^2*(4*g + 5*h*x) +
h*(-2*d*h*(2*g + h*x) + e*(g^2 + 2*g*h*x + 3*h^2*x^2))))/(3*h^3*(-2*c*g + b
*h)^3*(g + h*x)*Sqrt[(g + h*x)*(-(c*g) + b*h + c*h*x)])
```

fricas [B] time = 33.08, size = 465, normalized size = 2.24

$$\frac{2\left(8c^2fg^4-b^2dh^4+4\left(c^2e-4bcf\right)g^3h-2\left(2c^2d+bce-4b^2f\right)g^2h^2+2\left(4bcd-b^2e\right)gh^3-\left(4c^2fg^2h^2-4c^2e\right.\right.}{3\left(8c^4g^6h^3-20bc^3g^5h^4+18b^2c^2g^4h^5-7b^3cg^3h^6+b^4g^2h^7-\left(8c^4g^3h^6-12bc^3g^2h^7+6b^2c^2gh^8\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, al
gorithm="fricas")
```

```
[Out] 2/3*(8*c^2*f*g^4 - b^2*d*h^4 + 4*(c^2*e - 4*b*c*f)*g^3*h - 2*(2*c^2*d + b*c
*e - 4*b^2*f)*g^2*h^2 + 2*(4*b*c*d - b^2*e)*g*h^3 - (4*c^2*f*g^2*h^2 - 4*c^
2*e*g*h^3 - (8*c^2*d - 6*b*c*e + 3*b^2*f)*h^4)*x^2 + (8*c^2*f*g^3*h + 4*(c^
2*e - 5*b*c*f)*g^2*h^2 + 4*(2*c^2*d - b*c*e + 3*b^2*f)*g*h^3 + (4*b*c*d - 3
*b^2*e)*h^4)*x)*sqrt(c*h^2*x^2 + b*h^2*x - c*g^2 + b*g*h)/(8*c^4*g^6*h^3 -
20*b*c^3*g^5*h^4 + 18*b^2*c^2*g^4*h^5 - 7*b^3*c*g^3*h^6 + b^4*g^2*h^7 - (8*
c^4*g^3*h^6 - 12*b*c^3*g^2*h^7 + 6*b^2*c^2*g*h^8 - b^3*c*h^9)*x^3 - (8*c^4*
g^4*h^5 - 4*b*c^3*g^3*h^6 - 6*b^2*c^2*g^2*h^7 + 5*b^3*c*g*h^8 - b^4*h^9)*x^
2 + (8*c^4*g^5*h^4 - 28*b*c^3*g^4*h^5 + 30*b^2*c^2*g^3*h^6 - 13*b^3*c*g^2*h
^7 + 2*b^4*g*h^8)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx^2+ex+d}{\left(ch^2x^2+bh^2x-cg^2+bgh\right)^{\frac{3}{2}}(hx+g)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, al
gorithm="giac")
```

```
[Out] integrate((f*x^2 + e*x + d)/((c*h^2*x^2 + b*h^2*x - c*g^2 + b*g*h)^(3/2)*(h
*x + g)), x)
```

maple [A] time = 0.01, size = 324, normalized size = 1.56

$$-\frac{2\left(chx+hb-cg\right)\left(-3b^2f\,h^4x^2+6bce\,h^4x^2-8c^2d\,h^4x^2-4c^2eg\,h^3x^2+4c^2f\,g^2h^2x^2+3b^2e\,h^4x-12b^2fg\,h^3x-4b\right.}{3\left(b^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x)
```

```
[Out] -2/3*(c*h*x+b*h-c*g)*(-3*b^2*f*h^4*x^2+6*b*c*e*h^4*x^2-8*c^2*d*h^4*x^2-4*c^
2*e*g*h^3*x^2+4*c^2*f*g^2*h^2*x^2+3*b^2*e*h^4*x-12*b^2*f*g*h^3*x-4*b*c*d*h^
4*x+4*b*c*e*g*h^3*x+20*b*c*f*g^2*h^2*x-8*c^2*d*g*h^3*x-4*c^2*e*g^2*h^2*x-8*
c^2*f*g^3*h*x+b^2*d*h^4+2*b^2*e*g*h^3-8*b^2*f*g^2*h^2-8*b*c*d*g*h^3+2*b*c*e
*g^2*h^2+16*b*c*f*g^3*h+4*c^2*d*g^2*h^2-4*c^2*e*g^3*h-8*c^2*f*g^4)/(b^3*h^3
-6*b^2*c*g*h^2+12*b*c^2*g^2*h-8*c^3*g^3)/h^3/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2
)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re details)Is b*h-2*c*g zero or nonzero?
```

mupad [B] time = 5.75, size = 1089, normalized size = 5.24

$$\frac{16\,c^2\,f\,g^4\,\sqrt{-c\,g^2+b\,g\,h+c\,h^2\,x^2+b\,h^2\,x}-2\,b^2\,d\,h^4\,\sqrt{-c\,g^2+b\,g\,h+c\,h^2\,x^2+b\,h^2\,x}-8\,c^2\,d\,g^2\,h^2\,\sqrt{-c\,g^2+b\,g\,h}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(3/2
)),x)
```

```
[Out] (16*c^2*f*g^4*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 2*b^2*d*h^4*(b*
h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 8*c^2*d*g^2*h^2*(b*h^2*x - c*g^2
 + c*h^2*x^2 + b*g*h)^(1/2) + 16*b^2*f*g^2*h^2*(b*h^2*x - c*g^2 + c*h^2*x^2
 + b*g*h)^(1/2) + 16*c^2*d*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1
/2) + 6*b^2*f*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 4*b^2*e
*g*h^3*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 8*c^2*e*g^3*h*(b*h^2*x
 - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 6*b^2*e*h^4*x*(b*h^2*x - c*g^2 + c*h^
2*x^2 + b*g*h)^(1/2) + 8*b*c*d*h^4*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^
(1/2) - 8*c^2*f*g^2*h^2*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 4
*b*c*e*g^2*h^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 12*b*c*e*h^4*x
^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 16*c^2*d*g*h^3*x*(b*h^2*x
- c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 24*b^2*f*g*h^3*x*(b*h^2*x - c*g^2 + c*
h^2*x^2 + b*g*h)^(1/2) + 16*c^2*f*g^3*h*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*
g*h)^(1/2) + 8*c^2*e*g^2*h^2*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2)
```

```
+ 8*c^2*e*g*h^3*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 16*b*c*d*
g*h^3*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 32*b*c*f*g^3*h*(b*h^2*x
 - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 8*b*c*e*g*h^3*x*(b*h^2*x - c*g^2 + c*
h^2*x^2 + b*g*h)^(1/2) - 40*b*c*f*g^2*h^2*x*(b*h^2*x - c*g^2 + c*h^2*x^2 +
b*g*h)^(1/2))/(3*b^4*g^2*h^7 + 24*c^4*g^6*h^3 + 3*b^4*h^9*x^2 - 60*b*c^3*g^
5*h^4 - 21*b^3*c*g^3*h^6 + 3*b^3*c*h^9*x^3 + 24*c^4*g^5*h^4*x + 54*b^2*c^2*
g^4*h^5 - 24*c^4*g^4*h^5*x^2 - 24*c^4*g^3*h^6*x^3 + 6*b^4*g*h^8*x + 18*b^2*
c^2*g^2*h^7*x^2 - 84*b*c^3*g^4*h^5*x - 39*b^3*c*g^2*h^7*x - 15*b^3*c*g*h^8*
x^2 + 90*b^2*c^2*g^3*h^6*x + 12*b*c^3*g^3*h^6*x^2 + 36*b*c^3*g^2*h^7*x^3 -
18*b^2*c^2*g*h^8*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\left((g+hx)\left(bh-cg+chx\right)\right)^{\frac{3}{2}}\left(g+hx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*h**2*x**2+b*h**2*x+b*g*h-c*g**2)**(3/2)
,x)
```

```
[Out] Integral((d + e*x + f*x**2)/(((g + h*x)*(b*h - c*g + c*h*x))**(3/2)*(g + h*
x)), x)
```

3.259 $\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=906

$$\frac{2C(d+ex)^{3/2}\left(cx^2+bx+a\right)^{3/2}}{9ce}-\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\left(cx^2+bx+a\right)^{3/2}}{21c^2e}+\frac{2\sqrt{d+ex}\left(d\left(8Cd^2-3e(4Bd\right.\right.}{}$$

```
[Out] 2/9*C*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(3/2)/c/e-2/21*(-3*B*c*e+2*C*b*e+2*C*c*d)
*(c*x^2+b*x+a)^(3/2)*(e*x+d)^(1/2)/c^2/e+2/315*(8*b^3*C*e^3-3*b*c*e^2*(4*B*
b*e-C*a*e+C*b*d)+c^3*d*(8*C*d^2-3*e*(-7*A*e+4*B*d))+3*c^2*e*(a*e*(-5*B*e+C*
d)-b*(-7*A*e^2-2*B*d*e+C*d^2))+3*c*e*(8*b^2*C*e^2-c*e*(12*B*b*e+7*C*a*e+C*b
*d)-c^2*(2*C*d^2-3*e*(7*A*e+B*d)))*x)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3
/e^3+1/315*(2*(4*c^2*d^2-b^2*e^2-3/2*c*e*(-2*a*e+b*d))*(8*b^2*C*e^2-c*e*(12
*B*b*e+7*C*a*e+C*b*d)-c^2*(2*C*d^2-3*e*(7*A*e+B*d)))-5*c*e*(-b*e+2*c*d)*(6*
b^2*C*d*e+c*e*(21*A*c*d-3*B*a*e-5*C*a*d)+b*(2*a*C*e^2-c*d*(9*B*e+C*d))))*El
lipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)
,(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*
(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^4/
e^4/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-
2/315*(a*e^2-b*d*e+c*d^2)*(8*b^3*C*e^3-3*c^2*e^2*(-7*A*b*e-10*B*a*e+B*b*d+2
*C*a*d)+3*b*c*e^2*(-4*B*b*e-9*C*a*e+C*b*d)-2*c^3*d*(8*C*d^2-3*e*(-7*A*e+4*B
*d)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*
2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)
/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^4/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a
)^(1/2)
```

Rubi [A] time = 2.70, antiderivative size = 905, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {1653, 832, 814, 843, 718, 424, 419}

$$\frac{2C(d+ex)^{3/2}\left(cx^2+bx+a\right)^{3/2}}{9ce}-\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\left(cx^2+bx+a\right)^{3/2}}{21c^2e}+\frac{2\sqrt{d+ex}\left(\left(8Cd^3-3de(4Bd\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(8*b^3*C*e^3 - 3*b*c*e^2*(b*C*d + 4*b*B*e - a*C*e) + c^3*(
8*C*d^3 - 3*d*e*(4*B*d - 7*A*e)) - 3*c^2*e*(b*C*d^2 - b*e*(2*B*d + 7*A*e) -
 a*e*(C*d - 5*B*e)) + 3*c*e*(8*b^2*C*e^2 - c*e*(b*C*d + 12*b*B*e + 7*a*C*e)
 - c^2*(2*C*d^2 - 3*e*(B*d + 7*A*e)))*x)*Sqrt[a + b*x + c*x^2])/(315*c^3*e^
3) - (2*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2)
)/(21*c^2*e) + (2*C*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(9*c*e) - (Sqr
t[2]*Sqrt[b^2 - 4*a*c]*(5*c*e*(2*c*d - b*e)*(6*b^2*C*d*e + 2*a*b*C*e^2 - b*
c*d*(C*d + 9*B*e) + c*e*(21*A*c*d - 5*a*C*d - 3*a*B*e)) - 2*(4*c^2*d^2 - b^
2*e^2 - (3*c*e*(b*d - 2*a*e))/2)*(8*b^2*C*e^2 - c*e*(b*C*d + 12*b*B*e + 7*a
*C*e) - c^2*(2*C*d^2 - 3*e*(B*d + 7*A*e))))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
```

```
Sqrt[b^2 - 4*a*c])*e)])/(315*c^4*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*
d^2 - b*d*e + a*e^2)*(8*b^3*C*e^3 - 3*c^2*e^2*(b*B*d + 2*a*C*d - 7*A*b*e -
10*a*B*e) + 3*b*c*e^2*(b*C*d - 4*b*B*e - 9*a*C*e) - 2*c^3*(8*C*d^3 - 3*d*e*
(4*B*d - 7*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315*c^4*e^4*Sqrt[d + e*x]*Sqrt[a +
 b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)dx = \frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce} + \frac{2\int \sqrt{d+ex}\left(-\frac{3}{2}e(bCd-3Ac\right.}{}$$

$$= -\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{21c^2e} + \frac{2C(d+ex}{}$$

$$= \frac{2\sqrt{d+ex}\left(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3\left(8Cd^3-3de\right.\right.}{}$$

$$= \frac{2\sqrt{d+ex}\left(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3\left(8Cd^3-3de\right.\right.}{}$$

$$= \frac{2\sqrt{d+ex}\left(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3\left(8Cd^3-3de\right.\right.}{}$$

$$= \frac{2\sqrt{d+ex}\left(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3\left(8Cd^3-3de\right.\right.}{}$$

Mathematica [C] time = 14.99, size = 15669, normalized size = 17.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\sqrt{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\sqrt{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d), x)
```

maple [B] time = 0.19, size = 19955, normalized size = 22.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\sqrt{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\sqrt{d+e\,x}\,\left(C\,x^2+B\,x+A\right)\,\sqrt{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)*(A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2),x)
[Out] int((d + e*x)^(1/2)*(A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{d+ex}\left(A+Bx+Cx^2\right)\sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(sqrt(d + e*x)*(A + B*x + C*x**2)*sqrt(a + b*x + c*x**2), x)
```

3.260 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=668

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(ce(-10aCe-7bBe+8bCd)+c^2\left(48Cd^2-14\right.\right.}{105c^3e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] 2/7*C*(c*x^2+b*x+a)^(3/2)*(e*x+d)^(1/2)/c/e-2/105*(5*c*e*(-7*A*c*e+C*a*e+3*
C*b*d)-(-b*e+4*c*d)*(-7*B*c*e+4*C*b*e+6*C*c*d)+3*c*e*(-7*B*c*e+4*C*b*e+6*C*
c*d)*x)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/e^3+1/105*(5*c*e*(-b*e+2*c*d)
*(-7*A*c*e+C*a*e+3*C*b*d)-(-7*B*c*e+4*C*b*e+6*C*c*d)*(8*c^2*d^2-2*b^2*e^2-3
*c*e*(-2*a*e+b*d)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-
4*a*c+b^2))^(1/2)/c^3/e^4/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2))))^(1/2)+2/105*(a*e^2-b*d*e+c*d^2)*(4*b^2*C*e^2+c*e*(-7*B*b*e-
10*C*a*e+8*C*b*d)+c^2*(48*C*d^2-14*e*(-5*A*e+4*B*d)))*EllipticF(1/2*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)
^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*
(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(
1/2))))^(1/2)/c^3/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 1.19, antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1653, 814, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(ce(-10aCe-7bBe+8bCd)+c^2\left(48Cd^2-14\right.\right.}{105c^3e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/Sqrt[d + e*x],x]
```

```
[Out] (-2*Sqrt[d + e*x]*(5*c*e*(3*b*C*d - 7*A*c*e + a*C*e) - (4*c*d - b*e)*(6*c*C
*d - 7*B*c*e + 4*b*C*e) + 3*c*e*(6*c*C*d - 7*B*c*e + 4*b*C*e)*x)*Sqrt[a + b
*x + c*x^2])/(105*c^2*e^3) + (2*C*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2))/(7
*c*e) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(5*c*e*(2*c*d - b*e)*(3*b*C*d - 7*A*c*e
+ a*C*e) - (6*c*C*d - 7*B*c*e + 4*b*C*e)*(8*c^2*d^2 - 2*b^2*e^2 - 3*c*e*(b*
d - 2*a*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ell
ipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[
2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^
3*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
+ c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(4*b^2*C*e
^2 + c*e*(8*b*C*d - 7*b*B*e - 10*a*C*e) + c^2*(48*C*d^2 - 14*e*(4*B*d - 5*A
*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^3*e^4*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2
])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}}\,dx = \frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce} + \frac{2\int \frac{\left(-\frac{1}{2}e(3bCd-7Ace+aCe)-\frac{1}{2}e(6cCd-7Bce+4bCe)\right.}{\sqrt{d+ex}}}{7ce^2}$$

$$= -\frac{2\sqrt{d+ex}\,(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe)}{105c^2e^3}$$

$$= -\frac{2\sqrt{d+ex}\,(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe)}{105c^2e^3}$$

$$= -\frac{2\sqrt{d+ex}\,(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe)}{105c^2e^3}$$

$$= -\frac{2\sqrt{d+ex}\,(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe)}{105c^2e^3}$$

Mathematica [C] time = 14.41, size = 9965, normalized size = 14.92

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/Sqrt[d + e*x],x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}}{\sqrt{ex+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/sqrt(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/sqrt(e*x + d), x)
```

maple [B] time = 0.07, size = 12761, normalized size = 19.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/sqrt(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(C\,x^2 + B\,x + A\right)\,\sqrt{c\,x^2 + b\,x + a}}{\sqrt{d + e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(1/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

3.261 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=749

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(-ce\left(2ae(9Cd-5Be)-b\left(32Cd^2-5e(5Bd-3Ae)\right)\right)+bCe^2(bd\right.}{15c^2e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] -2*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(
1/2)-2/15*(b*C*e^2*(-a*e+b*d)+c^2*d*(24*C*d^2-5*e*(-3*A*e+4*B*d))+c*e*(a*e*
(-5*B*e+9*C*d)-5*b*(3*A*e^2-4*B*d*e+5*C*d^2))+3*c*e^2*(5*B*c*d+b*C*d-6*c*C*
d^2/e-5*A*c*e-a*C*e)*x)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e^3/(a*e^2-b*d*
e+c*d^2)-1/15*(2*b^2*C*e^2+c*e*(-5*B*b*e-6*C*a*e+8*C*b*d)-c^2*(48*C*d^2-10*
e*(-3*A*e+4*B*d)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/
2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4
*a*c+b^2))^(1/2)/c^2/e^4/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2))))^(1/2)+2/15*(b*C*e^2*(-a*e+b*d)-2*c^2*d*(24*C*d^2-5*e*(-3*A*e
+4*B*d))-c*e*(2*a*e*(-5*B*e+9*C*d)-b*(32*C*d^2-5*e*(-3*A*e+5*B*d))))*Ellipt
icF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2
*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*
a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b
+(-4*a*c+b^2)^(1/2))))^(1/2)/c^2/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 1.52, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1650, 814, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(ce\left(-2ae(9Cd-5Be)-5be(5Bd-3Ae)+32bCd^2\right)+bCe^2(bd-\right.}{15c^2e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(b*C*e^2*(b*d - a*e) + c^2*(24*C*d^3 - 5*d*e*(4*B*d - 3*A
*e)) + c*e*(a*e*(9*C*d - 5*B*e) - 5*b*(5*C*d^2 - 4*B*d*e + 3*A*e^2)) + 3*c*
e^2*(5*B*c*d + b*C*d - (6*c*C*d^2)/e - 5*A*c*e - a*C*e)*x)*Sqrt[a + b*x + c
*x^2])/(15*c*e^3*(c*d^2 - b*d*e + a*e^2)) - (2*(C*d^2 - e*(B*d - A*e))*(a +
 b*x + c*x^2)^(3/2))/(e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (Sqrt[2]*S
qrt[b^2 - 4*a*c]*(2*b^2*C*e^2 + c*e*(8*b*C*d - 5*b*B*e - 6*a*C*e) - c^2*(48
*C*d^2 - 10*e*(4*B*d - 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/
(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[
b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)])/(15*c^2*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(b*C*e^2*(b*d -
a*e) - 2*c^2*d*(24*C*d^2 - 5*e*(4*B*d - 3*A*e)) + c*e*(32*b*C*d^2 - 5*b*e*(
5*B*d - 3*A*e) - 2*a*e*(9*C*d - 5*B*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elliptic
F[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
```

```
(-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c^2*e^4*
Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{3/2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(a+bx+cx^2\right)^{3/2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{2\int \frac{\left(-\frac{3bCd^2-be(3Bd-2Ae)+e(Acd-aCd+a}{2e}\right.}{}}{c}$$

$$= -\frac{2\sqrt{d+ex}\left(bCe^2(bd-ae)+c^2\left(24Cd^3-5de(4Bd-3Ae)\right)+ce\left(ae(9Cd\right.\right.}{}$$

$$= -\frac{2\sqrt{d+ex}\left(bCe^2(bd-ae)+c^2\left(24Cd^3-5de(4Bd-3Ae)\right)+ce\left(ae(9Cd\right.\right.}{}$$

$$= -\frac{2\sqrt{d+ex}\left(bCe^2(bd-ae)+c^2\left(24Cd^3-5de(4Bd-3Ae)\right)+ce\left(ae(9Cd\right.\right.}{}$$

$$= -\frac{2\sqrt{d+ex}\left(bCe^2(bd-ae)+c^2\left(24Cd^3-5de(4Bd-3Ae)\right)+ce\left(ae(9Cd\right.\right.}{}$$

Mathematica [C] time = 14.04, size = 13240, normalized size = 17.68

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(3/2),x]
[Out] Result too large to show
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\,\sqrt{ex+d}}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="fri
cas")
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^2*x^2 + 2
*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(3/2), x)
```

maple [B] time = 0.08, size = 8221, normalized size = 10.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}}{(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(C\,x^2 + B\,x + A\right)\,\sqrt{c\,x^2 + b\,x + a}}{{\left(d + e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(3/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + Bx + Cx^2\right)\sqrt{a + bx + cx^2}}{(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(3/2), x)
```

3.262 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=712

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(e(-2aCe-3bBe+8bCd)-2c\left(8Cd^2-e(4Bd-Ae)\right)\right)F\left(\sin^{-1}\right.}{3ce^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] -2/3*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(3/2)-2/3*(e*(-a*e+b*d)*(-3*B*e+7*C*d)-c*d*(8*C*d^2-e*(-A*e+4*B*d))+e^2*(B
*c*d+b*C*d-2*c*C*d^2/e-A*c*e-a*C*e)*x)*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e
+c*d^2)/(e*x+d)^(1/2)+1/3*(2*(4*c*d-1/2*b*e)*(B*c*d+b*C*d-2*c*C*d^2/e-A*c*e
-a*C*e)+6*c*(b*d*(-B*e+C*d)+e*(A*c*d+B*a*e-C*a*d)))*EllipticE(1/2*((b+2*c*x
+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(
1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e
*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/e^3/(a*e^2-b*d*e+c*d^2)
/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3
*(e*(-3*B*b*e-2*C*a*e+8*C*b*d)-2*c*(8*C*d^2-e*(-A*e+4*B*d)))*EllipticF(1/2*
((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a
*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)
^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2))))^(1/2)/c/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 1.27, antiderivative size = 711, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1650, 812, 843, 718, 424, 419}

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(e(-2aCe-3bBe+8bCd)-2c\left(8Cd^2-e(4Bd-Ae)\right)\right)F\left(\sin^{-1}\right.}{3ce^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(5/2),x]
```

```
[Out] (2*((8*c*C*d^3)/e - c*d*(4*B*d - A*e) - (b*d - a*e)*(7*C*d - 3*B*e) - e*(B*
c*d + b*C*d - (2*c*C*d^2)/e - A*c*e - a*C*e)*x)*Sqrt[a + b*x + c*x^2])/(3*e
^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (2*(C*d^2 - e*(B*d - A*e))*(a +
 b*x + c*x^2)^(3/2))/(3*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) + (Sqrt[
2]*Sqrt[b^2 - 4*a*c]*(2*(4*c*d - (b*e)/2)*(B*c*d + b*C*d - (2*c*C*d^2)/e -
A*c*e - a*C*e) + 6*c*(b*d*(C*d - B*e) + e*(A*c*d - a*C*d + a*B*e)))*Sqrt[d
+ e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^3*(c*d^2 - b*d*e + a
*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
+ c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*(8*b*C*d - 3*b*B*e - 2*a*C*e) -
 2*c*(8*C*d^2 - e*(4*B*d - A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSi
n[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqr
t[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^4*Sqrt[d + e
*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{5/2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(a+bx+cx^2\right)^{3/2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2\int \frac{\left(-\frac{3(bd(Cd-Be)+e(Acd-aCd+aBe))}{2e}+\frac{3}{2}\right.}{}}{3\left(cd^2\right.}$$

$$= \frac{2\left(\frac{8cCd^3}{e}-cd(4Bd-Ae)-(bd-ae)(7Cd-3Be)-e\left(Bcd+bCd-\frac{2cCd^2}{e}\right.\right.}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$= \frac{2\left(\frac{8cCd^3}{e}-cd(4Bd-Ae)-(bd-ae)(7Cd-3Be)-e\left(Bcd+bCd-\frac{2cCd^2}{e}\right.\right.}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$= \frac{2\left(\frac{8cCd^3}{e}-cd(4Bd-Ae)-(bd-ae)(7Cd-3Be)-e\left(Bcd+bCd-\frac{2cCd^2}{e}\right.\right.}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$= \frac{2\left(\frac{8cCd^3}{e}-cd(4Bd-Ae)-(bd-ae)(7Cd-3Be)-e\left(Bcd+bCd-\frac{2cCd^2}{e}\right.\right.}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

Mathematica [C] time = 14.27, size = 8456, normalized size = 11.88

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(5/2),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\,\sqrt{ex+d}}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^3*x^3 + 3
*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(5/2), x)
```

maple [B] time = 0.13, size = 21038, normalized size = 29.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}}{(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(C\,x^2 + B\,x + A\right)\,\sqrt{c\,x^2 + b\,x + a}}{(d + e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(5/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + Bx + Cx^2\right)\sqrt{a + bx + cx^2}}{(d + ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(5/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(5/2), x)
```

3.263 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=992

$$-\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}-\frac{2\left(c^2\left(24Cd^2-e(4Bd+Ae)\right)d^3-ce\left(bd\left(41Cd^2-6Bed+Ae^2\right)-ae\left(37\right.\right.\right.}{}$$

```
[Out] -2/5*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(5/2)-2/15*(c^2*d^3*(24*C*d^2-e*(A*e+4*B*d))+e^2*(15*b^2*C*d^3+5*a^2*e^2*(
B*e+C*d)-a*b*e*(2*A*e^2+3*B*d*e+22*C*d^2))-c*d*e*(b*d*(A*e^2-6*B*d*e+41*C*d
^2)-a*e*(7*A*e^2-7*B*d*e+37*C*d^2))+e*(5*c^2*d^2*(6*C*d^2-e*(A*e+B*d))+e^2*
(15*a^2*C*e^2-5*a*b*e*(-B*e+8*C*d)+b^2*(-2*A*e^2-3*B*d*e+23*C*d^2))-c*e*(5*
b*d*(-A*e^2-2*B*d*e+11*C*d^2)-a*e*(3*A*e^2-13*B*d*e+53*C*d^2)))*x)*(c*x^2+b
*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(3/2)+1/15*(2*c^2*d^2*(24*C*d
^2-e*(A*e+4*B*d))+e^2*(30*a^2*C*e^2-5*a*b*e*(-B*e+14*C*d)+b^2*(-2*A*e^2-3*B
*d*e+38*C*d^2))-c*e*(b*d*(-2*A*e^2-13*B*d*e+88*C*d^2)-2*a*e*(3*A*e^2-8*B*d*
e+43*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))
^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+
b^2))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d
-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/15*(15*b*C*e^2*(-a*e+b*d)+2*c^2*d*(24*C
*d^2-e*(A*e+4*B*d))+c*e*(10*a*e*(-B*e+5*C*d)-b*(-A*e^2-9*B*d*e+64*C*d^2)))*
EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^4/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(1/2)/
(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 1.92, antiderivative size = 989, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1650, 810, 843, 718, 424, 419}

$$-\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}-\frac{2\left(\left(24Cd^5-d^3e(4Bd+Ae)\right)c^2-de\left(bd\left(41Cd^2-6Bed+Ae^2\right)-ae\left(37\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(c^2*(24*C*d^5 - d^3*e*(4*B*d + A*e)) + e^2*(15*b^2*C*d^3 + 5*a^2*e^2*(
C*d + B*e) - a*b*e*(22*C*d^2 + 3*B*d*e + 2*A*e^2)) - c*d*e*(b*d*(41*C*d^2 -
 6*B*d*e + A*e^2) - a*e*(37*C*d^2 - 7*B*d*e + 7*A*e^2)) + e^2*((30*c^2*C*d^
4)/e + 15*a^2*C*e^3 - 5*c^2*d^2*(B*d + A*e) - 5*a*b*e^2*(8*C*d - B*e) + a*c
*e*(53*C*d^2 - e*(13*B*d - 3*A*e)) - 5*b*c*d*(11*C*d^2 - e*(2*B*d + A*e)) +
 b^2*e*(23*C*d^2 - e*(3*B*d + 2*A*e)))*x)*Sqrt[a + b*x + c*x^2])/(15*e^3*(c
*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(3/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a +
b*x + c*x^2)^(3/2))/(5*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(5/2)) + (Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(c^2*(48*C*d^4 - 2*d^2*e*(4*B*d + A*e)) + e^2*(30*a^2*C
*e^2 - 5*a*b*e*(14*C*d - B*e) + b^2*(38*C*d^2 - 3*B*d*e - 2*A*e^2)) - c*e*(
```

```
b*d*(88*C*d^2 - 13*B*d*e - 2*A*e^2) - 2*a*e*(43*C*d^2 - 8*B*d*e + 3*A*e^2))
)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*e^4*(c*d^2 - b
*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqr
t[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(15*b*C*e^2*(b*d - a*e)
+ c^2*(48*C*d^3 - 2*d*e*(4*B*d + A*e)) - c*e*(64*b*C*d^2 - b*e*(9*B*d + A*e
) - 10*a*e*(5*C*d - B*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c*e^4*(c*d^2 - b*d*e +
 a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{7/2}}dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(a+bx+cx^2\right)^{3/2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} - \frac{2\int \frac{\left(-\frac{3bCd^2-be(3Bd+2Ae)+5e(Acd-aCd+}{2e}\right.}{\cdots}}{5\left(\cdots\right.}$$

$$= -\frac{2\left(c^2\left(24Cd^5-d^3e(4Bd+Ae)\right)+e^2\left(15b^2Cd^3+5a^2e^2(Cd+Be)-abe\right.\right.}{\cdots}$$

$$= -\frac{2\left(c^2\left(24Cd^5-d^3e(4Bd+Ae)\right)+e^2\left(15b^2Cd^3+5a^2e^2(Cd+Be)-abe\right.\right.}{\cdots}$$

$$= -\frac{2\left(c^2\left(24Cd^5-d^3e(4Bd+Ae)\right)+e^2\left(15b^2Cd^3+5a^2e^2(Cd+Be)-abe\right.\right.}{\cdots}$$

$$= -\frac{2\left(c^2\left(24Cd^5-d^3e(4Bd+Ae)\right)+e^2\left(15b^2Cd^3+5a^2e^2(Cd+Be)-abe\right.\right.}{\cdots}$$

Mathematica [C] time = 14.67, size = 12997, normalized size = 13.10

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(7/2),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\,\sqrt{ex+d}}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^4*x^4 + 4
*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(d*exp(1)+x*exp(1
)^2)]integrate()  Bad Argument Typeintegrate()  Bad Argument TypeEvaluation
 time: 23.8Unable to transpose Error: Bad Argument Value
```

maple [B] time = 0.25, size = 48427, normalized size = 48.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}}{(ex + d)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(C\,x^2 + B\,x + A\right)\,\sqrt{c\,x^2 + b\,x + a}}{{\left(d + e\,x\right)}^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(7/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + Bx + Cx^2\right)\sqrt{a + bx + cx^2}}{(d + ex)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(7/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(7/2), x)
```

3.264 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=1363

$$-\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{7e\left(cd^2-bed+ae^2\right)(d+ex)^{7/2}}-\frac{2\left(c^2\left(24Cd^2+e(4Bd+3Ae)\right)d^3-ce\left(bd\left(43Cd^2+6Bed+15Ae^2\right)-a\right.\right.}{}$$

```
[Out] -2/7*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(7/2)-2/105*(c^2*d^3*(24*C*d^2+e*(3*A*e+4*B*d))-e^2*(7*a^2*e^2*(-3*B*e+C*d
)-b^2*d*(8*A*e^2+6*B*d*e+15*C*d^2)+a*b*e*(12*A*e^2+23*B*d*e+12*C*d^2))-c*d*
e*(b*d*(15*A*e^2+6*B*d*e+43*C*d^2)-a*e*(19*A*e^2+9*B*d*e+33*C*d^2))+e*(7*c^
2*d^2*(6*C*d^2+e*(-3*A*e+B*d))+e^2*(35*a^2*C*e^2-7*a*b*e*(-B*e+12*C*d)+b^2*
(-4*A*e^2-3*B*d*e+45*C*d^2))+c*e*(a*e*(-5*A*e^2-9*B*d*e+93*C*d^2)-b*(-21*A*
d*e^2+91*C*d^3)))*x)*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^
(5/2)+2/105*(2*c^3*d^3*(24*C*d^2+e*(3*A*e+4*B*d))-b*e^3*(35*a^2*C*e^2-14*a*
b*e*(B*e+3*C*d)+b^2*(8*A*e^2+6*B*d*e+15*C*d^2))+c^2*d*e*(2*a*e*(69*C*d^2+e*
(-29*A*e+15*B*d))-b*d*(128*C*d^2+e*(9*A*e+19*B*d)))+c*e^2*(14*a^2*e^2*(-3*B
*e+11*C*d)-a*b*e*(237*C*d^2+e*(-29*A*e+B*d))+b^2*d*(103*C*d^2+e*(19*A*e+9*B
*d))))*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^(1/2)-1/105*(2
*c^3*d^3*(24*C*d^2+e*(3*A*e+4*B*d))-b*e^3*(35*a^2*C*e^2-14*a*b*e*(B*e+3*C*d
)+b^2*(8*A*e^2+6*B*d*e+15*C*d^2))+c^2*d*e*(2*a*e*(69*C*d^2+e*(-29*A*e+15*B*
d))-b*d*(128*C*d^2+e*(9*A*e+19*B*d)))+c*e^2*(14*a^2*e^2*(-3*B*e+11*C*d)-a*b
*e*(237*C*d^2+e*(-29*A*e+B*d))+b^2*d*(103*C*d^2+e*(19*A*e+9*B*d))))*Ellipti
cE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*
e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a
*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^4/(a*e^
2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)+2/105*(2*c^2*d^2*(24*C*d^2+e*(3*A*e+4*B*d))+c*e*(2*a*e*(51*C*d
^2+e*(-5*A*e+12*B*d))-b*d*(104*C*d^2+3*e*(2*A*e+5*B*d)))+e^2*(70*a^2*C*e^2-
7*a*b*e*(B*e+18*C*d)+b^2*(60*C*d^2+e*(4*A*e+3*B*d))))*EllipticF(1/2*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)
^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*
(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(
1/2))))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 4.20, antiderivative size = 1363, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {1650, 810, 834, 843, 718, 424, 419}

$$-\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{7e\left(cd^2-bed+ae^2\right)(d+ex)^{7/2}}-\frac{2\left(\left(24Cd^5+e(4Bd+3Ae)d^3\right)c^2-de\left(bd\left(43Cd^2+6Bed+15Ae^2\right)-a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(9/2),x]
```

```
[Out] (2*(c^3*(48*C*d^5 + 2*d^3*e*(4*B*d + 3*A*e)) - b*e^3*(35*a^2*C*e^2 - 14*a*b
*e*(3*C*d + B*e) + b^2*(15*C*d^2 + 6*B*d*e + 8*A*e^2)) + c^2*d*e*(2*a*e*(69
*C*d^2 + e*(15*B*d - 29*A*e)) - b*(128*C*d^3 + d*e*(19*B*d + 9*A*e))) + c*e
```

```
^2*(14*a^2*e^2*(11*C*d - 3*B*e) - a*b*e*(237*C*d^2 + e*(B*d - 29*A*e)) + b^
2*(103*C*d^3 + d*e*(9*B*d + 19*A*e))))*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d
^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x]) - (2*(c^2*(24*C*d^5 + d^3*e*(4*B*d + 3
*A*e)) - e^2*(7*a^2*e^2*(C*d - 3*B*e) - b^2*d*(15*C*d^2 + 6*B*d*e + 8*A*e^2
) + a*b*e*(12*C*d^2 + 23*B*d*e + 12*A*e^2)) - c*d*e*(b*d*(43*C*d^2 + 6*B*d*
e + 15*A*e^2) - a*e*(33*C*d^2 + 9*B*d*e + 19*A*e^2)) + e*(7*c^2*(6*C*d^4 +
d^2*e*(B*d - 3*A*e)) + e^2*(35*a^2*C*e^2 - 7*a*b*e*(12*C*d - B*e) + b^2*(45
*C*d^2 - 3*B*d*e - 4*A*e^2)) + c*e*(a*e*(93*C*d^2 - 9*B*d*e - 5*A*e^2) - b*
(91*C*d^3 - 21*A*d*e^2)))*x)*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d^2 - b*d*e
 + a*e^2)^2*(d + e*x)^(5/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)
^(3/2))/(7*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(7/2)) - (Sqrt[2]*Sqrt[b^2 -
 4*a*c]*(2*c^3*(24*C*d^5 + d^3*e*(4*B*d + 3*A*e)) - b*e^3*(35*a^2*C*e^2 - 1
4*a*b*e*(3*C*d + B*e) + b^2*(15*C*d^2 + 6*B*d*e + 8*A*e^2)) + c^2*d*e*(2*a*
e*(69*C*d^2 + e*(15*B*d - 29*A*e)) - b*(128*C*d^3 + d*e*(19*B*d + 9*A*e)))
+ c*e^2*(14*a^2*e^2*(11*C*d - 3*B*e) - a*b*e*(237*C*d^2 + e*(B*d - 29*A*e))
 + b^2*(103*C*d^3 + d*e*(9*B*d + 19*A*e))))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)])/(105*e^4*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[(c*(d + e*x
))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]
*Sqrt[b^2 - 4*a*c]*(c^2*(48*C*d^4 + 2*d^2*e*(4*B*d + 3*A*e)) + c*e*(2*a*e*(
51*C*d^2 + e*(12*B*d - 5*A*e)) - b*(104*C*d^3 + 3*d*e*(5*B*d + 2*A*e))) + e
^2*(70*a^2*C*e^2 - 7*a*b*e*(18*C*d + B*e) + b^2*(60*C*d^2 + e*(3*B*d + 4*A*
e))))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(105*e^4*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x
]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
```

```
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{9/2}}dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(a+bx+cx^2\right)^{3/2}}{7e\left(cd^2-bde+ae^2\right)(d+ex)^{7/2}} - \frac{2\int \frac{\left(-\frac{3bCd^2-be(3Bd+4Ae)+7e(Acd-}{2e}\right.}{}}{}$$

$$= -\frac{2\left(c^2\left(24Cd^5+d^3e(4Bd+3Ae)\right)-e^2\left(7a^2e^2(Cd-3Be)-b^2d\left(15Cd\right.\right.\right.}{}$$

$$= \frac{2\left(c^3\left(48Cd^5+2d^3e(4Bd+3Ae)\right)-be^3\left(35a^2Ce^2-14abe(3Cd+Be)\right.\right.}{}$$

$$= \frac{2\left(c^3\left(48Cd^5+2d^3e(4Bd+3Ae)\right)-be^3\left(35a^2Ce^2-14abe(3Cd+Be)\right.\right.}{}$$

$$= \frac{2\left(c^3\left(48Cd^5+2d^3e(4Bd+3Ae)\right)-be^3\left(35a^2Ce^2-14abe(3Cd+Be)\right.\right.}{}$$

$$= \frac{2\left(c^3\left(48Cd^5+2d^3e(4Bd+3Ae)\right)-be^3\left(35a^2Ce^2-14abe(3Cd+Be)\right.\right.}{}$$

Mathematica [C] time = 16.02, size = 19853, normalized size = 14.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(9/2),x]
[Out] Result too large to show
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\,\sqrt{ex+d}}{e^5x^5+5\,de^4x^4+10\,d^2e^3x^3+10\,d^3e^2x^2+5\,d^4ex+d^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="fri
cas")
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^5*x^5 + 5
*d*e^4*x^4 + 10*d^2*e^3*x^3 + 10*d^3*e^2*x^2 + 5*d^4*e*x + d^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(9/2), x)
```

maple [B] time = 0.43, size = 88790, normalized size = 65.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Cx^2 + Bx + A)\sqrt{cx^2 + bx + a}}{(ex + d)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(C\,x^2 + B\,x + A\right)\,\sqrt{c\,x^2 + b\,x + a}}{(d + e\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(9/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(9/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx + Cx^2)\sqrt{a + bx + cx^2}}{(d + ex)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(9/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(9/2), x)
```

3.265 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{11/2}}\,dx$

Optimal. Leaf size=1904

$$-\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{9e\left(cd^2-bed+ae^2\right)(d+ex)^{9/2}}-\frac{2\left(c^2\left(8Cd^2+e(4Bd+5Ae)\right)d^3-ce\left(3bd\left(5Cd^2+2Bed+5Ae^2\right)-a\right.\right.}{}$$

```
[Out] -2/9*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(9/2)+2/315*(2*c^3*d^3*(8*C*d^2+e*(5*A*e+4*B*d))+3*c^2*d*e*(2*a*e*(-9*A*e^
2+7*B*d*e+9*C*d^2)-b*d*(5*A*e^2+7*B*d*e+16*C*d^2))+3*c*e^2*(2*a^2*e^2*(-5*B
*e+17*C*d)-a*b*e*(-9*A*e^2+5*B*d*e+41*C*d^2)+b^2*d*(7*A*e^2+3*B*d*e+15*C*d^
2))-b*e^3*(21*a^2*C*e^2-6*a*b*e*(2*B*e+3*C*d)+b^2*(8*A*e^2+4*B*d*e+5*C*d^2)
))*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^(3/2)-2/105*(c^2*d
^3*(8*C*d^2+e*(5*A*e+4*B*d))-e^2*(3*a^2*e^2*(-5*B*e+3*C*d)-a*b*e*(-10*A*e^2
-17*B*d*e+2*C*d^2)-b^2*d*(8*A*e^2+4*B*d*e+5*C*d^2))-c*d*e*(3*b*d*(5*A*e^2+2
*B*d*e+5*C*d^2)-a*e*(13*A*e^2+11*B*d*e+7*C*d^2))+e*(3*c^2*d^2*(6*C*d^2+e*(-
5*A*e+3*B*d))+c*e*(a*e*(-7*A*e^2+B*d*e+47*C*d^2)-3*b*d*(-5*A*e^2+2*B*d*e+15
*C*d^2))+e^2*(21*a^2*C*e^2-3*a*b*e*(-B*e+16*C*d)+b^2*(25*C*d^2-e*(2*A*e+B*d
))))*x)*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(7/2)+2/315*(
2*c^4*d^4*(8*C*d^2+e*(5*A*e+4*B*d))+2*b^2*e^4*(21*a^2*C*e^2-6*a*b*e*(2*B*e+
3*C*d)+b^2*(8*A*e^2+4*B*d*e+5*C*d^2))-6*c^2*e^2*(a*b*d*e*(-34*A*e^2-5*B*d*e
+30*C*d^2)-a^2*e^2*(7*A*e^2-36*B*d*e+30*C*d^2)-b^2*d^2*(11*A*e^2+3*B*d*e+11
*C*d^2))-c*e^3*(126*a^3*C*e^3-3*a^2*b*e^2*(29*B*e+12*C*d)-6*a*b^2*e*(-12*A*
e^2+7*B*d*e+5*C*d^2)+b^3*d*(56*A*e^2+25*B*d*e+20*C*d^2))+c^3*d^2*e*(6*a*e*(
-34*A*e^2+8*B*d*e+11*C*d^2)-b*d*(56*C*d^2+5*e*(4*A*e+5*B*d))))*(c*x^2+b*x+a
)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^4/(e*x+d)^(1/2)-1/315*(2*c^4*d^4*(8*C*d^2+e
*(5*A*e+4*B*d))+2*b^2*e^4*(21*a^2*C*e^2-6*a*b*e*(2*B*e+3*C*d)+b^2*(8*A*e^2+
4*B*d*e+5*C*d^2))-6*c^2*e^2*(a*b*d*e*(-34*A*e^2-5*B*d*e+30*C*d^2)-a^2*e^2*(
7*A*e^2-36*B*d*e+30*C*d^2)-b^2*d^2*(11*A*e^2+3*B*d*e+11*C*d^2))-c*e^3*(126*
a^3*C*e^3-3*a^2*b*e^2*(29*B*e+12*C*d)-6*a*b^2*e*(-12*A*e^2+7*B*d*e+5*C*d^2)
+b^3*d*(56*A*e^2+25*B*d*e+20*C*d^2))+c^3*d^2*e*(6*a*e*(-34*A*e^2+8*B*d*e+11
*C*d^2)-b*d*(56*C*d^2+5*e*(4*A*e+5*B*d))))*EllipticE(1/2*((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c
*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/
2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b
*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/315*(2*c^3
*d^3*(8*C*d^2+e*(5*A*e+4*B*d))+3*c^2*d*e*(2*a*e*(-9*A*e^2+7*B*d*e+9*C*d^2)-
b*d*(5*A*e^2+7*B*d*e+16*C*d^2))+3*c*e^2*(2*a^2*e^2*(-5*B*e+17*C*d)-a*b*e*(-
9*A*e^2+5*B*d*e+41*C*d^2)+b^2*d*(7*A*e^2+3*B*d*e+15*C*d^2))-b*e^3*(21*a^2*C
*e^2-6*a*b*e*(2*B*e+3*C*d)+b^2*(8*A*e^2+4*B*d*e+5*C*d^2)))*EllipticF(1/2*((
b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c
+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(
1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b
^2)^(1/2))))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1
/2)
```

Rubi [A] time = 6.24, antiderivative size = 1904, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {1650, 810, 834, 843, 718, 424, 419}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(11/2),x]
```

```
[Out] (2*(2*c^3*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) + 3*c^2*d*e*(2*a*e*(9*C*d^2 + 7
*B*d*e - 9*A*e^2) - b*d*(16*C*d^2 + 7*B*d*e + 5*A*e^2)) + 3*c*e^2*(2*a^2*e^
2*(17*C*d - 5*B*e) - a*b*e*(41*C*d^2 + 5*B*d*e - 9*A*e^2) + b^2*d*(15*C*d^2
 + 3*B*d*e + 7*A*e^2)) - b*e^3*(21*a^2*C*e^2 - 6*a*b*e*(3*C*d + 2*B*e) + b^
2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)))*Sqrt[a + b*x + c*x^2])/(315*e^3*(c*d^2 -
b*d*e + a*e^2)^3*(d + e*x)^(3/2)) + (2*(2*c^4*(8*C*d^6 + d^4*e*(4*B*d + 5*A
*e)) - c^3*d^2*e*(56*b*C*d^3 + 5*b*d*e*(5*B*d + 4*A*e) - 6*a*e*(11*C*d^2 +
8*B*d*e - 34*A*e^2)) + 2*b^2*e^4*(21*a^2*C*e^2 - 6*a*b*e*(3*C*d + 2*B*e) +
b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)) - 6*c^2*e^2*(a*b*d*e*(30*C*d^2 - 5*B*d*e
 - 34*A*e^2) - a^2*e^2*(30*C*d^2 - 36*B*d*e + 7*A*e^2) - b^2*d^2*(11*C*d^2
+ 3*B*d*e + 11*A*e^2)) - c*e^3*(126*a^3*C*e^3 - 3*a^2*b*e^2*(12*C*d + 29*B*
e) - 6*a*b^2*e*(5*C*d^2 + 7*B*d*e - 12*A*e^2) + b^3*d*(20*C*d^2 + 25*B*d*e
+ 56*A*e^2)))*Sqrt[a + b*x + c*x^2])/(315*e^3*(c*d^2 - b*d*e + a*e^2)^4*Sqr
t[d + e*x]) - (2*(c^2*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) - e^2*(3*a^2*e^2*(3
*C*d - 5*B*e) - a*b*e*(2*C*d^2 - 17*B*d*e - 10*A*e^2) - b^2*d*(5*C*d^2 + 4*
B*d*e + 8*A*e^2)) - c*d*e*(3*b*d*(5*C*d^2 + 2*B*d*e + 5*A*e^2) - a*e*(7*C*d
^2 + 11*B*d*e + 13*A*e^2)) + e^2*((3*c^2*(6*C*d^4 + d^2*e*(3*B*d - 5*A*e)))
/e + c*(a*e*(47*C*d^2 + e*(B*d - 7*A*e)) - 3*b*(15*C*d^3 + d*e*(2*B*d - 5*A
*e))) + e*(21*a^2*C*e^2 - 3*a*b*e*(16*C*d - B*e) + b^2*(25*C*d^2 - e*(B*d +
 2*A*e))))*x)*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d^2 - b*d*e + a*e^2)^2*(d
+ e*x)^(7/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)^(3/2))/(9*e*(c
*d^2 - b*d*e + a*e^2)*(d + e*x)^(9/2)) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c^4*
(8*C*d^6 + d^4*e*(4*B*d + 5*A*e)) - c^3*d^2*e*(56*b*C*d^3 + 5*b*d*e*(5*B*d
+ 4*A*e) - 6*a*e*(11*C*d^2 + 8*B*d*e - 34*A*e^2)) + 2*b^2*e^4*(21*a^2*C*e^2
 - 6*a*b*e*(3*C*d + 2*B*e) + b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)) - 6*c^2*e^2
*(a*b*d*e*(30*C*d^2 - 5*B*d*e - 34*A*e^2) - a^2*e^2*(30*C*d^2 - 36*B*d*e +
7*A*e^2) - b^2*d^2*(11*C*d^2 + 3*B*d*e + 11*A*e^2)) - c*e^3*(126*a^3*C*e^3
- 3*a^2*b*e^2*(12*C*d + 29*B*e) - 6*a*b^2*e*(5*C*d^2 + 7*B*d*e - 12*A*e^2)
+ b^3*d*(20*C*d^2 + 25*B*d*e + 56*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)])/(315*e^4*(c*d^2 - b*d*e + a*e^2)^4*Sqrt[(c*(d + e*x)
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*
Sqrt[b^2 - 4*a*c]*(2*c^3*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) + 3*c^2*d*e*(2*a
*e*(9*C*d^2 + 7*B*d*e - 9*A*e^2) - b*d*(16*C*d^2 + 7*B*d*e + 5*A*e^2)) + 3*
c*e^2*(2*a^2*e^2*(17*C*d - 5*B*e) - a*b*e*(41*C*d^2 + 5*B*d*e - 9*A*e^2) +
b^2*d*(15*C*d^2 + 3*B*d*e + 7*A*e^2)) - b*e^3*(21*a^2*C*e^2 - 6*a*b*e*(3*C*
d + 2*B*e) + b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)))*Sqrt[(c*(d + e*x))/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*
EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sq
rt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315
*e^4*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
```

```
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{11/2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(a+bx+cx^2\right)^{3/2}}{9e\left(cd^2-bde+ae^2\right)(d+ex)^{9/2}} - \frac{2\int \frac{\left(-\frac{3\left(bCd^2-be(Bd+2Ae)+3e(Acd-aCd+\cdots\right.}{2e}\cdots\right.}{\cdots}}{9\cdots}$$

$$= -\frac{2\left(c^2\left(8Cd^5+d^3e(4Bd+5Ae)\right)-e^2\left(3a^2e^2(3Cd-5Be)-abe\left(2Cd^2-1\cdots\right.\right.\right.}{\cdots}$$

$$= \frac{2\left(2c^3\left(8Cd^5+d^3e(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-b\cdots\right.\right.}{\cdots}$$

$$= \frac{2\left(2c^3\left(8Cd^5+d^3e(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-b\cdots\right.\right.}{\cdots}$$

$$= \frac{2\left(2c^3\left(8Cd^5+d^3e(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-b\cdots\right.\right.}{\cdots}$$

$$= \frac{2\left(2c^3\left(8Cd^5+d^3e(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-b\cdots\right.\right.}{\cdots}$$

$$= \frac{2\left(2c^3\left(8Cd^5+d^3e(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-b\cdots\right.\right.}{\cdots}$$

Mathematica [C] time = 18.39, size = 29140, normalized size = 15.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(11/2),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\,\sqrt{ex+d}}{e^6x^6+6\,de^5x^5+15\,d^2e^4x^4+20\,d^3e^3x^3+15\,d^4e^2x^2+6\,d^5ex+d^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^6*x^6 + 6
*d*e^5*x^5 + 15*d^2*e^4*x^4 + 20*d^3*e^3*x^3 + 15*d^4*e^2*x^2 + 6*d^5*e*x +
 d^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(11/2), x)
```

maple [B] time = 0.80, size = 153623, normalized size = 80.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(C\,x^2+B\,x+A\right)\,\sqrt{c\,x^2+b\,x+a}}{{\left(d+e\,x\right)}^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(11/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(11/2),x)
```

```
[Out] Timed out
```

3.266 $\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=724

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(-ce(25aCe+28bBe+15bCd)-\left(c^2\left(6Cd^2-\right.\right.\right.}{105c^4e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] -2/35*(-7*B*c*e+6*C*b*e+2*C*c*d)*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/e+2/
7*C*(e*x+d)^(5/2)*(c*x^2+b*x+a)^(1/2)/c/e+2/105*(24*b^2*C*e^2-c*e*(28*B*b*e
+25*C*a*e+15*C*b*d)-c^2*(6*C*d^2-7*e*(5*A*e+3*B*d)))*(e*x+d)^(1/2)*(c*x^2+b
*x+a)^(1/2)/c^3/e-1/105*(48*b^3*C*e^3-8*b*c*e^2*(7*B*b*e+13*C*a*e+9*C*b*d)+
c^3*d*(6*C*d^2-7*e*(20*A*e+3*B*d))+c^2*e*(a*e*(63*B*e+82*C*d)+b*(70*A*e^2+9
1*B*d*e+12*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-
4*a*c+b^2))^(1/2)/c^4/e^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2))))^(1/2)-2/105*(a*e^2-b*d*e+c*d^2)*(24*b^2*C*e^2-c*e*(28*B*b*e
+25*C*a*e+15*C*b*d)-c^2*(6*C*d^2-7*e*(5*A*e+3*B*d)))*EllipticF(1/2*((b+2*c*
x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^
(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(
-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)/c^4/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 1.78, antiderivative size = 724, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1653, 832, 843, 718, 424, 419}

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(-ce(25aCe+28bBe+15bCd)+c^2\left(-\left(6Cd^2-7e(5Ae+3Bd)\right)\right)+24b^2Ce^2\right)}{105c^3e}-\frac{2\sqrt{2}\sqrt{b^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*(24*b^2*C*e^2 - c*e*(15*b*C*d + 28*b*B*e + 25*a*C*e) - c^2*(6*C*d^2 - 7*
e*(3*B*d + 5*A*e)))*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(105*c^3*e) - (2*(
2*c*C*d - 7*B*c*e + 6*b*C*e)*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(35*c^2
*e) + (2*C*(d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c*e) - (Sqrt[2]*Sqrt[b
^2 - 4*a*c]*(48*b^3*C*e^3 - 8*b*c*e^2*(9*b*C*d + 7*b*B*e + 13*a*C*e) + c^3*
(6*C*d^3 - 7*d*e*(3*B*d + 20*A*e)) + c^2*e*(a*e*(82*C*d + 63*B*e) + b*(12*C
*d^2 + 91*B*d*e + 70*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)])/(105*c^4*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
a*e^2)*(24*b^2*C*e^2 - c*e*(15*b*C*d + 28*b*B*e + 25*a*C*e) - c^2*(6*C*d^2
- 7*e*(3*B*d + 5*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*
```

```
a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^4*e^2*Sqrt[d + e*x]*Sq
rt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce} + \frac{2\int \frac{(d+ex)^{3/2}\left(-\frac{1}{2}e(bCd-7Ace+5aCe)-\frac{1}{2}e(2cCd-7Bce+6bC\ldots}{\sqrt{a+bx+cx^2}}}{7ce^2}$$

$$= -\frac{2(2cCd-7Bce+6bCe)(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{35c^2e} + \frac{2C(d+ex)^{5/2}\sqrt{a+bx+\ldots}}{7ce}$$

$$= \frac{2\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2\left(6Cd^2-7e(3Bd+5Ae)\right)\right)\sqrt{d\ldots}}{105c^3e}$$

$$= \frac{2\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2\left(6Cd^2-7e(3Bd+5Ae)\right)\right)\sqrt{d\ldots}}{105c^3e}$$

$$= \frac{2\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2\left(6Cd^2-7e(3Bd+5Ae)\right)\right)\sqrt{d\ldots}}{105c^3e}$$

$$= \frac{2\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2\left(6Cd^2-7e(3Bd+5Ae)\right)\right)\sqrt{d\ldots}}{105c^3e}$$

Mathematica [C] time = 14.55, size = 9972, normalized size = 13.77

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cex^3+(Cd+Be)x^2+Ad+(Bd+Ae)x\right)\sqrt{ex+d}}{\sqrt{cx^2+bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*e*x^3 + (C*d + B*e)*x^2 + A*d + (B*d + A*e)*x)*sqrt(e*x + d)/sq
rt(c*x^2 + b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2+Bx+A\right)(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*(e*x + d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

maple [B] time = 0.08, size = 14084, normalized size = 19.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)(ex + d)^{\frac{3}{2}}}{\sqrt{cx^2 + bx + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(e*x + d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d+e\,x\right)}^{3/2}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{c\,x^2+b\,x+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(3/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int(((d + e*x)^(3/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^{\frac{3}{2}} \left(A + Bx + Cx^{2}\right)}{\sqrt{a + bx + cx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(C*x**2+B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(A + B*x + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

3.267 $\int \frac{\sqrt{d+ex}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=557

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce(9aCe+10bBe+3bCd)-\left(c^2\left(2Cd^2-5e(3Ae+Bd)\right)\right)+8b^2Ce^2\right)E\left(\text{si}\right.}{15c^3e^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[Out] 2/5*C*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2)/c/e-2/15*(-5*B*c*e+4*C*b*e+2*C*c*d)
*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/e+1/15*(8*b^2*C*e^2-c*e*(10*B*b*e+9*
C*a*e+3*C*b*d)-c^2*(2*C*d^2-5*e*(3*A*e+B*d)))*EllipticE(1/2*((b+2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(
2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^
(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/e^2/(c*x^2+b*x+a)^(1/2)/(c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/15*(-5*B*c*e+4*C*b*e+2*C*
c*d)*(a*e^2-b*d*e+c*d^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^
2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2
))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^3/e^2/(e*x+d)
^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1653, 832, 843, 718, 424, 419}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce(9aCe+10bBe+3bCd)+c^2\left(-\left(2Cd^2-5e(3Ae+Bd)\right)\right)+8b^2Ce^2\right)E\left(\right.}{15c^3e^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-2*(2*c*C*d - 5*B*c*e + 4*b*C*e)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(15*
c^2*e) + (2*C*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c*e) + (Sqrt[2]*Sqr
t[b^2 - 4*a*c]*(8*b^2*C*e^2 - c*e*(3*b*C*d + 10*b*B*e + 9*a*C*e) - c^2*(2*C
*d^2 - 5*e*(B*d + 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)])/(15*c^3*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d - 5*B*c*e +
4*b*C*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c^3*e^2*Sqrt[d +
 e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
```

```
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}dx = \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce} + \frac{2\int \frac{\sqrt{d+ex}\left(-\frac{1}{2}e(bCd-5Ace+3aCe)-\frac{1}{2}e(2cCd-5Bce+4bCe)x\right)}{\sqrt{a+bx+cx^2}}}{5ce^2}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx}}{5ce}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx}}{5ce}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx}}{5ce}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx}}{5ce}$$

Mathematica [C] time = 11.59, size = 992, normalized size = 1.78

$$\frac{\left(\frac{2(cCd+5Bce-4bCe)}{15c^2e}+\frac{2Cx}{5c}\right)\sqrt{d+ex}\left(cx^2+bx+a\right)}{\sqrt{a+x(b+cx)}} - \frac{2(d+ex)^{3/2}\sqrt{cx^2+bx+a}\left(\left(\left(2Cd^2-5e(Bd+3Ae)\right)c^2+e(3bC\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (((2*(c*C*d + 5*B*c*e - 4*b*C*e))/(15*c^2*e) + (2*C*x)/(5*c))*Sqrt[d + e*x]
*(a + b*x + c*x^2))/Sqrt[a + x*(b + c*x)] - (2*(d + e*x)^(3/2)*Sqrt[a + b*x
 + c*x^2]*((-8*b^2*C*e^2 + c*e*(3*b*C*d + 10*b*B*e + 9*a*C*e) + c^2*(2*C*d^
2 - 5*e*(B*d + 3*A*e)))*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) +
 (a*e)/(d + e*x)))/(d + e*x)) + ((I/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e
)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2
 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]
*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(8*b^2*C*e^2 - c*e*(3*b*C*d + 10*
b*B*e + 9*a*C*e) + c^2*(-2*C*d^2 + 5*e*(B*d + 3*A*e)))*EllipticE[I*ArcSinh[
(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^
2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b
*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (8*b^3*C*e^3 - b^2*e^2*(11*c*C*d + 10*B*c
*e + 8*C*Sqrt[(b^2 - 4*a*c)*e^2]) + c*(c*d*Sqrt[(b^2 - 4*a*c)*e^2]*(2*C*d -
 5*B*e) - 15*A*c*e^2*(2*c*d + Sqrt[(b^2 - 4*a*c)*e^2]) + a*e^2*(14*c*C*d +
10*B*c*e + 9*C*Sqrt[(b^2 - 4*a*c)*e^2])) + b*c*e*(15*A*c*e^2 - 17*a*C*e^2 +
```

```
3*C*d*Sqrt[(b^2 - 4*a*c)*e^2] + 5*B*(3*c*d*e + 2*e*Sqrt[(b^2 - 4*a*c)*e^2]
)))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 -
 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*
d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[d +
e*x]))))/(15*c^3*e^3*Sqrt[a + x*(b + c*x)]*Sqrt[((d + e*x)^2*(c*(-1 + d/(d +
 e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{ex + d}}{\sqrt{cx^2 + bx + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{ex + d}}{\sqrt{cx^2 + bx + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

maple [B] time = 0.06, size = 8161, normalized size = 14.65

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{ex + d}}{\sqrt{cx^2 + bx + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d + e\,x}\,\left(C\,x^2 + B\,x + A\right)}{\sqrt{c\,x^2 + b\,x + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int(((d + e*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(C*x**2+B*x+A)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(sqrt(d + e*x)*(A + B*x + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

3.268 $\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=471

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(Ce(bd-ae)+c\left(2Cd^2-3e(Bd-Ae)\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-\ldots}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] 2/3*C*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e-1/3*(-3*B*c*e+2*C*b*e+2*C*c*d)*
EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^
2/e^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2
)+2/3*(C*e*(-a*e+b*d)+c*(2*C*d^2-3*e*(-A*e+B*d)))*EllipticF(1/2*((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/
2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*
(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)
)))^(1/2)/c^2/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1653, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(Ce(bd-ae)-3ce(Bd-Ae)+2cCd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4a\ldots}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*C*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) - (Sqrt[2]*Sqrt[b^2 - 4*a
*c]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)])/(3*c^2*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 + C*
e*(b*d - a*e) - 3*c*e*(B*d - A*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c^2*e^2*Sqrt[d
+ e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} + \frac{2\int \frac{-\frac{1}{2}e(bCd-3Ace+aCe)-\frac{1}{2}e(2cCd-3Bce+2bCe)x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{3ce^2}$$

$$= \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{(2cCd-3Bce+2bCe)\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx}{3ce^2} + \frac{\left(2cCd^2\right.}{}$$

$$= \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\,(2cCd-3Bce+2bCe)\sqrt{d+ex}\right.}{3c^2e^2\sqrt{\frac{}{2cd-}}}$$

$$= \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{\sqrt{2}\sqrt{b^2-4ac}\,(2cCd-3Bce+2bCe)\sqrt{d+ex}}{3c^2e^2\sqrt{\frac{c(}{2cd-(b+}}}$$

Mathematica [C] time = 12.50, size = 1080, normalized size = 2.29

$$\frac{\sqrt{cx^2+bx+a}\left(-4(2cCd-3Bce+2bCe)\sqrt{\frac{cd^2+e(ae-bd)}{-2cd+be+\sqrt{(b^2-4ac)e^2}}}\left(c\left(\frac{d}{d+ex}-1\right)^2+\frac{e\left(-\frac{db}{d+ex}+b+\frac{ae}{d+ex}\right)}{d+ex}\right)+\frac{i\sqrt{2}\,(2cCd-3Bce}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*C*Sqrt[d + e*x]*(a + b*x + c*x^2))/(3*c*e*Sqrt[a + x*(b + c*x)]) + ((d +
 e*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(-4*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[(c
*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c*(-1 +
 d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)) +
(I*Sqrt[2]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*
e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d/(
d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e
^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(-1 + d/(
d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b
*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] -
(I*Sqrt[2]*(-2*b^2*C*e^2 + b*e*(3*B*c*e + 2*C*Sqrt[(b^2 - 4*a*c)*e^2]) + c*
(-6*A*c*e^2 + 2*a*C*e^2 + Sqrt[(b^2 - 4*a*c)*e^2]*(2*C*d - 3*B*e)))*Sqrt[(S
qrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d/(d + e*x)) + b
*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(S
qrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(-1 + d/(d + e*x)) + b
```

```
*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Ellipti
cF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^
2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2
])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/(6*c^2*e^3*Sq
rt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqr
t[a + x*(b + c*x)]*Sqrt[((d + e*x)^2*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d
)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\,\sqrt{ex+d}}{cex^3+(cd+be)x^2+ad+(bd+ae)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e*x^3 + (
c*d + b*e)*x^2 + a*d + (b*d + a*e)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\,\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

maple [B] time = 0.05, size = 4251, normalized size = 9.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/3/c^2*(-2*C*x^3*c^2*e^3-2*C*x*b*c*d*e^2-2*C*x*a*c*e^3-2*C*x^2*b*c*e^3-2*
C*x^2*c^2*d*e^2+C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/
2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)
*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*El
lipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-
4*a*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*(-4*a*
c+b^2)^(1/2)*b*d*e^2-6*B*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*
d))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)
)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(
1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),
(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))
*b*c*d*e^2-4*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*
((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((
b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ellip
ticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a
*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*a*c*d*e^2
-3*B*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(
-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-
4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/
2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1
```

```
/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*(-4*a*c+b^2)^(1/2)*
c*d*e^2+3*B*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-
2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2
*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Elliptic
F(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*b*c*d*e^2+2*
C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*
a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-4*a
*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*
(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)
+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*(-4*a*c+b^2)^(1/2)*c*d
^2*e+6*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c
*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*
x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2
^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*a*c*d*e^2-2*C*a
*c*d*e^2-4*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((
-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+
2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ellipti
cE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c
+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*c^2*d^3+6*B
*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*a
*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-4*a*
c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(
-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+
b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*c^2*d^2*e-4*C*2^(1/2)*(
-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1
/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/
2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c
/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)
/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*a*b*e^3+4*C*2^(1/2)*(-(e*x+d)*c/(
e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*
(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4
*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+
b^2)^(1/2)-b*e+2*c*d))^(1/2))*b^2*d*e^2+3*A*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^
2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)
+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)
-b*e+2*c*d))^(1/2))*(-4*a*c+b^2)^(1/2)*c*e^3-6*A*2^(1/2)*(-(e*x+d)*c/(e*(-4
*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a
*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^
(1/2)-b*e+2*c*d))^(1/2))*c^2*d*e^2-6*B*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1
/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)
+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-
2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+
2*c*d))^(1/2))*a*c*e^3+6*B*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*
c*d))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*
d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))
^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2
),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2
))*a*c*e^3-C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((
-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+
2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ellipti
cF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c
```

```
+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*(-4*a*c+b^2
)^(1/2)*a*e^3+3*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/
2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)
*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*El
lipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-
4*a*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*a*b*e^
3-3*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+
(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(
-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1
/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(
1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*b^2*d*e^2+3*A*2^(1
/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*a*c+b^
2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2
)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x
+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2
*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*b*c*e^3)*(e*x+d)^(1/2)*(c*x^
2+b*x+a)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)/e^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^2 + Bx + A}{\sqrt{cx^2 + bx + a}\,\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\,x^2 + B\,x + A}{\sqrt{d + e\,x}\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{\sqrt{d + ex}\,\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.269 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=508

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\,\sqrt{d+ex}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\left(Ce(bd-ae)-c\left(2Cd^2-e(Bd-Ae)\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{\cdots}{2cd\cdots}\right.}{ce^2\sqrt{a+bx+cx^2}\,\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[Out] -2*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(
1/2)-(C*e*(-a*e+b*d)-c*(2*C*d^2-e*(-A*e+B*d)))*EllipticE(1/2*((b+2*c*x+(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)
^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/e^2/(a*e^2-b*d*e+c*d^2)/(c*x
^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2*(-B*e+
2*C*d)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))
*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d
)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)
^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1650, 843, 718, 424, 419}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\,\sqrt{d+ex}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\left(-Ce(bd-ae)-ce(Bd-Ae)+2cCd^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\cdots}{2cd-(\cdots}\right.}{ce^2\sqrt{a+bx+cx^2}\,\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(e*(c*d^2 - b*d*e + a*e^
2)*Sqrt[d + e*x]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 - C*e*(b*d - a*e)
 - c*e*(B*d - A*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c
))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/
(c*e^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*C*d -
B*e)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{2\int \frac{-\frac{bd(Cd-Be)+e(Acd-aCd+aBe)}{2e}+\frac{1}{2}\left(Bcd+bCd\right.}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}}{cd^2-bde+ae^2}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{e^2} -$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\left(Bcd+bCd-\frac{2cCd^2}{e}\right.\right.}{}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{\sqrt{2}\sqrt{b^2-4ac}\left(Bcd+bCd-\frac{2cCd^2}{e}\right.}{ce}$$

Mathematica [C] time = 7.28, size = 772, normalized size = 1.52

$$2\left(-\frac{i(d+ex)^{3/2}\sqrt{1-\frac{2\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2(b^2-4ac)}-be+2cd\right)}}\sqrt{\frac{2\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2(b^2-4ac)}+be-2cd\right)}+1}\left(\left(\sqrt{e^2(b^2-4ac)}-be+2cd\right)\left(Ce(ae-bd)+ce(Ae-Bd)+2cCd^2\right)E\left(i\sinh\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*(-(e^2*(C*d^2 + e*(-(B*d) + A*e))*(a + x*(b + c*x))) + (e^2*(2*c*C*d^2 +
 C*e*(-(b*d) + a*e) + c*e*(-(B*d) + A*e))*(a + x*(b + c*x)))/c - ((I/2)*(d
+ e*x)^(3/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(
b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2
*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2
 - 4*a*c)*e^2])*(2*c*C*d^2 + C*e*(-(b*d) + a*e) + c*e*(-(B*d) + A*e))*Ellip
ticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(
b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (-(b^2*C*d*e^2) + 2*a*c*C*
d*e^2 - 2*a*B*c*e^3 - 2*c*C*d^2*Sqrt[(b^2 - 4*a*c)*e^2] + B*c*d*e*Sqrt[(b^2
 - 4*a*c)*e^2] - a*C*e^2*Sqrt[(b^2 - 4*a*c)*e^2] - A*c*e^2*(2*c*d + Sqrt[(b
^2 - 4*a*c)*e^2]) + b*(B*c*d*e^2 + A*c*e^3 + a*C*e^3 + C*d*e*Sqrt[(b^2 - 4*
a*c)*e^2]))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqr
t[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*
c*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]
)))/(e^3*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^{2}+Bx+A\right)\sqrt{cx^{2}+bx+a}\,\sqrt{ex+d}}{ce^{2}x^{4}+\left(2\,cde+be^{2}\right)x^{3}+ad^{2}+\left(cd^{2}+2\,bde+ae^{2}\right)x^{2}+\left(bd^{2}+2\,ade\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e^2*x^4 +
 (2*c*d*e + b*e^2)*x^3 + a*d^2 + (c*d^2 + 2*b*d*e + a*e^2)*x^2 + (b*d^2 + 2
*a*d*e)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^{2}+Bx+A}{\sqrt{cx^{2}+bx+a}\,(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)), x)
```

maple [B] time = 0.07, size = 6053, normalized size = 11.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^{2}+Bx+A}{\sqrt{cx^{2}+bx+a}\,(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\,x^2+B\,x+A}{{\left(d+e\,x\right)}^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x + C*x^2)/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^{2}}{(d+ex)^{\frac{3}{2}}\,\sqrt{a+bx+cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((d + e*x)**(3/2)*sqrt(a + b*x + c*x**2)), x)
```

3.270 $\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=684

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(3Ce(bd-ae)-c\left(e(Bd-Ae)+2Cd^2\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

```
[Out] -2/3*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(3/2)+2/3*(c*d*(2*C*d^2+e*(-4*A*e+B*d))+e*(3*a*e*(-B*e+2*C*d)-b*(-2*A*e^2-
B*d*e+4*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(1/2)-
1/3*(c*d*(2*C*d^2+e*(-4*A*e+B*d))+e*(3*a*e*(-B*e+2*C*d)-b*(-2*A*e^2-B*d*e+4
*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/
2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2)
)^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3*(3*C*e*(-a*e+b*d)-c*(2*C*d^2+e*(-A*e+B*d)
))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(
1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2
*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(1/
2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 1.19, antiderivative size = 680, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1650, 834, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(-3Ce(bd-ae)+ce(Bd-Ae)+2cCd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(3*e*(c*d^2 - b*d*e + a*
e^2)*(d + e*x)^(3/2)) + (2*(2*c*C*d^3 + c*d*e*(B*d - 4*A*e) + 3*a*e^2*(2*C*
d - B*e) - b*e*(4*C*d^2 - e*(B*d + 2*A*e)))*Sqrt[a + b*x + c*x^2])/(3*e*(c*
d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d
^3 + c*d*e*(B*d - 4*A*e) + 3*a*e^2*(2*C*d - B*e) - b*e*(4*C*d^2 - e*(B*d +
2*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
 (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*e^2*(c*d
^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 - 3*C*e
*(b*d - a*e) + c*e*(B*d - A*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^2*(c*d^2 - b*d
*e + a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2\int \frac{-\frac{bCd^2-be(Bd+2Ae)+3e(Acd-aCd+aBe)}{2e}-\frac{1}{2}\left(Bcd-3\right.}{(d+ex)^{3/2}\sqrt{a+bx+cx}}}{3\left(cd^2-bde+ae\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{2\left(2cCd^3+cde(Bd-4Ae)+3ae^2(2C\right.}{3e\left(cd^2-\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{2\left(2cCd^3+cde(Bd-4Ae)+3ae^2(2C\right.}{3e\left(cd^2-\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{2\left(2cCd^3+cde(Bd-4Ae)+3ae^2(2C\right.}{3e\left(cd^2-\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{2\left(2cCd^3+cde(Bd-4Ae)+3ae^2(2C\right.}{3e\left(cd^2-\right.}$$

Mathematica [C] time = 12.16, size = 1194, normalized size = 1.75

$$2\sqrt{cx^2+bx+a}\left(\frac{i\sqrt{1-\frac{2\left(cd^2+e(ae-bd)\right)}{\left(2cd-be+\sqrt{\left(b^2-4ac\right)e^2}\right)(d+ex)}}\sqrt{\frac{2\left(cd^2+e(ae-bd)\right)}{\left(-2cd+be+\sqrt{\left(b^2-4ac\right)e^2}\right)(d+ex)}+1}\left(\left(2cd-be+\sqrt{\left(b^2-4ac\right)e^2}\right)\left(cd\left(2Cd^2+e(Bd-4Ae)\right)+e\left(-4bCd^2\right.\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(a + b*x + c*x^2)*((-2*(C*d^2 - B*d*e + A*e^2))/(3*e*(c*d^2
- b*d*e + a*e^2)*(d + e*x)^2) - (2*(-2*c*C*d^3 - B*c*d^2*e + 4*b*C*d^2*e -
b*B*d*e^2 + 4*A*c*d*e^2 - 6*a*C*d*e^2 - 2*A*b*e^3 + 3*a*B*e^3))/(3*e*(c*d^2
 - b*d*e + a*e^2)^2*(d + e*x))))/Sqrt[a + x*(b + c*x)] + (2*(d + e*x)^(3/2)
*Sqrt[a + b*x + c*x^2]*(-((2*c*C*d^3 + c*d*e*(B*d - 4*A*e) - 3*a*e^2*(-2*C*
d + B*e) + b*e*(-4*C*d^2 + e*(B*d + 2*A*e)))*(c*(-1 + d/(d + e*x))^2 + (e*(
b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x))) + ((I/2)*Sqrt[1 - (2*(c
*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x
))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*
a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(c*d*(2*C*d
^2 + e*(B*d - 4*A*e)) + e*(-4*b*C*d^2 + b*e*(B*d + 2*A*e) - 3*a*e*(-2*C*d +
 B*e)))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d +
```

```
 b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] - (2*a*c*C*d^2*
e^2 - 8*a*B*c*d*e^3 - 6*a^2*C*e^4 + 2*c*C*d^3*Sqrt[(b^2 - 4*a*c)*e^2] + B*c
*d^2*e*Sqrt[(b^2 - 4*a*c)*e^2] + 6*a*C*d*e^2*Sqrt[(b^2 - 4*a*c)*e^2] - 3*a*
B*e^3*Sqrt[(b^2 - 4*a*c)*e^2] + 2*A*c*e^2*(-3*c*d^2 + a*e^2 - 2*d*Sqrt[(b^2
 - 4*a*c)*e^2]) - b^2*e^2*(2*C*d^2 + e*(B*d + 2*A*e)) + b*e*(2*A*e^2*(3*c*d
 + Sqrt[(b^2 - 4*a*c)*e^2]) + 2*C*d*(3*a*e^2 - 2*d*Sqrt[(b^2 - 4*a*c)*e^2])
 + B*e*(3*c*d^2 + 3*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2])))*EllipticF[I*ArcSin
h[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*
e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d -
 b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*d^2 + e*(-(b*d) + a*e)
)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[d + e*x])))/(3*e^3*(c*d^2
- b*d*e + a*e^2)^2*Sqrt[a + x*(b + c*x)]*Sqrt[((d + e*x)^2*(c*(-1 + d/(d +
e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}\,\sqrt{ex + d}}{ce^3x^5 + \left(3\,cde^2 + be^3\right)x^4 + ad^3 + \left(3\,cd^2e + 3\,bde^2 + ae^3\right)x^3 + \left(cd^3 + 3\,bd^2e + 3\,ade^2\right)x^2 + \left(bd^3 + 3\,ad^2e\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e^3*x^5 +
 (3*c*d*e^2 + b*e^3)*x^4 + a*d^3 + (3*c*d^2*e + 3*b*d*e^2 + a*e^3)*x^3 + (c
*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^2 + (b*d^3 + 3*a*d^2*e)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^2 + Bx + A}{\sqrt{cx^2 + bx + a}\,(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(5/2)), x)
```

maple [B] time = 0.13, size = 20481, normalized size = 29.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^2 + Bx + A}{\sqrt{cx^2 + bx + a}\,(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\,x^2+B\,x+A}{{\left(d+e\,x\right)}^{5/2}\,\sqrt{c\,x^2+b\,x+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d + e*x)^(5/2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((A + B*x + C*x^2)/((d + e*x)^(5/2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^{2}}{\left(d + ex\right)^{\frac{5}{2}} \sqrt{a + bx + cx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((A + B*x + C*x**2)/((d + e*x)**(5/2)*sqrt(a + b*x + c*x**2)), x)
```

3.271 $\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=944

$$-\frac{2\sqrt{cx^2+bx+a}\left(Cd^2-e(Bd-Ae)\right)}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}-\frac{\sqrt{2}\sqrt{b^2-4ac}\left(c^2\left(2Cd^2+e(3Bd-23Ae)\right)d^2-e^2\left(\left(3Cd^2+2Bed+8A\right.\right.\right.}{}$$

```
[Out] -2/5*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(5/2)+2/15*(c*d*(2*C*d^2+e*(-8*A*e+3*B*d))+e*(5*a*e*(-B*e+2*C*d)-b*(-4*A*e
^2-B*d*e+6*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(3/
2)+2/15*(c^2*d^2*(2*C*d^2+e*(-23*A*e+3*B*d))-e^2*(15*a^2*C*e^2-10*a*b*e*(B*
e+C*d)+b^2*(8*A*e^2+2*B*d*e+3*C*d^2))-c*e*(b*d*(-23*A*e^2-7*B*d*e+7*C*d^2)-
a*e*(9*A*e^2-29*B*d*e+19*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)
^3/(e*x+d)^(1/2)-1/15*(c^2*d^2*(2*C*d^2+e*(-23*A*e+3*B*d))-e^2*(15*a^2*C*e^
2-10*a*b*e*(B*e+C*d)+b^2*(8*A*e^2+2*B*d*e+3*C*d^2))-c*e*(b*d*(-23*A*e^2-7*B
*d*e+7*C*d^2)-a*e*(9*A*e^2-29*B*d*e+19*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)
/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d
)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)^3/(c*
x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/15*(c
*d*(2*C*d^2+e*(-8*A*e+3*B*d))+e*(5*a*e*(-B*e+2*C*d)-b*(-4*A*e^2-B*d*e+6*C*d
^2)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*
2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)
/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^
(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 2.26, antiderivative size = 942, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1650, 834, 843, 718, 424, 419}

$$-\frac{2\sqrt{cx^2+bx+a}\left(Cd^2-e(Bd-Ae)\right)}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}-\frac{\sqrt{2}\sqrt{b^2-4ac}\left(\left(2Cd^4+e(3Bd-23Ae)d^2\right)c^2-e\left(bd\left(7Cd^2-7Bed-2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(5*e*(c*d^2 - b*d*e + a*
e^2)*(d + e*x)^(5/2)) + (2*(2*c*C*d^3 + c*d*e*(3*B*d - 8*A*e) + 5*a*e^2*(2*
C*d - B*e) - b*e*(6*C*d^2 - e*(B*d + 4*A*e)))*Sqrt[a + b*x + c*x^2])/(15*e*
(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(3/2)) + (2*(c^2*(2*C*d^4 + d^2*e*(3*B*
d - 23*A*e)) - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2*
B*d*e + 8*A*e^2)) - c*e*(b*d*(7*C*d^2 - 7*B*d*e - 23*A*e^2) - a*e*(19*C*d^2
 - 29*B*d*e + 9*A*e^2)))*Sqrt[a + b*x + c*x^2])/(15*e*(c*d^2 - b*d*e + a*e^
2)^3*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*(2*C*d^4 + d^2*e*(3*B
*d - 23*A*e)) - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2
*B*d*e + 8*A*e^2)) - c*e*(b*d*(7*C*d^2 - 7*B*d*e - 23*A*e^2) - a*e*(19*C*d^
2 - 29*B*d*e + 9*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
```

```
4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)])/(15*e^2*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(2*c*C*d^3 + c*d*e*(3*B*d - 8*A*e) + 5*a*e^2*(2*C*d - B*e) - b*e*(6*C*d^2 -
 e*(B*d + 4*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqr
t[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*
e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*e^2*(c*d^2 - b*d*e + a*e^2)^2*
Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
```

```
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} - \frac{2\int \frac{-\frac{bCd^2-be(Bd+4Ae)+5e(Acd-aCd+aBe)}{2e}-\frac{1}{2}\left(3b\ldots\right.}{(d+ex)^{5/2}\sqrt{a+b\ldots}}}{5\left(cd^2-bde\ldots\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2\left(2cCd^3+cde(3Bd-8Ae)+5ae^2\ldots\right.}{15e\left(c\ldots\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2\left(2cCd^3+cde(3Bd-8Ae)+5ae^2\ldots\right.}{15e\left(c\ldots\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2\left(2cCd^3+cde(3Bd-8Ae)+5ae^2\ldots\right.}{15e\left(c\ldots\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2\left(2cCd^3+cde(3Bd-8Ae)+5ae^2\ldots\right.}{15e\left(c\ldots\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2\left(2cCd^3+cde(3Bd-8Ae)+5ae^2\ldots\right.}{15e\left(c\ldots\right.}$$

Mathematica [C] time = 15.04, size = 12295, normalized size = 13.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
[Out] Result too large to show
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\,\sqrt{ex+d}}{ce^4x^6+\left(4\,cde^3+be^4\right)x^5+ad^4+\left(6\,cd^2e^2+4\,bde^3+ae^4\right)x^4+2\left(2\,cd^3e+3\,bd^2e^2+2\,ade^3\right)x^3+\left(cd^4\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e^4*x^6 +
 (4*c*d*e^3 + b*e^4)*x^5 + a*d^4 + (6*c*d^2*e^2 + 4*b*d*e^3 + a*e^4)*x^4 +
2*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^3 + (c*d^4 + 4*b*d^3*e + 6*a*d^2*
e^2)*x^2 + (b*d^4 + 4*a*d^3*e)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^2 + Bx + A}{\sqrt{cx^2 + bx + a}\,(ex + d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(7/2)), x)
```

maple [B] time = 0.26, size = 46697, normalized size = 49.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^2 + Bx + A}{\sqrt{cx^2 + bx + a}\,(ex + d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\,x^2 + B\,x + A}{(d + e\,x)^{7/2}\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d + e*x)^(7/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x + C*x^2)/((d + e*x)^(7/2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(d + ex)^{\frac{7}{2}}\,\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(7/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((d + e*x)**(7/2)*sqrt(a + b*x + c*x**2)), x)
```

3.272 $\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=510

$$\frac{(g+hx)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(g\cdots}{2cg-\left(b-\sqrt{\cdots}\right.}\right.}{ch^3(m+1)(m+2p+\cdots}$$

```
[Out] f*(h*x+g)^(1+m)*(c*x^2+b*x+a)^(1+p)/c/h/(3+m+2*p)+(f*h*(-a*h+b*g)*(1+m)+c*(
2*f*g^2*(1+p)-h*(-d*h+e*g)*(3+m+2*p)))*(h*x+g)^(1+m)*(c*x^2+b*x+a)^p*Appell
F1(1+m,-p,-p,2+m,2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(
2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))/c/h^3/(1+m)/(3+m+2*p)/((1-2*c*(h*x+g)/(2*c
*g-h*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(
1/2))))^p)-(b*f*h*(2+m+p)+c*(2*f*g*(1+p)-e*h*(3+m+2*p)))*(h*x+g)^(2+m)*(c*x
^2+b*x+a)^p*AppellF1(2+m,-p,-p,3+m,2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/
2))),2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))/c/h^3/(2+m)/(3+m+2*p)/((
1-2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(h*x+g)/(2*c*g-h
*(b+(-4*a*c+b^2)^(1/2))))^p)
```

Rubi [A] time = 0.83, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1653, 843, 759, 133}

$$\frac{(g+hx)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(g\cdots}{2cg-\left(b-\sqrt{\cdots}\right.}\right.}{ch^3(m+1)(m+2p+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^(1 + p))/(c*h*(3 + m + 2*p)) + ((f*h
*(b*g - a*h)*(1 + m) + 2*c*f*g^2*(1 + p) - c*h*(e*g - d*h)*(3 + m + 2*p))*(
g + h*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*(g
 + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b +
 Sqrt[b^2 - 4*a*c])*h)])/(c*h^3*(1 + m)*(3 + m + 2*p)*(1 - (2*c*(g + h*x))/
(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(2*c*g - (b + S
qrt[b^2 - 4*a*c])*h))^p) - ((2*c*f*g*(1 + p) + b*f*h*(2 + m + p) - c*e*h*(3
 + m + 2*p))*(g + h*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[2 + m, -p, -p,
3 + m, (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))
/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h)])/(c*h^3*(2 + m)*(3 + m + 2*p)*(1 - (2
*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(
2*c*g - (b + Sqrt[b^2 - 4*a*c])*h))^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
```

```
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^m\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)dx=\frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)}+\frac{\int (g+hx)^m(-h(afh(1+m)\ldots}{\ldots}$$

$$=\frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)}-\frac{(2cfg(1+p)+bfh(2+m+\ldots}{\ldots}$$

$$=\frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)}-\frac{\Big((2cfg(1+p)+bfh(2+m+\ldots}{\ldots}$$

$$=\frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)}+\frac{\left(fh(bg-ah)(1+m)+2cfg^{2}\ldots\right.}{\ldots}$$

Mathematica [F] time = 2.28, size = 0, normalized size = 0.00

$$\int (g+hx)^m\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x]
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p\left(hx+g\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
[Out] integral((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx^2 + ex + d\right)\left(cx^2 + bx + a\right)^p \left(hx + g\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(f\,x^2 + ex + d\right)\left(hx + g\right)^m \left(c\,x^2 + bx + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
[Out] int((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx^2 + ex + d\right)\left(cx^2 + bx + a\right)^p \left(hx + g\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g+h\,x\right)}^m\,{\left(c\,x^2+b\,x+a\right)}^p\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**m*(c*x**2+b*x+a)**p*(f*x**2+e*x+d),x)
[Out] Timed out
```

3.273 $\int (g+hx)^m\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=496

$$\frac{\sqrt{a+bx+cx^2}\,(g+hx)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)\left(fh(m+1)(bg-ah)+c\left(3\right.\right.}{ch^3(m+1)(m+4)\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[Out] f*(h*x+g)^(1+m)*(c*x^2+b*x+a)^(3/2)/c/h/(4+m)+(f*h*(-a*h+b*g)*(1+m)+c*(3*f*
g^2-h*(-d*h+e*g)*(4+m)))*(h*x+g)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,2*c*(h*x+
g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1
/2))))*(c*x^2+b*x+a)^(1/2)/c/h^3/(1+m)/(4+m)/(1-2*c*(h*x+g)/(2*c*g-h*(b-(-4
*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))^(
1/2)-1/2*(b*f*h*(5+2*m)+c*(6*f*g-2*e*h*(4+m)))*(h*x+g)^(2+m)*AppellF1(2+m,-
1/2,-1/2,3+m,2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(2*c*
g-h*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/c/h^3/(2+m)/(4+m)/(1-2*c*(
h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(h*x+g)/(2*c*g-h*(b+(
-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1653, 843, 759, 133}

$$\frac{\sqrt{a+bx+cx^2}\,(g+hx)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)\left(fh(m+1)(bg-ah)-ch(\right.}{ch^3(m+1)(m+4)\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^(3/2))/(c*h*(4 + m)) + ((3*c*f*g^2 +
 f*h*(b*g - a*h)*(1 + m) - c*h*(e*g - d*h)*(4 + m))*(g + h*x)^(1 + m)*Sqrt[
a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (2*c*(g + h*x))/(2*c*g
- (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*
c])*h)])/(c*h^3*(1 + m)*(4 + m)*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b - Sqrt
[b^2 - 4*a*c])*h)]*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c]
)*h)]) - ((6*c*f*g - 2*c*e*h*(4 + m) + b*f*h*(5 + 2*m))*(g + h*x)^(2 + m)*S
qrt[a + b*x + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (2*c*(g + h*x))/(2*
c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 -
4*a*c])*h)])/(2*c*h^3*(2 + m)*(4 + m)*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b
- Sqrt[b^2 - 4*a*c])*h)]*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 -
4*a*c])*h)])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
```

```
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\int (g+hx)^m\left(-\frac{1}{2}h(3bfg+2\right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\left(3cfg^2+fh(bg-ah)(1+\right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\left(\left(3cfg^2+fh(bg-ah)(1+\right.\right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\left(3cfg^2+fh(bg-ah)(1+\right.}{}$$

Mathematica [F] time = 1.46, size = 0, normalized size = 0.00

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2), x]
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)\left(hx+g\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)\left(hx+g\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\left(fx^2+ex+d\right)\sqrt{cx^2+bx+a}\,\left(hx+g\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)\left(hx+g\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int{\left(g+h\,x\right)}^m\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^m*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^m*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(g+hx\right)^m\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**m*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**m*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

3.274 $\int (g+hx)^{-3-2p} \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=590

$$-\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{2c(g}{2cg-(b-}\right.}{2h^3p}$$

```
[Out] -1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1+p)/h/(a*h^2-b*g*h+c*g^2)/(1+p)/(
(h*x+g)^(2+2*p))-1/2*(2*c*(-d*g*h^2+f*g^3)+h*(2*a*h*(-e*h+2*f*g)-b*(-d*h^2-
e*g*h+3*f*g^2)))*(h*x+g)^(-1-2*p)*(c*x^2+b*x+a)^p*hypergeom([-p, -1-2*p],[-
2*p],-4*c*(h*x+g)*(-4*a*c+b^2)^(1/2)/(b+2*c*x-(-4*a*c+b^2)^(1/2))/(2*c*g-h*
(b+(-4*a*c+b^2)^(1/2))))*(b+2*c*x-(-4*a*c+b^2)^(1/2))/h^2/(a*h^2-b*g*h+c*g^
2)/(1+2*p)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2)))/(((2*c*g-h*(b-(-4*a*c+b^2)^(1/2
)))*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+2*c*x-(-4*a*c+b^2)^(1/2))/(2*c*g-h*(b+(
-4*a*c+b^2)^(1/2))))^p)-1/2*f*(c*x^2+b*x+a)^p*AppellF1(-2*p,-p,-p,1-2*p,2*c
*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b
^2)^(1/2))))/h^3/p/((h*x+g)^(2*p))/((1-2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)
^(1/2))))^p)/((1-2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))^p)
```

Rubi [A] time = 0.76, antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1655, 759, 133, 806, 726}

$$-\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{2c(g}{2cg-(b-}\right.}{2h^3p}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] -((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(1 + p))/(2*h*(c*g^2 - b*g*h +
a*h^2)*(1 + p)*(g + h*x)^(2*(1 + p))) - (f*(a + b*x + c*x^2)^p*AppellF1[-2*
p, -p, -p, 1 - 2*p, (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2
*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h)])/(2*h^3*p*(g + h*x)^(2*p
)*(1 - (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g
+ h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h))^p) - ((2*c*(f*g^3 - d*g*h^2) -
 h*(3*b*f*g^2 - b*h*(e*g + d*h) - 2*a*h*(2*f*g - e*h)))*(b - Sqrt[b^2 - 4*a
*c] + 2*c*x)*(g + h*x)^(-1 - 2*p)*(a + b*x + c*x^2)^p*Hypergeometric2F1[-1
- 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(g + h*x))/((2*c*g - (b + Sqrt[b^2
 - 4*a*c])*h)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/(2*h^2*(2*c*g - (b - Sqrt[
b^2 - 4*a*c])*h)*(c*g^2 - b*g*h + a*h^2)*(1 + 2*p)*(((2*c*g - (b - Sqrt[b^2
 - 4*a*c])*h)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*g - (b + Sqrt[b^2 - 4*
a*c])*h)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 726

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((b - Rt[b^2 - 4*a*c, 2] + 2*c*x)*(d + e*x)^(m + 1)*(a + b*
```

```
x + c*x^2)^p*Hypergeometric2F1[m + 1, -p, m + 2, (-4*c*Rt[b^2 - 4*a*c, 2]*(
d + e*x))/((2*c*d - b*e - e*Rt[b^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2
*c*x))])/((m + 1)*(2*c*d - b*e + e*Rt[b^2 - 4*a*c, 2])*(((2*c*d - b*e + e*R
t[b^2 - 4*a*c, 2])*(b + Rt[b^2 - 4*a*c, 2] + 2*c*x))/((2*c*d - b*e - e*Rt[b
^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2*c*x)))^p), x] /; FreeQ[{a, b, c
, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1655

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> With[{q = Expon[Pq, x]}, Dist[Coeff[Pq, x, q]/e^q, Int[(d
 + e*x)^(m + q)*(a + b*x + c*x^2)^p, x], x] + Dist[1/e^q, Int[(d + e*x)^m*(
a + b*x + c*x^2)^p*ExpandToSum[e^q*Pq - Coeff[Pq, x, q]*(d + e*x)^q, x], x]
, x]] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ[m, 0] && RationalQ[a, b, c
, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)dx=\frac{\int (g+hx)^{-3-2p}\left(-fg^2+dh^2-h(2fg-eh)x\right)\left(a+bx+cx^2\right)^{p}}{h^2}$$

$$=-\frac{\left(fg^2-h(eg-dh)\right)(g+hx)^{-2(1+p)}\left(a+bx+cx^2\right)^{1+p}}{2h\left(cg^2-bgh+ah^2\right)(1+p)}-\frac{(2c}{}$$

$$=-\frac{\left(fg^2-h(eg-dh)\right)(g+hx)^{-2(1+p)}\left(a+bx+cx^2\right)^{1+p}}{2h\left(cg^2-bgh+ah^2\right)(1+p)}-\frac{f(g}{}$$

Mathematica [F] time = 3.46, size = 0, normalized size = 0.00

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] Integrate[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x]
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p\left(hx+g\right)^{-2\,p-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="fric
as")
```

```
[Out] integral((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p\left(hx+g\right)^{-2\,p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="giac
")
```

```
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(f\,x^2+ex+d\right)\left(hx+g\right)^{-2p-3}\left(c\,x^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^(-2*p-3)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

```
[Out] int((h*x+g)^(-2*p-3)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p\left(hx+g\right)^{-2\,p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="maxi
ma")
```

```
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^p\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^{2\,p+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3),x)
```

```
[Out] int(((a + b*x + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**(-3-2*p)*(c*x**2+b*x+a)**p*(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

3.275 $\int \left(d+fx^2\right)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x\right.$

Optimal. Leaf size=41

$$\frac{bf(2p+3)\left(d+fx^2\right)^{p+1}}{p+1}+2cfx\left(d+fx^2\right)^{p+1}$$

```
[Out] b*f*(3+2*p)*(f*x^2+d)^(1+p)/(1+p)+2*c*f*x*(f*x^2+d)^(1+p)
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1815, 12, 261}

$$\frac{bf(2p+3)\left(d+fx^2\right)^{p+1}}{p+1}+2cfx\left(d+fx^2\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + f*x^2)^p*(2*c*d*f + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x
]
```

```
[Out] (b*f*(3 + 2*p)*(d + f*x^2)^(1 + p))/(1 + p) + 2*c*f*x*(d + f*x^2)^(1 + p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(d+fx^2\right)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)dx &= 2cfx\left(d+fx^2\right)^{1+p}+\frac{\int 2bf^3(3+2p)^2x\left(d+fx^2\right.}{f(3+2p)}\\ &= 2cfx\left(d+fx^2\right)^{1+p}+\left(2bf^2(3+2p)\right)\int x\left(d+f\right.\\ &= \frac{bf(3+2p)\left(d+fx^2\right)^{1+p}}{1+p}+2cfx\left(d+fx^2\right)^{1+p}\end{aligned}$$

Mathematica [C] time = 0.11, size = 119, normalized size = 2.90

$$\frac{f\left(d+fx^2\right)^p\left(\frac{fx^2}{d}+1\right)^{-p}\left((2p+3)\left(3b\left(d+fx^2\right)\left(\frac{fx^2}{d}+1\right)^p+2cf(p+1)x^3\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{fx^2}{d}\right)\right)+6cd(p+1)x\right.}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + f*x^2)^p*(2*c*d*f + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*
x^2),x]
```

```
[Out] (f*(d + f*x^2)^p*(6*c*d*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((f*x^2)
/d)] + (3 + 2*p)*(3*b*(d + f*x^2)*(1 + (f*x^2)/d)^p + 2*c*f*(1 + p)*x^3*Hyp
ergeometric2F1[3/2, -p, 5/2, -((f*x^2)/d)])))/(3*(1 + p)*(1 + (f*x^2)/d)^p)
```

fricas [A] time = 0.58, size = 75, normalized size = 1.83

$$\frac{\left(2\,bdfp + 2\left(cf^2p + cf^2\right)x^3 + 3\,bdf + \left(2\,bf^2p + 3\,bf^2\right)x^2 + 2\left(cdfp + cdf\right)x\right)\left(fx^2 + d\right)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, al
gorithm="fricas")
```

```
[Out] (2*b*d*f*p + 2*(c*f^2*p + c*f^2)*x^3 + 3*b*d*f + (2*b*f^2*p + 3*b*f^2)*x^2
+ 2*(c*d*f*p + c*d*f)*x)*(f*x^2 + d)^p/(p + 1)
```

giac [B] time = 0.18, size = 141, normalized size = 3.44

$$\frac{2\left(fx^2 + d\right)^p cf^2px^3 + 2\left(fx^2 + d\right)^p bf^2px^2 + 2\left(fx^2 + d\right)^p cf^2x^3 + 2\left(fx^2 + d\right)^p cdfpx + 3\left(fx^2 + d\right)^p bf^2x^2 + 2\left(fx\right.}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, al
gorithm="giac")
```

```
[Out] (2*(f*x^2 + d)^p*c*f^2*p*x^3 + 2*(f*x^2 + d)^p*b*f^2*p*x^2 + 2*(f*x^2 + d)^
p*c*f^2*x^3 + 2*(f*x^2 + d)^p*c*d*f*p*x + 3*(f*x^2 + d)^p*b*f^2*x^2 + 2*(f*
x^2 + d)^p*b*d*f*p + 2*(f*x^2 + d)^p*c*d*f*x + 3*(f*x^2 + d)^p*b*d*f)/(p +
1)
```

maple [A] time = 0.00, size = 36, normalized size = 0.88

$$\frac{\left(2pcx + 2pb + 2cx + 3b\right) f \left(f\,x^2 + d\right)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x)
```

```
[Out] f*(f*x^2+d)^(1+p)*(2*c*p*x+2*b*p+2*c*x+3*b)/(1+p)
```

maxima [A] time = 0.58, size = 59, normalized size = 1.44

$$\frac{\left(2\,cf^2\left(p + 1\right)x^3 + bf^2\left(2\,p + 3\right)x^2 + 2\,cdf\left(p + 1\right)x + bdf\left(2\,p + 3\right)\right)\left(fx^2 + d\right)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, al
gorithm="maxima")
```

```
[Out] (2*c*f^2*(p + 1)*x^3 + b*f^2*(2*p + 3)*x^2 + 2*c*d*f*(p + 1)*x + b*d*f*(2*p
 + 3))*(f*x^2 + d)^p/(p + 1)
```

mupad [B] time = 4.25, size = 58, normalized size = 1.41

$$\left(f\,x^2+d\right)^p\,\left(2\,c\,f^2\,x^3+2\,c\,d\,f\,x+\frac{b\,f^2\,x^2\,\left(2\,p+3\right)}{p+1}+\frac{b\,d\,f\,\left(2\,p+3\right)}{p+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*x^2)^p*(2*c*d*f + 2*b*f^2*x*(2*p + 3) + 2*c*f^2*x^2*(2*p + 3)),x
)
```

```
[Out] (d + f*x^2)^p*(2*c*f^2*x^3 + 2*c*d*f*x + (b*f^2*x^2*(2*p + 3))/(p + 1) + (b
*d*f*(2*p + 3))/(p + 1))
```

sympy [B] time = 13.25, size = 221, normalized size = 5.39

$$\begin{cases} \frac{2bdfp(d+fx^2)^p}{p+1} + \frac{3bdf(d+fx^2)^p}{p+1} + \frac{2bf^2px^2(d+fx^2)^p}{p+1} + \frac{3bf^2x^2(d+fx^2)^p}{p+1} + \frac{2cdfpx(d+fx^2)^p}{p+1} + \frac{2cdfx(d+fx^2)^p}{p+1} + \frac{2cf^2px^3(d+fx^2)^p}{p+1} + \\ bf\log\left(-i\sqrt{d}\sqrt{\frac{1}{f}}+x\right) + bf\log\left(i\sqrt{d}\sqrt{\frac{1}{f}}+x\right) + 2cfx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+d)**p*(2*c*d*f+2*b*f**2*(3+2*p)*x+2*c*f**2*(3+2*p)*x**2),
x)
```

```
[Out] Piecewise((2*b*d*f*p*(d + f*x**2)**p/(p + 1) + 3*b*d*f*(d + f*x**2)**p/(p +
 1) + 2*b*f**2*p*x**2*(d + f*x**2)**p/(p + 1) + 3*b*f**2*x**2*(d + f*x**2)*
*p/(p + 1) + 2*c*d*f*p*x*(d + f*x**2)**p/(p + 1) + 2*c*d*f*x*(d + f*x**2)**
p/(p + 1) + 2*c*f**2*p*x**3*(d + f*x**2)**p/(p + 1) + 2*c*f**2*x**3*(d + f*
x**2)**p/(p + 1), Ne(p, -1)), (b*f*log(-I*sqrt(d)*sqrt(1/f) + x) + b*f*log(
I*sqrt(d)*sqrt(1/f) + x) + 2*c*f*x, True))
```

3.276 $\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)\right.$

Optimal. Leaf size=46

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{ce(p+2)\left(d+ex+fx^2\right)^{p+1}}{p+1}$$

```
[Out] -c*e*(2+p)*(f*x^2+e*x+d)^(1+p)/(1+p)+2*c*f*x*(f*x^2+e*x+d)^(1+p)
```

Rubi [A] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1661, 629}

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{ce(p+2)\left(d+ex+fx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f - c*e^2*p + 2*c*f^2*(3 + 2*p)*x
^2),x]
```

```
[Out] -((c*e*(2 + p)*(d + e*x + f*x^2)^(1 + p))/(1 + p)) + 2*c*f*x*(d + e*x + f*x
^2)^(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right) dx = 2cfx\left(d+ex+fx^2\right)^{1+p}+\frac{\int \left(-ce^2f(2+p)(3+\right.}{}$$

$$=-\frac{ce(2+p)\left(d+ex+fx^2\right)^{1+p}}{1+p}+2cfx\left(d+ex+\right.$$

Mathematica [A] time = 0.13, size = 34, normalized size = 0.74

$$\frac{c(2f(p+1)x-e(p+2))(d+x(e+fx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f - c*e^2*p + 2*c*f^2*(3 +
2*p)*x^2),x]
```

[Out] (c*(-(e*(2 + p)) + 2*f*(1 + p)*x)*(d + x*(e + f*x))^(1 + p))/(1 + p)

fricas [A] time = 0.63, size = 83, normalized size = 1.80

$$\frac{\left(cefpx^2-cdep+2\left(cf^2p+cf^2\right)x^3-2\,cde-\left(2\,ce^2-2\,cdf+\left(ce^2-2\,cdf\right)p\right)x\right)\left(fx^2+ex+d\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,
 algorithm="fricas")
```

```
[Out] (c*e*f*p*x^2 - c*d*e*p + 2*(c*f^2*p + c*f^2)*x^3 - 2*c*d*e - (2*c*e^2 - 2*c
*d*f + (c*e^2 - 2*c*d*f)*p)*x)*(f*x^2 + e*x + d)^p/(p + 1)
```

giac [B] time = 0.24, size = 191, normalized size = 4.15

$$\frac{2\left(fx^2+xe+d\right)^pcf^2px^3+2\left(fx^2+xe+d\right)^pcf^2x^3+\left(fx^2+xe+d\right)^pcfpx^2e+2\left(fx^2+xe+d\right)^pcdfpx+2\left(fx\right.}{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,
 algorithm="giac")
```

```
[Out] (2*(f*x^2 + x*e + d)^p*c*f^2*p*x^3 + 2*(f*x^2 + x*e + d)^p*c*f^2*x^3 + (f*x
^2 + x*e + d)^p*c*f*p*x^2*e + 2*(f*x^2 + x*e + d)^p*c*d*f*p*x + 2*(f*x^2 +
x*e + d)^p*c*d*f*x - (f*x^2 + x*e + d)^p*c*p*x*e^2 - (f*x^2 + x*e + d)^p*c*
d*p*e - 2*(f*x^2 + x*e + d)^p*c*x*e^2 - 2*(f*x^2 + x*e + d)^p*c*d*e)/(p + 1
)
```

maple [A] time = 0.00, size = 39, normalized size = 0.85

$$-\frac{\left(-2fpx+ep-2fx+2e\right)c\left(f\,x^2+ex+d\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x)
```

```
[Out] -c*(f*x^2+e*x+d)^(p+1)*(-2*f*p*x+e*p-2*f*x+2*e)/(p+1)
```

maxima [A] time = 0.58, size = 66, normalized size = 1.43

$$\frac{\left(2\,cf^2(p+1)x^3+cefpx^2-cde(p+2)-\left(e^2(p+2)-2\,df(p+1)\right)cx\right)\left(fx^2+ex+d\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,
 algorithm="maxima")
```

```
[Out] (2*c*f^2*(p + 1)*x^3 + c*e*f*p*x^2 - c*d*e*(p + 2) - (e^2*(p + 2) - 2*d*f*(
p + 1))*c*x)*(f*x^2 + e*x + d)^p/(p + 1)
```

mupad [B] time = 4.39, size = 78, normalized size = 1.70

$$\left(f\,x^2+e\,x+d\right)^p\,\left(2\,c\,f^2\,x^3+\frac{c\,x\,\left(2\,d\,f-e^2\,p-2\,e^2+2\,d\,f\,p\right)}{p+1}-\frac{c\,d\,e\,\left(p+2\right)}{p+1}+\frac{c\,e\,f\,p\,x^2}{p+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(d + e*x + f*x^2)^p*(2*c*e^2 - 2*c*d*f + c*e^2*p - 2*c*f^2*x^2*(2*p +
3)),x)
```

```
[Out] (d + e*x + f*x^2)^p*(2*c*f^2*x^3 + (c*x*(2*d*f - e^2*p - 2*e^2 + 2*d*f*p))/
(p + 1) - (c*d*e*(p + 2))/(p + 1) + (c*e*f*p*x^2)/(p + 1))
```

sympy [A] time = 173.95, size = 280, normalized size = 6.09

$$\begin{cases} -\frac{cdep(d+ex+fx^2)^p}{p+1} - \frac{2cde(d+ex+fx^2)^p}{p+1} + \frac{2cdfpx(d+ex+fx^2)^p}{p+1} + \frac{2cdfx(d+ex+fx^2)^p}{p+1} - \frac{ce^2px(d+ex+fx^2)^p}{p+1} - \frac{2ce^2x(d+ex+fx^2)^p}{p+1} + \frac{cefpx^2(}{} \\ -ce\log\left(\frac{e}{2f} + x - \frac{\sqrt{-4df+e^2}}{2f}\right) - ce\log\left(\frac{e}{2f} + x + \frac{\sqrt{-4df+e^2}}{2f}\right) + 2cfx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**p*(-2*c*e**2+2*c*d*f-c*e**2*p+2*c*f**2*(3+2*p)*x*
*2),x)
```

```
[Out] Piecewise((-c*d*e*p*(d + e*x + f*x**2)**p/(p + 1) - 2*c*d*e*(d + e*x + f*x*
*2)**p/(p + 1) + 2*c*d*f*p*x*(d + e*x + f*x**2)**p/(p + 1) + 2*c*d*f*x*(d +
 e*x + f*x**2)**p/(p + 1) - c*e**2*p*x*(d + e*x + f*x**2)**p/(p + 1) - 2*c*
e**2*x*(d + e*x + f*x**2)**p/(p + 1) + c*e*f*p*x**2*(d + e*x + f*x**2)**p/(
p + 1) + 2*c*f**2*p*x**3*(d + e*x + f*x**2)**p/(p + 1) + 2*c*f**2*x**3*(d +
 e*x + f*x**2)**p/(p + 1), Ne(p, -1)), (-c*e*log(e/(2*f) + x - sqrt(-4*d*f
+ e**2)/(2*f)) - c*e*log(e/(2*f) + x + sqrt(-4*d*f + e**2)/(2*f)) + 2*c*f*x
, True))
```

3.277 $\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf+3bef-ce^2p+2be\right.$

Optimal. Leaf size=57

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{(ce(p+2)-bf(2p+3))\left(d+ex+fx^2\right)^{p+1}}{p+1}$$

```
[Out] -(c*e*(2+p)-b*f*(3+2*p))*(f*x^2+e*x+d)^(1+p)/(1+p)+2*c*f*x*(f*x^2+e*x+d)^(1
+p)
```

Rubi [A] time = 0.12, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 69, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1661, 629}

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{(ce(p+2)-bf(2p+3))\left(d+ex+fx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f + 3*b*e*f - c*e^2*p + 2*b*e*f*p
 + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x]
```

```
[Out] -(((c*e*(2 + p) - b*f*(3 + 2*p))*(d + e*x + f*x^2)^(1 + p))/(1 + p)) + 2*c*
f*x*(d + e*x + f*x^2)^(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right) dx = 2cfx\left(d+\right.$$

$$=-\frac{(ce(2+p}{}$$

Mathematica [A] time = 0.31, size = 43, normalized size = 0.75

$$\frac{(d+x(e+fx))^{p+1}(bf(2p+3)-ce(p+2)+2cf(p+1)x)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f + 3*b*e*f - c*e^2*p + 2*b
*e*f*p + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x]
```

```
[Out] ((-(c*e*(2 + p)) + b*f*(3 + 2*p) + 2*c*f*(1 + p)*x)*(d + x*(e + f*x))^(1 +
p))/(1 + p)
```

fricas [B] time = 0.52, size = 123, normalized size = 2.16

$$\frac{\left(2\left(cf^2p+cf^2\right)x^3-2\,cde+3\,bdf+\left(3\,bf^2+\left(cef+2\,bf^2\right)p\right)x^2-\left(cde-2\,bdf\right)p-\left(2\,ce^2-\left(2\,cd+3\,be\right)f+\left(ce^2\right.\right.\right.}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f
^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="fricas")
```

```
[Out] (2*(c*f^2*p + c*f^2)*x^3 - 2*c*d*e + 3*b*d*f + (3*b*f^2 + (c*e*f + 2*b*f^2)
*p)*x^2 - (c*d*e - 2*b*d*f)*p - (2*c*e^2 - (2*c*d + 3*b*e)*f + (c*e^2 - 2*(
c*d + b*e)*f)*p)*x)*(f*x^2 + e*x + d)^p/(p + 1)
```

giac [B] time = 0.31, size = 314, normalized size = 5.51

$$2\left(fx^2+xe+d\right)^p cf^2px^3+2\left(fx^2+xe+d\right)^p bf^2px^2+2\left(fx^2+xe+d\right)^p cf^2x^3+\left(fx^2+xe+d\right)^p cfpx^2e+2\left(fx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f
^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="giac")
```

```
[Out] (2*(f*x^2 + x*e + d)^p*c*f^2*p*x^3 + 2*(f*x^2 + x*e + d)^p*b*f^2*p*x^2 + 2*
(f*x^2 + x*e + d)^p*c*f^2*x^3 + (f*x^2 + x*e + d)^p*c*f*p*x^2*e + 2*(f*x^2
+ x*e + d)^p*c*d*f*p*x + 3*(f*x^2 + x*e + d)^p*b*f^2*x^2 + 2*(f*x^2 + x*e +
 d)^p*b*f*p*x*e + 2*(f*x^2 + x*e + d)^p*b*d*f*p + 2*(f*x^2 + x*e + d)^p*c*d
*f*x - (f*x^2 + x*e + d)^p*c*p*x*e^2 - (f*x^2 + x*e + d)^p*c*d*p*e + 3*(f*x
^2 + x*e + d)^p*b*f*x*e + 3*(f*x^2 + x*e + d)^p*b*d*f - 2*(f*x^2 + x*e + d)
^p*c*x*e^2 - 2*(f*x^2 + x*e + d)^p*c*d*e)/(p + 1)
```

maple [A] time = 0.01, size = 51, normalized size = 0.89

$$\frac{\left(2cfxp+2bfp-cep+2cfx+3bf-2ce\right)\left(fx^2+ex+d\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f^2*(3+
2*p)*x+2*c*f^2*(3+2*p)*x^2),x)
```

```
[Out] (f*x^2+e*x+d)^(p+1)*(2*c*f*p*x+2*b*f*p-c*e*p+2*c*f*x+3*b*f-2*c*e)/(p+1)
```

maxima [A] time = 0.60, size = 98, normalized size = 1.72

$$\frac{\left(2\,cf^2\left(p+1\right)x^3+bdf\left(2\,p+3\right)-cde\left(p+2\right)+\left(bf^2\left(2\,p+3\right)+cefp\right)x^2+\left(bef\left(2\,p+3\right)-\left(e^2\left(p+2\right)-2\,df\left(p+1\right.\right.\right.\right.}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f
^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="maxima")
```

```
[Out] (2*c*f^2*(p + 1)*x^3 + b*d*f*(2*p + 3) - c*d*e*(p + 2) + (b*f^2*(2*p + 3) +
 c*e*f*p)*x^2 + (b*e*f*(2*p + 3) - (e^2*(p + 2) - 2*d*f*(p + 1))*c)*x)*(f*x
^2 + e*x + d)^p/(p + 1)
```

mupad [B] time = 4.46, size = 120, normalized size = 2.11

$$\left(f x^2+e x+d\right)^p \left(\frac{x^2 \left(3 b f^2+2 b f^2 p+c e f p\right)}{p+1}+2 c f^2 x^3+\frac{d \left(3 b f-2 c e+2 b f p-c e p\right)}{p+1}+\frac{x \left(3 b e f-2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^p*(3*b*e*f - 2*c*e^2 + 2*c*d*f - c*e^2*p + 2*b*f^2*x*
(2*p + 3) + 2*c*f^2*x^2*(2*p + 3) + 2*b*e*f*p),x)
```

```
[Out] (d + e*x + f*x^2)^p*((x^2*(3*b*f^2 + 2*b*f^2*p + c*e*f*p))/(p + 1) + 2*c*f^
2*x^3 + (d*(3*b*f - 2*c*e + 2*b*f*p - c*e*p))/(p + 1) + (x*(3*b*e*f - 2*c*e
^2 + 2*c*d*f - c*e^2*p + 2*b*e*f*p + 2*c*d*f*p))/(p + 1))
```

sympy [B] time = 171.17, size = 483, normalized size = 8.47

$$\begin{cases} \frac{2bdfp\left(d+ex+fx^2\right)^p}{p+1}+\frac{3bdf\left(d+ex+fx^2\right)^p}{p+1}+\frac{2befpx\left(d+ex+fx^2\right)^p}{p+1}+\frac{3befx\left(d+ex+fx^2\right)^p}{p+1}+\frac{2bf^2px^2\left(d+ex+fx^2\right)^p}{p+1}+\frac{3bf^2x^2\left(d+ex+fx^2\right)^p}{p+1}- \\ bf\log\left(\frac{e}{2f}+x-\frac{\sqrt{-4df+e^2}}{2f}\right)+bf\log\left(\frac{e}{2f}+x+\frac{\sqrt{-4df+e^2}}{2f}\right)-ce\log\left(\frac{e}{2f}+x-\frac{\sqrt{-4df+e^2}}{2f}\right)-ce\log\left(\frac{e}{2f}+x+\frac{\sqrt{-}}{}\right. \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**p*(-2*c*e**2+2*c*d*f+3*b*e*f-c*e**2*p+2*b*e*f*p+2
*b*f**2*(3+2*p)*x+2*c*f**2*(3+2*p)*x**2),x)
```

```
[Out] Piecewise((2*b*d*f*p*(d + e*x + f*x**2)**p/(p + 1) + 3*b*d*f*(d + e*x + f*x
**2)**p/(p + 1) + 2*b*e*f*p*x*(d + e*x + f*x**2)**p/(p + 1) + 3*b*e*f*x*(d
+ e*x + f*x**2)**p/(p + 1) + 2*b*f**2*p*x**2*(d + e*x + f*x**2)**p/(p + 1)
+ 3*b*f**2*x**2*(d + e*x + f*x**2)**p/(p + 1) - c*d*e*p*(d + e*x + f*x**2)*
*p/(p + 1) - 2*c*d*e*(d + e*x + f*x**2)**p/(p + 1) + 2*c*d*f*p*x*(d + e*x +
 f*x**2)**p/(p + 1) + 2*c*d*f*x*(d + e*x + f*x**2)**p/(p + 1) - c*e**2*p*x*
(d + e*x + f*x**2)**p/(p + 1) - 2*c*e**2*x*(d + e*x + f*x**2)**p/(p + 1) +
c*e*f*p*x**2*(d + e*x + f*x**2)**p/(p + 1) + 2*c*f**2*p*x**3*(d + e*x + f*x
**2)**p/(p + 1) + 2*c*f**2*x**3*(d + e*x + f*x**2)**p/(p + 1), Ne(p, -1)),
(b*f*log(e/(2*f) + x - sqrt(-4*d*f + e**2)/(2*f)) + b*f*log(e/(2*f) + x + s
qrt(-4*d*f + e**2)/(2*f)) - c*e*log(e/(2*f) + x - sqrt(-4*d*f + e**2)/(2*f)
) - c*e*log(e/(2*f) + x + sqrt(-4*d*f + e**2)/(2*f)) + 2*c*f*x, True))
```

3.278 $\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17b\right.\right.$

Optimal. Leaf size=20

$$(d+ex)^5 \left(a+bx+cx^2\right)^6$$

```
[Out] (e*x+d)^5*(c*x^2+b*x+a)^6
```

Rubi [A] time = 0.42, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 75, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {1624, 1590}

$$(d+ex)^5 \left(a+bx+cx^2\right)^6$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*x + c*x^2)^5*(d*(6*b*d + 5*a*e) + (12*c*d^2 + 17*b*d
*e + 5*a*e^2)*x + e*(29*c*d + 11*b*e)*x^2 + 17*c*e^2*x^3),x]
```

```
[Out] (d + e*x)^5*(a + b*x + c*x^2)^6
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rule 1624

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^(m + 1)*PolynomialQuotient[Pq, d + e*x, x]*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[PolynomialRemainder[Pq, d + e*x, x], 0]
```

Rubi steps

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x+e(29cd+11be)x^2+17ce^2x^3\right)\,dx = \int (d+$$

$$= (d+e$$

Mathematica [B] time = 0.45, size = 167, normalized size = 8.35

$$x\left(a^6e\left(5d^4+10d^3ex+10d^2e^2x^2+5de^3x^3+e^4x^4\right)+6a^5(b+cx)(d+ex)^5+15a^4x(b+cx)^2(d+ex)^5+20a^3x^2(b+$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*x + c*x^2)^5*(d*(6*b*d + 5*a*e) + (12*c*d^2 +
17*b*d*e + 5*a*e^2)*x + e*(29*c*d + 11*b*e)*x^2 + 17*c*e^2*x^3),x]
```

```
[Out] x*(6*a^5*(b + c*x)*(d + e*x)^5 + 15*a^4*x*(b + c*x)^2*(d + e*x)^5 + 20*a^3*
x^2*(b + c*x)^3*(d + e*x)^5 + 15*a^2*x^3*(b + c*x)^4*(d + e*x)^5 + 6*a*x^4*
(b + c*x)^5*(d + e*x)^5 + x^5*(b + c*x)^6*(d + e*x)^5 + a^6*e*(5*d^4 + 10*d
^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4))
```

fricas [B] time = 0.44, size = 2467, normalized size = 123.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c
*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="fricas")
```

```
[Out] x^17*e^5*c^6 + 5*x^16*e^4*d*c^6 + 6*x^16*e^5*c^5*b + 10*x^15*e^3*d^2*c^6 +
30*x^15*e^4*d*c^5*b + 15*x^15*e^5*c^4*b^2 + 6*x^15*e^5*c^5*a + 10*x^14*e^2*
d^3*c^6 + 60*x^14*e^3*d^2*c^5*b + 75*x^14*e^4*d*c^4*b^2 + 20*x^14*e^5*c^3*b
^3 + 30*x^14*e^4*d*c^5*a + 30*x^14*e^5*c^4*b*a + 5*x^13*e*d^4*c^6 + 60*x^13
*e^2*d^3*c^5*b + 150*x^13*e^3*d^2*c^4*b^2 + 100*x^13*e^4*d*c^3*b^3 + 15*x^1
3*e^5*c^2*b^4 + 60*x^13*e^3*d^2*c^5*a + 150*x^13*e^4*d*c^4*b*a + 60*x^13*e^
5*c^3*b^2*a + 15*x^13*e^5*c^4*a^2 + x^12*d^5*c^6 + 30*x^12*e*d^4*c^5*b + 15
0*x^12*e^2*d^3*c^4*b^2 + 200*x^12*e^3*d^2*c^3*b^3 + 75*x^12*e^4*d*c^2*b^4 +
 6*x^12*e^5*c*b^5 + 60*x^12*e^2*d^3*c^5*a + 300*x^12*e^3*d^2*c^4*b*a + 300*
x^12*e^4*d*c^3*b^2*a + 60*x^12*e^5*c^2*b^3*a + 75*x^12*e^4*d*c^4*a^2 + 60*x
^12*e^5*c^3*b*a^2 + 6*x^11*d^5*c^5*b + 75*x^11*e*d^4*c^4*b^2 + 200*x^11*e^2
*d^3*c^3*b^3 + 150*x^11*e^3*d^2*c^2*b^4 + 30*x^11*e^4*d*c*b^5 + x^11*e^5*b^
6 + 30*x^11*e*d^4*c^5*a + 300*x^11*e^2*d^3*c^4*b*a + 600*x^11*e^3*d^2*c^3*b
^2*a + 300*x^11*e^4*d*c^2*b^3*a + 30*x^11*e^5*c*b^4*a + 150*x^11*e^3*d^2*c^
4*a^2 + 300*x^11*e^4*d*c^3*b*a^2 + 90*x^11*e^5*c^2*b^2*a^2 + 20*x^11*e^5*c^
3*a^3 + 15*x^10*d^5*c^4*b^2 + 100*x^10*e*d^4*c^3*b^3 + 150*x^10*e^2*d^3*c^2
*b^4 + 60*x^10*e^3*d^2*c*b^5 + 5*x^10*e^4*d*b^6 + 6*x^10*d^5*c^5*a + 150*x^
10*e*d^4*c^4*b*a + 600*x^10*e^2*d^3*c^3*b^2*a + 600*x^10*e^3*d^2*c^2*b^3*a
+ 150*x^10*e^4*d*c*b^4*a + 6*x^10*e^5*b^5*a + 150*x^10*e^2*d^3*c^4*a^2 + 60
0*x^10*e^3*d^2*c^3*b*a^2 + 450*x^10*e^4*d*c^2*b^2*a^2 + 60*x^10*e^5*c*b^3*a
^2 + 100*x^10*e^4*d*c^3*a^3 + 60*x^10*e^5*c^2*b*a^3 + 20*x^9*d^5*c^3*b^3 +
75*x^9*e*d^4*c^2*b^4 + 60*x^9*e^2*d^3*c*b^5 + 10*x^9*e^3*d^2*b^6 + 30*x^9*d
^5*c^4*b*a + 300*x^9*e*d^4*c^3*b^2*a + 600*x^9*e^2*d^3*c^2*b^3*a + 300*x^9*
e^3*d^2*c*b^4*a + 30*x^9*e^4*d*b^5*a + 75*x^9*e*d^4*c^4*a^2 + 600*x^9*e^2*d
^3*c^3*b*a^2 + 900*x^9*e^3*d^2*c^2*b^2*a^2 + 300*x^9*e^4*d*c*b^3*a^2 + 15*x
^9*e^5*b^4*a^2 + 200*x^9*e^3*d^2*c^3*a^3 + 300*x^9*e^4*d*c^2*b*a^3 + 60*x^9
*e^5*c*b^2*a^3 + 15*x^9*e^5*c^2*a^4 + 15*x^8*d^5*c^2*b^4 + 30*x^8*e*d^4*c*b
^5 + 10*x^8*e^2*d^3*b^6 + 60*x^8*d^5*c^3*b^2*a + 300*x^8*e*d^4*c^2*b^3*a +
300*x^8*e^2*d^3*c*b^4*a + 60*x^8*e^3*d^2*b^5*a + 15*x^8*d^5*c^4*a^2 + 300*x
^8*e*d^4*c^3*b*a^2 + 900*x^8*e^2*d^3*c^2*b^2*a^2 + 600*x^8*e^3*d^2*c*b^3*a^
2 + 75*x^8*e^4*d*b^4*a^2 + 200*x^8*e^2*d^3*c^3*a^3 + 600*x^8*e^3*d^2*c^2*b*
a^3 + 300*x^8*e^4*d*c*b^2*a^3 + 20*x^8*e^5*b^3*a^3 + 75*x^8*e^4*d*c^2*a^4 +
 30*x^8*e^5*c*b*a^4 + 6*x^7*d^5*c*b^5 + 5*x^7*e*d^4*b^6 + 60*x^7*d^5*c^2*b^
3*a + 150*x^7*e*d^4*c*b^4*a + 60*x^7*e^2*d^3*b^5*a + 60*x^7*d^5*c^3*b*a^2 +
 450*x^7*e*d^4*c^2*b^2*a^2 + 600*x^7*e^2*d^3*c*b^3*a^2 + 150*x^7*e^3*d^2*b^
4*a^2 + 100*x^7*e*d^4*c^3*a^3 + 600*x^7*e^2*d^3*c^2*b*a^3 + 600*x^7*e^3*d^2
*c*b^2*a^3 + 100*x^7*e^4*d*b^3*a^3 + 150*x^7*e^3*d^2*c^2*a^4 + 150*x^7*e^4*
d*c*b*a^4 + 15*x^7*e^5*b^2*a^4 + 6*x^7*e^5*c*a^5 + x^6*d^5*b^6 + 30*x^6*d^5
*c*b^4*a + 30*x^6*e*d^4*b^5*a + 90*x^6*d^5*c^2*b^2*a^2 + 300*x^6*e*d^4*c*b^
3*a^2 + 150*x^6*e^2*d^3*b^4*a^2 + 20*x^6*d^5*c^3*a^3 + 300*x^6*e*d^4*c^2*b*
a^3 + 600*x^6*e^2*d^3*c*b^2*a^3 + 200*x^6*e^3*d^2*b^3*a^3 + 150*x^6*e^2*d^3
*c^2*a^4 + 300*x^6*e^3*d^2*c*b*a^4 + 75*x^6*e^4*d*b^2*a^4 + 30*x^6*e^4*d*c*
a^5 + 6*x^6*e^5*b*a^5 + 6*x^5*d^5*b^5*a + 60*x^5*d^5*c*b^3*a^2 + 75*x^5*e*d
^4*b^4*a^2 + 60*x^5*d^5*c^2*b*a^3 + 300*x^5*e*d^4*c*b^2*a^3 + 200*x^5*e^2*d
^3*b^3*a^3 + 75*x^5*e*d^4*c^2*a^4 + 300*x^5*e^2*d^3*c*b*a^4 + 150*x^5*e^3*d
^2*b^2*a^4 + 60*x^5*e^3*d^2*c*a^5 + 30*x^5*e^4*d*b*a^5 + x^5*e^5*a^6 + 15*x
^4*d^5*b^4*a^2 + 60*x^4*d^5*c*b^2*a^3 + 100*x^4*e*d^4*b^3*a^3 + 15*x^4*d^5*
c^2*a^4 + 150*x^4*e*d^4*c*b*a^4 + 150*x^4*e^2*d^3*b^2*a^4 + 60*x^4*e^2*d^3*
c*a^5 + 60*x^4*e^3*d^2*b*a^5 + 5*x^4*e^4*d*a^6 + 20*x^3*d^5*b^3*a^3 + 30*x^
3*d^5*c*b*a^4 + 75*x^3*e*d^4*b^2*a^4 + 30*x^3*e*d^4*c*a^5 + 60*x^3*e^2*d^3*
b*a^5 + 10*x^3*e^3*d^2*a^6 + 15*x^2*d^5*b^2*a^4 + 6*x^2*d^5*c*a^5 + 30*x^2*
e*d^4*b*a^5 + 10*x^2*e^2*d^3*a^6 + 6*x*d^5*b*a^5 + 5*x*e*d^4*a^6
```

giac [B] time = 0.23, size = 2383, normalized size = 119.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c
*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="giac")
```

```
[Out] c^6*x^17*e^5 + 5*c^6*d*x^16*e^4 + 10*c^6*d^2*x^15*e^3 + 10*c^6*d^3*x^14*e^2
 + 5*c^6*d^4*x^13*e + c^6*d^5*x^12 + 6*b*c^5*x^16*e^5 + 30*b*c^5*d*x^15*e^4
 + 60*b*c^5*d^2*x^14*e^3 + 60*b*c^5*d^3*x^13*e^2 + 30*b*c^5*d^4*x^12*e + 6*
b*c^5*d^5*x^11 + 15*b^2*c^4*x^15*e^5 + 6*a*c^5*x^15*e^5 + 75*b^2*c^4*d*x^14
*e^4 + 30*a*c^5*d*x^14*e^4 + 150*b^2*c^4*d^2*x^13*e^3 + 60*a*c^5*d^2*x^13*e
^3 + 150*b^2*c^4*d^3*x^12*e^2 + 60*a*c^5*d^3*x^12*e^2 + 75*b^2*c^4*d^4*x^11
*e + 30*a*c^5*d^4*x^11*e + 15*b^2*c^4*d^5*x^10 + 6*a*c^5*d^5*x^10 + 20*b^3*
c^3*x^14*e^5 + 30*a*b*c^4*x^14*e^5 + 100*b^3*c^3*d*x^13*e^4 + 150*a*b*c^4*d
*x^13*e^4 + 200*b^3*c^3*d^2*x^12*e^3 + 300*a*b*c^4*d^2*x^12*e^3 + 200*b^3*c
^3*d^3*x^11*e^2 + 300*a*b*c^4*d^3*x^11*e^2 + 100*b^3*c^3*d^4*x^10*e + 150*a
*b*c^4*d^4*x^10*e + 20*b^3*c^3*d^5*x^9 + 30*a*b*c^4*d^5*x^9 + 15*b^4*c^2*x^
13*e^5 + 60*a*b^2*c^3*x^13*e^5 + 15*a^2*c^4*x^13*e^5 + 75*b^4*c^2*d*x^12*e^
4 + 300*a*b^2*c^3*d*x^12*e^4 + 75*a^2*c^4*d*x^12*e^4 + 150*b^4*c^2*d^2*x^11
*e^3 + 600*a*b^2*c^3*d^2*x^11*e^3 + 150*a^2*c^4*d^2*x^11*e^3 + 150*b^4*c^2*
d^3*x^10*e^2 + 600*a*b^2*c^3*d^3*x^10*e^2 + 150*a^2*c^4*d^3*x^10*e^2 + 75*b
^4*c^2*d^4*x^9*e + 300*a*b^2*c^3*d^4*x^9*e + 75*a^2*c^4*d^4*x^9*e + 15*b^4*
c^2*d^5*x^8 + 60*a*b^2*c^3*d^5*x^8 + 15*a^2*c^4*d^5*x^8 + 6*b^5*c*x^12*e^5
+ 60*a*b^3*c^2*x^12*e^5 + 60*a^2*b*c^3*x^12*e^5 + 30*b^5*c*d*x^11*e^4 + 300
*a*b^3*c^2*d*x^11*e^4 + 300*a^2*b*c^3*d*x^11*e^4 + 60*b^5*c*d^2*x^10*e^3 +
600*a*b^3*c^2*d^2*x^10*e^3 + 600*a^2*b*c^3*d^2*x^10*e^3 + 60*b^5*c*d^3*x^9*
e^2 + 600*a*b^3*c^2*d^3*x^9*e^2 + 600*a^2*b*c^3*d^3*x^9*e^2 + 30*b^5*c*d^4*
x^8*e + 300*a*b^3*c^2*d^4*x^8*e + 300*a^2*b*c^3*d^4*x^8*e + 6*b^5*c*d^5*x^7
 + 60*a*b^3*c^2*d^5*x^7 + 60*a^2*b*c^3*d^5*x^7 + b^6*x^11*e^5 + 30*a*b^4*c*
x^11*e^5 + 90*a^2*b^2*c^2*x^11*e^5 + 20*a^3*c^3*x^11*e^5 + 5*b^6*d*x^10*e^4
 + 150*a*b^4*c*d*x^10*e^4 + 450*a^2*b^2*c^2*d*x^10*e^4 + 100*a^3*c^3*d*x^10
*e^4 + 10*b^6*d^2*x^9*e^3 + 300*a*b^4*c*d^2*x^9*e^3 + 900*a^2*b^2*c^2*d^2*x
^9*e^3 + 200*a^3*c^3*d^2*x^9*e^3 + 10*b^6*d^3*x^8*e^2 + 300*a*b^4*c*d^3*x^8
*e^2 + 900*a^2*b^2*c^2*d^3*x^8*e^2 + 200*a^3*c^3*d^3*x^8*e^2 + 5*b^6*d^4*x^
7*e + 150*a*b^4*c*d^4*x^7*e + 450*a^2*b^2*c^2*d^4*x^7*e + 100*a^3*c^3*d^4*x
^7*e + b^6*d^5*x^6 + 30*a*b^4*c*d^5*x^6 + 90*a^2*b^2*c^2*d^5*x^6 + 20*a^3*c
^3*d^5*x^6 + 6*a*b^5*x^10*e^5 + 60*a^2*b^3*c*x^10*e^5 + 60*a^3*b*c^2*x^10*e
^5 + 30*a*b^5*d*x^9*e^4 + 300*a^2*b^3*c*d*x^9*e^4 + 300*a^3*b*c^2*d*x^9*e^4
 + 60*a*b^5*d^2*x^8*e^3 + 600*a^2*b^3*c*d^2*x^8*e^3 + 600*a^3*b*c^2*d^2*x^8
*e^3 + 60*a*b^5*d^3*x^7*e^2 + 600*a^2*b^3*c*d^3*x^7*e^2 + 600*a^3*b*c^2*d^3
*x^7*e^2 + 30*a*b^5*d^4*x^6*e + 300*a^2*b^3*c*d^4*x^6*e + 300*a^3*b*c^2*d^4
*x^6*e + 6*a*b^5*d^5*x^5 + 60*a^2*b^3*c*d^5*x^5 + 60*a^3*b*c^2*d^5*x^5 + 15
*a^2*b^4*x^9*e^5 + 60*a^3*b^2*c*x^9*e^5 + 15*a^4*c^2*x^9*e^5 + 75*a^2*b^4*d
*x^8*e^4 + 300*a^3*b^2*c*d*x^8*e^4 + 75*a^4*c^2*d*x^8*e^4 + 150*a^2*b^4*d^2
*x^7*e^3 + 600*a^3*b^2*c*d^2*x^7*e^3 + 150*a^4*c^2*d^2*x^7*e^3 + 150*a^2*b^
4*d^3*x^6*e^2 + 600*a^3*b^2*c*d^3*x^6*e^2 + 150*a^4*c^2*d^3*x^6*e^2 + 75*a^
2*b^4*d^4*x^5*e + 300*a^3*b^2*c*d^4*x^5*e + 75*a^4*c^2*d^4*x^5*e + 15*a^2*b
^4*d^5*x^4 + 60*a^3*b^2*c*d^5*x^4 + 15*a^4*c^2*d^5*x^4 + 20*a^3*b^3*x^8*e^5
 + 30*a^4*b*c*x^8*e^5 + 100*a^3*b^3*d*x^7*e^4 + 150*a^4*b*c*d*x^7*e^4 + 200
*a^3*b^3*d^2*x^6*e^3 + 300*a^4*b*c*d^2*x^6*e^3 + 200*a^3*b^3*d^3*x^5*e^2 +
300*a^4*b*c*d^3*x^5*e^2 + 100*a^3*b^3*d^4*x^4*e + 150*a^4*b*c*d^4*x^4*e + 2
0*a^3*b^3*d^5*x^3 + 30*a^4*b*c*d^5*x^3 + 15*a^4*b^2*x^7*e^5 + 6*a^5*c*x^7*e
^5 + 75*a^4*b^2*d*x^6*e^4 + 30*a^5*c*d*x^6*e^4 + 150*a^4*b^2*d^2*x^5*e^3 +
60*a^5*c*d^2*x^5*e^3 + 150*a^4*b^2*d^3*x^4*e^2 + 60*a^5*c*d^3*x^4*e^2 + 75*
a^4*b^2*d^4*x^3*e + 30*a^5*c*d^4*x^3*e + 15*a^4*b^2*d^5*x^2 + 6*a^5*c*d^5*x
^2 + 6*a^5*b*x^6*e^5 + 30*a^5*b*d*x^5*e^4 + 60*a^5*b*d^2*x^4*e^3 + 60*a^5*b
*d^3*x^3*e^2 + 30*a^5*b*d^4*x^2*e + 6*a^5*b*d^5*x + a^6*x^5*e^5 + 5*a^6*d*x
^4*e^4 + 10*a^6*d^2*x^3*e^3 + 10*a^6*d^3*x^2*e^2 + 5*a^6*d^4*x*e
```

maple [B] time = 0.00, size = 8419, normalized size = 420.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c*d^2)*
x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.50, size = 1779, normalized size = 88.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c
*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="maxima")
```

```
[Out] c^6*e^5*x^17 + (5*c^6*d*e^4 + 6*b*c^5*e^5)*x^16 + (10*c^6*d^2*e^3 + 30*b*c^
5*d*e^4 + 3*(5*b^2*c^4 + 2*a*c^5)*e^5)*x^15 + 5*(2*c^6*d^3*e^2 + 12*b*c^5*d
^2*e^3 + 3*(5*b^2*c^4 + 2*a*c^5)*d*e^4 + 2*(2*b^3*c^3 + 3*a*b*c^4)*e^5)*x^1
4 + 5*(c^6*d^4*e + 12*b*c^5*d^3*e^2 + 6*(5*b^2*c^4 + 2*a*c^5)*d^2*e^3 + 10*
(2*b^3*c^3 + 3*a*b*c^4)*d*e^4 + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*e^5)*x^
13 + (c^6*d^5 + 30*b*c^5*d^4*e + 30*(5*b^2*c^4 + 2*a*c^5)*d^3*e^2 + 100*(2*
b^3*c^3 + 3*a*b*c^4)*d^2*e^3 + 75*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d*e^4 +
 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*e^5)*x^12 + (6*b*c^5*d^5 + 15*(5*b
^2*c^4 + 2*a*c^5)*d^4*e + 100*(2*b^3*c^3 + 3*a*b*c^4)*d^3*e^2 + 150*(b^4*c^
2 + 4*a*b^2*c^3 + a^2*c^4)*d^2*e^3 + 30*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^
3)*d*e^4 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*e^5)*x^11 + (3*
(5*b^2*c^4 + 2*a*c^5)*d^5 + 50*(2*b^3*c^3 + 3*a*b*c^4)*d^4*e + 150*(b^4*c^2
 + 4*a*b^2*c^3 + a^2*c^4)*d^3*e^2 + 60*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3
)*d^2*e^3 + 5*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d*e^4 + 6*(a
*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*e^5)*x^10 + 5*(2*(2*b^3*c^3 + 3*a*b*c^4
)*d^5 + 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^4*e + 12*(b^5*c + 10*a*b^3*c
^2 + 10*a^2*b*c^3)*d^3*e^2 + 2*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*
c^3)*d^2*e^3 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d*e^4 + 3*(a^2*b^4 +
 4*a^3*b^2*c + a^4*c^2)*e^5)*x^9 + 5*(3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d
^5 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^4*e + 2*(b^6 + 30*a*b^4*c +
90*a^2*b^2*c^2 + 20*a^3*c^3)*d^3*e^2 + 12*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*
c^2)*d^2*e^3 + 15*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d*e^4 + 2*(2*a^3*b^3 +
3*a^4*b*c)*e^5)*x^8 + (6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^5 + 5*(b^6
 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^4*e + 60*(a*b^5 + 10*a^2*b^3
*c + 10*a^3*b*c^2)*d^3*e^2 + 150*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^2*e^3
+ 50*(2*a^3*b^3 + 3*a^4*b*c)*d*e^4 + 3*(5*a^4*b^2 + 2*a^5*c)*e^5)*x^7 + (6*
a^5*b*e^5 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^5 + 30*(a*b^
5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d^4*e + 150*(a^2*b^4 + 4*a^3*b^2*c + a^4*c
^2)*d^3*e^2 + 100*(2*a^3*b^3 + 3*a^4*b*c)*d^2*e^3 + 15*(5*a^4*b^2 + 2*a^5*c
)*d*e^4)*x^6 + (30*a^5*b*d*e^4 + a^6*e^5 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3
*b*c^2)*d^5 + 75*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^4*e + 100*(2*a^3*b^3 +
 3*a^4*b*c)*d^3*e^2 + 30*(5*a^4*b^2 + 2*a^5*c)*d^2*e^3)*x^5 + 5*(12*a^5*b*d
^2*e^3 + a^6*d*e^4 + 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^5 + 10*(2*a^3*b^
3 + 3*a^4*b*c)*d^4*e + 6*(5*a^4*b^2 + 2*a^5*c)*d^3*e^2)*x^4 + 5*(12*a^5*b*d
^3*e^2 + 2*a^6*d^2*e^3 + 2*(2*a^3*b^3 + 3*a^4*b*c)*d^5 + 3*(5*a^4*b^2 + 2*a
^5*c)*d^4*e)*x^3 + (30*a^5*b*d^4*e + 10*a^6*d^3*e^2 + 3*(5*a^4*b^2 + 2*a^5*
c)*d^5)*x^2 + (6*a^5*b*d^5 + 5*a^6*d^4*e)*x
```

mupad [B] time = 4.87, size = 2026, normalized size = 101.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(a + b*x + c*x^2)^5*(d*(5*a*e + 6*b*d) + x*(5*a*e^2 + 12*c*
d^2 + 17*b*d*e) + e*x^2*(11*b*e + 29*c*d) + 17*c*e^2*x^3),x)
```

```
[Out] x^6*(b^6*d^5 + 6*a^5*b*e^5 + 20*a^3*c^3*d^5 + 75*a^4*b^2*d*e^4 + 90*a^2*b^2
*c^2*d^5 + 150*a^2*b^4*d^3*e^2 + 200*a^3*b^3*d^2*e^3 + 150*a^4*c^2*d^3*e^2
+ 30*a*b^4*c*d^5 + 30*a*b^5*d^4*e + 30*a^5*c*d*e^4 + 300*a^2*b^3*c*d^4*e +
300*a^3*b*c^2*d^4*e + 300*a^4*b*c*d^2*e^3 + 600*a^3*b^2*c*d^3*e^2) + x^11*(
b^6*e^5 + 6*b*c^5*d^5 + 20*a^3*c^3*e^5 + 75*b^2*c^4*d^4*e + 90*a^2*b^2*c^2*
e^5 + 150*a^2*c^4*d^2*e^3 + 200*b^3*c^3*d^3*e^2 + 150*b^4*c^2*d^2*e^3 + 30*
a*b^4*c*e^5 + 30*a*c^5*d^4*e + 30*b^5*c*d*e^4 + 300*a*b*c^4*d^3*e^2 + 300*a
*b^3*c^2*d*e^4 + 300*a^2*b*c^3*d*e^4 + 600*a*b^2*c^3*d^2*e^3) + x^5*(a^6*e^
5 + 6*a*b^5*d^5 + 60*a^2*b^3*c*d^5 + 60*a^3*b*c^2*d^5 + 75*a^2*b^4*d^4*e +
75*a^4*c^2*d^4*e + 60*a^5*c*d^2*e^3 + 200*a^3*b^3*d^3*e^2 + 150*a^4*b^2*d^2
*e^3 + 30*a^5*b*d*e^4 + 300*a^3*b^2*c*d^4*e + 300*a^4*b*c*d^3*e^2) + x^3*(2
0*a^3*b^3*d^5 + 10*a^6*d^2*e^3 + 75*a^4*b^2*d^4*e + 60*a^5*b*d^3*e^2 + 30*a
^4*b*c*d^5 + 30*a^5*c*d^4*e) + x^12*(c^6*d^5 + 6*b^5*c*e^5 + 60*a*b^3*c^2*e
^5 + 60*a^2*b*c^3*e^5 + 60*a*c^5*d^3*e^2 + 75*a^2*c^4*d*e^4 + 75*b^4*c^2*d*
e^4 + 150*b^2*c^4*d^3*e^2 + 200*b^3*c^3*d^2*e^3 + 30*b*c^5*d^4*e + 300*a*b*
c^4*d^2*e^3 + 300*a*b^2*c^3*d*e^4) + x^7*(6*a^5*c*e^5 + 6*b^5*c*d^5 + 5*b^6
*d^4*e + 15*a^4*b^2*e^5 + 60*a*b^3*c^2*d^5 + 60*a^2*b*c^3*d^5 + 60*a*b^5*d^
3*e^2 + 100*a^3*b^3*d*e^4 + 100*a^3*c^3*d^4*e + 150*a^2*b^4*d^2*e^3 + 150*a
^4*c^2*d^2*e^3 + 150*a*b^4*c*d^4*e + 150*a^4*b*c*d*e^4 + 450*a^2*b^2*c^2*d^
4*e + 600*a^2*b^3*c*d^3*e^2 + 600*a^3*b*c^2*d^3*e^2 + 600*a^3*b^2*c*d^2*e^3
) + x^10*(6*a*b^5*e^5 + 6*a*c^5*d^5 + 5*b^6*d*e^4 + 15*b^2*c^4*d^5 + 60*a^2
*b^3*c*e^5 + 60*a^3*b*c^2*e^5 + 100*a^3*c^3*d*e^4 + 100*b^3*c^3*d^4*e + 60*
b^5*c*d^2*e^3 + 150*a^2*c^4*d^3*e^2 + 150*b^4*c^2*d^3*e^2 + 150*a*b*c^4*d^4
*e + 150*a*b^4*c*d*e^4 + 600*a*b^2*c^3*d^3*e^2 + 600*a*b^3*c^2*d^2*e^3 + 60
0*a^2*b*c^3*d^2*e^3 + 450*a^2*b^2*c^2*d*e^4) + x^8*(15*a^2*c^4*d^5 + 20*a^3
*b^3*e^5 + 15*b^4*c^2*d^5 + 10*b^6*d^3*e^2 + 60*a*b^2*c^3*d^5 + 60*a*b^5*d^
2*e^3 + 75*a^2*b^4*d*e^4 + 75*a^4*c^2*d*e^4 + 200*a^3*c^3*d^3*e^2 + 30*a^4*
b*c*e^5 + 30*b^5*c*d^4*e + 900*a^2*b^2*c^2*d^3*e^2 + 300*a*b^3*c^2*d^4*e +
300*a*b^4*c*d^3*e^2 + 300*a^2*b*c^3*d^4*e + 300*a^3*b^2*c*d*e^4 + 600*a^2*b
^3*c*d^2*e^3 + 600*a^3*b*c^2*d^2*e^3) + x^9*(15*a^2*b^4*e^5 + 15*a^4*c^2*e^
5 + 20*b^3*c^3*d^5 + 10*b^6*d^2*e^3 + 60*a^3*b^2*c*e^5 + 75*a^2*c^4*d^4*e +
 75*b^4*c^2*d^4*e + 60*b^5*c*d^3*e^2 + 200*a^3*c^3*d^2*e^3 + 30*a*b*c^4*d^5
 + 30*a*b^5*d*e^4 + 900*a^2*b^2*c^2*d^2*e^3 + 300*a*b^2*c^3*d^4*e + 300*a*b
^4*c*d^2*e^3 + 300*a^2*b^3*c*d*e^4 + 300*a^3*b*c^2*d*e^4 + 600*a*b^3*c^2*d^
3*e^2 + 600*a^2*b*c^3*d^3*e^2) + x^4*(5*a^6*d*e^4 + 15*a^2*b^4*d^5 + 15*a^4
*c^2*d^5 + 60*a^3*b^2*c*d^5 + 100*a^3*b^3*d^4*e + 60*a^5*b*d^2*e^3 + 60*a^5
*c*d^3*e^2 + 150*a^4*b^2*d^3*e^2 + 150*a^4*b*c*d^4*e) + x^13*(5*c^6*d^4*e +
 15*a^2*c^4*e^5 + 15*b^4*c^2*e^5 + 60*a*b^2*c^3*e^5 + 60*a*c^5*d^2*e^3 + 60
*b*c^5*d^3*e^2 + 100*b^3*c^3*d*e^4 + 150*b^2*c^4*d^2*e^3 + 150*a*b*c^4*d*e^
4) + c^6*e^5*x^17 + a^5*d^4*x*(5*a*e + 6*b*d) + 5*c^3*e^2*x^14*(4*b^3*e^3 +
 2*c^3*d^3 + 6*a*b*c*e^3 + 6*a*c^2*d*e^2 + 12*b*c^2*d^2*e + 15*b^2*c*d*e^2)
 + c^5*e^4*x^16*(6*b*e + 5*c*d) + a^4*d^3*x^2*(10*a^2*e^2 + 15*b^2*d^2 + 6*
a*c*d^2 + 30*a*b*d*e) + c^4*e^3*x^15*(15*b^2*e^2 + 10*c^2*d^2 + 6*a*c*e^2 +
 30*b*c*d*e)
```

sympy [B] time = 1.53, size = 2281, normalized size = 114.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**5*(d*(5*a*e+6*b*d)+(5*a*e**2+17*b*d*e+
12*c*d**2)*x+e*(11*b*e+29*c*d)*x**2+17*c*e**2*x**3),x)
```

```
[Out] c**6*e**5*x**17 + x**16*(6*b*c**5*e**5 + 5*c**6*d*e**4) + x**15*(6*a*c**5*e
**5 + 15*b**2*c**4*e**5 + 30*b*c**5*d*e**4 + 10*c**6*d**2*e**3) + x**14*(30
```

```
*a*b*c**4*e**5 + 30*a*c**5*d*e**4 + 20*b**3*c**3*e**5 + 75*b**2*c**4*d*e**4
 + 60*b*c**5*d**2*e**3 + 10*c**6*d**3*e**2) + x**13*(15*a**2*c**4*e**5 + 60
*a*b**2*c**3*e**5 + 150*a*b*c**4*d*e**4 + 60*a*c**5*d**2*e**3 + 15*b**4*c**
2*e**5 + 100*b**3*c**3*d*e**4 + 150*b**2*c**4*d**2*e**3 + 60*b*c**5*d**3*e*
*2 + 5*c**6*d**4*e) + x**12*(60*a**2*b*c**3*e**5 + 75*a**2*c**4*d*e**4 + 60
*a*b**3*c**2*e**5 + 300*a*b**2*c**3*d*e**4 + 300*a*b*c**4*d**2*e**3 + 60*a*
c**5*d**3*e**2 + 6*b**5*c*e**5 + 75*b**4*c**2*d*e**4 + 200*b**3*c**3*d**2*e
**3 + 150*b**2*c**4*d**3*e**2 + 30*b*c**5*d**4*e + c**6*d**5) + x**11*(20*a
**3*c**3*e**5 + 90*a**2*b**2*c**2*e**5 + 300*a**2*b*c**3*d*e**4 + 150*a**2*
c**4*d**2*e**3 + 30*a*b**4*c*e**5 + 300*a*b**3*c**2*d*e**4 + 600*a*b**2*c**
3*d**2*e**3 + 300*a*b*c**4*d**3*e**2 + 30*a*c**5*d**4*e + b**6*e**5 + 30*b*
*5*c*d*e**4 + 150*b**4*c**2*d**2*e**3 + 200*b**3*c**3*d**3*e**2 + 75*b**2*c
**4*d**4*e + 6*b*c**5*d**5) + x**10*(60*a**3*b*c**2*e**5 + 100*a**3*c**3*d*
e**4 + 60*a**2*b**3*c*e**5 + 450*a**2*b**2*c**2*d*e**4 + 600*a**2*b*c**3*d*
*2*e**3 + 150*a**2*c**4*d**3*e**2 + 6*a*b**5*e**5 + 150*a*b**4*c*d*e**4 + 6
00*a*b**3*c**2*d**2*e**3 + 600*a*b**2*c**3*d**3*e**2 + 150*a*b*c**4*d**4*e
+ 6*a*c**5*d**5 + 5*b**6*d*e**4 + 60*b**5*c*d**2*e**3 + 150*b**4*c**2*d**3*
e**2 + 100*b**3*c**3*d**4*e + 15*b**2*c**4*d**5) + x**9*(15*a**4*c**2*e**5
+ 60*a**3*b**2*c*e**5 + 300*a**3*b*c**2*d*e**4 + 200*a**3*c**3*d**2*e**3 +
15*a**2*b**4*e**5 + 300*a**2*b**3*c*d*e**4 + 900*a**2*b**2*c**2*d**2*e**3 +
 600*a**2*b*c**3*d**3*e**2 + 75*a**2*c**4*d**4*e + 30*a*b**5*d*e**4 + 300*a
*b**4*c*d**2*e**3 + 600*a*b**3*c**2*d**3*e**2 + 300*a*b**2*c**3*d**4*e + 30
*a*b*c**4*d**5 + 10*b**6*d**2*e**3 + 60*b**5*c*d**3*e**2 + 75*b**4*c**2*d**
4*e + 20*b**3*c**3*d**5) + x**8*(30*a**4*b*c*e**5 + 75*a**4*c**2*d*e**4 + 2
0*a**3*b**3*e**5 + 300*a**3*b**2*c*d*e**4 + 600*a**3*b*c**2*d**2*e**3 + 200
*a**3*c**3*d**3*e**2 + 75*a**2*b**4*d*e**4 + 600*a**2*b**3*c*d**2*e**3 + 90
0*a**2*b**2*c**2*d**3*e**2 + 300*a**2*b*c**3*d**4*e + 15*a**2*c**4*d**5 + 6
0*a*b**5*d**2*e**3 + 300*a*b**4*c*d**3*e**2 + 300*a*b**3*c**2*d**4*e + 60*a
*b**2*c**3*d**5 + 10*b**6*d**3*e**2 + 30*b**5*c*d**4*e + 15*b**4*c**2*d**5)
 + x**7*(6*a**5*c*e**5 + 15*a**4*b**2*e**5 + 150*a**4*b*c*d*e**4 + 150*a**4
*c**2*d**2*e**3 + 100*a**3*b**3*d*e**4 + 600*a**3*b**2*c*d**2*e**3 + 600*a*
*3*b*c**2*d**3*e**2 + 100*a**3*c**3*d**4*e + 150*a**2*b**4*d**2*e**3 + 600*
a**2*b**3*c*d**3*e**2 + 450*a**2*b**2*c**2*d**4*e + 60*a**2*b*c**3*d**5 + 6
0*a*b**5*d**3*e**2 + 150*a*b**4*c*d**4*e + 60*a*b**3*c**2*d**5 + 5*b**6*d**
4*e + 6*b**5*c*d**5) + x**6*(6*a**5*b*e**5 + 30*a**5*c*d*e**4 + 75*a**4*b**
2*d*e**4 + 300*a**4*b*c*d**2*e**3 + 150*a**4*c**2*d**3*e**2 + 200*a**3*b**3
*d**2*e**3 + 600*a**3*b**2*c*d**3*e**2 + 300*a**3*b*c**2*d**4*e + 20*a**3*c
**3*d**5 + 150*a**2*b**4*d**3*e**2 + 300*a**2*b**3*c*d**4*e + 90*a**2*b**2*
c**2*d**5 + 30*a*b**5*d**4*e + 30*a*b**4*c*d**5 + b**6*d**5) + x**5*(a**6*e
**5 + 30*a**5*b*d*e**4 + 60*a**5*c*d**2*e**3 + 150*a**4*b**2*d**2*e**3 + 30
0*a**4*b*c*d**3*e**2 + 75*a**4*c**2*d**4*e + 200*a**3*b**3*d**3*e**2 + 300*
a**3*b**2*c*d**4*e + 60*a**3*b*c**2*d**5 + 75*a**2*b**4*d**4*e + 60*a**2*b*
*3*c*d**5 + 6*a*b**5*d**5) + x**4*(5*a**6*d*e**4 + 60*a**5*b*d**2*e**3 + 60
*a**5*c*d**3*e**2 + 150*a**4*b**2*d**3*e**2 + 150*a**4*b*c*d**4*e + 15*a**4
*c**2*d**5 + 100*a**3*b**3*d**4*e + 60*a**3*b**2*c*d**5 + 15*a**2*b**4*d**5
) + x**3*(10*a**6*d**2*e**3 + 60*a**5*b*d**3*e**2 + 30*a**5*c*d**4*e + 75*a
**4*b**2*d**4*e + 30*a**4*b*c*d**5 + 20*a**3*b**3*d**5) + x**2*(10*a**6*d**
3*e**2 + 30*a**5*b*d**4*e + 6*a**5*c*d**5 + 15*a**4*b**2*d**5) + x*(5*a**6*
d**4*e + 6*a**5*b*d**5)
```

3.279 $\int \frac{x^2+x^3}{-2+x+x^2} \, dx$

Optimal. Leaf size=26

$$\frac{x^2}{2}+\frac{2}{3}\log(1-x)+\frac{4}{3}\log(x+2)$$

```
[Out] 1/2*x^2+2/3*ln(1-x)+4/3*ln(2+x)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1593, 800, 632, 31}

$$\frac{x^2}{2}+\frac{2}{3}\log(1-x)+\frac{4}{3}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2 + x^3)/(-2 + x + x^2),x]
```

```
[Out] x^2/2 + (2*Log[1 - x])/3 + (4*Log[2 + x])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2+x^3}{-2+x+x^2} dx &= \int \frac{x^2(1+x)}{-2+x+x^2} dx \\
&= \int \left(x+\frac{2x}{-2+x+x^2}\right) dx \\
&= \frac{x^2}{2}+2\int \frac{x}{-2+x+x^2} dx \\
&= \frac{x^2}{2}+\frac{2}{3}\int \frac{1}{-1+x} dx+\frac{4}{3}\int \frac{1}{2+x} dx \\
&= \frac{x^2}{2}+\frac{2}{3}\log(1-x)+\frac{4}{3}\log(2+x)
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$\frac{x^2}{2}+\frac{2}{3}\log(1-x)+\frac{4}{3}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2 + x^3)/(-2 + x + x^2),x]
[Out] x^2/2 + (2*Log[1 - x])/3 + (4*Log[2 + x])/3
```

fricas [A] time = 0.54, size = 18, normalized size = 0.69

$$\frac{1}{2}x^2+\frac{4}{3}\log\left(x+2\right)+\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)/(x^2+x-2),x, algorithm="fricas")
[Out] 1/2*x^2 + 4/3*log(x + 2) + 2/3*log(x - 1)
```

giac [A] time = 0.16, size = 20, normalized size = 0.77

$$\frac{1}{2}x^2+\frac{4}{3}\log\left(|x+2|\right)+\frac{2}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)/(x^2+x-2),x, algorithm="giac")
[Out] 1/2*x^2 + 4/3*log(abs(x + 2)) + 2/3*log(abs(x - 1))
```

maple [A] time = 0.00, size = 19, normalized size = 0.73

$$\frac{x^2}{2}+\frac{4\ln\left(x+2\right)}{3}+\frac{2\ln\left(x-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2)/(x^2+x-2),x)
[Out] 1/2*x^2+4/3*ln(x+2)+2/3*ln(x-1)
```

maxima [A] time = 0.43, size = 18, normalized size = 0.69

$$\frac{1}{2}x^2+\frac{4}{3}\log\left(x+2\right)+\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)/(x^2+x-2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 4/3*log(x + 2) + 2/3*log(x - 1)
```

mupad [B] time = 0.05, size = 18, normalized size = 0.69

$$\frac{2\ \ln(x-1)}{3}+\frac{4\ \ln(x+2)}{3}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3)/(x + x^2 - 2),x)
```

```
[Out] (2*log(x - 1))/3 + (4*log(x + 2))/3 + x^2/2
```

sympy [A] time = 0.24, size = 20, normalized size = 0.77

$$\frac{x^2}{2}+\frac{2\log(x-1)}{3}+\frac{4\log(x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2)/(x**2+x-2),x)
```

```
[Out] x**2/2 + 2*log(x - 1)/3 + 4*log(x + 2)/3
```

3.280 $\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=346

$$\frac{x^2\sqrt{a+bx+cx^2}\left(-64acg+63b^2g-70bcf+80c^2e\right)}{240c^3}-\frac{\sqrt{a+bx+cx^2}\left(-2cx\left(-40c^2(9af+10be)+14bc(46ag\right.\right.}{}$$

```
[Out] 1/256*(70*b^4*c*f+48*b^2*c^2*(-5*a*f+2*c*d)-32*a*c^3*(-3*a*f+4*c*d)-63*b^5*
g-40*b^3*c*(-7*a*g+2*c*e)+48*a*b*c^2*(-5*a*g+4*c*e))*arctanh(1/2*(2*c*x+b)/
c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/240*(-64*a*c*g+63*b^2*g-70*b*c*f+80
*c^2*e)*x^2*(c*x^2+b*x+a)^(1/2)/c^3+1/40*(-9*b*g+10*c*f)*x^3*(c*x^2+b*x+a)^
(1/2)/c^2+1/5*g*x^4*(c*x^2+b*x+a)^(1/2)/c-1/1920*(1050*b^3*c*f+40*b*c^2*(-5
5*a*f+36*c*d)-945*b^4*g-60*b^2*c*(-49*a*g+20*c*e)+256*a*c^2*(-4*a*g+5*c*e)-
2*c*(480*c^3*d-40*c^2*(9*a*f+10*b*e)-315*b^3*g+14*b*c*(46*a*g+25*b*f))*x)*(
c*x^2+b*x+a)^(1/2)/c^5
```

Rubi [A] time = 0.81, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1653, 832, 779, 621, 206}

$$-\frac{\sqrt{a+bx+cx^2}\left(-2cx\left(-40c^2(9af+10be)+14bc(46ag+25bf)-315b^3g+480c^3d\right)-60b^2c(20ce-49ag)+\right.}{1920c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((80*c^2*e - 70*b*c*f + 63*b^2*g - 64*a*c*g)*x^2*Sqrt[a + b*x + c*x^2])/(24
0*c^3) + ((10*c*f - 9*b*g)*x^3*Sqrt[a + b*x + c*x^2])/(40*c^2) + (g*x^4*Sqr
t[a + b*x + c*x^2])/(5*c) - ((1050*b^3*c*f + 40*b*c^2*(36*c*d - 55*a*f) - 9
45*b^4*g - 60*b^2*c*(20*c*e - 49*a*g) + 256*a*c^2*(5*c*e - 4*a*g) - 2*c*(48
0*c^3*d - 40*c^2*(10*b*e + 9*a*f) - 315*b^3*g + 14*b*c*(25*b*f + 46*a*g))*x
)*Sqrt[a + b*x + c*x^2])/(1920*c^5) + ((70*b^4*c*f + 48*b^2*c^2*(2*c*d - 5*
a*f) - 32*a*c^3*(4*c*d - 3*a*f) - 63*b^5*g - 40*b^3*c*(2*c*e - 7*a*g) + 48*
a*b*c^2*(4*c*e - 5*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(256*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^2\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{gx^4\sqrt{a+bx+cx^2}}{5c} + \frac{\int \frac{x^2\left(5cd+(5ce-4ag)x+\frac{1}{2}(10cf-9bg)x^2\right)}{\sqrt{a+bx+cx^2}}\,dx}{5c}$$

$$= \frac{(10cf-9bg)x^3\sqrt{a+bx+cx^2}}{40c^2} + \frac{gx^4\sqrt{a+bx+cx^2}}{5c} + \frac{\int \frac{x^2\left(\frac{1}{2}\left(40c^2d-30acf+27abg\right)+\ldots\right.}{\sqrt{a+\ldots}}}{2\ldots}$$

$$= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a+bx+\ldots}}{40c^2}$$

$$= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a+bx+\ldots}}{40c^2}$$

$$= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a+bx+\ldots}}{40c^2}$$

$$= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a+bx+\ldots}}{40c^2}$$

Mathematica [A] time = 0.74, size = 282, normalized size = 0.82

$$\sqrt{a+x(b+cx)}\left(16c^2\left(64a^2g-ac(80e+x(45f+32gx))+2c^2x(30d+x(20e+3x(5f+4gx)))\right)+4b^2c(-735ag+\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(945*b^4*g - 210*b^3*c*(5*f + 3*g*x) + 4*b^2*c*(300*
c*e - 735*a*g + 7*c*x*(25*f + 18*g*x)) - 8*b*c^2*(-(a*(275*f + 161*g*x)) +
```

```
2*c*(90*d + x*(50*e + 35*f*x + 27*g*x^2))) + 16*c^2*(64*a^2*g - a*c*(80*e +
 x*(45*f + 32*g*x)) + 2*c^2*x*(30*d + x*(20*e + 3*x*(5*f + 4*g*x))))))/(192
0*c^5) - ((-70*b^4*c*f - 48*b^2*c^2*(2*c*d - 5*a*f) + 32*a*c^3*(4*c*d - 3*a
*f) + 63*b^5*g + 40*b^3*c*(2*c*e - 7*a*g) + 48*a*b*c^2*(-4*c*e + 5*a*g))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(256*c^(11/2))
```

fricas [A] time = 1.05, size = 701, normalized size = 2.03

$$\left[-\frac{15\left(32\left(3b^2c^3-4ac^4\right)d-16\left(5b^3c^2-12abc^3\right)e+2\left(35b^4c-120ab^2c^2+48a^2c^3\right)f-\left(63b^5-280ab^3c+2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/7680*(15*(32*(3*b^2*c^3 - 4*a*c^4)*d - 16*(5*b^3*c^2 - 12*a*b*c^3)*e +
2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f - (63*b^5 - 280*a*b^3*c + 240*a
^2*b*c^2)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*g*x^4 - 1440*b*c^4*d + 48*(10*
c^5*f - 9*b*c^4*g)*x^3 + 8*(80*c^5*e - 70*b*c^4*f + (63*b^2*c^3 - 64*a*c^4)
*g)*x^2 + 80*(15*b^2*c^3 - 16*a*c^4)*e - 50*(21*b^3*c^2 - 44*a*b*c^3)*f + (
945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*g + 2*(480*c^5*d - 400*b*c^4*e +
 10*(35*b^2*c^3 - 36*a*c^4)*f - 7*(45*b^3*c^2 - 92*a*b*c^3)*g)*x)*sqrt(c*x^
2 + b*x + a))/c^6, -1/3840*(15*(32*(3*b^2*c^3 - 4*a*c^4)*d - 16*(5*b^3*c^2
- 12*a*b*c^3)*e + 2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f - (63*b^5 - 2
80*a*b^3*c + 240*a^2*b*c^2)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(384*c^5*g*x^4 - 1440*b*c^4
*d + 48*(10*c^5*f - 9*b*c^4*g)*x^3 + 8*(80*c^5*e - 70*b*c^4*f + (63*b^2*c^3
 - 64*a*c^4)*g)*x^2 + 80*(15*b^2*c^3 - 16*a*c^4)*e - 50*(21*b^3*c^2 - 44*a*
b*c^3)*f + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*g + 2*(480*c^5*d - 4
00*b*c^4*e + 10*(35*b^2*c^3 - 36*a*c^4)*f - 7*(45*b^3*c^2 - 92*a*b*c^3)*g)*
x)*sqrt(c*x^2 + b*x + a))/c^6]
```

giac [A] time = 0.31, size = 330, normalized size = 0.95

$$\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(\frac{8gx}{c}+\frac{10c^4f-9bc^3g}{c^5}\right)x-\frac{70bc^3f-63b^2c^2g+64ac^3g-80c^4e}{c^5}\right)x+\frac{480c^4d+}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*g*x/c + (10*c^4*f - 9*b*c^3*g)/c^5
)*x - (70*b*c^3*f - 63*b^2*c^2*g + 64*a*c^3*g - 80*c^4*e)/c^5)*x + (480*c^4
*d + 350*b^2*c^2*f - 360*a*c^3*f - 315*b^3*c*g + 644*a*b*c^2*g - 400*b*c^3*
e)/c^5)*x - (1440*b*c^3*d + 1050*b^3*c*f - 2200*a*b*c^2*f - 945*b^4*g + 294
0*a*b^2*c*g - 1024*a^2*c^2*g - 1200*b^2*c^2*e + 1280*a*c^3*e)/c^5) - 1/256*
(96*b^2*c^3*d - 128*a*c^4*d + 70*b^4*c*f - 240*a*b^2*c^2*f + 96*a^2*c^3*f -
 63*b^5*g + 280*a*b^3*c*g - 240*a^2*b*c^2*g - 80*b^3*c^2*e + 192*a*b*c^3*e)
*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

maple [B] time = 0.01, size = 783, normalized size = 2.26

$$\frac{\sqrt{c\,x^2+bx+a}\;g\,x^4}{5c}-\frac{9\sqrt{c\,x^2+bx+a}\;bg\,x^3}{40c^2}+\frac{\sqrt{c\,x^2+bx+a}\;f\,x^3}{4c}-\frac{4\sqrt{c\,x^2+bx+a}\;ag\,x^2}{15c^2}+\frac{21\sqrt{c\,x^2+bx+a}\;b}{80c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 161/240*g/c^3*b*a*x*(c*x^2+b*x+a)^(1/2)+35/128*f/c^(9/2)*b^4*ln((c*x+1/2*b)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/8*f*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))+63/128*g/c^5*b^4*(c*x^2+b*x+a)^(1/2)-63/256*g/c^(11/2)*b^5
*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+8/15*g*a^2/c^3*(c*x^2+b*x+a)^(
1/2)+1/2*d*x/c*(c*x^2+b*x+a)^(1/2)-3/4*d/c^2*b*(c*x^2+b*x+a)^(1/2)+3/8*d/c^
(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*d*a/c^(3/2)*ln((c
*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/3*e*x^2/c*(c*x^2+b*x+a)^(1/2)+5/8*
e/c^3*b^2*(c*x^2+b*x+a)^(1/2)-5/16*e/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))-2/3*e*a/c^2*(c*x^2+b*x+a)^(1/2)+1/4*f*x^3/c*(c*x^2+b*x+a)
^(1/2)-35/64*f/c^4*b^3*(c*x^2+b*x+a)^(1/2)+1/5*g*x^4*(c*x^2+b*x+a)^(1/2)/c-
3/8*f*a/c^2*x*(c*x^2+b*x+a)^(1/2)-9/40*g/c^2*b*x^3*(c*x^2+b*x+a)^(1/2)+21/8
0*g/c^3*b^2*x^2*(c*x^2+b*x+a)^(1/2)-21/64*g/c^4*b^3*x*(c*x^2+b*x+a)^(1/2)+3
5/32*g/c^(9/2)*b^3*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-49/32*g/c^
4*b^2*a*(c*x^2+b*x+a)^(1/2)-7/24*f/c^2*b*x^2*(c*x^2+b*x+a)^(1/2)+35/96*f/c^
3*b^2*x*(c*x^2+b*x+a)^(1/2)-15/16*f/c^(7/2)*b^2*a*ln((c*x+1/2*b)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))-4/15*g*a/c^2*x^2*(c*x^2+b*x+a)^(1/2)+55/48*f/c^3*b*a*(c*
x^2+b*x+a)^(1/2)-15/16*g/c^(7/2)*b*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))+3/4*e/c^(5/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-5/12*
e/c^2*b*x*(c*x^2+b*x+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(g\,x^3+f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((x^2*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\left(d+ex+fx^{2}+gx^{3}\right)}{\sqrt{a+bx+cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(x**2*(d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x + c*x**2), x)
```

3.281 $\int \frac{x\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=245

$$\frac{\sqrt{a+bx+cx^2}\left(2cx\left(-36acg+35b^2g-40bcf+48c^2e\right)-16c^2(8af+9be)+20bc(11ag+6bf)-105b^3g+192\right.}{192c^4}$$

```
[Out] -1/128*(40*b^3*c*f+32*b*c^2*(-3*a*f+2*c*d)-35*b^4*g-24*b^2*c*(-5*a*g+2*c*e)
+16*a*c^2*(-3*a*g+4*c*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
)/c^(9/2)+1/24*(-7*b*g+8*c*f)*x^2*(c*x^2+b*x+a)^(1/2)/c^2+1/4*g*x^3*(c*x^2+
b*x+a)^(1/2)/c+1/192*(192*c^3*d-16*c^2*(8*a*f+9*b*e)-105*b^3*g+20*b*c*(11*a
*g+6*b*f)+2*c*(-36*a*c*g+35*b^2*g-40*b*c*f+48*c^2*e)*x)*(c*x^2+b*x+a)^(1/2)
/c^4
```

Rubi [A] time = 0.44, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1653, 779, 621, 206}

$$\frac{\sqrt{a+bx+cx^2}\left(2cx\left(-36acg+35b^2g-40bcf+48c^2e\right)-16c^2(8af+9be)+20bc(11ag+6bf)-105b^3g+192\right.}{192c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((8*c*f - 7*b*g)*x^2*Sqrt[a + b*x + c*x^2])/(24*c^2) + (g*x^3*Sqrt[a + b*x
+ c*x^2])/(4*c) + ((192*c^3*d - 16*c^2*(9*b*e + 8*a*f) - 105*b^3*g + 20*b*c
*(6*b*f + 11*a*g) + 2*c*(48*c^2*e - 40*b*c*f + 35*b^2*g - 36*a*c*g)*x)*Sqrt
[a + b*x + c*x^2])/(192*c^4) - ((40*b^3*c*f + 32*b*c^2*(2*c*d - 3*a*f) - 35
*b^4*g - 24*b^2*c*(2*c*e - 5*a*g) + 16*a*c^2*(4*c*e - 3*a*g))*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
```

```
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{x\left(4cd+(4ce-3ag)x+\frac{1}{2}(8cf-7bg)x^2\right)}{\sqrt{a+bx+cx^2}}\,dx}{4c}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{x\left(12c^2d-8acf+7abg+\frac{1}{4}\left(48c^2e-4\right.\right.}{\sqrt{a+bx+cx^2}}}{12c^2}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{\left(192c^3d-16c^2(9be+8af)\right.}{}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{\left(192c^3d-16c^2(9be+8af)\right.}{}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{\left(192c^3d-16c^2(9be+8af)\right.}{}$$

Mathematica [A] time = 0.44, size = 199, normalized size = 0.81

$$\frac{\sqrt{a+x(b+cx)}\left(-8c^2\left(16af+9agx+18be+10bfx+7bgx^2\right)+10bc(22ag+12bf+7bgx)-105b^3g+16c^3\left(12d\right.\right.}{192c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(-105*b^3*g + 10*b*c*(12*b*f + 22*a*g + 7*b*g*x) - 8
*c^2*(18*b*e + 16*a*f + 10*b*f*x + 9*a*g*x + 7*b*g*x^2) + 16*c^3*(12*d + x*
(6*e + 4*f*x + 3*g*x^2))))/(192*c^4) + ((-40*b^3*c*f + 32*b*c^2*(-2*c*d + 3
*a*f) + 35*b^4*g + 24*b^2*c*(2*c*e - 5*a*g) + 16*a*c^2*(-4*c*e + 3*a*g))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(128*c^(9/2))
```

fricas [A] time = 0.71, size = 499, normalized size = 2.04

$$\left[-\frac{3\left(64bc^3d-16\left(3b^2c^2-4ac^3\right)e+8\left(5b^3c-12abc^2\right)f-\left(35b^4-120ab^2c+48a^2c^2\right)g\right)\sqrt{c}\,\log\left(-8c^2x^2-8bc\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(64*b*c^3*d - 16*(3*b^2*c^2 - 4*a*c^3)*e + 8*(5*b^3*c - 12*a*b*c
^2)*f - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b
*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(48*c
^4*g*x^3 + 192*c^4*d - 144*b*c^3*e + 8*(8*c^4*f - 7*b*c^3*g)*x^2 + 8*(15*b^
```

```
2*c^2 - 16*a*c^3)*f - 5*(21*b^3*c - 44*a*b*c^2)*g + 2*(48*c^4*e - 40*b*c^3*
f + (35*b^2*c^2 - 36*a*c^3)*g)*x)*sqrt(c*x^2 + b*x + a))/c^5, 1/384*(3*(64*
b*c^3*d - 16*(3*b^2*c^2 - 4*a*c^3)*e + 8*(5*b^3*c - 12*a*b*c^2)*f - (35*b^4
 - 120*a*b^2*c + 48*a^2*c^2)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(
2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*g*x^3 + 192*c^4*d
- 144*b*c^3*e + 8*(8*c^4*f - 7*b*c^3*g)*x^2 + 8*(15*b^2*c^2 - 16*a*c^3)*f -
 5*(21*b^3*c - 44*a*b*c^2)*g + 2*(48*c^4*e - 40*b*c^3*f + (35*b^2*c^2 - 36*
a*c^3)*g)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

giac [**A**] time = 0.39, size = 228, normalized size = 0.93

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6\,gx}{c}+\frac{8\,c^3f-7\,bc^2g}{c^4}\right)x-\frac{40\,bc^2f-35\,b^2cg+36\,ac^2g-48\,c^3e}{c^4}\right)x+\frac{192\,c^3d+120\,b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*g*x/c + (8*c^3*f - 7*b*c^2*g)/c^4)*x -
 (40*b*c^2*f - 35*b^2*c*g + 36*a*c^2*g - 48*c^3*e)/c^4)*x + (192*c^3*d + 12
0*b^2*c*f - 128*a*c^2*f - 105*b^3*g + 220*a*b*c*g - 144*b*c^2*e)/c^4) + 1/1
28*(64*b*c^3*d + 40*b^3*c*f - 96*a*b*c^2*f - 35*b^4*g + 120*a*b^2*c*g - 48*
a^2*c^2*g - 48*b^2*c^2*e + 64*a*c^3*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*sqrt(c) - b))/c^(9/2)
```

maple [**B**] time = 0.01, size = 532, normalized size = 2.17

$$\frac{\sqrt{c\,x^2+bx+a}\;g\,x^3}{4c}-\frac{7\sqrt{c\,x^2+bx+a}\;bg\,x^2}{24c^2}+\frac{\sqrt{c\,x^2+bx+a}\;f\,x^2}{3c}+\frac{3a^2g\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{5}{2}}}-\frac{15a\,b^2g}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/4*g*x^3*(c*x^2+b*x+a)^(1/2)/c-7/24*g/c^2*b*x^2*(c*x^2+b*x+a)^(1/2)+35/96*
g/c^3*b^2*x*(c*x^2+b*x+a)^(1/2)-35/64*g/c^4*b^3*(c*x^2+b*x+a)^(1/2)+35/128*
g/c^(9/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-15/16*g/c^(7/2)*b
^2*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+55/48*g/c^3*b*a*(c*x^2+b*x
+a)^(1/2)-3/8*g*a/c^2*x*(c*x^2+b*x+a)^(1/2)+3/8*g*a^2/c^(5/2)*ln((c*x+1/2*b
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/3*f*x^2/c*(c*x^2+b*x+a)^(1/2)-5/12*f/c^2*b
*x*(c*x^2+b*x+a)^(1/2)+5/8*f/c^3*b^2*(c*x^2+b*x+a)^(1/2)-5/16*f/c^(7/2)*b^3
*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/4*f/c^(5/2)*b*a*ln((c*x+1/2*
b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-2/3*f*a/c^2*(c*x^2+b*x+a)^(1/2)+1/2*e*x/c*(
c*x^2+b*x+a)^(1/2)-3/4*e/c^2*b*(c*x^2+b*x+a)^(1/2)+3/8*e/c^(5/2)*b^2*ln((c*
x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*e*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))+d/c*(c*x^2+b*x+a)^(1/2)-1/2*d*b/c^(3/2)*ln((c*x+1/2*
b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(g\,x^3+f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2),x)
[Out] int((x*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(x*(d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x + c*x**2), x)
```

3.282 $\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=177

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)}{16c^{7/2}}+\frac{\sqrt{a+bx+cx^2}\left(-16acg+15b^2g-1\right.}{24c^3}$$

```
[Out] 1/16*(16*c^3*d-8*c^2*(a*f+b*e)-5*b^3*g+6*b*c*(2*a*g+b*f))*arctanh(1/2*(2*c*
x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/24*(-16*a*c*g+15*b^2*g-18*b*c*f
+24*c^2*e)*(c*x^2+b*x+a)^(1/2)/c^3+1/12*(-5*b*g+6*c*f)*x*(c*x^2+b*x+a)^(1/2
)/c^2+1/3*g*x^2*(c*x^2+b*x+a)^(1/2)/c
```

Rubi [A] time = 0.24, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)}{16c^{7/2}}+\frac{\sqrt{a+bx+cx^2}\left(-16acg+15b^2g-1\right.}{24c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((24*c^2*e - 18*b*c*f + 15*b^2*g - 16*a*c*g)*Sqrt[a + b*x + c*x^2])/(24*c^3
) + ((6*c*f - 5*b*g)*x*Sqrt[a + b*x + c*x^2])/(12*c^2) + (g*x^2*Sqrt[a + b*
x + c*x^2])/(3*c) + ((16*c^3*d - 8*c^2*(b*e + a*f) - 5*b^3*g + 6*b*c*(b*f +
 2*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(7/2
))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx = \frac{gx^2\sqrt{a+bx+cx^2}}{3c} + \frac{\int \frac{3cd+(3ce-2ag)x+\frac{1}{2}(6cf-5bg)x^2}{\sqrt{a+bx+cx^2}}\,dx}{3c}$$

$$= \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + \frac{gx^2\sqrt{a+bx+cx^2}}{3c} + \frac{\int \frac{\frac{1}{2}\left(12c^2d-6acf+5abg\right)+\frac{1}{4}\left(24c^2e-18bcf\right.}{\sqrt{a+bx+cx^2}}}{6c^2}$$

$$= \frac{\left(24c^2e-18bcf+15b^2g-16acg\right)\sqrt{a+bx+cx^2}}{24c^3} + \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + g$$

$$= \frac{\left(24c^2e-18bcf+15b^2g-16acg\right)\sqrt{a+bx+cx^2}}{24c^3} + \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + g$$

$$= \frac{\left(24c^2e-18bcf+15b^2g-16acg\right)\sqrt{a+bx+cx^2}}{24c^3} + \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + g$$

Mathematica [A] time = 0.26, size = 141, normalized size = 0.80

$$\frac{3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)+2\sqrt{c}\sqrt{a+x(b+cx)}\left(-2c(8ag+9bf\right.}{48c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^2*g - 2*c*(9*b*f + 8*a*g + 5*b*g*x)
+ 4*c^2*(6*e + x*(3*f + 2*g*x))) + 3*(16*c^3*d - 8*c^2*(b*e + a*f) - 5*b^3*
g + 6*b*c*(b*f + 2*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x
)])])/(48*c^(7/2))
```

fricas [A] time = 0.76, size = 341, normalized size = 1.93

$$\left[\frac{3\left(16c^3d-8bc^2e+2\left(3b^2c-4ac^2\right)f-\left(5b^3-12abc\right)g\right)\sqrt{c}\log\left(-8c^2x^2-8bcx-b^2-4\sqrt{cx^2+bx+a}\,(2cx+\right.}{96c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(16*c^3*d - 8*b*c^2*e + 2*(3*b^2*c - 4*a*c^2)*f - (5*b^3 - 12*a*b*
c)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c
*x + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*g*x^2 + 24*c^3*e - 18*b*c^2*f + (15*b^2
*c - 16*a*c^2)*g + 2*(6*c^3*f - 5*b*c^2*g)*x)*sqrt(c*x^2 + b*x + a))/c^4, -
1/48*(3*(16*c^3*d - 8*b*c^2*e + 2*(3*b^2*c - 4*a*c^2)*f - (5*b^3 - 12*a*b*c
)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^
2 + b*c*x + a*c)) - 2*(8*c^3*g*x^2 + 24*c^3*e - 18*b*c^2*f + (15*b^2*c - 16
*a*c^2)*g + 2*(6*c^3*f - 5*b*c^2*g)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

giac [A] time = 0.27, size = 149, normalized size = 0.84

$$\frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(\frac{4gx}{c}+\frac{6c^2f-5bcg}{c^3}\right)x-\frac{18bcf-15b^2g+16acg-24c^2e}{c^3}\right)-\frac{\left(16c^3d+6b^2cf-8ac^2f-5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*g*x/c + (6*c^2*f - 5*b*c*g)/c^3)*x - (18*b
*c*f - 15*b^2*g + 16*a*c*g - 24*c^2*e)/c^3) - 1/16*(16*c^3*d + 6*b^2*c*f -
8*a*c^2*f - 5*b^3*g + 12*a*b*c*g - 8*b*c^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

maple [B] time = 0.01, size = 333, normalized size = 1.88

$$\frac{\sqrt{c\,x^{2}+b x+a}\;g\,x^{2}}{3c}+\frac{3 a b g \ln\left(\frac{c x+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+a}\right)}{4 c^{\frac{5}{2}}}-\frac{a f \ln\left(\frac{c x+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+a}\right)}{2 c^{\frac{3}{2}}}-\frac{5 b^{3} g \ln\left(\frac{c x+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{2}}\right.}{16 c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/3*g*x^2*(c*x^2+b*x+a)^(1/2)/c-5/12*g/c^2*b*x*(c*x^2+b*x+a)^(1/2)+5/8*g/c^
3*b^2*(c*x^2+b*x+a)^(1/2)-5/16*g/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))+3/4*g/c^(5/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
-2/3*g*a/c^2*(c*x^2+b*x+a)^(1/2)+1/2*(c*x^2+b*x+a)^(1/2)/c*f*x-3/4*(c*x^2+b
*x+a)^(1/2)*b/c^2*f+3/8*b^2/c^(5/2)*f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))-1/2*a/c^(3/2)*f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+(c*x^2+b
*x+a)^(1/2)/c*e-1/2*b/c^(3/2)*e*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
+1/c^(1/2)*d*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*x^2 + g*x^3)/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^{2}+gx^{3}}{\sqrt{a+bx+cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x + c*x**2), x)
```

3.283 $\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=155

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(ag+bf)+3b^2g+8c^2e\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}\,(4cf-3bg)}{4c^2}-\frac{d\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}+\frac{gx}{}$$

```
[Out] 1/8*(8*c^2*e+3*b^2*g-4*c*(a*g+b*f))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*
x+a)^(1/2))/c^(5/2)-d*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^
(1/2)+1/4*(-3*b*g+4*c*f)*(c*x^2+b*x+a)^(1/2)/c^2+1/2*g*x*(c*x^2+b*x+a)^(1/2
)/c
```

Rubi [A] time = 0.26, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1653, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(ag+bf)+3b^2g+8c^2e\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}\,(4cf-3bg)}{4c^2}-\frac{d\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}+\frac{gx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((4*c*f - 3*b*g)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (g*x*Sqrt[a + b*x + c*x^2
])/(2*c) - (d*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[
a] + ((8*c^2*e + 3*b^2*g - 4*c*(b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \frac{gx\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{2cd+(2ce-ag)x+\frac{1}{2}(4cf-3bg)x^2}{x\sqrt{a+bx+cx^2}}\,dx}{2c}$$

$$= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2} + \frac{gx\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{2c^2d+\frac{1}{4}\left(8c^2e+3b^2g-4c(bf+ag)\right)x}{x\sqrt{a+bx+cx^2}}\,dx}{2c^2}$$

$$= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2} + \frac{gx\sqrt{a+bx+cx^2}}{2c} + d\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx + \frac{(8c^2}{}$$

$$= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2} + \frac{gx\sqrt{a+bx+cx^2}}{2c} - (2d)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{}{\sqrt{}}\right.$$

$$= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2} + \frac{gx\sqrt{a+bx+cx^2}}{2c} - \frac{d\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}} + \frac{(8c}{}$$

Mathematica [A] time = 0.37, size = 134, normalized size = 0.86

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-4c(ag+bf)+3b^2g+8c^2e\right)}{8c^{5/2}} + \frac{\sqrt{a+x(b+cx)}\,(-3bg+4cf+2cgx)}{4c^2} - \frac{d\tanh^{-1}\left(\frac{2}{2\sqrt{a}\sqrt{}}\right.}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((4*c*f - 3*b*g + 2*c*g*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) - (d*ArcTanh[(2*a
 + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + ((8*c^2*e + 3*b^2*g -
 4*c*(b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(
8*c^(5/2))
```

fricas [A] time = 5.78, size = 733, normalized size = 4.73

$$\left[\frac{8\sqrt{a}c^3d\log\left(-\frac{8abx+\left(b^2+4ac\right)x^2-4\sqrt{cx^2+bx+a}\,(bx+2a)\sqrt{a}+8a^2}{x^2}\right) - \left(8ac^2e-4abcf+\left(3ab^2-4a^2c\right)g\right)\sqrt{c}\log\left(-8c^2x\right.}{16ac^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(a)*c^3*d*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b
*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b
```

```
^2 - 4*a^2*c)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*
x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*
g)*sqrt(c*x^2 + b*x + a))/(a*c^3), 1/8*(4*sqrt(a)*c^3*d*log(-(8*a*b*x + (b^
2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2)
- (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a^2*c)*g)*sqrt(-c)*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(2*a*c^2
*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2 + b*x + a))/(a*c^3), 1/16*(16*sqrt
(-a)*c^3*d*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 +
 a*b*x + a^2)) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a^2*c)*g)*sqrt(c)*lo
g(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c)
- 4*a*c) + 4*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2 + b*x + a))/(
a*c^3), 1/8*(8*sqrt(-a)*c^3*d*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*
sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a
^2*c)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^
2*x^2 + b*c*x + a*c)) + 2*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2
+ b*x + a))/(a*c^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.index.cc index_m operator + Erro
r: Bad Argument Value
```

maple [A] time = 0.01, size = 220, normalized size = 1.42

$$-\frac{ag\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}-\frac{d\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\;\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{3b^2g\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{5}{2}}}-\frac{bf\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{}\right.}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*g*x*(c*x^2+b*x+a)^(1/2)/c-3/4*g/c^2*b*(c*x^2+b*x+a)^(1/2)+3/8*g/c^(5/2)
*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*g*a/c^(3/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+f/c*(c*x^2+b*x+a)^(1/2)-1/2*f*b/c^(3/2)*l
n((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+e*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))/c^(1/2)-d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/
x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{x\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2 + g*x^3)/(x*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x*sqrt(a + b*x + c*x**2)), x)
```

3.284 $\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=139

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}+\frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}-\frac{d\sqrt{a+bx+cx^2}}{ax}+\frac{g\sqrt{a+bx+cx^2}}{c}$$

```
[Out] 1/2*(-2*a*e+b*d)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)
+1/2*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2
)+g*(c*x^2+b*x+a)^(1/2)/c-d*(c*x^2+b*x+a)^(1/2)/a/x
```

Rubi [A] time = 0.24, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1650, 1653, 843, 621, 206, 724}

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}+\frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}-\frac{d\sqrt{a+bx+cx^2}}{ax}+\frac{g\sqrt{a+bx+cx^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g*Sqrt[a + b*x + c*x^2])/c - (d*Sqrt[a + b*x + c*x^2])/(a*x) + ((b*d - 2*a
*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)) + (
(2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^
(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
```

```
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx &= -\frac{d\sqrt{a+bx+cx^2}}{ax}-\frac{\int \frac{\frac{1}{2}(bd-2ae)-afx-agx^2}{x\sqrt{a+bx+cx^2}}\,dx}{a}\\
&= \frac{g\sqrt{a+bx+cx^2}}{c}-\frac{d\sqrt{a+bx+cx^2}}{ax}-\frac{\int \frac{\frac{1}{2}c(bd-2ae)-\frac{1}{2}a(2cf-bg)x}{x\sqrt{a+bx+cx^2}}\,dx}{ac}\\
&= \frac{g\sqrt{a+bx+cx^2}}{c}-\frac{d\sqrt{a+bx+cx^2}}{ax}-\frac{(bd-2ae)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2a}+\frac{(2cf-bg)\int
\end{aligned}$$

$$= \frac{g\sqrt{a+bx+cx^2}}{c}-\frac{d\sqrt{a+bx+cx^2}}{ax}+\frac{(bd-2ae)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{a}$$

$$= \frac{g\sqrt{a+bx+cx^2}}{c}-\frac{d\sqrt{a+bx+cx^2}}{ax}+\frac{(bd-2ae)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}+\frac{(2cf-$$

Mathematica [A] time = 0.40, size = 127, normalized size = 0.91

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{2a^{3/2}}+\frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{2c^{3/2}}+\frac{\sqrt{a+x(b+cx)}\,(agx-cd)}{acx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((-(c*d) + a*g*x)*Sqrt[a + x*(b + c*x)])/(a*c*x) + ((b*d - 2*a*e)*ArcTanh[(
2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(2*a^(3/2)) + ((2*c*f - b*g)
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(2*c^(3/2))
```

fricas [A] time = 3.72, size = 703, normalized size = 5.06

$$\left[-\frac{\left(2\,a^2cf-a^2bg\right)\sqrt{c}\,x\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{c}\,-4\,ac\right)+\left(bc^2d-2\,ac^2e\right)\sqrt{a}\,x}{4\,a^2c^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/4*((2*a^2*c*f - a^2*b*g)*sqrt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*s
qrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + (b*c^2*d - 2*a*c^2*e)*s
qrt(a)*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x +
 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^2 + b*x + a)
)/(a^2*c^2*x), -1/4*(2*(2*a^2*c*f - a^2*b*g)*sqrt(-c)*x*arctan(1/2*sqrt(c*x
^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + (b*c^2*d - 2*
a*c^2*e)*sqrt(a)*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x +
 a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^2
+ b*x + a))/(a^2*c^2*x), -1/4*(2*(b*c^2*d - 2*a*c^2*e)*sqrt(-a)*x*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + (2*
a^2*c*f - a^2*b*g)*sqrt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2
+ b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^
2 + b*x + a))/(a^2*c^2*x), -1/2*((b*c^2*d - 2*a*c^2*e)*sqrt(-a)*x*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + (2*
a^2*c*f - a^2*b*g)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*
sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^2 + b*
x + a))/(a^2*c^2*x)]
```

giac [A] time = 0.37, size = 171, normalized size = 1.23

$$\frac{\sqrt{cx^2+bx+a}\,g}{c}-\frac{(bd-2\,ae)\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{\left(2\,cf-bg\right)\log\left(\left|2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{3}{2}}}+\frac{(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x^2 + b*x + a)*g/c - (b*d - 2*a*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))/sqrt(-a))/(sqrt(-a)*a) - 1/2*(2*c*f - b*g)*log(abs(2*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2) + ((sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b*d + 2*a*sqrt(c)*d)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a
)*a)
```

maple [A] time = 0.01, size = 173, normalized size = 1.24

$$-\frac{e\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{bd\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{bg\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}+\frac{f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+\cdots}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] g*(c*x^2+b*x+a)^(1/2)/c-1/2*g*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))+f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-d*(c*x^2+b*x
+a)^(1/2)/a/x+1/2*d*b/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
-e/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 4.46, size = 166, normalized size = 1.19

$$\frac{g\sqrt{cx^2+bx+a}}{c}-\frac{e\ln\left(\frac{b}{2}+\frac{a}{x}+\frac{\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)}{\sqrt{a}}+\frac{f\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{\sqrt{c}}-\frac{bg\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx-}\right.}{2c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^2*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] (g*(a + b*x + c*x^2)^(1/2))/c - (e*log(b/2 + a/x + (a^(1/2)*(a + b*x + c*x^
2)^(1/2))/x))/a^(1/2) + (f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2
)))/c^(1/2) - (b*g*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/(2*c
^(3/2)) - (d*(a + b*x + c*x^2)^(1/2))/(a*x) + (b*d*atanh((a + (b*x)/2)/(a^(
1/2)*(a + b*x + c*x^2)^(1/2))))/(2*a^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**2*sqrt(a + b*x + c*x**2)), x)
```

3.285 $\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=159

$$\frac{\sqrt{a+bx+cx^2}\,(3bd-4ae)}{4a^2x}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)\left(8a^2f-4abe-4acd+3b^2d\right)}{8a^{5/2}}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}+\frac{g\tanh^{-1}\left(\frac{\quad}{2\cdot}\right.}{\sqrt{}}$$

```
[Out] -1/8*(8*a^2*f-4*a*b*e-4*a*c*d+3*b^2*d)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2
+b*x+a)^(1/2))/a^(5/2)+g*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))
/c^(1/2)-1/2*d*(c*x^2+b*x+a)^(1/2)/a/x^2+1/4*(-4*a*e+3*b*d)*(c*x^2+b*x+a)^(
1/2)/a^2/x
```

Rubi [A] time = 0.24, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1650, 843, 621, 206, 724}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)\left(8a^2f-4abe-4acd+3b^2d\right)}{8a^{5/2}}+\frac{\sqrt{a+bx+cx^2}\,(3bd-4ae)}{4a^2x}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}+\frac{g\tanh^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(d*Sqrt[a + b*x + c*x^2])/(2*a*x^2) + ((3*b*d - 4*a*e)*Sqrt[a + b*x + c*x^
2])/(4*a^2*x) - ((3*b^2*d - 4*a*c*d - 4*a*b*e + 8*a^2*f)*ArcTanh[(2*a + b*x
)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)) + (g*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{2ax^2}-\frac{\int \frac{\frac{1}{2}(3bd-4ae)+(cd-2af)x-2agx^2}{x^2\sqrt{a+bx+cx^2}}\,dx}{2a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2}+\frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x}+\frac{\int \frac{\frac{1}{4}\left(3b^2d-4abe-4a(cd-2af)\right)+2a^2gx}{x\sqrt{a+bx+cx^2}}\,dx}{2a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2}+\frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x}+\frac{\left(3b^2d-4acd-4abe+8a^2f\right)\int}{8a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2}+\frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x}-\frac{\left(3b^2d-4acd-4abe+8a^2f\right)\mathrm{Su}}{4a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2}+\frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x}-\frac{\left(3b^2d-4acd-4abe+8a^2f\right)\mathrm{ta}}{8a^{5/2}}$$

Mathematica [A] time = 0.36, size = 137, normalized size = 0.86

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)\left(4abe+4a(cd-2af)-3b^2d\right)}{8a^{5/2}}+\frac{\sqrt{a+x(b+cx)}\,(3bdx-2a(d+2ex))}{4a^2x^2}+\frac{g\tanh^{-1}\left(\frac{b}{2\sqrt{c}\sqrt{}}\right.}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(3*b*d*x - 2*a*(d + 2*e*x)))/(4*a^2*x^2) + ((-3*b^2*
d + 4*a*b*e + 4*a*(c*d - 2*a*f))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*
(b + c*x)])])/(8*a^(5/2)) + (g*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
 + c*x)])])/Sqrt[c]
```

fricas [A] time = 4.81, size = 783, normalized size = 4.92

$$\left[\frac{8a^3\sqrt{c}gx^2\log\left(-8c^2x^2-8bcx-b^2-4\sqrt{cx^2+bx+a}\,(2cx+b)\sqrt{c}-4ac\right)-\left(4abce-8a^2cf-\left(3b^2c-4ac\right.\right.}{16a^3c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(8*a^3*sqrt(c)*g*x^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 +
b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c -
```

```
 4*a*c^2)*d)*sqrt(a)*x^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 +
 b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(2*a^2*c*d - (3*a*b*c*d - 4
*a^2*c*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^2), -1/16*(16*a^3*sqrt(-c)*g*x
^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x +
 a*c)) + (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c - 4*a*c^2)*d)*sqrt(a)*x^2*log(-(
8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) +
 8*a^2)/x^2) + 4*(2*a^2*c*d - (3*a*b*c*d - 4*a^2*c*e)*x)*sqrt(c*x^2 + b*x +
 a))/(a^3*c*x^2), 1/8*(4*a^3*sqrt(c)*g*x^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 -
 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (4*a*b*c*e - 8*a^2*
c*f - (3*b^2*c - 4*a*c^2)*d)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*
(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(2*a^2*c*d - (3*a*b*c*d -
 4*a^2*c*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^2), -1/8*(8*a^3*sqrt(-c)*g*x
^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x +
 a*c)) + (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c - 4*a*c^2)*d)*sqrt(-a)*x^2*arcta
n(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) +
 2*(2*a^2*c*d - (3*a*b*c*d - 4*a^2*c*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^
2)]
```

giac [B] time = 0.36, size = 352, normalized size = 2.21

$$-\frac{g\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)c-b\sqrt{c}\right|\right)}{\sqrt{c}}+\frac{\left(3\,b^2d-4\,acd+8\,a^2f-4\,abe\right)\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}\,a^2}-\frac{3\left(\sqrt{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -g*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*c - b*sqrt(c)))/sqrt(c) +
 1/4*(3*b^2*d - 4*a*c*d + 8*a^2*f - 4*a*b*e)*arctan(-(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^2) - 1/4*(3*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*b^2*d - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d - 4*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*e - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*a^2*sqrt(c)*e - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d - 4*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^2*b*e - 8*a^2*b*sqrt(c)*d + 8*a^3*sqrt(c)*e)/(((sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^2 - a)^2*a^2)
```

maple [A] time = 0.01, size = 241, normalized size = 1.52

$$-\frac{f\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{be\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}+\frac{cd\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2d\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx}}{x}\right.}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] g*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/2*d*(c*x^2+b*x+a)^(
1/2)/a/x^2+3/4*d*b/a^2/x*(c*x^2+b*x+a)^(1/2)-3/8*d*b^2/a^(5/2)*ln((b*x+2*a+
2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+1/2*d*c/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+
a)^(1/2)*a^(1/2))/x)-e/a/x*(c*x^2+b*x+a)^(1/2)+1/2*e*b/a^(3/2)*ln((b*x+2*a+
2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-f/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/
2)*a^(1/2))/x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^3*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^3*(a + b*x + c*x^2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**3*sqrt(a + b*x + c*x**2)), x)
```

3.286 $\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=186

$$\frac{\sqrt{a+bx+cx^2}\,(5bd-6ae)}{12a^2x^2}+\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)\left(8a^2(ce-2ag)-6ab^2e-4ab(3cd-2af)+5b^3d\right)}{16a^{7/2}}-\frac{\sqrt{a+bx+}}{}$$

```
[Out] 1/16*(5*b^3*d-6*a*b^2*e-4*a*b*(-2*a*f+3*c*d)+8*a^2*(-2*a*g+c*e))*arctanh(1/
2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(7/2)-1/3*d*(c*x^2+b*x+a)^(1/2)/
a/x^3+1/12*(-6*a*e+5*b*d)*(c*x^2+b*x+a)^(1/2)/a^2/x^2-1/24*(24*a^2*f-18*a*b
*e-16*a*c*d+15*b^2*d)*(c*x^2+b*x+a)^(1/2)/a^3/x
```

Rubi [A] time = 0.32, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1650, 806, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)\left(8a^2(ce-2ag)-6ab^2e-4ab(3cd-2af)+5b^3d\right)}{16a^{7/2}}-\frac{\sqrt{a+bx+cx^2}\,\left(24a^2f-18abe-16ac\right.}{24a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^4*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(d*Sqrt[a + b*x + c*x^2])/(3*a*x^3) + ((5*b*d - 6*a*e)*Sqrt[a + b*x + c*x^
2])/(12*a^2*x^2) - ((15*b^2*d - 16*a*c*d - 18*a*b*e + 24*a^2*f)*Sqrt[a + b*
x + c*x^2])/(24*a^3*x) + ((5*b^3*d - 6*a*b^2*e - 4*a*b*(3*c*d - 2*a*f) + 8*
a^2*(c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/
(16*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
```

```
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} - \frac{\int \frac{\frac{1}{2}(5bd-6ae)+(2cd-3af)x-3agx^2}{x^3\sqrt{a+bx+cx^2}}\,dx}{3a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} + \frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2} + \frac{\int \frac{\frac{1}{4}\left(15b^2d-16acd-18abe+24a^2f\right)+\frac{1}{2}(5b}{x^2\sqrt{a+bx+cx^2}}}{6a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} + \frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2} - \frac{\left(15b^2d-16acd-18abe+24a^2\right.}{24a^3x}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} + \frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2} - \frac{\left(15b^2d-16acd-18abe+24a^2\right.}{24a^3x}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} + \frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2} - \frac{\left(15b^2d-16acd-18abe+24a^2\right.}{24a^3x}$$

Mathematica [A] time = 0.31, size = 150, normalized size = 0.81

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)\left(8a^2(ce-2ag)-6ab^2e+4ab(2af-3cd)+5b^3d\right)}{16a^{7/2}} - \sqrt{a+x(b+cx)}\left(4a^2(2d+3x(e+2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^4*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/24*(Sqrt[a + x*(b + c*x)]*(15*b^2*d*x^2 - 2*a*x*(5*b*d + 8*c*d*x + 9*b*e
*x) + 4*a^2*(2*d + 3*x*(e + 2*f*x))))/(a^3*x^3) + ((5*b^3*d - 6*a*b^2*e + 4
*a*b*(-3*c*d + 2*a*f) + 8*a^2*(c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]
*Sqrt[a + x*(b + c*x)]])])/(16*a^(7/2))
```

fricas [A] time = 5.55, size = 365, normalized size = 1.96

$$\left[-\frac{3\left(8a^2bf-16a^3g+\left(5b^3-12abc\right)d-2\left(3ab^2-4a^2c\right)e\right)\sqrt{a}\,x^3\log\left(-\frac{8abx+\left(b^2+4ac\right)x^2-4\sqrt{cx^2+bx+a}\,(bx+2a)\sqrt{a}+8}{x^2}\right.}{96a^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/96*(3*(8*a^2*b*f - 16*a^3*g + (5*b^3 - 12*a*b*c)*d - 2*(3*a*b^2 - 4*a^2
*c)*e)*sqrt(a)*x^3*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x +
 a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(8*a^3*d - (18*a^2*b*e - 24*a^3*f
 - (15*a*b^2 - 16*a^2*c)*d)*x^2 - 2*(5*a^2*b*d - 6*a^3*e)*x)*sqrt(c*x^2 + b
*x + a))/(a^4*x^3), -1/48*(3*(8*a^2*b*f - 16*a^3*g + (5*b^3 - 12*a*b*c)*d -
 2*(3*a*b^2 - 4*a^2*c)*e)*sqrt(-a)*x^3*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*
x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(8*a^3*d - (18*a^2*b*e - 24*
```

```
a^3*f - (15*a*b^2 - 16*a^2*c)*d)*x^2 - 2*(5*a^2*b*d - 6*a^3*e)*x)*sqrt(c*x^
2 + b*x + a))/(a^4*x^3)]
```

giac [B] time = 0.29, size = 689, normalized size = 3.70

$$-\frac{\left(5\,b^3d-12\,abcd+8\,a^2bf-16\,a^3g-6\,ab^2e+8\,a^2ce\right)\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^3}+\frac{15\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)^5b^3c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(5*b^3*d - 12*a*b*c*d + 8*a^2*b*f - 16*a^3*g - 6*a*b^2*e + 8*a^2*c*e)*
arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/24
*(15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*d - 36*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a*b*c*d + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*f
 - 18*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*e + 24*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^5*a^2*c*e + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*
sqrt(c)*f - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*d + 96*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*d - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*a^3*b*f + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*e + 96*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(3/2)*d - 96*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a^4*sqrt(c)*f + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
a^3*b*sqrt(c)*e + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*d + 36*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c*d + 24*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^4*b*f - 30*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*e - 24*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*c*e + 48*a^3*b^2*sqrt(c)*d - 32*a^4*c
^(3/2)*d + 48*a^5*sqrt(c)*f - 48*a^4*b*sqrt(c)*e)/(((sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2 - a)^3*a^3)
```

maple [B] time = 0.01, size = 375, normalized size = 2.02

$$-\frac{g\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{bf\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}+\frac{ce\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2e\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+}}{x}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/2*e/a/x^2*(c*x^2+b*x+a)^(1/2)+3/4*e*b/a^2/x*(c*x^2+b*x+a)^(1/2)-3/8*e*b^
2/a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+1/2*e*c/a^(3/2)*ln(
(b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-f/a/x*(c*x^2+b*x+a)^(1/2)+1/2*f*
b/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-g/a^(1/2)*ln((b*x+2
*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-1/3*d*(c*x^2+b*x+a)^(1/2)/a/x^3+5/12*d
*b/a^2/x^2*(c*x^2+b*x+a)^(1/2)-5/8*d*b^2/a^3/x*(c*x^2+b*x+a)^(1/2)+5/16*d*b
^3/a^(7/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-3/4*d*b/a^(5/2)*c*
ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+2/3*d*c/a^2/x*(c*x^2+b*x+a)^(
1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^4\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^4*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^4*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**4/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**4*sqrt(a + b*x + c*x**2)), x)
```

3.287 $\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=270

$$\frac{\sqrt{a+bx+cx^2}\,(7bd-8ae)}{24a^2x^3}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(32a^2b(3ce-2ag)+16a^2c(3cd-4af)-40ab^3e-24ab^2(5cd\right.}{128a^{9/2}}$$

```
[Out] -1/128*(35*b^4*d-40*a*b^3*e+16*a^2*c*(-4*a*f+3*c*d)-24*a*b^2*(-2*a*f+5*c*d)
+32*a^2*b*(-2*a*g+3*c*e))*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2)
)/a^(9/2)-1/4*d*(c*x^2+b*x+a)^(1/2)/a/x^4+1/24*(-8*a*e+7*b*d)*(c*x^2+b*x+a)
^(1/2)/a^2/x^3-1/96*(48*a^2*f-40*a*b*e-36*a*c*d+35*b^2*d)*(c*x^2+b*x+a)^(1/
2)/a^3/x^2+1/192*(105*b^3*d-120*a*b^2*e-4*a*b*(-36*a*f+55*c*d)+64*a^2*(-3*a
*g+2*c*e))*(c*x^2+b*x+a)^(1/2)/a^4/x
```

Rubi [A] time = 0.49, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1650, 834, 806, 724, 206}

$$\frac{\sqrt{a+bx+cx^2}\left(64a^2(2ce-3ag)-120ab^2e-4ab(55cd-36af)+105b^3d\right)}{192a^4x}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(32a^2b(3ce-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^5*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(d*Sqrt[a + b*x + c*x^2])/(4*a*x^4) + ((7*b*d - 8*a*e)*Sqrt[a + b*x + c*x^
2])/(24*a^2*x^3) - ((35*b^2*d - 36*a*c*d - 40*a*b*e + 48*a^2*f)*Sqrt[a + b*
x + c*x^2])/(96*a^3*x^2) + ((105*b^3*d - 120*a*b^2*e - 4*a*b*(55*c*d - 36*a
*f) + 64*a^2*(2*c*e - 3*a*g))*Sqrt[a + b*x + c*x^2])/(192*a^4*x) - ((35*b^4
*d - 40*a*b^3*e + 16*a^2*c*(3*c*d - 4*a*f) - 24*a*b^2*(5*c*d - 2*a*f) + 32*
a^2*b*(3*c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2]
)])/(128*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} - \frac{\int \frac{\frac{1}{2}(7bd-8ae)+(3cd-4af)x-4agx^2}{x^4\sqrt{a+bx+cx^2}}\,dx}{4a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} + \frac{\int \frac{\frac{1}{4}\left(35b^2d-40abe-12a(3cd-4af)\right)+\left(7bc\right.}{x^3\sqrt{a+bx+cx^2}}}{12a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{\left(35b^2d-36acd-40abe+48a^2\right.}{96a^3x^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{\left(35b^2d-36acd-40abe+48a^2\right.}{96a^3x^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{\left(35b^2d-36acd-40abe+48a^2\right.}{96a^3x^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{\left(35b^2d-36acd-40abe+48a^2\right.}{96a^3x^2}$$

Mathematica [A] time = 0.52, size = 212, normalized size = 0.79

$$\frac{\sqrt{a+x(b+cx)}\left(-16a^3\left(3d+4ex+6x^2(f+2gx)\right)+8a^2x(7bd+2bx(5e+9fx)+cx(9d+16ex))-10abx^2(7bc\right.}{192a^4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^5*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(105*b^3*d*x^3 - 10*a*b*x^2*(7*b*d + 22*c*d*x + 12*b
*e*x) + 8*a^2*x*(7*b*d + c*x*(9*d + 16*e*x) + 2*b*x*(5*e + 9*f*x)) - 16*a^3
*(3*d + 4*e*x + 6*x^2*(f + 2*g*x))))/(192*a^4*x^4) - ((35*b^4*d - 40*a*b^3*
e + 16*a^2*c*(3*c*d - 4*a*f) + 24*a*b^2*(-5*c*d + 2*a*f) + 32*a^2*b*(3*c*e
- 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(128*a^(9
/2))
```

fricas [A] time = 10.83, size = 525, normalized size = 1.94

$$\left[\frac{3\left(64\,a^3bg-\left(35\,b^4-120\,ab^2c+48\,a^2c^2\right)d+8\left(5\,ab^3-12\,a^2bc\right)e-16\left(3\,a^2b^2-4\,a^3c\right)f\right)\sqrt{a}\,x^4\log\left(-\frac{8\,abx+\left(b^2+4\,ac\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/768*(3*(64*a^3*b*g - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*d + 8*(5*a*b^3
- 12*a^2*b*c)*e - 16*(3*a^2*b^2 - 4*a^3*c)*f)*sqrt(a)*x^4*log(-(8*a*b*x + (
b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2
) - 4*(48*a^4*d - (144*a^3*b*f - 192*a^4*g + 5*(21*a*b^3 - 44*a^2*b*c)*d -
8*(15*a^2*b^2 - 16*a^3*c)*e)*x^3 - 2*(40*a^3*b*e - 48*a^4*f - (35*a^2*b^2 -
 36*a^3*c)*d)*x^2 - 8*(7*a^3*b*d - 8*a^4*e)*x)*sqrt(c*x^2 + b*x + a))/(a^5*
x^4), -1/384*(3*(64*a^3*b*g - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*d + 8*(5*
a*b^3 - 12*a^2*b*c)*e - 16*(3*a^2*b^2 - 4*a^3*c)*f)*sqrt(-a)*x^4*arctan(1/2
*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(4
8*a^4*d - (144*a^3*b*f - 192*a^4*g + 5*(21*a*b^3 - 44*a^2*b*c)*d - 8*(15*a^
2*b^2 - 16*a^3*c)*e)*x^3 - 2*(40*a^3*b*e - 48*a^4*f - (35*a^2*b^2 - 36*a^3*
c)*d)*x^2 - 8*(7*a^3*b*d - 8*a^4*e)*x)*sqrt(c*x^2 + b*x + a))/(a^5*x^4)]
```

giac [B] time = 0.30, size = 1448, normalized size = 5.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(35*b^4*d - 120*a*b^2*c*d + 48*a^2*c^2*d + 48*a^2*b^2*f - 64*a^3*c*f -
 64*a^3*b*g - 40*a*b^3*e + 96*a^2*b*c*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/192*(105*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^7*b^4*d - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^2*c*d + 14
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*c^2*d + 144*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^7*a^2*b^2*f - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^
3*c*f - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*g - 120*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^7*a*b^3*e + 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^7*a^2*b*c*e - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*sqrt(c)*g - 3
85*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^4*d + 1320*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^5*a^2*b^2*c*d - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*
a^3*c^2*d - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^2*f + 192*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*c*f + 576*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^5*a^4*b*g + 440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^3*e - 1
056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b*c*e - 384*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^4*a^4*b*sqrt(c)*f + 1152*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^4*a^5*sqrt(c)*g - 768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*c^(3/2
)*e + 511*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^4*d - 1752*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*a^3*b^2*c*d - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*a^4*c^2*d + 624*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^2*f +
192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*c*f - 576*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^3*a^5*b*g - 584*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*
b^3*e + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b*c*e - 2048*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*a^4*b*c^(3/2)*d + 768*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*a^5*b*sqrt(c)*f - 1152*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*a^6*sqrt(c)*g - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^2*sqrt(c)*
e + 1024*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*c^(3/2)*e - 279*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*a^3*b^4*d - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x +
```

```
a))*a^4*b^2*c*d + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*c^2*d - 240*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^2*f - 192*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*a^6*c*f + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*g + 26
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^3*e + 288*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a^5*b*c*e - 384*a^4*b^3*sqrt(c)*d + 512*a^5*b*c^(3/2)*d - 38
4*a^6*b*sqrt(c)*f + 384*a^7*sqrt(c)*g + 384*a^5*b^2*sqrt(c)*e - 256*a^6*c^(
3/2)*e)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^4*a^4)
```

maple [B] time = 0.01, size = 591, normalized size = 2.19

$$\frac{bg\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}+\frac{cf\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2f\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}}-\frac{3bce\ln\left(\frac{bx+2a+2\sqrt{c}\cdots}{\cdots}\right)}{4a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/2*f/a/x^2*(c*x^2+b*x+a)^(1/2)+3/4*f*b/a^2/x*(c*x^2+b*x+a)^(1/2)-3/8*f*b^
2/a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+1/2*f*c/a^(3/2)*ln(
(b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-1/3*e/a/x^3*(c*x^2+b*x+a)^(1/2)+
5/12*e*b/a^2/x^2*(c*x^2+b*x+a)^(1/2)-5/8*e*b^2/a^3/x*(c*x^2+b*x+a)^(1/2)+5/
16*e*b^3/a^(7/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-3/4*e*b/a^(5
/2)*c*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+2/3*e*c/a^2/x*(c*x^2+b*
x+a)^(1/2)-1/4*d*(c*x^2+b*x+a)^(1/2)/a/x^4+7/24*d*b/a^2/x^3*(c*x^2+b*x+a)^(
1/2)-35/96*d*b^2/a^3/x^2*(c*x^2+b*x+a)^(1/2)+35/64*d*b^3/a^4/x*(c*x^2+b*x+a
)^(1/2)-35/128*d*b^4/a^(9/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+
15/16*d*b^2/a^(7/2)*c*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-55/48*d
*b/a^3*c/x*(c*x^2+b*x+a)^(1/2)+3/8*d*c/a^2/x^2*(c*x^2+b*x+a)^(1/2)-3/8*d*c^
2/a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-g/a/x*(c*x^2+b*x+a)
^(1/2)+1/2*g*b/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^5\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^5*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^5*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**5/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**5*sqrt(a + b*x + c*x**2)), x)
```

3.288 $\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=371

$$\frac{\sqrt{a+bx+cx^2}\,(9bd-10ae)}{40a^2x^4}-\frac{\sqrt{a+bx+cx^2}\,\left(40a^2b(55ce-36ag)+256a^2c(4cd-5af)-1050ab^3e-60ab^2(49\right.}{1920a^5x}$$

```
[Out] 1/256*(63*b^5*d-70*a*b^4*e+48*a^2*b*c*(-4*a*f+5*c*d)-40*a*b^3*(-2*a*f+7*c*d
)-32*a^3*c*(-4*a*g+3*c*e)+48*a^2*b^2*(-2*a*g+5*c*e))*arctanh(1/2*(b*x+2*a)/
a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(11/2)-1/5*d*(c*x^2+b*x+a)^(1/2)/a/x^5+1/40*
(-10*a*e+9*b*d)*(c*x^2+b*x+a)^(1/2)/a^2/x^4-1/240*(80*a^2*f-70*a*b*e-64*a*c
*d+63*b^2*d)*(c*x^2+b*x+a)^(1/2)/a^3/x^3+1/960*(315*b^3*d-350*a*b^2*e-4*a*b
*(-100*a*f+161*c*d)+120*a^2*(-4*a*g+3*c*e))*(c*x^2+b*x+a)^(1/2)/a^4/x^2-1/1
920*(945*b^4*d-1050*a*b^3*e-60*a*b^2*(-20*a*f+49*c*d)+256*a^2*c*(-5*a*f+4*c
*d)+40*a^2*b*(-36*a*g+55*c*e))*(c*x^2+b*x+a)^(1/2)/a^5/x
```

Rubi [A] time = 0.82, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1650, 834, 806, 724, 206}

$$-\frac{\sqrt{a+bx+cx^2}\,\left(40a^2b(55ce-36ag)+256a^2c(4cd-5af)-60ab^2(49cd-20af)-1050ab^3e+945b^4d\right)}{1920a^5x}+\frac{\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^6*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(d*Sqrt[a + b*x + c*x^2])/(5*a*x^5) + ((9*b*d - 10*a*e)*Sqrt[a + b*x + c*x
^2])/(40*a^2*x^4) - ((63*b^2*d - 64*a*c*d - 70*a*b*e + 80*a^2*f)*Sqrt[a + b
*x + c*x^2])/(240*a^3*x^3) + ((315*b^3*d - 350*a*b^2*e - 4*a*b*(161*c*d - 1
00*a*f) + 120*a^2*(3*c*e - 4*a*g))*Sqrt[a + b*x + c*x^2])/(960*a^4*x^2) - (
(945*b^4*d - 1050*a*b^3*e - 60*a*b^2*(49*c*d - 20*a*f) + 256*a^2*c*(4*c*d -
 5*a*f) + 40*a^2*b*(55*c*e - 36*a*g))*Sqrt[a + b*x + c*x^2])/(1920*a^5*x) +
 ((63*b^5*d - 70*a*b^4*e + 48*a^2*b*c*(5*c*d - 4*a*f) - 40*a*b^3*(7*c*d - 2
*a*f) - 32*a^3*c*(3*c*e - 4*a*g) + 48*a^2*b^2*(5*c*e - 2*a*g))*ArcTanh[(2*a
 + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(256*a^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} - \frac{\int \frac{\frac{1}{2}(9bd-10ae)+(4cd-5af)x-5agx^2}{x^5\sqrt{a+bx+cx^2}}\,dx}{5a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} + \frac{\int \frac{\frac{1}{4}\left(63b^2d-64acd-70abe+80a^2f\right)+\frac{1}{2}\left(27bc\right.}{x^4\sqrt{a+bx+cx^2}}}{20a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{\left(63b^2d-64acd-70abe+80a^2f\right.}{240a^3x^3}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{\left(63b^2d-64acd-70abe+80a^2f\right.}{240a^3x^3}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{\left(63b^2d-64acd-70abe+80a^2f\right.}{240a^3x^3}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{\left(63b^2d-64acd-70abe+80a^2f\right.}{240a^3x^3}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{\left(63b^2d-64acd-70abe+80a^2f\right.}{240a^3x^3}$$

Mathematica [A] time = 0.73, size = 299, normalized size = 0.81

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)\left(32a^3c(4ag-3ce)-48a^2b^2(2ag-5ce)-48a^2bc(4af-5cd)-70ab^4e+40ab^3(2af-7cd\right.}{256a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^6*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/1920*(Sqrt[a + x*(b + c*x)]*(945*b^4*d*x^4 - 210*a*b^2*x^3*(3*b*d + 14*c
*d*x + 5*b*e*x) + 32*a^4*(12*d + 5*x*(3*e + 4*f*x + 6*g*x^2)) + 4*a^2*x^2*(
256*c^2*d*x^2 + 2*b*c*x*(161*d + 275*e*x) + b^2*(126*d + 25*x*(7*e + 12*f*x
))) - 16*a^3*x*(c*x*(32*d + 5*x*(9*e + 16*f*x)) + b*(27*d + 5*x*(7*e + 2*x*
(5*f + 9*g*x))))))/(a^5*x^5) + ((63*b^5*d - 70*a*b^4*e + 40*a*b^3*(-7*c*d +
 2*a*f) - 48*a^2*b*c*(-5*c*d + 4*a*f) - 48*a^2*b^2*(-5*c*e + 2*a*g) + 32*a^
3*c*(-3*c*e + 4*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])
])/(256*a^(11/2))
```

fricas [A] time = 42.47, size = 727, normalized size = 1.96

$$\left[\frac{15\left(\left(63\,b^5-280\,ab^3c+240\,a^2bc^2\right)d-2\left(35\,ab^4-120\,a^2b^2c+48\,a^3c^2\right)e+16\left(5\,a^2b^3-12\,a^3bc\right)f-32\left(3\,a^3b^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/7680*(15*((63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*d - 2*(35*a*b^4 - 120*a
^2*b^2*c + 48*a^3*c^2)*e + 16*(5*a^2*b^3 - 12*a^3*b*c)*f - 32*(3*a^3*b^2 -
4*a^4*c)*g)*sqrt(a)*x^5*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 +
b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(384*a^5*d - (1440*a^4*b*g -
 (945*a*b^4 - 2940*a^2*b^2*c + 1024*a^3*c^2)*d + 50*(21*a^2*b^3 - 44*a^3*b*
c)*e - 80*(15*a^3*b^2 - 16*a^4*c)*f)*x^4 - 2*(400*a^4*b*f - 480*a^5*g + 7*(
45*a^2*b^3 - 92*a^3*b*c)*d - 10*(35*a^3*b^2 - 36*a^4*c)*e)*x^3 - 8*(70*a^4*
b*e - 80*a^5*f - (63*a^3*b^2 - 64*a^4*c)*d)*x^2 - 48*(9*a^4*b*d - 10*a^5*e)
*x)*sqrt(c*x^2 + b*x + a))/(a^6*x^5), -1/3840*(15*((63*b^5 - 280*a*b^3*c +
240*a^2*b*c^2)*d - 2*(35*a*b^4 - 120*a^2*b^2*c + 48*a^3*c^2)*e + 16*(5*a^2*
b^3 - 12*a^3*b*c)*f - 32*(3*a^3*b^2 - 4*a^4*c)*g)*sqrt(-a)*x^5*arctan(1/2*s
qrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(384
*a^5*d - (1440*a^4*b*g - (945*a*b^4 - 2940*a^2*b^2*c + 1024*a^3*c^2)*d + 50
*(21*a^2*b^3 - 44*a^3*b*c)*e - 80*(15*a^3*b^2 - 16*a^4*c)*f)*x^4 - 2*(400*a
^4*b*f - 480*a^5*g + 7*(45*a^2*b^3 - 92*a^3*b*c)*d - 10*(35*a^3*b^2 - 36*a^
4*c)*e)*x^3 - 8*(70*a^4*b*e - 80*a^5*f - (63*a^3*b^2 - 64*a^4*c)*d)*x^2 - 4
8*(9*a^4*b*d - 10*a^5*e)*x)*sqrt(c*x^2 + b*x + a))/(a^6*x^5)]
```

giac [B] time = 0.51, size = 2177, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/128*(63*b^5*d - 280*a*b^3*c*d + 240*a^2*b*c^2*d + 80*a^2*b^3*f - 192*a^3
*b*c*f - 96*a^3*b^2*g + 128*a^4*c*g - 70*a*b^4*e + 240*a^2*b^2*c*e - 96*a^3
*c^2*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^5
) + 1/1920*(945*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*d - 4200*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c*d + 3600*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^9*a^2*b*c^2*d + 1200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^3*
f - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b*c*f - 1440*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^9*a^3*b^2*g + 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^9*a^4*c*g - 1050*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^4*e + 3600*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^2*c*e - 1440*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^9*a^3*c^2*e - 4410*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*
a*b^5*d + 19600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c*d - 16800*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^2*d - 5600*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^7*a^3*b^3*f + 13440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*
```

```
a^4*b*c*f + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b^2*g - 3840*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*c*g + 4900*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^7*a^2*b^4*e - 16800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^
2*c*e + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*c^2*e + 7680*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^6*a^5*c^(3/2)*f + 3840*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^6*a^5*b*sqrt(c)*g + 8064*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5
*a^2*b^5*d - 35840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^3*c*d + 3072
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b*c^2*d + 10240*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^5*a^4*b^3*f - 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^5*a^5*b*c*f - 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^2*g - 896
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^4*e + 30720*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^5*a^4*b^2*c*e + 20480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^4*a^5*c^(5/2)*d + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^2*sqrt
(c)*f - 17920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*c^(3/2)*f - 11520*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b*sqrt(c)*g + 20480*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*a^5*b*c^(3/2)*e - 7110*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a^3*b^5*d + 31600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c
*d + 16800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*c^2*d - 8480*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^3*f + 1920*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*a^6*b*c*f + 8640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b^2*g
 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*c*g + 7900*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*a^4*b^4*e - 13920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^3*a^5*b^2*c*e - 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*c^2*e + 3
8400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^2*c^(3/2)*d - 10240*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*c^(5/2)*d - 7680*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*a^6*b^2*sqrt(c)*f + 12800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*a^7*c^(3/2)*f + 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b*sqrt
(c)*g + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^3*sqrt(c)*e - 2560
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b*c^(3/2)*e + 2895*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*a^4*b^5*d + 4200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*a^5*b^3*c*d - 3600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*c^2*d + 2640
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^3*f + 2880*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a^7*b*c*f - 2400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b^2
*g - 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8*c*g - 2790*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^5*b^4*e - 3600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
a^6*b^2*c*e + 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*c^2*e + 3840*a^5
*b^4*sqrt(c)*d - 7680*a^6*b^2*c^(3/2)*d + 2048*a^7*c^(5/2)*d + 3840*a^7*b^2
*sqrt(c)*f - 2560*a^8*c^(3/2)*f - 3840*a^8*b*sqrt(c)*g - 3840*a^6*b^3*sqrt(
c)*e + 5120*a^7*b*c^(3/2)*e)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^5
*a^5)
```

maple [B] time = 0.02, size = 859, normalized size = 2.32

$$\frac{cg\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\ \sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2g\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\ \sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}}-\frac{3bcf\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\ \sqrt{a}}{x}\right)}{4a^{\frac{5}{2}}}-\frac{3c^2e\ln\left(\frac{bx+2a+2\sqrt{c\,x^2}}{x}\right.}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/4*e/a/x^4*(c*x^2+b*x+a)^(1/2)-35/128*e*b^4/a^(9/2)*ln((b*x+2*a+2*(c*x^2+
b*x+a)^(1/2)*a^(1/2))/x)-3/8*e*c^2/a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2
)*a^(1/2))/x)-1/3*f/a/x^3*(c*x^2+b*x+a)^(1/2)+5/16*f*b^3/a^(7/2)*ln((b*x+2*
a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+63/256*d*b^5/a^(11/2)*ln((b*x+2*a+2*(c*
x^2+b*x+a)^(1/2)*a^(1/2))/x)-1/2*g/a/x^2*(c*x^2+b*x+a)^(1/2)-3/8*g*b^2/a^(5
/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+1/2*g*c/a^(3/2)*ln((b*x+2
*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-1/5*d*(c*x^2+b*x+a)^(1/2)/a/x^5-161/24
0*d*b/a^3*c/x^2*(c*x^2+b*x+a)^(1/2)-55/48*e*b/a^3*c/x*(c*x^2+b*x+a)^(1/2)+4
9/32*d*b^2/a^4*c/x*(c*x^2+b*x+a)^(1/2)+3/8*e*c/a^2/x^2*(c*x^2+b*x+a)^(1/2)+
```

```
9/40*d*b/a^2/x^4*(c*x^2+b*x+a)^(1/2)-21/80*d*b^2/a^3/x^3*(c*x^2+b*x+a)^(1/2
)+21/64*d*b^3/a^4/x^2*(c*x^2+b*x+a)^(1/2)-63/128*d*b^4/a^5/x*(c*x^2+b*x+a)^
(1/2)-35/32*d*b^3/a^(9/2)*c*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+1
5/16*d*b/a^(7/2)*c^2*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+4/15*d*c
/a^2/x^3*(c*x^2+b*x+a)^(1/2)-8/15*d*c^2/a^3/x*(c*x^2+b*x+a)^(1/2)+5/12*f*b/
a^2/x^2*(c*x^2+b*x+a)^(1/2)-5/8*f*b^2/a^3/x*(c*x^2+b*x+a)^(1/2)-3/4*f*b/a^(
5/2)*c*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+2/3*f*c/a^2/x*(c*x^2+b
*x+a)^(1/2)+7/24*e*b/a^2/x^3*(c*x^2+b*x+a)^(1/2)-35/96*e*b^2/a^3/x^2*(c*x^2
+b*x+a)^(1/2)+3/4*g*b/a^2/x*(c*x^2+b*x+a)^(1/2)+35/64*e*b^3/a^4/x*(c*x^2+b*
x+a)^(1/2)+15/16*e*b^2/a^(7/2)*c*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))
/x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^6\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^6*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^6*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**6/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**6*sqrt(a + b*x + c*x**2)), x)
```

3.289 $\int (d+ex)^3 \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx$

Optimal. Leaf size=258

$$\frac{\left(300d^2+85de+17e^2\right)(d+ex)^8}{8e^7}-\frac{2\left(200d^3+85d^2e+34de^2+2e^3\right)(d+ex)^7}{7e^7}+\frac{\left(300d^4+170d^3e+102d^2e^2+12de\right.}{6e^7}$$

```
[Out] 1/4*(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^4/e^7
-1/5*(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*(e*x+d)^5/e^7+
1/6*(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e^4)*(e*x+d)^6/e^7-2/7*(200*
d^3+85*d^2*e+34*d*e^2+2*e^3)*(e*x+d)^7/e^7+1/8*(300*d^2+85*d*e+17*e^2)*(e*x
+d)^8/e^7-1/9*(120*d+17*e)*(e*x+d)^9/e^7+2*(e*x+d)^10/e^7
```

Rubi [A] time = 0.26, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1628}

$$\frac{\left(300d^2+85de+17e^2\right)(d+ex)^8}{8e^7}-\frac{2\left(85d^2e+200d^3+34de^2+2e^3\right)(d+ex)^7}{7e^7}+\frac{\left(102d^2e^2+170d^3e+300d^4+12de\right.}{6e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d +
 e*x)^4)/(4*e^7) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^
4 - 7*e^5)*(d + e*x)^5)/(5*e^7) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*
d*e^3 + 21*e^4)*(d + e*x)^6)/(6*e^7) - (2*(200*d^3 + 85*d^2*e + 34*d*e^2 +
2*e^3)*(d + e*x)^7)/(7*e^7) + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^8)/(8*
e^7) - ((120*d + 17*e)*(d + e*x)^9)/(9*e^7) + (2*(d + e*x)^10)/e^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)dx=\int\left(\frac{\left(20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5\right.}{e^6}\right.$$

$$=\frac{\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d}{4e^7}$$

Mathematica [A] time = 0.04, size = 212, normalized size = 0.82

$$6d^3x+\frac{1}{8}ex^8\left(60d^2-51de+17e^2\right)+dx^3\left(7d^2+7de+6e^2\right)+\frac{1}{2}d^2x^2(7d+18e)+\frac{1}{7}x^7\left(20d^3-51d^2e+51de^2-4e^3\right)+\frac{1}{6}x$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d^3*x + (d^2*(7*d + 18*e)*x^2)/2 + d*(7*d^2 + 7*d*e + 6*e^2)*x^3 + ((-4*d
^3 + 63*d^2*e + 21*d*e^2 + 6*e^3)*x^4)/4 + ((17*d^3 - 12*d^2*e + 63*d*e^2 +
 7*e^3)*x^5)/5 + ((-17*d^3 + 51*d^2*e - 12*d*e^2 + 21*e^3)*x^6)/6 + ((20*d^
```

```
3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7)/7 + (e*(60*d^2 - 51*d*e + 17*e^2)*x^8
)/8 + ((60*d - 17*e)*e^2*x^9)/9 + 2*e^3*x^10
```

fricas [A] time = 0.74, size = 237, normalized size = 0.92

$$2x^{10}e^3-\frac{17}{9}x^9e^3+\frac{20}{3}x^9e^2d+\frac{17}{8}x^8e^3-\frac{51}{8}x^8e^2d+\frac{15}{2}x^8ed^2-\frac{4}{7}x^7e^3+\frac{51}{7}x^7e^2d-\frac{51}{7}x^7ed^2+\frac{20}{7}x^7d^3+\frac{7}{2}x^6e^3-2x^6e^2d+\frac{17}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 2*x^10*e^3 - 17/9*x^9*e^3 + 20/3*x^9*e^2*d + 17/8*x^8*e^3 - 51/8*x^8*e^2*d
+ 15/2*x^8*e*d^2 - 4/7*x^7*e^3 + 51/7*x^7*e^2*d - 51/7*x^7*e*d^2 + 20/7*x^7
*d^3 + 7/2*x^6*e^3 - 2*x^6*e^2*d + 17/2*x^6*e*d^2 - 17/6*x^6*d^3 + 7/5*x^5*
e^3 + 63/5*x^5*e^2*d - 12/5*x^5*e*d^2 + 17/5*x^5*d^3 + 3/2*x^4*e^3 + 21/4*x
^4*e^2*d + 63/4*x^4*e*d^2 - x^4*d^3 + 6*x^3*e^2*d + 7*x^3*e*d^2 + 7*x^3*d^3
 + 9*x^2*e*d^2 + 7/2*x^2*d^3 + 6*x*d^3
```

giac [A] time = 0.17, size = 230, normalized size = 0.89

$$2\,x^{10}e^3+\frac{20}{3}\,dx^9e^2+\frac{15}{2}\,d^2x^8e+\frac{20}{7}\,d^3x^7-\frac{17}{9}\,x^9e^3-\frac{51}{8}\,dx^8e^2-\frac{51}{7}\,d^2x^7e-\frac{17}{6}\,d^3x^6+\frac{17}{8}\,x^8e^3+\frac{51}{7}\,dx^7e^2+\frac{17}{2}\,d^2x^6e+\frac{17}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 2*x^10*e^3 + 20/3*d*x^9*e^2 + 15/2*d^2*x^8*e + 20/7*d^3*x^7 - 17/9*x^9*e^3
- 51/8*d*x^8*e^2 - 51/7*d^2*x^7*e - 17/6*d^3*x^6 + 17/8*x^8*e^3 + 51/7*d*x^
7*e^2 + 17/2*d^2*x^6*e + 17/5*d^3*x^5 - 4/7*x^7*e^3 - 2*d*x^6*e^2 - 12/5*d^
2*x^5*e - d^3*x^4 + 7/2*x^6*e^3 + 63/5*d*x^5*e^2 + 63/4*d^2*x^4*e + 7*d^3*x
^3 + 7/5*x^5*e^3 + 21/4*d*x^4*e^2 + 7*d^2*x^3*e + 7/2*d^3*x^2 + 3/2*x^4*e^3
 + 6*d*x^3*e^2 + 9*d^2*x^2*e + 6*d^3*x
```

maple [A] time = 0.00, size = 208, normalized size = 0.81

$$2e^3x^{10}+\frac{\left(60d\,e^2-17e^3\right)x^9}{9}+\frac{\left(60d^2e-51d\,e^2+17e^3\right)x^8}{8}+\frac{\left(20d^3-51d^2e+51d\,e^2-4e^3\right)x^7}{7}+\frac{\left(-17d^3+51d^2e-1\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 2*e^3*x^10+1/9*(60*d*e^2-17*e^3)*x^9+1/8*(60*d^2*e-51*d*e^2+17*e^3)*x^8+1/7
*(20*d^3-51*d^2*e+51*d*e^2-4*e^3)*x^7+1/6*(-17*d^3+51*d^2*e-12*d*e^2+21*e^3
)*x^6+1/5*(17*d^3-12*d^2*e+63*d*e^2+7*e^3)*x^5+1/4*(-4*d^3+63*d^2*e+21*d*e^
2+6*e^3)*x^4+1/3*(21*d^3+21*d^2*e+18*d*e^2)*x^3+1/2*(7*d^3+18*d^2*e)*x^2+6*
d^3*x
```

maxima [A] time = 0.43, size = 206, normalized size = 0.80

$$2\,e^3x^{10}+\frac{1}{9}\left(60\,de^2-17\,e^3\right)x^9+\frac{1}{8}\left(60\,d^2e-51\,de^2+17\,e^3\right)x^8+\frac{1}{7}\left(20\,d^3-51\,d^2e+51\,de^2-4\,e^3\right)x^7-\frac{1}{6}\left(17\,d^3-51\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 2*e^3*x^10 + 1/9*(60*d*e^2 - 17*e^3)*x^9 + 1/8*(60*d^2*e - 51*d*e^2 + 17*e^
3)*x^8 + 1/7*(20*d^3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7 - 1/6*(17*d^3 - 51*
```

```
d^2*e + 12*d*e^2 - 21*e^3)*x^6 + 1/5*(17*d^3 - 12*d^2*e + 63*d*e^2 + 7*e^3)
*x^5 - 1/4*(4*d^3 - 63*d^2*e - 21*d*e^2 - 6*e^3)*x^4 + 6*d^3*x + (7*d^3 + 7
*d^2*e + 6*d*e^2)*x^3 + 1/2*(7*d^3 + 18*d^2*e)*x^2
```

mupad [B] time = 4.20, size = 196, normalized size = 0.76

$$6\,d^3\,x+x^8\,\left(\frac{15\,d^2\,e}{2}-\frac{51\,d\,e^2}{8}+\frac{17\,e^3}{8}\right)-x^6\,\left(\frac{17\,d^3}{6}-\frac{17\,d^2\,e}{2}+2\,d\,e^2-\frac{7\,e^3}{2}\right)+x^4\,\left(-d^3+\frac{63\,d^2\,e}{4}+\frac{21\,d\,e^2}{4}+\frac{3\,e^3}{2}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] 6*d^3*x + x^8*((15*d^2*e)/2 - (51*d*e^2)/8 + (17*e^3)/8) - x^6*(2*d*e^2 - (
17*d^2*e)/2 + (17*d^3)/6 - (7*e^3)/2) + x^4*((21*d*e^2)/4 + (63*d^2*e)/4 -
d^3 + (3*e^3)/2) + x^5*((63*d*e^2)/5 - (12*d^2*e)/5 + (17*d^3)/5 + (7*e^3)/
5) + x^7*((51*d*e^2)/7 - (51*d^2*e)/7 + (20*d^3)/7 - (4*e^3)/7) + 2*e^3*x^1
0 + d*x^3*(7*d*e + 7*d^2 + 6*e^2) + (d^2*x^2*(7*d + 18*e))/2 + (e^2*x^9*(60
*d - 17*e))/9
```

sympy [A] time = 0.52, size = 230, normalized size = 0.89

$$6d^3x+2e^3x^{10}+x^9\left(\frac{20de^2}{3}-\frac{17e^3}{9}\right)+x^8\left(\frac{15d^2e}{2}-\frac{51de^2}{8}+\frac{17e^3}{8}\right)+x^7\left(\frac{20d^3}{7}-\frac{51d^2e}{7}+\frac{51de^2}{7}-\frac{4e^3}{7}\right)+x^6\left(-\frac{17d^3}{6}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 6*d**3*x + 2*e**3*x**10 + x**9*(20*d*e**2/3 - 17*e**3/9) + x**8*(15*d**2*e/
2 - 51*d*e**2/8 + 17*e**3/8) + x**7*(20*d**3/7 - 51*d**2*e/7 + 51*d*e**2/7
- 4*e**3/7) + x**6*(-17*d**3/6 + 17*d**2*e/2 - 2*d*e**2 + 7*e**3/2) + x**5*
(17*d**3/5 - 12*d**2*e/5 + 63*d*e**2/5 + 7*e**3/5) + x**4*(-d**3 + 63*d**2*
e/4 + 21*d*e**2/4 + 3*e**3/2) + x**3*(7*d**3 + 7*d**2*e + 6*d*e**2) + x**2*
(7*d**3/2 + 9*d**2*e)
```

3.290 $\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) d$

Optimal. Leaf size=157

$$\frac{1}{7}x^7 \left(20d^2-34de+17e^2\right) - \frac{1}{6}x^6 \left(17d^2-34de+4e^2\right) + \frac{1}{5}x^5 \left(17d^2-8de+21e^2\right) - \frac{1}{4}x^4 \left(4d^2-42de-7e^2\right) + \frac{1}{3}x^3 \left(2\right.$$

```
[Out] 6*d^2*x+1/2*d*(7*d+12*e)*x^2+1/3*(21*d^2+14*d*e+6*e^2)*x^3-1/4*(4*d^2-42*d*
e-7*e^2)*x^4+1/5*(17*d^2-8*d*e+21*e^2)*x^5-1/6*(17*d^2-34*d*e+4*e^2)*x^6+1/
7*(20*d^2-34*d*e+17*e^2)*x^7+1/8*(40*d-17*e)*e*x^8+20/9*e^2*x^9
```

Rubi [A] time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1628}

$$\frac{1}{7}x^7 \left(20d^2-34de+17e^2\right) - \frac{1}{6}x^6 \left(17d^2-34de+4e^2\right) + \frac{1}{5}x^5 \left(17d^2-8de+21e^2\right) - \frac{1}{4}x^4 \left(4d^2-42de-7e^2\right) + \frac{1}{3}x^3 \left(2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d^2*x + (d*(7*d + 12*e)*x^2)/2 + ((21*d^2 + 14*d*e + 6*e^2)*x^3)/3 - ((4*
d^2 - 42*d*e - 7*e^2)*x^4)/4 + ((17*d^2 - 8*d*e + 21*e^2)*x^5)/5 - ((17*d^2
 - 34*d*e + 4*e^2)*x^6)/6 + ((20*d^2 - 34*d*e + 17*e^2)*x^7)/7 + ((40*d - 1
7*e)*e*x^8)/8 + (20*e^2*x^9)/9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx = \int \left(6d^2+d(7d+12e)x+\left(21d^2+14de+6e^2\right)x^2-\right.$$

$$= 6d^2x+\frac{1}{2}d(7d+12e)x^2+\frac{1}{3}\left(21d^2+14de+6e^2\right)x^3$$

Mathematica [A] time = 0.03, size = 136, normalized size = 0.87

$$d^2\left(\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x\right)+de\left(5x^8-\frac{34x^7}{7}+\frac{17x^6}{3}-\frac{8x^5}{5}+\frac{21x^4}{2}+\frac{14x^3}{3}+6x^2\right)+\frac{e^2\left(5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] (e^2*x^3*(5040 + 4410*x + 10584*x^2 - 1680*x^3 + 6120*x^4 - 5355*x^5 + 5600
*x^6))/2520 + d^2*(6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6
+ (20*x^7)/7) + d*e*(6*x^2 + (14*x^3)/3 + (21*x^4)/2 - (8*x^5)/5 + (17*x^6)
/3 - (34*x^7)/7 + 5*x^8)
```

fricas [A] time = 0.67, size = 160, normalized size = 1.02

$$\frac{20}{9}x^9e^2-\frac{17}{8}x^8e^2+5x^8ed+\frac{17}{7}x^7e^2-\frac{34}{7}x^7ed+\frac{20}{7}x^7d^2-\frac{2}{3}x^6e^2+\frac{17}{3}x^6ed-\frac{17}{6}x^6d^2+\frac{21}{5}x^5e^2-\frac{8}{5}x^5ed+\frac{17}{5}x^5d^2+\frac{7}{4}x^4e^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 20/9*x^9*e^2 - 17/8*x^8*e^2 + 5*x^8*e*d + 17/7*x^7*e^2 - 34/7*x^7*e*d + 20/
7*x^7*d^2 - 2/3*x^6*e^2 + 17/3*x^6*e*d - 17/6*x^6*d^2 + 21/5*x^5*e^2 - 8/5*
x^5*e*d + 17/5*x^5*d^2 + 7/4*x^4*e^2 + 21/2*x^4*e*d - x^4*d^2 + 2*x^3*e^2 +
 14/3*x^3*e*d + 7*x^3*d^2 + 6*x^2*e*d + 7/2*x^2*d^2 + 6*x*d^2
```

giac [A] time = 0.16, size = 160, normalized size = 1.02

$$\frac{20}{9}x^9e^2+5dx^8e+\frac{20}{7}d^2x^7-\frac{17}{8}x^8e^2-\frac{34}{7}dx^7e-\frac{17}{6}d^2x^6+\frac{17}{7}x^7e^2+\frac{17}{3}dx^6e+\frac{17}{5}d^2x^5-\frac{2}{3}x^6e^2-\frac{8}{5}dx^5e-d^2x^4+\frac{21}{5}x^5e^2-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 20/9*x^9*e^2 + 5*d*x^8*e + 20/7*d^2*x^7 - 17/8*x^8*e^2 - 34/7*d*x^7*e - 17/
6*d^2*x^6 + 17/7*x^7*e^2 + 17/3*d*x^6*e + 17/5*d^2*x^5 - 2/3*x^6*e^2 - 8/5*
d*x^5*e - d^2*x^4 + 21/5*x^5*e^2 + 21/2*d*x^4*e + 7*d^2*x^3 + 7/4*x^4*e^2 +
 14/3*d*x^3*e + 7/2*d^2*x^2 + 2*x^3*e^2 + 6*d*x^2*e + 6*d^2*x
```

maple [A] time = 0.00, size = 146, normalized size = 0.93

$$\frac{20e^2x^9}{9}+\frac{\left(40de-17e^2\right)x^8}{8}+\frac{\left(20d^2-34de+17e^2\right)x^7}{7}+\frac{\left(-17d^2+34de-4e^2\right)x^6}{6}+\frac{\left(17d^2-8de+21e^2\right)x^5}{5}+\frac{\left(-4d^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 20/9*e^2*x^9+1/8*(40*d*e-17*e^2)*x^8+1/7*(20*d^2-34*d*e+17*e^2)*x^7+1/6*(-1
7*d^2+34*d*e-4*e^2)*x^6+1/5*(17*d^2-8*d*e+21*e^2)*x^5+1/4*(-4*d^2+42*d*e+7*
e^2)*x^4+1/3*(21*d^2+14*d*e+6*e^2)*x^3+1/2*(7*d^2+12*d*e)*x^2+6*d^2*x
```

maxima [A] time = 0.43, size = 145, normalized size = 0.92

$$\frac{20}{9}e^2x^9+\frac{1}{8}\left(40de-17e^2\right)x^8+\frac{1}{7}\left(20d^2-34de+17e^2\right)x^7-\frac{1}{6}\left(17d^2-34de+4e^2\right)x^6+\frac{1}{5}\left(17d^2-8de+21e^2\right)x^5-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 20/9*e^2*x^9 + 1/8*(40*d*e - 17*e^2)*x^8 + 1/7*(20*d^2 - 34*d*e + 17*e^2)*x
^7 - 1/6*(17*d^2 - 34*d*e + 4*e^2)*x^6 + 1/5*(17*d^2 - 8*d*e + 21*e^2)*x^5
- 1/4*(4*d^2 - 42*d*e - 7*e^2)*x^4 + 1/3*(21*d^2 + 14*d*e + 6*e^2)*x^3 + 6*
d^2*x + 1/2*(7*d^2 + 12*d*e)*x^2
```

mupad [B] time = 4.11, size = 137, normalized size = 0.87

$$x^3\left(7d^2+\frac{14de}{3}+2e^2\right)+x^4\left(-d^2+\frac{21de}{2}+\frac{7e^2}{4}\right)-x^6\left(\frac{17d^2}{6}-\frac{17de}{3}+\frac{2e^2}{3}\right)+x^5\left(\frac{17d^2}{5}-\frac{8de}{5}+\frac{21e^2}{5}\right)+x^7\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] x^3*((14*d*e)/3 + 7*d^2 + 2*e^2) + x^4*((21*d*e)/2 - d^2 + (7*e^2)/4) - x^6
*((17*d^2)/6 - (17*d*e)/3 + (2*e^2)/3) + x^5*((17*d^2)/5 - (8*d*e)/5 + (21*
```

```
e^2)/5) + x^7*((20*d^2)/7 - (34*d*e)/7 + (17*e^2)/7) + 6*d^2*x + (20*e^2*x^
9)/9 + (d*x^2*(7*d + 12*e))/2 + (e*x^8*(40*d - 17*e))/8
```

sympy [A] time = 0.15, size = 158, normalized size = 1.01

$$6d^2x+\frac{20e^2x^9}{9}+x^8\left(5de-\frac{17e^2}{8}\right)+x^7\left(\frac{20d^2}{7}-\frac{34de}{7}+\frac{17e^2}{7}\right)+x^6\left(-\frac{17d^2}{6}+\frac{17de}{3}-\frac{2e^2}{3}\right)+x^5\left(\frac{17d^2}{5}-\frac{8de}{5}+\frac{21e^2}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 6*d**2*x + 20*e**2*x**9/9 + x**8*(5*d*e - 17*e**2/8) + x**7*(20*d**2/7 - 34
*d*e/7 + 17*e**2/7) + x**6*(-17*d**2/6 + 17*d*e/3 - 2*e**2/3) + x**5*(17*d*
*2/5 - 8*d*e/5 + 21*e**2/5) + x**4*(-d**2 + 21*d*e/2 + 7*e**2/4) + x**3*(7*
d**2 + 14*d*e/3 + 2*e**2) + x**2*(7*d**2/2 + 6*d*e)
```

3.291 $\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx$

Optimal. Leaf size=93

$$\frac{1}{7}x^7(20d-17e)-\frac{17}{6}x^6(d-e)+\frac{1}{5}x^5(17d-4e)-\frac{1}{4}x^4(4d-21e)+\frac{7}{3}x^3(3d+e)+\frac{1}{2}x^2(7d+6e)+6dx+\frac{5ex^8}{2}$$

```
[Out] 6*d*x+1/2*(7*d+6*e)*x^2+7/3*(3*d+e)*x^3-1/4*(4*d-21*e)*x^4+1/5*(17*d-4*e)*x
^5-17/6*(d-e)*x^6+1/7*(20*d-17*e)*x^7+5/2*e*x^8
```

Rubi [A] time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1628}

$$\frac{1}{7}x^7(20d-17e)-\frac{17}{6}x^6(d-e)+\frac{1}{5}x^5(17d-4e)-\frac{1}{4}x^4(4d-21e)+\frac{7}{3}x^3(3d+e)+\frac{1}{2}x^2(7d+6e)+6dx+\frac{5ex^8}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d*x + ((7*d + 6*e)*x^2)/2 + (7*(3*d + e)*x^3)/3 - ((4*d - 21*e)*x^4)/4 +
((17*d - 4*e)*x^5)/5 - (17*(d - e)*x^6)/6 + ((20*d - 17*e)*x^7)/7 + (5*e*x^
8)/2
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx = \int \left(6d+(7d+6e)x+7(3d+e)x^2-(4d-21e)x^3+(17\right.$$
$$= 6dx+\frac{1}{2}(7d+6e)x^2+\frac{7}{3}(3d+e)x^3-\frac{1}{4}(4d-21e)x^4+$$

Mathematica [A] time = 0.01, size = 93, normalized size = 1.00

$$\frac{1}{7}x^7(20d-17e)-\frac{17}{6}x^6(d-e)+\frac{1}{5}x^5(17d-4e)+\frac{1}{4}x^4(21e-4d)+\frac{7}{3}x^3(3d+e)+\frac{1}{2}x^2(7d+6e)+6dx+\frac{5ex^8}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d*x + ((7*d + 6*e)*x^2)/2 + (7*(3*d + e)*x^3)/3 + ((-4*d + 21*e)*x^4)/4 +
 ((17*d - 4*e)*x^5)/5 - (17*(d - e)*x^6)/6 + ((20*d - 17*e)*x^7)/7 + (5*e*x
^8)/2
```

fricas [A] time = 0.61, size = 83, normalized size = 0.89

$$\frac{5}{2}x^8e-\frac{17}{7}x^7e+\frac{20}{7}x^7d+\frac{17}{6}x^6e-\frac{17}{6}x^6d-\frac{4}{5}x^5e+\frac{17}{5}x^5d+\frac{21}{4}x^4e-x^4d+\frac{7}{3}x^3e+7x^3d+3x^2e+\frac{7}{2}x^2d+6xd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="frica
s")
```

```
[Out] 5/2*x^8*e - 17/7*x^7*e + 20/7*x^7*d + 17/6*x^6*e - 17/6*x^6*d - 4/5*x^5*e +
 17/5*x^5*d + 21/4*x^4*e - x^4*d + 7/3*x^3*e + 7*x^3*d + 3*x^2*e + 7/2*x^2*
d + 6*x*d
```

giac [A] time = 0.15, size = 90, normalized size = 0.97

$$\frac{5}{2}x^8e+\frac{20}{7}dx^7-\frac{17}{7}x^7e-\frac{17}{6}dx^6+\frac{17}{6}x^6e+\frac{17}{5}dx^5-\frac{4}{5}x^5e-dx^4+\frac{21}{4}x^4e+7dx^3+\frac{7}{3}x^3e+\frac{7}{2}dx^2+3x^2e+6dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac"
)
```

```
[Out] 5/2*x^8*e + 20/7*d*x^7 - 17/7*x^7*e - 17/6*d*x^6 + 17/6*x^6*e + 17/5*d*x^5
- 4/5*x^5*e - d*x^4 + 21/4*x^4*e + 7*d*x^3 + 7/3*x^3*e + 7/2*d*x^2 + 3*x^2*
e + 6*d*x
```

maple [A] time = 0.00, size = 84, normalized size = 0.90

$$\frac{5ex^8}{2}+\frac{(20d-17e)x^7}{7}+\frac{(-17d+17e)x^6}{6}+\frac{(17d-4e)x^5}{5}+\frac{(-4d+21e)x^4}{4}+\frac{(21d+7e)x^3}{3}+6dx+\frac{(7d+6e)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 5/2*e*x^8+1/7*(20*d-17*e)*x^7+1/6*(-17*d+17*e)*x^6+1/5*(17*d-4*e)*x^5+1/4*(
-4*d+21*e)*x^4+1/3*(21*d+7*e)*x^3+1/2*(7*d+6*e)*x^2+6*d*x
```

maxima [A] time = 0.43, size = 79, normalized size = 0.85

$$\frac{5}{2}ex^8+\frac{1}{7}(20d-17e)x^7-\frac{17}{6}(d-e)x^6+\frac{1}{5}(17d-4e)x^5-\frac{1}{4}(4d-21e)x^4+\frac{7}{3}(3d+e)x^3+\frac{1}{2}(7d+6e)x^2+6dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxim
a")
```

```
[Out] 5/2*e*x^8 + 1/7*(20*d - 17*e)*x^7 - 17/6*(d - e)*x^6 + 1/5*(17*d - 4*e)*x^5
 - 1/4*(4*d - 21*e)*x^4 + 7/3*(3*d + e)*x^3 + 1/2*(7*d + 6*e)*x^2 + 6*d*x
```

mupad [B] time = 0.05, size = 77, normalized size = 0.83

$$\frac{5ex^8}{2}+\left(\frac{20d}{7}-\frac{17e}{7}\right)x^7+\left(\frac{17e}{6}-\frac{17d}{6}\right)x^6+\left(\frac{17d}{5}-\frac{4e}{5}\right)x^5+\left(\frac{21e}{4}-d\right)x^4+\left(7d+\frac{7e}{3}\right)x^3+\left(\frac{7d}{2}+3e\right)x^2+6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] x^2*((7*d)/2 + 3*e) + x^3*(7*d + (7*e)/3) + x^5*((17*d)/5 - (4*e)/5) - x^6*
((17*d)/6 - (17*e)/6) + x^7*((20*d)/7 - (17*e)/7) + 6*d*x + (5*e*x^8)/2 - x
^4*(d - (21*e)/4)
```

sympy [A] time = 1.91, size = 87, normalized size = 0.94

$$6dx+\frac{5ex^8}{2}+x^7\left(\frac{20d}{7}-\frac{17e}{7}\right)+x^6\left(-\frac{17d}{6}+\frac{17e}{6}\right)+x^5\left(\frac{17d}{5}-\frac{4e}{5}\right)+x^4\left(-d+\frac{21e}{4}\right)+x^3\left(7d+\frac{7e}{3}\right)+x^2\left(\frac{7d}{2}+3e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 6*d*x + 5*e*x**8/2 + x**7*(20*d/7 - 17*e/7) + x**6*(-17*d/6 + 17*e/6) + x**
5*(17*d/5 - 4*e/5) + x**4*(-d + 21*e/4) + x**3*(7*d + 7*e/3) + x**2*(7*d/2
+ 3*e)
```

3.292 $\int \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx$

Optimal. Leaf size=42

$$\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

```
[Out] 6*x+7/2*x^2+7*x^3-x^4+17/5*x^5-17/6*x^6+20/7*x^7
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1657}

$$\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
[Out] 6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx &= \int \left(6+7x+21x^2-4x^3+17x^4-17x^5+20x^6\right)\,dx\\ &= 6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 42, normalized size = 1.00

$$\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
[Out] 6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

fricas [A] time = 0.64, size = 34, normalized size = 0.81

$$\frac{20}{7}x^7-\frac{17}{6}x^6+\frac{17}{5}x^5-x^4+7x^3+\frac{7}{2}x^2+6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fricas")
[Out] 20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

giac [A] time = 0.15, size = 34, normalized size = 0.81

$$\frac{20}{7}x^7-\frac{17}{6}x^6+\frac{17}{5}x^5-x^4+7x^3+\frac{7}{2}x^2+6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac")
[Out] 20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

maple [A] time = 0.00, size = 35, normalized size = 0.83

$$\frac{20}{7}x^7 - \frac{17}{6}x^6 + \frac{17}{5}x^5 - x^4 + 7x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
[Out] 6*x+7/2*x^2+7*x^3-x^4+17/5*x^5-17/6*x^6+20/7*x^7
```

maxima [A] time = 0.42, size = 34, normalized size = 0.81

$$\frac{20}{7}\,x^7 - \frac{17}{6}\,x^6 + \frac{17}{5}\,x^5 - x^4 + 7\,x^3 + \frac{7}{2}\,x^2 + 6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")
[Out] 20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

mupad [B] time = 0.03, size = 34, normalized size = 0.81

$$\frac{20\,x^7}{7} - \frac{17\,x^6}{6} + \frac{17\,x^5}{5} - x^4 + 7\,x^3 + \frac{7\,x^2}{2} + 6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
[Out] 6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

sympy [A] time = 0.29, size = 37, normalized size = 0.88

$$\frac{20x^7}{7} - \frac{17x^6}{6} + \frac{17x^5}{5} - x^4 + 7x^3 + \frac{7x^2}{2} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
[Out] 20*x**7/7 - 17*x**6/6 + 17*x**5/5 - x**4 + 7*x**3 + 7*x**2/2 + 6*x
```

3.293 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$

Optimal. Leaf size=228

$$\frac{x^4(20d^2+17de+17e^2)}{4e^3}-\frac{x^3(20d^3+17d^2e+17de^2+4e^3)}{3e^4}+\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\,\mathrm{l}}{e^7}$$

```
[Out] -(20*d^5+17*d^4*e+17*d^3*e^2+4*d^2*e^3+21*d*e^4-7*e^5)*x/e^6+1/2*(20*d^4+17
*d^3*e+17*d^2*e^2+4*d*e^3+21*e^4)*x^2/e^5-1/3*(20*d^3+17*d^2*e+17*d*e^2+4*e
^3)*x^3/e^4+1/4*(20*d^2+17*d*e+17*e^2)*x^4/e^3-1/5*(20*d+17*e)*x^5/e^2+10/3
*x^6/e+(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/
e^7
```

Rubi [A] time = 0.19, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1628}

$$\frac{x^4(20d^2+17de+17e^2)}{4e^3}-\frac{x^3(17d^2e+20d^3+17de^2+4e^3)}{3e^4}+\frac{x^2(17d^2e^2+17d^3e+20d^4+4de^3+21e^4)}{2e^5}-\frac{x(17d^3e^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

```
[Out] -(((20*d^5 + 17*d^4*e + 17*d^3*e^2 + 4*d^2*e^3 + 21*d*e^4 - 7*e^5)*x)/e^6)
+ ((20*d^4 + 17*d^3*e + 17*d^2*e^2 + 4*d*e^3 + 21*e^4)*x^2)/(2*e^5) - ((20*
d^3 + 17*d^2*e + 17*d*e^2 + 4*e^3)*x^3)/(3*e^4) + ((20*d^2 + 17*d*e + 17*e^
2)*x^4)/(4*e^3) - ((20*d + 17*e)*x^5)/(5*e^2) + (10*x^6)/(3*e) + ((5*d^2 -
2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/
e^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx = \int \left(\frac{-20d^5-17d^4e-17d^3e^2-4d^2e^3-21de^4+7e^5}{e^6}+\frac{(20d^4+}{}\right.$$
$$= -\frac{(20d^5+17d^4e+17d^3e^2+4d^2e^3+21de^4-7e^5)x}{e^6}+\frac{(20d^4+}{}$$

Mathematica [A] time = 0.06, size = 179, normalized size = 0.79

$$\frac{ex\left(-1200d^5+60d^4e(10x-17)-10d^3e^2(40x^2-51x+102)+10d^2e^3(30x^3-34x^2+51x-24)-5de^4(48x^4-5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

```
[Out] (e*x*(-1200*d^5 + 60*d^4*e*(-17 + 10*x) - 10*d^3*e^2*(102 - 51*x + 40*x^2)
+ 10*d^2*e^3*(-24 + 51*x - 34*x^2 + 30*x^3) - 5*d*e^4*(252 - 24*x + 68*x^2
- 51*x^3 + 48*x^4) + e^5*(420 + 630*x - 80*x^2 + 255*x^3 - 204*x^4 + 200*x^
```

```
5)) + 60*(20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5
 + 6*e^6)*Log[d + e*x])/(60*e^7)
```

fricas [A] time = 0.84, size = 230, normalized size = 1.01

$$\frac{200\,e^{6}x^{6}-12\,\left(20\,de^{5}+17\,e^{6}\right)x^{5}+15\,\left(20\,d^{2}e^{4}+17\,de^{5}+17\,e^{6}\right)x^{4}-20\,\left(20\,d^{3}e^{3}+17\,d^{2}e^{4}+17\,de^{5}+4\,e^{6}\right)x^{3}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="frica
s")
```

```
[Out] 1/60*(200*e^6*x^6 - 12*(20*d*e^5 + 17*e^6)*x^5 + 15*(20*d^2*e^4 + 17*d*e^5
+ 17*e^6)*x^4 - 20*(20*d^3*e^3 + 17*d^2*e^4 + 17*d*e^5 + 4*e^6)*x^3 + 30*(2
0*d^4*e^2 + 17*d^3*e^3 + 17*d^2*e^4 + 4*d*e^5 + 21*e^6)*x^2 - 60*(20*d^5*e
+ 17*d^4*e^2 + 17*d^3*e^3 + 4*d^2*e^4 + 21*d*e^5 - 7*e^6)*x + 60*(20*d^6 +
17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*log(e*x +
 d))/e^7
```

giac [A] time = 0.16, size = 228, normalized size = 1.00

$$\left(20\,d^{6}+17\,d^{5}e+17\,d^{4}e^{2}+4\,d^{3}e^{3}+21\,d^{2}e^{4}-7\,de^{5}+6\,e^{6}\right)e^{(-7)}\log\left(|xe+d|\right)+\frac{1}{60}\left(200\,x^{6}e^{5}-240\,dx^{5}e^{4}+300\,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="giac"
)
```

```
[Out] (20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)
*e^(-7)*log(abs(x*e + d)) + 1/60*(200*x^6*e^5 - 240*d*x^5*e^4 + 300*d^2*x^4
*e^3 - 400*d^3*x^3*e^2 + 600*d^4*x^2*e - 1200*d^5*x - 204*x^5*e^5 + 255*d*x
^4*e^4 - 340*d^2*x^3*e^3 + 510*d^3*x^2*e^2 - 1020*d^4*x*e + 255*x^4*e^5 - 3
40*d*x^3*e^4 + 510*d^2*x^2*e^3 - 1020*d^3*x*e^2 - 80*x^3*e^5 + 120*d*x^2*e^
4 - 240*d^2*x*e^3 + 630*x^2*e^5 - 1260*d*x*e^4 + 420*x*e^5)*e^(-6)
```

maple [A] time = 0.01, size = 286, normalized size = 1.25

$$\frac{10x^{6}}{3e}-\frac{4d\,x^{5}}{e^{2}}-\frac{17x^{5}}{5e}+\frac{5d^{2}x^{4}}{e^{3}}+\frac{17d\,x^{4}}{4e^{2}}+\frac{17x^{4}}{4e}-\frac{20d^{3}x^{3}}{3e^{4}}-\frac{17d^{2}x^{3}}{3e^{3}}-\frac{17d\,x^{3}}{3e^{2}}-\frac{4x^{3}}{3e}+\frac{10d^{4}x^{2}}{e^{5}}+\frac{17d^{3}x^{2}}{2e^{4}}+\frac{17d^{2}x^{2}}{2e^{3}}+\frac{2d\,x^{2}}{e^{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x)
```

```
[Out] 21/2*x^2/e+10/3*x^6/e+6/e*ln(e*x+d)+17/4/e*x^4-4/3/e*x^3+7/e*x-17/5/e*x^5+1
7/4/e^2*x^4*d-20/3/e^4*x^3*d^3-17/3/e^3*x^3*d^2-17/3/e^2*x^3*d+10/e^5*x^2*d
^4+17/2/e^4*x^2*d^3+17/2/e^3*x^2*d^2-4/e^2*x^5*d+5/e^3*x^4*d^2+4/e^4*ln(e*x
+d)*d^3-17/e^4*x*d^3-4/e^3*x*d^2-21/e^2*x*d+2/e^2*x^2*d-20/e^6*x*d^5-17/e^5
*x*d^4+21/e^3*ln(e*x+d)*d^2-7/e^2*ln(e*x+d)*d+20/e^7*ln(e*x+d)*d^6+17/e^6*l
n(e*x+d)*d^5+17/e^5*ln(e*x+d)*d^4
```

maxima [A] time = 0.43, size = 228, normalized size = 1.00

$$\frac{200\,e^{5}x^{6}-12\,\left(20\,de^{4}+17\,e^{5}\right)x^{5}+15\,\left(20\,d^{2}e^{3}+17\,de^{4}+17\,e^{5}\right)x^{4}-20\,\left(20\,d^{3}e^{2}+17\,d^{2}e^{3}+17\,de^{4}+4\,e^{5}\right)x^{3}+}{60\,e^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="maxim
a")
```

```
[Out] 1/60*(200*e^5*x^6 - 12*(20*d*e^4 + 17*e^5)*x^5 + 15*(20*d^2*e^3 + 17*d*e^4
+ 17*e^5)*x^4 - 20*(20*d^3*e^2 + 17*d^2*e^3 + 17*d*e^4 + 4*e^5)*x^3 + 30*(2
0*d^4*e + 17*d^3*e^2 + 17*d^2*e^3 + 4*d*e^4 + 21*e^5)*x^2 - 60*(20*d^5 + 17
*d^4*e + 17*d^3*e^2 + 4*d^2*e^3 + 21*d*e^4 - 7*e^5)*x)/e^6 + (20*d^6 + 17*d
^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*log(e*x + d)/
e^7
```

mupad [B] time = 4.14, size = 260, normalized size = 1.14

$$x\left(\frac{7}{e}-\frac{d\left(\frac{21}{e}+\frac{d\left(\frac{4}{e}+\frac{d\left(\frac{17}{e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)-x^5\left(\frac{4\,d}{e^2}+\frac{17}{5\,e}\right)+x^4\left(\frac{17}{4\,e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{4\,e}\right)-x^3\left(\frac{4}{3\,e}+\frac{d\left(\frac{17}{e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{e}\right)}{3\,e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x),x)
```

```
[Out] x*(7/e - (d*(21/e + (d*(4/e + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/e))/e)
)/e) - x^5*((4*d)/e^2 + 17/(5*e)) + x^4*(17/(4*e) + (d*((20*d)/e^2 + 17/e))
/(4*e)) - x^3*(4/(3*e) + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/(3*e)) + x^
2*(21/(2*e) + (d*(4/e + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/e))/(2*e)) +
 (10*x^6)/(3*e) + (log(d + e*x)*(17*d^5*e - 7*d*e^5 + 20*d^6 + 6*e^6 + 21*d
^2*e^4 + 4*d^3*e^3 + 17*d^4*e^2))/e^7
```

sympy [A] time = 1.17, size = 235, normalized size = 1.03

$$x^5\left(-\frac{4d}{e^2}-\frac{17}{5e}\right)+x^4\left(\frac{5d^2}{e^3}+\frac{17d}{4e^2}+\frac{17}{4e}\right)+x^3\left(-\frac{20d^3}{3e^4}-\frac{17d^2}{3e^3}-\frac{17d}{3e^2}-\frac{4}{3e}\right)+x^2\left(\frac{10d^4}{e^5}+\frac{17d^3}{2e^4}+\frac{17d^2}{2e^3}+\frac{2d}{e^2}+\frac{21}{2e}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d),x)
```

```
[Out] x**5*(-4*d/e**2 - 17/(5*e)) + x**4*(5*d**2/e**3 + 17*d/(4*e**2) + 17/(4*e))
 + x**3*(-20*d**3/(3*e**4) - 17*d**2/(3*e**3) - 17*d/(3*e**2) - 4/(3*e)) +
x**2*(10*d**4/e**5 + 17*d**3/(2*e**4) + 17*d**2/(2*e**3) + 2*d/e**2 + 21/(2
*e)) + x*(-20*d**5/e**6 - 17*d**4/e**5 - 17*d**3/e**4 - 4*d**2/e**3 - 21*d/
e**2 + 7/e) + 10*x**6/(3*e) + (5*d**2 - 2*d*e + 3*e**2)*(4*d**4 + 5*d**3*e
+ 3*d**2*e**2 - d*e**3 + 2*e**4)*log(d + e*x)/e**7
```

3.294 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2} dx$

Optimal. Leaf size=228

$$\frac{x^3(60d^2+34de+17e^2)}{3e^4}-\frac{x^2(80d^3+51d^2e+34de^2+4e^3)}{2e^5}-\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^7(d+ex)}$$

```
[Out] (100*d^4+68*d^3*e+51*d^2*e^2+8*d*e^3+21*e^4)*x/e^6-1/2*(80*d^3+51*d^2*e+34*
d*e^2+4*e^3)*x^2/e^5+1/3*(60*d^2+34*d*e+17*e^2)*x^3/e^4-1/4*(40*d+17*e)*x^4
/e^3+4*x^5/e^2-(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/e^
7/(e*x+d)-(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*ln(e*x+d)
/e^7
```

Rubi [A] time = 0.19, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1628}

$$\frac{x^3(60d^2+34de+17e^2)}{3e^4}-\frac{x^2(51d^2e+80d^3+34de^2+4e^3)}{2e^5}+\frac{x(51d^2e^2+68d^3e+100d^4+8de^3+21e^4)}{e^6}-\frac{(5d^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x]
```

```
[Out] ((100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x)/e^6 - ((80*d^3 + 5
1*d^2*e + 34*d*e^2 + 4*e^3)*x^2)/(2*e^5) + ((60*d^2 + 34*d*e + 17*e^2)*x^3)
/(3*e^4) - ((40*d + 17*e)*x^4)/(4*e^3) + (4*x^5)/e^2 - ((5*d^2 - 2*d*e + 3*
e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e^7*(d + e*x)) - ((120
*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*Log[d + e*x])
/e^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2} dx = \int \left(\frac{100d^4+68d^3e+51d^2e^2+8de^3+21e^4}{e^6}-\frac{(80d^3+51d^2e}{}\right.$$

$$=\frac{(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)x}{e^6}-\frac{(80d^3+51d^2e}{}$$

Mathematica [A] time = 0.09, size = 223, normalized size = 0.98

$$\frac{4e^3x^3(60d^2+34de+17e^2)-6e^2x^2(80d^3+51d^2e+34de^2+4e^3)+12ex(100d^4+68d^3e+51d^2e^2+8de^3+2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x
]
```

```
[Out] (12*e*(100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x - 6*e^2*(80*d^
3 + 51*d^2*e + 34*d*e^2 + 4*e^3)*x^2 + 4*e^3*(60*d^2 + 34*d*e + 17*e^2)*x^3
 - 3*e^4*(40*d + 17*e)*x^4 + 48*e^5*x^5 - (12*(20*d^6 + 17*d^5*e + 17*d^4*e
^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6))/(d + e*x) - 12*(120*d^5 + 8
5*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*Log[d + e*x])/(12*e^7
)
```

fricas [A] time = 0.78, size = 319, normalized size = 1.40

$$\frac{48\,e^6x^6 - 240\,d^6 - 204\,d^5e - 204\,d^4e^2 - 48\,d^3e^3 - 252\,d^2e^4 + 84\,de^5 - 72\,e^6 - 3\left(24\,de^5 + 17\,e^6\right)x^5 + \left(120\,d^2e^4 + 8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="fri
cas")
```

```
[Out] 1/12*(48*e^6*x^6 - 240*d^6 - 204*d^5*e - 204*d^4*e^2 - 48*d^3*e^3 - 252*d^2
*e^4 + 84*d*e^5 - 72*e^6 - 3*(24*d*e^5 + 17*e^6)*x^5 + (120*d^2*e^4 + 85*d*
e^5 + 68*e^6)*x^4 - 2*(120*d^3*e^3 + 85*d^2*e^4 + 68*d*e^5 + 12*e^6)*x^3 +
6*(120*d^4*e^2 + 85*d^3*e^3 + 68*d^2*e^4 + 12*d*e^5 + 42*e^6)*x^2 + 12*(100
*d^5*e + 68*d^4*e^2 + 51*d^3*e^3 + 8*d^2*e^4 + 21*d*e^5)*x - 12*(120*d^6 +
85*d^5*e + 68*d^4*e^2 + 12*d^3*e^3 + 42*d^2*e^4 - 7*d*e^5 + (120*d^5*e + 85
*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5 - 7*e^6)*x)*log(e*x + d))/(e^
8*x + d*e^7)
```

giac [A] time = 0.17, size = 308, normalized size = 1.35

$$-\frac{1}{12}\,(xe+d)^5\left(\frac{3\left(120\,de+17\,e^2\right)e^{(-1)}}{xe+d} - \frac{4\left(300\,d^2e^2+85\,de^3+17\,e^4\right)e^{(-2)}}{(xe+d)^2} + \frac{12\left(200\,d^3e^3+85\,d^2e^4+34\,de^5+2\,e^6\right.}{(xe+d)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="gia
c")
```

```
[Out] -1/12*(x*e + d)^5*(3*(120*d*e + 17*e^2)*e^(-1)/(x*e + d) - 4*(300*d^2*e^2 +
 85*d*e^3 + 17*e^4)*e^(-2)/(x*e + d)^2 + 12*(200*d^3*e^3 + 85*d^2*e^4 + 34*
d*e^5 + 2*e^6)*e^(-3)/(x*e + d)^3 - 12*(300*d^4*e^4 + 170*d^3*e^5 + 102*d^2
*e^6 + 12*d*e^7 + 21*e^8)*e^(-4)/(x*e + d)^4 - 48)*e^(-7) + (120*d^5 + 85*d
^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*e^(-7)*log(abs(x*e + d)*
e^(-1)/(x*e + d)^2) - (20*d^6*e^5/(x*e + d) + 17*d^5*e^6/(x*e + d) + 17*d^4
*e^7/(x*e + d) + 4*d^3*e^8/(x*e + d) + 21*d^2*e^9/(x*e + d) - 7*d*e^10/(x*e
 + d) + 6*e^11/(x*e + d))*e^(-12)
```

maple [A] time = 0.01, size = 313, normalized size = 1.37

$$\frac{4x^5}{e^2} - \frac{10d\,x^4}{e^3} - \frac{17x^4}{4e^2} + \frac{20d^2x^3}{e^4} + \frac{34d\,x^3}{3e^3} + \frac{17x^3}{3e^2} - \frac{40d^3x^2}{e^5} - \frac{51d^2x^2}{2e^4} - \frac{17d\,x^2}{e^3} - \frac{2x^2}{e^2} - \frac{20d^6}{(ex+d)\,e^7} - \frac{17d^5}{(ex+d)\,e^6} - \frac{120d^5\ln(e}{e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x)
```

```
[Out] 21*x/e^2+4*x^5/e^2-2/e^2*x^2-6/e/(e*x+d)+7/e^2*ln(e*x+d)+17/3/e^2*x^3-17/4/
e^2*x^4-42/e^3*ln(e*x+d)*d-120/e^7*ln(e*x+d)*d^5-85/e^6*ln(e*x+d)*d^4-68/e^
5*ln(e*x+d)*d^3-12/e^4*ln(e*x+d)*d^2-21/e^3/(e*x+d)*d^2+7/e^2/(e*x+d)*d-20/
e^7/(e*x+d)*d^6-17/e^6/(e*x+d)*d^5-17/e^5/(e*x+d)*d^4-4/e^4/(e*x+d)*d^3+8/e
^3*x*d+34/3/e^3*x^3*d-40/e^5*x^2*d^3-51/2/e^4*x^2*d^2-17/e^3*x^2*d+100/e^6*
d^4*x+68/e^5*x*d^3+51/e^4*x*d^2-10/e^3*x^4*d+20/e^4*x^3*d^2
```

maxima [A] time = 0.43, size = 234, normalized size = 1.03

$$-\frac{20\,d^6+17\,d^5e+17\,d^4e^2+4\,d^3e^3+21\,d^2e^4-7\,de^5+6\,e^6}{e^8x+de^7}+\frac{48\,e^4x^5-3\left(40\,de^3+17\,e^4\right)x^4+4\left(60\,d^2e^2+34\,de\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="max
ima")
```

```
[Out] -(20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6
)/(e^8*x + d*e^7) + 1/12*(48*e^4*x^5 - 3*(40*d*e^3 + 17*e^4)*x^4 + 4*(60*d^
2*e^2 + 34*d*e^3 + 17*e^4)*x^3 - 6*(80*d^3*e + 51*d^2*e^2 + 34*d*e^3 + 4*e^
4)*x^2 + 12*(100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x)/e^6 - (
120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*log(e*x +
d)/e^7
```

mupad [B] time = 4.18, size = 363, normalized size = 1.59

$$x^3\left(\frac{17}{3\,e^2}-\frac{20\,d^2}{3\,e^4}+\frac{2\,d\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{3\,e}\right)-x^2\left(\frac{2}{e^2}+\frac{d\left(\frac{17}{e^2}-\frac{20\,d^2}{e^4}+\frac{2\,d\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{2\,e^2}\right)-x^4\left(\frac{10\,d}{e^3}+\frac{17}{4\,e^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^2,x)
```

```
[Out] x^3*(17/(3*e^2) - (20*d^2)/(3*e^4) + (2*d*((40*d)/e^3 + 17/e^2))/(3*e)) - x
^2*(2/e^2 + (d*(17/e^2 - (20*d^2)/e^4 + (2*d*((40*d)/e^3 + 17/e^2))/e))/e -
 (d^2*((40*d)/e^3 + 17/e^2))/(2*e^2)) - x^4*((10*d)/e^3 + 17/(4*e^2)) + x*(
21/e^2 + (2*d*(4/e^2 + (2*d*(17/e^2 - (20*d^2)/e^4 + (2*d*((40*d)/e^3 + 17/
e^2))/e))/e - (d^2*((40*d)/e^3 + 17/e^2))/e^2))/e - (d^2*(17/e^2 - (20*d^2)
/e^4 + (2*d*((40*d)/e^3 + 17/e^2))/e))/e^2) + (4*x^5)/e^2 - (log(d + e*x)*(
42*d*e^4 + 85*d^4*e + 120*d^5 - 7*e^5 + 12*d^2*e^3 + 68*d^3*e^2))/e^7 - (17
*d^5*e - 7*d*e^5 + 20*d^6 + 6*e^6 + 21*d^2*e^4 + 4*d^3*e^3 + 17*d^4*e^2)/(e
*(d*e^6 + e^7*x))
```

sympy [A] time = 1.13, size = 238, normalized size = 1.04

$$x^4\left(-\frac{10d}{e^3}-\frac{17}{4e^2}\right)+x^3\left(\frac{20d^2}{e^4}+\frac{34d}{3e^3}+\frac{17}{3e^2}\right)+x^2\left(-\frac{40d^3}{e^5}-\frac{51d^2}{2e^4}-\frac{17d}{e^3}-\frac{2}{e^2}\right)+x\left(\frac{100d^4}{e^6}+\frac{68d^3}{e^5}+\frac{51d^2}{e^4}+\frac{8d}{e^3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2,x)
```

```
[Out] x**4*(-10*d/e**3 - 17/(4*e**2)) + x**3*(20*d**2/e**4 + 34*d/(3*e**3) + 17/(
3*e**2)) + x**2*(-40*d**3/e**5 - 51*d**2/(2*e**4) - 17*d/e**3 - 2/e**2) + x
*(100*d**4/e**6 + 68*d**3/e**5 + 51*d**2/e**4 + 8*d/e**3 + 21/e**2) + (-20*
d**6 - 17*d**5*e - 17*d**4*e**2 - 4*d**3*e**3 - 21*d**2*e**4 + 7*d*e**5 - 6
*e**6)/(d*e**7 + e**8*x) + 4*x**5/e**2 - (120*d**5 + 85*d**4*e + 68*d**3*e*
*2 + 12*d**2*e**3 + 42*d*e**4 - 7*e**5)*log(d + e*x)/e**7
```

3.295 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3} \, dx$

Optimal. Leaf size=231

$$\frac{x^2(120d^2+51de+17e^2)}{2e^5}-\frac{x(200d^3+102d^2e+51de^2+4e^3)}{e^6}-\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^7(d+ex)^2}$$

```
[Out] -(200*d^3+102*d^2*e+51*d*e^2+4*e^3)*x/e^6+1/2*(120*d^2+51*d*e+17*e^2)*x^2/e
^5-1/3*(60*d+17*e)*x^3/e^4+5*x^4/e^3-1/2*(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e
+3*d^2*e^2-d*e^3+2*e^4)/e^7/(e*x+d)^2+(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e
^3+42*d*e^4-7*e^5)/e^7/(e*x+d)+(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e
^4)*ln(e*x+d)/e^7
```

Rubi [A] time = 0.20, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1628}

$$\frac{x^2(120d^2+51de+17e^2)}{2e^5}-\frac{x(102d^2e+200d^3+51de^2+4e^3)}{e^6}+\frac{68d^3e^2+12d^2e^3+85d^4e+120d^5+42de^4-7e^5}{e^7(d+ex)}-$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x]
```

```
[Out] -(((200*d^3 + 102*d^2*e + 51*d*e^2 + 4*e^3)*x)/e^6) + ((120*d^2 + 51*d*e +
17*e^2)*x^2)/(2*e^5) - ((60*d + 17*e)*x^3)/(3*e^4) + (5*x^4)/e^3 - ((5*d^2
- 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(2*e^7*(d +
 e*x)^2) + (120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5
)/(e^7*(d + e*x)) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4
)*Log[d + e*x])/e^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3} \, dx = \int \left(\frac{-200d^3-102d^2e-51de^2-4e^3}{e^6}+\frac{(120d^2+51de+17e^2)x}{e^5}\right.$$

$$= -\frac{(200d^3+102d^2e+51de^2+4e^3)x}{e^6}+\frac{(120d^2+51de+17e^2)x}{2e^5}$$

Mathematica [A] time = 0.07, size = 204, normalized size = 0.88

$$660d^6+d^5e(459-480x)-51d^4e^2(40x^2+2x-7)-3d^3e^3(200x^3+357x^2-34x-20)+d^2e^4(150x^4-340x^3-$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x
]
```

```
[Out] (660*d^6 + d^5*e*(459 - 480*x) - 51*d^4*e^2*(-7 + 2*x + 40*x^2) - 3*d^3*e^3
*(-20 - 34*x + 357*x^2 + 200*x^3) + d^2*e^4*(189 + 48*x - 561*x^2 - 340*x^3
```

```
 + 150*x^4) - d*e^5*(21 - 252*x + 48*x^2 + 204*x^3 - 85*x^4 + 60*x^5) + e^6
*(-18 - 42*x - 24*x^3 + 51*x^4 - 34*x^5 + 30*x^6) + 6*(300*d^4 + 170*d^3*e
+ 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^2*Log[d + e*x])/(6*e^7*(d + e*
x)^2)
```

fricas [A] time = 0.57, size = 360, normalized size = 1.56

$$\frac{30\,e^{6}x^{6} + 660\,d^{6} + 459\,d^{5}e + 357\,d^{4}e^{2} + 60\,d^{3}e^{3} + 189\,d^{2}e^{4} - 21\,de^{5} - 18\,e^{6} - 2\left(30\,de^{5} + 17\,e^{6}\right)x^{5} + \left(150\,d^{2}e^{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="fri
cas")
```

```
[Out] 1/6*(30*e^6*x^6 + 660*d^6 + 459*d^5*e + 357*d^4*e^2 + 60*d^3*e^3 + 189*d^2*
e^4 - 21*d*e^5 - 18*e^6 - 2*(30*d*e^5 + 17*e^6)*x^5 + (150*d^2*e^4 + 85*d*e
^5 + 51*e^6)*x^4 - 4*(150*d^3*e^3 + 85*d^2*e^4 + 51*d*e^5 + 6*e^6)*x^3 - 3*
(680*d^4*e^2 + 357*d^3*e^3 + 187*d^2*e^4 + 16*d*e^5)*x^2 - 6*(80*d^5*e + 17
*d^4*e^2 - 17*d^3*e^3 - 8*d^2*e^4 - 42*d*e^5 + 7*e^6)*x + 6*(300*d^6 + 170*
d^5*e + 102*d^4*e^2 + 12*d^3*e^3 + 21*d^2*e^4 + (300*d^4*e^2 + 170*d^3*e^3
+ 102*d^2*e^4 + 12*d*e^5 + 21*e^6)*x^2 + 2*(300*d^5*e + 170*d^4*e^2 + 102*d
^3*e^3 + 12*d^2*e^4 + 21*d*e^5)*x)*log(e*x + d))/(e^9*x^2 + 2*d*e^8*x + d^2
*e^7)
```

giac [A] time = 0.18, size = 216, normalized size = 0.94

$$\left(300\,d^{4} + 170\,d^{3}e + 102\,d^{2}e^{2} + 12\,de^{3} + 21\,e^{4}\right)e^{(-7)}\log\left(|xe + d|\right) + \frac{1}{6}\left(30\,x^{4}e^{9} - 120\,dx^{3}e^{8} + 360\,d^{2}x^{2}e^{7} - 1200\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="gia
c")
```

```
[Out] (300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*e^(-7)*log(abs(x*e
+ d)) + 1/6*(30*x^4*e^9 - 120*d*x^3*e^8 + 360*d^2*x^2*e^7 - 1200*d^3*x*e^6
- 34*x^3*e^9 + 153*d*x^2*e^8 - 612*d^2*x*e^7 + 51*x^2*e^9 - 306*d*x*e^8 - 2
4*x*e^9)*e^(-12) + 1/2*(220*d^6 + 153*d^5*e + 119*d^4*e^2 + 20*d^3*e^3 + 63
*d^2*e^4 + 2*(120*d^5*e + 85*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5 -
 7*e^6)*x - 7*d*e^5 - 6*e^6)*e^(-7)/(x*e + d)^2
```

maple [A] time = 0.01, size = 336, normalized size = 1.45

$$\frac{5x^{4}}{e^{3}} - \frac{20d\,x^{3}}{e^{4}} - \frac{17x^{3}}{3e^{3}} - \frac{10d^{6}}{\left(ex+d\right)^{2}e^{7}} - \frac{17d^{5}}{2\left(ex+d\right)^{2}e^{6}} - \frac{17d^{4}}{2\left(ex+d\right)^{2}e^{5}} - \frac{2d^{3}}{\left(ex+d\right)^{2}e^{4}} - \frac{21d^{2}}{2\left(ex+d\right)^{2}e^{3}} + \frac{60d^{2}x^{2}}{e^{5}} + \frac{7d}{2\left(ex+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x)
```

```
[Out] -200/e^6*d^3*x-102/e^5*x*d^2-51/e^4*x*d+5*x^4/e^3-3/e/(e*x+d)^2+21/e^3*ln(e
*x+d)+17/2/e^3*x^2-4/e^3*x-7/e^2/(e*x+d)-17/3/e^3*x^3+120/e^7/(e*x+d)*d^5+8
5/e^6/(e*x+d)*d^4+68/e^5/(e*x+d)*d^3+12/e^4/(e*x+d)*d^2+42/e^3/(e*x+d)*d-20
/e^4*x^3*d+60/e^5*x^2*d^2+51/2/e^4*x^2*d-10/e^7/(e*x+d)^2*d^6-17/2/e^6/(e*x
+d)^2*d^5-17/2/e^5/(e*x+d)^2*d^4-2/e^4/(e*x+d)^2*d^3-21/2/e^3/(e*x+d)^2*d^2
+7/2/e^2/(e*x+d)^2*d+300/e^7*ln(e*x+d)*d^4+170/e^6*ln(e*x+d)*d^3+102/e^5*ln
(e*x+d)*d^2+12/e^4*ln(e*x+d)*d
```

maxima [A] time = 0.44, size = 240, normalized size = 1.04

$$\frac{220\,d^{6} + 153\,d^{5}e + 119\,d^{4}e^{2} + 20\,d^{3}e^{3} + 63\,d^{2}e^{4} - 7\,de^{5} - 6\,e^{6} + 2\left(120\,d^{5}e + 85\,d^{4}e^{2} + 68\,d^{3}e^{3} + 12\,d^{2}e^{4} + 42\,d\right.}{2\left(e^{9}x^{2} + 2\,de^{8}x + d^{2}e^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="max
ima")
```

```
[Out] 1/2*(220*d^6 + 153*d^5*e + 119*d^4*e^2 + 20*d^3*e^3 + 63*d^2*e^4 - 7*d*e^5
- 6*e^6 + 2*(120*d^5*e + 85*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5 -
7*e^6)*x)/(e^9*x^2 + 2*d*e^8*x + d^2*e^7) + 1/6*(30*e^3*x^4 - 2*(60*d*e^2 +
 17*e^3)*x^3 + 3*(120*d^2*e + 51*d*e^2 + 17*e^3)*x^2 - 6*(200*d^3 + 102*d^2
*e + 51*d*e^2 + 4*e^3)*x)/e^6 + (300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e
^3 + 21*e^4)*log(e*x + d)/e^7
```

mupad [B] time = 0.09, size = 297, normalized size = 1.29

$$x^2\left(\frac{17}{2\,e^3}-\frac{30\,d^2}{e^5}+\frac{3\,d\,\left(\frac{60\,d}{e^4}+\frac{17}{e^3}\right)}{2\,e}\right)-x^3\,\left(\frac{20\,d}{e^4}+\frac{17}{3\,e^3}\right)+\frac{x\,\left(120\,d^5+85\,d^4\,e+68\,d^3\,e^2+12\,d^2\,e^3+42\,d\,e^4-7\,e^5\right)\cdots}{d^2\,e^6+2\,d\,e^7\,x+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^3,x)
```

```
[Out] x^2*(17/(2*e^3) - (30*d^2)/e^5 + (3*d*((60*d)/e^4 + 17/e^3))/(2*e)) - x^3*(
(20*d)/e^4 + 17/(3*e^3)) + (x*(42*d*e^4 + 85*d^4*e + 120*d^5 - 7*e^5 + 12*d
^2*e^3 + 68*d^3*e^2) + (153*d^5*e - 7*d*e^5 + 220*d^6 - 6*e^6 + 63*d^2*e^4
+ 20*d^3*e^3 + 119*d^4*e^2)/(2*e))/(d^2*e^6 + e^8*x^2 + 2*d*e^7*x) - x*(4/e
^3 + (20*d^3)/e^6 + (3*d*(17/e^3 - (60*d^2)/e^5 + (3*d*((60*d)/e^4 + 17/e^3
))/e))/e - (3*d^2*((60*d)/e^4 + 17/e^3))/e^2) + (5*x^4)/e^3 + (log(d + e*x)
*(12*d*e^3 + 170*d^3*e + 300*d^4 + 21*e^4 + 102*d^2*e^2))/e^7
```

sympy [A] time = 2.63, size = 248, normalized size = 1.07

$$x^3\left(-\frac{20d}{e^4}-\frac{17}{3e^3}\right)+x^2\left(\frac{60d^2}{e^5}+\frac{51d}{2e^4}+\frac{17}{2e^3}\right)+x\left(-\frac{200d^3}{e^6}-\frac{102d^2}{e^5}-\frac{51d}{e^4}-\frac{4}{e^3}\right)+\frac{220d^6+153d^5e+119d^4e^2+20d^3e\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3,x)
```

```
[Out] x**3*(-20*d/e**4 - 17/(3*e**3)) + x**2*(60*d**2/e**5 + 51*d/(2*e**4) + 17/(
2*e**3)) + x*(-200*d**3/e**6 - 102*d**2/e**5 - 51*d/e**4 - 4/e**3) + (220*d
**6 + 153*d**5*e + 119*d**4*e**2 + 20*d**3*e**3 + 63*d**2*e**4 - 7*d*e**5 -
 6*e**6 + x*(240*d**5*e + 170*d**4*e**2 + 136*d**3*e**3 + 24*d**2*e**4 + 84
*d*e**5 - 14*e**6))/(2*d**2*e**7 + 4*d*e**8*x + 2*e**9*x**2) + 5*x**4/e**3
+ (300*d**4 + 170*d**3*e + 102*d**2*e**2 + 12*d*e**3 + 21*e**4)*log(d + e*x
)/e**7
```

3.296 $\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal. Leaf size=391

$$\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{10}}{10e^9}-\frac{\left(5600d^3+945d^2e+666de^2+37e^3\right)(d+ex)^9}{9e^9}+\frac{\left(7000d^4+1575d^3e+16\right.}{}$$

```
[Out] 1/4*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^4/e
^9-1/5*(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*d^3*e^2-4*d^2*e^3+64*d*e^4
-11*e^5)*(e*x+d)^5/e^9+1/6*(2800*d^6+945*d^5*e+1665*d^4*e^2+370*d^3*e^3+888
*d^2*e^4-195*d*e^5+107*e^6)*(e*x+d)^6/e^9-1/7*(5600*d^5+1575*d^4*e+2220*d^3
*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*(e*x+d)^7/e^9+1/8*(7000*d^4+1575*d^3*e+1
665*d^2*e^2+185*d*e^3+148*e^4)*(e*x+d)^8/e^9-1/9*(5600*d^3+945*d^2*e+666*d*
e^2+37*e^3)*(e*x+d)^9/e^9+1/10*(2800*d^2+315*d*e+111*e^2)*(e*x+d)^10/e^9-5/
11*(160*d+9*e)*(e*x+d)^11/e^9+25/3*(e*x+d)^12/e^9
```

Rubi [A] time = 0.39, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1628}

$$\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{10}}{10e^9}-\frac{\left(945d^2e+5600d^3+666de^2+37e^3\right)(d+ex)^9}{9e^9}+\frac{\left(1665d^2e^2+1575d^3e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^4)/(4*e^9) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d^3
*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x)^5)/(5*e^9) + ((2800*d^6 + 9
45*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*
(d + e*x)^6)/(6*e^9) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3
 + 592*d*e^4 - 65*e^5)*(d + e*x)^7)/(7*e^9) + ((7000*d^4 + 1575*d^3*e + 166
5*d^2*e^2 + 185*d*e^3 + 148*e^4)*(d + e*x)^8)/(8*e^9) - ((5600*d^3 + 945*d^
2*e + 666*d*e^2 + 37*e^3)*(d + e*x)^9)/(9*e^9) + ((2800*d^2 + 315*d*e + 111
*e^2)*(d + e*x)^10)/(10*e^9) - (5*(160*d + 9*e)*(d + e*x)^11)/(11*e^9) + (2
5*(d + e*x)^12)/(3*e^9)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx = \int \left(\frac{\left(5d^2-2de+3e^2\right)^2 \left(4d^4+5d^3e+3d^2e^2-de^3\right.}{e^8}\right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)^2 \left(4d^4+5d^3e+3d^2e^2-de^3+2\right.}{4e^9}$$

Mathematica [A] time = 0.04, size = 277, normalized size = 0.71

$$18d^3x+\frac{3}{10}ex^{10}\left(100d^2-45de+37e^2\right)+\frac{1}{3}dx^3\left(107d^2+99de+54e^2\right)+\frac{3}{2}d^2x^2(11d+18e)+\frac{1}{9}x^9\left(100d^3-135d^2e+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] 18*d^3*x + (3*d^2*(11*d + 18*e)*x^2)/2 + (d*(107*d^2 + 99*d*e + 54*e^2)*x^3
)/3 + ((65*d^3 + 321*d^2*e + 99*d*e^2 + 18*e^3)*x^4)/4 + ((148*d^3 + 195*d^
2*e + 321*d*e^2 + 33*e^3)*x^5)/5 + ((-37*d^3 + 444*d^2*e + 195*d*e^2 + 107*
e^3)*x^6)/6 + ((111*d^3 - 111*d^2*e + 444*d*e^2 + 65*e^3)*x^7)/7 + ((-45*d^
3 + 333*d^2*e - 111*d*e^2 + 148*e^3)*x^8)/8 + ((100*d^3 - 135*d^2*e + 333*d
*e^2 - 37*e^3)*x^9)/9 + (3*e*(100*d^2 - 45*d*e + 37*e^2)*x^10)/10 + (15*(20
*d - 3*e)*e^2*x^11)/11 + (25*e^3*x^12)/3
```

fricas [A] time = 0.70, size = 305, normalized size = 0.78

$$\frac{25}{3}x^{12}e^3-\frac{45}{11}x^{11}e^3+\frac{300}{11}x^{11}e^2d+\frac{111}{10}x^{10}e^3-\frac{27}{2}x^{10}e^2d+30x^{10}ed^2-\frac{37}{9}x^9e^3+37x^9e^2d-15x^9ed^2+\frac{100}{9}x^9d^3+\frac{37}{2}x^8e^3-\frac{11}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] 25/3*x^12*e^3 - 45/11*x^11*e^3 + 300/11*x^11*e^2*d + 111/10*x^10*e^3 - 27/2
*x^10*e^2*d + 30*x^10*e*d^2 - 37/9*x^9*e^3 + 37*x^9*e^2*d - 15*x^9*e*d^2 +
100/9*x^9*d^3 + 37/2*x^8*e^3 - 111/8*x^8*e^2*d + 333/8*x^8*e*d^2 - 45/8*x^8
*d^3 + 65/7*x^7*e^3 + 444/7*x^7*e^2*d - 111/7*x^7*e*d^2 + 111/7*x^7*d^3 + 1
07/6*x^6*e^3 + 65/2*x^6*e^2*d + 74*x^6*e*d^2 - 37/6*x^6*d^3 + 33/5*x^5*e^3
+ 321/5*x^5*e^2*d + 39*x^5*e*d^2 + 148/5*x^5*d^3 + 9/2*x^4*e^3 + 99/4*x^4*e
^2*d + 321/4*x^4*e*d^2 + 65/4*x^4*d^3 + 18*x^3*e^2*d + 33*x^3*e*d^2 + 107/3
*x^3*d^3 + 27*x^2*e*d^2 + 33/2*x^2*d^3 + 18*x*d^3
```

giac [A] time = 0.16, size = 296, normalized size = 0.76

$$\frac{25}{3}x^{12}e^3+\frac{300}{11}dx^{11}e^2+30d^2x^{10}e+\frac{100}{9}d^3x^9-\frac{45}{11}x^{11}e^3-\frac{27}{2}dx^{10}e^2-15d^2x^9e-\frac{45}{8}d^3x^8+\frac{111}{10}x^{10}e^3+37dx^9e^2+\frac{333}{8}d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] 25/3*x^12*e^3 + 300/11*d*x^11*e^2 + 30*d^2*x^10*e + 100/9*d^3*x^9 - 45/11*x
^11*e^3 - 27/2*d*x^10*e^2 - 15*d^2*x^9*e - 45/8*d^3*x^8 + 111/10*x^10*e^3 +
 37*d*x^9*e^2 + 333/8*d^2*x^8*e + 111/7*d^3*x^7 - 37/9*x^9*e^3 - 111/8*d*x^
8*e^2 - 111/7*d^2*x^7*e - 37/6*d^3*x^6 + 37/2*x^8*e^3 + 444/7*d*x^7*e^2 + 7
4*d^2*x^6*e + 148/5*d^3*x^5 + 65/7*x^7*e^3 + 65/2*d*x^6*e^2 + 39*d^2*x^5*e
+ 65/4*d^3*x^4 + 107/6*x^6*e^3 + 321/5*d*x^5*e^2 + 321/4*d^2*x^4*e + 107/3*
d^3*x^3 + 33/5*x^5*e^3 + 99/4*d*x^4*e^2 + 33*d^2*x^3*e + 33/2*d^3*x^2 + 9/2
*x^4*e^3 + 18*d*x^3*e^2 + 27*d^2*x^2*e + 18*d^3*x
```

maple [A] time = 0.00, size = 264, normalized size = 0.68

$$\frac{25e^3x^{12}}{3}+\frac{\left(300d\,e^2-45e^3\right)x^{11}}{11}+\frac{\left(300d^2e-135d\,e^2+111e^3\right)x^{10}}{10}+\frac{\left(100d^3-135d^2e+333d\,e^2-37e^3\right)x^9}{9}+\frac{\left(-45d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 25/3*e^3*x^12+1/11*(300*d*e^2-45*e^3)*x^11+1/10*(300*d^2*e-135*d*e^2+111*e^
3)*x^10+1/9*(100*d^3-135*d^2*e+333*d*e^2-37*e^3)*x^9+1/8*(-45*d^3+333*d^2*e
-111*d*e^2+148*e^3)*x^8+1/7*(111*d^3-111*d^2*e+444*d*e^2+65*e^3)*x^7+1/6*(-
```

```
37*d^3+444*d^2*e+195*d*e^2+107*e^3)*x^6+1/5*(148*d^3+195*d^2*e+321*d*e^2+33
*e^3)*x^5+1/4*(65*d^3+321*d^2*e+99*d*e^2+18*e^3)*x^4+1/3*(107*d^3+99*d^2*e+
54*d*e^2)*x^3+1/2*(33*d^3+54*d^2*e)*x^2+18*d^3*x
```

maxima [A] time = 0.43, size = 263, normalized size = 0.67

$$\frac{25}{3}e^3x^{12}+\frac{15}{11}\left(20\,de^2-3\,e^3\right)x^{11}+\frac{3}{10}\left(100\,d^2e-45\,de^2+37\,e^3\right)x^{10}+\frac{1}{9}\left(100\,d^3-135\,d^2e+333\,de^2-37\,e^3\right)x^9-\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 25/3*e^3*x^12 + 15/11*(20*d*e^2 - 3*e^3)*x^11 + 3/10*(100*d^2*e - 45*d*e^2
+ 37*e^3)*x^10 + 1/9*(100*d^3 - 135*d^2*e + 333*d*e^2 - 37*e^3)*x^9 - 1/8*(
45*d^3 - 333*d^2*e + 111*d*e^2 - 148*e^3)*x^8 + 1/7*(111*d^3 - 111*d^2*e +
444*d*e^2 + 65*e^3)*x^7 - 1/6*(37*d^3 - 444*d^2*e - 195*d*e^2 - 107*e^3)*x^
6 + 1/5*(148*d^3 + 195*d^2*e + 321*d*e^2 + 33*e^3)*x^5 + 1/4*(65*d^3 + 321*
d^2*e + 99*d*e^2 + 18*e^3)*x^4 + 18*d^3*x + 1/3*(107*d^3 + 99*d^2*e + 54*d*
e^2)*x^3 + 3/2*(11*d^3 + 18*d^2*e)*x^2
```

mupad [B] time = 4.25, size = 251, normalized size = 0.64

$$18\,d^3\,x+x^3\,\left(\frac{107\,d^3}{3}+33\,d^2\,e+18\,d\,e^2\right)+x^9\,\left(\frac{100\,d^3}{9}-15\,d^2\,e+37\,d\,e^2-\frac{37\,e^3}{9}\right)+x^6\,\left(-\frac{37\,d^3}{6}+74\,d^2\,e+\frac{65\,d\,e}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] 18*d^3*x + x^3*(18*d*e^2 + 33*d^2*e + (107*d^3)/3) + x^9*(37*d*e^2 - 15*d^2
*e + (100*d^3)/9 - (37*e^3)/9) + x^6*((65*d*e^2)/2 + 74*d^2*e - (37*d^3)/6
+ (107*e^3)/6) + x^4*((99*d*e^2)/4 + (321*d^2*e)/4 + (65*d^3)/4 + (9*e^3)/2
) - x^8*((111*d*e^2)/8 - (333*d^2*e)/8 + (45*d^3)/8 - (37*e^3)/2) + x^5*((3
21*d*e^2)/5 + 39*d^2*e + (148*d^3)/5 + (33*e^3)/5) + x^7*((444*d*e^2)/7 - (
111*d^2*e)/7 + (111*d^3)/7 + (65*e^3)/7) + (25*e^3*x^12)/3 + (3*e*x^10*(100
*d^2 - 45*d*e + 37*e^2))/10 + (3*d^2*x^2*(11*d + 18*e))/2 + (15*e^2*x^11*(2
0*d - 3*e))/11
```

sympy [A] time = 0.20, size = 298, normalized size = 0.76

$$18d^3x+\frac{25e^3x^{12}}{3}+x^{11}\left(\frac{300de^2}{11}-\frac{45e^3}{11}\right)+x^{10}\left(30d^2e-\frac{27de^2}{2}+\frac{111e^3}{10}\right)+x^9\left(\frac{100d^3}{9}-15d^2e+37de^2-\frac{37e^3}{9}\right)+x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 18*d**3*x + 25*e**3*x**12/3 + x**11*(300*d*e**2/11 - 45*e**3/11) + x**10*(3
0*d**2*e - 27*d*e**2/2 + 111*e**3/10) + x**9*(100*d**3/9 - 15*d**2*e + 37*d
*e**2 - 37*e**3/9) + x**8*(-45*d**3/8 + 333*d**2*e/8 - 111*d*e**2/8 + 37*e*
*3/2) + x**7*(111*d**3/7 - 111*d**2*e/7 + 444*d*e**2/7 + 65*e**3/7) + x**6*
(-37*d**3/6 + 74*d**2*e + 65*d*e**2/2 + 107*e**3/6) + x**5*(148*d**3/5 + 39
*d**2*e + 321*d*e**2/5 + 33*e**3/5) + x**4*(65*d**3/4 + 321*d**2*e/4 + 99*d
*e**2/4 + 9*e**3/2) + x**3*(107*d**3/3 + 33*d**2*e + 18*d*e**2) + x**2*(33*
d**3/2 + 27*d**2*e)
```

3.297 $\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx$

Optimal. Leaf size=201

$$\frac{1}{9}x^9\left(100d^2-90de+111e^2\right)-\frac{1}{8}x^8\left(45d^2-222de+37e^2\right)+\frac{37}{7}x^7\left(3d^2-2de+4e^2\right)-\frac{1}{6}x^6\left(37d^2-296de-65e^2\right)+\frac{1}{5}$$

```
[Out] 18*d^2*x+3/2*d*(11*d+12*e)*x^2+1/3*(107*d^2+66*d*e+18*e^2)*x^3+1/4*(65*d^2+
214*d*e+33*e^2)*x^4+1/5*(148*d^2+130*d*e+107*e^2)*x^5-1/6*(37*d^2-296*d*e-6
5*e^2)*x^6+37/7*(3*d^2-2*d*e+4*e^2)*x^7-1/8*(45*d^2-222*d*e+37*e^2)*x^8+1/9
*(100*d^2-90*d*e+111*e^2)*x^9+1/2*(40*d-9*e)*e*x^10+100/11*e^2*x^11
```

Rubi [A] time = 0.24, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{1}{9}x^9\left(100d^2-90de+111e^2\right)-\frac{1}{8}x^8\left(45d^2-222de+37e^2\right)+\frac{37}{7}x^7\left(3d^2-2de+4e^2\right)-\frac{1}{6}x^6\left(37d^2-296de-65e^2\right)+\frac{1}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*d^2*x + (3*d*(11*d + 12*e)*x^2)/2 + ((107*d^2 + 66*d*e + 18*e^2)*x^3)/3
+ ((65*d^2 + 214*d*e + 33*e^2)*x^4)/4 + ((148*d^2 + 130*d*e + 107*e^2)*x^5)
/5 - ((37*d^2 - 296*d*e - 65*e^2)*x^6)/6 + (37*(3*d^2 - 2*d*e + 4*e^2)*x^7)
/7 - ((45*d^2 - 222*d*e + 37*e^2)*x^8)/8 + ((100*d^2 - 90*d*e + 111*e^2)*x^
9)/9 + ((40*d - 9*e)*e*x^10)/2 + (100*e^2*x^11)/11
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^2\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)dx=\int\left(18d^2+3d(11d+12e)x+\left(107d^2+66de+18e^2\right)\right.$$

$$=18d^2x+\frac{3}{2}d(11d+12e)x^2+\frac{1}{3}\left(107d^2+66de+18e^2\right)$$

Mathematica [A] time = 0.03, size = 201, normalized size = 1.00

$$\frac{1}{9}x^9\left(100d^2-90de+111e^2\right)+\frac{1}{8}x^8\left(-45d^2+222de-37e^2\right)+\frac{37}{7}x^7\left(3d^2-2de+4e^2\right)+\frac{1}{6}x^6\left(-37d^2+296de+65e^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] 18*d^2*x + (3*d*(11*d + 12*e)*x^2)/2 + ((107*d^2 + 66*d*e + 18*e^2)*x^3)/3
+ ((65*d^2 + 214*d*e + 33*e^2)*x^4)/4 + ((148*d^2 + 130*d*e + 107*e^2)*x^5)
/5 + ((-37*d^2 + 296*d*e + 65*e^2)*x^6)/6 + (37*(3*d^2 - 2*d*e + 4*e^2)*x^7
)/7 + ((-45*d^2 + 222*d*e - 37*e^2)*x^8)/8 + ((100*d^2 - 90*d*e + 111*e^2)*
x^9)/9 + ((40*d - 9*e)*e*x^10)/2 + (100*e^2*x^11)/11
```

fricas [A] time = 0.70, size = 206, normalized size = 1.02

$$\frac{100}{11}x^{11}e^2-\frac{9}{2}x^{10}e^2+20x^{10}ed+\frac{37}{3}x^9e^2-10x^9ed+\frac{100}{9}x^9d^2-\frac{37}{8}x^8e^2+\frac{111}{4}x^8ed-\frac{45}{8}x^8d^2+\frac{148}{7}x^7e^2-\frac{74}{7}x^7ed+\frac{111}{7}x^7d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] 100/11*x^11*e^2 - 9/2*x^10*e^2 + 20*x^10*e*d + 37/3*x^9*e^2 - 10*x^9*e*d +
100/9*x^9*d^2 - 37/8*x^8*e^2 + 111/4*x^8*e*d - 45/8*x^8*d^2 + 148/7*x^7*e^2
 - 74/7*x^7*e*d + 111/7*x^7*d^2 + 65/6*x^6*e^2 + 148/3*x^6*e*d - 37/6*x^6*d
^2 + 107/5*x^5*e^2 + 26*x^5*e*d + 148/5*x^5*d^2 + 33/4*x^4*e^2 + 107/2*x^4*
e*d + 65/4*x^4*d^2 + 6*x^3*e^2 + 22*x^3*e*d + 107/3*x^3*d^2 + 18*x^2*e*d +
33/2*x^2*d^2 + 18*x*d^2
```

giac [A] time = 0.16, size = 206, normalized size = 1.02

$$\frac{100}{11}x^{11}e^2+20\,dx^{10}e+\frac{100}{9}d^2x^9-\frac{9}{2}x^{10}e^2-10\,dx^9e-\frac{45}{8}d^2x^8+\frac{37}{3}x^9e^2+\frac{111}{4}dx^8e+\frac{111}{7}d^2x^7-\frac{37}{8}x^8e^2-\frac{74}{7}dx^7e-\frac{37}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] 100/11*x^11*e^2 + 20*d*x^10*e + 100/9*d^2*x^9 - 9/2*x^10*e^2 - 10*d*x^9*e -
 45/8*d^2*x^8 + 37/3*x^9*e^2 + 111/4*d*x^8*e + 111/7*d^2*x^7 - 37/8*x^8*e^2
 - 74/7*d*x^7*e - 37/6*d^2*x^6 + 148/7*x^7*e^2 + 148/3*d*x^6*e + 148/5*d^2*
x^5 + 65/6*x^6*e^2 + 26*d*x^5*e + 65/4*d^2*x^4 + 107/5*x^5*e^2 + 107/2*d*x^
4*e + 107/3*d^2*x^3 + 33/4*x^4*e^2 + 22*d*x^3*e + 33/2*d^2*x^2 + 6*x^3*e^2
+ 18*d*x^2*e + 18*d^2*x
```

maple [A] time = 0.00, size = 186, normalized size = 0.93

$$\frac{100e^2x^{11}}{11}+\frac{\left(200de-45e^2\right)x^{10}}{10}+\frac{\left(100d^2-90de+111e^2\right)x^9}{9}+\frac{\left(-45d^2+222de-37e^2\right)x^8}{8}+\frac{\left(111d^2-74de+148e\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 100/11*e^2*x^11+1/10*(200*d*e-45*e^2)*x^10+1/9*(100*d^2-90*d*e+111*e^2)*x^9
+1/8*(-45*d^2+222*d*e-37*e^2)*x^8+1/7*(111*d^2-74*d*e+148*e^2)*x^7+1/6*(-37
*d^2+296*d*e+65*e^2)*x^6+1/5*(148*d^2+130*d*e+107*e^2)*x^5+1/4*(65*d^2+214*
d*e+33*e^2)*x^4+1/3*(107*d^2+66*d*e+18*e^2)*x^3+1/2*(33*d^2+36*d*e)*x^2+18*
d^2*x
```

maxima [A] time = 0.43, size = 185, normalized size = 0.92

$$\frac{100}{11}e^2x^{11}+\frac{1}{2}\left(40\,de-9\,e^2\right)x^{10}+\frac{1}{9}\left(100\,d^2-90\,de+111\,e^2\right)x^9-\frac{1}{8}\left(45\,d^2-222\,de+37\,e^2\right)x^8+\frac{37}{7}\left(3\,d^2-2\,de+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 100/11*e^2*x^11 + 1/2*(40*d*e - 9*e^2)*x^10 + 1/9*(100*d^2 - 90*d*e + 111*e
^2)*x^9 - 1/8*(45*d^2 - 222*d*e + 37*e^2)*x^8 + 37/7*(3*d^2 - 2*d*e + 4*e^2
)*x^7 - 1/6*(37*d^2 - 296*d*e - 65*e^2)*x^6 + 1/5*(148*d^2 + 130*d*e + 107*
```

```
e^2)*x^5 + 1/4*(65*d^2 + 214*d*e + 33*e^2)*x^4 + 1/3*(107*d^2 + 66*d*e + 18
*e^2)*x^3 + 18*d^2*x + 3/2*(11*d^2 + 12*d*e)*x^2
```

mupad [B] time = 0.11, size = 175, normalized size = 0.87

$$x^3\left(\frac{107\,d^2}{3}+22\,d\,e+6\,e^2\right)+x^9\left(\frac{100\,d^2}{9}-10\,d\,e+\frac{37\,e^2}{3}\right)+x^4\left(\frac{65\,d^2}{4}+\frac{107\,d\,e}{2}+\frac{33\,e^2}{4}\right)-x^8\left(\frac{45\,d^2}{8}-\frac{111\,d\,e}{4}+\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] x^3*(22*d*e + (107*d^2)/3 + 6*e^2) + x^9*((100*d^2)/9 - 10*d*e + (37*e^2)/3
) + x^4*((107*d*e)/2 + (65*d^2)/4 + (33*e^2)/4) - x^8*((45*d^2)/8 - (111*d*
e)/4 + (37*e^2)/8) + x^6*((148*d*e)/3 - (37*d^2)/6 + (65*e^2)/6) + x^5*(26*
d*e + (148*d^2)/5 + (107*e^2)/5) + x^7*((111*d^2)/7 - (74*d*e)/7 + (148*e^2
)/7) + 18*d^2*x + (100*e^2*x^11)/11 + (3*d*x^2*(11*d + 12*e))/2 + (e*x^10*(
40*d - 9*e))/2
```

sympy [A] time = 0.15, size = 206, normalized size = 1.02

$$18d^2x+\frac{100e^2x^{11}}{11}+x^{10}\left(20de-\frac{9e^2}{2}\right)+x^9\left(\frac{100d^2}{9}-10de+\frac{37e^2}{3}\right)+x^8\left(-\frac{45d^2}{8}+\frac{111de}{4}-\frac{37e^2}{8}\right)+x^7\left(\frac{111d^2}{7}-\frac{74de}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 18*d**2*x + 100*e**2*x**11/11 + x**10*(20*d*e - 9*e**2/2) + x**9*(100*d**2/
9 - 10*d*e + 37*e**2/3) + x**8*(-45*d**2/8 + 111*d*e/4 - 37*e**2/8) + x**7*
(111*d**2/7 - 74*d*e/7 + 148*e**2/7) + x**6*(-37*d**2/6 + 148*d*e/3 + 65*e*
*2/6) + x**5*(148*d**2/5 + 26*d*e + 107*e**2/5) + x**4*(65*d**2/4 + 107*d*e
/2 + 33*e**2/4) + x**3*(107*d**2/3 + 22*d*e + 6*e**2) + x**2*(33*d**2/2 + 1
8*d*e)
```

3.298 $\int (d+ex) \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) d$

Optimal. Leaf size=121

$$\frac{5}{9}x^9(20d-9e)-\frac{3}{8}x^8(15d-37e)+\frac{37}{7}x^7(3d-e)-\frac{37}{6}x^6(d-4e)+\frac{1}{5}x^5(148d+65e)+\frac{1}{4}x^4(65d+107e)+\frac{1}{3}x^3(107d+33e)+$$

```
[Out] 18*d*x+3/2*(11*d+6*e)*x^2+1/3*(107*d+33*e)*x^3+1/4*(65*d+107*e)*x^4+1/5*(14
8*d+65*e)*x^5-37/6*(d-4*e)*x^6+37/7*(3*d-e)*x^7-3/8*(15*d-37*e)*x^8+5/9*(20
*d-9*e)*x^9+10*e*x^10
```

Rubi [A] time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1628}

$$\frac{5}{9}x^9(20d-9e)-\frac{3}{8}x^8(15d-37e)+\frac{37}{7}x^7(3d-e)-\frac{37}{6}x^6(d-4e)+\frac{1}{5}x^5(148d+65e)+\frac{1}{4}x^4(65d+107e)+\frac{1}{3}x^3(107d+33e)+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*d*x + (3*(11*d + 6*e)*x^2)/2 + ((107*d + 33*e)*x^3)/3 + ((65*d + 107*e)*
x^4)/4 + ((148*d + 65*e)*x^5)/5 - (37*(d - 4*e)*x^6)/6 + (37*(3*d - e)*x^7)
/7 - (3*(15*d - 37*e)*x^8)/8 + (5*(20*d - 9*e)*x^9)/9 + 10*e*x^10
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)dx = \int \left(18d+3(11d+6e)x+(107d+33e)x^2+(65d+\right.$$
$$= 18dx+\frac{3}{2}(11d+6e)x^2+\frac{1}{3}(107d+33e)x^3+\frac{1}{4}(65d$$

Mathematica [A] time = 0.02, size = 121, normalized size = 1.00

$$\frac{5}{9}x^9(20d-9e)-\frac{3}{8}x^8(15d-37e)+\frac{37}{7}x^7(3d-e)-\frac{37}{6}x^6(d-4e)+\frac{1}{5}x^5(148d+65e)+\frac{1}{4}x^4(65d+107e)+\frac{1}{3}x^3(107d+33e)+$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*d*x + (3*(11*d + 6*e)*x^2)/2 + ((107*d + 33*e)*x^3)/3 + ((65*d + 107*e)*
x^4)/4 + ((148*d + 65*e)*x^5)/5 - (37*(d - 4*e)*x^6)/6 + (37*(3*d - e)*x^7)
/7 - (3*(15*d - 37*e)*x^8)/8 + (5*(20*d - 9*e)*x^9)/9 + 10*e*x^10
```

fricas [A] time = 0.67, size = 107, normalized size = 0.88

$$10x^{10}e-5x^9e+\frac{100}{9}x^9d+\frac{111}{8}x^8e-\frac{45}{8}x^8d-\frac{37}{7}x^7e+\frac{111}{7}x^7d+\frac{74}{3}x^6e-\frac{37}{6}x^6d+13x^5e+\frac{148}{5}x^5d+\frac{107}{4}x^4e+\frac{65}{4}x^4d+11x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 10*x^10*e - 5*x^9*e + 100/9*x^9*d + 111/8*x^8*e - 45/8*x^8*d - 37/7*x^7*e +
 111/7*x^7*d + 74/3*x^6*e - 37/6*x^6*d + 13*x^5*e + 148/5*x^5*d + 107/4*x^4
*e + 65/4*x^4*d + 11*x^3*e + 107/3*x^3*d + 9*x^2*e + 33/2*x^2*d + 18*x*d
```

giac [A] time = 0.16, size = 116, normalized size = 0.96

$$10\,x^{10}e+\frac{100}{9}\,dx^{9}-5\,x^{9}e-\frac{45}{8}\,dx^{8}+\frac{111}{8}\,x^{8}e+\frac{111}{7}\,dx^{7}-\frac{37}{7}\,x^{7}e-\frac{37}{6}\,dx^{6}+\frac{74}{3}\,x^{6}e+\frac{148}{5}\,dx^{5}+13\,x^{5}e+\frac{65}{4}\,dx^{4}+\frac{107}{4}\,x^{4}e+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 10*x^10*e + 100/9*d*x^9 - 5*x^9*e - 45/8*d*x^8 + 111/8*x^8*e + 111/7*d*x^7
- 37/7*x^7*e - 37/6*d*x^6 + 74/3*x^6*e + 148/5*d*x^5 + 13*x^5*e + 65/4*d*x^
4 + 107/4*x^4*e + 107/3*d*x^3 + 11*x^3*e + 33/2*d*x^2 + 9*x^2*e + 18*d*x
```

maple [A] time = 0.00, size = 108, normalized size = 0.89

$$10e\,x^{10}+\frac{(100d-45e)\,x^{9}}{9}+\frac{(-45d+111e)\,x^{8}}{8}+\frac{(111d-37e)\,x^{7}}{7}+\frac{(-37d+148e)\,x^{6}}{6}+\frac{(148d+65e)\,x^{5}}{5}+\frac{(65d+107e)\,x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 10*e*x^10+1/9*(100*d-45*e)*x^9+1/8*(-45*d+111*e)*x^8+1/7*(111*d-37*e)*x^7+1
/6*(-37*d+148*e)*x^6+1/5*(148*d+65*e)*x^5+1/4*(65*d+107*e)*x^4+1/3*(107*d+3
3*e)*x^3+1/2*(33*d+18*e)*x^2+18*d*x
```

maxima [A] time = 0.43, size = 105, normalized size = 0.87

$$10\,ex^{10}+\frac{5}{9}\,(20\,d-9\,e)x^{9}-\frac{3}{8}\,(15\,d-37\,e)x^{8}+\frac{37}{7}\,(3\,d-e)x^{7}-\frac{37}{6}\,(d-4\,e)x^{6}+\frac{1}{5}\,(148\,d+65\,e)x^{5}+\frac{1}{4}\,(65\,d+107\,e)x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 10*e*x^10 + 5/9*(20*d - 9*e)*x^9 - 3/8*(15*d - 37*e)*x^8 + 37/7*(3*d - e)*x
^7 - 37/6*(d - 4*e)*x^6 + 1/5*(148*d + 65*e)*x^5 + 1/4*(65*d + 107*e)*x^4 +
 1/3*(107*d + 33*e)*x^3 + 3/2*(11*d + 6*e)*x^2 + 18*d*x
```

mupad [B] time = 4.17, size = 101, normalized size = 0.83

$$10\,e\,x^{10}+\left(\frac{100\,d}{9}-5\,e\right)\,x^{9}+\left(\frac{111\,e}{8}-\frac{45\,d}{8}\right)\,x^{8}+\left(\frac{111\,d}{7}-\frac{37\,e}{7}\right)\,x^{7}+\left(\frac{74\,e}{3}-\frac{37\,d}{6}\right)\,x^{6}+\left(\frac{148\,d}{5}+13\,e\right)\,x^{5}+\left(\frac{65\,d}{4}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] x^2*((33*d)/2 + 9*e) + x^9*((100*d)/9 - 5*e) + x^3*((107*d)/3 + 11*e) - x^6
*((37*d)/6 - (74*e)/3) + x^7*((111*d)/7 - (37*e)/7) + x^5*((148*d)/5 + 13*e
) - x^8*((45*d)/8 - (111*e)/8) + x^4*((65*d)/4 + (107*e)/4) + 18*d*x + 10*e
*x^10
```

sympy [A] time = 0.14, size = 112, normalized size = 0.93

$$18dx+10ex^{10}+x^{9}\left(\frac{100d}{9}-5e\right)+x^{8}\left(-\frac{45d}{8}+\frac{111e}{8}\right)+x^{7}\left(\frac{111d}{7}-\frac{37e}{7}\right)+x^{6}\left(-\frac{37d}{6}+\frac{74e}{3}\right)+x^{5}\left(\frac{148d}{5}+13e\right)+x^{4}\left(\frac{65}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 18*d*x + 10*e*x**10 + x**9*(100*d/9 - 5*e) + x**8*(-45*d/8 + 111*e/8) + x**
7*(111*d/7 - 37*e/7) + x**6*(-37*d/6 + 74*e/3) + x**5*(148*d/5 + 13*e) + x*
*4*(65*d/4 + 107*e/4) + x**3*(107*d/3 + 11*e) + x**2*(33*d/2 + 9*e)
```

3.299 $\int \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx$

Optimal. Leaf size=60

$$\frac{100x^9}{9}-\frac{45x^8}{8}+\frac{111x^7}{7}-\frac{37x^6}{6}+\frac{148x^5}{5}+\frac{65x^4}{4}+\frac{107x^3}{3}+\frac{33x^2}{2}+18x$$

```
[Out] 18*x+33/2*x^2+107/3*x^3+65/4*x^4+148/5*x^5-37/6*x^6+111/7*x^7-45/8*x^8+100/
9*x^9
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1657}

$$\frac{100x^9}{9}-\frac{45x^8}{8}+\frac{111x^7}{7}-\frac{37x^6}{6}+\frac{148x^5}{5}+\frac{65x^4}{4}+\frac{107x^3}{3}+\frac{33x^2}{2}+18x$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 + (
111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx = \int \left(18+33x+107x^2+65x^3+148x^4-37x^5+111x^6-45x^7\right.$$
$$= 18x+\frac{33x^2}{2}+\frac{107x^3}{3}+\frac{65x^4}{4}+\frac{148x^5}{5}-\frac{37x^6}{6}+\frac{111x^7}{7}-\frac{45x}{8}$$

Mathematica [A] time = 0.00, size = 60, normalized size = 1.00

$$\frac{100x^9}{9}-\frac{45x^8}{8}+\frac{111x^7}{7}-\frac{37x^6}{6}+\frac{148x^5}{5}+\frac{65x^4}{4}+\frac{107x^3}{3}+\frac{33x^2}{2}+18x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 + (
111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

fricas [A] time = 0.72, size = 44, normalized size = 0.73

$$\frac{100}{9}x^9-\frac{45}{8}x^8+\frac{111}{7}x^7-\frac{37}{6}x^6+\frac{148}{5}x^5+\frac{65}{4}x^4+\frac{107}{3}x^3+\frac{33}{2}x^2+18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*
x^3 + 33/2*x^2 + 18*x
```

giac [A] time = 0.15, size = 44, normalized size = 0.73

$$\frac{100}{9}\,x^{9} - \frac{45}{8}\,x^{8} + \frac{111}{7}\,x^{7} - \frac{37}{6}\,x^{6} + \frac{148}{5}\,x^{5} + \frac{65}{4}\,x^{4} + \frac{107}{3}\,x^{3} + \frac{33}{2}\,x^{2} + 18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*
x^3 + 33/2*x^2 + 18*x
```

maple [A] time = 0.00, size = 45, normalized size = 0.75

$$\frac{100}{9}x^{9} - \frac{45}{8}x^{8} + \frac{111}{7}x^{7} - \frac{37}{6}x^{6} + \frac{148}{5}x^{5} + \frac{65}{4}x^{4} + \frac{107}{3}x^{3} + \frac{33}{2}x^{2} + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 18*x+33/2*x^2+107/3*x^3+65/4*x^4+148/5*x^5-37/6*x^6+111/7*x^7-45/8*x^8+100/
9*x^9
```

maxima [A] time = 0.42, size = 44, normalized size = 0.73

$$\frac{100}{9}\,x^{9} - \frac{45}{8}\,x^{8} + \frac{111}{7}\,x^{7} - \frac{37}{6}\,x^{6} + \frac{148}{5}\,x^{5} + \frac{65}{4}\,x^{4} + \frac{107}{3}\,x^{3} + \frac{33}{2}\,x^{2} + 18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*
x^3 + 33/2*x^2 + 18*x
```

mupad [B] time = 0.03, size = 44, normalized size = 0.73

$$\frac{100\,x^{9}}{9} - \frac{45\,x^{8}}{8} + \frac{111\,x^{7}}{7} - \frac{37\,x^{6}}{6} + \frac{148\,x^{5}}{5} + \frac{65\,x^{4}}{4} + \frac{107\,x^{3}}{3} + \frac{33\,x^{2}}{2} + 18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] 18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 + (
111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

sympy [A] time = 0.15, size = 56, normalized size = 0.93

$$\frac{100x^{9}}{9} - \frac{45x^{8}}{8} + \frac{111x^{7}}{7} - \frac{37x^{6}}{6} + \frac{148x^{5}}{5} + \frac{65x^{4}}{4} + \frac{107x^{3}}{3} + \frac{33x^{2}}{2} + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 100*x**9/9 - 45*x**8/8 + 111*x**7/7 - 37*x**6/6 + 148*x**5/5 + 65*x**4/4 +
107*x**3/3 + 33*x**2/2 + 18*x
```

3.300 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$

Optimal. Leaf size=352

$$\frac{x^6(100d^2+45de+111e^2)}{6e^3}-\frac{x^5(100d^3+45d^2e+111de^2+37e^3)}{5e^4}+\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+$$

```
[Out] -(100*d^7+45*d^6*e+111*d^5*e^2+37*d^4*e^3+148*d^3*e^4-65*d^2*e^5+107*d*e^6-
33*e^7)*x/e^8+1/2*(100*d^6+45*d^5*e+111*d^4*e^2+37*d^3*e^3+148*d^2*e^4-65*d
*e^5+107*e^6)*x^2/e^7-1/3*(100*d^5+45*d^4*e+111*d^3*e^2+37*d^2*e^3+148*d*e^
4-65*e^5)*x^3/e^6+1/4*(100*d^4+45*d^3*e+111*d^2*e^2+37*d*e^3+148*e^4)*x^4/e
^5-1/5*(100*d^3+45*d^2*e+111*d*e^2+37*e^3)*x^5/e^4+1/6*(100*d^2+45*d*e+111*
e^2)*x^6/e^3-5/7*(20*d+9*e)*x^7/e^2+25/2*x^8/e+(5*d^2-2*d*e+3*e^2)^2*(4*d^4
+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/e^9
```

Rubi [A] time = 0.32, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{x^6(100d^2+45de+111e^2)}{6e^3}-\frac{x^5(45d^2e+100d^3+111de^2+37e^3)}{5e^4}+\frac{x^4(111d^2e^2+45d^3e+100d^4+37de^3+148e^4)}{4e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

```
[Out] -(((100*d^7 + 45*d^6*e + 111*d^5*e^2 + 37*d^4*e^3 + 148*d^3*e^4 - 65*d^2*e^
5 + 107*d*e^6 - 33*e^7)*x)/e^8) + ((100*d^6 + 45*d^5*e + 111*d^4*e^2 + 37*d
^3*e^3 + 148*d^2*e^4 - 65*d*e^5 + 107*e^6)*x^2)/(2*e^7) - ((100*d^5 + 45*d^
4*e + 111*d^3*e^2 + 37*d^2*e^3 + 148*d*e^4 - 65*e^5)*x^3)/(3*e^6) + ((100*d
^4 + 45*d^3*e + 111*d^2*e^2 + 37*d*e^3 + 148*e^4)*x^4)/(4*e^5) - ((100*d^3
+ 45*d^2*e + 111*d*e^2 + 37*e^3)*x^5)/(5*e^4) + ((100*d^2 + 45*d*e + 111*e^
2)*x^6)/(6*e^3) - (5*(20*d + 9*e)*x^7)/(7*e^2) + (25*x^8)/(2*e) + ((5*d^2 -
 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x
])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx = \int \left(\frac{-100d^7-45d^6e-111d^5e^2-37d^4e^3-148d^3e^4+65d^2e^5-}{e^8}\right.$$

$$= -\frac{(100d^7+45d^6e+111d^5e^2+37d^4e^3+148d^3e^4-65d^2e^5+10}{e^8}$$

Mathematica [A] time = 0.12, size = 262, normalized size = 0.74

$$\frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4)(5d^2-2de+3e^2)^2\log(d+ex)}{e^9}+\frac{x(-42000d^7+2100d^6e(10x-9)-70d^5e^2(200$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x
]
```

```
[Out] (x*(-42000*d^7 + 2100*d^6*e*(-9 + 10*x) - 70*d^5*e^2*(666 - 135*x + 200*x^2
) + 210*d^4*e^3*(-74 + 111*x - 30*x^2 + 50*x^3) - 105*d^3*e^4*(592 - 74*x +
 148*x^2 - 45*x^3 + 80*x^4) + 35*d^2*e^5*(780 + 888*x - 148*x^2 + 333*x^3 -
 108*x^4 + 200*x^5) - d*e^6*(44940 + 13650*x + 20720*x^2 - 3885*x^3 + 9324*
x^4 - 3150*x^5 + 6000*x^6) + 2*e^7*(6930 + 11235*x + 4550*x^2 + 7770*x^3 -
1554*x^4 + 3885*x^5 - 1350*x^6 + 2625*x^7)))/(420*e^8) + ((5*d^2 - 2*d*e +
3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/e^9
```

fricas [A] time = 0.87, size = 368, normalized size = 1.05

$$5250\,e^{8}x^{8} - 300\left(20\,de^{7} + 9\,e^{8}\right)x^{7} + 70\left(100\,d^{2}e^{6} + 45\,de^{7} + 111\,e^{8}\right)x^{6} - 84\left(100\,d^{3}e^{5} + 45\,d^{2}e^{6} + 111\,de^{7} + 37\,e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] 1/420*(5250*e^8*x^8 - 300*(20*d*e^7 + 9*e^8)*x^7 + 70*(100*d^2*e^6 + 45*d*e
^7 + 111*e^8)*x^6 - 84*(100*d^3*e^5 + 45*d^2*e^6 + 111*d*e^7 + 37*e^8)*x^5
+ 105*(100*d^4*e^4 + 45*d^3*e^5 + 111*d^2*e^6 + 37*d*e^7 + 148*e^8)*x^4 - 1
40*(100*d^5*e^3 + 45*d^4*e^4 + 111*d^3*e^5 + 37*d^2*e^6 + 148*d*e^7 - 65*e^
8)*x^3 + 210*(100*d^6*e^2 + 45*d^5*e^3 + 111*d^4*e^4 + 37*d^3*e^5 + 148*d^2
*e^6 - 65*d*e^7 + 107*e^8)*x^2 - 420*(100*d^7*e + 45*d^6*e^2 + 111*d^5*e^3
+ 37*d^4*e^4 + 148*d^3*e^5 - 65*d^2*e^6 + 107*d*e^7 - 33*e^8)*x + 420*(100*
d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 + 107*
d^2*e^6 - 33*d*e^7 + 18*e^8)*log(e*x + d))/e^9
```

giac [A] time = 0.17, size = 378, normalized size = 1.07

$$\left(100\,d^{8} + 45\,d^{7}e + 111\,d^{6}e^{2} + 37\,d^{5}e^{3} + 148\,d^{4}e^{4} - 65\,d^{3}e^{5} + 107\,d^{2}e^{6} - 33\,de^{7} + 18\,e^{8}\right)e^{(-9)}\log\left(|xe + d|\right) + \frac{1}{420}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] (100*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 +
 107*d^2*e^6 - 33*d*e^7 + 18*e^8)*e^(-9)*log(abs(x*e + d)) + 1/420*(5250*x^
8*e^7 - 6000*d*x^7*e^6 + 7000*d^2*x^6*e^5 - 8400*d^3*x^5*e^4 + 10500*d^4*x^
4*e^3 - 14000*d^5*x^3*e^2 + 21000*d^6*x^2*e - 42000*d^7*x - 2700*x^7*e^7 +
3150*d*x^6*e^6 - 3780*d^2*x^5*e^5 + 4725*d^3*x^4*e^4 - 6300*d^4*x^3*e^3 + 9
450*d^5*x^2*e^2 - 18900*d^6*x*e + 7770*x^6*e^7 - 9324*d*x^5*e^6 + 11655*d^2
*x^4*e^5 - 15540*d^3*x^3*e^4 + 23310*d^4*x^2*e^3 - 46620*d^5*x*e^2 - 3108*x
^5*e^7 + 3885*d*x^4*e^6 - 5180*d^2*x^3*e^5 + 7770*d^3*x^2*e^4 - 15540*d^4*x
*e^3 + 15540*x^4*e^7 - 20720*d*x^3*e^6 + 31080*d^2*x^2*e^5 - 62160*d^3*x*e^
4 + 9100*x^3*e^7 - 13650*d*x^2*e^6 + 27300*d^2*x*e^5 + 22470*x^2*e^7 - 4494
0*d*x*e^6 + 13860*x*e^7)*e^(-8)
```

maple [A] time = 0.01, size = 465, normalized size = 1.32

$$\frac{25x^{8}}{2e} - \frac{100d\,x^{7}}{7e^{2}} - \frac{45x^{7}}{7e} + \frac{50d^{2}x^{6}}{3e^{3}} + \frac{15d\,x^{6}}{2e^{2}} + \frac{37x^{6}}{2e} - \frac{20d^{3}x^{5}}{e^{4}} - \frac{9d^{2}x^{5}}{e^{3}} - \frac{111d\,x^{5}}{5e^{2}} - \frac{37x^{5}}{5e} + \frac{25d^{4}x^{4}}{e^{5}} + \frac{45d^{3}x^{4}}{4e^{4}} + \frac{111d^{2}x^{4}}{4e^{3}} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x)
```

```
[Out] 107/2/e*x^2+25/2*x^8/e+37/2/e*x^6-45/7/e*x^7+18/e*ln(e*x+d)+37/e*x^4+65/3/e
*x^3+33/e*x-37/5/e*x^5-15/e^5*x^3*d^4+25/e^5*x^4*d^4-20/e^4*x^5*d^3+45/4/e^
4*x^4*d^3+50/3/e^3*x^6*d^2-9/e^3*x^5*d^2-100/7/e^2*x^7*d+15/2/e^2*x^6*d-100
/e^8*x*d^7-45/e^7*x*d^6+45/e^8*ln(e*x+d)*d^7-100/3/e^6*x^3*d^5+45/2/e^6*x^2
*d^5+50/e^7*x^2*d^6+100/e^9*ln(e*x+d)*d^8+37/4*d/e^2*x^4-37*d^3/e^4*x^3-37/
3*d^2/e^3*x^3-148/3*d/e^2*x^3+111/2*d^4/e^5*x^2+37/2*d^3/e^4*x^2+74*d^2/e^3
*x^2-111/5*d/e^2*x^5+111/4*d^2/e^3*x^4-65*d^3/e^4*ln(e*x+d)-148*d^3/e^4*x+6
5*d^2/e^3*x-107*d/e^2*x-65/2*d/e^2*x^2-111*d^5/e^6*x-37*d^4/e^5*x+107*d^2/e
^3*ln(e*x+d)-33*d/e^2*ln(e*x+d)+111*d^6/e^7*ln(e*x+d)+37*d^5/e^6*ln(e*x+d)+
148*d^4/e^5*ln(e*x+d)
```

maxima [A] time = 0.44, size = 366, normalized size = 1.04

$$\frac{5250\, e^{7} x^{8} - 300 \left(20\, d e^{6} + 9\, e^{7}\right) x^{7} + 70 \left(100\, d^{2} e^{5} + 45\, d e^{6} + 111\, e^{7}\right) x^{6} - 84 \left(100\, d^{3} e^{4} + 45\, d^{2} e^{5} + 111\, d e^{6} + 37\, e^{7}\right) x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] 1/420*(5250*e^7*x^8 - 300*(20*d*e^6 + 9*e^7)*x^7 + 70*(100*d^2*e^5 + 45*d*e
^6 + 111*e^7)*x^6 - 84*(100*d^3*e^4 + 45*d^2*e^5 + 111*d*e^6 + 37*e^7)*x^5
+ 105*(100*d^4*e^3 + 45*d^3*e^4 + 111*d^2*e^5 + 37*d*e^6 + 148*e^7)*x^4 - 1
40*(100*d^5*e^2 + 45*d^4*e^3 + 111*d^3*e^4 + 37*d^2*e^5 + 148*d*e^6 - 65*e^
7)*x^3 + 210*(100*d^6*e + 45*d^5*e^2 + 111*d^4*e^3 + 37*d^3*e^4 + 148*d^2*e
^5 - 65*d*e^6 + 107*e^7)*x^2 - 420*(100*d^7 + 45*d^6*e + 111*d^5*e^2 + 37*d
^4*e^3 + 148*d^3*e^4 - 65*d^2*e^5 + 107*d*e^6 - 33*e^7)*x)/e^8 + (100*d^8 +
 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 + 107*d^2*e
^6 - 33*d*e^7 + 18*e^8)*log(e*x + d)/e^9
```

mupad [B] time = 0.08, size = 434, normalized size = 1.23

$$x\left(\frac{33}{e}-\frac{d\left(\frac{107}{e}-\frac{d\left(\frac{65}{e}-\frac{d\left(\frac{148}{e}+\frac{d\left(\frac{37}{e}+\frac{d\left(\frac{111}{e}+\frac{d\left(\frac{100\,d}{e^2}+\frac{45}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)-x^7\left(\frac{100\,d}{7\,e^2}+\frac{45}{7\,e}\right)+x^6\left(\frac{37}{2\,e}+\frac{d\left(\frac{100\,d}{e^2}+\frac{45}{e}\right)}{6\,e}\right)-x^5\left(\frac{37}{5\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x),x)
```

```
[Out] x*(33/e - (d*(107/e - (d*(65/e - (d*(148/e + (d*(37/e + (d*(111/e + (d*((10
0*d)/e^2 + 45/e))/e))/e))/e))/e))/e))/e) - x^7*((100*d)/(7*e^2) + 45/(7*e))
 + x^6*(37/(2*e) + (d*((100*d)/e^2 + 45/e))/(6*e)) - x^5*(37/(5*e) + (d*(11
1/e + (d*((100*d)/e^2 + 45/e))/e))/(5*e)) + x^4*(37/e + (d*(37/e + (d*(111/
e + (d*((100*d)/e^2 + 45/e))/e))/e))/(4*e)) + x^3*(65/(3*e) - (d*(148/e + (
d*(37/e + (d*(111/e + (d*((100*d)/e^2 + 45/e))/e))/e))/e))/(3*e)) + x^2*(10
7/(2*e) - (d*(65/e - (d*(148/e + (d*(37/e + (d*(111/e + (d*((100*d)/e^2 + 4
5/e))/e))/e))/e))/e))/(2*e)) + (25*x^8)/(2*e) + (log(d + e*x)*(45*d^7*e - 3
3*d*e^7 + 100*d^8 + 18*e^8 + 107*d^2*e^6 - 65*d^3*e^5 + 148*d^4*e^4 + 37*d^
5*e^3 + 111*d^6*e^2))/e^9
```

sympy [A] time = 1.00, size = 372, normalized size = 1.06

$$x^7\left(-\frac{100d}{7e^2}-\frac{45}{7e}\right)+x^6\left(\frac{50d^2}{3e^3}+\frac{15d}{2e^2}+\frac{37}{2e}\right)+x^5\left(-\frac{20d^3}{e^4}-\frac{9d^2}{e^3}-\frac{111d}{5e^2}-\frac{37}{5e}\right)+x^4\left(\frac{25d^4}{e^5}+\frac{45d^3}{4e^4}+\frac{111d^2}{4e^3}+\frac{37d}{4e^2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d),x)
```

```
[Out] x**7*(-100*d/(7*e**2) - 45/(7*e)) + x**6*(50*d**2/(3*e**3) + 15*d/(2*e**2)
+ 37/(2*e)) + x**5*(-20*d**3/e**4 - 9*d**2/e**3 - 111*d/(5*e**2) - 37/(5*e)
) + x**4*(25*d**4/e**5 + 45*d**3/(4*e**4) + 111*d**2/(4*e**3) + 37*d/(4*e**
2) + 37/e) + x**3*(-100*d**5/(3*e**6) - 15*d**4/e**5 - 37*d**3/e**4 - 37*d*
*2/(3*e**3) - 148*d/(3*e**2) + 65/(3*e)) + x**2*(50*d**6/e**7 + 45*d**5/(2*
e**6) + 111*d**4/(2*e**5) + 37*d**3/(2*e**4) + 74*d**2/e**3 - 65*d/(2*e**2)
 + 107/(2*e)) + x*(-100*d**7/e**8 - 45*d**6/e**7 - 111*d**5/e**6 - 37*d**4/
e**5 - 148*d**3/e**4 + 65*d**2/e**3 - 107*d/e**2 + 33/e) + 25*x**8/(2*e) +
(5*d**2 - 2*d*e + 3*e**2)**2*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*
e**4)*log(d + e*x)/e**9
```

3.301 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$

Optimal. Leaf size=353

$$\frac{3x^5(100d^2+30de+37e^2)}{5e^4}-\frac{x^4(400d^3+135d^2e+222de^2+37e^3)}{4e^5}-\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-}{e^9(d+ex)}$$

```
[Out] (700*d^6+270*d^5*e+555*d^4*e^2+148*d^3*e^3+444*d^2*e^4-130*d*e^5+107*e^6)*x
/e^8-1/2*(600*d^5+225*d^4*e+444*d^3*e^2+111*d^2*e^3+296*d*e^4-65*e^5)*x^2/e
^7+1/3*(500*d^4+180*d^3*e+333*d^2*e^2+74*d*e^3+148*e^4)*x^3/e^6-1/4*(400*d^
3+135*d^2*e+222*d*e^2+37*e^3)*x^4/e^5+3/5*(100*d^2+30*d*e+37*e^2)*x^5/e^4-5
/6*(40*d+9*e)*x^6/e^3+100/7*x^7/e^2-(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*
d^2*e^2-d*e^3+2*e^4)/e^9/(e*x+d)-(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*
d^3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)*ln(e*x+d)/e^9
```

Rubi [A] time = 0.33, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{3x^5(100d^2+30de+37e^2)}{5e^4}-\frac{x^4(135d^2e+400d^3+222de^2+37e^3)}{4e^5}+\frac{x^3(333d^2e^2+180d^3e+500d^4+74de^3+}{3e^6}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x]
```

```
[Out] ((700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130*d*e^5
 + 107*e^6)*x)/e^8 - ((600*d^5 + 225*d^4*e + 444*d^3*e^2 + 111*d^2*e^3 + 29
6*d*e^4 - 65*e^5)*x^2)/(2*e^7) + ((500*d^4 + 180*d^3*e + 333*d^2*e^2 + 74*d
*e^3 + 148*e^4)*x^3)/(3*e^6) - ((400*d^3 + 135*d^2*e + 222*d*e^2 + 37*e^3)*
x^4)/(4*e^5) + (3*(100*d^2 + 30*d*e + 37*e^2)*x^5)/(5*e^4) - (5*(40*d + 9*e
)*x^6)/(6*e^3) + (100*x^7)/(7*e^2) - ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*
d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e^9*(d + e*x)) - ((5*d^2 - 2*d*e + 3*e
^2)*(160*d^5 + 127*d^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*Log[
d + e*x])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx=\int \left(\frac{700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130}{e^8}\right.$$

$$=\frac{(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5}{e^8}$$

Mathematica [A] time = 0.14, size = 342, normalized size = 0.97

$$252e^5x^5(100d^2+30de+37e^2)-105e^4x^4(400d^3+135d^2e+222de^2+37e^3)+140e^3x^3(500d^4+180d^3e+33$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2
,x]
```

```
[Out] (420*e*(700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130
*d*e^5 + 107*e^6)*x - 210*e^2*(600*d^5 + 225*d^4*e + 444*d^3*e^2 + 111*d^2*
e^3 + 296*d*e^4 - 65*e^5)*x^2 + 140*e^3*(500*d^4 + 180*d^3*e + 333*d^2*e^2
+ 74*d*e^3 + 148*e^4)*x^3 - 105*e^4*(400*d^3 + 135*d^2*e + 222*d*e^2 + 37*e
^3)*x^4 + 252*e^5*(100*d^2 + 30*d*e + 37*e^2)*x^5 - 350*e^6*(40*d + 9*e)*x^
6 + 6000*e^7*x^7 - (420*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*
e^2 - d*e^3 + 2*e^4))/(d + e*x) - 420*(800*d^7 + 315*d^6*e + 666*d^5*e^2 +
185*d^4*e^3 + 592*d^3*e^4 - 195*d^2*e^5 + 214*d*e^6 - 33*e^7)*Log[d + e*x])
/(420*e^9)
```

fricas [A] time = 1.00, size = 490, normalized size = 1.39

$$\frac{6000\,e^{8}x^{8} - 42000\,d^{8} - 18900\,d^{7}e - 46620\,d^{6}e^{2} - 15540\,d^{5}e^{3} - 62160\,d^{4}e^{4} + 27300\,d^{3}e^{5} - 44940\,d^{2}e^{6} + 13860\,de}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="f
ricas")
```

```
[Out] 1/420*(6000*e^8*x^8 - 42000*d^8 - 18900*d^7*e - 46620*d^6*e^2 - 15540*d^5*e
^3 - 62160*d^4*e^4 + 27300*d^3*e^5 - 44940*d^2*e^6 + 13860*d*e^7 - 7560*e^8
 - 50*(160*d*e^7 + 63*e^8)*x^7 + 14*(800*d^2*e^6 + 315*d*e^7 + 666*e^8)*x^6
 - 21*(800*d^3*e^5 + 315*d^2*e^6 + 666*d*e^7 + 185*e^8)*x^5 + 35*(800*d^4*e
^4 + 315*d^3*e^5 + 666*d^2*e^6 + 185*d*e^7 + 592*e^8)*x^4 - 70*(800*d^5*e^3
 + 315*d^4*e^4 + 666*d^3*e^5 + 185*d^2*e^6 + 592*d*e^7 - 195*e^8)*x^3 + 210
*(800*d^6*e^2 + 315*d^5*e^3 + 666*d^4*e^4 + 185*d^3*e^5 + 592*d^2*e^6 - 195
*d*e^7 + 214*e^8)*x^2 + 420*(700*d^7*e + 270*d^6*e^2 + 555*d^5*e^3 + 148*d^
4*e^4 + 444*d^3*e^5 - 130*d^2*e^6 + 107*d*e^7)*x - 420*(800*d^8 + 315*d^7*e
 + 666*d^6*e^2 + 185*d^5*e^3 + 592*d^4*e^4 - 195*d^3*e^5 + 214*d^2*e^6 - 33
*d*e^7 + (800*d^7*e + 315*d^6*e^2 + 666*d^5*e^3 + 185*d^4*e^4 + 592*d^3*e^5
 - 195*d^2*e^6 + 214*d*e^7 - 33*e^8)*x)*log(e*x + d))/(e^10*x + d*e^9)
```

giac [A] time = 0.18, size = 459, normalized size = 1.30

$$-\frac{1}{420}\,(xe+d)^{7}\left(\frac{350\left(160\,de+9\,e^{2}\right)e^{(-1)}}{xe+d} - \frac{84\left(2800\,d^{2}e^{2}+315\,de^{3}+111\,e^{4}\right)e^{(-2)}}{(xe+d)^{2}} + \frac{105\left(5600\,d^{3}e^{3}+945\,d^{2}e^{4}+6\right.}{(xe+d)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="g
iac")
```

```
[Out] -1/420*(x*e + d)^7*(350*(160*d*e + 9*e^2)*e^(-1)/(x*e + d) - 84*(2800*d^2*e
^2 + 315*d*e^3 + 111*e^4)*e^(-2)/(x*e + d)^2 + 105*(5600*d^3*e^3 + 945*d^2*
e^4 + 666*d*e^5 + 37*e^6)*e^(-3)/(x*e + d)^3 - 140*(7000*d^4*e^4 + 1575*d^3
*e^5 + 1665*d^2*e^6 + 185*d*e^7 + 148*e^8)*e^(-4)/(x*e + d)^4 + 210*(5600*d
^5*e^5 + 1575*d^4*e^6 + 2220*d^3*e^7 + 370*d^2*e^8 + 592*d*e^9 - 65*e^10)*e
^(-5)/(x*e + d)^5 - 420*(2800*d^6*e^6 + 945*d^5*e^7 + 1665*d^4*e^8 + 370*d^
3*e^9 + 888*d^2*e^10 - 195*d*e^11 + 107*e^12)*e^(-6)/(x*e + d)^6 - 6000)*e^
(-9) + (800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195
*d^2*e^5 + 214*d*e^6 - 33*e^7)*e^(-9)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)
- (100*d^8*e^7/(x*e + d) + 45*d^7*e^8/(x*e + d) + 111*d^6*e^9/(x*e + d) + 3
7*d^5*e^10/(x*e + d) + 148*d^4*e^11/(x*e + d) - 65*d^3*e^12/(x*e + d) + 107
*d^2*e^13/(x*e + d) - 33*d*e^14/(x*e + d) + 18*e^15/(x*e + d))*e^(-16)
```

maple [A] time = 0.01, size = 500, normalized size = 1.42

$$\frac{100x^7}{7e^2}-\frac{100d\,x^6}{3e^3}-\frac{15x^6}{2e^2}+\frac{60d^2x^5}{e^4}+\frac{18d\,x^5}{e^3}+\frac{111x^5}{5e^2}-\frac{100d^3x^4}{e^5}-\frac{135d^2x^4}{4e^4}-\frac{111d\,x^4}{2e^3}-\frac{37x^4}{4e^2}+\frac{500d^4x^3}{3e^6}+\frac{60d^3x^3}{e^5}+\frac{111d}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x)
```

```
[Out] 700/e^8*d^6*x-300/e^7*x^2*d^5-225/2/e^6*x^2*d^4+500/3/e^6*x^3*d^4+60/e^5*x^
3*d^3+270/e^7*x*d^5-100/e^5*x^4*d^3-135/4/e^4*x^4*d^2+60/e^4*x^5*d^2-100/3/
e^3*x^6*d+18/e^3*x^5*d-100/e^9/(e*x+d)*d^8-45/e^8/(e*x+d)*d^7-800/e^9*ln(e*
x+d)*d^7-315/e^8*ln(e*x+d)*d^6+107/e^2*x+100/7*x^7/e^2+111/5/e^2*x^5-15/2/e
^2*x^6+65/2/e^2*x^2-18/(e*x+d)/e+33/e^2*ln(e*x+d)+148/3/e^2*x^3-37/4/e^2*x^
4-214*d/e^3*ln(e*x+d)-666*d^5/e^7*ln(e*x+d)-185*d^4/e^6*ln(e*x+d)-592*d^3/e
^5*ln(e*x+d)+195*d^2/e^4*ln(e*x+d)-107/(e*x+d)*d^2/e^3+33/(e*x+d)*d/e^2-111
/(e*x+d)*d^6/e^7-37/(e*x+d)*d^5/e^6-148/(e*x+d)*d^4/e^5+65/(e*x+d)*d^3/e^4-
130*d/e^3*x+74/3*d/e^3*x^3-222*d^3/e^5*x^2-111/2*d^2/e^4*x^2-148*d/e^3*x^2+
555*d^4/e^6*x+148*d^3/e^5*x+444*d^2/e^4*x-111/2*d/e^3*x^4+111*d^2/e^4*x^3
```

maxima [A] time = 0.44, size = 372, normalized size = 1.05

$$-\frac{100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8}{e^{10}x+de^9}+\frac{6000\,e^6x^7-350\left(40\,d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="m
axima")
```

```
[Out] -(100*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5
+ 107*d^2*e^6 - 33*d*e^7 + 18*e^8)/(e^10*x + d*e^9) + 1/420*(6000*e^6*x^7 -
 350*(40*d*e^5 + 9*e^6)*x^6 + 252*(100*d^2*e^4 + 30*d*e^5 + 37*e^6)*x^5 - 1
05*(400*d^3*e^3 + 135*d^2*e^4 + 222*d*e^5 + 37*e^6)*x^4 + 140*(500*d^4*e^2
+ 180*d^3*e^3 + 333*d^2*e^4 + 74*d*e^5 + 148*e^6)*x^3 - 210*(600*d^5*e + 22
5*d^4*e^2 + 444*d^3*e^3 + 111*d^2*e^4 + 296*d*e^5 - 65*e^6)*x^2 + 420*(700*
d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130*d*e^5 + 107
*e^6)*x)/e^8 - (800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e
^4 - 195*d^2*e^5 + 214*d*e^6 - 33*e^7)*log(e*x + d)/e^9
```

mupad [B] time = 4.22, size = 939, normalized size = 2.66

$$x^2 \left(\frac{65}{2e^2} - \frac{d \left(\frac{148}{e^2} + \frac{2d \left(\frac{37}{e^2} + \frac{2d \left(\frac{111}{e^2} - \frac{100d^2}{e^4} + \frac{2d \left(\frac{200d}{e^3} + \frac{45}{e^2} \right)}{e} \right)}{e} - \frac{d^2 \left(\frac{200d}{e^3} + \frac{45}{e^2} \right)}{e^2} \right)}{e} - \frac{d^2 \left(\frac{111}{e^2} - \frac{100d^2}{e^4} + \frac{2d \left(\frac{200d}{e^3} + \frac{45}{e^2} \right)}{e} \right)}{e^2} \right)}{e} + \frac{d^2 \left(\frac{37}{e^2} + \frac{2d \left(\frac{111}{e^2} - \frac{100}{e} \cdots \right.}{\cdots} \right.}{\cdots} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^2,x)
```

```
[Out] x^2*(65/(2*e^2) - (d*(148/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^
4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))
/e - (d^2*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^2))
/e + (d^2*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/
e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/(2*e^2)) + x^3*(148/(3*e^2
) + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e
^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/(3*e) - (d^2*(111/e^2 - (10
0*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/(3*e^2)) - x^4*(37/(4*e^2) +
(d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/(2*e) - (d^2
*((200*d)/e^3 + 45/e^2))/(4*e^2)) + x^5*(111/(5*e^2) - (20*d^2)/e^4 + (2*d*
((200*d)/e^3 + 45/e^2))/(5*e)) - x^6*((100*d)/(3*e^3) + 15/(2*e^2)) - x*((2
```

```
*d*(65/e^2 - (2*d*(148/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 +
 (2*d*((200*d)/e^3 + 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/e
- (d^2*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^2))/e
+ (d^2*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2
))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/e^2))/e - 107/e^2 + (d^2*(148
/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 4
5/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/e - (d^2*(111/e^2 - (100
*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^2))/e^2) + (100*x^7)/(7*e^2)
 - (45*d^7*e - 33*d*e^7 + 100*d^8 + 18*e^8 + 107*d^2*e^6 - 65*d^3*e^5 + 148
*d^4*e^4 + 37*d^5*e^3 + 111*d^6*e^2)/(e*(d*e^8 + e^9*x)) - (log(d + e*x)*(2
14*d*e^6 + 315*d^6*e + 800*d^7 - 33*e^7 - 195*d^2*e^5 + 592*d^3*e^4 + 185*d
^4*e^3 + 666*d^5*e^2))/e^9
```

sympy [A] time = 2.31, size = 393, normalized size = 1.11

$$x^6\left(-\frac{100d}{3e^3}-\frac{15}{2e^2}\right)+x^5\left(\frac{60d^2}{e^4}+\frac{18d}{e^3}+\frac{111}{5e^2}\right)+x^4\left(-\frac{100d^3}{e^5}-\frac{135d^2}{4e^4}-\frac{111d}{2e^3}-\frac{37}{4e^2}\right)+x^3\left(\frac{500d^4}{3e^6}+\frac{60d^3}{e^5}+\frac{111d^2}{e^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2,x)
```

```
[Out] x**6*(-100*d/(3*e**3) - 15/(2*e**2)) + x**5*(60*d**2/e**4 + 18*d/e**3 + 111
/(5*e**2)) + x**4*(-100*d**3/e**5 - 135*d**2/(4*e**4) - 111*d/(2*e**3) - 37
/(4*e**2)) + x**3*(500*d**4/(3*e**6) + 60*d**3/e**5 + 111*d**2/e**4 + 74*d/
(3*e**3) + 148/(3*e**2)) + x**2*(-300*d**5/e**7 - 225*d**4/(2*e**6) - 222*d
**3/e**5 - 111*d**2/(2*e**4) - 148*d/e**3 + 65/(2*e**2)) + x*(700*d**6/e**8
 + 270*d**5/e**7 + 555*d**4/e**6 + 148*d**3/e**5 + 444*d**2/e**4 - 130*d/e*
*3 + 107/e**2) + (-100*d**8 - 45*d**7*e - 111*d**6*e**2 - 37*d**5*e**3 - 14
8*d**4*e**4 + 65*d**3*e**5 - 107*d**2*e**6 + 33*d*e**7 - 18*e**8)/(d*e**9 +
 e**10*x) + 100*x**7/(7*e**2) - (5*d**2 - 2*d*e + 3*e**2)*(160*d**5 + 127*d
**4*e + 88*d**3*e**2 - 4*d**2*e**3 + 64*d*e**4 - 11*e**5)*log(d + e*x)/e**9
```

3.302 $\int \frac{\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)}{(d+ex)^3}\,dx$

Optimal. Leaf size=354

$$\frac{3x^4\left(200d^2+45de+37e^2\right)}{4e^5}-\frac{x^3\left(1000d^3+270d^2e+333de^2+37e^3\right)}{3e^6}-\frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de\right.}{2e^9(d+ex)^2}$$

```
[Out] -(2100*d^5+675*d^4*e+1110*d^3*e^2+222*d^2*e^3+444*d*e^4-65*e^5)*x/e^8+1/2*(
1500*d^4+450*d^3*e+666*d^2*e^2+111*d*e^3+148*e^4)*x^2/e^7-1/3*(1000*d^3+270
*d^2*e+333*d*e^2+37*e^3)*x^3/e^6+3/4*(200*d^2+45*d*e+37*e^2)*x^4/e^5-3*(20*
d+3*e)*x^5/e^4+50/3*x^6/e^3-1/2*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^2*
e^2-d*e^3+2*e^4)/e^9/(e*x+d)^2+(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*d^
3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)/e^9/(e*x+d)+(2800*d^6+945*d^5*e+1665*d^4*e
^2+370*d^3*e^3+888*d^2*e^4-195*d*e^5+107*e^6)*ln(e*x+d)/e^9
```

Rubi [A] time = 0.34, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1628}

$$\frac{3x^4\left(200d^2+45de+37e^2\right)}{4e^5}-\frac{x^3\left(270d^2e+1000d^3+333de^2+37e^3\right)}{3e^6}+\frac{x^2\left(666d^2e^2+450d^3e+1500d^4+111de^3+\right.}{2e^7}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x]
```

```
[Out] -(((2100*d^5 + 675*d^4*e + 1110*d^3*e^2 + 222*d^2*e^3 + 444*d*e^4 - 65*e^5)
*x)/e^8) + ((1500*d^4 + 450*d^3*e + 666*d^2*e^2 + 111*d*e^3 + 148*e^4)*x^2)
/(2*e^7) - ((1000*d^3 + 270*d^2*e + 333*d*e^2 + 37*e^3)*x^3)/(3*e^6) + (3*(
200*d^2 + 45*d*e + 37*e^2)*x^4)/(4*e^5) - (3*(20*d + 3*e)*x^5)/e^4 + (50*x^
6)/(3*e^3) - ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^
3 + 2*e^4))/(2*e^9*(d + e*x)^2) + ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d
^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5))/(e^9*(d + e*x)) + ((280
0*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 +
107*e^6)*Log[d + e*x])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)}{(d+ex)^3}\,dx = \int \left(\frac{-2100d^5-675d^4e-1110d^3e^2-222d^2e^3-444de^4+65e^5}{e^8}\right.$$

$$= -\frac{\left(2100d^5+675d^4e+1110d^3e^2+222d^2e^3+444de^4-65e^5\right)x}{e^8}$$

Mathematica [A] time = 0.10, size = 311, normalized size = 0.88

$$\frac{9000d^8-390d^7e(40x-9)-18d^6e^2\left(2300x^2+240x-407\right)-2d^5e^3\left(5600x^3+6750x^2+2664x-999\right)+4d^4e^4\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3
,x]
```

```
[Out] (9000*d^8 - 390*d^7*e*(-9 + 40*x) - 18*d^6*e^2*(-407 + 240*x + 2300*x^2) -
2*d^5*e^3*(-999 + 2664*x + 6750*x^2 + 5600*x^3) + 4*d^4*e^4*(1554 - 111*x -
 5661*x^2 - 945*x^3 + 700*x^4) - d^3*e^5*(1950 - 1776*x + 4662*x^2 + 6660*x
^3 - 945*x^4 + 1120*x^5) + d^2*e^6*(1926 - 1560*x - 9768*x^2 - 1480*x^3 + 1
665*x^4 - 378*x^5 + 560*x^6) + d*e^7*(-198 + 2568*x + 1560*x^2 - 3552*x^3 +
 370*x^4 - 666*x^5 + 189*x^6 - 320*x^7) + e^8*(-108 - 396*x + 780*x^3 + 888
*x^4 - 148*x^5 + 333*x^6 - 108*x^7 + 200*x^8) + 12*(2800*d^6 + 945*d^5*e +
1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*(d + e*x)^2
*Log[d + e*x])/(12*e^9*(d + e*x)^2)
```

fricas [A] time = 0.81, size = 545, normalized size = 1.54

$$\frac{200 e^8 x^8 + 9000 d^8 + 3510 d^7 e + 7326 d^6 e^2 + 1998 d^5 e^3 + 6216 d^4 e^4 - 1950 d^3 e^5 + 1926 d^2 e^6 - 198 d e^7 - 108 e^8 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="f
ricas")
```

```
[Out] 1/12*(200*e^8*x^8 + 9000*d^8 + 3510*d^7*e + 7326*d^6*e^2 + 1998*d^5*e^3 + 6
216*d^4*e^4 - 1950*d^3*e^5 + 1926*d^2*e^6 - 198*d*e^7 - 108*e^8 - 4*(80*d*e
^7 + 27*e^8)*x^7 + (560*d^2*e^6 + 189*d*e^7 + 333*e^8)*x^6 - 2*(560*d^3*e^5
 + 189*d^2*e^6 + 333*d*e^7 + 74*e^8)*x^5 + (2800*d^4*e^4 + 945*d^3*e^5 + 16
65*d^2*e^6 + 370*d*e^7 + 888*e^8)*x^4 - 4*(2800*d^5*e^3 + 945*d^4*e^4 + 166
5*d^3*e^5 + 370*d^2*e^6 + 888*d*e^7 - 195*e^8)*x^3 - 6*(6900*d^6*e^2 + 2250
*d^5*e^3 + 3774*d^4*e^4 + 777*d^3*e^5 + 1628*d^2*e^6 - 260*d*e^7)*x^2 - 12*
(1300*d^7*e + 360*d^6*e^2 + 444*d^5*e^3 + 37*d^4*e^4 - 148*d^3*e^5 + 130*d^
2*e^6 - 214*d*e^7 + 33*e^8)*x + 12*(2800*d^8 + 945*d^7*e + 1665*d^6*e^2 + 3
70*d^5*e^3 + 888*d^4*e^4 - 195*d^3*e^5 + 107*d^2*e^6 + (2800*d^6*e^2 + 945*
d^5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x
^2 + 2*(2800*d^7*e + 945*d^6*e^2 + 1665*d^5*e^3 + 370*d^4*e^4 + 888*d^3*e^5
 - 195*d^2*e^6 + 107*d*e^7)*x)*log(e*x + d))/(e^11*x^2 + 2*d*e^10*x + d^2*e
^9)
```

giac [A] time = 0.16, size = 354, normalized size = 1.00

$$\left(2800 d^6 + 945 d^5 e + 1665 d^4 e^2 + 370 d^3 e^3 + 888 d^2 e^4 - 195 d e^5 + 107 e^6\right) e^{(-9)} \log\left(|xe + d|\right) + \frac{1}{12}\left(200 x^6 e^{15} - 7 \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="g
iac")
```

```
[Out] (2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^
5 + 107*e^6)*e^(-9)*log(abs(x*e + d)) + 1/12*(200*x^6*e^15 - 720*d*x^5*e^14
 + 1800*d^2*x^4*e^13 - 4000*d^3*x^3*e^12 + 9000*d^4*x^2*e^11 - 25200*d^5*x*
e^10 - 108*x^5*e^15 + 405*d*x^4*e^14 - 1080*d^2*x^3*e^13 + 2700*d^3*x^2*e^1
2 - 8100*d^4*x*e^11 + 333*x^4*e^15 - 1332*d*x^3*e^14 + 3996*d^2*x^2*e^13 -
13320*d^3*x*e^12 - 148*x^3*e^15 + 666*d*x^2*e^14 - 2664*d^2*x*e^13 + 888*x^
2*e^15 - 5328*d*x*e^14 + 780*x*e^15)*e^(-18) + 1/2*(1500*d^8 + 585*d^7*e +
1221*d^6*e^2 + 333*d^5*e^3 + 1036*d^4*e^4 - 325*d^3*e^5 + 321*d^2*e^6 + 2*(
800*d^7*e + 315*d^6*e^2 + 666*d^5*e^3 + 185*d^4*e^4 + 592*d^3*e^5 - 195*d^2
*e^6 + 214*d*e^7 - 33*e^8)*x - 33*d*e^7 - 18*e^8)*e^(-9)/(x*e + d)^2
```

maple [A] time = 0.01, size = 531, normalized size = 1.50

$$\frac{50x^6}{3e^3}-\frac{60d\,x^5}{e^4}-\frac{9x^5}{e^3}+\frac{150d^2x^4}{e^5}+\frac{135d\,x^4}{4e^4}+\frac{111x^4}{4e^3}-\frac{1000d^3x^3}{3e^6}-\frac{90d^2x^3}{e^5}-\frac{111d\,x^3}{e^4}-\frac{37x^3}{3e^3}-\frac{50d^8}{(ex+d)^2\,e^9}-\frac{45d^7}{2\,(ex+d)^2\,e^8}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x)
```

```
[Out] -1110*d^3/e^6*x-222*d^2/e^5*x-444*d/e^4*x+50/3*x^6/e^3+111/4/e^3*x^4-9/e^3*
x^5-9/(e*x+d)^2/e+107/e^3*ln(e*x+d)+74/e^3*x^2+65/e^3*x-33/(e*x+d)/e^2-37/3
/e^3*x^3-50/e^9/(e*x+d)^2*d^8-45/2/e^8/(e*x+d)^2*d^7+2800/e^9*ln(e*x+d)*d^6
+945/e^8*ln(e*x+d)*d^5-60/e^4*x^5*d+150/e^5*x^4*d^2+135/4/e^4*x^4*d-1000/3/
e^6*x^3*d^3-90/e^5*x^3*d^2+750/e^7*x^2*d^4+225/e^6*x^2*d^3-2100/e^8*x*d^5-6
75/e^7*x*d^4+800/e^9/(e*x+d)*d^7+315/e^8/(e*x+d)*d^6+666/(e*x+d)*d^5/e^7+18
5/(e*x+d)*d^4/e^6+592/(e*x+d)*d^3/e^5-195/(e*x+d)*d^2/e^4+214/(e*x+d)*d/e^3
-111*d/e^4*x^3+333*d^2/e^5*x^2+111/2*d/e^4*x^2-111/2/(e*x+d)^2*d^6/e^7-37/2
/(e*x+d)^2*d^5/e^6-74/(e*x+d)^2*d^4/e^5+65/2/(e*x+d)^2*d^3/e^4-107/2/(e*x+d
)^2*d^2/e^3+33/2/(e*x+d)^2*d/e^2+1665*d^4/e^7*ln(e*x+d)+370*d^3/e^6*ln(e*x+
d)+888*d^2/e^5*ln(e*x+d)-195*d/e^4*ln(e*x+d)
```

maxima [A] time = 0.45, size = 378, normalized size = 1.07

$$\frac{1500\,d^8+585\,d^7e+1221\,d^6e^2+333\,d^5e^3+1036\,d^4e^4-325\,d^3e^5+321\,d^2e^6-33\,de^7-18\,e^8+2\left(800\,d^7e+315\,d^6\right.}{2\left(e^{11}x^2+2\,de^{10}x+d^2e^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="m
axima")
```

```
[Out] 1/2*(1500*d^8 + 585*d^7*e + 1221*d^6*e^2 + 333*d^5*e^3 + 1036*d^4*e^4 - 325
*d^3*e^5 + 321*d^2*e^6 - 33*d*e^7 - 18*e^8 + 2*(800*d^7*e + 315*d^6*e^2 + 6
66*d^5*e^3 + 185*d^4*e^4 + 592*d^3*e^5 - 195*d^2*e^6 + 214*d*e^7 - 33*e^8)*
x)/(e^11*x^2 + 2*d*e^10*x + d^2*e^9) + 1/12*(200*e^5*x^6 - 36*(20*d*e^4 + 3
*e^5)*x^5 + 9*(200*d^2*e^3 + 45*d*e^4 + 37*e^5)*x^4 - 4*(1000*d^3*e^2 + 270
*d^2*e^3 + 333*d*e^4 + 37*e^5)*x^3 + 6*(1500*d^4*e + 450*d^3*e^2 + 666*d^2*
e^3 + 111*d*e^4 + 148*e^5)*x^2 - 12*(2100*d^5 + 675*d^4*e + 1110*d^3*e^2 +
222*d^2*e^3 + 444*d*e^4 - 65*e^5)*x)/e^8 + (2800*d^6 + 945*d^5*e + 1665*d^4
*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*log(e*x + d)/e^9
```

mupad [B] time = 0.13, size = 771, normalized size = 2.18

$$x^4\left(\frac{111}{4e^3}-\frac{75d^2}{e^5}+\frac{3d\left(\frac{300d}{e^4}+\frac{45}{e^3}\right)}{4e}\right)-x^3\left(\frac{37}{3e^3}+\frac{100d^3}{3e^6}+\frac{d\left(\frac{111}{e^3}-\frac{300d^2}{e^5}+\frac{3d\left(\frac{300d}{e^4}+\frac{45}{e^3}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{300d}{e^4}+\frac{45}{e^3}\right)}{e^2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^3,x)
```

```
[Out] x^4*(111/(4*e^3) - (75*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/(4*e)) - x^3
*(37/(3*e^3) + (100*d^3)/(3*e^6) + (d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300
*d)/e^4 + 45/e^3))/e))/e - (d^2*((300*d)/e^4 + 45/e^3))/e^2) - x^5*((60*d)/
e^4 + 9/e^3) + x*(65/e^3 - (3*d*(148/e^3 + (3*d*(37/e^3 + (100*d^3)/e^6 + (
3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/e - (3*d^2*
((300*d)/e^4 + 45/e^3))/e^2))/e - (3*d^2*(111/e^3 - (300*d^2)/e^5 + (3*d*((
300*d)/e^4 + 45/e^3))/e))/e^2 + (d^3*((300*d)/e^4 + 45/e^3))/e^3))/e + (3*d
^2*(37/e^3 + (100*d^3)/e^6 + (3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/
e^4 + 45/e^3))/e))/e - (3*d^2*((300*d)/e^4 + 45/e^3))/e^2))/e^2 - (d^3*(111
/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/e^3) + (x*(214*d*e^
6 + 315*d^6*e + 800*d^7 - 33*e^7 - 195*d^2*e^5 + 592*d^3*e^4 + 185*d^4*e^3
+ 666*d^5*e^2) + (585*d^7*e - 33*d*e^7 + 1500*d^8 - 18*e^8 + 321*d^2*e^6 -
325*d^3*e^5 + 1036*d^4*e^4 + 333*d^5*e^3 + 1221*d^6*e^2)/(2*e))/(d^2*e^8 +
e^10*x^2 + 2*d*e^9*x) + (50*x^6)/(3*e^3) + x^2*(74/e^3 + (3*d*(37/e^3 + (10
0*d^3)/e^6 + (3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e
))/e - (3*d^2*((300*d)/e^4 + 45/e^3))/e^2))/(2*e) - (3*d^2*(111/e^3 - (300*
d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/(2*e^2) + (d^3*((300*d)/e^4 + 4
5/e^3))/(2*e^3)) + (log(d + e*x)*(945*d^5*e - 195*d*e^5 + 2800*d^6 + 107*e^
6 + 888*d^2*e^4 + 370*d^3*e^3 + 1665*d^4*e^2))/e^9
```

sympy [A] time = 4.87, size = 394, normalized size = 1.11

$$x^5\left(-\frac{60d}{e^4}-\frac{9}{e^3}\right)+x^4\left(\frac{150d^2}{e^5}+\frac{135d}{4e^4}+\frac{111}{4e^3}\right)+x^3\left(-\frac{1000d^3}{3e^6}-\frac{90d^2}{e^5}-\frac{111d}{e^4}-\frac{37}{3e^3}\right)+x^2\left(\frac{750d^4}{e^7}+\frac{225d^3}{e^6}+\frac{333d}{e^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3,x)
```

```
[Out] x**5*(-60*d/e**4 - 9/e**3) + x**4*(150*d**2/e**5 + 135*d/(4*e**4) + 111/(4*
e**3)) + x**3*(-1000*d**3/(3*e**6) - 90*d**2/e**5 - 111*d/e**4 - 37/(3*e**3
)) + x**2*(750*d**4/e**7 + 225*d**3/e**6 + 333*d**2/e**5 + 111*d/(2*e**4) +
 74/e**3) + x*(-2100*d**5/e**8 - 675*d**4/e**7 - 1110*d**3/e**6 - 222*d**2/
```

```
e**5 - 444*d/e**4 + 65/e**3) + (1500*d**8 + 585*d**7*e + 1221*d**6*e**2 + 3
33*d**5*e**3 + 1036*d**4*e**4 - 325*d**3*e**5 + 321*d**2*e**6 - 33*d*e**7 -
 18*e**8 + x*(1600*d**7*e + 630*d**6*e**2 + 1332*d**5*e**3 + 370*d**4*e**4
+ 1184*d**3*e**5 - 390*d**2*e**6 + 428*d*e**7 - 66*e**8))/(2*d**2*e**9 + 4*
d*e**10*x + 2*e**11*x**2) + 50*x**6/(3*e**3) + (2800*d**6 + 945*d**5*e + 16
65*d**4*e**2 + 370*d**3*e**3 + 888*d**2*e**4 - 195*d*e**5 + 107*e**6)*log(d
 + e*x)/e**9
```

3.303 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4} dx$

Optimal. Leaf size=360

$$\frac{x^3(1000d^2+180de+111e^2)}{3e^6}-\frac{x^2(2000d^3+450d^2e+444de^2+37e^3)}{2e^7}-\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2}{3e^9(d+ex)^3}$$

```
[Out] 2*(1750*d^4+450*d^3*e+555*d^2*e^2+74*d*e^3+74*e^4)*x/e^8-1/2*(2000*d^3+450*
d^2*e+444*d*e^2+37*e^3)*x^2/e^7+1/3*(1000*d^2+180*d*e+111*e^2)*x^3/e^6-5/4*
(80*d+9*e)*x^4/e^5+20*x^5/e^4-1/3*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^
2*e^2-d*e^3+2*e^4)/e^9/(e*x+d)^3+1/2*(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e
+88*d^3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)/e^9/(e*x+d)^2+(-2800*d^6-945*d^5*e-1
665*d^4*e^2-370*d^3*e^3-888*d^2*e^4+195*d*e^5-107*e^6)/e^9/(e*x+d)-(5600*d^
5+1575*d^4*e+2220*d^3*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*ln(e*x+d)/e^9
```

Rubi [A] time = 0.36, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{x^3(1000d^2+180de+111e^2)}{3e^6}-\frac{x^2(450d^2e+2000d^3+444de^2+37e^3)}{2e^7}+\frac{2x(555d^2e^2+450d^3e+1750d^4+74d}{e^8}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^4,x]
```

```
[Out] (2*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4)*x)/e^8 - ((2000
*d^3 + 450*d^2*e + 444*d*e^2 + 37*e^3)*x^2)/(2*e^7) + ((1000*d^2 + 180*d*e
+ 111*e^2)*x^3)/(3*e^6) - (5*(80*d + 9*e)*x^4)/(4*e^5) + (20*x^5)/e^4 - ((5
*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(3*e
^9*(d + e*x)^3) + ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d^3*e^
2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5))/(2*e^9*(d + e*x)^2) - (2800*d^6 + 945*d
^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)/(e^9
*(d + e*x)) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*
e^4 - 65*e^5)*Log[d + e*x])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4} dx = \int \left(\frac{2(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)}{e^8}-\frac{(200}{}\right.$$

$$=\frac{2(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)x}{e^8}-\frac{(2000}{}$$

Mathematica [A] time = 0.12, size = 344, normalized size = 0.96

$$\frac{4e^3x^3(1000d^2+180de+111e^2)-6e^2x^2(2000d^3+450d^2e+444de^2+37e^3)+24ex(1750d^4+450d^3e+555d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^4
,x]
```

```
[Out] (24*e*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4)*x - 6*e^2*(2
000*d^3 + 450*d^2*e + 444*d*e^2 + 37*e^3)*x^2 + 4*e^3*(1000*d^2 + 180*d*e +
 111*e^2)*x^3 - 15*e^4*(80*d + 9*e)*x^4 + 240*e^5*x^5 - (4*(5*d^2 - 2*d*e +
 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(d + e*x)^3 + (6*(
800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195*d^2*e^5
 + 214*d*e^6 - 33*e^7))/(d + e*x)^2 - (12*(2800*d^6 + 945*d^5*e + 1665*d^4*
e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6))/(d + e*x) - 12*(560
0*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*Log[d
 + e*x])/(12*e^9)
```

fricas [A] time = 0.65, size = 587, normalized size = 1.63

$$\frac{240 e^8x^8 - 29200 d^8 - 9630 d^7e - 16428 d^6e^2 - 3478 d^5e^3 - 7696 d^4e^4 + 1430 d^3e^5 - 428 d^2e^6 - 66 de^7 - 72 e^8 - 1\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="f
ricas")
```

```
[Out] 1/12*(240*e^8*x^8 - 29200*d^8 - 9630*d^7*e - 16428*d^6*e^2 - 3478*d^5*e^3 -
 7696*d^4*e^4 + 1430*d^3*e^5 - 428*d^2*e^6 - 66*d*e^7 - 72*e^8 - 15*(32*d*e
^7 + 9*e^8)*x^7 + (1120*d^2*e^6 + 315*d*e^7 + 444*e^8)*x^6 - 3*(1120*d^3*e^
5 + 315*d^2*e^6 + 444*d*e^7 + 74*e^8)*x^5 + 3*(5600*d^4*e^4 + 1575*d^3*e^5
+ 2220*d^2*e^6 + 370*d*e^7 + 592*e^8)*x^4 + 2*(47000*d^5*e^3 + 12510*d^4*e^
4 + 16206*d^3*e^5 + 2331*d^2*e^6 + 2664*d*e^7)*x^3 + 6*(13400*d^6*e^2 + 306
0*d^5*e^3 + 2886*d^4*e^4 + 111*d^3*e^5 - 888*d^2*e^6 + 390*d*e^7 - 214*e^8)
*x^2 - 6*(3400*d^7*e + 1665*d^6*e^2 + 3774*d^5*e^3 + 999*d^4*e^4 + 2664*d^3
*e^5 - 585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x - 12*(5600*d^8 + 1575*d^7*e + 22
20*d^6*e^2 + 370*d^5*e^3 + 592*d^4*e^4 - 65*d^3*e^5 + (5600*d^5*e^3 + 1575*
d^4*e^4 + 2220*d^3*e^5 + 370*d^2*e^6 + 592*d*e^7 - 65*e^8)*x^3 + 3*(5600*d^
6*e^2 + 1575*d^5*e^3 + 2220*d^4*e^4 + 370*d^3*e^5 + 592*d^2*e^6 - 65*d*e^7)
*x^2 + 3*(5600*d^7*e + 1575*d^6*e^2 + 2220*d^5*e^3 + 370*d^4*e^4 + 592*d^3*
e^5 - 65*d^2*e^6)*x)*log(e*x + d))/(e^12*x^3 + 3*d*e^11*x^2 + 3*d^2*e^10*x
+ d^3*e^9)
```

giac [A] time = 0.16, size = 345, normalized size = 0.96

$$-\left(5600 d^5 + 1575 d^4e + 2220 d^3e^2 + 370 d^2e^3 + 592 de^4 - 65 e^5\right)e^{(-9)} \log\left(|xe + d|\right) + \frac{1}{12}\left(240 x^5e^{16} - 1200 dx^4e^{15} + \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="g
iac")
```

```
[Out] -(5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*
e^(-9)*log(abs(x*e + d)) + 1/12*(240*x^5*e^16 - 1200*d*x^4*e^15 + 4000*d^2*
x^3*e^14 - 12000*d^3*x^2*e^13 + 42000*d^4*x*e^12 - 135*x^4*e^16 + 720*d*x^3
*e^15 - 2700*d^2*x^2*e^14 + 10800*d^3*x*e^13 + 444*x^3*e^16 - 2664*d*x^2*e^
15 + 13320*d^2*x*e^14 - 222*x^2*e^16 + 1776*d*x*e^15 + 1776*x*e^16)*e^(-20)
 - 1/6*(14600*d^8 + 4815*d^7*e + 8214*d^6*e^2 + 1739*d^5*e^3 + 3848*d^4*e^4
 - 715*d^3*e^5 + 6*(2800*d^6*e^2 + 945*d^5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5
 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x^2 + 214*d^2*e^6 + 3*(10400*d^7*e +
3465*d^6*e^2 + 5994*d^5*e^3 + 1295*d^4*e^4 + 2960*d^3*e^5 - 585*d^2*e^6 + 2
14*d*e^7 + 33*e^8)*x + 33*d*e^7 + 36*e^8)*e^(-9)/(x*e + d)^3
```

maple [A] time = 0.01, size = 558, normalized size = 1.55

$$\frac{20x^5}{e^4}-\frac{100d\,x^4}{e^5}-\frac{45x^4}{4e^4}-\frac{100d^8}{3\left(ex+d\right)^3e^9}-\frac{15d^7}{\left(ex+d\right)^3e^8}-\frac{37d^6}{\left(ex+d\right)^3e^7}-\frac{37d^5}{3\left(ex+d\right)^3e^6}-\frac{148d^4}{3\left(ex+d\right)^3e^5}+\frac{65d^3}{3\left(ex+d\right)^3e^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x)
```

```
[Out] 20*x^5/e^4-107/e^3/(e*x+d)-6/e/(e*x+d)^3+65/e^4*ln(e*x+d)-33/2/e^2/(e*x+d)^
2+37/e^4*x^3-37/2/e^4*x^2+148/e^4*x-45/4/e^4*x^4-37/3/e^6/(e*x+d)^3*d^5-148
/3/e^5/(e*x+d)^3*d^4+65/3/e^4/(e*x+d)^3*d^3-107/3/e^3/(e*x+d)^3*d^2+11/e^2/
(e*x+d)^3*d-100/e^5*x^4*d+1000/3/e^6*x^3*d^2-5600/e^9*ln(e*x+d)*d^5-1575/e^
8*ln(e*x+d)*d^4-2220/e^7*ln(e*x+d)*d^3-370/e^6*ln(e*x+d)*d^2-592/e^5*ln(e*x
+d)*d+400/e^9/(e*x+d)^2*d^7+315/2/e^8/(e*x+d)^2*d^6+333/e^7/(e*x+d)^2*d^5+1
85/2/e^6/(e*x+d)^2*d^4+296/e^5/(e*x+d)^2*d^3-195/2/e^4/(e*x+d)^2*d^2+107/e^
3/(e*x+d)^2*d+60/e^5*x^3*d-1000/e^7*x^2*d^3-225/e^6*x^2*d^2-222/e^5*x^2*d+3
500/e^8*d^4*x+900/e^7*x*d^3+1110/e^6*x*d^2+148/e^5*x*d-2800/e^9/(e*x+d)*d^6
-945/e^8/(e*x+d)*d^5-1665/e^7/(e*x+d)*d^4-370/e^6/(e*x+d)*d^3-888/e^5/(e*x+
d)*d^2+195/e^4/(e*x+d)*d-100/3/e^9/(e*x+d)^3*d^8-15/e^8/(e*x+d)^3*d^7-37/e^
7/(e*x+d)^3*d^6
```

maxima [A] time = 0.46, size = 390, normalized size = 1.08

$$-\frac{14600\,d^8+4815\,d^7e+8214\,d^6e^2+1739\,d^5e^3+3848\,d^4e^4-715\,d^3e^5+214\,d^2e^6+33\,de^7+36\,e^8+6\left(2800\,d^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="m
axima")
```

```
[Out] -1/6*(14600*d^8 + 4815*d^7*e + 8214*d^6*e^2 + 1739*d^5*e^3 + 3848*d^4*e^4 -
 715*d^3*e^5 + 214*d^2*e^6 + 33*d*e^7 + 36*e^8 + 6*(2800*d^6*e^2 + 945*d^5*
e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x^2 +
 3*(10400*d^7*e + 3465*d^6*e^2 + 5994*d^5*e^3 + 1295*d^4*e^4 + 2960*d^3*e^5
 - 585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x)/(e^12*x^3 + 3*d*e^11*x^2 + 3*d^2*e^
10*x + d^3*e^9) + 1/12*(240*e^4*x^5 - 15*(80*d*e^3 + 9*e^4)*x^4 + 4*(1000*d
^2*e^2 + 180*d*e^3 + 111*e^4)*x^3 - 6*(2000*d^3*e + 450*d^2*e^2 + 444*d*e^3
 + 37*e^4)*x^2 + 24*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4
)*x)/e^8 - (5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4
- 65*e^5)*log(e*x + d)/e^9
```

mupad [B] time = 4.28, size = 560, normalized size = 1.56

$$x^3\left(\frac{37}{e^4}-\frac{200\,d^2}{e^6}+\frac{4\,d\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{3\,e}\right)-x^2\left(\frac{37}{2\,e^4}+\frac{200\,d^3}{e^7}+\frac{2\,d\left(\frac{111}{e^4}-\frac{600\,d^2}{e^6}+\frac{4\,d\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e}\right)}{e}-\frac{3\,d^2\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^4,x)
```

```
[Out] x^3*(37/e^4 - (200*d^2)/e^6 + (4*d*((400*d)/e^5 + 45/e^4))/(3*e)) - x^2*(37
/(2*e^4) + (200*d^3)/e^7 + (2*d*(111/e^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^
5 + 45/e^4))/e))/e - (3*d^2*((400*d)/e^5 + 45/e^4))/e^2) - (x*(107*d*e^6 +
(3465*d^6*e)/2 + 5200*d^7 + (33*e^7)/2 - (585*d^2*e^5)/2 + 1480*d^3*e^4 + (
1295*d^4*e^3)/2 + 2997*d^5*e^2) + (33*d*e^7 + 4815*d^7*e + 14600*d^8 + 36*e
^8 + 214*d^2*e^6 - 715*d^3*e^5 + 3848*d^4*e^4 + 1739*d^5*e^3 + 8214*d^6*e^2
)/(6*e) + x^2*(2800*d^6*e - 195*d*e^6 + 107*e^7 + 888*d^2*e^5 + 370*d^3*e^4
 + 1665*d^4*e^3 + 945*d^5*e^2))/(d^3*e^8 + e^11*x^3 + 3*d^2*e^9*x + 3*d*e^1
0*x^2) - x^4*((100*d)/e^5 + 45/(4*e^4)) + x*(148/e^4 - (100*d^4)/e^8 + (4*d
*(37/e^4 + (400*d^3)/e^7 + (4*d*(111/e^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^
5 + 45/e^4))/e))/e - (6*d^2*((400*d)/e^5 + 45/e^4))/e^2))/e - (6*d^2*(111/e
^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^5 + 45/e^4))/e))/e^2 + (4*d^3*((400*d)
/e^5 + 45/e^4))/e^3) + (20*x^5)/e^4 - (log(d + e*x)*(592*d*e^4 + 1575*d^4*e
 + 5600*d^5 - 65*e^5 + 370*d^2*e^3 + 2220*d^3*e^2))/e^9
```

sympy [A] time = 8.09, size = 401, normalized size = 1.11

$$x^4\left(-\frac{100d}{e^5}-\frac{45}{4e^4}\right)+x^3\left(\frac{1000d^2}{3e^6}+\frac{60d}{e^5}+\frac{37}{e^4}\right)+x^2\left(-\frac{1000d^3}{e^7}-\frac{225d^2}{e^6}-\frac{222d}{e^5}-\frac{37}{2e^4}\right)+x\left(\frac{3500d^4}{e^8}+\frac{900d^3}{e^7}+\frac{1110}{e^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**4,x)
```

```
[Out] x**4*(-100*d/e**5 - 45/(4*e**4)) + x**3*(1000*d**2/(3*e**6) + 60*d/e**5 + 3
7/e**4) + x**2*(-1000*d**3/e**7 - 225*d**2/e**6 - 222*d/e**5 - 37/(2*e**4))
 + x*(3500*d**4/e**8 + 900*d**3/e**7 + 1110*d**2/e**6 + 148*d/e**5 + 148/e*
*4) + (-14600*d**8 - 4815*d**7*e - 8214*d**6*e**2 - 1739*d**5*e**3 - 3848*d
**4*e**4 + 715*d**3*e**5 - 214*d**2*e**6 - 33*d*e**7 - 36*e**8 + x**2*(-168
00*d**6*e**2 - 5670*d**5*e**3 - 9990*d**4*e**4 - 2220*d**3*e**5 - 5328*d**2
*e**6 + 1170*d*e**7 - 642*e**8) + x*(-31200*d**7*e - 10395*d**6*e**2 - 1798
2*d**5*e**3 - 3885*d**4*e**4 - 8880*d**3*e**5 + 1755*d**2*e**6 - 642*d*e**7
 - 99*e**8))/(6*d**3*e**9 + 18*d**2*e**10*x + 18*d*e**11*x**2 + 6*e**12*x**
3) + 20*x**5/e**4 - (5600*d**5 + 1575*d**4*e + 2220*d**3*e**2 + 370*d**2*e*
*3 + 592*d*e**4 - 65*e**5)*log(d + e*x)/e**9
```

3.304 $\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx$

Optimal. Leaf size=221

$$\frac{3}{500}ex^4\left(100d^2-165de+27e^2\right)+\frac{x^3\left(500d^3-2475d^2e+1215de^2+458e^3\right)}{1875}-\frac{x^2\left(4125d^3-6075d^2e-6870de^2\right.}{6250}$$

```
[Out] 1/15625*(10125*d^3+34350*d^2*e-13215*d*e^2-5108*e^3)*x-1/6250*(4125*d^3-607
5*d^2*e-6870*d*e^2+881*e^3)*x^2+1/1875*(500*d^3-2475*d^2*e+1215*d*e^2+458*e
^3)*x^3+3/500*e*(100*d^2-165*d*e+27*e^2)*x^4+3/125*(20*d-11*e)*e^2*x^5+2/15
*e^3*x^6+1/156250*(57250*d^3-66075*d^2*e-76620*d*e^2+23431*e^3)*ln(5*x^2+2*
x+3)-1/1093750*(52875*d^3+449175*d^2*e-274845*d*e^2-53189*e^3)*arctan(1/14*
(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1628, 634, 618, 204, 628}

$$\frac{3}{500}ex^4\left(100d^2-165de+27e^2\right)+\frac{x^3\left(-2475d^2e+500d^3+1215de^2+458e^3\right)}{1875}-\frac{x^2\left(-6075d^2e+4125d^3-6870d\right.}{6250}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] ((10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3)*x)/15625 - ((4125*d^3 -
 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2)/6250 + ((500*d^3 - 2475*d^2*e + 12
15*d*e^2 + 458*e^3)*x^3)/1875 + (3*e*(100*d^2 - 165*d*e + 27*e^2)*x^4)/500
+ (3*(20*d - 11*e)*e^2*x^5)/125 + (2*e^3*x^6)/15 - ((52875*d^3 + 449175*d^2
*e - 274845*d*e^2 - 53189*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(78125*Sqrt[14])
 + ((57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*Log[3 + 2*x + 5*x^2
])/156250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \int \left(\frac{10125d^3+34350d^2e-13215de^2-5108e^3}{15625} - \frac{\left(4125d^3-6075d^2e\right.}{3}\right.$$

$$= \frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625} - \frac{\left(4125d^3-6075d^2e\right.}{6}$$

$$= \frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625} - \frac{\left(4125d^3-6075d^2e\right.}{6}$$

$$= \frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625} - \frac{\left(4125d^3-6075d^2e\right.}{6}$$

$$= \frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625} - \frac{\left(4125d^3-6075d^2e\right.}{6}$$

Mathematica [A] time = 0.12, size = 178, normalized size = 0.81

$$\frac{42\left(57250d^3-66075d^2e-76620de^2+23431e^3\right)\log\left(5x^2+2x+3\right)+35x\left(250d^3\left(200x^2-495x+486\right)+450d^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x
]
```

```
[Out] (35*x*(250*d^3*(486 - 495*x + 200*x^2) + 450*d^2*e*(916 + 405*x - 550*x^2 +
 250*x^3) + 45*d*e^2*(-3524 + 4580*x + 2700*x^2 - 4125*x^3 + 2000*x^4) + e^
3*(-61296 - 26430*x + 45800*x^2 + 30375*x^3 - 49500*x^4 + 25000*x^5)) - 6*S
qrt[14]*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*ArcTan[(1 + 5
*x)/Sqrt[14]] + 42*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*Log[
3 + 2*x + 5*x^2])/6562500
```

fricas [A] time = 0.84, size = 206, normalized size = 0.93

$$\frac{2}{15}e^3x^6+\frac{3}{125}\left(20\,de^2-11\,e^3\right)x^5+\frac{3}{500}\left(100\,d^2e-165\,de^2+27\,e^3\right)x^4+\frac{1}{1875}\left(500\,d^3-2475\,d^2e+1215\,de^2+458\,e^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] 2/15*e^3*x^6 + 3/125*(20*d*e^2 - 11*e^3)*x^5 + 3/500*(100*d^2*e - 165*d*e^2
 + 27*e^3)*x^4 + 1/1875*(500*d^3 - 2475*d^2*e + 1215*d*e^2 + 458*e^3)*x^3 -
 1/6250*(4125*d^3 - 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2 - 1/1093750*sqrt
(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*arctan(1/14*sqrt
(14)*(5*x + 1)) + 1/15625*(10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3
)*x + 1/156250*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^
2 + 2*x + 3)
```

giac [A] time = 0.17, size = 212, normalized size = 0.96

$$\frac{2}{15}x^6e^3+\frac{12}{25}dx^5e^2+\frac{3}{5}d^2x^4e+\frac{4}{15}d^3x^3-\frac{33}{125}x^5e^3-\frac{99}{100}dx^4e^2-\frac{33}{25}d^2x^3e-\frac{33}{50}d^3x^2+\frac{81}{500}x^4e^3+\frac{81}{125}dx^3e^2+\frac{243}{250}d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] 2/15*x^6*e^3 + 12/25*d*x^5*e^2 + 3/5*d^2*x^4*e + 4/15*d^3*x^3 - 33/125*x^5*
e^3 - 99/100*d*x^4*e^2 - 33/25*d^2*x^3*e - 33/50*d^3*x^2 + 81/500*x^4*e^3 +
 81/125*d*x^3*e^2 + 243/250*d^2*x^2*e + 81/125*d^3*x + 458/1875*x^3*e^3 + 6
87/625*d*x^2*e^2 + 1374/625*d^2*x*e - 881/6250*x^2*e^3 - 2643/3125*d*x*e^2
- 1/1093750*sqrt(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*
arctan(1/14*sqrt(14)*(5*x + 1)) - 5108/15625*x*e^3 + 1/156250*(57250*d^3 -
66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^2 + 2*x + 3)
```

maple [A] time = 0.01, size = 291, normalized size = 1.32

$$\frac{2e^3x^6}{15}+\frac{12d\,e^2x^5}{25}-\frac{33e^3x^5}{125}+\frac{3d^2e\,x^4}{5}-\frac{99d\,e^2x^4}{100}+\frac{81e^3x^4}{500}+\frac{4d^3x^3}{15}-\frac{33d^2e\,x^3}{25}+\frac{81d\,e^2x^3}{125}+\frac{458e^3x^3}{1875}-\frac{33d^3x^2}{50}+\frac{243d^2}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] -881/6250*e^3*x^2+2/15*e^3*x^6+81/500*e^3*x^4-17967/43750*14^(1/2)*arctan(1
/28*(10*x+2)*14^(1/2))*d^2*e+54969/218750*14^(1/2)*arctan(1/28*(10*x+2)*14^
(1/2))*d*e^2+4/15*x^3*d^3-5108/15625*x*e^3-33/125*x^5*e^3-33/50*x^2*d^3+458
/1875*x^3*e^3+229/625*ln(5*x^2+2*x+3)*d^3+23431/156250*ln(5*x^2+2*x+3)*e^3+
81/125*d^3*x-2643/3125*x*d*e^2-423/8750*14^(1/2)*arctan(1/28*(10*x+2)*14^(1
/2))*d^3+53189/1093750*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^3+3/5*x^4*
d^2*e-99/100*x^4*d*e^2-33/25*x^3*d^2*e+687/625*x^2*d*e^2+1374/625*x*d^2*e+8
1/125*x^3*d*e^2+243/250*x^2*d^2*e-7662/15625*ln(5*x^2+2*x+3)*d*e^2+12/25*x^
5*d*e^2-2643/6250*ln(5*x^2+2*x+3)*d^2*e
```

maxima [A] time = 0.96, size = 206, normalized size = 0.93

$$\frac{2}{15}e^3x^6+\frac{3}{125}\left(20\,de^2-11\,e^3\right)x^5+\frac{3}{500}\left(100\,d^2e-165\,de^2+27\,e^3\right)x^4+\frac{1}{1875}\left(500\,d^3-2475\,d^2e+1215\,de^2+458\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] 2/15*e^3*x^6 + 3/125*(20*d*e^2 - 11*e^3)*x^5 + 3/500*(100*d^2*e - 165*d*e^2
 + 27*e^3)*x^4 + 1/1875*(500*d^3 - 2475*d^2*e + 1215*d*e^2 + 458*e^3)*x^3 -
 1/6250*(4125*d^3 - 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2 - 1/1093750*sqrt
(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*arctan(1/14*sqrt
(14)*(5*x + 1)) + 1/15625*(10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3
)*x + 1/156250*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^
2 + 2*x + 3)
```

mupad [B] time = 4.18, size = 397, normalized size = 1.80

$$x^2\left(\frac{26\,e^2\,\left(12\,d-5\,e\right)}{625}-\frac{33\,e\,\left(4\,d^2-5\,d\,e+e^2\right)}{250}-\frac{3\,d\,e^2}{50}+\frac{3\,d^2\,e}{2}-\frac{33\,d^3}{50}+\frac{622\,e^3}{3125}\right)-x^3\left(\frac{11\,e^2\,\left(12\,d-5\,e\right)}{375}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

```
[Out] x^2*((26*e^2*(12*d - 5*e))/625 - (33*e*(4*d^2 - 5*d*e + e^2))/250 - (3*d*e^
2)/50 + (3*d^2*e)/2 - (33*d^3)/50 + (622*e^3)/3125) - x^3*((11*e^2*(12*d -
5*e))/375 + (2*e*(4*d^2 - 5*d*e + e^2))/25 - (3*d*e^2)/5 + d^2*e - (4*d^3)/
15 - (111*e^3)/625) + x^5*((e^2*(12*d - 5*e))/25 - (8*e^3)/125) - log(2*x +
 5*x^2 + 3)*((7662*d*e^2)/15625 + (2643*d^2*e)/6250 - (229*d^3)/625 - (2343
1*e^3)/156250) - x^4*((e^2*(12*d - 5*e))/50 - (3*e*(4*d^2 - 5*d*e + e^2))/2
0 + (11*e^3)/125) + (2*e^3*x^6)/15 + x*((61*e^2*(12*d - 5*e))/3125 + (3*d*(
d*e + d^2 + 2*e^2))/5 + (156*e*(4*d^2 - 5*d*e + e^2))/625 - (129*d*e^2)/125
 + (3*d^2*e)/5 + (6*d^3)/125 - (7483*e^3)/15625) + (14^(1/2)*atan(((14^(1/2
)*(274845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/1093750 + (14^(1/2
)*x*(274845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/218750)/((54969*
d*e^2)/15625 - (17967*d^2*e)/3125 - (423*d^3)/625 + (53189*e^3)/78125))*(27
4845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/1093750
```

sympy [C] time = 2.58, size = 450, normalized size = 2.04

$$\frac{2e^3x^6}{15}+x^5\left(\frac{12de^2}{25}-\frac{33e^3}{125}\right)+x^4\left(\frac{3d^2e}{5}-\frac{99de^2}{100}+\frac{81e^3}{500}\right)+x^3\left(\frac{4d^3}{15}-\frac{33d^2e}{25}+\frac{81de^2}{125}+\frac{458e^3}{1875}\right)+x^2\left(-\frac{33d^3}{50}+\frac{243d^2}{250}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] 2*e**3*x**6/15 + x**5*(12*d*e**2/25 - 33*e**3/125) + x**4*(3*d**2*e/5 - 99*
d*e**2/100 + 81*e**3/500) + x**3*(4*d**3/15 - 33*d**2*e/25 + 81*d*e**2/125
+ 458*e**3/1875) + x**2*(-33*d**3/50 + 243*d**2*e/250 + 687*d*e**2/625 - 88
1*e**3/6250) + x*(81*d**3/125 + 1374*d**2*e/625 - 2643*d*e**2/3125 - 5108*e
**3/15625) + (229*d**3/625 - 2643*d**2*e/6250 - 7662*d*e**2/15625 + 23431*e
**3/156250 - sqrt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189
*e**3)/2187500)*log(x + (10575*d**3 + 89835*d**2*e - 54969*d*e**2 - 53189*e
**3/5 + sqrt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3
)/5)/(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)) + (229*d**3
/625 - 2643*d**2*e/6250 - 7662*d*e**2/15625 + 23431*e**3/156250 + sqrt(14)*
I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)/2187500)*log(x
+ (10575*d**3 + 89835*d**2*e - 54969*d*e**2 - 53189*e**3/5 - sqrt(14)*I*(52
875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)/5)/(52875*d**3 + 449
175*d**2*e - 274845*d*e**2 - 53189*e**3))
```

3.305 $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2} dx$

Optimal. Leaf size=156

$$\frac{1}{375}x^3\left(100d^2-330de+81e^2\right)-\frac{x^2\left(825d^2-810de-458e^2\right)}{1250}+\frac{\left(5725d^2-4405de-2554e^2\right)\log\left(5x^2+2x+3\right)}{15625}$$

```
[Out] 1/3125*(2025*d^2+4580*d*e-881*e^2)*x-1/1250*(825*d^2-810*d*e-458*e^2)*x^2+1
/375*(100*d^2-330*d*e+81*e^2)*x^3+1/100*(40*d-33*e)*e*x^4+4/25*e^2*x^5+1/15
625*(5725*d^2-4405*d*e-2554*e^2)*ln(5*x^2+2*x+3)-1/218750*(10575*d^2+59890*
d*e-18323*e^2)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1628, 634, 618, 204, 628}

$$\frac{1}{375}x^3\left(100d^2-330de+81e^2\right)-\frac{x^2\left(825d^2-810de-458e^2\right)}{1250}+\frac{\left(5725d^2-4405de-2554e^2\right)\log\left(5x^2+2x+3\right)}{15625}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] ((2025*d^2 + 4580*d*e - 881*e^2)*x)/3125 - ((825*d^2 - 810*d*e - 458*e^2)*x
^2)/1250 + ((100*d^2 - 330*d*e + 81*e^2)*x^3)/375 + ((40*d - 33*e)*e*x^4)/1
00 + (4*e^2*x^5)/25 - ((10575*d^2 + 59890*d*e - 18323*e^2)*ArcTan[(1 + 5*x)
/Sqrt[14]])/(15625*Sqrt[14]) + ((5725*d^2 - 4405*d*e - 2554*e^2)*Log[3 + 2*
x + 5*x^2])/15625
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \int\left(\frac{2025d^2+4580de-881e^2}{3125}-\frac{1}{625}\left(825d^2-810de-458e^2\right)x+\right.$$

$$=\frac{\left(2025d^2+4580de-881e^2\right)x}{3125}-\frac{\left(825d^2-810de-458e^2\right)x^2}{1250}+\frac{1}{375}\left(\right.$$

$$=\frac{\left(2025d^2+4580de-881e^2\right)x}{3125}-\frac{\left(825d^2-810de-458e^2\right)x^2}{1250}+\frac{1}{375}\left(\right.$$

$$=\frac{\left(2025d^2+4580de-881e^2\right)x}{3125}-\frac{\left(825d^2-810de-458e^2\right)x^2}{1250}+\frac{1}{375}\left(\right.$$

$$=\frac{\left(2025d^2+4580de-881e^2\right)x}{3125}-\frac{\left(825d^2-810de-458e^2\right)x^2}{1250}+\frac{1}{375}\left(\right.$$

Mathematica [A] time = 0.08, size = 130, normalized size = 0.83

$$\frac{84\left(5725d^2-4405de-2554e^2\right)\log\left(5x^2+2x+3\right)+35x\left(50d^2\left(200x^2-495x+486\right)+60de\left(250x^3-550x^2+\right.\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x
]
```

```
[Out] (35*x*(50*d^2*(486 - 495*x + 200*x^2) + 60*d*e*(916 + 405*x - 550*x^2 + 250
*x^3) + 3*e^2*(-3524 + 4580*x + 2700*x^2 - 4125*x^3 + 2000*x^4)) - 6*Sqrt[1
4]*(10575*d^2 + 59890*d*e - 18323*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]] + 84*(572
5*d^2 - 4405*d*e - 2554*e^2)*Log[3 + 2*x + 5*x^2])/1312500
```

fricas [A] time = 0.79, size = 141, normalized size = 0.90

$$\frac{4}{25}e^2x^5+\frac{1}{100}\left(40\,de-33\,e^2\right)x^4+\frac{1}{375}\left(100\,d^2-330\,de+81\,e^2\right)x^3-\frac{1}{1250}\left(825\,d^2-810\,de-458\,e^2\right)x^2-\frac{1}{218750}\sqrt{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] 4/25*e^2*x^5 + 1/100*(40*d*e - 33*e^2)*x^4 + 1/375*(100*d^2 - 330*d*e + 81*
e^2)*x^3 - 1/1250*(825*d^2 - 810*d*e - 458*e^2)*x^2 - 1/218750*sqrt(14)*(10
575*d^2 + 59890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/3125*(
2025*d^2 + 4580*d*e - 881*e^2)*x + 1/15625*(5725*d^2 - 4405*d*e - 2554*e^2)
*log(5*x^2 + 2*x + 3)
```

giac [A] time = 0.16, size = 145, normalized size = 0.93

$$\frac{4}{25}x^5e^2+\frac{2}{5}dx^4e+\frac{4}{15}d^2x^3-\frac{33}{100}x^4e^2-\frac{22}{25}dx^3e-\frac{33}{50}d^2x^2+\frac{27}{125}x^3e^2+\frac{81}{125}dx^2e+\frac{81}{125}d^2x+\frac{229}{625}x^2e^2+\frac{916}{625}dxe-\frac{1}{21875}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] 4/25*x^5*e^2 + 2/5*d*x^4*e + 4/15*d^2*x^3 - 33/100*x^4*e^2 - 22/25*d*x^3*e
- 33/50*d^2*x^2 + 27/125*x^3*e^2 + 81/125*d*x^2*e + 81/125*d^2*x + 229/625*
x^2*e^2 + 916/625*d*x*e - 1/218750*sqrt(14)*(10575*d^2 + 59890*d*e - 18323*
e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) - 881/3125*x*e^2 + 1/15625*(5725*d^2 -
 4405*d*e - 2554*e^2)*log(5*x^2 + 2*x + 3)
```

maple [A] time = 0.01, size = 191, normalized size = 1.22

$$\frac{4e^2x^5}{25}+\frac{2de\,x^4}{5}-\frac{33e^2x^4}{100}+\frac{4d^2x^3}{15}-\frac{22de\,x^3}{25}+\frac{27e^2x^3}{125}-\frac{33d^2x^2}{50}+\frac{81de\,x^2}{125}+\frac{229e^2x^2}{625}+\frac{81d^2x}{125}-\frac{423\sqrt{14}\,d^2\arctan\left(\frac{(10}{}\right.}{8750}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] 4/25*e^2*x^5+2/5*x^4*d*e-33/100*x^4*e^2+4/15*x^3*d^2-22/25*x^3*d*e+27/125*e
^2*x^3-33/50*x^2*d^2+81/125*x^2*d*e+229/625*x^2*e^2+81/125*d^2*x+916/625*x*
d*e-881/3125*e^2*x+229/625*ln(5*x^2+2*x+3)*d^2-881/3125*ln(5*x^2+2*x+3)*d*e
-2554/15625*ln(5*x^2+2*x+3)*e^2-423/8750*14^(1/2)*arctan(1/28*(10*x+2)*14^(
1/2))*d^2-5989/21875*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d*e+18323/2187
50*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^2
```

maxima [A] time = 0.96, size = 141, normalized size = 0.90

$$\frac{4}{25}\,e^2x^5+\frac{1}{100}\left(40\,de-33\,e^2\right)x^4+\frac{1}{375}\left(100\,d^2-330\,de+81\,e^2\right)x^3-\frac{1}{1250}\left(825\,d^2-810\,de-458\,e^2\right)x^2-\frac{1}{218750}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] 4/25*e^2*x^5 + 1/100*(40*d*e - 33*e^2)*x^4 + 1/375*(100*d^2 - 330*d*e + 81*
e^2)*x^3 - 1/1250*(825*d^2 - 810*d*e - 458*e^2)*x^2 - 1/218750*sqrt(14)*(10
575*d^2 + 59890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/3125*(
2025*d^2 + 4580*d*e - 881*e^2)*x + 1/15625*(5725*d^2 - 4405*d*e - 2554*e^2)
*log(5*x^2 + 2*x + 3)
```

mupad [B] time = 0.10, size = 223, normalized size = 1.43

$$x\left(\frac{4\,d\,e}{5}+\frac{52\,e\,\left(8\,d-5\,e\right)}{625}+\frac{81\,d^2}{125}+\frac{419\,e^2}{3125}\right)-\ln\left(5\,x^2+2\,x+3\right)\left(-\frac{229\,d^2}{625}+\frac{881\,d\,e}{3125}+\frac{2554\,e^2}{15625}\right)+x^4\left(\frac{e\,\left(8\,d-\right.}{20}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

```
[Out] x*((4*d*e)/5 + (52*e*(8*d - 5*e))/625 + (81*d^2)/125 + (419*e^2)/3125) - lo
g(2*x + 5*x^2 + 3)*((881*d*e)/3125 - (229*d^2)/625 + (2554*e^2)/15625) + x^
4*((e*(8*d - 5*e))/20 - (2*e^2)/25) - x^3*((2*d*e)/3 + (2*e*(8*d - 5*e))/75
 - (4*d^2)/15 - (31*e^2)/375) + x^2*(d*e - (11*e*(8*d - 5*e))/250 - (33*d^2
)/50 + (183*e^2)/1250) + (4*e^2*x^5)/25 - (14^(1/2)*atan(((14^(1/2)*(59890*
d*e + 10575*d^2 - 18323*e^2))/218750 + (14^(1/2)*x*(59890*d*e + 10575*d^2 -
 18323*e^2))/43750)/((11978*d*e)/3125 + (423*d^2)/625 - (18323*e^2)/15625))
*(59890*d*e + 10575*d^2 - 18323*e^2))/218750
```

sympy [C] time = 1.72, size = 303, normalized size = 1.94

$$\frac{4e^2x^5}{25}+x^4\left(\frac{2de}{5}-\frac{33e^2}{100}\right)+x^3\left(\frac{4d^2}{15}-\frac{22de}{25}+\frac{27e^2}{125}\right)+x^2\left(-\frac{33d^2}{50}+\frac{81de}{125}+\frac{229e^2}{625}\right)+x\left(\frac{81d^2}{125}+\frac{916de}{625}-\frac{881e^2}{3125}\right)+\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] 4*e**2*x**5/25 + x**4*(2*d*e/5 - 33*e**2/100) + x**3*(4*d**2/15 - 22*d*e/25
 + 27*e**2/125) + x**2*(-33*d**2/50 + 81*d*e/125 + 229*e**2/625) + x*(81*d*
*2/125 + 916*d*e/625 - 881*e**2/3125) + (229*d**2/625 - 881*d*e/3125 - 2554
*e**2/15625 - sqrt(14)*I*(10575*d**2 + 59890*d*e - 18323*e**2)/437500)*log(
x + (2115*d**2 + 11978*d*e - 18323*e**2/5 + sqrt(14)*I*(10575*d**2 + 59890*
d*e - 18323*e**2)/5)/(10575*d**2 + 59890*d*e - 18323*e**2)) + (229*d**2/625
 - 881*d*e/3125 - 2554*e**2/15625 + sqrt(14)*I*(10575*d**2 + 59890*d*e - 18
323*e**2)/437500)*log(x + (2115*d**2 + 11978*d*e - 18323*e**2/5 - sqrt(14)*
I*(10575*d**2 + 59890*d*e - 18323*e**2)/5)/(10575*d**2 + 59890*d*e - 18323*
e**2))
```

3.306 $\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2} \, dx$

Optimal. Leaf size=99

$$\frac{1}{75}x^3(20d-33e)-\frac{3}{250}x^2(55d-27e)+\frac{(2290d-881e)\log\left(5x^2+2x+3\right)}{6250}+\frac{1}{625}x(405d+458e)-\frac{(2115d+5989e)\text{ t}}{3125\sqrt{1}}$$

```
[Out] 1/625*(405*d+458*e)*x-3/250*(55*d-27*e)*x^2+1/75*(20*d-33*e)*x^3+1/5*e*x^4+
1/6250*(2290*d-881*e)*ln(5*x^2+2*x+3)-1/43750*(2115*d+5989*e)*arctan(1/14*(
1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {1628, 634, 618, 204, 628}

$$\frac{1}{75}x^3(20d-33e)-\frac{3}{250}x^2(55d-27e)+\frac{(2290d-881e)\log\left(5x^2+2x+3\right)}{6250}+\frac{1}{625}x(405d+458e)-\frac{(2115d+5989e)\text{ t}}{3125\sqrt{1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] ((405*d + 458*e)*x)/625 - (3*(55*d - 27*e)*x^2)/250 + ((20*d - 33*e)*x^3)/7
5 + (e*x^4)/5 - ((2115*d + 5989*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(3125*Sqrt[1
4]) + ((2290*d - 881*e)*Log[3 + 2*x + 5*x^2])/6250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \int\left(\frac{1}{625}(405d+458e)-\frac{3}{125}(55d-27e)x+\frac{1}{25}(20d-33e)x^2+\frac{4ex^3}{5}\right.$$

$$=\frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2+\frac{1}{75}(20d-33e)x^3+\frac{ex^4}{5}+$$

$$=\frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2+\frac{1}{75}(20d-33e)x^3+\frac{ex^4}{5}+$$

$$=\frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2+\frac{1}{75}(20d-33e)x^3+\frac{ex^4}{5}+$$

$$=\frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2+\frac{1}{75}(20d-33e)x^3+\frac{ex^4}{5}-$$

Mathematica [A] time = 0.05, size = 86, normalized size = 0.87

$$\frac{21(2290d-881e)\log\left(5x^2+2x+3\right)+35x\left(5d\left(200x^2-495x+486\right)+3e\left(250x^3-550x^2+405x+916\right)\right)-3\sqrt{}}{131250}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] (35*x*(5*d*(486 - 495*x + 200*x^2) + 3*e*(916 + 405*x - 550*x^2 + 250*x^3))
 - 3*Sqrt[14]*(2115*d + 5989*e)*ArcTan[(1 + 5*x)/Sqrt[14]] + 21*(2290*d - 8
81*e)*Log[3 + 2*x + 5*x^2])/131250
```

fricas [A] time = 0.78, size = 84, normalized size = 0.85

$$\frac{1}{5}ex^4+\frac{1}{75}(20d-33e)x^3-\frac{3}{250}(55d-27e)x^2-\frac{1}{43750}\sqrt{14}(2115d+5989e)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)+\frac{1}{625}(405$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="frica
s")
```

```
[Out] 1/5*e*x^4 + 1/75*(20*d - 33*e)*x^3 - 3/250*(55*d - 27*e)*x^2 - 1/43750*sqrt
(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(405*d + 458
*e)*x + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

giac [A] time = 0.16, size = 88, normalized size = 0.89

$$\frac{1}{5}x^4e+\frac{4}{15}dx^3-\frac{11}{25}x^3e-\frac{33}{50}dx^2+\frac{81}{250}x^2e-\frac{1}{43750}\sqrt{14}(2115d+5989e)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)+\frac{81}{125}dx+\frac{458}{625}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="giac"
)
```

```
[Out] 1/5*x^4*e + 4/15*d*x^3 - 11/25*x^3*e - 33/50*d*x^2 + 81/250*x^2*e - 1/43750
*sqrt(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 81/125*d*x +
458/625*x*e + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

maple [A] time = 0.00, size = 102, normalized size = 1.03

$$\frac{ex^4}{5}+\frac{4dx^3}{15}-\frac{11ex^3}{25}-\frac{33dx^2}{50}+\frac{81ex^2}{250}+\frac{81dx}{125}-\frac{423\sqrt{14}\,d\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{8750}+\frac{229d\ln\left(5x^2+2x+3\right)}{625}+\frac{458ex}{625}-\frac{598}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] 1/5*e*x^4+4/15*x^3*d-11/25*x^3*e-33/50*x^2*d+81/250*e*x^2+81/125*d*x+458/62
5*e*x+229/625*ln(5*x^2+2*x+3)*d-881/6250*e*ln(5*x^2+2*x+3)-423/8750*14^(1/2
)*arctan(1/28*(10*x+2)*14^(1/2))*d-5989/43750*14^(1/2)*arctan(1/28*(10*x+2)
*14^(1/2))*e
```

maxima [A] time = 0.96, size = 84, normalized size = 0.85

$$\frac{1}{5}\,ex^4+\frac{1}{75}\,(20\,d-33\,e)x^3-\frac{3}{250}\,(55\,d-27\,e)x^2-\frac{1}{43750}\,\sqrt{14}\,(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{1}{625}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="maxim
a")
```

```
[Out] 1/5*e*x^4 + 1/75*(20*d - 33*e)*x^3 - 3/250*(55*d - 27*e)*x^2 - 1/43750*sqrt
(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(405*d + 458
*e)*x + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

mupad [B] time = 0.07, size = 107, normalized size = 1.08

$$x^3\,\left(\frac{4\,d}{15}-\frac{11\,e}{25}\right)-x^2\,\left(\frac{33\,d}{50}-\frac{81\,e}{250}\right)+\ln\left(5\,x^2+2\,x+3\right)\,\left(\frac{229\,d}{625}-\frac{881\,e}{6250}\right)+\frac{e\,x^4}{5}+x\,\left(\frac{81\,d}{125}+\frac{458\,e}{625}\right)-\frac{\sqrt{14}\,\mathrm{atan}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

```
[Out] x^3*((4*d)/15 - (11*e)/25) - x^2*((33*d)/50 - (81*e)/250) + log(2*x + 5*x^2
 + 3)*((229*d)/625 - (881*e)/6250) + (e*x^4)/5 + x*((81*d)/125 + (458*e)/62
5) - (14^(1/2)*atan(((14^(1/2)*(2115*d + 5989*e))/43750 + (14^(1/2)*x*(2115
*d + 5989*e))/8750)/((423*d)/625 + (5989*e)/3125))*(2115*d + 5989*e))/43750
```

sympy [C] time = 0.85, size = 163, normalized size = 1.65

$$\frac{ex^4}{5}+x^3\left(\frac{4d}{15}-\frac{11e}{25}\right)+x^2\left(-\frac{33d}{50}+\frac{81e}{250}\right)+x\left(\frac{81d}{125}+\frac{458e}{625}\right)+\left(\frac{229d}{625}-\frac{881e}{6250}-\frac{\sqrt{14}\,i\,(2115d+5989e)}{87500}\right)\log\left(x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] e*x**4/5 + x**3*(4*d/15 - 11*e/25) + x**2*(-33*d/50 + 81*e/250) + x*(81*d/1
25 + 458*e/625) + (229*d/625 - 881*e/6250 - sqrt(14)*I*(2115*d + 5989*e)/87
500)*log(x + (423*d + 5989*e/5 + sqrt(14)*I*(2115*d + 5989*e)/5)/(2115*d +
5989*e)) + (229*d/625 - 881*e/6250 + sqrt(14)*I*(2115*d + 5989*e)/87500)*lo
g(x + (423*d + 5989*e/5 - sqrt(14)*I*(2115*d + 5989*e)/5)/(2115*d + 5989*e)
)
```

3.307 $\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2} \, dx$

Optimal. Leaf size=56

$$\frac{4x^3}{15} - \frac{33x^2}{50} + \frac{229}{625}\log\left(5x^2+2x+3\right) + \frac{81x}{125} - \frac{423\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{625\sqrt{14}}$$

```
[Out] 81/125*x-33/50*x^2+4/15*x^3+229/625*ln(5*x^2+2*x+3)-423/8750*arctan(1/14*(1
+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1657, 634, 618, 204, 628}

$$\frac{4x^3}{15} - \frac{33x^2}{50} + \frac{229}{625}\log\left(5x^2+2x+3\right) + \frac{81x}{125} - \frac{423\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{625\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2),x]
```

```
[Out] (81*x)/125 - (33*x^2)/50 + (4*x^3)/15 - (423*ArcTan[(1 + 5*x)/Sqrt[14]])/(6
25*Sqrt[14]) + (229*Log[3 + 2*x + 5*x^2])/625
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx &= \int \left(\frac{81}{125}-\frac{33x}{25}+\frac{4x^2}{5}+\frac{7+458x}{125\left(3+2x+5x^2\right)}\right)dx \\
&= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}+\frac{1}{125}\int \frac{7+458x}{3+2x+5x^2}\,dx \\
&= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}+\frac{229}{625}\int \frac{2+10x}{3+2x+5x^2}\,dx-\frac{423}{625}\int \frac{1}{3+2x+5x^2}\,dx \\
&= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}+\frac{229}{625}\log\left(3+2x+5x^2\right)+\frac{846}{625}\,\mathrm{Subst}\left(\int \frac{1}{-56-x^2}\,dx,
\end{aligned}
$$

$$
= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}-\frac{423\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{625\sqrt{14}}+\frac{229}{625}\log\left(3+2x+5x^2\right)
$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.89

$$
\frac{35x\left(200x^2-495x+486\right)+9618\log\left(5x^2+2x+3\right)-1269\sqrt{14}\,\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{26250}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2),x]
```

```
[Out] (35*x*(486 - 495*x + 200*x^2) - 1269*Sqrt[14]*ArcTan[(1 + 5*x)/Sqrt[14]] +
9618*Log[3 + 2*x + 5*x^2])/26250
```

fricas [A] time = 0.65, size = 43, normalized size = 0.77

$$
\frac{4}{15}\,x^3-\frac{33}{50}\,x^2-\frac{423}{8750}\,\sqrt{14}\,\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{81}{125}\,x+\frac{229}{625}\,\log\left(5\,x^2+2\,x+3\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fricas")
```

```
[Out] 4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

giac [A] time = 0.16, size = 43, normalized size = 0.77

$$
\frac{4}{15}\,x^3-\frac{33}{50}\,x^2-\frac{423}{8750}\,\sqrt{14}\,\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{81}{125}\,x+\frac{229}{625}\,\log\left(5\,x^2+2\,x+3\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="giac")
```

```
[Out] 4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

maple [A] time = 0.00, size = 44, normalized size = 0.79

$$
\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{81x}{125}-\frac{423\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{8750}+\frac{229\ln\left(5x^2+2x+3\right)}{625}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] 4/15*x^3-33/50*x^2+81/125*x+229/625*ln(5*x^2+2*x+3)-423/8750*14^(1/2)*arcta
n(1/28*(10*x+2)*14^(1/2))
```

maxima [A] time = 0.97, size = 43, normalized size = 0.77

$$\frac{4}{15}x^3 - \frac{33}{50}x^2 - \frac{423}{8750}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{81}{125}x + \frac{229}{625}\log\left(5x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="maxima")
```

```
[Out] 4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

mupad [B] time = 0.04, size = 45, normalized size = 0.80

$$\frac{81x}{125} + \frac{229\ln\left(5x^2+2x+3\right)}{625} - \frac{423\sqrt{14}\,\mathrm{atan}\left(\frac{5\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{8750} - \frac{33x^2}{50} + \frac{4x^3}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3),x)
```

```
[Out] (81*x)/125 + (229*log(2*x + 5*x^2 + 3))/625 - (423*14^(1/2)*atan((5*14^(1/2
)*x)/14 + 14^(1/2)/14))/8750 - (33*x^2)/50 + (4*x^3)/15
```

sympy [A] time = 0.23, size = 61, normalized size = 1.09

$$\frac{4x^3}{15} - \frac{33x^2}{50} + \frac{81x}{125} + \frac{229\log\left(x^2+\frac{2x}{5}+\frac{3}{5}\right)}{625} - \frac{423\sqrt{14}\,\mathrm{atan}\left(\frac{5\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{8750}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] 4*x**3/15 - 33*x**2/50 + 81*x/125 + 229*log(x**2 + 2*x/5 + 3/5)/625 - 423*s
qrt(14)*atan(5*sqrt(14)*x/14 + sqrt(14)/14)/8750
```

3.308 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)}\,dx$

Optimal. Leaf size=168

$$\frac{(458d-7e)\log\left(5x^2+2x+3\right)}{250\left(5d^2-2de+3e^2\right)}-\frac{(423d-1367e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{125\sqrt{14}\left(5d^2-2de+3e^2\right)}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}-$$

```
[Out] -1/25*(20*d+33*e)*x/e^2+2/5*x^2/e+(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(
e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)+1/250*(458*d-7*e)*ln(5*x^2+2*x+3)/(5*d^2-2*d
*e+3*e^2)-1/1750*(423*d-1367*e)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+
3*e^2)*14^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1628, 634, 618, 204, 628}

$$\frac{(458d-7e)\log\left(5x^2+2x+3\right)}{250\left(5d^2-2de+3e^2\right)}+\frac{\left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}-\frac{(423d-1367e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{125\sqrt{14}\left(5d^2-2de+3e^2\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)),x]
```

```
[Out] -((20*d + 33*e)*x)/(25*e^2) + (2*x^2)/(5*e) - ((423*d - 1367*e)*ArcTan[(1 +
 5*x)/Sqrt[14]])/(125*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)) + ((4*d^4 + 5*d^3*e
 + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)) +
 ((458*d - 7*e)*Log[3 + 2*x + 5*x^2])/(250*(5*d^2 - 2*d*e + 3*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)}\,dx = \int \left(\frac{-20d-33e}{25e^2}+\frac{4x}{5e}+\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2\left(5d^2-2de+3e^2\right)(d+ex)}+\frac{7d+272e+(45}{25\left(5d^2-2de+3e^2\right)}\right.$$

$$= -\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}+\frac{\int \frac{7d+27}{}}{25\left(5d\right.}$$

$$= -\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}-\frac{(423d-}{125}$$

$$= -\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}+\frac{(458d-}{25}$$

$$= -\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}-\frac{(423d-1367e)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{125\sqrt{14}\left(5d^2-2de+3e^2\right)}+\frac{\left(4d^4+5d^3e+3d^2e^2-d\right.}{e^3\left(5d^2-2d\right.}$$

Mathematica [A] time = 0.11, size = 146, normalized size = 0.87

$$\frac{70ex\left(5d^2-2de+3e^2\right)(e(10x-33)-20d)+1750\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)+7e^3(458d-7e)}{1750e^3\left(5d^2-2de+3e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)),x]
```

```
[Out] (70*e*(5*d^2 - 2*d*e + 3*e^2)*x*(-20*d + e*(-33 + 10*x)) - Sqrt[14]*(423*d
- 1367*e)*e^3*ArcTan[(1 + 5*x)/Sqrt[14]] + 1750*(4*d^4 + 5*d^3*e + 3*d^2*e^
2 - d*e^3 + 2*e^4)*Log[d + e*x] + 7*(458*d - 7*e)*e^3*Log[3 + 2*x + 5*x^2])
/(1750*e^3*(5*d^2 - 2*d*e + 3*e^2))
```

fricas [A] time = 1.00, size = 171, normalized size = 1.02

$$\frac{700\left(5\,d^2e^2-2\,de^3+3\,e^4\right)x^2-\sqrt{14}\left(423\,de^3-1367\,e^4\right)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)-70\left(100\,d^3e+125\,d^2e^2-6\,de^3\right.}{1750\left(5\,d^2e^3-2\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="frica
s")
```

```
[Out] 1/1750*(700*(5*d^2*e^2 - 2*d*e^3 + 3*e^4)*x^2 - sqrt(14)*(423*d*e^3 - 1367*
e^4)*arctan(1/14*sqrt(14)*(5*x + 1)) - 70*(100*d^3*e + 125*d^2*e^2 - 6*d*e^
3 + 99*e^4)*x + 1750*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e*x
+ d) + 7*(458*d*e^3 - 7*e^4)*log(5*x^2 + 2*x + 3))/(5*d^2*e^3 - 2*d*e^4 + 3
*e^5)
```

giac [A] time = 0.22, size = 158, normalized size = 0.94

$$\frac{1}{25}\left(10\,x^2e-20\,dx-33\,xe\right)e^{(-2)}-\frac{\sqrt{14}\,(423\,d-1367\,e)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{1750\left(5\,d^2-2\,de+3\,e^2\right)}+\frac{(458\,d-7\,e)\log\left(5\,x^2+2\,x+3\right.}{250\left(5\,d^2-2\,de+3\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="giac"
)
```

```
[Out] 1/25*(10*x^2*e - 20*d*x - 33*x*e)*e^(-2) - 1/1750*sqrt(14)*(423*d - 1367*e)
*arctan(1/14*sqrt(14)*(5*x + 1))/(5*d^2 - 2*d*e + 3*e^2) + 1/250*(458*d - 7
*e)*log(5*x^2 + 2*x + 3)/(5*d^2 - 2*d*e + 3*e^2) + (4*d^4 + 5*d^3*e + 3*d^2
*e^2 - d*e^3 + 2*e^4)*log(abs(x*e + d))/(5*d^2*e^3 - 2*d*e^4 + 3*e^5)
```

maple [A] time = 0.01, size = 298, normalized size = 1.77

$$\frac{4d^4\ln(ex+d)}{(5d^2-2de+3e^2)e^3}+\frac{5d^3\ln(ex+d)}{(5d^2-2de+3e^2)e^2}+\frac{3d^2\ln(ex+d)}{(5d^2-2de+3e^2)e}-\frac{423\sqrt{14}\,d\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{70(125d^2-50de+75e^2)}-\frac{d\ln(ex+d)}{5d^2-2de+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x)
```

```
[Out] 2/5/e*x^2-4/5*d/e^2*x-33/25/e*x+229/5/(125*d^2-50*d*e+75*e^2)*ln(5*x^2+2*x+
3)*d-7/10/(125*d^2-50*d*e+75*e^2)*ln(5*x^2+2*x+3)*e-423/70/(125*d^2-50*d*e+
75*e^2)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d+1367/70/(125*d^2-50*d*e+7
5*e^2)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e+4/e^3/(5*d^2-2*d*e+3*e^2)*
ln(e*x+d)*d^4+5/e^2/(5*d^2-2*d*e+3*e^2)*ln(e*x+d)*d^3+3/e/(5*d^2-2*d*e+3*e^
2)*ln(e*x+d)*d^2-1/(5*d^2-2*d*e+3*e^2)*ln(e*x+d)*d+2*e/(5*d^2-2*d*e+3*e^2)*
ln(e*x+d)
```

maxima [A] time = 0.96, size = 160, normalized size = 0.95

$$-\frac{\sqrt{14}\,(423\,d-1367\,e)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{1750\left(5\,d^2-2\,de+3\,e^2\right)}+\frac{\left(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4\right)\log(ex+d)}{5\,d^2e^3-2\,de^4+3\,e^5}+\frac{(458\,d-7}{250\,(5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="maxim
a")
```

```
[Out] -1/1750*sqrt(14)*(423*d - 1367*e)*arctan(1/14*sqrt(14)*(5*x + 1))/(5*d^2 -
2*d*e + 3*e^2) + (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e*x + d)
/(5*d^2*e^3 - 2*d*e^4 + 3*e^5) + 1/250*(458*d - 7*e)*log(5*x^2 + 2*x + 3)/(
5*d^2 - 2*d*e + 3*e^2) + 1/25*(10*e*x^2 - (20*d + 33*e)*x)/e^2
```

mupad [B] time = 6.39, size = 713, normalized size = 4.24

$$\frac{2\,x^2}{5\,e}-\ln(d+e\,x)\left(\frac{\frac{458\,d}{125}-\frac{7\,e}{125}}{5\,d^2-2\,d\,e+3\,e^2}-\frac{100\,d^2+165\,d\,e+81\,e^2}{125\,e^3}\right)-x\left(\frac{4\,(5\,d+2\,e)}{25\,e^2}+\frac{1}{e}\right)-\frac{\ln\left(\frac{-28\,d^3+1053\,d^2\,e+1791}{25\,e^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)),x)
```

```
[Out] (2*x^2)/(5*e) - log(d + e*x)*(((458*d)/125 - (7*e)/125)/(5*d^2 - 2*d*e + 3*
e^2) - (165*d*e + 100*d^2 + 81*e^2)/(125*e^3)) - x*((4*(5*d + 2*e))/(25*e^2
) + 1/e) - (log((1791*d*e^2 + 1053*d^2*e - 28*d^3 + 916*e^3)/(25*e^2) - (x*
```

```
(321*d*e^2 + 2318*d^2*e + 1832*d^3 - 2249*e^3))/(25*e^2) + ((d*((423*14^(1/
2))/3500 - 229i/125) - e*((1367*14^(1/2))/3500 - 7i/250))*((4751*d*e^3 + 43
50*d^3*e - 1000*d^4 + 874*e^4 + 8490*d^2*e^2)/(25*e^2) + (x*(8200*d*e^3 - 6
250*d^3*e - 5000*d^4 + 2917*e^4 + 1850*d^2*e^2))/(25*e^2) - (((750*e^5 - 14
500*d*e^4 + 1250*d^2*e^3)/(25*e^2) - (x*(2500*d*e^4 + 10250*e^5 - 6250*d^2*
e^3))/(25*e^2))*(d*((423*14^(1/2))/3500 - 229i/125) - e*((1367*14^(1/2))/35
00 - 7i/250)))/(d^2*5i - d*e*2i + e^2*3i)))/(d^2*5i - d*e*2i + e^2*3i))*(d*
((423*14^(1/2))/3500 - 229i/125) - e*((1367*14^(1/2))/3500 - 7i/250)))/(d^2
*5i - d*e*2i + e^2*3i) + (log((1791*d*e^2 + 1053*d^2*e - 28*d^3 + 916*e^3)/
(25*e^2) - (x*(321*d*e^2 + 2318*d^2*e + 1832*d^3 - 2249*e^3))/(25*e^2) - ((
d*((423*14^(1/2))/3500 + 229i/125) - e*((1367*14^(1/2))/3500 + 7i/250))*((4
751*d*e^3 + 4350*d^3*e - 1000*d^4 + 874*e^4 + 8490*d^2*e^2)/(25*e^2) + (x*(
8200*d*e^3 - 6250*d^3*e - 5000*d^4 + 2917*e^4 + 1850*d^2*e^2))/(25*e^2) + (
((750*e^5 - 14500*d*e^4 + 1250*d^2*e^3)/(25*e^2) - (x*(2500*d*e^4 + 10250*e
^5 - 6250*d^2*e^3))/(25*e^2))*(d*((423*14^(1/2))/3500 + 229i/125) - e*((136
7*14^(1/2))/3500 + 7i/250)))/(d^2*5i - d*e*2i + e^2*3i)))/(d^2*5i - d*e*2i
+ e^2*3i))*(d*((423*14^(1/2))/3500 + 229i/125) - e*((1367*14^(1/2))/3500 +
7i/250)))/(d^2*5i - d*e*2i + e^2*3i)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

3.309 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx$

Optimal. Leaf size=233

$$\frac{(229d^2-7de-136e^2)\log(5x^2+2x+3)}{25(5d^2-2de+3e^2)^2}-\frac{(423d^2-2734de+293e^2)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{14}\,(5d^2-2de+3e^2)^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3}{e^3(5d^2-2de+3e^2)(d}$$

```
[Out] 4/5*x/e^2+(-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e^3/(5*d^2-2*d*e+3*e^2)/(e
*x+d)-(40*d^5+d^4*e+28*d^3*e^2+44*d^2*e^3-2*d*e^4+e^5)*ln(e*x+d)/e^3/(5*d^2
-2*d*e+3*e^2)^2+1/25*(229*d^2-7*d*e-136*e^2)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3
*e^2)^2-1/350*(423*d^2-2734*d*e+293*e^2)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d
^2-2*d*e+3*e^2)^2*14^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1628, 634, 618, 204, 628}

$$\frac{(229d^2-7de-136e^2)\log(5x^2+2x+3)}{25(5d^2-2de+3e^2)^2}-\frac{3d^2e^2+5d^3e+4d^4-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)}-\frac{(28d^3e^2+44d^2e^3+d^4e+40d^5-}{e^3(5d^2-2de+}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)),x]
```

```
[Out] (4*x)/(5*e^2) - (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e^3*(5*d^2 -
 2*d*e + 3*e^2)*(d + e*x)) - ((423*d^2 - 2734*d*e + 293*e^2)*ArcTan[(1 + 5*
x)/Sqrt[14]])/(25*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) - ((40*d^5 + d^4*e +
28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e
+ 3*e^2)^2) + ((229*d^2 - 7*d*e - 136*e^2)*Log[3 + 2*x + 5*x^2])/(25*(5*d^2
 - 2*d*e + 3*e^2)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx = \int \left(\frac{4}{5e^2} + \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2(5d^2-2de+3e^2)(d+ex)^2} + \frac{-40d^5-d^4e-28d^3e^2-44d^2e^3+2}{e^2(5d^2-2de+3e^2)^2(d+e}\right.$$

$$= \frac{4x}{5e^2} - \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} - \frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4}{e^3(5d^2-2de+3e^2)^2}$$

$$= \frac{4x}{5e^2} - \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} - \frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4}{e^3(5d^2-2de+3e^2)^2}$$

$$= \frac{4x}{5e^2} - \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} - \frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4}{e^3(5d^2-2de+3e^2)^2}$$

$$= \frac{4x}{5e^2} - \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} - \frac{(423d^2-2734de+293e^2)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right.}{25\sqrt{14}(5d^2-2de+3e^2)^2}$$

Mathematica [A] time = 0.16, size = 233, normalized size = 1.00

$$\frac{(229d^2-7de-136e^2)\log(5x^2+2x+3)}{25(5d^2-2de+3e^2)^2} + \frac{(-423d^2+2734de-293e^2)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{14}(5d^2-2de+3e^2)^2} + \frac{-4d^4-5d^3e-3d^2e^2+de^3}{e^3(5d^2-2de+3e^2)(d+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)),x
]
```

```
[Out] (4*x)/(5*e^2) + (-4*d^4 - 5*d^3*e - 3*d^2*e^2 + d*e^3 - 2*e^4)/(e^3*(5*d^2
- 2*d*e + 3*e^2)*(d + e*x)) + ((-423*d^2 + 2734*d*e - 293*e^2)*ArcTan[(1 +
5*x)/Sqrt[14]])/(25*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) + ((-40*d^5 - d^4*e
 - 28*d^3*e^2 - 44*d^2*e^3 + 2*d*e^4 - e^5)*Log[d + e*x])/(e^3*(5*d^2 - 2*d
*e + 3*e^2)^2) + ((229*d^2 - 7*d*e - 136*e^2)*Log[3 + 2*x + 5*x^2])/(25*(5*
d^2 - 2*d*e + 3*e^2)^2)
```

fricas [A] time = 1.05, size = 416, normalized size = 1.79

$$-\frac{7000\,d^6+5950\,d^5e+5950\,d^4e^2+1400\,d^3e^3+7350\,d^2e^4-2450\,de^5+2100\,e^6-280\left(25\,d^4e^2-20\,d^3e^3+34\,d^2e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] -1/350*(7000*d^6 + 5950*d^5*e + 5950*d^4*e^2 + 1400*d^3*e^3 + 7350*d^2*e^4
- 2450*d*e^5 + 2100*e^6 - 280*(25*d^4*e^2 - 20*d^3*e^3 + 34*d^2*e^4 - 12*d*
e^5 + 9*e^6)*x^2 + sqrt(14)*(423*d^3*e^3 - 2734*d^2*e^4 + 293*d*e^5 + (423*
d^2*e^4 - 2734*d*e^5 + 293*e^6)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) - 280*(2
```

```
5*d^5*e - 20*d^4*e^2 + 34*d^3*e^3 - 12*d^2*e^4 + 9*d*e^5)*x + 350*(40*d^6 +
 d^5*e + 28*d^4*e^2 + 44*d^3*e^3 - 2*d^2*e^4 + d*e^5 + (40*d^5*e + d^4*e^2
+ 28*d^3*e^3 + 44*d^2*e^4 - 2*d*e^5 + e^6)*x)*log(e*x + d) - 14*(229*d^3*e^
3 - 7*d^2*e^4 - 136*d*e^5 + (229*d^2*e^4 - 7*d*e^5 - 136*e^6)*x)*log(5*x^2
+ 2*x + 3))/(25*d^5*e^3 - 20*d^4*e^4 + 34*d^3*e^5 - 12*d^2*e^6 + 9*d*e^7 +
(25*d^4*e^4 - 20*d^3*e^5 + 34*d^2*e^6 - 12*d*e^7 + 9*e^8)*x)
```

giac [A] time = 0.18, size = 355, normalized size = 1.52

$$\frac{1}{25}(40d+33e)e^{(-3)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)-\frac{\sqrt{14}\left(423d^2e^2-2734de^3+293e^4\right)\arctan\left(\frac{1}{14}\sqrt{14}\left(5d-\frac{5d^2}{xe+d}+\frac{2de}{xe+d}\right.\right.}{350\left(25d^4-20d^3e+34d^2e^2-12de^3+9e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] 1/25*(40*d + 33*e)*e^(-3)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) - 1/350*sqrt
(14)*(423*d^2*e^2 - 2734*d*e^3 + 293*e^4)*arctan(1/14*sqrt(14)*(5*d - 5*d^2
/(x*e + d) + 2*d*e/(x*e + d) - 3*e^2/(x*e + d) - e)*e^(-1))*e^(-2)/(25*d^4
- 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) + 4/5*(x*e + d)*e^(-3) + 1/25*(
229*d^2 - 7*d*e - 136*e^2)*log(-10*d/(x*e + d) + 5*d^2/(x*e + d)^2 + 2*e/(x
*e + d) - 2*d*e/(x*e + d)^2 + 3*e^2/(x*e + d)^2 + 5)/(25*d^4 - 20*d^3*e + 3
4*d^2*e^2 - 12*d*e^3 + 9*e^4) - (4*d^4*e^3/(x*e + d) + 5*d^3*e^4/(x*e + d)
+ 3*d^2*e^5/(x*e + d) - d*e^6/(x*e + d) + 2*e^7/(x*e + d))/(5*d^2*e^6 - 2*d
*e^7 + 3*e^8)
```

maple [B] time = 0.02, size = 538, normalized size = 2.31

$$-\frac{40d^5\ln(ex+d)}{\left(5d^2-2de+3e^2\right)^2e^3}-\frac{d^4\ln(ex+d)}{\left(5d^2-2de+3e^2\right)^2e^2}-\frac{28d^3\ln(ex+d)}{\left(5d^2-2de+3e^2\right)^2e}-\frac{423\sqrt{14}\,d^2\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{350\left(5d^2-2de+3e^2\right)^2}-\frac{44d^2}{\left(5d^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x)
```

```
[Out] 4/5/e^2*x+229/25/(5*d^2-2*d*e+3*e^2)^2*ln(5*x^2+2*x+3)*d^2-7/25/(5*d^2-2*d*
e+3*e^2)^2*ln(5*x^2+2*x+3)*d*e-136/25/(5*d^2-2*d*e+3*e^2)^2*ln(5*x^2+2*x+3)
*e^2-423/350/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*
d^2+1367/175/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*
d*e-293/350/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e
^2-4/e^3/(5*d^2-2*d*e+3*e^2)/(e*x+d)*d^4-5/e^2/(5*d^2-2*d*e+3*e^2)/(e*x+d)*
d^3-3/e/(5*d^2-2*d*e+3*e^2)/(e*x+d)*d^2+1/(5*d^2-2*d*e+3*e^2)/(e*x+d)*d-2*e
/(5*d^2-2*d*e+3*e^2)/(e*x+d)-40/e^3/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)*d^5-1/e
^2/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)*d^4-28/e/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)
*d^3-44/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)*d^2+2*e/(5*d^2-2*d*e+3*e^2)^2*ln(e*
x+d)*d-e^2/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)
```

maxima [A] time = 0.98, size = 294, normalized size = 1.26

$$-\frac{\sqrt{14}\left(423d^2-2734de+293e^2\right)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)}{350\left(25d^4-20d^3e+34d^2e^2-12de^3+9e^4\right)}-\frac{\left(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5\right)\log}{25d^4e^3-20d^3e^4+34d^2e^5-12de^6+9\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] -1/350*sqrt(14)*(423*d^2 - 2734*d*e + 293*e^2)*arctan(1/14*sqrt(14)*(5*x +
1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) - (40*d^5 + d^4*e +
 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5)*log(e*x + d)/(25*d^4*e^3 - 20*d^3
*e^4 + 34*d^2*e^5 - 12*d*e^6 + 9*e^7) + 1/25*(229*d^2 - 7*d*e - 136*e^2)*lo
g(5*x^2 + 2*x + 3)/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) - (4
*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(5*d^3*e^3 - 2*d^2*e^4 + 3*d*e^
5 + (5*d^2*e^4 - 2*d*e^5 + 3*e^6)*x) + 4/5*x/e^2
```

mupad [B] time = 4.67, size = 312, normalized size = 1.34

$$\frac{4\,x}{5\,e^2}-\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\left(\left(\frac{423\,\sqrt{14}}{700}-\frac{229}{25}\mathrm{i}\right)\,d^2+\left(-\frac{1367\,\sqrt{14}}{350}+\frac{7}{25}\mathrm{i}\right)\,d\,e+\left(\frac{293\,\sqrt{14}}{700}+\frac{136}{25}\mathrm{i}\right)\,e^2\right)}{d^4\,25\mathrm{i}-d^3\,e\,20\mathrm{i}+d^2\,e^2\,34\mathrm{i}-d\,e^3\,12\mathrm{i}+e^4\,9\mathrm{i}}+\frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)),x)
```

```
[Out] (4*x)/(5*e^2) - (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^2*((423*14^(1/2))/700 -
229i/25) + e^2*((293*14^(1/2))/700 + 136i/25) - d*e*((1367*14^(1/2))/350 -
7i/25)))/(d^4*25i - d^3*e*20i - d*e^3*12i + e^4*9i + d^2*e^2*34i) + (log(x
+ (14^(1/2)*1i)/5 + 1/5)*(d^2*((423*14^(1/2))/700 + 229i/25) + e^2*((293*14
^(1/2))/700 - 136i/25) - d*e*((1367*14^(1/2))/350 + 7i/25)))/(d^4*25i - d^3
*e*20i - d*e^3*12i + e^4*9i + d^2*e^2*34i) - (5*(5*d^3*e - d*e^3 + 4*d^4 +
2*e^4 + 3*d^2*e^2))/(e*(5*d*e^2 + 5*e^3*x)*(5*d^2 - 2*d*e + 3*e^2)) - (log(
d + e*x)*(d^4*e - 2*d*e^4 + 40*d^5 + e^5 + 44*d^2*e^3 + 28*d^3*e^2))/(e^3*(
5*d^2 - 2*d*e + 3*e^2)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

3.310 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)} dx$

Optimal. Leaf size=317

$$\frac{\left(458d^3-21d^2e-816de^2+113e^3\right)\log\left(5x^2+2x+3\right)}{10\left(5d^2-2de+3e^2\right)^3}-\frac{\left(423d^3-4101d^2e+879de^2+703e^3\right)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{5\sqrt{14}\left(5d^2-2de+3e^2\right)^3}-\frac{4d}{2e}$$

```
[Out] 1/2*(-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e^3/(5*d^2-2*d*e+3*e^2)/(e*x+d)^
2+(40*d^5+d^4*e+28*d^3*e^2+44*d^2*e^3-2*d*e^4+e^5)/e^3/(5*d^2-2*d*e+3*e^2)^
2/(e*x+d)+(100*d^6-120*d^5*e+228*d^4*e^2-242*d^3*e^3+141*d^2*e^4+120*d*e^5-
e^6)*ln(e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)^3+1/10*(458*d^3-21*d^2*e-816*d*e^2+1
13*e^3)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^3-1/70*(423*d^3-4101*d^2*e+879*
d*e^2+703*e^3)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1628, 634, 618, 204, 628}

$$\frac{\left(-21d^2e+458d^3-816de^2+113e^3\right)\log\left(5x^2+2x+3\right)}{10\left(5d^2-2de+3e^2\right)^3}-\frac{3d^2e^2+5d^3e+4d^4-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2}+\frac{28d^3e^2+44d^2e^3+d}{e^3\left(5d^2-2de\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)),x]
```

```
[Out] -(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(2*e^3*(5*d^2 - 2*d*e + 3*e^
2)*(d + e*x)^2) + (40*d^5 + d^4*e + 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5
)/(e^3*(5*d^2 - 2*d*e + 3*e^2)^2*(d + e*x)) - ((423*d^3 - 4101*d^2*e + 879*
d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(5*Sqrt[14]*(5*d^2 - 2*d*e + 3
*e^2)^3) + ((100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d^3*e^3 + 141*d^2*e^4
+ 120*d*e^5 - e^6)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)^3) + ((458*d^
3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*Log[3 + 2*x + 5*x^2])/(10*(5*d^2 - 2*d*
e + 3*e^2)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)}\,dx = \int \left(\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2\left(5d^2-2de+3e^2\right)(d+ex)^3} + \frac{-40d^5-d^4e-28d^3e^2-44d^2e^3+2de^4-}{e^2\left(5d^2-2de+3e^2\right)^2(d+ex)^2}\right.$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2} + \frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)} +$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2} + \frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)} +$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2} + \frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)} +$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2} + \frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)} -$$

Mathematica [A] time = 0.43, size = 278, normalized size = 0.88

$$-\frac{-7\left(458d^3-21d^2e-816de^2+113e^3\right)\log\left(5x^2+2x+3\right)+\sqrt{14}\left(423d^3-4101d^2e+879de^2+703e^3\right)\tan^{-1}\left(\frac{5x+}{\sqrt{1}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)),x
]
```

```
[Out] -1/70*((35*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 +
 2*e^4))/(e^3*(d + e*x)^2) - (70*(5*d^2 - 2*d*e + 3*e^2)*(40*d^5 + d^4*e +
28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5))/(e^3*(d + e*x)) + Sqrt[14]*(423*d
^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + (70*(-1
00*d^6 + 120*d^5*e - 228*d^4*e^2 + 242*d^3*e^3 - 141*d^2*e^4 - 120*d*e^5 +
e^6)*Log[d + e*x])/e^3 - 7*(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*Log[3
 + 2*x + 5*x^2])/(5*d^2 - 2*d*e + 3*e^2)^3
```

fricas [B] time = 1.36, size = 698, normalized size = 2.20

$$\frac{10500\,d^8-6825\,d^7e+14175\,d^6e^2+10395\,d^5e^3-6160\,d^4e^4+12145\,d^3e^5-4305\,d^2e^6+1365\,de^7-630\,e^8-\sqrt{14}\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] 1/70*(10500*d^8 - 6825*d^7*e + 14175*d^6*e^2 + 10395*d^5*e^3 - 6160*d^4*e^4
 + 12145*d^3*e^5 - 4305*d^2*e^6 + 1365*d*e^7 - 630*e^8 - sqrt(14)*(423*d^5*
e^3 - 4101*d^4*e^4 + 879*d^3*e^5 + 703*d^2*e^6 + (423*d^3*e^5 - 4101*d^2*e^
6 + 879*d*e^7 + 703*e^8)*x^2 + 2*(423*d^4*e^4 - 4101*d^3*e^5 + 879*d^2*e^6
+ 703*d*e^7)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 70*(200*d^7*e - 75*d^6*e^
2 + 258*d^5*e^3 + 167*d^4*e^4 - 14*d^3*e^5 + 141*d^2*e^6 - 8*d*e^7 + 3*e^8)
*x + 70*(100*d^8 - 120*d^7*e + 228*d^6*e^2 - 242*d^5*e^3 + 141*d^4*e^4 + 12
0*d^3*e^5 - d^2*e^6 + (100*d^6*e^2 - 120*d^5*e^3 + 228*d^4*e^4 - 242*d^3*e^
5 + 141*d^2*e^6 + 120*d*e^7 - e^8)*x^2 + 2*(100*d^7*e - 120*d^6*e^2 + 228*d
^5*e^3 - 242*d^4*e^4 + 141*d^3*e^5 + 120*d^2*e^6 - d*e^7)*x)*log(e*x + d) +
 7*(458*d^5*e^3 - 21*d^4*e^4 - 816*d^3*e^5 + 113*d^2*e^6 + (458*d^3*e^5 - 2
1*d^2*e^6 - 816*d*e^7 + 113*e^8)*x^2 + 2*(458*d^4*e^4 - 21*d^3*e^5 - 816*d^
2*e^6 + 113*d*e^7)*x)*log(5*x^2 + 2*x + 3))/(125*d^8*e^3 - 150*d^7*e^4 + 28
5*d^6*e^5 - 188*d^5*e^6 + 171*d^4*e^7 - 54*d^3*e^8 + 27*d^2*e^9 + (125*d^6*
e^5 - 150*d^5*e^6 + 285*d^4*e^7 - 188*d^3*e^8 + 171*d^2*e^9 - 54*d*e^10 + 2
7*e^11)*x^2 + 2*(125*d^7*e^4 - 150*d^6*e^5 + 285*d^5*e^6 - 188*d^4*e^7 + 17
1*d^3*e^8 - 54*d^2*e^9 + 27*d*e^10)*x)
```

giac [A] time = 0.18, size = 406, normalized size = 1.28

$$-\frac{\sqrt{14}\left(423\,d^3-4101\,d^2e+879\,de^2+703\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{70\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}+\frac{\left(458\,d^3-21\,d^2e-816\,de^2+\right.}{10\left(125\,d^6-150\,d^5e+285\,d^4e^2-18\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] -1/70*sqrt(14)*(423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*arctan(1/14*sqr
t(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2
*e^4 - 54*d*e^5 + 27*e^6) + 1/10*(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)
*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 17
1*d^2*e^4 - 54*d*e^5 + 27*e^6) + (100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d
^3*e^3 + 141*d^2*e^4 + 120*d*e^5 - e^6)*log(abs(x*e + d))/(125*d^6*e^3 - 15
0*d^5*e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8 + 27*e^9) +
1/2*(2*(200*d^7 - 75*d^6*e + 258*d^5*e^2 + 167*d^4*e^3 - 14*d^3*e^4 + 141*d
^2*e^5 - 8*d*e^6 + 3*e^7)*x + (300*d^8 - 195*d^7*e + 405*d^6*e^2 + 297*d^5*
e^3 - 176*d^4*e^4 + 347*d^3*e^5 - 123*d^2*e^6 + 39*d*e^7 - 18*e^8)*e^(-1))*
e^(-2)/((5*d^2 - 2*d*e + 3*e^2)^3*(x*e + d)^2)
```

maple [B] time = 0.02, size = 819, normalized size = 2.58

$$\frac{100d^6\ln(ex+d)}{\left(5d^2-2de+3e^2\right)^3e^3}-\frac{120d^5\ln(ex+d)}{\left(5d^2-2de+3e^2\right)^3e^2}+\frac{228d^4\ln(ex+d)}{\left(5d^2-2de+3e^2\right)^3e}-\frac{423\sqrt{14}\,d^3\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{70\left(5d^2-2de+3e^2\right)^3}-\frac{242d^3\ln}{\left(5d^2-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x)
```

```
[Out] -423/70/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^3-7
03/70/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^3-2/e
^3/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2*d^4-5/2/e^2/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2*
d^3-3/2/e/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2*d^2+40/e^3/(5*d^2-2*d*e+3*e^2)^2/(e
*x+d)*d^5+1/e^2/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)*d^4-2*e/(5*d^2-2*d*e+3*e^2)^2
/(e*x+d)*d-21/10/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d^2*e-408/5/(5*d^2-2
*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d*e^2+120/(5*d^2-2*d*e+3*e^2)^3*e^2*ln(e*x+d)
*d+100/(5*d^2-2*d*e+3*e^2)^3/e^3*ln(e*x+d)*d^6-120/(5*d^2-2*d*e+3*e^2)^3/e^
2*ln(e*x+d)*d^5+228/(5*d^2-2*d*e+3*e^2)^3/e*ln(e*x+d)*d^4+141/(5*d^2-2*d*e+
```

```
3*e^2)^3*e*ln(e*x+d)*d^2+28/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)*d^3+44/(5*d^2-2
*d*e+3*e^2)^2/(e*x+d)*d^2+1/2/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2*d-e/(5*d^2-2*d*
e+3*e^2)/(e*x+d)^2+e^2/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)-242/(5*d^2-2*d*e+3*e^2
)^3*ln(e*x+d)*d^3-1/(5*d^2-2*d*e+3*e^2)^3*e^3*ln(e*x+d)+229/5/(5*d^2-2*d*e+
3*e^2)^3*ln(5*x^2+2*x+3)*d^3+113/10/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*e
^3+4101/70/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^
2*e-879/70/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d*
e^2
```

maxima [A] time = 1.00, size = 498, normalized size = 1.57

$$-\frac{\sqrt{14}\left(423\,d^3-4101\,d^2e+879\,de^2+703\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{70\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}+\frac{\left(100\,d^6-120\,d^5e+228\,d^4e^2-242\,d^3e\right.}{125\,d^6e^3-150\,d^5e^4+285\,d^4e^5-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] -1/70*sqrt(14)*(423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*arctan(1/14*sqr
t(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2
*e^4 - 54*d*e^5 + 27*e^6) + (100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d^3*e^
3 + 141*d^2*e^4 + 120*d*e^5 - e^6)*log(e*x + d)/(125*d^6*e^3 - 150*d^5*e^4
+ 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8 + 27*e^9) + 1/10*(458*
d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d
^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + 1/2*(
60*d^6 - 15*d^5*e + 39*d^4*e^2 + 84*d^3*e^3 - 25*d^2*e^4 + 9*d*e^5 - 6*e^6
+ 2*(40*d^5*e + d^4*e^2 + 28*d^3*e^3 + 44*d^2*e^4 - 2*d*e^5 + e^6)*x)/(25*d
^6*e^3 - 20*d^5*e^4 + 34*d^4*e^5 - 12*d^3*e^6 + 9*d^2*e^7 + (25*d^4*e^5 - 2
0*d^3*e^6 + 34*d^2*e^7 - 12*d*e^8 + 9*e^9)*x^2 + 2*(25*d^5*e^4 - 20*d^4*e^5
 + 34*d^3*e^6 - 12*d^2*e^7 + 9*d*e^8)*x)
```

mupad [B] time = 4.76, size = 493, normalized size = 1.56

$$\frac{\frac{60\,d^6-15\,d^5\,e+39\,d^4\,e^2+84\,d^3\,e^3-25\,d^2\,e^4+9\,d\,e^5-6\,e^6}{2\,e^3\left(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4\right)}+\frac{x\left(40\,d^5+d^4\,e+28\,d^3\,e^2+44\,d^2\,e^3-2\,d\,e^4+e^5\right)}{e^2\left(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4\right)}}{d^2+2\,d\,e\,x+e^2\,x^2}-\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(\left(\frac{423\,\sqrt{14}}{140}-\frac{229}{5}\mathrm{i}\right.\right.}{d^6\,125\mathrm{i}-d^5\,e\,150}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^3*(2*x + 5*x^2 + 3)),x)
```

```
[Out] ((9*d*e^5 - 15*d^5*e + 60*d^6 - 6*e^6 - 25*d^2*e^4 + 84*d^3*e^3 + 39*d^4*e^
2)/(2*e^3*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x*(d^4*e
- 2*d*e^4 + 40*d^5 + e^5 + 44*d^2*e^3 + 28*d^3*e^2))/(e^2*(25*d^4 - 20*d^3*
e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)))/(d^2 + e^2*x^2 + 2*d*e*x) - (log(x - (
14^(1/2)*1i)/5 + 1/5)*(d^3*((423*14^(1/2))/140 - 229i/5) + e^3*((703*14^(1/
2))/140 - 113i/10) + d*e^2*((879*14^(1/2))/140 + 408i/5) - d^2*e*((4101*14^
(1/2))/140 - 21i/10)))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e
^4*171i - d^3*e^3*188i + d^4*e^2*285i) + (log(x + (14^(1/2)*1i)/5 + 1/5)*(d
^3*((423*14^(1/2))/140 + 229i/5) + e^3*((703*14^(1/2))/140 + 113i/10) + d*e
^2*((879*14^(1/2))/140 - 408i/5) - d^2*e*((4101*14^(1/2))/140 + 21i/10)))/(
d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*188i +
 d^4*e^2*285i) + (log(d + e*x)*(120*d*e^5 - 120*d^5*e + 100*d^6 - e^6 + 141
*d^2*e^4 - 242*d^3*e^3 + 228*d^4*e^2))/(e^3*(5*d^2 - 2*d*e + 3*e^2)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

3.311 $\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$

Optimal. Leaf size=189

$$\frac{ex^2\left(840d^2-1722de+373e^2\right)}{3500}-\frac{\left(1025d^3-1545d^2e-2601de^2+832e^3\right)\log\left(5x^2+2x+3\right)}{6250}+\frac{x\left(2800d^3-17220d^2\right.}{17}$$

```
[Out] 1/17500*(2800*d^3-17220*d^2*e+9921*d*e^2+6053*e^3)*x+1/3500*e*(840*d^2-1722
*d*e+373*e^2)*x^2+1/375*(60*d-41*e)*e^2*x^3+1/25*e^3*x^4-1/3500*(1367+423*x
)*(e*x+d)^3/(5*x^2+2*x+3)-1/6250*(1025*d^3-1545*d^2*e-2601*d*e^2+832*e^3)*l
n(5*x^2+2*x+3)+1/1225000*(32825*d^3+317565*d^2*e-221643*d*e^2-67499*e^3)*ar
ctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1644, 1628, 634, 618, 204, 628}

$$\frac{ex^2\left(840d^2-1722de+373e^2\right)}{3500}-\frac{\left(-1545d^2e+1025d^3-2601de^2+832e^3\right)\log\left(5x^2+2x+3\right)}{6250}+x\left(-17220d^2e+28\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] ((2800*d^3 - 17220*d^2*e + 9921*d*e^2 + 6053*e^3)*x)/17500 + (e*(840*d^2 -
1722*d*e + 373*e^2)*x^2)/3500 + ((60*d - 41*e)*e^2*x^3)/375 + (e^3*x^4)/25
- ((1367 + 423*x)*(d + e*x)^3)/(3500*(3 + 2*x + 5*x^2)) + ((32825*d^3 + 317
565*d^2*e - 221643*d*e^2 - 67499*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(87500*Sq
rt[14]) - ((1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)*Log[3 + 2*x + 5*x
^2])/6250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx = -\frac{(1367+423x)(d+ex)^3}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int \frac{(d+ex)^2\left(\frac{6}{125}(615d+1367e)-\right.}{}$$

$$= -\frac{(1367+423x)(d+ex)^3}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int \left(\frac{2}{625}\left(2800d^3-17220d^2e+99\right.\right.$$

$$= \frac{\left(2800d^3-17220d^2e+9921de^2+6053e^3\right)x}{17500}+\frac{e\left(840d^2-1722de\right.}{3500}$$

$$= \frac{\left(2800d^3-17220d^2e+9921de^2+6053e^3\right)x}{17500}+\frac{e\left(840d^2-1722de\right.}{3500}$$

$$= \frac{\left(2800d^3-17220d^2e+9921de^2+6053e^3\right)x}{17500}+\frac{e\left(840d^2-1722de\right.}{3500}$$

$$= \frac{\left(2800d^3-17220d^2e+9921de^2+6053e^3\right)x}{17500}+\frac{e\left(840d^2-1722de\right.}{3500}$$

Mathematica [A] time = 0.16, size = 209, normalized size = 1.11

$$\frac{14700ex^2\left(300d^2-615de+103e^2\right)-\frac{42\left(125d^3(423x+1367)+75d^2e(5989x-1269)-15de^2(18323x+17967)+e^3(54969-53189x)\right)}{5x^2+2x+3}+29}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2
,x]
```

```
[Out] (5880*(500*d^3 - 3075*d^2*e + 1545*d*e^2 + 867*e^3)*x + 14700*e*(300*d^2 -
615*d*e + 103*e^2)*x^2 + 49000*(60*d - 41*e)*e^2*x^3 + 735000*e^3*x^4 - (42
*(e^3*(54969 - 53189*x) + 125*d^3*(1367 + 423*x) + 75*d^2*e*(-1269 + 5989*x
```

```
) - 15*d*e^2*(17967 + 18323*x)))/(3 + 2*x + 5*x^2) + 15*Sqrt[14]*(32825*d^3
 + 317565*d^2*e - 221643*d*e^2 - 67499*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + 29
40*(-1025*d^3 + 1545*d^2*e + 2601*d*e^2 - 832*e^3)*Log[3 + 2*x + 5*x^2])/18
375000
```

fricas [B] time = 0.87, size = 350, normalized size = 1.85

$$\frac{3675000\,e^3x^6+1225000\left(12\,de^2-7\,e^3\right)x^5+122500\left(180\,d^2e-321\,de^2+47\,e^3\right)x^4+147000\left(100\,d^3-555\,d^2e+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] 1/18375000*(3675000*e^3*x^6 + 1225000*(12*d*e^2 - 7*e^3)*x^5 + 122500*(180*
d^2*e - 321*d*e^2 + 47*e^3)*x^4 + 147000*(100*d^3 - 555*d^2*e + 246*d*e^2 +
 153*e^3)*x^3 - 7176750*d^3 + 3997350*d^2*e + 11319210*d*e^2 - 2308698*e^3
+ 2940*(2000*d^3 - 7800*d^2*e - 3045*d*e^2 + 5013*e^3)*x^2 + 15*sqrt(14)*(9
8475*d^3 + 952695*d^2*e - 664929*d*e^2 - 202497*e^3 + 5*(32825*d^3 + 317565
*d^2*e - 221643*d*e^2 - 67499*e^3)*x^2 + 2*(32825*d^3 + 317565*d^2*e - 2216
43*d*e^2 - 67499*e^3)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 42*(157125*d^3 -
 1740675*d^2*e + 923745*d*e^2 + 417329*e^3)*x - 2940*(3075*d^3 - 4635*d^2*e
 - 7803*d*e^2 + 2496*e^3 + 5*(1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)
*x^2 + 2*(1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)*x)*log(5*x^2 + 2*x
+ 3))/(5*x^2 + 2*x + 3)
```

giac [A] time = 0.16, size = 206, normalized size = 1.09

$$\frac{1}{25}\,x^4e^3+\frac{4}{25}\,dx^3e^2+\frac{6}{25}\,d^2x^2e+\frac{4}{25}\,d^3x-\frac{41}{375}\,x^3e^3-\frac{123}{250}\,dx^2e^2-\frac{123}{125}\,d^2xe+\frac{103}{1250}\,x^2e^3+\frac{309}{625}\,dxe^2+\frac{1}{1225000}\,\sqrt{14}\left(328\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 1/25*x^4*e^3 + 4/25*d*x^3*e^2 + 6/25*d^2*x^2*e + 4/25*d^3*x - 41/375*x^3*e^
3 - 123/250*d*x^2*e^2 - 123/125*d^2*x*e + 103/1250*x^2*e^3 + 309/625*d*x*e^
2 + 1/1225000*sqrt(14)*(32825*d^3 + 317565*d^2*e - 221643*d*e^2 - 67499*e^3
)*arctan(1/14*sqrt(14)*(5*x + 1)) + 867/3125*x*e^3 - 1/6250*(1025*d^3 - 154
5*d^2*e - 2601*d*e^2 + 832*e^3)*log(5*x^2 + 2*x + 3) - 1/437500*(170875*d^3
 - 95175*d^2*e + (52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*x -
269505*d*e^2 + 54969*e^3)/(5*x^2 + 2*x + 3)
```

maple [A] time = 0.02, size = 283, normalized size = 1.50

$$\frac{e^3x^4}{25}+\frac{4d\,e^2x^3}{25}-\frac{41e^3x^3}{375}+\frac{6d^2e\,x^2}{25}-\frac{123d\,e^2x^2}{250}+\frac{103e^3x^2}{1250}+\frac{4d^3x}{25}+\frac{1313\sqrt{14}\,d^3\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{49000}-\frac{41d^3\ln\left(5x^2+2\right.}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] 1/25*e^3*x^4+4/25*d*e^2*x^3-41/375*e^3*x^3+6/25*d^2*e*x^2-123/250*d*e^2*x^2
+103/1250*e^3*x^2+4/25*d^3*x-123/125*d^2*e*x+309/625*d*e^2*x+867/3125*e^3*x
-1/3125*((2115/28*d^3+17967/28*d^2*e-54969/140*d*e^2-53189/700*e^3)*x+6835/
28*d^3-3807/28*d^2*e-53901/140*d*e^2+54969/700*e^3)/(x^2+2/5*x+3/5)-41/250*
d^3*ln(5*x^2+2*x+3)+309/1250*d^2*e*ln(5*x^2+2*x+3)+2601/6250*d*e^2*ln(5*x^2
```

```
+2*x+3)-416/3125*e^3*ln(5*x^2+2*x+3)+1313/49000*14^(1/2)*d^3*arctan(1/28*(1
0*x+2)*14^(1/2))+63513/245000*14^(1/2)*d^2*e*arctan(1/28*(10*x+2)*14^(1/2))
-221643/1225000*14^(1/2)*d*e^2*arctan(1/28*(10*x+2)*14^(1/2))-67499/1225000
*14^(1/2)*e^3*arctan(1/28*(10*x+2)*14^(1/2))
```

maxima [A] time = 0.96, size = 212, normalized size = 1.12

$$\frac{1}{25}e^3x^4+\frac{1}{375}\left(60\,de^2-41\,e^3\right)x^3+\frac{1}{1250}\left(300\,d^2e-615\,de^2+103\,e^3\right)x^2+\frac{1}{1225000}\sqrt{14}\left(32825\,d^3+317565\,d^2e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 1/25*e^3*x^4 + 1/375*(60*d*e^2 - 41*e^3)*x^3 + 1/1250*(300*d^2*e - 615*d*e^
2 + 103*e^3)*x^2 + 1/1225000*sqrt(14)*(32825*d^3 + 317565*d^2*e - 221643*d*
e^2 - 67499*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/3125*(500*d^3 - 3075*d
^2*e + 1545*d*e^2 + 867*e^3)*x - 1/6250*(1025*d^3 - 1545*d^2*e - 2601*d*e^2
 + 832*e^3)*log(5*x^2 + 2*x + 3) - 1/437500*(170875*d^3 - 95175*d^2*e - 269
505*d*e^2 + 54969*e^3 + (52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^
3)*x)/(5*x^2 + 2*x + 3)
```

mupad [B] time = 0.15, size = 333, normalized size = 1.76

$$\frac{\frac{53901\,d\,e^2}{28}+\frac{19035\,d^2\,e}{28}+x\,\left(-\frac{10575\,d^3}{28}-\frac{89835\,d^2\,e}{28}+\frac{54969\,d\,e^2}{28}+\frac{53189\,e^3}{140}\right)-\frac{34175\,d^3}{28}-\frac{54969\,e^3}{140}}{15625\,x^2+6250\,x+9375}+x^3\,\left(\frac{e^2\,\left(12\,d-5\,e\right)}{75}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] ((53901*d*e^2)/28 + (19035*d^2*e)/28 + x*((54969*d*e^2)/28 - (89835*d^2*e)/
28 - (10575*d^3)/28 + (53189*e^3)/140) - (34175*d^3)/28 - (54969*e^3)/140)/
(6250*x + 15625*x^2 + 9375) + x^3*((e^2*(12*d - 5*e))/75 - (16*e^3)/375) -
x*((18*e^2*(12*d - 5*e))/625 + (12*e*(4*d^2 - 5*d*e + e^2))/125 - (9*d*e^2)
/25 + (3*d^2*e)/5 - (4*d^3)/25 - (717*e^3)/3125) + log(2*x + 5*x^2 + 3)*((2
601*d*e^2)/6250 + (309*d^2*e)/1250 - (41*d^3)/250 - (416*e^3)/3125) - x^2*(
(2*e^2*(12*d - 5*e))/125 - (3*e*(4*d^2 - 5*d*e + e^2))/50 + (36*e^3)/625) +
 (e^3*x^4)/25 - (14^(1/2)*atan(((14^(1/2)*(221643*d*e^2 - 317565*d^2*e - 32
825*d^3 + 67499*e^3))/1225000 + (14^(1/2)*x*(221643*d*e^2 - 317565*d^2*e -
32825*d^3 + 67499*e^3))/245000)/((221643*d*e^2)/87500 - (63513*d^2*e)/17500
 - (1313*d^3)/3500 + (67499*e^3)/87500))*(221643*d*e^2 - 317565*d^2*e - 328
25*d^3 + 67499*e^3))/1225000
```

sympy [C] time = 2.77, size = 444, normalized size = 2.35

$$\frac{e^3x^4}{25}+x^3\left(\frac{4de^2}{25}-\frac{41e^3}{375}\right)+x^2\left(\frac{6d^2e}{25}-\frac{123de^2}{250}+\frac{103e^3}{1250}\right)+x\left(\frac{4d^3}{25}-\frac{123d^2e}{125}+\frac{309de^2}{625}+\frac{867e^3}{3125}\right)+\left(-\frac{41d^3}{250}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] e**3*x**4/25 + x**3*(4*d*e**2/25 - 41*e**3/375) + x**2*(6*d**2*e/25 - 123*d
*e**2/250 + 103*e**3/1250) + x*(4*d**3/25 - 123*d**2*e/125 + 309*d*e**2/625
 + 867*e**3/3125) + (-41*d**3/250 + 309*d**2*e/1250 + 2601*d*e**2/6250 - 41
6*e**3/3125 - sqrt(14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 6749
```

```
9*e**3)/2450000)*log(x + (6565*d**3 + 63513*d**2*e - 221643*d*e**2/5 - 6749
9*e**3/5 - sqrt(14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e
**3)/5)/(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)) + (-41*d
**3/250 + 309*d**2*e/1250 + 2601*d*e**2/6250 - 416*e**3/3125 + sqrt(14)*I*(
32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)/2450000)*log(x + (
6565*d**3 + 63513*d**2*e - 221643*d*e**2/5 - 67499*e**3/5 + sqrt(14)*I*(328
25*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)/5)/(32825*d**3 + 3175
65*d**2*e - 221643*d*e**2 - 67499*e**3)) + (-170875*d**3 + 95175*d**2*e + 2
69505*d*e**2 - 54969*e**3 + x*(-52875*d**3 - 449175*d**2*e + 274845*d*e**2
+ 53189*e**3))/(2187500*x**2 + 875000*x + 1312500)
```

3.312 $\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$

Optimal. Leaf size=140

$$-\frac{\left(1025d^2-1030de-867e^2\right)\log\left(5x^2+2x+3\right)}{6250}+\frac{x\left(2800d^2-11480de+3307e^2\right)}{17500}+\frac{\left(32825d^2+211710de-73\right.}{87500\sqrt{}}$$

```
[Out] 1/17500*(2800*d^2-11480*d*e+3307*e^2)*x+1/250*(40*d-41*e)*e*x^2+4/75*e^2*x^
3-1/3500*(1367+423*x)*(e*x+d)^2/(5*x^2+2*x+3)-1/6250*(1025*d^2-1030*d*e-867
*e^2)*ln(5*x^2+2*x+3)+1/1225000*(32825*d^2+211710*d*e-73881*e^2)*arctan(1/1
4*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1644, 1628, 634, 618, 204, 628}

$$-\frac{\left(1025d^2-1030de-867e^2\right)\log\left(5x^2+2x+3\right)}{6250}+\frac{x\left(2800d^2-11480de+3307e^2\right)}{17500}+\frac{\left(32825d^2+211710de-73\right.}{87500\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] ((2800*d^2 - 11480*d*e + 3307*e^2)*x)/17500 + ((40*d - 41*e)*e*x^2)/250 + (
4*e^2*x^3)/75 - ((1367 + 423*x)*(d + e*x)^2)/(3500*(3 + 2*x + 5*x^2)) + ((3
2825*d^2 + 211710*d*e - 73881*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/(87500*Sqrt[
14]) - ((1025*d^2 - 1030*d*e - 867*e^2)*Log[3 + 2*x + 5*x^2])/6250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx = -\frac{(1367+423x)(d+ex)^2}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int \frac{(d+ex)\left(\frac{2}{125}(1845d+2734e)-\frac{6}{12}\right.}{3}$$

$$= -\frac{(1367+423x)(d+ex)^2}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int\left(\frac{2}{625}\left(2800d^2-11480de+3307e^2\right.\right.$$

$$= \frac{\left(2800d^2-11480de+3307e^2\right)x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x^3}{75}-\frac{(13}{35}$$

$$= \frac{\left(2800d^2-11480de+3307e^2\right)x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x^3}{75}-\frac{(13}{35}$$

$$= \frac{\left(2800d^2-11480de+3307e^2\right)x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x^3}{75}-\frac{(13}{35}$$

$$= \frac{\left(2800d^2-11480de+3307e^2\right)x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x^3}{75}-\frac{(13}{35}$$

Mathematica [A] time = 0.11, size = 150, normalized size = 1.07

$$\frac{-\frac{42\left(25d^2(423x+1367)+10de(5989x-1269)-e^2(18323x+17967)\right)}{5x^2+2x+3}+588\left(-1025d^2+1030de+867e^2\right)\log\left(5x^2+2x+3\right)+5880x}{36}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2
,x]
```

```
[Out] (5880*(100*d^2 - 410*d*e + 103*e^2)*x + 14700*(40*d - 41*e)*e*x^2 + 196000*
e^2*x^3 - (42*(25*d^2*(1367 + 423*x) + 10*d*e*(-1269 + 5989*x) - e^2*(17967
 + 18323*x)))/(3 + 2*x + 5*x^2) + 3*Sqrt[14]*(32825*d^2 + 211710*d*e - 7388
1*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]] + 588*(-1025*d^2 + 1030*d*e + 867*e^2)*Lo
g[3 + 2*x + 5*x^2])/3675000
```

fricas [A] time = 0.80, size = 245, normalized size = 1.75

$$\frac{980000\,e^2x^5 + 24500\left(120\,de - 107\,e^2\right)x^4 + 58800\left(50\,d^2 - 185\,de + 41\,e^2\right)x^3 + 2940\left(400\,d^2 - 1040\,de - 203\,e^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] 1/3675000*(980000*e^2*x^5 + 24500*(120*d*e - 107*e^2)*x^4 + 58800*(50*d^2 -
 185*d*e + 41*e^2)*x^3 + 2940*(400*d^2 - 1040*d*e - 203*e^2)*x^2 + 3*sqrt(1
4)*(5*(32825*d^2 + 211710*d*e - 73881*e^2)*x^2 + 98475*d^2 + 635130*d*e - 2
21643*e^2 + 2*(32825*d^2 + 211710*d*e - 73881*e^2)*x)*arctan(1/14*sqrt(14)*
(5*x + 1)) - 1435350*d^2 + 532980*d*e + 754614*e^2 + 42*(31425*d^2 - 232090
*d*e + 61583*e^2)*x - 588*(5*(1025*d^2 - 1030*d*e - 867*e^2)*x^2 + 3075*d^2
 - 3090*d*e - 2601*e^2 + 2*(1025*d^2 - 1030*d*e - 867*e^2)*x)*log(5*x^2 + 2
*x + 3))/(5*x^2 + 2*x + 3)
```

giac [A] time = 0.16, size = 145, normalized size = 1.04

$$\frac{4}{75}\,x^3e^2+\frac{4}{25}\,dx^2e+\frac{4}{25}\,d^2x-\frac{41}{250}\,x^2e^2-\frac{82}{125}\,dxe+\frac{1}{1225000}\,\sqrt{14}\left(32825\,d^2 + 211710\,de - 73881\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 4/75*x^3*e^2 + 4/25*d*x^2*e + 4/25*d^2*x - 41/250*x^2*e^2 - 82/125*d*x*e +
1/1225000*sqrt(14)*(32825*d^2 + 211710*d*e - 73881*e^2)*arctan(1/14*sqrt(14
)*(5*x + 1)) + 103/625*x*e^2 - 1/6250*(1025*d^2 - 1030*d*e - 867*e^2)*log(5
*x^2 + 2*x + 3) - 1/87500*(34175*d^2 + (10575*d^2 + 59890*d*e - 18323*e^2)*
x - 12690*d*e - 17967*e^2)/(5*x^2 + 2*x + 3)
```

maple [A] time = 0.01, size = 189, normalized size = 1.35

$$\frac{4e^2x^3}{75}+\frac{4de\,x^2}{25}-\frac{41e^2x^2}{250}+\frac{4d^2x}{25}+\frac{1313\sqrt{14}\;d^2\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{49000}-\frac{41d^2\ln\left(5x^2+2x+3\right)}{250}-\frac{82dex}{125}+\frac{21171\sqrt{14}\;\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] 4/75*e^2*x^3+4/25*d*e*x^2-41/250*e^2*x^2+4/25*d^2*x-82/125*d*e*x+103/625*e^
2*x-1/625*((423/28*d^2+5989/70*d*e-18323/700*e^2)*x+1367/28*d^2-1269/70*d*e
-17967/700*e^2)/(x^2+2/5*x+3/5)-41/250*d^2*ln(5*x^2+2*x+3)+103/625*d*e*ln(5
*x^2+2*x+3)+867/6250*e^2*ln(5*x^2+2*x+3)+1313/49000*14^(1/2)*d^2*arctan(1/2
8*(10*x+2)*14^(1/2))+21171/122500*14^(1/2)*d*e*arctan(1/28*(10*x+2)*14^(1/2
))-73881/1225000*14^(1/2)*e^2*arctan(1/28*(10*x+2)*14^(1/2))
```

maxima [A] time = 0.96, size = 147, normalized size = 1.05

$$\frac{4}{75}\,e^2x^3+\frac{1}{250}\left(40\,de - 41\,e^2\right)x^2+\frac{1}{1225000}\,\sqrt{14}\left(32825\,d^2 + 211710\,de - 73881\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x + 1)\right)+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 4/75*e^2*x^3 + 1/250*(40*d*e - 41*e^2)*x^2 + 1/1225000*sqrt(14)*(32825*d^2
+ 211710*d*e - 73881*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(100*d^2
- 410*d*e + 103*e^2)*x - 1/6250*(1025*d^2 - 1030*d*e - 867*e^2)*log(5*x^2 +
 2*x + 3) - 1/87500*(34175*d^2 - 12690*d*e - 17967*e^2 + (10575*d^2 + 59890
*d*e - 18323*e^2)*x)/(5*x^2 + 2*x + 3)
```

mupad [B] time = 0.11, size = 211, normalized size = 1.51

$$\ln\left(5\,x^2+2\,x+3\right)\,\left(-\frac{41\,d^2}{250}+\frac{103\,d\,e}{625}+\frac{867\,e^2}{6250}\right)-x\,\left(\frac{2\,d\,e}{5}+\frac{4\,e\,\left(8\,d-5\,e\right)}{125}-\frac{4\,d^2}{25}-\frac{3\,e^2}{625}\right)+x^2\,\left(\frac{e\,\left(8\,d-5\,e\right)}{50}-\frac{8\,e^2}{125}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] log(2*x + 5*x^2 + 3)*((103*d*e)/625 - (41*d^2)/250 + (867*e^2)/6250) - x*((
2*d*e)/5 + (4*e*(8*d - 5*e))/125 - (4*d^2)/25 - (3*e^2)/625) + x^2*((e*(8*d
 - 5*e))/50 - (8*e^2)/125) + ((1269*d*e)/14 - x*((5989*d*e)/14 + (2115*d^2)
/28 - (18323*e^2)/140) - (6835*d^2)/28 + (17967*e^2)/140)/(1250*x + 3125*x^
2 + 1875) + (4*e^2*x^3)/75 + (14^(1/2)*atan(((14^(1/2)*(211710*d*e + 32825*
d^2 - 73881*e^2))/1225000 + (14^(1/2)*x*(211710*d*e + 32825*d^2 - 73881*e^2
))/245000)/((21171*d*e)/8750 + (1313*d^2)/3500 - (73881*e^2)/87500))*(21171
0*d*e + 32825*d^2 - 73881*e^2))/1225000
```

sympy [C] time = 1.96, size = 298, normalized size = 2.13

$$\frac{4e^2x^3}{75}+x^2\left(\frac{4de}{25}-\frac{41e^2}{250}\right)+x\left(\frac{4d^2}{25}-\frac{82de}{125}+\frac{103e^2}{625}\right)+\left(-\frac{41d^2}{250}+\frac{103de}{625}+\frac{867e^2}{6250}-\frac{\sqrt{14}\,i\left(32825d^2+211710de-73\right.}{2450000}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] 4*e**2*x**3/75 + x**2*(4*d*e/25 - 41*e**2/250) + x*(4*d**2/25 - 82*d*e/125
+ 103*e**2/625) + (-41*d**2/250 + 103*d*e/625 + 867*e**2/6250 - sqrt(14)*I*
(32825*d**2 + 211710*d*e - 73881*e**2)/2450000)*log(x + (6565*d**2 + 42342*
d*e - 73881*e**2/5 - sqrt(14)*I*(32825*d**2 + 211710*d*e - 73881*e**2)/5)/(
32825*d**2 + 211710*d*e - 73881*e**2)) + (-41*d**2/250 + 103*d*e/625 + 867*
e**2/6250 + sqrt(14)*I*(32825*d**2 + 211710*d*e - 73881*e**2)/2450000)*log(
x + (6565*d**2 + 42342*d*e - 73881*e**2/5 + sqrt(14)*I*(32825*d**2 + 211710
*d*e - 73881*e**2)/5)/(32825*d**2 + 211710*d*e - 73881*e**2)) + (-34175*d**
2 + 12690*d*e + 17967*e**2 + x*(-10575*d**2 - 59890*d*e + 18323*e**2))/(437
500*x**2 + 175000*x + 262500)
```

3.313 $\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{(423x+1367)(d+ex)}{3500\left(5x^2+2x+3\right)}-\frac{(205d-103e)\log\left(5x^2+2x+3\right)}{1250}+\frac{1}{125}x(20d-41e)+\frac{(6565d+21171e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{17500\sqrt{14}}+$$

```
[Out] 1/125*(20*d-41*e)*x+2/25*e*x^2-1/3500*(1367+423*x)*(e*x+d)/(5*x^2+2*x+3)-1/
1250*(205*d-103*e)*ln(5*x^2+2*x+3)+1/245000*(6565*d+21171*e)*arctan(1/14*(1
+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1644, 1657, 634, 618, 204, 628}

$$-\frac{(423x+1367)(d+ex)}{3500\left(5x^2+2x+3\right)}-\frac{(205d-103e)\log\left(5x^2+2x+3\right)}{1250}+\frac{1}{125}x(20d-41e)+\frac{(6565d+21171e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{17500\sqrt{14}}+$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] ((20*d - 41*e)*x)/125 + (2*e*x^2)/25 - ((1367 + 423*x)*(d + e*x))/(3500*(3
+ 2*x + 5*x^2)) + ((6565*d + 21171*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(17500*Sq
rt[14]) - ((205*d - 103*e)*Log[3 + 2*x + 5*x^2])/1250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
```

```
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}dx = -\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)} + \frac{1}{56}\int \frac{\frac{2}{125}(1845d+1367e)-\frac{168}{125}(55d-27e \cdots}{3+2x+\cdots}$$

$$= -\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)} + \frac{1}{56}\int\left(\frac{56}{125}(20d-41e)+\frac{224ex}{25}+\frac{2(165d \cdots}{\cdots}\right.$$

$$= \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}-\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)}+\frac{\int \frac{165d+4811e-28(2\cdots}{3+2x+5x\cdots}}{3500\cdots}$$

$$= \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}-\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)}+\frac{(-205d+103e)\cdots}{125\cdots}$$

$$= \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}-\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)}-\frac{(205d-103e)\log\cdots}{12\cdots}$$

$$= \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}-\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)}+\frac{(6565d+21171e\cdots}{17500\cdots}$$

Mathematica [A] time = 0.07, size = 96, normalized size = 0.99

$$\frac{-\frac{14(5d(423x+1367)+e(5989x-1269))}{5x^2+2x+3}+196(103e-205d)\log\left(5x^2+2x+3\right)+1960x(20d-41e)+\sqrt{14}\,(6565d+21171e\cdots}{245000}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x
]
```

```
[Out] (1960*(20*d - 41*e)*x + 19600*e*x^2 - (14*(5*d*(1367 + 423*x) + e*(-1269 +
5989*x)))/(3 + 2*x + 5*x^2) + Sqrt[14]*(6565*d + 21171*e)*ArcTan[(1 + 5*x)/
Sqrt[14]] + 196*(-205*d + 103*e)*Log[3 + 2*x + 5*x^2])/245000
```

fricas [A] time = 0.82, size = 147, normalized size = 1.52

$$98000\,ex^4+9800\,(20\,d-37\,e)x^3+7840\,(10\,d-13\,e)x^2+\sqrt{14}\left(5\,(6565\,d+21171\,e)x^2+2\,(6565\,d+21171\,e)x+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="fri
cas")
```

```
[Out] 1/245000*(98000*e*x^4 + 9800*(20*d - 37*e)*x^3 + 7840*(10*d - 13*e)*x^2 + s
qrt(14)*(5*(6565*d + 21171*e)*x^2 + 2*(6565*d + 21171*e)*x + 19695*d + 6351
3*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(6285*d - 23209*e)*x - 196*(5*(20
5*d - 103*e)*x^2 + 2*(205*d - 103*e)*x + 615*d - 309*e)*log(5*x^2 + 2*x + 3
) - 95690*d + 17766*e)/(5*x^2 + 2*x + 3)
```

giac [A] time = 0.16, size = 94, normalized size = 0.97

$$\frac{2}{25}x^2e+\frac{1}{245000}\sqrt{14}\,(6565\,d+21171\,e)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+\frac{4}{25}dx-\frac{41}{125}xe-\frac{1}{1250}(205\,d-103\,e)\log\left(5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="gia
c")
```

```
[Out] 2/25*x^2*e + 1/245000*sqrt(14)*(6565*d + 21171*e)*arctan(1/14*sqrt(14)*(5*x
 + 1)) + 4/25*d*x - 41/125*x*e - 1/1250*(205*d - 103*e)*log(5*x^2 + 2*x + 3
) - 1/17500*((2115*d + 5989*e)*x + 6835*d - 1269*e)/(5*x^2 + 2*x + 3)
```

maple [A] time = 0.01, size = 106, normalized size = 1.09

$$\frac{2e\,x^2}{25}+\frac{4dx}{25}+\frac{1313\sqrt{14}\,d\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{49000}-\frac{41d\ln\left(5x^2+2x+3\right)}{250}-\frac{41ex}{125}+\frac{21171\sqrt{14}\,e\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{245000}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] 2/25*e*x^2+4/25*d*x-41/125*e*x-1/125*((423/140*d+5989/700*e)*x+1367/140*d-1
269/700*e)/(x^2+2/5*x+3/5)-41/250*d*ln(5*x^2+2*x+3)+103/1250*e*ln(5*x^2+2*x
+3)+1313/49000*14^(1/2)*d*arctan(1/28*(10*x+2)*14^(1/2))+21171/245000*14^(1
/2)*e*arctan(1/28*(10*x+2)*14^(1/2))
```

maxima [A] time = 0.96, size = 90, normalized size = 0.93

$$\frac{2}{25}ex^2+\frac{1}{245000}\sqrt{14}\,(6565\,d+21171\,e)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+\frac{1}{125}(20\,d-41\,e)x-\frac{1}{1250}(205\,d-103\,e)\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="max
ima")
```

```
[Out] 2/25*e*x^2 + 1/245000*sqrt(14)*(6565*d + 21171*e)*arctan(1/14*sqrt(14)*(5*x
 + 1)) + 1/125*(20*d - 41*e)*x - 1/1250*(205*d - 103*e)*log(5*x^2 + 2*x + 3
) - 1/17500*((2115*d + 5989*e)*x + 6835*d - 1269*e)/(5*x^2 + 2*x + 3)
```

mupad [B] time = 4.15, size = 115, normalized size = 1.19

$$\frac{2\,e\,x^2}{25}-\ln\left(5\,x^2+2\,x+3\right)\left(\frac{41\,d}{250}-\frac{103\,e}{1250}\right)+x\left(\frac{4\,d}{25}-\frac{41\,e}{125}\right)-\frac{\frac{1367\,d}{28}-\frac{1269\,e}{140}+x\left(\frac{423\,d}{28}+\frac{5989\,e}{140}\right)}{625\,x^2+250\,x+375}+\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\sqrt{14}\,(\ldots}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] (2*e*x^2)/25 - log(2*x + 5*x^2 + 3)*((41*d)/250 - (103*e)/1250) + x*((4*d)/
25 - (41*e)/125) - ((1367*d)/28 - (1269*e)/140 + x*((423*d)/28 + (5989*e)/1
40))/(250*x + 625*x^2 + 375) + (14^(1/2)*atan(((14^(1/2)*(6565*d + 21171*e)
)/245000 + (14^(1/2)*x*(6565*d + 21171*e))/49000)/((1313*d)/3500 + (21171*e
)/17500))*(6565*d + 21171*e))/245000
```

sympy [C] time = 1.02, size = 165, normalized size = 1.70

$$\frac{2ex^2}{25}+x\left(\frac{4d}{25}-\frac{41e}{125}\right)+\frac{-6835d+1269e+x\left(-2115d-5989e\right)}{87500x^2+35000x+52500}+\left(-\frac{41d}{250}+\frac{103e}{1250}-\frac{\sqrt{14}\,i\left(6565d+21171e\right)}{490000}\right)\log\left(x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] 2*e*x**2/25 + x*(4*d/25 - 41*e/125) + (-6835*d + 1269*e + x*(-2115*d - 5989
*e))/(87500*x**2 + 35000*x + 52500) + (-41*d/250 + 103*e/1250 - sqrt(14)*I*
(6565*d + 21171*e)/490000)*log(x + (1313*d + 21171*e/5 - sqrt(14)*I*(6565*d
 + 21171*e)/5)/(6565*d + 21171*e)) + (-41*d/250 + 103*e/1250 + sqrt(14)*I*(
6565*d + 21171*e)/490000)*log(x + (1313*d + 21171*e/5 + sqrt(14)*I*(6565*d
+ 21171*e)/5)/(6565*d + 21171*e))
```

3.314 $\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^2}\,dx$

Optimal. Leaf size=63

$$-\frac{423x+1367}{3500\left(5x^2+2x+3\right)}-\frac{41}{250}\log\left(5x^2+2x+3\right)+\frac{4x}{25}+\frac{1313\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{3500\sqrt{14}}$$

```
[Out] 4/25*x+1/3500*(-1367-423*x)/(5*x^2+2*x+3)-41/250*ln(5*x^2+2*x+3)+1313/49000
*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1660, 1657, 634, 618, 204, 628}

$$-\frac{423x+1367}{3500\left(5x^2+2x+3\right)}-\frac{41}{250}\log\left(5x^2+2x+3\right)+\frac{4x}{25}+\frac{1313\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{3500\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] (4*x)/25 - (1367 + 423*x)/(3500*(3 + 2*x + 5*x^2)) + (1313*ArcTan[(1 + 5*x)
/Sqrt[14]])/(3500*Sqrt[14]) - (41*Log[3 + 2*x + 5*x^2])/250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^2}\,dx &= -\frac{1367+423x}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int \frac{\frac{738}{25}-\frac{1848x}{25}+\frac{224x^2}{5}}{3+2x+5x^2}\,dx \\
&= -\frac{1367+423x}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int \left(\frac{224}{25}+\frac{2(33-1148x)}{25\left(3+2x+5x^2\right)}\right)dx \\
&= \frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}+\frac{1}{700}\int \frac{33-1148x}{3+2x+5x^2}\,dx \\
&= \frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}-\frac{41}{250}\int \frac{2+10x}{3+2x+5x^2}\,dx+\frac{1313\int \frac{1}{3+2x+5x^2}\,dx}{3500} \\
&= \frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}-\frac{41}{250}\log\left(3+2x+5x^2\right)-\frac{1313\,\mathrm{Subst}\left(\int \frac{1}{-56-x^2}\,dx,\right.}{1750} \\
&= \frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}+\frac{1313\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{3500\sqrt{14}}-\frac{41}{250}\log\left(3+2x+5x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 0.94

$$\frac{-\frac{14(423x+1367)}{5x^2+2x+3}-8036\log\left(5x^2+2x+3\right)+7840x+1313\sqrt{14}\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{49000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] (7840*x - (14*(1367 + 423*x))/(3 + 2*x + 5*x^2) + 1313*Sqrt[14]*ArcTan[(1 +
 5*x)/Sqrt[14]] - 8036*Log[3 + 2*x + 5*x^2])/49000
```

fricas [A] time = 0.77, size = 78, normalized size = 1.24

$$\frac{39200\,x^3+1313\,\sqrt{14}\left(5\,x^2+2\,x+3\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+15680\,x^2-8036\left(5\,x^2+2\,x+3\right)\log\left(5\,x^2+2\,x\right.}{49000\left(5\,x^2+2\,x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/49000*(39200*x^3 + 1313*sqrt(14)*(5*x^2 + 2*x + 3)*arctan(1/14*sqrt(14)*(
5*x + 1)) + 15680*x^2 - 8036*(5*x^2 + 2*x + 3)*log(5*x^2 + 2*x + 3) + 17598
*x - 19138)/(5*x^2 + 2*x + 3)
```

giac [A] time = 0.15, size = 52, normalized size = 0.83

$$\frac{1313}{49000}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)+\frac{4}{25}x-\frac{423x+1367}{3500(5x^2+2x+3)}-\frac{41}{250}\log\left(5x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="giac")
```

```
[Out] 1313/49000*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4/25*x - 1/3500*(423*
x + 1367)/(5*x^2 + 2*x + 3) - 41/250*log(5*x^2 + 2*x + 3)
```

maple [A] time = 0.01, size = 51, normalized size = 0.81

$$\frac{4x}{25}+\frac{1313\sqrt{14}\ \arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{49000}-\frac{41\ln\left(5x^2+2x+3\right)}{250}-\frac{\frac{423x}{700}+\frac{1367}{700}}{25\left(x^2+\frac{2}{5}x+\frac{3}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] 4/25*x-1/25*(423/700*x+1367/700)/(x^2+2/5*x+3/5)-41/250*ln(5*x^2+2*x+3)+131
3/49000*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

maxima [A] time = 0.95, size = 52, normalized size = 0.83

$$\frac{1313}{49000}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)+\frac{4}{25}x-\frac{423x+1367}{3500(5x^2+2x+3)}-\frac{41}{250}\log\left(5x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="maxima")
```

```
[Out] 1313/49000*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4/25*x - 1/3500*(423*
x + 1367)/(5*x^2 + 2*x + 3) - 41/250*log(5*x^2 + 2*x + 3)
```

mupad [B] time = 4.15, size = 52, normalized size = 0.83

$$\frac{4x}{25}-\frac{41\ \ln\left(5x^2+2x+3\right)}{250}-\frac{\frac{423x}{17500}+\frac{1367}{17500}}{x^2+\frac{2x}{5}+\frac{3}{5}}+\frac{1313\sqrt{14}\ \mathrm{atan}\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{49000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] (4*x)/25 - (41*log(2*x + 5*x^2 + 3))/250 - ((423*x)/17500 + 1367/17500)/((2
*x)/5 + x^2 + 3/5) + (1313*14^(1/2)*atan((5*14^(1/2)*x)/14 + 14^(1/2)/14))/
49000
```

sympy [A] time = 0.19, size = 65, normalized size = 1.03

$$\frac{4x}{25}+\frac{-423x-1367}{17500x^2+7000x+10500}-\frac{41\log\left(x^2+\frac{2x}{5}+\frac{3}{5}\right)}{250}+\frac{1313\sqrt{14}\ \mathrm{atan}\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{49000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] 4*x/25 + (-423*x - 1367)/(17500*x**2 + 7000*x + 10500) - 41*log(x**2 + 2*x/
5 + 3/5)/250 + 1313*sqrt(14)*atan(5*sqrt(14)*x/14 + sqrt(14)/14)/49000
```

3.315 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx$

Optimal. Leaf size=224

$$-\frac{x(423d-1367e)+1367d-293e}{700\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)}-\frac{\left(205d^3-61d^2e+23de^2+14e^3\right)\log\left(5x^2+2x+3\right)}{50\left(5d^2-2de+3e^2\right)^2}+\frac{\left(6565d^3-26423d^2\right.}{700\sqrt{}}$$

```
[Out] 1/700*(-1367*d+293*e-(423*d-1367*e)*x)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)+(4
*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/e/(5*d^2-2*d*e+3*e^2)^2-1/50*
(205*d^3-61*d^2*e+23*d*e^2+14*e^3)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^2+1/
9800*(6565*d^3-26423*d^2*e+11089*d*e^2-6623*e^3)*arctan(1/14*(1+5*x)*14^(1/
2))/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1646, 1628, 634, 618, 204, 628}

$$-\frac{x(423d-1367e)+1367d-293e}{700\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)}-\frac{\left(-61d^2e+205d^3+23de^2+14e^3\right)\log\left(5x^2+2x+3\right)}{50\left(5d^2-2de+3e^2\right)^2}+\frac{\left(3d^2e^2+5d^3e+4\right.}{e\left(5d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^2),x]
```

```
[Out] -(1367*d - 293*e + (423*d - 1367*e)*x)/(700*(5*d^2 - 2*d*e + 3*e^2)*(3 + 2*
x + 5*x^2)) + ((6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*ArcTan[(1
+ 5*x)/Sqrt[14]])/(700*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) + ((4*d^4 + 5*d^
3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/(e*(5*d^2 - 2*d*e + 3*e^2)^2
) - ((205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*Log[3 + 2*x + 5*x^2])/(50*(5*
d^2 - 2*d*e + 3*e^2)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx = -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{1}{56}\int \frac{\frac{2\left(369d^2-421de+280e^2\right)}{5\left(5d^2-2de+3e^2\right)}-\frac{2\left(924d^2-\cdots\right.}{5\left(5d\cdots\right.}}{(d+ex)\left(3+2x\cdots\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{1}{56}\int \left(\frac{56\left(4d^4+5d^3e+3d^2e^2-d\cdots\right.}{\left(5d^2-2de+3e^2\right)^2(d+\cdots}\right.$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log\cdots}{e\left(5d^2-2de+3e^2\right)^2}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log\cdots}{e\left(5d^2-2de+3e^2\right)^2}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log\cdots}{e\left(5d^2-2de+3e^2\right)^2}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{\left(6565d^3-26423d^2e+11089de^2-\cdots\right.}{700\sqrt{14}\left(5d^2-2de\cdots\right.}$$

Mathematica [A] time = 0.16, size = 186, normalized size = 0.83

$$\frac{\frac{14\left(5d^2-2de+3e^2\right)\left(e(1367x+293)-d(423x+1367)\right)}{5x^2+2x+3}-196\left(205d^3-61d^2e+23de^2+14e^3\right)\log\left(5x^2+2x+3\right)+\sqrt{14}\left(6565d\cdots\right.}{9800\left(5d^2-2de+3e^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^2),x
]
```

```
[Out] ((14*(5*d^2 - 2*d*e + 3*e^2)*(-(d*(1367 + 423*x)) + e*(293 + 1367*x)))/(3 +
 2*x + 5*x^2) + Sqrt[14]*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*
ArcTan[(1 + 5*x)/Sqrt[14]] + (9800*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2
```

```
*e^4)*Log[d + e*x])/e - 196*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*Log[3
+ 2*x + 5*x^2])/(9800*(5*d^2 - 2*d*e + 3*e^2)^2)
```

fricas [B] time = 0.94, size = 479, normalized size = 2.14

$$-\frac{95690\,d^3e - 58786\,d^2e^2 + 65618\,de^3 - 12306\,e^4 - \sqrt{14}\left(19695\,d^3e - 79269\,d^2e^2 + 33267\,de^3 - 19869\,e^4 + 5\left(656\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="fri
cas")
```

```
[Out] -1/9800*(95690*d^3*e - 58786*d^2*e^2 + 65618*d*e^3 - 12306*e^4 - sqrt(14)*(
19695*d^3*e - 79269*d^2*e^2 + 33267*d*e^3 - 19869*e^4 + 5*(6565*d^3*e - 264
23*d^2*e^2 + 11089*d*e^3 - 6623*e^4)*x^2 + 2*(6565*d^3*e - 26423*d^2*e^2 +
11089*d*e^3 - 6623*e^4)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(2115*d^3*e
 - 7681*d^2*e^2 + 4003*d*e^3 - 4101*e^4)*x - 9800*(12*d^4 + 15*d^3*e + 9*d^
2*e^2 - 3*d*e^3 + 6*e^4 + 5*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*x
^2 + 2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*x)*log(e*x + d) + 196*
(615*d^3*e - 183*d^2*e^2 + 69*d*e^3 + 42*e^4 + 5*(205*d^3*e - 61*d^2*e^2 +
23*d*e^3 + 14*e^4)*x^2 + 2*(205*d^3*e - 61*d^2*e^2 + 23*d*e^3 + 14*e^4)*x)*
log(5*x^2 + 2*x + 3))/(75*d^4*e - 60*d^3*e^2 + 102*d^2*e^3 - 36*d*e^4 + 27*
e^5 + 5*(25*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5)*x^2 + 2*(25
*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5)*x)
```

giac [A] time = 0.17, size = 284, normalized size = 1.27

$$\frac{\sqrt{14}\left(6565\,d^3 - 26423\,d^2e + 11089\,de^2 - 6623\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{9800\left(25\,d^4 - 20\,d^3e + 34\,d^2e^2 - 12\,de^3 + 9\,e^4\right)} - \frac{\left(205\,d^3 - 61\,d^2e + 23\,de^2 + 14\,e^3\right)\mathrm{lo}}{50\left(25\,d^4 - 20\,d^3e + 34\,d^2e^2 - 1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="gia
c")
```

```
[Out] 1/9800*sqrt(14)*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*arctan(1/
14*sqrt(14)*(5*x + 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)
- 1/50*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*log(5*x^2 + 2*x + 3)/(25*d^
4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) + (4*d^4 + 5*d^3*e + 3*d^2*e^
2 - d*e^3 + 2*e^4)*log(abs(x*e + d))/(25*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 -
12*d*e^4 + 9*e^5) - 1/700*(6835*d^3 - 4199*d^2*e + (2115*d^3 - 7681*d^2*e +
 4003*d*e^2 - 4101*e^3)*x + 4687*d*e^2 - 879*e^3)/((5*d^2 - 2*d*e + 3*e^2)^
2*(5*x^2 + 2*x + 3))
```

maple [B] time = 0.02, size = 691, normalized size = 3.08

$$\frac{4d^4\ln(ex+d)}{\left(5d^2-2de+3e^2\right)^2 e} - \frac{423d^3x}{700\left(5d^2-2de+3e^2\right)^2\left(x^2+\frac{2}{5}x+\frac{3}{5}\right)} + \frac{1313\sqrt{14}\,d^3\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{1960\left(5d^2-2de+3e^2\right)^2} + \frac{5d^3\ln(ex+d)}{\left(5d^2-2de+3e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x)
```

```
[Out] -423/700/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*d^3*x+7681/3500/(5*d^2-2*d*e
+3*e^2)^2/(x^2+2/5*x+3/5)*x*d^2*e-4003/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*
x+3/5)*x*d*e^2+4101/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*x*e^3-1367/7
00/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*d^3+4199/3500/(5*d^2-2*d*e+3*e^2)^
```

```
2/(x^2+2/5*x+3/5)*d^2*e-4687/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*d*e
^2+879/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*e^3-41/10/(5*d^2-2*d*e+3*
e^2)^2*ln(5*x^2+2*x+3)*d^3+61/50/(5*d^2-2*d*e+3*e^2)^2*ln(5*x^2+2*x+3)*d^2*
e-23/50/(5*d^2-2*d*e+3*e^2)^2*ln(5*x^2+2*x+3)*d*e^2-7/25/(5*d^2-2*d*e+3*e^2
)^2*ln(5*x^2+2*x+3)*e^3+1313/1960/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arctan(1/2
8*(10*x+2)*14^(1/2))*d^3-26423/9800/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arctan(1
/28*(10*x+2)*14^(1/2))*d^2*e+11089/9800/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arct
an(1/28*(10*x+2)*14^(1/2))*d*e^2-6623/9800/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*a
rctan(1/28*(10*x+2)*14^(1/2))*e^3+4/(5*d^2-2*d*e+3*e^2)^2/e*ln(e*x+d)*d^4+5
/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)*d^3+3/(5*d^2-2*d*e+3*e^2)^2*e*ln(e*x+d)*d^
2-1/(5*d^2-2*d*e+3*e^2)^2*e^2*ln(e*x+d)*d+2/(5*d^2-2*d*e+3*e^2)^2*e^3*ln(e*
x+d)
```

maxima [A] time = 0.98, size = 289, normalized size = 1.29

$$\frac{\sqrt{14}\left(6565\,d^3-26423\,d^2e+11089\,de^2-6623\,e^3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)}{9800\,\left(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4\right)}+\frac{\left(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e\right.}{25\,d^4e-20\,d^3e^2+34\,d^2e^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="max
ima")
```

```
[Out] 1/9800*sqrt(14)*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*arctan(1/
14*sqrt(14)*(5*x + 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)
+ (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e*x + d)/(25*d^4*e - 20
*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5) - 1/50*(205*d^3 - 61*d^2*e + 23*d
*e^2 + 14*e^3)*log(5*x^2 + 2*x + 3)/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*
e^3 + 9*e^4) - 1/700*((423*d - 1367*e)*x + 1367*d - 293*e)/(5*(5*d^2 - 2*d*
e + 3*e^2)*x^2 + 15*d^2 - 6*d*e + 9*e^2 + 2*(5*d^2 - 2*d*e + 3*e^2)*x)
```

mupad [B] time = 4.61, size = 330, normalized size = 1.47

$$\frac{\ln\left(d+e\,x\right)\,\left(4\,d^4+5\,d^3\,e+3\,d^2\,e^2-d\,e^3+2\,e^4\right)}{e\,{\left(5\,d^2-2\,d\,e+3\,e^2\right)}^2}+\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1{}\mathrm{i}}{5}\right)\,\left(\left(\frac{1313\,\sqrt{14}}{3920}-\frac{41}{10}{}\mathrm{i}\right)\,d^3+\left(-\frac{26423\,\sqrt{14}}{19600}+\frac{61}{50}{}\mathrm{i}\right.\right.}{d^4\,25{}\mathrm{i}-d^3\,e\,20{}\mathrm{i}+d^2\,e^2\,3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)^2),x)
```

```
[Out] (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^3*((1313*14^(1/2))/3920 - 41i/10) - e^3*
((6623*14^(1/2))/19600 + 7i/25) + d*e^2*((11089*14^(1/2))/19600 - 23i/50) -
 d^2*e*((26423*14^(1/2))/19600 - 61i/50)))/(d^4*25i - d^3*e*20i - d*e^3*12i
 + e^4*9i + d^2*e^2*34i) - ((1367*d - 293*e)/(700*(5*d^2 - 2*d*e + 3*e^2))
+ (x*(423*d - 1367*e))/(700*(5*d^2 - 2*d*e + 3*e^2)))/(2*x + 5*x^2 + 3) - (
log(x + (14^(1/2)*1i)/5 + 1/5)*(d^3*((1313*14^(1/2))/3920 + 41i/10) - e^3*(
(6623*14^(1/2))/19600 - 7i/25) + d*e^2*((11089*14^(1/2))/19600 + 23i/50) -
d^2*e*((26423*14^(1/2))/19600 + 61i/50)))/(d^4*25i - d^3*e*20i - d*e^3*12i
+ e^4*9i + d^2*e^2*34i) + (log(d + e*x)*(5*d^3*e - d*e^3 + 4*d^4 + 2*e^4 +
3*d^2*e^2))/(e*(5*d^2 - 2*d*e + 3*e^2)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3)**2,x)
```

```
[Out] Timed out
```

3.316 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx$

Optimal. Leaf size=313

$$-\frac{x\left(423d^2-2734de+293e^2\right)+1367d^2-586de-703e^2}{140\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^2}-\frac{\left(41d^4-8d^3e-60d^2e^2+24de^3-5e^4\right)\log\left(5x^2+2x+\right.}{2\left(5d^2-2de+3e^2\right)^3}$$

```
[Out] (-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)+1/14
0*(-1367*d^2+586*d*e+703*e^2-(423*d^2-2734*d*e+293*e^2)*x)/(5*d^2-2*d*e+3*e
^2)^2/(5*x^2+2*x+3)+(41*d^4-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)*ln(e*x+d)/(5
*d^2-2*d*e+3*e^2)^3-1/2*(41*d^4-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)*ln(5*x^2
+2*x+3)/(5*d^2-2*d*e+3*e^2)^3+1/392*(1313*d^4-10044*d^3*e+4290*d^2*e^2+156*
d*e^3-271*e^4)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1646, 1628, 634, 618, 204, 628}

$$-\frac{x\left(423d^2-2734de+293e^2\right)+1367d^2-586de-703e^2}{140\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^2}-\frac{\left(-60d^2e^2-8d^3e+41d^4+24de^3-5e^4\right)\log\left(5x^2+2x\right.}{2\left(5d^2-2de+3e^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^2),x]
```

```
[Out] -((4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e*(5*d^2 - 2*d*e + 3*e^2)^
2*(d + e*x))) - (1367*d^2 - 586*d*e - 703*e^2 + (423*d^2 - 2734*d*e + 293*e
^2)*x)/(140*(5*d^2 - 2*d*e + 3*e^2)^2*(3 + 2*x + 5*x^2)) + ((1313*d^4 - 100
44*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4)*ArcTan[(1 + 5*x)/Sqrt[14]])/
(28*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^3) + ((41*d^4 - 8*d^3*e - 60*d^2*e^2 +
 24*d*e^3 - 5*e^4)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^3 - ((41*d^4 - 8*d
^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d
*e + 3*e^2)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx = -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}+\frac{1}{56}\int \frac{2\left(369d^4-84\right.}{}$$

$$= -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}+\frac{1}{56}\int\left(\frac{56\left(4d^4\right.}{\left(5d^2\right.}\right.$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-273\right.}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x\right.}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-273\right.}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x\right.}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-273\right.}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x\right.}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-273\right.}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x\right.}$$

Mathematica [A] time = 0.26, size = 270, normalized size = 0.86

$$\frac{-\frac{14\left(5d^2-2de+3e^2\right)\left(d^2(423x+1367)-2de(1367x+293)+e^2(293x-703)\right)}{5x^2+2x+3}+980\left(-41d^4+8d^3e+60d^2e^2-24de^3+5e^4\right)\log\left(5x^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^2)
,x]
```

```
[Out] ((-1960*(5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^
4))/(e*(d + e*x)) - (14*(5*d^2 - 2*d*e + 3*e^2)*(e^2*(-703 + 293*x) + d^2*(
1367 + 423*x) - 2*d*e*(293 + 1367*x)))/(3 + 2*x + 5*x^2) + 5*Sqrt[14]*(1313
*d^4 - 10044*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4)*ArcTan[(1 + 5*x)/S
qrt[14]] + 1960*(41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log[d +
e*x] + 980*(-41*d^4 + 8*d^3*e + 60*d^2*e^2 - 24*d*e^3 + 5*e^4)*Log[3 + 2*x
+ 5*x^2])/(1960*(5*d^2 - 2*d*e + 3*e^2)^3)
```

fricas [B] time = 1.12, size = 910, normalized size = 2.91

$$-\frac{117600\,d^6+195650\,d^5e+20664\,d^4e^2+48132\,d^3e^3+118552\,d^2e^4-70686\,de^5+35280\,e^6+14\left(14000\,d^6+119\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] -1/1960*(117600*d^6 + 195650*d^5*e + 20664*d^4*e^2 + 48132*d^3*e^3 + 118552
*d^2*e^4 - 70686*d*e^5 + 35280*e^6 + 14*(14000*d^6 + 11900*d^5*e + 14015*d^
4*e^2 - 11716*d^3*e^3 + 22902*d^2*e^4 - 13688*d*e^5 + 5079*e^6)*x^2 - 5*sqr
t(14)*(3939*d^5*e - 30132*d^4*e^2 + 12870*d^3*e^3 + 468*d^2*e^4 - 813*d*e^5
 + 5*(1313*d^4*e^2 - 10044*d^3*e^3 + 4290*d^2*e^4 + 156*d*e^5 - 271*e^6)*x^
3 + (6565*d^5*e - 47594*d^4*e^2 + 1362*d^3*e^3 + 9360*d^2*e^4 - 1043*d*e^5
- 542*e^6)*x^2 + (2626*d^5*e - 16149*d^4*e^2 - 21552*d^3*e^3 + 13182*d^2*e^
4 - 74*d*e^5 - 813*e^6)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(5600*d^6 +
 6875*d^5*e - 2921*d^4*e^2 + 3658*d^3*e^3 - 1150*d^2*e^4 - 1433*d*e^5 - 429
*e^6)*x - 1960*(123*d^5*e - 24*d^4*e^2 - 180*d^3*e^3 + 72*d^2*e^4 - 15*d*e^
5 + 5*(41*d^4*e^2 - 8*d^3*e^3 - 60*d^2*e^4 + 24*d*e^5 - 5*e^6)*x^3 + (205*d
^5*e + 42*d^4*e^2 - 316*d^3*e^3 + 23*d*e^5 - 10*e^6)*x^2 + (82*d^5*e + 107*
d^4*e^2 - 144*d^3*e^3 - 132*d^2*e^4 + 62*d*e^5 - 15*e^6)*x)*log(e*x + d) +
980*(123*d^5*e - 24*d^4*e^2 - 180*d^3*e^3 + 72*d^2*e^4 - 15*d*e^5 + 5*(41*d
^4*e^2 - 8*d^3*e^3 - 60*d^2*e^4 + 24*d*e^5 - 5*e^6)*x^3 + (205*d^5*e + 42*d
^4*e^2 - 316*d^3*e^3 + 23*d*e^5 - 10*e^6)*x^2 + (82*d^5*e + 107*d^4*e^2 - 1
44*d^3*e^3 - 132*d^2*e^4 + 62*d*e^5 - 15*e^6)*x)*log(5*x^2 + 2*x + 3))/(375
*d^7*e - 450*d^6*e^2 + 855*d^5*e^3 - 564*d^4*e^4 + 513*d^3*e^5 - 162*d^2*e^
6 + 81*d*e^7 + 5*(125*d^6*e^2 - 150*d^5*e^3 + 285*d^4*e^4 - 188*d^3*e^5 + 1
71*d^2*e^6 - 54*d*e^7 + 27*e^8)*x^3 + (625*d^7*e - 500*d^6*e^2 + 1125*d^5*e
^3 - 370*d^4*e^4 + 479*d^3*e^5 + 72*d^2*e^6 + 27*d*e^7 + 54*e^8)*x^2 + (250
*d^7*e + 75*d^6*e^2 + 120*d^5*e^3 + 479*d^4*e^4 - 222*d^3*e^5 + 405*d^2*e^6
 - 108*d*e^7 + 81*e^8)*x)
```

giac [A] time = 0.20, size = 571, normalized size = 1.82

$$\frac{\sqrt{14}\left(1313\,d^4e^2-10044\,d^3e^3+4290\,d^2e^4+156\,de^5-271\,e^6\right)\arctan\left(\frac{1}{14}\sqrt{14}\left(5\,d-\frac{5\,d^2}{xe+d}+\frac{2\,de}{xe+d}-\frac{3\,e^2}{xe+d}-e\right)e^{(-1)}\right)e}{392\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 1/392*sqrt(14)*(1313*d^4*e^2 - 10044*d^3*e^3 + 4290*d^2*e^4 + 156*d*e^5 - 2
71*e^6)*arctan(1/14*sqrt(14)*(5*d - 5*d^2/(x*e + d) + 2*d*e/(x*e + d) - 3*e
^2/(x*e + d) - e)*e^(-1))*e^(-2)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d
^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(41*d^4 - 8*d^3*e - 60*d^2*
e^2 + 24*d*e^3 - 5*e^4)*log(-10*d/(x*e + d) + 5*d^2/(x*e + d)^2 + 2*e/(x*e
+ d) - 2*d*e/(x*e + d)^2 + 3*e^2/(x*e + d)^2 + 5)/(125*d^6 - 150*d^5*e + 28
```

```
5*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - (4*d^4*e^3/(x*
e + d) + 5*d^3*e^4/(x*e + d) + 3*d^2*e^5/(x*e + d) - d*e^6/(x*e + d) + 2*e^
7/(x*e + d))/(25*d^4*e^4 - 20*d^3*e^5 + 34*d^2*e^6 - 12*d*e^7 + 9*e^8) + 1/
28*((423*d^3*e - 4101*d^2*e^2 + 879*d*e^3 + 703*e^4)/(5*d^2 - 2*d*e + 3*e^2
) - (423*d^4*e^2 - 5468*d^3*e^3 + 1758*d^2*e^4 + 2812*d*e^5 - 457*e^6)*e^(-
1)/((5*d^2 - 2*d*e + 3*e^2)*(x*e + d)))/((5*d^2 - 2*d*e + 3*e^2)^2*(10*d/(x
*e + d) - 5*d^2/(x*e + d)^2 - 2*e/(x*e + d) + 2*d*e/(x*e + d)^2 - 3*e^2/(x*
e + d)^2 - 5))
```

maple [B] time = 0.02, size = 986, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x)
```

```
[Out] 1/(5*d^2-2*d*e+3*e^2)^2*e^2/(e*x+d)*d-8/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d^3
*e-60/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d^2*e^2+24/(5*d^2-2*d*e+3*e^2)^3*ln(e
*x+d)*d*e^3-423/140/(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*d^4*x-879/700/(5*
d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*x*e^4+1416/175/(5*d^2-2*d*e+3*e^2)^3/(x^
2+2/5*x+3/5)*d^3*e-879/350/(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*d^2*e^2+88
/175/(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*d*e^3+4/(5*d^2-2*d*e+3*e^2)^3*ln
(5*x^2+2*x+3)*d^3*e+30/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d^2*e^2-12/(5*
d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d*e^3+1313/392/(5*d^2-2*d*e+3*e^2)^3*14^
(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^4-271/392/(5*d^2-2*d*e+3*e^2)^3*14^(
1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^4-4/(5*d^2-2*d*e+3*e^2)^2/e/(e*x+d)*d
^4-3/(5*d^2-2*d*e+3*e^2)^2*e/(e*x+d)*d^2-5/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)*d^
3-1367/140/(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*d^4+2109/700/(5*d^2-2*d*e+
3*e^2)^3/(x^2+2/5*x+3/5)*e^4-41/2/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d^4
+5/2/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*e^4-2/(5*d^2-2*d*e+3*e^2)^2*e^3/
(e*x+d)+41/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d^4-5/(5*d^2-2*d*e+3*e^2)^3*ln(e
*x+d)*e^4+3629/175/(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*x*d^3*e-4101/350/(
5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*x*d^2*e^2+2197/175/(5*d^2-2*d*e+3*e^2)
^3/(x^2+2/5*x+3/5)*x*d*e^3-2511/98/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/
28*(10*x+2)*14^(1/2))*d^3*e+2145/196/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(
1/28*(10*x+2)*14^(1/2))*d^2*e^2+39/98/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan
(1/28*(10*x+2)*14^(1/2))*d*e^3
```

maxima [A] time = 1.02, size = 548, normalized size = 1.75

$$\frac{\sqrt{14}\left(1313\,d^4-10044\,d^3e+4290\,d^2e^2+156\,de^3-271\,e^4\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{392\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}+\frac{\left(41\,d^4-8\,d^3e-6\right.}{125\,d^6-150\,d^5e+285\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 1/392*sqrt(14)*(1313*d^4 - 10044*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4
)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*
d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + (41*d^4 - 8*d^3*e - 60*d^2*e^2
 + 24*d*e^3 - 5*e^4)*log(e*x + d)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*
d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(41*d^4 - 8*d^3*e - 60*d^2
*e^2 + 24*d*e^3 - 5*e^4)*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d^5*e + 285*d^
4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/140*(1680*d^4 +
3467*d^3*e + 674*d^2*e^2 - 1123*d*e^3 + 840*e^4 + (2800*d^4 + 3500*d^3*e +
2523*d^2*e^2 - 3434*d*e^3 + 1693*e^4)*x^2 + (1120*d^4 + 1823*d^3*e - 527*d^
2*e^2 - 573*d*e^3 - 143*e^4)*x)/(75*d^5*e - 60*d^4*e^2 + 102*d^3*e^3 - 36*d
^2*e^4 + 27*d*e^5 + 5*(25*d^4*e^2 - 20*d^3*e^3 + 34*d^2*e^4 - 12*d*e^5 + 9*
```

```
e^6)*x^3 + (125*d^5*e - 50*d^4*e^2 + 130*d^3*e^3 + 8*d^2*e^4 + 21*d*e^5 + 1
8*e^6)*x^2 + (50*d^5*e + 35*d^4*e^2 + 8*d^3*e^3 + 78*d^2*e^4 - 18*d*e^5 + 2
7*e^6)*x)
```

mupad [B] time = 4.84, size = 601, normalized size = 1.92

$$\ln(d+ex)\left(\frac{41}{25\left(5d^2-2de+3e^2\right)}-\frac{4e^3\,(423d-1367e)}{125\left(5d^2-2de+3e^2\right)^3}+\frac{2e\,(310d-1323e)}{125\left(5d^2-2de+3e^2\right)^2}\right)-\frac{\frac{1680d^4+3467d^3e+674d}{140e\left(25d^4-20d^3e+34\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)^2),x)
```

```
[Out] log(d + e*x)*(41/(25*(5*d^2 - 2*d*e + 3*e^2)) - (4*e^3*(423*d - 1367*e))/(1
25*(5*d^2 - 2*d*e + 3*e^2)^3) + (2*e*(310*d - 1323*e))/(125*(5*d^2 - 2*d*e
+ 3*e^2)^2)) - ((3467*d^3*e - 1123*d*e^3 + 1680*d^4 + 840*e^4 + 674*d^2*e^2
)/(140*e*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) - (x*(573*d*e
^3 - 1823*d^3*e - 1120*d^4 + 143*e^4 + 527*d^2*e^2))/(140*e*(25*d^4 - 20*d^
3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x^2*(3500*d^3*e - 3434*d*e^3 + 280
0*d^4 + 1693*e^4 + 2523*d^2*e^2))/(140*e*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*
e^4 + 34*d^2*e^2)))/(3*d + x^2*(5*d + 2*e) + 5*e*x^3 + x*(2*d + 3*e)) + (lo
g(x - (14^(1/2)*1i)/5 + 1/5)*(d^4*((1313*14^(1/2))/784 - 41i/2) - e^4*((271
*14^(1/2))/784 - 5i/2) + d^2*e^2*((2145*14^(1/2))/392 + 30i) + d*e^3*((39*1
4^(1/2))/196 - 12i) - d^3*e*((2511*14^(1/2))/196 - 4i)))/(d^6*125i - d^5*e*
150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*188i + d^4*e^2*285i) -
(log(x + (14^(1/2)*1i)/5 + 1/5)*(d^4*((1313*14^(1/2))/784 + 41i/2) - e^4*((
271*14^(1/2))/784 + 5i/2) + d^2*e^2*((2145*14^(1/2))/392 - 30i) + d*e^3*((3
9*14^(1/2))/196 + 12i) - d^3*e*((2511*14^(1/2))/196 + 4i)))/(d^6*125i - d^5
*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*188i + d^4*e^2*285i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3)**2,x)
```

```
[Out] Timed out
```

3.317 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2}\,dx$

Optimal. Leaf size=412

$$-\frac{1367d^3-879d^2e+x\left(423d^3-4101d^2e+879de^2+703e^3\right)-2109de^2+457e^3}{28\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^3}-\frac{41d^4-8d^3e-60d^2e^2+24de^{\ldots}}{\left(5d^2-2de+3e^2\right)^3(d+e\ldots}$$

```
[Out] 1/2*(-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)^
2+(-41*d^4+8*d^3*e+60*d^2*e^2-24*d*e^3+5*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)
+1/28*(-1367*d^3+879*d^2*e+2109*d*e^2-457*e^3-(423*d^3-4101*d^2*e+879*d*e^2
+703*e^3)*x)/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)+(205*d^5-19*d^4*e-846*d^3*
e^2+396*d^2*e^3+57*d*e^4-21*e^5)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^4-1/2*(205*d
^5-19*d^4*e-846*d^3*e^2+396*d^2*e^3+57*d*e^4-21*e^5)*ln(5*x^2+2*x+3)/(5*d^2
-2*d*e+3*e^2)^4+1/392*(6565*d^5-74017*d^4*e+35022*d^3*e^2+42858*d^2*e^3-172
47*d*e^4+579*e^5)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^4*14^(1
/2)
```

Rubi [A] time = 0.71, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1646, 1628, 634, 618, 204, 628}

$$-\frac{x\left(-4101d^2e+423d^3+879de^2+703e^3\right)-879d^2e+1367d^3-2109de^2+457e^3}{28\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^3}-\frac{\left(-846d^3e^2+396d^2e^3-19d\ldots\right.}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)^2),x]
```

```
[Out] -(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(2*e*(5*d^2 - 2*d*e + 3*e^2)
^2*(d + e*x)^2) - (41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)/((5*d^
2 - 2*d*e + 3*e^2)^3*(d + e*x)) - (1367*d^3 - 879*d^2*e - 2109*d*e^2 + 457*
e^3 + (423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*x)/(28*(5*d^2 - 2*d*e +
3*e^2)^3*(3 + 2*x + 5*x^2)) + ((6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42
858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]])/(28*Sqrt[1
4]*(5*d^2 - 2*d*e + 3*e^2)^4) + ((205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^
2*e^3 + 57*d*e^4 - 21*e^5)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^4 - ((205*
d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*Log[3 + 2*x
 + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^4)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)^2}\,dx = -\frac{1367d^3-879d^2e-2109de^2+457e^3+\left(423d^3-4101d^2e+879de^2+703e^3\right)x}{28\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)}$$

$$= -\frac{1367d^3-879d^2e-2109de^2+457e^3+\left(423d^3-4101d^2e+879de^2+703e^3\right)x}{28\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2}-\frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{136}{}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2}-\frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{136}{}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2}-\frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{136}{}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2}-\frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{136}{}$$

Mathematica [A] time = 0.40, size = 363, normalized size = 0.88

$$-\frac{14\left(5d^2-2de+3e^2\right)\left(d^3(423x+1367)-3d^2e(1367x+293)+3de^2(293x-703)+e^3(703x+457)\right)}{5x^2+2x+3}-\frac{196\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\left(5d^2-2de+3e^2\right)^2}{e(d+ex)^2}+\frac{392}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)^2)
,x]
```

```
[Out] ((-196*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e
^4))/(e*(d + e*x)^2) + (392*(5*d^2 - 2*d*e + 3*e^2)*(-41*d^4 + 8*d^3*e + 60
*d^2*e^2 - 24*d*e^3 + 5*e^4))/(d + e*x) - (14*(5*d^2 - 2*d*e + 3*e^2)*(3*d*
e^2*(-703 + 293*x) + d^3*(1367 + 423*x) + e^3*(457 + 703*x) - 3*d^2*e*(293
+ 1367*x)))/(3 + 2*x + 5*x^2) + Sqrt[14]*(6565*d^5 - 74017*d^4*e + 35022*d^
3*e^2 + 42858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]] +
 392*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*L
og[d + e*x] + 196*(-205*d^5 + 19*d^4*e + 846*d^3*e^2 - 396*d^2*e^3 - 57*d*e
^4 + 21*e^5)*Log[3 + 2*x + 5*x^2])/(392*(5*d^2 - 2*d*e + 3*e^2)^4)
```

fricas [B] time = 1.46, size = 1499, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] -1/392*(58800*d^8 + 363230*d^7*e - 178010*d^6*e^2 - 233184*d^5*e^3 + 395164
*d^4*e^4 - 437122*d^3*e^5 + 178542*d^2*e^6 - 37044*d*e^7 + 10584*e^8 + 14*(
28700*d^6*e^2 - 14965*d^5*e^3 - 43891*d^4*e^4 + 44106*d^3*e^5 - 45966*d^2*e
^6 + 12711*d*e^7 + 9*e^8)*x^3 + 14*(7000*d^8 + 31850*d^7*e + 6400*d^6*e^2 -
 62649*d^5*e^3 + 52187*d^4*e^4 - 53652*d^3*e^5 + 11130*d^2*e^6 - 2841*d*e^7
 + 1791*e^8)*x^2 - sqrt(14)*(19695*d^7*e - 222051*d^6*e^2 + 105066*d^5*e^3
+ 128574*d^4*e^4 - 51741*d^3*e^5 + 1737*d^2*e^6 + 5*(6565*d^5*e^3 - 74017*d
^4*e^4 + 35022*d^3*e^5 + 42858*d^2*e^6 - 17247*d*e^7 + 579*e^8)*x^4 + 2*(32
825*d^6*e^2 - 363520*d^5*e^3 + 101093*d^4*e^4 + 249312*d^3*e^5 - 43377*d^2*
e^6 - 14352*d*e^7 + 579*e^8)*x^3 + (32825*d^7*e - 343825*d^6*e^2 - 101263*d
^5*e^3 + 132327*d^4*e^4 + 190263*d^3*e^5 + 62481*d^2*e^6 - 49425*d*e^7 + 17
37*e^8)*x^2 + 2*(6565*d^7*e - 54322*d^6*e^2 - 187029*d^5*e^3 + 147924*d^4*e
^4 + 111327*d^3*e^5 - 51162*d^2*e^6 + 1737*d*e^7)*x)*arctan(1/14*sqrt(14)*(
5*x + 1)) + 14*(2800*d^8 + 14855*d^7*e + 5815*d^6*e^2 - 18620*d^5*e^3 - 172
02*d^4*e^4 + 11119*d^3*e^5 - 26037*d^2*e^6 + 7866*d*e^7 - 756*e^8)*x - 392*
(615*d^7*e - 57*d^6*e^2 - 2538*d^5*e^3 + 1188*d^4*e^4 + 171*d^3*e^5 - 63*d^
2*e^6 + 5*(205*d^5*e^3 - 19*d^4*e^4 - 846*d^3*e^5 + 396*d^2*e^6 + 57*d*e^7
- 21*e^8)*x^4 + 2*(1025*d^6*e^2 + 110*d^5*e^3 - 4249*d^4*e^4 + 1134*d^3*e^5
 + 681*d^2*e^6 - 48*d*e^7 - 21*e^8)*x^3 + (1025*d^7*e + 725*d^6*e^2 - 3691*
d^5*e^3 - 1461*d^4*e^4 - 669*d^3*e^5 + 1311*d^2*e^6 + 87*d*e^7 - 63*e^8)*x^
2 + 2*(205*d^7*e + 596*d^6*e^2 - 903*d^5*e^3 - 2142*d^4*e^4 + 1245*d^3*e^5
+ 150*d^2*e^6 - 63*d*e^7)*x)*log(e*x + d) + 196*(615*d^7*e - 57*d^6*e^2 - 2
538*d^5*e^3 + 1188*d^4*e^4 + 171*d^3*e^5 - 63*d^2*e^6 + 5*(205*d^5*e^3 - 19
*d^4*e^4 - 846*d^3*e^5 + 396*d^2*e^6 + 57*d*e^7 - 21*e^8)*x^4 + 2*(1025*d^6
*e^2 + 110*d^5*e^3 - 4249*d^4*e^4 + 1134*d^3*e^5 + 681*d^2*e^6 - 48*d*e^7 -
 21*e^8)*x^3 + (1025*d^7*e + 725*d^6*e^2 - 3691*d^5*e^3 - 1461*d^4*e^4 - 66
9*d^3*e^5 + 1311*d^2*e^6 + 87*d*e^7 - 63*e^8)*x^2 + 2*(205*d^7*e + 596*d^6*
e^2 - 903*d^5*e^3 - 2142*d^4*e^4 + 1245*d^3*e^5 + 150*d^2*e^6 - 63*d*e^7)*x
)*log(5*x^2 + 2*x + 3))/(1875*d^10*e - 3000*d^9*e^2 + 6300*d^8*e^3 - 5880*d
^7*e^4 + 6258*d^6*e^5 - 3528*d^5*e^6 + 2268*d^4*e^7 - 648*d^3*e^8 + 243*d^2
*e^9 + 5*(625*d^8*e^3 - 1000*d^7*e^4 + 2100*d^6*e^5 - 1960*d^5*e^6 + 2086*d
^4*e^7 - 1176*d^3*e^8 + 756*d^2*e^9 - 216*d*e^10 + 81*e^11)*x^4 + 2*(3125*d
^9*e^2 - 4375*d^8*e^3 + 9500*d^7*e^4 - 7700*d^6*e^5 + 8470*d^5*e^6 - 3794*d
^4*e^7 + 2604*d^3*e^8 - 324*d^2*e^9 + 189*d*e^10 + 81*e^11)*x^3 + (3125*d^1
0*e - 2500*d^9*e^2 + 8375*d^8*e^3 - 4400*d^7*e^4 + 8890*d^6*e^5 - 3416*d^5*
e^6 + 5334*d^4*e^7 - 1584*d^3*e^8 + 1809*d^2*e^9 - 324*d*e^10 + 243*e^11)*x
^2 + 2*(625*d^10*e + 875*d^9*e^2 - 900*d^8*e^3 + 4340*d^7*e^4 - 3794*d^6*e^
5 + 5082*d^5*e^6 - 2772*d^4*e^7 + 2052*d^3*e^8 - 567*d^2*e^9 + 243*d*e^10)*
x)
```

giac [A] time = 0.20, size = 595, normalized size = 1.44

$$\frac{\sqrt{14}\left(6565\,d^{5}-74017\,d^{4}e+35022\,d^{3}e^{2}+42858\,d^{2}e^{3}-17247\,de^{4}+579\,e^{5}\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{392\left(625\,d^{8}-1000\,d^{7}e+2100\,d^{6}e^{2}-1960\,d^{5}e^{3}+2086\,d^{4}e^{4}-1176\,d^{3}e^{5}+756\,d^{2}e^{6}-216\,de^{7}+81\,e^{8}\right)}-\frac{}{2\left(625\,d^{8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 1/392*sqrt(14)*(6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 17
247*d*e^4 + 579*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(625*d^8 - 1000*d^7*e
+ 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 -
 216*d*e^7 + 81*e^8) - 1/2*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3
+ 57*d*e^4 - 21*e^5)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6*
e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7
+ 81*e^8) + (205*d^5*e - 19*d^4*e^2 - 846*d^3*e^3 + 396*d^2*e^4 + 57*d*e^5
- 21*e^6)*log(abs(x*e + d))/(625*d^8*e - 1000*d^7*e^2 + 2100*d^6*e^3 - 1960
*d^5*e^4 + 2086*d^4*e^5 - 1176*d^3*e^6 + 756*d^2*e^7 - 216*d*e^8 + 81*e^9)
- 1/28*(4200*d^8 + 25945*d^7*e - 12715*d^6*e^2 - 16656*d^5*e^3 + 28226*d^4*
e^4 + (28700*d^6*e^2 - 14965*d^5*e^3 - 43891*d^4*e^4 + 44106*d^3*e^5 - 4596
6*d^2*e^6 + 12711*d*e^7 + 9*e^8)*x^3 - 31223*d^3*e^5 + (7000*d^8 + 31850*d^
7*e + 6400*d^6*e^2 - 62649*d^5*e^3 + 52187*d^4*e^4 - 53652*d^3*e^5 + 11130*
d^2*e^6 - 2841*d*e^7 + 1791*e^8)*x^2 + 12753*d^2*e^6 + (2800*d^8 + 14855*d^
7*e + 5815*d^6*e^2 - 18620*d^5*e^3 - 17202*d^4*e^4 + 11119*d^3*e^5 - 26037*
d^2*e^6 + 7866*d*e^7 - 756*e^8)*x - 2646*d*e^7 + 756*e^8)*e^(-1)/((5*d^2 -
2*d*e + 3*e^2)^4*(5*x^2 + 2*x + 3)*(x*e + d)^2)
```

maple [B] time = 0.03, size = 1314, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x)
```

```
[Out] -3/2/(5*d^2-2*d*e+3*e^2)^2*e/(e*x+d)^2*d^2+1/2/(5*d^2-2*d*e+3*e^2)^2*e^2/(e
*x+d)^2*d+8/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d^3*e+60/(5*d^2-2*d*e+3*e^2)^3/(e
*x+d)*d^2*e^2-24/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d*e^3-19/(5*d^2-2*d*e+3*e^2)
^4*ln(e*x+d)*d^4*e-846/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d^3*e^2+396/(5*d^2-2
*d*e+3*e^2)^4*ln(e*x+d)*d^2*e^3+57/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d*e^4-42
3/28/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*x*d^5-2109/140/(5*d^2-2*d*e+3*e^
2)^4/(x^2+2/5*x+3/5)*x*e^5+7129/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d
^4*e+2343/70/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d^3*e^2-1933/70/(5*d^2-2
*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d^2*e^3+7241/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2
/5*x+3/5)*d*e^4+19/2/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^4*e+423/(5*d^2
-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^3*e^2-198/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+
2*x+3)*d^2*e^3-57/2/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d*e^4+6565/392/(5
*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^5+579/392/(5*
d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^5-2/(5*d^2-2*d
*e+3*e^2)^2/e/(e*x+d)^2*d^4-5/2/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)^2*d^3+205/(5*
d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d^5-21/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*e^5-136
7/28/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d^5-1371/140/(5*d^2-2*d*e+3*e^2)
^4/(x^2+2/5*x+3/5)*e^5-205/2/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^5+21/2
/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*e^5-1/(5*d^2-2*d*e+3*e^2)^2*e^3/(e*x
+d)^2-41/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d^4+5/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*
e^4+21429/196/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
*d^2*e^3-17247/392/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(
1/2))*d*e^4+21351/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*x*d^4*e-6933/70
```

```
/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*x*d^3*e^2+5273/70/(5*d^2-2*d*e+3*e^2
)^4/(x^2+2/5*x+3/5)*x*d^2*e^3-1231/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5
)*x*d*e^4-74017/392/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^
(1/2))*d^4*e+17511/196/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*
14^(1/2))*d^3*e^2
```

maxima [B] time = 1.09, size = 851, normalized size = 2.07

$$\frac{\sqrt{14}\left(6565\,d^5-74017\,d^4e+35022\,d^3e^2+42858\,d^2e^3-17247\,de^4+579\,e^5\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{392\left(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8\right)}+\frac{}{625\,d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 1/392*sqrt(14)*(6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 17
247*d*e^4 + 579*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(625*d^8 - 1000*d^7*e
+ 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 -
 216*d*e^7 + 81*e^8) + (205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57
*d*e^4 - 21*e^5)*log(e*x + d)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d
^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) -
1/2*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*lo
g(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 20
86*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - 1/28*(840*d
^6 + 5525*d^5*e - 837*d^4*e^2 - 6981*d^3*e^3 + 3355*d^2*e^4 - 714*d*e^5 + 2
52*e^6 + (5740*d^4*e^2 - 697*d^3*e^3 - 12501*d^2*e^4 + 4239*d*e^5 + 3*e^6)*
x^3 + (1400*d^6 + 6930*d^5*e + 3212*d^4*e^2 - 15403*d^3*e^3 + 2349*d^2*e^4
- 549*d*e^5 + 597*e^6)*x^2 + (560*d^6 + 3195*d^5*e + 2105*d^4*e^2 - 4799*d^
3*e^3 - 6623*d^2*e^4 + 2454*d*e^5 - 252*e^6)*x)/(375*d^8*e - 450*d^7*e^2 +
855*d^6*e^3 - 564*d^5*e^4 + 513*d^4*e^5 - 162*d^3*e^6 + 81*d^2*e^7 + 5*(125
*d^6*e^3 - 150*d^5*e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8
 + 27*e^9)*x^4 + 2*(625*d^7*e^2 - 625*d^6*e^3 + 1275*d^5*e^4 - 655*d^4*e^5
+ 667*d^3*e^6 - 99*d^2*e^7 + 81*d*e^8 + 27*e^9)*x^3 + (625*d^8*e - 250*d^7*
e^2 + 1200*d^6*e^3 - 250*d^5*e^4 + 958*d^4*e^5 - 150*d^3*e^6 + 432*d^2*e^7
- 54*d*e^8 + 81*e^9)*x^2 + 2*(125*d^8*e + 225*d^7*e^2 - 165*d^6*e^3 + 667*d
^5*e^4 - 393*d^4*e^5 + 459*d^3*e^6 - 135*d^2*e^7 + 81*d*e^8)*x)
```

mupad [B] time = 4.94, size = 887, normalized size = 2.15

$$\ln(d+e\,x)\left(\frac{\frac{41\,d}{5}+\frac{29\,e}{5}}{\left(5\,d^2-2\,d\,e+3\,e^2\right)^2}+\frac{168\,e^4\,(458\,d-7\,e)}{125\left(5\,d^2-2\,d\,e+3\,e^2\right)^4}-\frac{2\,e^2\,(12610\,d+1329\,e)}{125\left(5\,d^2-2\,d\,e+3\,e^2\right)^3}\right)-\frac{\frac{840\,d^6+5525\,d^5\,e-837\,d^4}{28\,e\left(125\,d^6-150\,d^5\,e+28\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^3*(2*x + 5*x^2 + 3)^2),x)
```

```
[Out] log(d + e*x)*(((41*d)/5 + (29*e)/5)/(5*d^2 - 2*d*e + 3*e^2)^2 + (168*e^4*(4
58*d - 7*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^4) - (2*e^2*(12610*d + 1329*e))/(
125*(5*d^2 - 2*d*e + 3*e^2)^3)) - ((5525*d^5*e - 714*d*e^5 + 840*d^6 + 252*
e^6 + 3355*d^2*e^4 - 6981*d^3*e^3 - 837*d^4*e^2)/(28*e*(125*d^6 - 150*d^5*e
 - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x^3*(42
39*d*e^4 + 5740*d^4*e + 3*e^5 - 12501*d^2*e^3 - 697*d^3*e^2))/(28*(125*d^6
- 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2))
 + (x^2*(6930*d^5*e - 549*d*e^5 + 1400*d^6 + 597*e^6 + 2349*d^2*e^4 - 15403
*d^3*e^3 + 3212*d^4*e^2))/(28*e*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 +
171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x*(2454*d*e^5 + 3195*d^5*e + 5
60*d^6 - 252*e^6 - 6623*d^2*e^4 - 4799*d^3*e^3 + 2105*d^4*e^2))/(28*e*(125*
d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e
```

```
^2)))/(x^2*(4*d*e + 5*d^2 + 3*e^2) + x*(6*d*e + 2*d^2) + 3*d^2 + x^3*(10*d*
e + 2*e^2) + 5*e^2*x^4) + (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^5*((6565*14^(1
/2))/784 - 205i/2) + e^5*((579*14^(1/2))/784 + 21i/2) + d^3*e^2*((17511*14^
(1/2))/392 + 423i) + d^2*e^3*((21429*14^(1/2))/392 - 198i) - d*e^4*((17247*
14^(1/2))/784 + 57i/2) - d^4*e*((74017*14^(1/2))/784 - 19i/2)))/(d^8*625i -
 d^7*e*1000i - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*e^5*1176i + d^4*e^
4*2086i - d^5*e^3*1960i + d^6*e^2*2100i) - (log(x + (14^(1/2)*1i)/5 + 1/5)*
(d^5*((6565*14^(1/2))/784 + 205i/2) + e^5*((579*14^(1/2))/784 - 21i/2) + d^
3*e^2*((17511*14^(1/2))/392 - 423i) + d^2*e^3*((21429*14^(1/2))/392 + 198i)
 - d*e^4*((17247*14^(1/2))/784 - 57i/2) - d^4*e*((74017*14^(1/2))/784 + 19i
/2)))/(d^8*625i - d^7*e*1000i - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*e
^5*1176i + d^4*e^4*2086i - d^5*e^3*1960i + d^6*e^2*2100i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3/(5*x**2+2*x+3)**2,x)
[Out] Timed out
```

3.318 $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$

Optimal. Leaf size=171

$$\frac{3e\left(100d^2-245de+47e^2\right)\log\left(5x^2+2x+3\right)}{6250}+\frac{3\left(353125d^3-855175d^2e+74085de^2+556349e^3\right)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{4900000\sqrt{14}}$$

```
[Out] 1/980000*(83065*d-126009*e)*e^2*x+2/125*e^3*x^2-1/7000*(1367+423*x)*(e*x+d)
^3/(5*x^2+2*x+3)^2+1/196000*(e*x+d)^2*(34347*d-6315*e+(11015*d+49177*e)*x)/
(5*x^2+2*x+3)+3/6250*e*(100*d^2-245*d*e+47*e^2)*ln(5*x^2+2*x+3)+3/68600000*
(353125*d^3-855175*d^2*e+74085*d*e^2+556349*e^3)*arctan(1/14*(1+5*x)*14^(1/
2))*14^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1644, 1628, 634, 618, 204, 628}

$$\frac{3e\left(100d^2-245de+47e^2\right)\log\left(5x^2+2x+3\right)}{6250}+\frac{3\left(-855175d^2e+353125d^3+74085de^2+556349e^3\right)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{4900000\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] ((83065*d - 126009*e)*e^2*x)/980000 + (2*e^3*x^2)/125 - ((1367 + 423*x)*(d
+ e*x)^3)/(7000*(3 + 2*x + 5*x^2)^2) + ((d + e*x)^2*(3*(11449*d - 2105*e) +
 (11015*d + 49177*e)*x))/(196000*(3 + 2*x + 5*x^2)) + (3*(353125*d^3 - 8551
75*d^2*e + 74085*d*e^2 + 556349*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(4900000*S
qrt[14]) + (3*e*(100*d^2 - 245*d*e + 47*e^2)*Log[3 + 2*x + 5*x^2])/6250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}dx = -\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int \frac{(d+ex)^2\left(\frac{6}{125}(1089d+1367e)-\right.}{(3}$$

$$= -\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)^2(3(11449d-2105e)+(11015d}{196000\left(3+2x+5x^2\right)}$$

$$= -\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)^2(3(11449d-2105e)+(11015d}{196000\left(3+2x+5x^2\right)}$$

$$= \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+e}{}$$

$$= \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+e}{}$$

$$= \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+e}{}$$

$$= \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+e}{}$$

Mathematica [A] time = 0.20, size = 209, normalized size = 1.22

$$164640e\left(100d^2-245de+47e^2\right)\log\left(5x^2+2x+3\right)-\frac{392\left(125d^3(423x+1367)+75d^2e(5989x-1269)-15de^2(18323x+17967)+e^3(5496\right.}{\left(5x^2+2x+3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3
,x]
```

```
[Out] (548800*(60*d - 49*e)*e^2*x + 5488000*e^3*x^2 - (392*(e^3*(54969 - 53189*x)
 + 125*d^3*(1367 + 423*x) + 75*d^2*e*(-1269 + 5989*x) - 15*d*e^2*(17967 + 1
8323*x)))/(3 + 2*x + 5*x^2)^2 + (14*(e^3*(2639639 - 3109005*x) + 125*d^3*(3
4347 + 11015*x) + 75*d^2*e*(-44399 + 181765*x) - 15*d*e^2*(809167 + 647195*
x)))/(3 + 2*x + 5*x^2) + 15*Sqrt[14]*(353125*d^3 - 855175*d^2*e + 74085*d*e
^2 + 556349*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + 164640*e*(100*d^2 - 245*d*e +
 47*e^2)*Log[3 + 2*x + 5*x^2])/343000000
```

fricas [B] time = 0.84, size = 441, normalized size = 2.58

$$\frac{27440000\, e^3x^6 + 2744000\left(60\, de^2 - 41\, e^3\right)x^5 + 8780800\left(15\, de^2 - 8\, e^3\right)x^4 + 70\left(275375\, d^3 + 2726475\, d^2e + 12\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="f
ricas")
```

```
[Out] 1/68600000*(27440000*e^3*x^6 + 2744000*(60*d*e^2 - 41*e^3)*x^5 + 8780800*(1
5*d*e^2 - 8*e^3)*x^4 + 70*(275375*d^3 + 2726475*d^2*e + 1257135*d*e^2 - 304
5929*e^3)*x^3 + 22667750*d^3 - 20509650*d^2*e - 80825850*d*e^2 + 17863398*e
^3 + 14*(4844125*d^3 + 2123025*d^2*e - 10375875*d*e^2 - 2508283*e^3)*x^2 +
3*sqrt(14)*(25*(353125*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*x^4 +
 20*(353125*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*x^3 + 3178125*d^
3 - 7696575*d^2*e + 666765*d*e^2 + 5007141*e^3 + 34*(353125*d^3 - 855175*d^
2*e + 74085*d*e^2 + 556349*e^3)*x^2 + 12*(353125*d^3 - 855175*d^2*e + 74085
*d*e^2 + 556349*e^3)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 42*(749125*d^3 +
1444025*d^2*e - 1635675*d*e^2 - 1323043*e^3)*x + 32928*(25*(100*d^2*e - 245
*d*e^2 + 47*e^3)*x^4 + 20*(100*d^2*e - 245*d*e^2 + 47*e^3)*x^3 + 900*d^2*e
- 2205*d*e^2 + 423*e^3 + 34*(100*d^2*e - 245*d*e^2 + 47*e^3)*x^2 + 12*(100*
d^2*e - 245*d*e^2 + 47*e^3)*x)*log(5*x^2 + 2*x + 3))/(25*x^4 + 20*x^3 + 34*
x^2 + 12*x + 9)
```

giac [A] time = 0.21, size = 201, normalized size = 1.18

$$\frac{2}{125}\, x^2e^3 + \frac{12}{125}\, dxe^2 + \frac{3}{68600000}\, \sqrt{14}\left(353125\, d^3 - 855175\, d^2e + 74085\, de^2 + 556349\, e^3\right) \arctan\left(\frac{1}{14}\, \sqrt{14}\,(5\, x + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="g
iac")
```

```
[Out] 2/125*x^2*e^3 + 12/125*d*x*e^2 + 3/68600000*sqrt(14)*(353125*d^3 - 855175*d
^2*e + 74085*d*e^2 + 556349*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) - 49/625*x
*e^3 + 3/6250*(100*d^2*e - 245*d*e^2 + 47*e^3)*log(5*x^2 + 2*x + 3) + 1/490
0000*(5*(275375*d^3 + 2726475*d^2*e - 1941585*d*e^2 - 621801*e^3)*x^3 + 161
9125*d^3 + (4844125*d^3 + 2123025*d^2*e - 16020675*d*e^2 + 1396037*e^3)*x^2
 - 1464975*d^2*e + 3*(749125*d^3 + 1444025*d^2*e - 3046875*d*e^2 - 170563*e
^3)*x - 5773275*d*e^2 + 1275957*e^3)/(5*x^2 + 2*x + 3)^2
```

maple [A] time = 0.02, size = 267, normalized size = 1.56

$$\frac{2e^3x^2}{125} + \frac{339\sqrt{14}\, d^3 \arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{21952} - \frac{102621\sqrt{14}\, d^2e \arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{2744000} + \frac{6d^2e\ln\left(5x^2+2x+3\right)}{125} + \frac{12d\,e^2\cdots}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x)
```

```
[Out] 2/125*e^3*x^2+12/125*d*e^2*x-49/625*e^3*x+1/25*((11015/1568*d^3+109059/1568
*d^2*e-388317/7840*d*e^2-621801/39200*e^3)*x^3+(38753/1568*d^3+84921/7840*d
^2*e-640827/7840*d*e^2+1396037/196000*e^3)*x^2+(17979/1568*d^3+173283/7840*
d^2*e-73125/1568*d*e^2-511689/196000*e^3)*x+12953/1568*d^3-58599/7840*d^2*e
-230931/7840*d*e^2+1275957/196000*e^3)/(5*x^2+2*x+3)^2+6/125*d^2*e*ln(5*x^2
+2*x+3)-147/1250*d*e^2*ln(5*x^2+2*x+3)+141/6250*e^3*ln(5*x^2+2*x+3)+339/219
52*14^(1/2)*d^3*arctan(1/28*(10*x+2)*14^(1/2))-102621/2744000*14^(1/2)*d^2*
e*arctan(1/28*(10*x+2)*14^(1/2))+44451/13720000*14^(1/2)*d*e^2*arctan(1/28*
(10*x+2)*14^(1/2))+1669047/68600000*14^(1/2)*e^3*arctan(1/28*(10*x+2)*14^(1
/2))
```

maxima [A] time = 0.97, size = 222, normalized size = 1.30

$$\frac{2}{125}\,e^3x^2+\frac{3}{68600000}\,\sqrt{14}\left(353125\,d^3-855175\,d^2e+74085\,de^2+556349\,e^3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{1}{625}\,(60$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="m
axima")
```

```
[Out] 2/125*e^3*x^2 + 3/68600000*sqrt(14)*(353125*d^3 - 855175*d^2*e + 74085*d*e^
2 + 556349*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(60*d*e^2 - 49*e^3)
*x + 3/6250*(100*d^2*e - 245*d*e^2 + 47*e^3)*log(5*x^2 + 2*x + 3) + 1/49000
00*(5*(275375*d^3 + 2726475*d^2*e - 1941585*d*e^2 - 621801*e^3)*x^3 + 16191
25*d^3 - 1464975*d^2*e - 5773275*d*e^2 + 1275957*e^3 + (4844125*d^3 + 21230
25*d^2*e - 16020675*d*e^2 + 1396037*e^3)*x^2 + 3*(749125*d^3 + 1444025*d^2*
e - 3046875*d*e^2 - 170563*e^3)*x)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

mupad [B] time = 0.15, size = 299, normalized size = 1.75

$$x\left(\frac{e^2\,\left(12\,d-5\,e\right)}{125}-\frac{24\,e^3}{625}\right)-\frac{\frac{1154655\,d\,e^2}{1568}+\frac{292995\,d^2\,e}{1568}+x\,\left(-\frac{449475\,d^3}{1568}-\frac{866415\,d^2\,e}{1568}+\frac{1828125\,d\,e^2}{1568}+\frac{511689\,e^3}{7840}\right)-\frac{323825\,d^3}{1568}}{15625\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

```
[Out] x*((e^2*(12*d - 5*e))/125 - (24*e^3)/625) - ((1154655*d*e^2)/1568 + (292995
*d^2*e)/1568 + x*((1828125*d*e^2)/1568 - (866415*d^2*e)/1568 - (449475*d^3)
/1568 + (511689*e^3)/7840) - (323825*d^3)/1568 - (1275957*e^3)/7840 + x^3*(
(1941585*d*e^2)/1568 - (2726475*d^2*e)/1568 - (275375*d^3)/1568 + (621801*e
^3)/1568) - x^2*((424605*d^2*e)/1568 - (3204135*d*e^2)/1568 + (968825*d^3)/
1568 + (1396037*e^3)/7840))/(7500*x + 21250*x^2 + 12500*x^3 + 15625*x^4 + 5
625) + log(2*x + 5*x^2 + 3)*((6*d^2*e)/125 - (147*d*e^2)/1250 + (141*e^3)/6
250) + (2*e^3*x^2)/125 + (3*14^(1/2)*atan(((3*14^(1/2)*(74085*d*e^2 - 85517
5*d^2*e + 353125*d^3 + 556349*e^3))/68600000 + (3*14^(1/2)*x*(74085*d*e^2 -
 855175*d^2*e + 353125*d^3 + 556349*e^3))/13720000)/((44451*d*e^2)/980000 -
 (102621*d^2*e)/196000 + (339*d^3)/1568 + (1669047*e^3)/4900000))*(74085*d*
e^2 - 855175*d^2*e + 353125*d^3 + 556349*e^3))/68600000
```

sympy [C] time = 8.08, size = 469, normalized size = 2.74

$$\frac{2e^3x^2}{125}+x\left(\frac{12de^2}{125}-\frac{49e^3}{625}\right)+\left(\frac{3e\left(100d^2-245de+47e^2\right)}{6250}-\frac{3\sqrt{14}\,i\left(353125d^3-855175d^2e+74085de^2+556349e^3\right.}{137200000}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] 2*e**3*x**2/125 + x*(12*d*e**2/125 - 49*e**3/625) + (3*e*(100*d**2 - 245*d*
e + 47*e**2)/6250 - 3*sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**
2 + 556349*e**3)/137200000)*log(x + (211875*d**3 - 1830225*d**2*e + 3271395
*d*e**2 - 285237*e**3 + 65856*e*(100*d**2 - 245*d*e + 47*e**2)/5 - 3*sqrt(1
4)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**2 + 556349*e**3)/5)/(1059375
*d**3 - 2565525*d**2*e + 222255*d*e**2 + 1669047*e**3)) + (3*e*(100*d**2 -
245*d*e + 47*e**2)/6250 + 3*sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085
*d*e**2 + 556349*e**3)/137200000)*log(x + (211875*d**3 - 1830225*d**2*e + 3
271395*d*e**2 - 285237*e**3 + 65856*e*(100*d**2 - 245*d*e + 47*e**2)/5 + 3*
sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**2 + 556349*e**3)/5)/(1
059375*d**3 - 2565525*d**2*e + 222255*d*e**2 + 1669047*e**3)) + (1619125*d*
*3 - 1464975*d**2*e - 5773275*d*e**2 + 1275957*e**3 + x**3*(1376875*d**3 +
13632375*d**2*e - 9707925*d*e**2 - 3109005*e**3) + x**2*(4844125*d**3 + 212
3025*d**2*e - 16020675*d*e**2 + 1396037*e**3) + x*(2247375*d**3 + 4332075*d
**2*e - 9140625*d*e**2 - 511689*e**3))/(122500000*x**4 + 98000000*x**3 + 16
6600000*x**2 + 58800000*x + 44100000)
```

3.319 $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$

Optimal. Leaf size=134

$$\frac{(211875d^2-342070de+14817e^2)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{980000\sqrt{14}}+\frac{(d+ex)(5x(2203d+8553e)+34347d-6413e)}{196000(5x^2+2x+3)}-\frac{(423x+1367}{7000(5x^2+}$$

[Out] 4/125*x*e^2-1/7000*(1367+423*x)*(e*x+d)^2/(5*x^2+2*x+3)^2+1/196000*(e*x+d)*(34347*d-6413*e+5*(2203*d+8553*e)*x)/(5*x^2+2*x+3)+1/1250*(40*d-49*e)*e*ln(5*x^2+2*x+3)+1/13720000*(211875*d^2-342070*d*e+14817*e^2)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)

Rubi [A] time = 0.24, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1644, 1657, 634, 618, 204, 628}

$$\frac{(211875d^2-342070de+14817e^2)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{980000\sqrt{14}}+\frac{(d+ex)(5x(2203d+8553e)+34347d-6413e)}{196000(5x^2+2x+3)}-\frac{(423x+1367}{7000(5x^2+}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]

[Out] (4*e^2*x)/125 - ((1367 + 423*x)*(d + e*x)^2)/(7000*(3 + 2*x + 5*x^2)^2) + ((d + e*x)*(34347*d - 6413*e + 5*(2203*d + 8553*e)*x))/(196000*(3 + 2*x + 5*x^2)) + ((211875*d^2 - 342070*d*e + 14817*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/(980000*Sqrt[14]) + ((40*d - 49*e)*e*Log[3 + 2*x + 5*x^2])/1250

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
```

```
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx = -\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int \frac{(d+ex)\left(\frac{2}{125}(3267d+2734e)\right.}{}$$

$$= -\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)(34347d-6413e+5(2203d}{196000\left(3+2x+5x^2\right)}$$

$$= -\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)(34347d-6413e+5(2203d}{196000\left(3+2x+5x^2\right)}$$

$$= \frac{4e^2x}{125}-\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)(34347d-6413e+5(2}{196000\left(3+2x+5\right.}$$

$$= \frac{4e^2x}{125}-\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)(34347d-6413e+5(2}{196000\left(3+2x+5\right.}$$

$$= \frac{4e^2x}{125}-\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)(34347d-6413e+5(2}{196000\left(3+2x+5\right.}$$

$$= \frac{4e^2x}{125}-\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)(34347d-6413e+5(2}{196000\left(3+2x+5\right.}$$

Mathematica [A] time = 0.18, size = 146, normalized size = 1.09

$$\frac{70\left(\frac{5\left(5d^2\left(11015x^3+38753x^2+17979x+12953\right)+2de\left(181765x^3+28307x^2+57761x-19533\right)+e^2\left(156800x^5+125440x^4+83809x^3-138345x^2-65427x\right.\right.}{\left(5x^2+2x+3\right)^2}\right.}{6860000}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3
,x]
```

```
[Out] (5*Sqrt[14]*(211875*d^2 - 342070*d*e + 14817*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]
] + 70*((5*(5*d^2*(12953 + 17979*x + 38753*x^2 + 11015*x^3) + 2*d*e*(-19533
 + 57761*x + 28307*x^2 + 181765*x^3) + e^2*(-76977 - 65427*x - 138345*x^2 +
 83809*x^3 + 125440*x^4 + 156800*x^5)))/(3 + 2*x + 5*x^2)^2 + 784*(40*d - 4
9*e)*e*Log[3 + 2*x + 5*x^2]))/68600000
```

fricas [B] time = 0.83, size = 302, normalized size = 2.25

$$\frac{10976000\,e^2x^5 + 8780800\,e^2x^4 + 70\left(55075\,d^2 + 363530\,de + 83809\,e^2\right)x^3 + 70\left(193765\,d^2 + 56614\,de - 138345\,e^2\right)\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="f
ricas")
```

```
[Out] 1/13720000*(10976000*e^2*x^5 + 8780800*e^2*x^4 + 70*(55075*d^2 + 363530*d*e
 + 83809*e^2)*x^3 + 70*(193765*d^2 + 56614*d*e - 138345*e^2)*x^2 + sqrt(14)
*(25*(211875*d^2 - 342070*d*e + 14817*e^2)*x^4 + 20*(211875*d^2 - 342070*d*
e + 14817*e^2)*x^3 + 34*(211875*d^2 - 342070*d*e + 14817*e^2)*x^2 + 1906875
*d^2 - 3078630*d*e + 133353*e^2 + 12*(211875*d^2 - 342070*d*e + 14817*e^2)*
x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4533550*d^2 - 2734620*d*e - 5388390*e^
2 + 70*(89895*d^2 + 115522*d*e - 65427*e^2)*x + 10976*(25*(40*d*e - 49*e^2)
*x^4 + 20*(40*d*e - 49*e^2)*x^3 + 34*(40*d*e - 49*e^2)*x^2 + 360*d*e - 441*
e^2 + 12*(40*d*e - 49*e^2)*x)*log(5*x^2 + 2*x + 3))/(25*x^4 + 20*x^3 + 34*x
^2 + 12*x + 9)
```

giac [A] time = 0.16, size = 144, normalized size = 1.07

$$\frac{1}{13720000}\,\sqrt{14}\left(211875\,d^2 - 342070\,de + 14817\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{4}{125}\,xe^2+\frac{1}{1250}\left(40\,de - 49\,e^2\right)\log\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="g
iac")
```

```
[Out] 1/13720000*sqrt(14)*(211875*d^2 - 342070*d*e + 14817*e^2)*arctan(1/14*sqrt(
14)*(5*x + 1)) + 4/125*x*e^2 + 1/1250*(40*d*e - 49*e^2)*log(5*x^2 + 2*x + 3
) + 1/196000*((55075*d^2 + 363530*d*e - 129439*e^2)*x^3 + (193765*d^2 + 566
14*d*e - 213609*e^2)*x^2 + 64765*d^2 + (89895*d^2 + 115522*d*e - 121875*e^2
)*x - 39066*d*e - 76977*e^2)/(5*x^2 + 2*x + 3)^2
```

maple [A] time = 0.01, size = 179, normalized size = 1.34

$$\frac{339\sqrt{14}\,d^2\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{21952}-\frac{34207\sqrt{14}\,de\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{1372000}+\frac{4de\ln\left(5x^2+2x+3\right)}{125}+\frac{4e^2x}{125}+\frac{14817\sqrt{14}\,e^2\ldots}{13\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x)
```

```
[Out] 4/125*e^2*x+1/5*((2203/1568*d^2+36353/3920*d*e-129439/39200*e^2)*x^3+(38753
/7840*d^2+28307/19600*d*e-213609/39200*e^2)*x^2+(17979/7840*d^2+57761/19600
*d*e-4875/1568*e^2)*x+12953/7840*d^2-19533/19600*d*e-76977/39200*e^2)/(5*x^
2+2*x+3)^2+4/125*d*e*ln(5*x^2+2*x+3)-49/1250*e^2*ln(5*x^2+2*x+3)+339/21952*
14^(1/2)*d^2*arctan(1/28*(10*x+2)*14^(1/2))-34207/1372000*14^(1/2)*d*e*arct
an(1/28*(10*x+2)*14^(1/2))+14817/13720000*14^(1/2)*e^2*arctan(1/28*(10*x+2)
*14^(1/2))
```

maxima [A] time = 0.96, size = 155, normalized size = 1.16

$$\frac{4}{125}\,e^2x+\frac{1}{13720000}\,\sqrt{14}\left(211875\,d^2-342070\,de+14817\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{1}{1250}\left(40\,de-49\,e^2\right)\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="m
axima")
```

```
[Out] 4/125*e^2*x + 1/13720000*sqrt(14)*(211875*d^2 - 342070*d*e + 14817*e^2)*arc
tan(1/14*sqrt(14)*(5*x + 1)) + 1/1250*(40*d*e - 49*e^2)*log(5*x^2 + 2*x + 3
) + 1/196000*((55075*d^2 + 363530*d*e - 129439*e^2)*x^3 + (193765*d^2 + 566
14*d*e - 213609*e^2)*x^2 + 64765*d^2 - 39066*d*e - 76977*e^2 + (89895*d^2 +
 115522*d*e - 121875*e^2)*x)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

mupad [B] time = 4.21, size = 203, normalized size = 1.51

$$\frac{x^3\left(\frac{55075\,d^2}{1568}+\frac{181765\,d\,e}{784}-\frac{129439\,e^2}{1568}\right)+x^2\left(\frac{193765\,d^2}{1568}+\frac{28307\,d\,e}{784}-\frac{213609\,e^2}{1568}\right)-\frac{19533\,d\,e}{784}+x\left(\frac{89895\,d^2}{1568}+\frac{57761\,d\,e}{784}-\frac{1218}{15}\right.}{3125\,x^4+2500\,x^3+4250\,x^2+1500\,x+1125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

```
[Out] (x^3*((181765*d*e)/784 + (55075*d^2)/1568 - (129439*e^2)/1568) + x^2*((2830
7*d*e)/784 + (193765*d^2)/1568 - (213609*e^2)/1568) - (19533*d*e)/784 + x*(
(57761*d*e)/784 + (89895*d^2)/1568 - (121875*e^2)/1568) + (64765*d^2)/1568
- (76977*e^2)/1568)/(1500*x + 4250*x^2 + 2500*x^3 + 3125*x^4 + 1125) + (4*e
^2*x)/125 + log(2*x + 5*x^2 + 3)*((4*d*e)/125 - (49*e^2)/1250) + (14^(1/2)*
atan(((14^(1/2)*(211875*d^2 - 342070*d*e + 14817*e^2))/13720000 + (14^(1/2)
*x*(211875*d^2 - 342070*d*e + 14817*e^2))/2744000)/((339*d^2)/1568 - (34207
*d*e)/98000 + (14817*e^2)/980000))*(211875*d^2 - 342070*d*e + 14817*e^2))/1
3720000
```

sympy [C] time = 3.96, size = 304, normalized size = 2.27

$$\frac{4e^2x}{125}+\left(\frac{e\,(40d-49e)}{1250}-\frac{\sqrt{14}\,i\left(211875d^2-342070de+14817e^2\right)}{27440000}\right)\log\left(x+\frac{42375d^2-244030de+218093e^2+}{211875d^2-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] 4*e**2*x/125 + (e*(40*d - 49*e)/1250 - sqrt(14)*I*(211875*d**2 - 342070*d*e
 + 14817*e**2)/27440000)*log(x + (42375*d**2 - 244030*d*e + 218093*e**2 + 2
1952*e*(40*d - 49*e)/5 - sqrt(14)*I*(211875*d**2 - 342070*d*e + 14817*e**2)
/5)/(211875*d**2 - 342070*d*e + 14817*e**2)) + (e*(40*d - 49*e)/1250 + sqrt
(14)*I*(211875*d**2 - 342070*d*e + 14817*e**2)/27440000)*log(x + (42375*d**
2 - 244030*d*e + 218093*e**2 + 21952*e*(40*d - 49*e)/5 + sqrt(14)*I*(211875
*d**2 - 342070*d*e + 14817*e**2)/5)/(211875*d**2 - 342070*d*e + 14817*e**2)
) + (64765*d**2 - 39066*d*e - 76977*e**2 + x**3*(55075*d**2 + 363530*d*e -
129439*e**2) + x**2*(193765*d**2 + 56614*d*e - 213609*e**2) + x*(89895*d**2
 + 115522*d*e - 121875*e**2))/(4900000*x**4 + 3920000*x**3 + 6664000*x**2 +
 2352000*x + 1764000)
```

3.320 $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$

Optimal. Leaf size=103

$$-\frac{(423x+1367)(d+ex)}{7000(5x^2+2x+3)^2}+\frac{x(11015d+36353e)+34347d-6511e}{196000(5x^2+2x+3)}+\frac{(42375d-34207e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{196000\sqrt{14}}+\frac{2}{125}e\log(5x$$

```
[Out] -1/7000*(1367+423*x)*(e*x+d)/(5*x^2+2*x+3)^2+1/196000*(34347*d-6511*e+(1101
5*d+36353*e)*x)/(5*x^2+2*x+3)+2/125*e*ln(5*x^2+2*x+3)+1/2744000*(42375*d-34
207*e)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1644, 1660, 634, 618, 204, 628}

$$-\frac{(423x+1367)(d+ex)}{7000(5x^2+2x+3)^2}+\frac{x(11015d+36353e)+34347d-6511e}{196000(5x^2+2x+3)}+\frac{(42375d-34207e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{196000\sqrt{14}}+\frac{2}{125}e\log(5x$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] -((1367 + 423*x)*(d + e*x))/(7000*(3 + 2*x + 5*x^2)^2) + (34347*d - 6511*e
+ (11015*d + 36353*e)*x)/(196000*(3 + 2*x + 5*x^2)) + ((42375*d - 34207*e)*
ArcTan[(1 + 5*x)/Sqrt[14]])/(196000*Sqrt[14]) + (2*e*Log[3 + 2*x + 5*x^2])/
125
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
```

```
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx = -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int \frac{\frac{2}{125}(3267d+1367e)-\frac{12}{25}(308d}{(3+2}$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)}$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)}$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)}$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)}$$

Mathematica [A] time = 0.09, size = 107, normalized size = 1.04

$$\frac{-2115dx-6835d-5989ex+1269e}{35000\left(5x^2+2x+3\right)^2}+\frac{55075dx+171735d+181765ex-44399e}{980000\left(5x^2+2x+3\right)}+\frac{(42375d-34207e)\tan^{-1}\left(\right.}{196000\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x
]
```

```
[Out] (-6835*d + 1269*e - 2115*d*x - 5989*e*x)/(35000*(3 + 2*x + 5*x^2)^2) + (171
735*d - 44399*e + 55075*d*x + 181765*e*x)/(980000*(3 + 2*x + 5*x^2)) + ((42
375*d - 34207*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(196000*Sqrt[14]) + (2*e*Log[3
 + 2*x + 5*x^2])/125
```

fricas [A] time = 0.76, size = 172, normalized size = 1.67

$$\frac{70\,(11015\,d + 36353\,e)x^3 + 14\,(193765\,d + 28307\,e)x^2 + \sqrt{14}\left(25\,(42375\,d - 34207\,e)x^4 + 20\,(42375\,d - 34207\,e)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="fri
cas")
```

```
[Out] 1/2744000*(70*(11015*d + 36353*e)*x^3 + 14*(193765*d + 28307*e)*x^2 + sqrt(
14)*(25*(42375*d - 34207*e)*x^4 + 20*(42375*d - 34207*e)*x^3 + 34*(42375*d
- 34207*e)*x^2 + 12*(42375*d - 34207*e)*x + 381375*d - 307863*e)*arctan(1/1
4*sqrt(14)*(5*x + 1)) + 14*(89895*d + 57761*e)*x + 43904*(25*e*x^4 + 20*e*x
^3 + 34*e*x^2 + 12*e*x + 9*e)*log(5*x^2 + 2*x + 3) + 906710*d - 273462*e)/(
25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

giac [A] time = 0.19, size = 97, normalized size = 0.94

$$\frac{1}{2744000}\,\sqrt{14}\,(42375\,d - 34207\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x + 1)\right) + \frac{2}{125}\,e\log\left(5\,x^2 + 2\,x + 3\right) + \frac{5\,(11015\,d + 36353\,e)x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="gia
c")
```

```
[Out] 1/2744000*sqrt(14)*(42375*d - 34207*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 2/
125*e*log(5*x^2 + 2*x + 3) + 1/196000*(5*(11015*d + 36353*e)*x^3 + (193765*
d + 28307*e)*x^2 + (89895*d + 57761*e)*x + 64765*d - 19533*e)/(5*x^2 + 2*x
+ 3)^2
```

maple [A] time = 0.01, size = 102, normalized size = 0.99

$$\frac{339\sqrt{14}\,d\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{21952} - \frac{34207\sqrt{14}\,e\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{2744000} + \frac{2e\ln\left(5x^2 + 2x + 3\right)}{125} + \frac{25\left(\frac{2203d}{196000} + \frac{36353e}{980000}\right)x^3 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x)
```

```
[Out] 25*((36353/980000*e+2203/196000*d)*x^3+(28307/4900000*e+38753/980000*d)*x^2
+(57761/4900000*e+17979/980000*d)*x+12953/980000*d-19533/4900000*e)/(5*x^2+
2*x+3)^2+2/125*e*ln(5*x^2+2*x+3)+339/21952*14^(1/2)*d*arctan(1/28*(10*x+2)*
14^(1/2))-34207/2744000*14^(1/2)*e*arctan(1/28*(10*x+2)*14^(1/2))
```

maxima [A] time = 0.96, size = 101, normalized size = 0.98

$$\frac{1}{2744000}\,\sqrt{14}\,(42375\,d - 34207\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x + 1)\right) + \frac{2}{125}\,e\log\left(5\,x^2 + 2\,x + 3\right) + \frac{5\,(11015\,d + 36353\,e)x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="max
ima")
```

```
[Out] 1/2744000*sqrt(14)*(42375*d - 34207*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 2/
125*e*log(5*x^2 + 2*x + 3) + 1/196000*(5*(11015*d + 36353*e)*x^3 + (193765*
```

```
d + 28307*e)*x^2 + (89895*d + 57761*e)*x + 64765*d - 19533*e)/(25*x^4 + 20*
x^3 + 34*x^2 + 12*x + 9)
```

mupad [B] time = 0.12, size = 125, normalized size = 1.21

$$\frac{\left(\frac{2203\,d}{7840}+\frac{36353\,e}{39200}\right)x^3+\left(\frac{38753\,d}{39200}+\frac{28307\,e}{196000}\right)x^2+\left(\frac{17979\,d}{39200}+\frac{57761\,e}{196000}\right)x+\frac{12953\,d}{39200}-\frac{19533\,e}{196000}}{25\,x^4+20\,x^3+34\,x^2+12\,x+9}+\frac{2\,e\,\ln\left(5\,x^2+2\,x+3\right)}{125}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

```
[Out] ((12953*d)/39200 - (19533*e)/196000 + x^3*((2203*d)/7840 + (36353*e)/39200)
 + x^2*((38753*d)/39200 + (28307*e)/196000) + x*((17979*d)/39200 + (57761*e
)/196000))/(12*x + 34*x^2 + 20*x^3 + 25*x^4 + 9) + (2*e*log(2*x + 5*x^2 + 3
))/125 + (14^(1/2)*atan(((14^(1/2)*(42375*d - 34207*e))/2744000 + (14^(1/2)
*x*(42375*d - 34207*e))/548800)/((339*d)/1568 - (34207*e)/196000))*(42375*d
 - 34207*e))/2744000
```

sympy [C] time = 2.31, size = 163, normalized size = 1.58

$$\left(\frac{2e}{125}-\frac{\sqrt{14}\,i\,(42375d-34207e)}{5488000}\right)\log\left(x+\frac{8475d-\frac{34207e}{5}-\frac{\sqrt{14}\,i(42375d-34207e)}{5}}{42375d-34207e}\right)+\left(\frac{2e}{125}+\frac{\sqrt{14}\,i\,(42375d-342\cdots}{5488000}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] (2*e/125 - sqrt(14)*I*(42375*d - 34207*e)/5488000)*log(x + (8475*d - 34207*
e/5 - sqrt(14)*I*(42375*d - 34207*e)/5)/(42375*d - 34207*e)) + (2*e/125 + s
qrt(14)*I*(42375*d - 34207*e)/5488000)*log(x + (8475*d - 34207*e/5 + sqrt(1
4)*I*(42375*d - 34207*e)/5)/(42375*d - 34207*e)) + (64765*d - 19533*e + x**
3*(55075*d + 181765*e) + x**2*(193765*d + 28307*e) + x*(89895*d + 57761*e))
/(4900000*x**4 + 3920000*x**3 + 6664000*x**2 + 2352000*x + 1764000)
```

3.321 $\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^3}\,dx$

Optimal. Leaf size=64

$$-\frac{423x+1367}{7000\left(5x^2+2x+3\right)^2}+\frac{11015x+34347}{196000\left(5x^2+2x+3\right)}+\frac{339\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{1568\sqrt{14}}$$

```
[Out] 1/7000*(-1367-423*x)/(5*x^2+2*x+3)^2+1/196000*(34347+11015*x)/(5*x^2+2*x+3)
+339/21952*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1660, 12, 618, 204}

$$-\frac{423x+1367}{7000\left(5x^2+2x+3\right)^2}+\frac{11015x+34347}{196000\left(5x^2+2x+3\right)}+\frac{339\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{1568\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] -(1367 + 423*x)/(7000*(3 + 2*x + 5*x^2)^2) + (34347 + 11015*x)/(196000*(3 +
 2*x + 5*x^2)) + (339*ArcTan[(1 + 5*x)/Sqrt[14]])/(1568*Sqrt[14])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^3}\,dx = -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2} + \frac{1}{112}\int \frac{\frac{6534}{125}-\frac{3696x}{25}+\frac{448x^2}{5}}{\left(3+2x+5x^2\right)^2}\,dx$$

$$= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2} + \frac{34347+11015x}{196000\left(3+2x+5x^2\right)} + \frac{\int \frac{1356}{3+2x+5x^2}\,dx}{6272}$$

$$= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2} + \frac{34347+11015x}{196000\left(3+2x+5x^2\right)} + \frac{339\int \frac{1}{3+2x+5x^2}\,dx}{1568}$$

$$= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2} + \frac{34347+11015x}{196000\left(3+2x+5x^2\right)} - \frac{339}{784}\text{Subst}\left(\int \frac{1}{-56-x^2}\right.$$

$$= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2} + \frac{34347+11015x}{196000\left(3+2x+5x^2\right)} + \frac{339\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1568\sqrt{14}}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.83

$$\frac{\frac{14\left(11015x^3+38753x^2+17979x+12953\right)}{\left(5x^2+2x+3\right)^2}+8475\sqrt{14}\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{548800}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] ((14*(12953 + 17979*x + 38753*x^2 + 11015*x^3))/(3 + 2*x + 5*x^2)^2 + 8475*
Sqrt[14]*ArcTan[(1 + 5*x)/Sqrt[14]])/548800
```

fricas [A] time = 0.73, size = 75, normalized size = 1.17

$$\frac{154210\,x^3+8475\,\sqrt{14}\left(25\,x^4+20\,x^3+34\,x^2+12\,x+9\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+542542\,x^2+251706\,x+}{548800\left(25\,x^4+20\,x^3+34\,x^2+12\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/548800*(154210*x^3 + 8475*sqrt(14)*(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)*
arctan(1/14*sqrt(14)*(5*x + 1)) + 542542*x^2 + 251706*x + 181342)/(25*x^4 +
 20*x^3 + 34*x^2 + 12*x + 9)
```

giac [A] time = 0.16, size = 46, normalized size = 0.72

$$\frac{339}{21952}\,\sqrt{14}\,\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{11015\,x^3+38753\,x^2+17979\,x+12953}{39200\left(5\,x^2+2\,x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="giac")
```

```
[Out] 339/21952*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/39200*(11015*x^3 + 3
8753*x^2 + 17979*x + 12953)/(5*x^2 + 2*x + 3)^2
```

maple [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{339\sqrt{14}\ \arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{21952}+\frac{\frac{2203}{7840}x^3+\frac{38753}{39200}x^2+\frac{17979}{39200}x+\frac{12953}{39200}}{\left(5x^2+2x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x)
```

```
[Out] 25*(2203/196000*x^3+38753/980000*x^2+17979/980000*x+12953/980000)/(5*x^2+2*
x+3)^2+339/21952*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

maxima [A] time = 0.96, size = 56, normalized size = 0.88

$$\frac{339}{21952}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+\frac{11015\,x^3+38753\,x^2+17979\,x+12953}{39200\,\left(25\,x^4+20\,x^3+34\,x^2+12\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="maxima")
```

```
[Out] 339/21952*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/39200*(11015*x^3 + 3
8753*x^2 + 17979*x + 12953)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

mupad [B] time = 0.05, size = 55, normalized size = 0.86

$$\frac{339\,\sqrt{14}\ \mathrm{atan}\left(\frac{5\,\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{21952}+\frac{\frac{2203\,x^3}{196000}+\frac{38753\,x^2}{980000}+\frac{17979\,x}{980000}+\frac{12953}{980000}}{x^4+\frac{4\,x^3}{5}+\frac{34\,x^2}{25}+\frac{12\,x}{25}+\frac{9}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3)^3,x)
```

```
[Out] (339*14^(1/2)*atan((5*14^(1/2)*x)/14 + 14^(1/2)/14))/21952 + ((17979*x)/980
000 + (38753*x^2)/980000 + (2203*x^3)/196000 + 12953/980000)/((12*x)/25 + (
34*x^2)/25 + (4*x^3)/5 + x^4 + 9/25)
```

sympy [A] time = 0.20, size = 61, normalized size = 0.95

$$\frac{11015x^3+38753x^2+17979x+12953}{980000x^4+784000x^3+1332800x^2+470400x+352800}+\frac{339\sqrt{14}\ \mathrm{atan}\left(\frac{5\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{21952}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] (11015*x**3 + 38753*x**2 + 17979*x + 12953)/(980000*x**4 + 784000*x**3 + 13
32800*x**2 + 470400*x + 352800) + 339*sqrt(14)*atan(5*sqrt(14)*x/14 + sqrt(
14)/14)/21952
```

3.322 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx$

Optimal. Leaf size=329

$$-\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}+\frac{171735d^3-92989d^2e+25x\left(2203d^3-9033d^2e+3635de^2-1829e\right.}{39200\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^2}$$

```
[Out] 1/1400*(-1367*d+293*e-(423*d-1367*e)*x)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)^2
+1/39200*(171735*d^3-92989*d^2*e+36207*d*e^2+1831*e^3+25*(2203*d^3-9033*d^2
*e+3635*d*e^2-1829*e^3)*x)/(5*d^2-2*d*e+3*e^2)^2/(5*x^2+2*x+3)+e*(4*d^4+5*d
^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^3-1/2*e*(4*d^4+5*
d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^3+1/21952*
(42375*d^5-16643*d^4*e+58530*d^3*e^2-56058*d^2*e^3+31811*d*e^4-8623*e^5)*ar
ctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1646, 800, 634, 618, 204, 628}

$$-\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}+\frac{25x\left(-9033d^2e+2203d^3+3635de^2-1829e^3\right)-92989d^2e+17173}{39200\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^3),x]
```

```
[Out] -(1367*d - 293*e + (423*d - 1367*e)*x)/(1400*(5*d^2 - 2*d*e + 3*e^2)*(3 + 2
*x + 5*x^2)^2) + (171735*d^3 - 92989*d^2*e + 36207*d*e^2 + 1831*e^3 + 25*(2
203*d^3 - 9033*d^2*e + 3635*d*e^2 - 1829*e^3)*x)/(39200*(5*d^2 - 2*d*e + 3*
e^2)^2*(3 + 2*x + 5*x^2)) + ((42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 560
58*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]])/(1568*Sqrt
[14]*(5*d^2 - 2*d*e + 3*e^2)^3) + (e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 +
 2*e^4)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^3 - (e*(4*d^4 + 5*d^3*e + 3*d
^2*e^2 - d*e^3 + 2*e^4)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx = -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int \frac{\frac{2\left(3267d^2-2843de+2800e^2\right)}{25\left(5d^2-2de+3e^2\right)}-\frac{6(308}{2}}{(d+ex)\left(3+2\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+36207de^2+}{39200\left(5d\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+36207de^2+}{39200\left(5d\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+36207de^2+}{39200\left(5d\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+36207de^2+}{39200\left(5d\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+36207de^2+}{39200\left(5d\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+36207de^2+}{39200\left(5d\right.}$$

Mathematica [A] time = 0.31, size = 282, normalized size = 0.86

$$\frac{\frac{392(5d^2-2de+3e^2)^2(e(1367x+293)-d(423x+1367))}{(5x^2+2x+3)^2}+\frac{14(5d^2-2de+3e^2)(5d^3(11015x+34347)-d^2e(225825x+92989)+de^2(90875x+36207)+e^3(183}{5x^2+2x+3}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^3),x
]
```

```
[Out] ((392*(5*d^2 - 2*d*e + 3*e^2)^2*(-(d*(1367 + 423*x)) + e*(293 + 1367*x)))/(
3 + 2*x + 5*x^2)^2 + (14*(5*d^2 - 2*d*e + 3*e^2)*(e^3*(1831 - 45725*x) + 5*
d^3*(34347 + 11015*x) + d*e^2*(36207 + 90875*x) - d^2*e*(92989 + 225825*x))
)/(3 + 2*x + 5*x^2) + 25*Sqrt[14]*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2
- 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]] + 5488
00*e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x] - 274400*e*
(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[3 + 2*x + 5*x^2])/(548800
*(5*d^2 - 2*d*e + 3*e^2)^3)
```

fricas [B] time = 1.13, size = 1052, normalized size = 3.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="fri
cas")
```

```
[Out] 1/109760*(4533550*d^5 - 4072950*d^4*e + 3307332*d^3*e^2 - 807604*d^2*e^3 -
358554*d*e^4 + 252882*e^5 + 350*(11015*d^5 - 49571*d^4*e + 42850*d^3*e^2 -
43514*d^2*e^3 + 14563*d*e^4 - 5487*e^5)*x^3 + 14*(968825*d^5 - 1304125*d^4*
e + 1310718*d^3*e^2 - 777366*d^2*e^3 + 250589*d*e^4 - 49377*e^5)*x^2 + 5*sq
rt(14)*(381375*d^5 - 149787*d^4*e + 526770*d^3*e^2 - 504522*d^2*e^3 + 28629
9*d*e^4 - 77607*e^5 + 25*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d
^2*e^3 + 31811*d*e^4 - 8623*e^5)*x^4 + 20*(42375*d^5 - 16643*d^4*e + 58530*
d^3*e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*x^3 + 34*(42375*d^5 - 166
43*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*x^2 + 12
*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8
623*e^5)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(449475*d^5 - 828175*d^4*e
 + 761994*d^3*e^2 - 500898*d^2*e^3 + 147247*d*e^4 - 11211*e^5)*x + 109760*(
36*d^4*e + 45*d^3*e^2 + 27*d^2*e^3 - 9*d*e^4 + 18*e^5 + 25*(4*d^4*e + 5*d^3
*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^4 + 20*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3
 - d*e^4 + 2*e^5)*x^3 + 34*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5
)*x^2 + 12*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x)*log(e*x + d
) - 54880*(36*d^4*e + 45*d^3*e^2 + 27*d^2*e^3 - 9*d*e^4 + 18*e^5 + 25*(4*d^
4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^4 + 20*(4*d^4*e + 5*d^3*e^2
+ 3*d^2*e^3 - d*e^4 + 2*e^5)*x^3 + 34*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*
e^4 + 2*e^5)*x^2 + 12*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x)*
log(5*x^2 + 2*x + 3))/(1125*d^6 - 1350*d^5*e + 2565*d^4*e^2 - 1692*d^3*e^3
+ 1539*d^2*e^4 - 486*d*e^5 + 243*e^6 + 25*(125*d^6 - 150*d^5*e + 285*d^4*e^
2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)*x^4 + 20*(125*d^6 - 150*
d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)*x^3 +
34*(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^
5 + 27*e^6)*x^2 + 12*(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171
*d^2*e^4 - 54*d*e^5 + 27*e^6)*x)
```

giac [A] time = 0.25, size = 460, normalized size = 1.40

$$\frac{\sqrt{14}\left(42375\,d^5-16643\,d^4e+58530\,d^3e^2-56058\,d^2e^3+31811\,de^4-8623\,e^5\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{21952\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}-\frac{}{2\,(12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="gia
c")
```

```
[Out] 1/21952*sqrt(14)*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 +
 31811*d*e^4 - 8623*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5
*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(4*
d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(5*x^2 + 2*x + 3)/(125*d^
6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6
) + (4*d^4*e^2 + 5*d^3*e^3 + 3*d^2*e^4 - d*e^5 + 2*e^6)*log(abs(x*e + d))/(
125*d^6*e - 150*d^5*e^2 + 285*d^4*e^3 - 188*d^3*e^4 + 171*d^2*e^5 - 54*d*e^
6 + 27*e^7) + 1/7840*(323825*d^5 - 290925*d^4*e + 25*(11015*d^5 - 49571*d^4
*e + 42850*d^3*e^2 - 43514*d^2*e^3 + 14563*d*e^4 - 5487*e^5)*x^3 + 236238*d
^3*e^2 + (968825*d^5 - 1304125*d^4*e + 1310718*d^3*e^2 - 777366*d^2*e^3 + 2
50589*d*e^4 - 49377*e^5)*x^2 - 57686*d^2*e^3 + (449475*d^5 - 828175*d^4*e +
 761994*d^3*e^2 - 500898*d^2*e^3 + 147247*d*e^4 - 11211*e^5)*x - 25611*d*e^
4 + 18063*e^5)/((5*d^2 - 2*d*e + 3*e^2)^3*(5*x^2 + 2*x + 3)^2)
```

maple [B] time = 0.02, size = 1437, normalized size = 4.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x)
```

```
[Out] 5*e^2/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d^3+3*e^3/(5*d^2-2*d*e+3*e^2)^3*ln(e*
x+d)*d^2-e^4/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d+193765/1568/(5*d^2-2*d*e+3*e
^2)^3/(5*x^2+2*x+3)^2*x^2*d^5-49377/7840/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3
)^2*x^2*e^5+89895/1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x*d^5-11211/78
40/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x*e^5-58185/1568/(5*d^2-2*d*e+3*e^
2)^3/(5*x^2+2*x+3)^2*d^4*e+118119/3920/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^
2*d^3*e^2-28843/3920/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*d^2*e^3-25611/78
40/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*d*e^4-8623/21952/(5*d^2-2*d*e+3*e^
2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^5+42375/21952/(5*d^2-2*d*e+3
*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^5-5/2/(5*d^2-2*d*e+3*e^2)
^3*ln(5*x^2+2*x+3)*d^3*e^2-3/2/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d^2*e^
3+1/2/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d*e^4-2/(5*d^2-2*d*e+3*e^2)^3*l
n(5*x^2+2*x+3)*d^4*e+55075/1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^3*d
^5-27435/1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^3*e^5+4*e/(5*d^2-2*d*
e+3*e^2)^3*ln(e*x+d)*d^4+64765/1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*d
^5+18063/7840/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*e^5-1/(5*d^2-2*d*e+3*e^
2)^3*ln(5*x^2+2*x+3)*e^5+2*e^5/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)-260825/1568/
(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^2*d^4*e+655359/3920/(5*d^2-2*d*e+3*
e^2)^3/(5*x^2+2*x+3)^2*x^2*d^3*e^2-388683/3920/(5*d^2-2*d*e+3*e^2)^3/(5*x^2
+2*x+3)^2*x^2*d^2*e^3+250589/7840/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^2
*d*e^4-165635/1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x*d^4*e+380997/392
0/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x*d^3*e^2-250449/3920/(5*d^2-2*d*e+
3*e^2)^3/(5*x^2+2*x+3)^2*x*d^2*e^3+147247/7840/(5*d^2-2*d*e+3*e^2)^3/(5*x^2
+2*x+3)^2*x*d*e^4-16643/21952/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(1
0*x+2)*14^(1/2))*d^4*e+29265/10976/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/
28*(10*x+2)*14^(1/2))*d^3*e^2-28029/10976/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*ar
ctan(1/28*(10*x+2)*14^(1/2))*d^2*e^3+31811/21952/(5*d^2-2*d*e+3*e^2)^3*14^(
1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d*e^4-247855/1568/(5*d^2-2*d*e+3*e^2)^3
/(5*x^2+2*x+3)^2*x^3*d^4*e-108785/784/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2
*x^3*d^2*e^3+107125/784/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^3*d^3*e^2+7
2815/1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^3*d*e^4
```

maxima [A] time = 1.03, size = 571, normalized size = 1.74

$$\frac{\sqrt{14}\left(42375\,d^5-16643\,d^4e+58530\,d^3e^2-56058\,d^2e^3+31811\,de^4-8623\,e^5\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{21952\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}+\frac{}{125\,\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="max
ima")
```

```
[Out] 1/21952*sqrt(14)*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 +
 31811*d*e^4 - 8623*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5
*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + (4*d^4*
e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(e*x + d)/(125*d^6 - 150*d^5*
e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(4*d
^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(5*x^2 + 2*x + 3)/(125*d^6
 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)
 + 1/7840*(25*(2203*d^3 - 9033*d^2*e + 3635*d*e^2 - 1829*e^3)*x^3 + 64765*d
^3 - 32279*d^2*e - 4523*d*e^2 + 6021*e^3 + (193765*d^3 - 183319*d^2*e + 725
57*d*e^2 - 16459*e^3)*x^2 + (89895*d^3 - 129677*d^2*e + 46591*d*e^2 - 3737*
e^3)*x)/(25*(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*x^4 + 225*d
^4 - 180*d^3*e + 306*d^2*e^2 - 108*d*e^3 + 81*e^4 + 20*(25*d^4 - 20*d^3*e +
 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*x^3 + 34*(25*d^4 - 20*d^3*e + 34*d^2*e^2 -
12*d*e^3 + 9*e^4)*x^2 + 12*(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e
^4)*x)
```

mupad [B] time = 4.79, size = 641, normalized size = 1.95

$$\frac{\frac{x\left(89895\,d^3-129677\,d^2\,e+46591\,d\,e^2-3737\,e^3\right)}{7840\left(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4\right)}-\frac{-64765\,d^3+32279\,d^2\,e+4523\,d\,e^2-6021\,e^3}{7840\left(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4\right)}+\frac{5\,x^3\left(2203\,d^3-9033\,d^2\,e+3635\,d\,e^2-1829\,e^3\right)}{1568\left(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4\right)}+\ldots}{25\,x^4+20\,x^3+34\,x^2+12\,x+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)^3),x)
```

```
[Out] ((x*(46591*d*e^2 - 129677*d^2*e + 89895*d^3 - 3737*e^3))/(7840*(25*d^4 - 20
*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) - (4523*d*e^2 + 32279*d^2*e - 6476
5*d^3 - 6021*e^3)/(7840*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)
) + (5*x^3*(3635*d*e^2 - 9033*d^2*e + 2203*d^3 - 1829*e^3))/(1568*(25*d^4 -
 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x^2*(72557*d*e^2 - 183319*d^
2*e + 193765*d^3 - 16459*e^3))/(7840*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4
+ 34*d^2*e^2)))/(12*x + 34*x^2 + 20*x^3 + 25*x^4 + 9) + log(d + e*x)*((4*e)
/(25*(5*d^2 - 2*d*e + 3*e^2)) + (e^2*(205*d + 21*e))/(125*(5*d^2 - 2*d*e +
3*e^2)^2) - (e^4*(458*d - 7*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^3)) - (log(x -
 (14^(1/2)*1i)/5 + 1/5)*(e^5*((8623*14^(1/2))/43904 + 1i) - (42375*14^(1/2)
*d^5)/43904 + d^2*e^3*((28029*14^(1/2))/21952 + 3i/2) - d^3*e^2*((29265*14^
(1/2))/21952 - 5i/2) + d^4*e*((16643*14^(1/2))/43904 + 2i) - d*e^4*((31811*
14^(1/2))/43904 + 1i/2)))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^
2*e^4*171i - d^3*e^3*188i + d^4*e^2*285i) + (log(x + (14^(1/2)*1i)/5 + 1/5)
*(e^5*((8623*14^(1/2))/43904 - 1i) - (42375*14^(1/2)*d^5)/43904 + d^2*e^3*(
(28029*14^(1/2))/21952 - 3i/2) - d^3*e^2*((29265*14^(1/2))/21952 + 5i/2) +
d^4*e*((16643*14^(1/2))/43904 - 2i) - d*e^4*((31811*14^(1/2))/43904 - 1i/2)
))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*18
8i + d^4*e^2*285i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3)**3,x)
```

```
[Out] Timed out
```

3.323 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx$

Optimal. Leaf size=443

$$-\frac{x\left(423d^2-2734de+293e^2\right)+1367d^2-586de-703e^2}{280\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)^2}+\frac{171735d^4-117284d^3e-200502d^2e^2+5x\left(11015d^4\right.}{7840\left(5x\right.}$$

```
[Out] -e*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)+1/28
0*(-1367*d^2+586*d*e+703*e^2-(423*d^2-2734*d*e+293*e^2)*x)/(5*d^2-2*d*e+3*e
^2)^2/(5*x^2+2*x+3)^2+1/7840*(171735*d^4-117284*d^3*e-200502*d^2*e^2+104428
*d*e^3-23189*e^4+5*(11015*d^4-85924*d^3*e+34698*d^2*e^2+10348*d*e^3-3589*e^
4)*x)/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)+e*(40*d^5+83*d^4*e+12*d^3*e^2-76*
d^2*e^3+46*d*e^4-9*e^5)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^4-1/2*e*(40*d^5+83*d^
4*e+12*d^3*e^2-76*d^2*e^3+46*d*e^4-9*e^5)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^
2)^4+1/21952*(211875*d^6+3070*d^5*e+209039*d^4*e^2-921444*d^3*e^3+380621*d^
2*e^4-49586*d*e^5-43695*e^6)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e
^2)^4*14^(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1646, 1628, 634, 618, 204, 628}

$$\frac{5x\left(34698d^2e^2-85924d^3e+11015d^4+10348de^3-3589e^4\right)-200502d^2e^2-117284d^3e+171735d^4+104428}{7840\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^3),x]
```

```
[Out] -((e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/((5*d^2 - 2*d*e + 3*e^2
)^3*(d + e*x))) - (1367*d^2 - 586*d*e - 703*e^2 + (423*d^2 - 2734*d*e + 293
*e^2)*x)/(280*(5*d^2 - 2*d*e + 3*e^2)^2*(3 + 2*x + 5*x^2)^2) + (171735*d^4
- 117284*d^3*e - 200502*d^2*e^2 + 104428*d*e^3 - 23189*e^4 + 5*(11015*d^4 -
 85924*d^3*e + 34698*d^2*e^2 + 10348*d*e^3 - 3589*e^4)*x)/(7840*(5*d^2 - 2*
d*e + 3*e^2)^3*(3 + 2*x + 5*x^2)) + ((211875*d^6 + 3070*d^5*e + 209039*d^4*
e^2 - 921444*d^3*e^3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*ArcTan[(1
+ 5*x)/Sqrt[14]])/(1568*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^4) + (e*(40*d^5 +
83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Log[d + e*x])/(5*d^2
 - 2*d*e + 3*e^2)^4 - (e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 + 46*
d*e^4 - 9*e^5)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^4)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx = -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int \frac{2\left(3267d^4-568\cdots\right.}{\cdots}$$

$$= -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)^2}+\frac{171735d^4-117284\cdots}{\cdots}$$

$$= -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)^2}+\frac{171735d^4-117284\cdots}{\cdots}$$

$$= -\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-273\cdots\right.}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x\cdots\right.}$$

$$= -\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-273\cdots\right.}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x\cdots\right.}$$

$$= -\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-273\cdots\right.}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x\cdots\right.}$$

$$= -\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-273\cdots\right.}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x\cdots\right.}$$

Mathematica [A] time = 0.53, size = 389, normalized size = 0.88

$$-\frac{\frac{392\left(5d^2-2de+3e^2\right)^2\left(d^2(423x+1367)-2de(1367x+293)+e^2(293x-703)\right)}{\left(5x^2+2x+3\right)^2}+\frac{14\left(5d^2-2de+3e^2\right)\left(5d^4(11015x+34347)-4d^3e(107405x+29321)+6d^2e^2\cdots\right.}{5x^2+2x+\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^3)
,x]
```

```
[Out] ((-109760*e*(5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 +
2*e^4))/(d + e*x) - (392*(5*d^2 - 2*d*e + 3*e^2)^2*(e^2*(-703 + 293*x) + d^
2*(1367 + 423*x) - 2*d*e*(293 + 1367*x)))/(3 + 2*x + 5*x^2)^2 + (14*(5*d^2
- 2*d*e + 3*e^2)*(5*d^4*(34347 + 11015*x) + 4*d*e^3*(26107 + 12935*x) - e^4
*(23189 + 17945*x) + 6*d^2*e^2*(-33417 + 28915*x) - 4*d^3*e*(29321 + 107405
*x)))/(3 + 2*x + 5*x^2) + 5*Sqrt[14]*(211875*d^6 + 3070*d^5*e + 209039*d^4*
e^2 - 921444*d^3*e^3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*ArcTan[(1
+ 5*x)/Sqrt[14]] + 109760*e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 +
46*d*e^4 - 9*e^5)*Log[d + e*x] - 54880*e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 -
76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Log[3 + 2*x + 5*x^2])/(109760*(5*d^2 - 2*d*e
 + 3*e^2)^4)
```

fricas [B] time = 1.56, size = 1734, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="f
ricas")
```

```
[Out] 1/21952*(4533550*d^7 - 8470420*d^6*e - 8666490*d^5*e^2 + 3186008*d^4*e^3 -
8213198*d^3*e^4 - 1375668*d^2*e^5 + 1294650*d*e^6 - 1185408*e^7 - 70*(10172
5*d^6*e + 584930*d^5*e^2 - 245103*d^4*e^3 + 306788*d^3*e^4 + 99187*d^2*e^5
- 93102*d*e^6 + 57807*e^7)*x^4 + 14*(275375*d^7 - 1916625*d^6*e - 474395*d^
5*e^2 - 1406231*d^4*e^3 + 222261*d^3*e^4 - 1262851*d^2*e^5 + 601791*d*e^6 -
 279261*e^7)*x^3 + 14*(968825*d^7 - 2449955*d^6*e - 1699045*d^5*e^2 - 27958
1*d^4*e^3 - 1024621*d^3*e^4 - 1118441*d^2*e^5 + 698097*d*e^6 - 394767*e^7)*
x^2 + sqrt(14)*(1906875*d^7 + 27630*d^6*e + 1881351*d^5*e^2 - 8292996*d^4*e
^3 + 3425589*d^3*e^4 - 446274*d^2*e^5 - 393255*d*e^6 + 25*(211875*d^6*e + 3
070*d^5*e^2 + 209039*d^4*e^3 - 921444*d^3*e^4 + 380621*d^2*e^5 - 49586*d*e^
6 - 43695*e^7)*x^5 + 5*(1059375*d^7 + 862850*d^6*e + 1057475*d^5*e^2 - 3771
064*d^4*e^3 - 1782671*d^3*e^4 + 1274554*d^2*e^5 - 416819*d*e^6 - 174780*e^7
)*x^4 + 2*(2118750*d^7 + 3632575*d^6*e + 2142580*d^5*e^2 - 5660777*d^4*e^3
- 11858338*d^3*e^4 + 5974697*d^2*e^5 - 1279912*d*e^6 - 742815*e^7)*x^3 + 2*
(3601875*d^7 + 1323440*d^6*e + 3572083*d^5*e^2 - 14410314*d^4*e^3 + 941893*
d^3*e^4 + 1440764*d^2*e^5 - 1040331*d*e^6 - 262170*e^7)*x^2 + 3*(847500*d^7
 + 647905*d^6*e + 845366*d^5*e^2 - 3058659*d^4*e^3 - 1241848*d^3*e^4 + 9435
19*d^2*e^5 - 323538*d*e^6 - 131085*e^7)*x)*arctan(1/14*sqrt(14)*(5*x + 1))
+ 42*(149825*d^7 - 449755*d^6*e - 12125*d^5*e^2 - 238325*d^4*e^3 - 14261*d^
3*e^4 - 169777*d^2*e^5 + 84969*d*e^6 - 39735*e^7)*x + 21952*(360*d^6*e + 74
7*d^5*e^2 + 108*d^4*e^3 - 684*d^3*e^4 + 414*d^2*e^5 - 81*d*e^6 + 25*(40*d^5
*e^2 + 83*d^4*e^3 + 12*d^3*e^4 - 76*d^2*e^5 + 46*d*e^6 - 9*e^7)*x^5 + 5*(20
0*d^6*e + 575*d^5*e^2 + 392*d^4*e^3 - 332*d^3*e^4 - 74*d^2*e^5 + 139*d*e^6
- 36*e^7)*x^4 + 2*(400*d^6*e + 1510*d^5*e^2 + 1531*d^4*e^3 - 556*d^3*e^4 -
832*d^2*e^5 + 692*d*e^6 - 153*e^7)*x^3 + 2*(680*d^6*e + 1651*d^5*e^2 + 702*
d^4*e^3 - 1220*d^3*e^4 + 326*d^2*e^5 + 123*d*e^6 - 54*e^7)*x^2 + 3*(160*d^6
*e + 452*d^5*e^2 + 297*d^4*e^3 - 268*d^3*e^4 - 44*d^2*e^5 + 102*d*e^6 - 27*
e^7)*x)*log(e*x + d) - 10976*(360*d^6*e + 747*d^5*e^2 + 108*d^4*e^3 - 684*d
```

```
^3*e^4 + 414*d^2*e^5 - 81*d*e^6 + 25*(40*d^5*e^2 + 83*d^4*e^3 + 12*d^3*e^4
- 76*d^2*e^5 + 46*d*e^6 - 9*e^7)*x^5 + 5*(200*d^6*e + 575*d^5*e^2 + 392*d^4
*e^3 - 332*d^3*e^4 - 74*d^2*e^5 + 139*d*e^6 - 36*e^7)*x^4 + 2*(400*d^6*e +
1510*d^5*e^2 + 1531*d^4*e^3 - 556*d^3*e^4 - 832*d^2*e^5 + 692*d*e^6 - 153*e
^7)*x^3 + 2*(680*d^6*e + 1651*d^5*e^2 + 702*d^4*e^3 - 1220*d^3*e^4 + 326*d^
2*e^5 + 123*d*e^6 - 54*e^7)*x^2 + 3*(160*d^6*e + 452*d^5*e^2 + 297*d^4*e^3
- 268*d^3*e^4 - 44*d^2*e^5 + 102*d*e^6 - 27*e^7)*x)*log(5*x^2 + 2*x + 3))/(
5625*d^9 - 9000*d^8*e + 18900*d^7*e^2 - 17640*d^6*e^3 + 18774*d^5*e^4 - 105
84*d^4*e^5 + 6804*d^3*e^6 - 1944*d^2*e^7 + 729*d*e^8 + 25*(625*d^8*e - 1000
*d^7*e^2 + 2100*d^6*e^3 - 1960*d^5*e^4 + 2086*d^4*e^5 - 1176*d^3*e^6 + 756*
d^2*e^7 - 216*d*e^8 + 81*e^9)*x^5 + 5*(3125*d^9 - 2500*d^8*e + 6500*d^7*e^2
 - 1400*d^6*e^3 + 2590*d^5*e^4 + 2464*d^4*e^5 - 924*d^3*e^6 + 1944*d^2*e^7
- 459*d*e^8 + 324*e^9)*x^4 + 2*(6250*d^9 + 625*d^8*e + 4000*d^7*e^2 + 16100
*d^6*e^3 - 12460*d^5*e^4 + 23702*d^4*e^5 - 12432*d^3*e^6 + 10692*d^2*e^7 -
2862*d*e^8 + 1377*e^9)*x^3 + 2*(10625*d^9 - 13250*d^8*e + 29700*d^7*e^2 - 2
0720*d^6*e^3 + 23702*d^5*e^4 - 7476*d^4*e^5 + 5796*d^3*e^6 + 864*d^2*e^7 +
81*d*e^8 + 486*e^9)*x^2 + 3*(2500*d^9 - 2125*d^8*e + 5400*d^7*e^2 - 1540*d^
6*e^3 + 2464*d^5*e^4 + 1554*d^4*e^5 - 504*d^3*e^6 + 1404*d^2*e^7 - 324*d*e^
8 + 243*e^9)*x)
```

giac [A] time = 0.26, size = 762, normalized size = 1.72

$$\frac{\sqrt{14}\left(211875\,d^6e^2+3070\,d^5e^3+209039\,d^4e^4-921444\,d^3e^5+380621\,d^2e^6-49586\,de^7-43695\,e^8\right)\arctan\left(\frac{1}{14}\sqrt{\right.}{21952\left(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="g
iac")
```

```
[Out] 1/21952*sqrt(14)*(211875*d^6*e^2 + 3070*d^5*e^3 + 209039*d^4*e^4 - 921444*d
^3*e^5 + 380621*d^2*e^6 - 49586*d*e^7 - 43695*e^8)*arctan(1/14*sqrt(14)*(5*
d - 5*d^2/(x*e + d) + 2*d*e/(x*e + d) - 3*e^2/(x*e + d) - e)*e^(-1))*e^(-2)
/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*
d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - 1/2*(40*d^5*e + 83*d^4*e^2 +
12*d^3*e^3 - 76*d^2*e^4 + 46*d*e^5 - 9*e^6)*log(-10*d/(x*e + d) + 5*d^2/(x*
e + d)^2 + 2*e/(x*e + d) - 2*d*e/(x*e + d)^2 + 3*e^2/(x*e + d)^2 + 5)/(625*
d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^
5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - (4*d^4*e^7/(x*e + d) + 5*d^3*e^8/(x
*e + d) + 3*d^2*e^9/(x*e + d) - d*e^10/(x*e + d) + 2*e^11/(x*e + d))/(125*d
^6*e^6 - 150*d^5*e^7 + 285*d^4*e^8 - 188*d^3*e^9 + 171*d^2*e^10 - 54*d*e^11
 + 27*e^12) + 1/1568*(275375*d^5*e - 3006775*d^4*e^2 + 1394650*d^3*e^3 + 18
35350*d^2*e^4 - 734925*d*e^5 - 5*(165225*d^6*e^2 - 1997830*d^5*e^3 + 121842
1*d^4*e^4 + 1520564*d^3*e^5 - 947049*d^2*e^6 + 93386*d*e^7 + 7963*e^8)*e^(-
1)/(x*e + d) + (826125*d^7*e^3 - 10957975*d^6*e^4 + 8449735*d^5*e^5 + 82111
75*d^4*e^6 - 7879025*d^3*e^7 + 2996315*d^2*e^8 - 443947*d*e^9 - 67267*e^10)
*e^(-2)/(x*e + d)^2 - (275375*d^8*e^4 - 3975600*d^7*e^5 + 3752280*d^6*e^6 +
 2119880*d^5*e^7 - 3655050*d^4*e^8 + 4008480*d^3*e^9 - 1453312*d^2*e^10 - 1
97784*d*e^11 + 66483*e^12)*e^(-3)/(x*e + d)^3 + 17525*e^6)/((5*d^2 - 2*d*e
+ 3*e^2)^4*(10*d/(x*e + d) - 5*d^2/(x*e + d)^2 - 2*e/(x*e + d) + 2*d*e/(x*e
 + d)^2 - 3*e^2/(x*e + d)^2 - 5)^2)
```

maple [B] time = 0.03, size = 1850, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x)
```

```
[Out] 99045/784/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*d*e^5-161395/784/(5*d^2-2*d
*e+3*e^2)^4/(5*x^2+2*x+3)^2*d^5*e-379131/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+
2*x+3)^2*d^4*e^2+116869/392/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*d^3*e^3-2
0/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^5*e-83/2/(5*d^2-2*d*e+3*e^2)^4*ln
(5*x^2+2*x+3)*d^4*e^2-6/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^3*e^3+38/(5
*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^2*e^4-23/(5*d^2-2*d*e+3*e^2)^4*ln(5*x
^2+2*x+3)*d*e^5+211875/21952/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10
*x+2)*14^(1/2))*d^6-43695/21952/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*
(10*x+2)*14^(1/2))*e^6+40*e/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d^5+83*e^2/(5*d
^2-2*d*e+3*e^2)^4*ln(e*x+d)*d^4+12*e^3/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d^3-
76*e^4/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d^2+46*e^5/(5*d^2-2*d*e+3*e^2)^4*ln(
e*x+d)*d-4*e/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d^4-5*e^2/(5*d^2-2*d*e+3*e^2)^3/
(e*x+d)*d^3-3*e^3/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d^2+e^4/(5*d^2-2*d*e+3*e^2)
^3/(e*x+d)*d+968825/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^2*d^6+2753
75/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^3*d^6+449475/1568/(5*d^2-2*
d*e+3*e^2)^4/(5*x^2+2*x+3)^2*d^6*x-53835/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+
2*x+3)^2*x^3*e^6-91101/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^2*e^6-7
4895/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x*e^6-530209/1568/(5*d^2-2*
d*e+3*e^2)^4/(5*x^2+2*x+3)^2*d^2*e^4-648385/784/(5*d^2-2*d*e+3*e^2)^4/(5*x^
2+2*x+3)^2*x*d^5*e+218053/784/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^2*d*e
^5-795401/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^2*d^2*e^4+95555/784/
(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^3*d*e^5-916595/784/(5*d^2-2*d*e+3*e
^2)^4/(5*x^2+2*x+3)^2*x^2*d^5*e+504029/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*
x+3)^2*x^2*d^4*e^2+5109/392/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^2*d^3*e
^3+1891915/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^3*d^4*e^2-344285/39
2/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^3*d^3*e^3+327265/1568/(5*d^2-2*d*
e+3*e^2)^4/(5*x^2+2*x+3)^2*x^3*d^2*e^4-1129125/784/(5*d^2-2*d*e+3*e^2)^4/(5
*x^2+2*x+3)^2*x^3*d^5*e-434995/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x
*d^2*e^4+323825/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*d^6-6309/1568/(5
*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*e^6+9/2/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+
2*x+3)*e^6-9*e^6/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)-2*e^5/(5*d^2-2*d*e+3*e^2)^
3/(e*x+d)+208007/784/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x*d*e^5+606287/1
568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x*d^4*e^2-3993/392/(5*d^2-2*d*e+3
*e^2)^4/(5*x^2+2*x+3)^2*x*d^3*e^3+1535/10976/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)
*arctan(1/28*(10*x+2)*14^(1/2))*d^5*e+209039/21952/(5*d^2-2*d*e+3*e^2)^4*14
^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^4*e^2-230361/5488/(5*d^2-2*d*e+3*e^
2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^3*e^3+380621/21952/(5*d^2-2*
d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^2*e^4-24793/10976/(5
*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d*e^5
```

maxima [B] time = 1.13, size = 916, normalized size = 2.07

$$\frac{\sqrt{14}\left(211875\,d^{6}+3070\,d^{5}e+209039\,d^{4}e^{2}-921444\,d^{3}e^{3}+380621\,d^{2}e^{4}-49586\,de^{5}-43695\,e^{6}\right)\arctan\left(\frac{1}{14}\sqrt{\cdots}\right.}{21952\left(625\,d^{8}-1000\,d^{7}e+2100\,d^{6}e^{2}-1960\,d^{5}e^{3}+2086\,d^{4}e^{4}-1176\,d^{3}e^{5}+756\,d^{2}e^{6}-216\,de^{7}+81\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="m
axima")
```

```
[Out] 1/21952*sqrt(14)*(211875*d^6 + 3070*d^5*e + 209039*d^4*e^2 - 921444*d^3*e^3
 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*arctan(1/14*sqrt(14)*(5*x + 1)
)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176
*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) + (40*d^5*e + 83*d^4*e^2 + 12*
d^3*e^3 - 76*d^2*e^4 + 46*d*e^5 - 9*e^6)*log(e*x + d)/(625*d^8 - 1000*d^7*e
 + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6
- 216*d*e^7 + 81*e^8) - 1/2*(40*d^5*e + 83*d^4*e^2 + 12*d^3*e^3 - 76*d^2*e^
4 + 46*d*e^5 - 9*e^6)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6
*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7
 + 81*e^8) + 1/1568*(64765*d^5 - 95100*d^4*e - 200706*d^3*e^2 + 22292*d^2*e
```

```
^3 + 12009*d*e^4 - 28224*e^5 - 5*(20345*d^4*e + 125124*d^3*e^2 - 11178*d^2*
e^3 - 18188*d*e^4 + 19269*e^5)*x^4 + (55075*d^5 - 361295*d^4*e - 272442*d^3
*e^2 - 173446*d^2*e^3 + 138539*d*e^4 - 93087*e^5)*x^3 + (193765*d^5 - 41248
5*d^4*e - 621062*d^3*e^2 - 56850*d^2*e^3 + 144973*d*e^4 - 131589*e^5)*x^2 +
 3*(29965*d^5 - 77965*d^4*e - 51590*d^3*e^2 - 21522*d^2*e^3 + 19493*d*e^4 -
 13245*e^5)*x)/(1125*d^7 - 1350*d^6*e + 2565*d^5*e^2 - 1692*d^4*e^3 + 1539*
d^3*e^4 - 486*d^2*e^5 + 243*d*e^6 + 25*(125*d^6*e - 150*d^5*e^2 + 285*d^4*e
^3 - 188*d^3*e^4 + 171*d^2*e^5 - 54*d*e^6 + 27*e^7)*x^5 + 5*(625*d^7 - 250*
d^6*e + 825*d^5*e^2 + 200*d^4*e^3 + 103*d^3*e^4 + 414*d^2*e^5 - 81*d*e^6 +
108*e^7)*x^4 + 2*(1250*d^7 + 625*d^6*e + 300*d^5*e^2 + 2965*d^4*e^3 - 1486*
d^3*e^4 + 2367*d^2*e^5 - 648*d*e^6 + 459*e^7)*x^3 + 2*(2125*d^7 - 1800*d^6*
e + 3945*d^5*e^2 - 1486*d^4*e^3 + 1779*d^3*e^4 + 108*d^2*e^5 + 135*d*e^6 +
162*e^7)*x^2 + 3*(500*d^7 - 225*d^6*e + 690*d^5*e^2 + 103*d^4*e^3 + 120*d^3
*e^4 + 297*d^2*e^5 - 54*d*e^6 + 81*e^7)*x)
```

mupad [B] time = 4.99, size = 965, normalized size = 2.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)^3),x)
```

```
[Out] log(d + e*x)*((2*e^3*(620*d - 2417*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^3) - (6
*e^5*(423*d - 1367*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^4) + (e*(8*d + 23*e))/(
5*(5*d^2 - 2*d*e + 3*e^2)^2)) - ((3*x*(77965*d^4*e - 19493*d*e^4 - 29965*d^
5 + 13245*e^5 + 21522*d^2*e^3 + 51590*d^3*e^2))/(1568*(125*d^6 - 150*d^5*e
- 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) - (12009*d*
e^4 - 95100*d^4*e + 64765*d^5 - 28224*e^5 + 22292*d^2*e^3 - 200706*d^3*e^2)
/(1568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3
 + 285*d^4*e^2)) + (5*x^4*(20345*d^4*e - 18188*d*e^4 + 19269*e^5 - 11178*d^
2*e^3 + 125124*d^3*e^2))/(1568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 1
71*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x^3*(361295*d^4*e - 138539*d*e^
4 - 55075*d^5 + 93087*e^5 + 173446*d^2*e^3 + 272442*d^3*e^2))/(1568*(125*d^
6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2
)) + (x^2*(412485*d^4*e - 144973*d*e^4 - 193765*d^5 + 131589*e^5 + 56850*d^
2*e^3 + 621062*d^3*e^2))/(1568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 1
71*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)))/(9*d + x^2*(34*d + 12*e) + x^4*(2
5*d + 20*e) + x^3*(20*d + 34*e) + 25*e*x^5 + x*(12*d + 9*e)) + (log(x - (14
^(1/2)*1i)/5 + 1/5)*((211875*14^(1/2)*d^6)/43904 - e^6*((43695*14^(1/2))/43
904 - 9i/2) - d^3*e^3*((230361*14^(1/2))/10976 + 6i) + d^4*e^2*((209039*14^
(1/2))/43904 - 83i/2) + d^2*e^4*((380621*14^(1/2))/43904 + 38i) + d^5*e*((1
535*14^(1/2))/21952 - 20i) - d*e^5*((24793*14^(1/2))/21952 + 23i)))/(d^8*62
5i - d^7*e*1000i - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*e^5*1176i + d^
4*e^4*2086i - d^5*e^3*1960i + d^6*e^2*2100i) - (log(x + (14^(1/2)*1i)/5 + 1
/5)*((211875*14^(1/2)*d^6)/43904 - e^6*((43695*14^(1/2))/43904 + 9i/2) - d^
3*e^3*((230361*14^(1/2))/10976 - 6i) + d^4*e^2*((209039*14^(1/2))/43904 + 8
3i/2) + d^2*e^4*((380621*14^(1/2))/43904 - 38i) + d^5*e*((1535*14^(1/2))/21
952 + 20i) - d*e^5*((24793*14^(1/2))/21952 - 23i)))/(d^8*625i - d^7*e*1000i
 - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*e^5*1176i + d^4*e^4*2086i - d^
5*e^3*1960i + d^6*e^2*2100i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3)**3,x)
```

```
[Out] Timed out
```

3.324 $\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)dx$

Optimal. Leaf size=143

$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4-\frac{823\left(2x^2-x+3\right)^{3/2}(2x+5)^3}{1344}+\frac{11433\left(2x^2-x+3\right)^{3/2}(2x+5)^2}{4480}-\frac{(295276x+10}{}$$

```
[Out] 11433/4480*(5+2*x)^2*(2*x^2-x+3)^(3/2)-823/1344*(5+2*x)^3*(2*x^2-x+3)^(3/2)
+5/112*(5+2*x)^4*(2*x^2-x+3)^(3/2)-1/71680*(1005757+295276*x)*(2*x^2-x+3)^(
3/2)-1183005/131072*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-51435/32768*(1-4
*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1653, 779, 612, 619, 215}

$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4-\frac{823\left(2x^2-x+3\right)^{3/2}(2x+5)^3}{1344}+\frac{11433\left(2x^2-x+3\right)^{3/2}(2x+5)^2}{4480}-\frac{(295276x+10}{}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x)*Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-51435*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/32768 + (11433*(5 + 2*x)^2*(3 - x +
2*x^2)^(3/2))/4480 - (823*(5 + 2*x)^3*(3 - x + 2*x^2)^(3/2))/1344 + (5*(5 +
 2*x)^4*(3 - x + 2*x^2)^(3/2))/112 - ((1005757 + 295276*x)*(3 - x + 2*x^2)^
(3/2))/71680 - (1183005*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
```

```
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)dx = \frac{5}{112}(5+2x)^4\left(3-x+2x^2\right)^{3/2}+\frac{1}{224}\int (5+2x)\sqrt{3-x+$$

$$= -\frac{823(5+2x)^3\left(3-x+2x^2\right)^{3/2}}{1344}+\frac{5}{112}(5+2x)^4\left(3-x+2\right.$$

$$= \frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480}-\frac{823(5+2x)^3\left(3-x+\right.}{1344}$$

$$= \frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480}-\frac{823(5+2x)^3\left(3-x+\right.}{1344}$$

$$= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768}+\frac{11433(5+2x)^2\left(3-x+\right.}{4480}$$

$$= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768}+\frac{11433(5+2x)^2\left(3-x+\right.}{4480}$$

$$= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768}+\frac{11433(5+2x)^2\left(3-x+\right.}{4480}$$

Mathematica [A] time = 0.15, size = 70, normalized size = 0.49

$$\frac{4\sqrt{2x^2-x+3}\left(4915200x^6+12984320x^5+1390592x^4+20304768x^3+11357024x^2+14742332x+6231117\right)-}{13762560}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x)*Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(6231117 + 14742332*x + 11357024*x^2 + 20304768*x^3
+ 1390592*x^4 + 12984320*x^5 + 4915200*x^6) - 124215525*Sqrt[2]*ArcSinh[(1
- 4*x)/Sqrt[23]])/13762560
```

fricas [A] time = 0.81, size = 83, normalized size = 0.58

$$\frac{1}{3440640}\left(4915200\,x^6+12984320\,x^5+1390592\,x^4+20304768\,x^3+11357024\,x^2+14742332\,x+6231117\right)\sqrt{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/3440640*(4915200*x^6 + 12984320*x^5 + 1390592*x^4 + 20304768*x^3 + 113570
24*x^2 + 14742332*x + 6231117)*sqrt(2*x^2 - x + 3) + 1183005/262144*sqrt(2)
*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.20, size = 78, normalized size = 0.55

$$\frac{1}{3440640}\,(4\,(8\,(4\,(16\,(20\,(120\,x+317)x+679)x+158631)x+354907)x+3685583)x+6231117)\sqrt{2\,x^2-x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/3440640*(4*(8*(4*(16*(20*(120*x + 317)*x + 679)*x + 158631)*x + 354907)*x
 + 3685583)*x + 6231117)*sqrt(2*x^2 - x + 3) - 1183005/131072*sqrt(2)*log(-
2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.02, size = 115, normalized size = 0.80

$$\frac{5\left(2x^2-x+3\right)^{\frac{3}{2}}x^4}{7}+\frac{377\left(2x^2-x+3\right)^{\frac{3}{2}}x^3}{168}+\frac{283\left(2x^2-x+3\right)^{\frac{3}{2}}x^2}{1120}-\frac{5179\left(2x^2-x+3\right)^{\frac{3}{2}}x}{17920}+\frac{1183005\sqrt{2}\ \mathrm{arcs}}{131}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/7*x^4*(2*x^2-x+3)^(3/2)+377/168*x^3*(2*x^2-x+3)^(3/2)+283/1120*x^2*(2*x^2
-x+3)^(3/2)-5179/17920*x*(2*x^2-x+3)^(3/2)+1183005/131072*2^(1/2)*arcsinh(4
/23*23^(1/2)*(x-1/4))+51435/32768*(4*x-1)*(2*x^2-x+3)^(1/2)+242329/215040*(
2*x^2-x+3)^(3/2)
```

maxima [A] time = 0.97, size = 126, normalized size = 0.88

$$\frac{5}{7}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^4+\frac{377}{168}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{283}{1120}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{5179}{17920}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x+\frac{242329}{215040}\left(2\,x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="max
ima")
```

```
[Out] 5/7*(2*x^2 - x + 3)^(3/2)*x^4 + 377/168*(2*x^2 - x + 3)^(3/2)*x^3 + 283/112
0*(2*x^2 - x + 3)^(3/2)*x^2 - 5179/17920*(2*x^2 - x + 3)^(3/2)*x + 242329/2
15040*(2*x^2 - x + 3)^(3/2) + 51435/8192*sqrt(2*x^2 - x + 3)*x + 1183005/13
1072*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 51435/32768*sqrt(2*x^2 - x
+ 3)
```

mupad [B] time = 1.72, size = 170, normalized size = 1.19

$$\frac{283\,x^2\left(2\,x^2-x+3\right)^{3/2}}{1120}+\frac{377\,x^3\left(2\,x^2-x+3\right)^{3/2}}{168}+\frac{5\,x^4\left(2\,x^2-x+3\right)^{3/2}}{7}+\frac{4478951\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}}{}\right.}{573440}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5)*(2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

```
[Out] (283*x^2*(2*x^2 - x + 3)^(3/2))/1120 + (377*x^3*(2*x^2 - x + 3)^(3/2))/168
+ (5*x^4*(2*x^2 - x + 3)^(3/2))/7 + (4478951*2^(1/2)*log((2*x^2 - x + 3)^(1
/2) + (2^(1/2)*(2*x - 1/2))/2))/573440 + (194737*(x/2 - 1/8)*(2*x^2 - x + 3
)^(1/2))/17920 + (242329*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*x + 45))/3440640
 - (5179*x*(2*x^2 - x + 3)^(3/2))/17920 + (5573567*2^(1/2)*log(2*(2*x^2 - x
 + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/4587520
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+5)\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((2*x + 5)*sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2), x
)
```

3.325 $\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right) dx$

Optimal. Leaf size=124

$$\frac{7}{80}\left(2x^2-x+3\right)^{3/2}x^2-\frac{71\left(2x^2-x+3\right)^{3/2}x}{1280}+\frac{287\left(2x^2-x+3\right)^{3/2}}{5120}-\frac{4609(1-4x)\sqrt{2x^2-x+3}}{16384}+\frac{5}{12}\left(2x^2-x+\right.$$

```
[Out] 287/5120*(2*x^2-x+3)^(3/2)-71/1280*x*(2*x^2-x+3)^(3/2)+7/80*x^2*(2*x^2-x+3)
^(3/2)+5/12*x^3*(2*x^2-x+3)^(3/2)-106007/65536*arcsinh(1/23*(1-4*x)*23^(1/2
))*2^(1/2)-4609/16384*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3+\frac{7}{80}\left(2x^2-x+3\right)^{3/2}x^2-\frac{71\left(2x^2-x+3\right)^{3/2}x}{1280}+\frac{287\left(2x^2-x+3\right)^{3/2}}{5120}-\frac{4609(1-4x)\sqrt{2x^2}}{16384}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-4609*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16384 + (287*(3 - x + 2*x^2)^(3/2))/5
120 - (71*x*(3 - x + 2*x^2)^(3/2))/1280 + (7*x^2*(3 - x + 2*x^2)^(3/2))/80
+ (5*x^3*(3 - x + 2*x^2)^(3/2))/12 - (106007*ArcSinh[(1 - 4*x)/Sqrt[23]])/(
32768*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
```

```
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)dx = \frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2}+\frac{1}{12}\int \sqrt{3-x+2x^2}\left(24+12x-9x^2+\frac{2}{}\right.$$
$$=\frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2}+\frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2}+\frac{1}{120}\int\left(240+5\right.$$
$$=-\frac{71x\left(3-x+2x^2\right)^{3/2}}{1280}+\frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2}+\frac{5}{12}x^3\left(3-x+2\right.$$
$$=\frac{287\left(3-x+2x^2\right)^{3/2}}{5120}-\frac{71x\left(3-x+2x^2\right)^{3/2}}{1280}+\frac{7}{80}x^2\left(3-x+2x^2\right)$$
$$=-\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384}+\frac{287\left(3-x+2x^2\right)^{3/2}}{5120}-\frac{71x\left(3-x\right.}{12}$$
$$=-\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384}+\frac{287\left(3-x+2x^2\right)^{3/2}}{5120}-\frac{71x\left(3-x\right.}{12}$$
$$=-\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384}+\frac{287\left(3-x+2x^2\right)^{3/2}}{5120}-\frac{71x\left(3-x\right.}{12}$$

Mathematica [A] time = 0.09, size = 65, normalized size = 0.52

$$\frac{4\sqrt{2x^2-x+3}\left(204800x^5-59392x^4+258432x^3+105696x^2+221868x-27807\right)-1590105\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{983040}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-27807 + 221868*x + 105696*x^2 + 258432*x^3 - 59392
*x^4 + 204800*x^5) - 1590105*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/983040
```

fricas [A] time = 0.85, size = 78, normalized size = 0.63

$$\frac{1}{245760}\left(204800\,x^5-59392\,x^4+258432\,x^3+105696\,x^2+221868\,x-27807\right)\sqrt{2\,x^2-x+3}+\frac{106007}{131072}\sqrt{2}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/245760*(204800*x^5 - 59392*x^4 + 258432*x^3 + 105696*x^2 + 221868*x - 278
07)*sqrt(2*x^2 - x + 3) + 106007/131072*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 -
 x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.20, size = 73, normalized size = 0.59

$$\frac{1}{245760}\left(4\left(8\left(4\left(16\left(100\,x-29\right)x+2019\right)x+3303\right)x+55467\right)x-27807\right)\sqrt{2\,x^2-x+3}-\frac{106007}{65536}\sqrt{2}\log\left(-2\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/245760*(4*(8*(4*(16*(100*x - 29)*x + 2019)*x + 3303)*x + 55467)*x - 27807
)*sqrt(2*x^2 - x + 3) - 106007/65536*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sq
rt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 98, normalized size = 0.79

$$\frac{5\left(2x^2-x+3\right)^{\frac{3}{2}}x^3}{12}+\frac{7\left(2x^2-x+3\right)^{\frac{3}{2}}x^2}{80}-\frac{71\left(2x^2-x+3\right)^{\frac{3}{2}}x}{1280}+\frac{106007\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536}+\frac{287\left(2x^2-\right.}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/12*(2*x^2-x+3)^(3/2)*x^3+7/80*(2*x^2-x+3)^(3/2)*x^2-71/1280*(2*x^2-x+3)^(
3/2)*x+287/5120*(2*x^2-x+3)^(3/2)+4609/16384*(4*x-1)*(2*x^2-x+3)^(1/2)+1060
07/65536*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 0.95, size = 109, normalized size = 0.88

$$\frac{5}{12}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{7}{80}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{71}{1280}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x+\frac{287}{5120}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{4609}{4096}\sqrt{2\,x^2-x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/12*(2*x^2 - x + 3)^(3/2)*x^3 + 7/80*(2*x^2 - x + 3)^(3/2)*x^2 - 71/1280*(
2*x^2 - x + 3)^(3/2)*x + 287/5120*(2*x^2 - x + 3)^(3/2) + 4609/4096*sqrt(2*
x^2 - x + 3)*x + 106007/65536*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 46
09/16384*sqrt(2*x^2 - x + 3)
```

mupad [B] time = 0.77, size = 153, normalized size = 1.23

$$\frac{7\,x^2\left(2\,x^2-x+3\right)^{3/2}}{80}+\frac{5\,x^3\left(2\,x^2-x+3\right)^{3/2}}{12}+\frac{63779\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\left(2\,x-\frac{1}{2}\right)}{2}\right)}{40960}+\frac{2773\left(\frac{x}{2}-\frac{1}{8}\right)\sqrt{2\,x^2}}{1280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

```
[Out] (7*x^2*(2*x^2 - x + 3)^(3/2))/80 + (5*x^3*(2*x^2 - x + 3)^(3/2))/12 + (6377
9*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/40960 + (27
73*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/1280 + (287*(2*x^2 - x + 3)^(1/2)*(32
*x^2 - 4*x + 45))/81920 - (71*x*(2*x^2 - x + 3)^(3/2))/1280 + (19803*2^(1/2
)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/327680
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{2x^2-x+3}\,\left(5x^4-x^3+3x^2+x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2), x)
```

3.326 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx$

Optimal. Leaf size=149

$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2-\frac{127}{128}\left(2x^2-x+3\right)^{3/2}(2x+5)+\frac{4535}{768}\left(2x^2-x+3\right)^{3/2}+\frac{(489587-80844x)\sqrt{2x^2-x+}}{4096}$$

```
[Out] 4535/768*(2*x^2-x+3)^(3/2)-127/128*(5+2*x)*(2*x^2-x+3)^(3/2)+1/16*(5+2*x)^2
*(2*x^2-x+3)^(3/2)+5627989/16384*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-110
01/32*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/4096*(489
587-80844*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2-\frac{127}{128}\left(2x^2-x+3\right)^{3/2}(2x+5)+\frac{4535}{768}\left(2x^2-x+3\right)^{3/2}+\frac{(489587-80844x)\sqrt{2x^2-x+}}{4096}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

```
[Out] ((489587 - 80844*x)*Sqrt[3 - x + 2*x^2])/4096 + (4535*(3 - x + 2*x^2)^(3/2)
)/768 - (127*(5 + 2*x)*(3 - x + 2*x^2)^(3/2))/128 + ((5 + 2*x)^2*(3 - x + 2
*x^2)^(3/2))/16 + (5627989*ArcSinh[(1 - 4*x)/Sqrt[23]])/(8192*Sqrt[2]) - (1
1001*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(16*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
```

```
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx = \frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2}+\frac{1}{160}\int \frac{\sqrt{3-x+2x^2}\left(-805-\right.}{5}$$

$$= -\frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2}+\frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2}$$

$$= \frac{4535}{768}\left(3-x+2x^2\right)^{3/2}-\frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2}+\frac{1}{16}(5$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096}+\frac{4535}{768}\left(3-x+2x^2\right)^{3/2}-\frac{1}{1}$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096}+\frac{4535}{768}\left(3-x+2x^2\right)^{3/2}-\frac{1}{1}$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096}+\frac{4535}{768}\left(3-x+2x^2\right)^{3/2}-\frac{1}{1}$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096}+\frac{4535}{768}\left(3-x+2x^2\right)^{3/2}-\frac{1}{1}$$

Mathematica [A] time = 0.15, size = 91, normalized size = 0.61

$$\frac{-16897536\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+4\sqrt{2x^2-x+3}\left(6144x^4-21120x^3+79840x^2-300404x+1561161\right)+1\cdots}{49152}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(1561161 - 300404*x + 79840*x^2 - 21120*x^3 + 6144*x
^4) + 16883967*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - 16897536*Sqrt[2]*ArcTa
nh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/49152
```

fricas [A] time = 0.89, size = 125, normalized size = 0.84

$$\frac{1}{12288}\left(6144\,x^4-21120\,x^3+79840\,x^2-300404\,x+1561161\right)\sqrt{2\,x^2-x+3}+\frac{5627989}{32768}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2\,x^2-x+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="fri
cas")
```

```
[Out] 1/12288*(6144*x^4 - 21120*x^3 + 79840*x^2 - 300404*x + 1561161)*sqrt(2*x^2
- x + 3) + 5627989/32768*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1
) - 32*x^2 + 16*x - 25) + 11001/64*sqrt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x
+ 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))
```

giac [A] time = 0.22, size = 129, normalized size = 0.87

$$\frac{1}{12288}\left(4\left(8\left(12\left(16\,x-55\right)x+2495\right)x-75101\right)x+1561161\right)\sqrt{2\,x^2-x+3}+\frac{5627989}{16384}\sqrt{2}\log\left(-4\sqrt{2}x+\sqrt{2}+4\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="gia
c")
```

```
[Out] 1/12288*(4*(8*(12*(16*x - 55)*x + 2495)*x - 75101)*x + 1561161)*sqrt(2*x^2
- x + 3) + 5627989/16384*sqrt(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2
- x + 3)) - 11001/32*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2
- x + 3))) + 11001/32*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*
x^2 - x + 3)))
```

maple [A] time = 0.01, size = 127, normalized size = 0.85

$$\frac{\left(2x^2-x+3\right)^{\frac{3}{2}}x^2}{4}-\frac{47\left(2x^2-x+3\right)^{\frac{3}{2}}x}{64}-\frac{5627989\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{16384}-\frac{11001\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2\cdots}}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x)
```

```
[Out] 1/4*(2*x^2-x+3)^(3/2)*x^2-47/64*(2*x^2-x+3)^(3/2)*x+1925/768*(2*x^2-x+3)^(3
/2)-20211/4096*(4*x-1)*(2*x^2-x+3)^(1/2)-5627989/16384*2^(1/2)*arcsinh(4/23
*23^(1/2)*(x-1/4))+3667/32*(2*(x+5/2)^2-11*x-19/2)^(1/2)-11001/32*2^(1/2)*a
rctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

maxima [A] time = 1.01, size = 128, normalized size = 0.86

$$\frac{1}{4}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{47}{64}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x+\frac{1925}{768}\left(2\,x^2-x+3\right)^{\frac{3}{2}}-\frac{20211}{1024}\sqrt{2\,x^2-x+3}\,x-\frac{5627989}{16384}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4\cdots}{23\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="max
ima")
```

```
[Out] 1/4*(2*x^2 - x + 3)^(3/2)*x^2 - 47/64*(2*x^2 - x + 3)^(3/2)*x + 1925/768*(2
*x^2 - x + 3)^(3/2) - 20211/1024*sqrt(2*x^2 - x + 3)*x - 5627989/16384*sqrt
(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 11001/32*sqrt(2)*arcsinh(22/
23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 489587/4096*sqr
t(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{2\,x+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5),x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2-x+3}\,\left(5x^4-x^3+3x^2+x+2\right)}{2x+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5), x
)
```

3.327 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx$

Optimal. Leaf size=149

$$\frac{5}{64}(2x+5)\left(2x^2-x+3\right)^{3/2}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}-\frac{541}{384}\left(2x^2-x+3\right)^{3/2}-\frac{(1996953-333380x)\sqrt{2x^2-x+3}}{18432}+$$

[Out] -541/384*(2*x^2-x+3)^(3/2)-3667/576*(2*x^2-x+3)^(3/2)/(5+2*x)+5/64*(5+2*x)*(2*x^2-x+3)^(3/2)-2551847/8192*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+239201/768*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-1/18432*(1996953-333380*x)*(2*x^2-x+3)^(1/2)

Rubi [A] time = 0.24, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1650, 1653, 814, 843, 619, 215, 724, 206}

$$\frac{5}{64}(2x+5)\left(2x^2-x+3\right)^{3/2}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}-\frac{541}{384}\left(2x^2-x+3\right)^{3/2}-\frac{(1996953-333380x)\sqrt{2x^2-x+3}}{18432}+$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x]

[Out] -((1996953 - 333380*x)*Sqrt[3 - x + 2*x^2])/18432 - (541*(3 - x + 2*x^2)^(3/2))/384 - (3667*(3 - x + 2*x^2)^(3/2))/(576*(5 + 2*x)) + (5*(5 + 2*x)*(3 - x + 2*x^2)^(3/2))/64 - (2551847*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4096*Sqrt[2]) + (239201*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(384*Sqrt[2])

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
```

```
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{576(5+2x)} - \frac{1}{72}\int \frac{\sqrt{3-x+2x^2}\left(\frac{19341}{16}-\frac{6313x}{2}+4\right.}{5+2x}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{576(5+2x)} + \frac{5}{64}(5+2x)\left(3-x+2x^2\right)^{3/2} - \int \frac{\sqrt{3-x+}}{}$$

$$= -\frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{576(5+2x)} + \frac{5}{64}(5+2x)\left(3-\right.$$

$$= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3}{}$$

$$= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3}{}$$

$$= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3}{}$$

$$= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3}{}$$

Mathematica [A] time = 0.16, size = 98, normalized size = 0.66

$$\frac{7654432\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{4\sqrt{2x^2-x+3}\left(3840x^4-17344x^3+94936x^2-728410x-3539439\right)}{2x+5} - 7655541\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{24576}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x
]
```

```
[Out] ((4*Sqrt[3 - x + 2*x^2]*(-3539439 - 728410*x + 94936*x^2 - 17344*x^3 + 3840
*x^4))/(5 + 2*x) - 7655541*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 7654432*Sq
rt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/24576
```

fricas [A] time = 0.92, size = 143, normalized size = 0.96

$$\frac{7655541\sqrt{2}(2x+5)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+7654432\sqrt{2}(2x+5)\log\left(\frac{24\sqrt{2}\sqrt{}}{}\right.}{49152(2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="f
ricas")
```

```
[Out] 1/49152*(7655541*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25) + 7654432*sqrt(2)*(2*x + 5)*log((24*sqrt(2)*sqrt
(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)
) + 8*(3840*x^4 - 17344*x^3 + 94936*x^2 - 728410*x - 3539439)*sqrt(2*x^2 -
x + 3))/(2*x + 5)
```

giac [B] time = 0.53, size = 531, normalized size = 3.56

$$\frac{1}{24576}\sqrt{2}\left(7654432\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)\mathrm{sgn}\left(\frac{1}{2x+5}\right)+7655541\log\left(\left|\sqrt{-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="g
iac")
```

```
[Out] 1/24576*sqrt(2)*(7654432*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) +
72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 7655541*log(abs(sqrt(-11/(2*x + 5) +
36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 7655541*log(abs(
sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*sgn(1/(2*x + 5
)) - 1408128*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)*sgn(1/(2*x + 5)) + 2*
(16367883*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^7*sgn(1/
(2*x + 5)) - 34896384*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
5))^6*sgn(1/(2*x + 5)) - 93395*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) +
6/(2*x + 5))^5*sgn(1/(2*x + 5)) + 25574400*(sqrt(-11/(2*x + 5) + 36/(2*x +
5)^2 + 1) + 6/(2*x + 5))^4*sgn(1/(2*x + 5)) + 19752365*(sqrt(-11/(2*x + 5)
+ 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3*sgn(1/(2*x + 5)) - 31921920*(sqrt(-1
1/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2*sgn(1/(2*x + 5)) - 24458
13*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x + 5)
) + 7663104*sgn(1/(2*x + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) +
6/(2*x + 5))^2 - 1)^4)
```

maple [A] time = 0.01, size = 152, normalized size = 1.02

$$\frac{5\left(2x^2-x+3\right)^{\frac{3}{2}}x}{32}+\frac{2551847\sqrt{2}\ \mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8192}+\frac{239201\sqrt{2}\ \mathrm{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{768}-\frac{391\left(2x^2\right.}{38}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x)
```

```
[Out] 5/32*(2*x^2-x+3)^(3/2)*x-391/384*(2*x^2-x+3)^(3/2)+6001/2048*(4*x-1)*(2*x^2
-x+3)^(1/2)+2551847/8192*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-3667/1152/(
x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-239201/2304*(-11*x+2*(x+5/2)^2-19/2)^
(1/2)+239201/768*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)
^2-19/2)^(1/2))+3667/2304*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)
```

maxima [A] time = 1.00, size = 132, normalized size = 0.89

$$\frac{5}{32}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{391}{384}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{6001}{512}\sqrt{2x^2-x+3}\,x+\frac{2551847}{8192}\sqrt{2}\ \mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="m
axima")
```

```
[Out] 5/32*(2*x^2 - x + 3)^(3/2)*x - 391/384*(2*x^2 - x + 3)^(3/2) + 6001/512*sqr
t(2*x^2 - x + 3)*x + 2551847/8192*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sq
rt(23)) - 239201/768*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
```

```
sqrt(23)/abs(2*x + 5)) - 182769/2048*sqrt(2*x^2 - x + 3) - 3667/32*sqrt(2*x
^2 - x + 3)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^{2} - x + 3} \left(5x^{4} - x^{3} + 3x^{2} + x + 2\right)}{\left(2x + 5\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**2,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**2
, x)
```

3.328 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^3}\,dx$

Optimal. Leaf size=151

$$\frac{357391\left(2x^2-x+3\right)^{3/2}}{82944(2x+5)}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{1152(2x+5)^2}+\frac{5}{48}\left(2x^2-x+3\right)^{3/2}+\frac{5(661065-110099x)\sqrt{2x^2-x+3}}{82944}-\frac{12\cdots}{\cdots}$$

```
[Out] 5/48*(2*x^2-x+3)^(3/2)-3667/1152*(2*x^2-x+3)^(3/2)/(5+2*x)^2+357391/82944*(
2*x^2-x+3)^(3/2)/(5+2*x)+117315/1024*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
-12670805/110592*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+
5/82944*(661065-110099*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1650, 1653, 814, 843, 619, 215, 724, 206}

$$\frac{357391\left(2x^2-x+3\right)^{3/2}}{82944(2x+5)}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{1152(2x+5)^2}+\frac{5}{48}\left(2x^2-x+3\right)^{3/2}+\frac{5(661065-110099x)\sqrt{2x^2-x+3}}{82944}-\frac{12\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x]
```

```
[Out] (5*(661065 - 110099*x)*Sqrt[3 - x + 2*x^2])/82944 + (5*(3 - x + 2*x^2)^(3/2
))/48 - (3667*(3 - x + 2*x^2)^(3/2))/(1152*(5 + 2*x)^2) + (357391*(3 - x +
2*x^2)^(3/2))/(82944*(5 + 2*x)) + (117315*ArcSinh[(1 - 4*x)/Sqrt[23]])/(512
*Sqrt[2]) - (12670805*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])]
)/(55296*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
```

```
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\,\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^3}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{1152(5+2x)^2} - \frac{1}{144}\int \frac{\sqrt{3-x+2x^2}\,\left(\frac{27681}{16}-\frac{14251}{4}\right.}{(5+2x)^2}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{1152(5+2x)^2} + \frac{357391\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)} + \int \frac{\sqrt{3-x+2}}{}$$

$$= \frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{1152(5+2x)^2} + \frac{357391\left(3-x\right.}{82944(5+}$$

$$= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3}{}$$

$$= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3}{}$$

$$= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3}{}$$

$$= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3}{}$$

Mathematica [A] time = 0.15, size = 98, normalized size = 0.65

$$\frac{-12670805\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{24\sqrt{2x^2-x+3}\left(3840x^4-25632x^3+272520x^2+2959330x+4880551\right)}{(2x+5)^2} + 12670020\sqrt{2}\,\mathrm{sinh}}{110592}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x
]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(4880551 + 2959330*x + 272520*x^2 - 25632*x^3 + 38
40*x^4))/(5 + 2*x)^2 + 12670020*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - 12670
805*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/110592
```

fricas [A] time = 0.84, size = 159, normalized size = 1.05

$$12670020\sqrt{2}\left(4x^2+20x+25\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)+12670805\sqrt{2}\left(4x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="f
ricas")
```

```
[Out] 1/221184*(12670020*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 12670805*sqrt(2)*(4*x^2 + 20*x + 25
)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 11
53)/(4*x^2 + 20*x + 25)) + 48*(3840*x^4 - 25632*x^3 + 272520*x^2 + 2959330*
x + 4880551)*sqrt(2*x^2 - x + 3))/(4*x^2 + 20*x + 25)
```

giac [B] time = 0.24, size = 258, normalized size = 1.71

$$\frac{1}{768}\left(4\left(40\,x-467\right)x+19695\right)\sqrt{2\,x^2-x+3}+\frac{117315}{1024}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)-\frac{12670805}{110592}\,\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="g
iac")
```

```
[Out] 1/768*(4*(40*x - 467)*x + 19695)*sqrt(2*x^2 - x + 3) + 117315/1024*sqrt(2)*
log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 12670805/110592*sqr
t(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 12670805/11
0592*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) +
1/9216*sqrt(2)*(10693526*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 7989
5946*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 124044603*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3)) + 80334011)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10
*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2
```

maple [A] time = 0.02, size = 158, normalized size = 1.05

$$-\frac{117315\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{1024}-\frac{12670805\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{110592}+\frac{5\left(2x^2-x+3\right)^{\frac{3}{2}}}{48}-\frac{149\left(4x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x)
```

```
[Out] 5/48*(2*x^2-x+3)^(3/2)-149/256*(4*x-1)*(2*x^2-x+3)^(1/2)-117315/1024*2^(1/2
)*arcsinh(4/23*23^(1/2)*(x-1/4))+357391/165888/(x+5/2)*(-11*x+2*(x+5/2)^2-1
9/2)^(3/2)+12670805/331776*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-12670805/110592*2
^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))-35
7391/331776*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-3667/4608/(x+5/2)^2*(-11
*x+2*(x+5/2)^2-19/2)^(3/2)
```

maxima [A] time = 1.00, size = 143, normalized size = 0.95

$$\frac{5}{48}\left(2\,x^2-x+3\right)^{\frac{3}{2}}-\frac{149}{64}\,\sqrt{2\,x^2-x+3}\,x-\frac{117315}{1024}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\,\sqrt{23}\,x-\frac{1}{23}\,\sqrt{23}\right)+\frac{12670805}{110592}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{22}{23\,|2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="m
axima")
```

```
[Out] 5/48*(2*x^2 - x + 3)^(3/2) - 149/64*sqrt(2*x^2 - x + 3)*x - 117315/1024*sqr
t(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 12670805/110592*sqrt(2)*arc
sinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 3877/14
4*sqrt(2*x^2 - x + 3) - 3667/1152*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25)
 + 357391/4608*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}\left(5x^4 - x^3 + 3x^2 + x + 2\right)}{(2x + 5)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**3,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**3
, x)
```

3.329 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx$

Optimal. Leaf size=158

$$-\frac{6467659\left(2x^2-x+3\right)^{3/2}}{5971968(2x+5)}+\frac{158527\left(2x^2-x+3\right)^{3/2}}{82944(2x+5)^2}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{1728(2x+5)^3}-\frac{(44378877-7400779x)\sqrt{2x^2-x+}}{5971968}$$

```
[Out] -3667/1728*(2*x^2-x+3)^(3/2)/(5+2*x)^3+158527/82944*(2*x^2-x+3)^(3/2)/(5+2*
x)^2-6467659/5971968*(2*x^2-x+3)^(3/2)/(5+2*x)-10939/512*arcsinh(1/23*(1-4*
x)*23^(1/2))*2^(1/2)+170114729/7962624*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^
2-x+3)^(1/2))*2^(1/2)-1/5971968*(44378877-7400779*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{6467659\left(2x^2-x+3\right)^{3/2}}{5971968(2x+5)}+\frac{158527\left(2x^2-x+3\right)^{3/2}}{82944(2x+5)^2}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{1728(2x+5)^3}-\frac{(44378877-7400779x)\sqrt{2x^2-x+}}{5971968}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x]
```

```
[Out] -((44378877 - 7400779*x)*Sqrt[3 - x + 2*x^2])/5971968 - (3667*(3 - x + 2*x^
2)^(3/2))/(1728*(5 + 2*x)^3) + (158527*(3 - x + 2*x^2)^(3/2))/(82944*(5 + 2
*x)^2) - (6467659*(3 - x + 2*x^2)^(3/2))/(5971968*(5 + 2*x)) - (10939*ArcSi
nh[(1 - 4*x)/Sqrt[23]])/(256*Sqrt[2]) + (170114729*ArcTanh[(17 - 22*x)/(12*
Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(3981312*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
```

```
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3} - \frac{1}{216}\int \frac{\sqrt{3-x+2x^2}\left(\frac{36021}{16}-3969.\right.}{(5+2x)}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3} + \frac{158527\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)^2} + \int \frac{\sqrt{3-x+2}}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3} + \frac{158527\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)^2} - \frac{6467659}{5971}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667\left(3-x+2x^2\right)}{1728(5+2x)^3}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667\left(3-x+2x^2\right)}{1728(5+2x)^3}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667\left(3-x+2x^2\right)}{1728(5+2x)^3}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667\left(3-x+2x^2\right)}{1728(5+2x)^3}$$

Mathematica [A] time = 0.16, size = 98, normalized size = 0.62

$$\frac{170114729\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+\frac{24\sqrt{2x^2-x+3}\left(414720x^4-5453568x^3-97682900x^2-329667508x-327735797\right)}{(2x+5)^3}-170123328\sqrt{}}{7962624}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x
]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(-327735797 - 329667508*x - 97682900*x^2 - 5453568
*x^3 + 414720*x^4))/(5 + 2*x)^3 - 170123328*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[
23]] + 170114729*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/7
962624
```

fricas [A] time = 0.66, size = 173, normalized size = 1.09

$$170123328\sqrt{2}\left(8x^3+60x^2+150x+125\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+170114729$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="f
ricas")
```

```
[Out] 1/15925248*(170123328*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)
*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 170114729*sqrt(2)*(8
*x^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17
) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(414720*x^4 - 54535
68*x^3 - 97682900*x^2 - 329667508*x - 327735797)*sqrt(2*x^2 - x + 3))/(8*x^
3 + 60*x^2 + 150*x + 125)
```

giac [B] time = 0.26, size = 304, normalized size = 1.92

$$\frac{1}{128}\sqrt{2x^2-x+3}(20x-413)-\frac{10939}{512}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{170114729}{7962624}\sqrt{2}\log\left(\left|-2\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="g
iac")
```

```
[Out] 1/128*sqrt(2*x^2 - x + 3)*(20*x - 413) - 10939/512*sqrt(2)*log(-2*sqrt(2)*(
sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 170114729/7962624*sqrt(2)*log(abs(-
2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 170114729/7962624*sqrt(2)
*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/663552*sqr
t(2)*(575810908*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 9206213116*(s
qrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 9688786604*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^3 - 73157325092*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 4948195
2947*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 20269228621)/(2*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) -
11)^3
```

maple [A] time = 0.01, size = 165, normalized size = 1.04

$$\frac{10939\sqrt{2}\,\mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{512}+\frac{170114729\sqrt{2}\,\mathrm{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{7962624}+\frac{5(4x-1)\sqrt{2x^2-x+3}}{128}-\frac{64676}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x)
```

```
[Out] 5/128*(4*x-1)*(2*x^2-x+3)^(1/2)+10939/512*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-
1/4))-6467659/11943936/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-170114729/238
87872*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+170114729/7962624*2^(1/2)*arctanh(1/12
*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))+6467659/23887872*(4*x
-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+158527/331776/(x+5/2)^2*(-11*x+2*(x+5/2)
^2-19/2)^(3/2)-3667/13824/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(3/2)
```

maxima [A] time = 0.98, size = 160, normalized size = 1.01

$$\frac{5}{32}\sqrt{2x^2-x+3}\,x+\frac{10939}{512}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{170114729}{7962624}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="m
axima")
```

```
[Out] 5/32*sqrt(2*x^2 - x + 3)*x + 10939/512*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/
23*sqrt(23)) - 170114729/7962624*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x +
 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 693775/165888*sqrt(2*x^2 - x + 3) - 36
67/1728*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 158527/82944
*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 6467659/331776*sqrt(2*x^2 - x
+ 3)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2x^2-x+3}\,\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2-x+3}\,\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**4,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**4
, x)
```

3.330 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx$

Optimal. Leaf size=165

$$-\frac{9363383\left(2x^2-x+3\right)^{3/2}}{23887872(2x+5)^2}+\frac{593771\left(2x^2-x+3\right)^{3/2}}{497664(2x+5)^3}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}+\frac{7(9616196x+52836655)\sqrt{2x^2-x}}{95551488(2x+5)}$$

```
[Out] -3667/2304*(2*x^2-x+3)^(3/2)/(5+2*x)^4+593771/497664*(2*x^2-x+3)^(3/2)/(5+2
*x)^3-9363383/23887872*(2*x^2-x+3)^(3/2)/(5+2*x)^2+259/128*arcsinh(1/23*(1-
4*x)*23^(1/2))*2^(1/2)-4640586097/2293235712*arctanh(1/24*(17-22*x)*2^(1/2)
/(2*x^2-x+3)^(1/2))*2^(1/2)+7/95551488*(52836655+9616196*x)*(2*x^2-x+3)^(1/
2)/(5+2*x)
```

Rubi [A] time = 0.23, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 812, 843, 619, 215, 724, 206}

$$-\frac{9363383\left(2x^2-x+3\right)^{3/2}}{23887872(2x+5)^2}+\frac{593771\left(2x^2-x+3\right)^{3/2}}{497664(2x+5)^3}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}+\frac{7(9616196x+52836655)\sqrt{2x^2-x}}{95551488(2x+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x]
```

```
[Out] (7*(52836655 + 9616196*x)*Sqrt[3 - x + 2*x^2])/(95551488*(5 + 2*x)) - (3667
*(3 - x + 2*x^2)^(3/2))/(2304*(5 + 2*x)^4) + (593771*(3 - x + 2*x^2)^(3/2))
/(497664*(5 + 2*x)^3) - (9363383*(3 - x + 2*x^2)^(3/2))/(23887872*(5 + 2*x)
^2) + (259*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2]) - (4640586097*ArcTanh[
(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1146617856*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
```

```
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4} - \frac{1}{288}\int \frac{\sqrt{3-x+2x^2}\left(\frac{44361}{16}-\frac{17501}{4}\right.}{(5+2x)}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4} + \frac{593771\left(3-x+2x^2\right)^{3/2}}{497664(5+2x)^3} + \int \frac{\sqrt{3-x+2}}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4} + \frac{593771\left(3-x+2x^2\right)^{3/2}}{497664(5+2x)^3} - \frac{9363383}{23887}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667\left(3-x+2x^2\right)}{2304(5+2x)^4}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667\left(3-x+2x^2\right)}{2304(5+2x)^4}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667\left(3-x+2x^2\right)}{2304(5+2x)^4}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667\left(3-x+2x^2\right)}{2304(5+2x)^4}$$

Mathematica [A] time = 0.18, size = 98, normalized size = 0.59

$$\frac{-4640586097\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+\frac{24\sqrt{2x^2-x+3}\left(238878720x^4+6105343976x^3+31323229164x^2+62847867486x+44676885233\right)}{(2x+5)^4}}{2293235712}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x
]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(44676885233 + 62847867486*x + 31323229164*x^2 + 6
105343976*x^3 + 238878720*x^4))/(5 + 2*x)^4 + 4640219136*Sqrt[2]*ArcSinh[(1
 - 4*x)/Sqrt[23]] - 4640586097*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x
 + 4*x^2])])/2293235712
```

fricas [A] time = 0.84, size = 189, normalized size = 1.15

$$\frac{4640219136\sqrt{2}\left(16x^4+160x^3+600x^2+1000x+625\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="f
ricas")
```

```
[Out] 1/4586471424*(4640219136*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625
)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 46405
86097*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*log(-(24*sqrt(2)*
sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x +
 25)) + 48*(238878720*x^4 + 6105343976*x^3 + 31323229164*x^2 + 62847867486*
x + 44676885233)*sqrt(2*x^2 - x + 3))/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x
+ 625)
```

giac [B] time = 0.37, size = 327, normalized size = 1.98

$$-\frac{1}{2293235712}\sqrt{2}\left(4640586097\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)+464021913\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="g
iac")
```

```
[Out] -1/2293235712*sqrt(2)*(4640586097*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^
2 + 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 4640219136*log(abs(sqrt(-11/
(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 4640
219136*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))
*sgn(1/(2*x + 5)) + 12*(24*(144*(792072*sgn(1/(2*x + 5))/(2*x + 5) - 835793
*sgn(1/(2*x + 5)))/(2*x + 5) + 57384361*sgn(1/(2*x + 5)))/(2*x + 5) - 46456
9597*sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 179159040
*(11*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x +
5)) - 12*sgn(1/(2*x + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(
2*x + 5))^2 - 1))
```

maple [A] time = 0.01, size = 167, normalized size = 1.01

$$-\frac{259\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128}-\frac{4640586097\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^{2}-\frac{19}{2}}}\right)}{2293235712}+\frac{4640586097\sqrt{-11x+2\left(x-\right.}}{6879707136}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x)
```

```
[Out] 4640586097/6879707136*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+593771/3981312/(x+5/2)
^3*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-3667/36864/(x+5/2)^4*(-11*x+2*(x+5/2)^2-1
9/2)^(3/2)-9363383/95551488/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-201573
155/6879707136*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+201573155/3439853568/
(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-4640586097/2293235712*2^(1/2)*arctan
h(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))-259/128*2^(1/2)
*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 1.02, size = 181, normalized size = 1.10

$$-\frac{259}{128}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)+\frac{4640586097}{2293235712}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|}-\frac{17\sqrt{23}}{23|2x+5|}\right)+\frac{16828343}{47775744}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="m
axima")
```

```
[Out] -259/128*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 4640586097/2293
235712*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2
*x + 5)) + 16828343/47775744*sqrt(2*x^2 - x + 3) - 3667/2304*(2*x^2 - x + 3
)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 593771/497664*(2*x^2
- x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 9363383/23887872*(2*x^2 - x
 + 3)^(3/2)/(4*x^2 + 20*x + 25) + 201573155/95551488*sqrt(2*x^2 - x + 3)/(2
*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^{2}-x+3}\left(5x^{4}-x^{3}+3x^{2}+x+2\right)}{\left(2x+5\right)^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**5,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**5
, x)
```

3.331 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx$

Optimal. Leaf size=165

$$-\frac{38732321\left(2x^2-x+3\right)^{3/2}}{179159040(2x+5)^3}+\frac{711961\left(2x^2-x+3\right)^{3/2}}{829440(2x+5)^4}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{2880(2x+5)^5}-\frac{(3174439702x+4583087983)\sqrt{2x}}{6879707136(2x+5)^2}$$

```
[Out] -3667/2880*(2*x^2-x+3)^(3/2)/(5+2*x)^5+711961/829440*(2*x^2-x+3)^(3/2)/(5+2
*x)^4-38732321/179159040*(2*x^2-x+3)^(3/2)/(5+2*x)^3-5/64*arcsinh(1/23*(1-4
*x)*23^(1/2))*2^(1/2)+12895597463/165112971264*arctanh(1/24*(17-22*x)*2^(1/
2)/(2*x^2-x+3)^(1/2))*2^(1/2)-1/6879707136*(4583087983+3174439702*x)*(2*x^2
-x+3)^(1/2)/(5+2*x)^2
```

Rubi [A] time = 0.23, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 810, 843, 619, 215, 724, 206}

$$-\frac{38732321\left(2x^2-x+3\right)^{3/2}}{179159040(2x+5)^3}+\frac{711961\left(2x^2-x+3\right)^{3/2}}{829440(2x+5)^4}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{2880(2x+5)^5}-\frac{(3174439702x+4583087983)\sqrt{2x}}{6879707136(2x+5)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x]
```

```
[Out] -((4583087983 + 3174439702*x)*Sqrt[3 - x + 2*x^2])/(6879707136*(5 + 2*x)^2)
 - (3667*(3 - x + 2*x^2)^(3/2))/(2880*(5 + 2*x)^5) + (711961*(3 - x + 2*x^2
)^(3/2))/(829440*(5 + 2*x)^4) - (38732321*(3 - x + 2*x^2)^(3/2))/(179159040
*(5 + 2*x)^3) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sqrt[2]) + (12895597463
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(82556485632*Sqrt[2
])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5} - \frac{1}{360}\int \frac{\sqrt{3-x+2x^2}\left(\frac{52701}{16}-\frac{9563x}{2}+\right.}{(5+2x)^5}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5} + \frac{711961\left(3-x+2x^2\right)^{3/2}}{829440(5+2x)^4} + \frac{\int \frac{\sqrt{3-x+2x^2}\left(\right.}{}}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5} + \frac{711961\left(3-x+2x^2\right)^{3/2}}{829440(5+2x)^4} - \frac{38732321\left(3\right.}{17915904}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667\left(3-x+2\right.}{2880(5+2x}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667\left(3-x+2\right.}{2880(5+2x}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667\left(3-x+2\right.}{2880(5+2x}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667\left(3-x+2\right.}{2880(5+2x}$$

Mathematica [A] time = 0.20, size = 98, normalized size = 0.59

$$\frac{64477987315\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) - \frac{24\sqrt{2x^2-x+3}\left(186470433136x^4+1285267446304x^3+3919478861832x^2+5608297138216x+311\right.}{(2x+5)^5}}{825564856320}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x
]
```

```
[Out] ((-24*Sqrt[3 - x + 2*x^2]*(3110673952831 + 5608297138216*x + 3919478861832*
x^2 + 1285267446304*x^3 + 186470433136*x^4))/(5 + 2*x)^5 - 64497254400*Sqrt
[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 64477987315*Sqrt[2]*ArcTanh[(17 - 22*x)/(
12*Sqrt[6 - 2*x + 4*x^2])])/825564856320
```

fricas [A] time = 0.89, size = 203, normalized size = 1.23

$$\frac{64497254400\sqrt{2}\left(32x^5+400x^4+2000x^3+5000x^2+6250x+3125\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="f
ricas")
```

```
[Out] 1/1651129712640*(64497254400*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^
2 + 6250*x + 3125)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 +
16*x - 25) + 64477987315*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 +
6250*x + 3125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 +
 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(186470433136*x^4 + 1285267446304
```

```
*x^3 + 3919478861832*x^2 + 5608297138216*x + 3110673952831)*sqrt(2*x^2 - x
+ 3))/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125)
```

giac [B] time = 0.28, size = 387, normalized size = 2.35

$$-\frac{5}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{12895597463}{165112971264}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="g
iac")
```

```
[Out] -5/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 12895
597463/165112971264*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) - 12895597463/165112971264*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt
(2) + 2*sqrt(2*x^2 - x + 3))) - 1/68797071360*sqrt(2)*(4368922304720*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 124570969998480*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^8 + 637804348664160*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^7 + 1828845222532320*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 37631893001870
16*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 - 10794416351958120*(sqrt(2)
*x - sqrt(2*x^2 - x + 3))^4 + 25049834283305880*sqrt(2)*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^3 - 34708488692384520*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 +
10654664764755165*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 2507056315485
767)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(
2*x^2 - x + 3)) - 11)^5
```

maple [A] time = 0.01, size = 188, normalized size = 1.14

$$\frac{5\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{64}+\frac{12895597463\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{165112971264}-\frac{12895597463\sqrt{-11x+2\left(x+\right.}}{495338913792}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x)
```

```
[Out] -12895597463/495338913792*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-3667/92160/(x+5/2)
^5*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-38732321/1433272320/(x+5/2)^3*(-11*x+2*(x
+5/2)^2-19/2)^(3/2)+711961/13271040/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^(3/2
)+46569601/6879707136/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+562688629/49
5338913792*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-562688629/247669456896/(x
+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+12895597463/165112971264*2^(1/2)*arcta
nh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))+5/64*2^(1/2)*a
rcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 1.04, size = 222, normalized size = 1.35

$$\frac{5}{64}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)-\frac{12895597463}{165112971264}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|}-\frac{17\sqrt{23}}{23|2x+5|}\right)-\frac{46569601}{3439853568}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="m
axima")
```

```
[Out] 5/64*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 12895597463/1651129
71264*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*
x + 5)) - 46569601/3439853568*sqrt(2*x^2 - x + 3) - 3667/2880*(2*x^2 - x +
3)^(3/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 711961/
829440*(2*x^2 - x + 3)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) -
38732321/179159040*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 4
6569601/1719926784*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 562688629/68
79707136*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x+5\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**6,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**6
, x)
```

3.332 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx$

Optimal. Leaf size=169

$$\frac{87677717\left(2x^2-x+3\right)^{3/2}}{8599633920(2x+5)^3}-\frac{5703277\left(2x^2-x+3\right)^{3/2}}{39813120(2x+5)^4}+\frac{92239\left(2x^2-x+3\right)^{3/2}}{138240(2x+5)^5}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{3456(2x+5)^6}-\frac{117272}{33}$$

```
[Out] -3667/3456*(2*x^2-x+3)^(3/2)/(5+2*x)^6+92239/138240*(2*x^2-x+3)^(3/2)/(5+2*
x)^5-5703277/39813120*(2*x^2-x+3)^(3/2)/(5+2*x)^4+87677717/8599633920*(2*x^
2-x+3)^(3/2)/(5+2*x)^3-26972675/7925422620672*arctanh(1/24*(17-22*x)*2^(1/2
)/(2*x^2-x+3)^(1/2))*2^(1/2)-1172725/330225942528*(17-22*x)*(2*x^2-x+3)^(1/
2)/(5+2*x)^2
```

Rubi [A] time = 0.22, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1650, 806, 720, 724, 206}

$$\frac{87677717\left(2x^2-x+3\right)^{3/2}}{8599633920(2x+5)^3}-\frac{5703277\left(2x^2-x+3\right)^{3/2}}{39813120(2x+5)^4}+\frac{92239\left(2x^2-x+3\right)^{3/2}}{138240(2x+5)^5}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{3456(2x+5)^6}-\frac{117272}{33}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x]
```

```
[Out] (-1172725*(17 - 22*x)*Sqrt[3 - x + 2*x^2])/(330225942528*(5 + 2*x)^2) - (36
67*(3 - x + 2*x^2)^(3/2))/(3456*(5 + 2*x)^6) + (92239*(3 - x + 2*x^2)^(3/2)
)/(138240*(5 + 2*x)^5) - (5703277*(3 - x + 2*x^2)^(3/2))/(39813120*(5 + 2*x
)^4) + (87677717*(3 - x + 2*x^2)^(3/2))/(8599633920*(5 + 2*x)^3) - (2697267
5*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(3962711310336*Sqr
t[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
```

```
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} - \frac{1}{432}\int \frac{\sqrt{3-x+2x^2}\left(\frac{61041}{16}-\frac{20751x}{4}+\right.}{(5+2x)^6}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{92239\left(3-x+2x^2\right)^{3/2}}{138240(5+2x)^5} + \frac{\int \frac{\sqrt{3-x+2x^2}\left(\right.}{}}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{92239\left(3-x+2x^2\right)^{3/2}}{138240(5+2x)^5} - \frac{5703277\left(3-\right.}{39813120}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{92239\left(3-x+2x^2\right)^{3/2}}{138240(5+2x)^5} - \frac{5703277\left(3-\right.}{39813120}$$

$$= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{92}{}$$

$$= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{92}{}$$

$$= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{92}{}$$

Mathematica [A] time = 0.17, size = 91, normalized size = 0.54

$$\frac{24\sqrt{2x^2-x+3}\left(271409942624x^5+12256250416x^4+397498825328x^3+158340720344x^2+27245373694x-\right.}{39627113103360(2x+5)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x
]
```

```
[Out] (24*Sqrt[3 - x + 2*x^2]*(-219337079305 + 27245373694*x + 158340720344*x^2 +
 397498825328*x^3 + 12256250416*x^4 + 271409942624*x^5) - 134863375*Sqrt[2]
*(5 + 2*x)^6*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/(396271131033
60*(5 + 2*x)^6)
```

fricas [A] time = 0.82, size = 156, normalized size = 0.92

$$\frac{134863375\sqrt{2}\left(64x^6+960x^5+6000x^4+20000x^3+37500x^2+37500x+15625\right)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x}{4x^2}\right.}{79254226206720\,(6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="f
ricas")
```

```
[Out] 1/79254226206720*(134863375*sqrt(2)*(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^
3 + 37500*x^2 + 37500*x + 15625)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x
 - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)) + 48*(271409942624*
x^5 + 12256250416*x^4 + 397498825328*x^3 + 158340720344*x^2 + 27245373694*x
 - 219337079305)*sqrt(2*x^2 - x + 3))/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*
x^3 + 37500*x^2 + 37500*x + 15625)
```

giac [B] time = 0.26, size = 405, normalized size = 2.40

$$-\frac{26972675}{7925422620672}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{26972675}{7925422620672}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="g
iac")
```

```
[Out] -26972675/7925422620672*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x
^2 - x + 3))) + 26972675/7925422620672*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sq
rt(2) + 2*sqrt(2*x^2 - x + 3))) + 1/3302259425280*sqrt(2)*(16506981498400*s
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^11 + 389429252643040*(sqrt(2)*x -
sqrt(2*x^2 - x + 3))^10 + 2263923918689840*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
- x + 3))^9 + 11663651054548560*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 90221
2326134736*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 - 84192729519861840*
(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 4317200555009448*sqrt(2)*(sqrt(2)*x -
 sqrt(2*x^2 - x + 3))^5 + 351543414066518760*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^4 - 376787166452923830*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 35
6306707647610982*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 82348353128195465*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 15499394004553969)/(2*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 1
1)^6
```

maple [A] time = 0.02, size = 195, normalized size = 1.15

$$-\frac{26972675\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{7925422620672}+\frac{26972675\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{23776267862016}-\frac{3667\left(-11x+2\left(x+\frac{5}{2}\right)\right.}{221184\left(x+\frac{5}{2}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x)
```

```
[Out] 26972675/23776267862016*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-3667/221184/(x+5/2)^
6*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+92239/4423680/(x+5/2)^5*(-11*x+2*(x+5/2)^2
-19/2)^(3/2)+87677717/68797071360/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-
5703277/637009920/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-1172725/33022594
```

```
2528/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+12899975/23776267862016*(4*x-
1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-12899975/11888133931008/(x+5/2)*(-11*x+2*
(x+5/2)^2-19/2)^(3/2)-26972675/7925422620672*2^(1/2)*arctanh(1/12*(-11*x+17
/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))
```

maxima [A] time = 1.05, size = 250, normalized size = 1.48

$$\frac{26972675}{7925422620672}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{1172725}{165112971264}\sqrt{2\,x^2-x+3}-\frac{}{3456\left(64\,x^6+960\,x^5+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="m
axima")
```

```
[Out] 26972675/7925422620672*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/2
3*sqrt(23)/abs(2*x + 5)) + 1172725/165112971264*sqrt(2*x^2 - x + 3) - 3667/
3456*(2*x^2 - x + 3)^(3/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500
*x^2 + 37500*x + 15625) + 92239/138240*(2*x^2 - x + 3)^(3/2)/(32*x^5 + 400*
x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) - 5703277/39813120*(2*x^2 - x +
3)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 87677717/8599633920*
(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 1172725/82556485632*
(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 12899975/330225942528*sqrt(2*x^
2 - x + 3)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2-x+3}\,\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x+5\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**7,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**7
, x)
```

3.333 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^8}\,dx$

Optimal. Leaf size=194

$$\frac{246159769\left(2x^2-x+3\right)^{3/2}}{866843099136(2x+5)^3}+\frac{19414831\left(2x^2-x+3\right)^{3/2}}{4013162496(2x+5)^4}-\frac{1464037\left(2x^2-x+3\right)^{3/2}}{13934592(2x+5)^5}+\frac{948341\left(2x^2-x+3\right)^{3/2}}{1741824(2x+5)^6}-$$

```
[Out] -3667/4032*(2*x^2-x+3)^(3/2)/(5+2*x)^7+948341/1741824*(2*x^2-x+3)^(3/2)/(5+
2*x)^6-1464037/13934592*(2*x^2-x+3)^(3/2)/(5+2*x)^5+19414831/4013162496*(2*
x^2-x+3)^(3/2)/(5+2*x)^4+246159769/866843099136*(2*x^2-x+3)^(3/2)/(5+2*x)^3
-289071245/570630428688384*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2)
)*2^(1/2)-12568315/23776267862016*(17-22*x)*(2*x^2-x+3)^(1/2)/(5+2*x)^2
```

Rubi [A] time = 0.27, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1650, 834, 806, 720, 724, 206}

$$\frac{246159769\left(2x^2-x+3\right)^{3/2}}{866843099136(2x+5)^3}+\frac{19414831\left(2x^2-x+3\right)^{3/2}}{4013162496(2x+5)^4}-\frac{1464037\left(2x^2-x+3\right)^{3/2}}{13934592(2x+5)^5}+\frac{948341\left(2x^2-x+3\right)^{3/2}}{1741824(2x+5)^6}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x]
```

```
[Out] (-12568315*(17 - 22*x)*Sqrt[3 - x + 2*x^2])/(23776267862016*(5 + 2*x)^2) -
(3667*(3 - x + 2*x^2)^(3/2))/(4032*(5 + 2*x)^7) + (948341*(3 - x + 2*x^2)^(
3/2))/(1741824*(5 + 2*x)^6) - (1464037*(3 - x + 2*x^2)^(3/2))/(13934592*(5
+ 2*x)^5) + (19414831*(3 - x + 2*x^2)^(3/2))/(4013162496*(5 + 2*x)^4) + (24
6159769*(3 - x + 2*x^2)^(3/2))/(866843099136*(5 + 2*x)^3) - (289071245*ArcT
anh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(285315214344192*Sqrt[2]
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^8}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} - \frac{1}{504}\int \frac{\sqrt{3-x+2x^2}\left(\frac{69381}{16} - 5594\right.}{(5+2x}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{948341\left(3-x+2x^2\right)^{3/2}}{1741824(5+2x)^6} + \int \frac{\sqrt{3-x+2}}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{948341\left(3-x+2x^2\right)^{3/2}}{1741824(5+2x)^6} - \frac{1464037}{13934}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{948341\left(3-x+2x^2\right)^{3/2}}{1741824(5+2x)^6} - \frac{1464037}{13934}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{948341\left(3-x+2x^2\right)^{3/2}}{1741824(5+2x)^6} - \frac{1464037}{13934}$$

$$= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} +$$

$$= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} +$$

$$= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} +$$

Mathematica [A] time = 0.20, size = 96, normalized size = 0.49

$$\frac{24\sqrt{2x^2-x+3}\left(1574342277056x^6 + 27976951397184x^5 + 4982916071952x^4 + 41058010262368x^3 + 14716\right.}{399441300081868}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x
]
```

```
[Out] (24*Sqrt[3 - x + 2*x^2]*(-20465234808721 + 590492177460*x + 14716683780036*
x^2 + 41058010262368*x^3 + 4982916071952*x^4 + 27976951397184*x^5 + 1574342
277056*x^6) - 2023498715*Sqrt[2]*(5 + 2*x)^7*ArcTanh[(17 - 22*x)/(12*Sqrt[6
 - 2*x + 4*x^2])])/(3994413000818688*(5 + 2*x)^7)
```

fricas [A] time = 0.71, size = 171, normalized size = 0.88

$$\frac{2023498715\sqrt{2}\left(128x^7 + 2240x^6 + 16800x^5 + 70000x^4 + 175000x^3 + 262500x^2 + 218750x + 78125\right)\log\left(\right.}{798}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="f
ricas")
```

```
[Out] 1/7988826001637376*(2023498715*sqrt(2)*(128*x^7 + 2240*x^6 + 16800*x^5 + 70
000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125)*log(-(24*sqrt(2)*sqrt
(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)
) + 48*(1574342277056*x^6 + 27976951397184*x^5 + 4982916071952*x^4 + 410580
```

```
10262368*x^3 + 14716683780036*x^2 + 590492177460*x - 20465234808721)*sqrt(2
*x^2 - x + 3))/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 2
62500*x^2 + 218750*x + 78125)
```

giac [B] time = 0.29, size = 456, normalized size = 2.35

$$-\frac{289071245}{570630428688384}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{289071245}{570630428688384}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="g
iac")
```

```
[Out] -289071245/570630428688384*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(
2*x^2 - x + 3))) + 289071245/570630428688384*sqrt(2)*log(abs(-2*sqrt(2)*x -
 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/332867750068224*sqrt(2)*(12950391
7760*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^13 - 3320259746027840*(sqrt(
2)*x - sqrt(2*x^2 - x + 3))^12 - 23966708071916736*sqrt(2)*(sqrt(2)*x - sqr
t(2*x^2 - x + 3))^11 - 186055342532355520*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
^10 - 274256644494948976*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 7961
35370176031760*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 2531523139171005408*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 - 4610393811900786336*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^6 - 7997126854300052364*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^5 + 30842713619423538868*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4
- 21873571601855032556*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 162047
0696060468100*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 3196254593191113265*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 536799032216117911)/(2*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) -
11)^7
```

maple [A] time = 0.02, size = 216, normalized size = 1.11

$$-\frac{289071245\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{570630428688384}+\frac{289071245\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{1711891286065152}-\frac{3667\left(-11x+2\left(x+\frac{5}{2}\right)^2\right.}{516096\left(x+\frac{5}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x)
```

```
[Out] 289071245/1711891286065152*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-3667/516096/(x+5/
2)^7*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+948341/111476736/(x+5/2)^6*(-11*x+2*(x+
5/2)^2-19/2)^(3/2)-1464037/445906944/(x+5/2)^5*(-11*x+2*(x+5/2)^2-19/2)^(3/
2)+246159769/6934744793088/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+1941483
1/64210599936/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-12568315/23776267862
016/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+138251465/1711891286065152*(4*
x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-138251465/855945643032576/(x+5/2)*(-11*
x+2*(x+5/2)^2-19/2)^(3/2)-289071245/570630428688384*2^(1/2)*arctanh(1/12*(-
11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))
```

maxima [A] time = 1.04, size = 301, normalized size = 1.55

$$\frac{289071245}{570630428688384}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|}-\frac{17\sqrt{23}}{23|2x+5|}\right)+\frac{12568315}{11888133931008}\sqrt{2x^2-x+3}-\frac{}{4032\left(128x^7+224\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="m
axima")
```

```
[Out] 289071245/570630428688384*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 1
7/23*sqrt(23)/abs(2*x + 5)) + 12568315/11888133931008*sqrt(2*x^2 - x + 3) -
 3667/4032*(2*x^2 - x + 3)^(3/2)/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^
4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125) + 948341/1741824*(2*x^2 - x
 + 3)^(3/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x
+ 15625) - 1464037/13934592*(2*x^2 - x + 3)^(3/2)/(32*x^5 + 400*x^4 + 2000*
x^3 + 5000*x^2 + 6250*x + 3125) + 19414831/4013162496*(2*x^2 - x + 3)^(3/2)
/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 246159769/866843099136*(2*x^
2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 12568315/5944066965504*(2
*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 138251465/23776267862016*sqrt(2*x
^2 - x + 3)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^{2} - x + 3} \left(5x^{4} - x^{3} + 3x^{2} + x + 2\right)}{\left(2x + 5\right)^{8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**8,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**8
, x)
```

3.334 $\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$

Optimal. Leaf size=166

$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4-\frac{1121\left(2x^2-x+3\right)^{5/2}(2x+5)^3}{2304}+\frac{69415\left(2x^2-x+3\right)^{5/2}(2x+5)^2}{32256}-\frac{3(215900x+66$$

```
[Out] -92727/131072*(1-4*x)*(2*x^2-x+3)^(3/2)+69415/32256*(5+2*x)^2*(2*x^2-x+3)^(
5/2)-1121/2304*(5+2*x)^3*(2*x^2-x+3)^(5/2)+5/144*(5+2*x)^4*(2*x^2-x+3)^(5/2
)-3/143360*(661397+215900*x)*(2*x^2-x+3)^(5/2)-147157749/8388608*arcsinh(1/
23*(1-4*x)*23^(1/2))*2^(1/2)-6398163/2097152*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1653, 779, 612, 619, 215}

$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4-\frac{1121\left(2x^2-x+3\right)^{5/2}(2x+5)^3}{2304}+\frac{69415\left(2x^2-x+3\right)^{5/2}(2x+5)^2}{32256}-\frac{3(215900x+66$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x)*(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-6398163*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2097152 - (92727*(1 - 4*x)*(3 - x
+ 2*x^2)^(3/2))/131072 + (69415*(5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))/32256 -
(1121*(5 + 2*x)^3*(3 - x + 2*x^2)^(5/2))/2304 + (5*(5 + 2*x)^4*(3 - x + 2*x
^2)^(5/2))/144 - (3*(661397 + 215900*x)*(3 - x + 2*x^2)^(5/2))/143360 - (14
7157749*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4194304*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)dx = \frac{5}{144}(5+2x)^4\left(3-x+2x^2\right)^{5/2}+\frac{1}{288}\int (5+2x)\left(3-\right.$$

$$= -\frac{1121(5+2x)^3\left(3-x+2x^2\right)^{5/2}}{2304}+\frac{5}{144}(5+2x)^4\left(3\right.$$

$$= \frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256}-\frac{1121(5+2x)^3\left(3\right.}{230}$$

$$= \frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256}-\frac{1121(5+2x)^3\left(3\right.}{230}$$

$$= -\frac{92727(1-4x)\left(3-x+2x^2\right)^{3/2}}{131072}+\frac{69415(5+2x)^2}{322}$$

$$= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{92727(1-4x)\left(3\right.}{1310}$$

$$= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{92727(1-4x)\left(3\right.}{1310}$$

$$= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{92727(1-4x)\left(3\right.}{1310}$$

Mathematica [A] time = 0.18, size = 80, normalized size = 0.48

$$\frac{4\sqrt{2x^2-x+3}\left(1468006400x^8+2926837760x^7+1033175040x^6+12117893120x^5+379086848x^4+126692\right.}{2642411520}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x)*(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(1592737263 + 12357760788*x + 4870637856*x^2 + 12669
290112*x^3 + 379086848*x^4 + 12117893120*x^5 + 1033175040*x^6 + 2926837760*
x^7 + 1468006400*x^8) - 46354690935*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/26
42411520
```

fricas [A] time = 0.81, size = 93, normalized size = 0.56

$$\frac{1}{660602880}\left(1468006400\,x^8+2926837760\,x^7+1033175040\,x^6+12117893120\,x^5+379086848\,x^4+126692\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 1/660602880*(1468006400*x^8 + 2926837760*x^7 + 1033175040*x^6 + 12117893120
*x^5 + 379086848*x^4 + 12669290112*x^3 + 4870637856*x^2 + 12357760788*x + 1
592737263)*sqrt(2*x^2 - x + 3) + 147157749/16777216*sqrt(2)*log(-4*sqrt(2)*
sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.19, size = 88, normalized size = 0.53

$$\frac{1}{660602880}\,(4\,(8\,(4\,(16\,(20\,(8\,(28\,(160\,x+319)x+3153)x+295847)x+185101)x+98978829)x+152207433)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 1/660602880*(4*(8*(4*(16*(20*(8*(28*(160*x + 319)*x + 3153)*x + 295847)*x +
 185101)*x + 98978829)*x + 152207433)*x + 3089440197)*x + 1592737263)*sqrt(
2*x^2 - x + 3) - 147157749/8388608*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.02, size = 134, normalized size = 0.81

$$\frac{5\left(2x^2-x+3\right)^{\frac{5}{2}}x^4}{9}+\frac{479\left(2x^2-x+3\right)^{\frac{5}{2}}x^3}{288}+\frac{2005\left(2x^2-x+3\right)^{\frac{5}{2}}x^2}{8064}+\frac{5645\left(2x^2-x+3\right)^{\frac{5}{2}}x}{21504}+\frac{147157749\sqrt{2}\ \mathrm{arc}}{8388}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x)
```

```
[Out] 120809/143360*(2*x^2-x+3)^(5/2)+5/9*x^4*(2*x^2-x+3)^(5/2)+479/288*x^3*(2*x^
2-x+3)^(5/2)+2005/8064*x^2*(2*x^2-x+3)^(5/2)+5645/21504*x*(2*x^2-x+3)^(5/2)
+92727/131072*(4*x-1)*(2*x^2-x+3)^(3/2)+147157749/8388608*2^(1/2)*arcsinh(4
/23*23^(1/2)*(x-1/4))+6398163/2097152*(4*x-1)*(2*x^2-x+3)^(1/2)
```

maxima [A] time = 0.98, size = 155, normalized size = 0.93

$$\frac{5}{9}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^4+\frac{479}{288}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^3+\frac{2005}{8064}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^2+\frac{5645}{21504}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x+\frac{120809}{143360}\left(2\,x^2-x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 5/9*(2*x^2 - x + 3)^(5/2)*x^4 + 479/288*(2*x^2 - x + 3)^(5/2)*x^3 + 2005/80
64*(2*x^2 - x + 3)^(5/2)*x^2 + 5645/21504*(2*x^2 - x + 3)^(5/2)*x + 120809/
143360*(2*x^2 - x + 3)^(5/2) + 92727/32768*(2*x^2 - x + 3)^(3/2)*x - 92727/
131072*(2*x^2 - x + 3)^(3/2) + 6398163/524288*sqrt(2*x^2 - x + 3)*x + 14715
7749/8388608*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 6398163/2097152*sqr
t(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2\,x+5)\,\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5)*(2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

```
[Out] int((2*x + 5)*(2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x + 5) \left(2x^2 - x + 3\right)^{\frac{3}{2}} \left(5x^4 - x^3 + 3x^2 + x + 2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2),x)
```

```
[Out] Integral((2*x + 5)*(2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)
, x)
```

3.335 $\int \left(3-x+2x^2\right)^{3/2} \left(2+x+3x^2-x^3+5x^4\right) dx$

Optimal. Leaf size=147

$$\frac{23}{448}\left(2x^2-x+3\right)^{5/2}x^2+\frac{125\left(2x^2-x+3\right)^{5/2}x}{3584}+\frac{1167\left(2x^2-x+3\right)^{5/2}}{14336}-\frac{8597(1-4x)\left(2x^2-x+3\right)^{3/2}}{65536}-\frac{593193(1$$

```
[Out] -8597/65536*(1-4*x)*(2*x^2-x+3)^(3/2)+1167/14336*(2*x^2-x+3)^(5/2)+125/3584
*x*(2*x^2-x+3)^(5/2)+23/448*x^2*(2*x^2-x+3)^(5/2)+5/16*x^3*(2*x^2-x+3)^(5/2
)-13643439/4194304*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-593193/1048576*(1
-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3+\frac{23}{448}\left(2x^2-x+3\right)^{5/2}x^2+\frac{125\left(2x^2-x+3\right)^{5/2}x}{3584}+\frac{1167\left(2x^2-x+3\right)^{5/2}}{14336}-\frac{8597(1-4x)\left(2x^2\right.}{65536}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-593193*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1048576 - (8597*(1 - 4*x)*(3 - x +
2*x^2)^(3/2))/65536 + (1167*(3 - x + 2*x^2)^(5/2))/14336 + (125*x*(3 - x +
2*x^2)^(5/2))/3584 + (23*x^2*(3 - x + 2*x^2)^(5/2))/448 + (5*x^3*(3 - x + 2
*x^2)^(5/2))/16 - (13643439*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2097152*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
```

```
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+x+3x^2-x^3+5x^4\right) dx = \frac{5}{16}x^3 \left(3-x+2x^2\right)^{5/2} + \frac{1}{16}\int \left(3-x+2x^2\right)^{3/2} \left(32+16x+\right.$$

$$= \frac{23}{448}x^2 \left(3-x+2x^2\right)^{5/2} + \frac{5}{16}x^3 \left(3-x+2x^2\right)^{5/2} + \frac{1}{224}\int (3$$

$$= \frac{125x \left(3-x+2x^2\right)^{5/2}}{3584} + \frac{23}{448}x^2 \left(3-x+2x^2\right)^{5/2} + \frac{5}{16}x^3 (3-$$

$$= \frac{1167 \left(3-x+2x^2\right)^{5/2}}{14336} + \frac{125x \left(3-x+2x^2\right)^{5/2}}{3584} + \frac{23}{448}x^2 (3-$$

$$= -\frac{8597(1-4x) \left(3-x+2x^2\right)^{3/2}}{65536} + \frac{1167 \left(3-x+2x^2\right)^{5/2}}{14336} + \frac{1}{}$$

$$= -\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576} - \frac{8597(1-4x) \left(3-x+2x^2\right.}{65536}$$

$$= -\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576} - \frac{8597(1-4x) \left(3-x+2x^2\right.}{65536}$$

$$= -\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576} - \frac{8597(1-4x) \left(3-x+2x^2\right.}{65536}$$

Mathematica [A] time = 0.12, size = 75, normalized size = 0.51

$$\frac{4\sqrt{2x^2-x+3} \left(9175040x^7 - 7667712x^6 + 29335552x^5 - 7497728x^4 + 27023744x^3 + 3845856x^2 + 2784561\right.}{29360128}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-1663407 + 27845612*x + 3845856*x^2 + 27023744*x^3
- 7497728*x^4 + 29335552*x^5 - 7667712*x^6 + 9175040*x^7) - 95504073*Sqrt[2
]*ArcSinh[(1 - 4*x)/Sqrt[23]])/29360128
```

fricas [A] time = 0.82, size = 88, normalized size = 0.60

$$\frac{1}{7340032} \left(9175040\, x^7 - 7667712\, x^6 + 29335552\, x^5 - 7497728\, x^4 + 27023744\, x^3 + 3845856\, x^2 + 27845612\, x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 1/7340032*(9175040*x^7 - 7667712*x^6 + 29335552*x^5 - 7497728*x^4 + 2702374
4*x^3 + 3845856*x^2 + 27845612*x - 1663407)*sqrt(2*x^2 - x + 3) + 13643439/
8388608*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*
x - 25)
```

giac [A] time = 0.33, size = 83, normalized size = 0.56

$$\frac{1}{7340032}(4(8(4(16(4(8(140x-117)x+3581)x-3661)x+211123)x+120183)x+6961403)x-1663407)\sqrt{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/7340032*(4*(8*(4*(16*(4*(8*(140*x - 117)*x + 3581)*x - 3661)*x + 211123)*
x + 120183)*x + 6961403)*x - 1663407)*sqrt(2*x^2 - x + 3) - 13643439/419430
4*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.00, size = 117, normalized size = 0.80

$$\frac{5\left(2x^2-x+3\right)^{\frac{5}{2}}x^3}{16}+\frac{23\left(2x^2-x+3\right)^{\frac{5}{2}}x^2}{448}+\frac{125\left(2x^2-x+3\right)^{\frac{5}{2}}x}{3584}+\frac{13643439\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4194304}+\frac{1167\left(2x^2\right.}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x)
```

```
[Out] 1167/14336*(2*x^2-x+3)^(5/2)+5/16*(2*x^2-x+3)^(5/2)*x^3+23/448*(2*x^2-x+3)^
(5/2)*x^2+125/3584*(2*x^2-x+3)^(5/2)*x+8597/65536*(4*x-1)*(2*x^2-x+3)^(3/2)
+13643439/4194304*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+593193/1048576*(4*
x-1)*(2*x^2-x+3)^(1/2)
```

maxima [A] time = 1.00, size = 138, normalized size = 0.94

$$\frac{5}{16}\left(2x^2-x+3\right)^{\frac{5}{2}}x^3+\frac{23}{448}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2+\frac{125}{3584}\left(2x^2-x+3\right)^{\frac{5}{2}}x+\frac{1167}{14336}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{8597}{16384}\left(2x^2-x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 5/16*(2*x^2 - x + 3)^(5/2)*x^3 + 23/448*(2*x^2 - x + 3)^(5/2)*x^2 + 125/358
4*(2*x^2 - x + 3)^(5/2)*x + 1167/14336*(2*x^2 - x + 3)^(5/2) + 8597/16384*(
2*x^2 - x + 3)^(3/2)*x - 8597/65536*(2*x^2 - x + 3)^(3/2) + 593193/262144*s
qrt(2*x^2 - x + 3)*x + 13643439/4194304*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x
- 1)) - 593193/1048576*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(2x^2-x+3\right)^{3/2}\left(5x^4-x^3+3x^2+x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^4-x^3+3x^2+x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2), x)
```

3.336 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$

Optimal. Leaf size=172

$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}-\frac{311}{448}(2x+5)\left(2x^2-x+3\right)^{5/2}+\frac{3505}{896}\left(2x^2-x+3\right)^{5/2}+\frac{(500141-123060x)\left(2x^2-\right.}{12288}$$

```
[Out] 1/12288*(500141-123060*x)*(2*x^2-x+3)^(3/2)+3505/896*(2*x^2-x+3)^(5/2)-311/
448*(5+2*x)*(2*x^2-x+3)^(5/2)+5/112*(5+2*x)^2*(2*x^2-x+3)^(5/2)+1622009981/
262144*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-99009/16*arctanh(1/24*(17-22*
x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/65536*(141051019-23482924*x)*(2*x^2
-x+3)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}-\frac{311}{448}(2x+5)\left(2x^2-x+3\right)^{5/2}+\frac{3505}{896}\left(2x^2-x+3\right)^{5/2}+\frac{(500141-123060x)\left(2x^2-\right.}{12288}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

```
[Out] ((141051019 - 23482924*x)*Sqrt[3 - x + 2*x^2])/65536 + ((500141 - 123060*x)
*(3 - x + 2*x^2)^(3/2))/12288 + (3505*(3 - x + 2*x^2)^(5/2))/896 - (311*(5
+ 2*x)*(3 - x + 2*x^2)^(5/2))/448 + (5*(5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))/1
12 + (1622009981*ArcSinh[(1 - 4*x)/Sqrt[23]])/(131072*Sqrt[2]) - (99009*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(8*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
```

```
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx = \frac{5}{112}(5+2x)^2\left(3-x+2x^2\right)^{5/2} + \frac{1}{224}\int \frac{\left(3-x+2x^2\right)^{3/2}(57$$

$$= -\frac{311}{448}(5+2x)\left(3-x+2x^2\right)^{5/2} + \frac{5}{112}(5+2x)^2\left(3-x+2x^2\right)$$

$$= \frac{3505}{896}\left(3-x+2x^2\right)^{5/2} - \frac{311}{448}(5+2x)\left(3-x+2x^2\right)^{5/2} + \frac{5}{112}$$

$$= \frac{(500141-123060x)\left(3-x+2x^2\right)^{3/2}}{12288} + \frac{3505}{896}\left(3-x+2x^2\right)^{5}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536} + \frac{(500141-12306}{1}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536} + \frac{(500141-12306}{1}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536} + \frac{(500141-12306}{1}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536} + \frac{(500141-12306}{1}$$

Mathematica [A] time = 0.18, size = 101, normalized size = 0.59

$$\frac{-34065432576\sqrt{2}\,\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + 4\sqrt{2x^2-x+3}\left(983040x^6-3710976x^5+14493696x^4-4647667\right.}{5505024}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x
]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(3149403255 - 609499532*x + 159973408*x^2 - 46476672
*x^3 + 14493696*x^4 - 3710976*x^5 + 983040*x^6) + 34062209601*Sqrt[2]*ArcSi
nh[(1 - 4*x)/Sqrt[23]] - 34065432576*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6
 - 2*x + 4*x^2])])/5505024
```

fricas [A] time = 0.62, size = 135, normalized size = 0.78

$$\frac{1}{1376256}\left(983040\,x^6-3710976\,x^5+14493696\,x^4-46476672\,x^3+159973408\,x^2-609499532\,x+31494032\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="fri
cas")
```

```
[Out] 1/1376256*(983040*x^6 - 3710976*x^5 + 14493696*x^4 - 46476672*x^3 + 1599734
08*x^2 - 609499532*x + 3149403255)*sqrt(2*x^2 - x + 3) + 1622009981/524288*
sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) +
 99009/32*sqrt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x
^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))
```

giac [A] time = 0.23, size = 139, normalized size = 0.81

$$\frac{1}{1376256}\left(4\left(8\left(12\left(16\left(4\left(40x-151\right)x+2359\right)x-121033\right)x+4999169\right)x-152374883\right)x+3149403255\right)\sqrt{2x^2-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="gia
c")
```

```
[Out] 1/1376256*(4*(8*(12*(16*(4*(40*x - 151)*x + 2359)*x - 121033)*x + 4999169)*
x - 152374883)*x + 3149403255)*sqrt(2*x^2 - x + 3) + 1622009981/262144*sqrt
(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 99009/16*sqrt(2)*
log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 99009/16*sqrt(2)
*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
```

maple [A] time = 0.01, size = 183, normalized size = 1.06

$$\frac{5\left(2x^2-x+3\right)^{\frac{5}{2}}x^2}{28}-\frac{111\left(2x^2-x+3\right)^{\frac{5}{2}}x}{224}-\frac{1622009981\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{262144}-\frac{99009\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\right.}{12\sqrt{-11x+2}}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x)
```

```
[Out] 5/28*(2*x^2-x+3)^(5/2)*x^2-111/224*(2*x^2-x+3)^(5/2)*x+1395/896*(2*x^2-x+3)
^(5/2)-10255/4096*(4*x-1)*(2*x^2-x+3)^(3/2)-707595/65536*(4*x-1)*(2*x^2-x+3
)^(1/2)-1622009981/262144*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+3667/96*(-
11*x+2*(x+5/2)^2-19/2)^(3/2)-40337/512*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/
2)+33003/16*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-99009/16*2^(1/2)*arctanh(1/12*(-
11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))
```

maxima [A] time = 1.00, size = 157, normalized size = 0.91

$$\frac{5}{28}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2-\frac{111}{224}\left(2x^2-x+3\right)^{\frac{5}{2}}x+\frac{1395}{896}\left(2x^2-x+3\right)^{\frac{5}{2}}-\frac{10255}{1024}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{500141}{12288}\left(2x^2-x+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="max
ima")
```

```
[Out] 5/28*(2*x^2 - x + 3)^(5/2)*x^2 - 111/224*(2*x^2 - x + 3)^(5/2)*x + 1395/896
*(2*x^2 - x + 3)^(5/2) - 10255/1024*(2*x^2 - x + 3)^(3/2)*x + 500141/12288*
(2*x^2 - x + 3)^(3/2) - 5870731/16384*sqrt(2*x^2 - x + 3)*x - 1622009981/26
2144*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 99009/16*sqrt(2)*ar
csinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 141051
019/65536*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2x^2-x+3\right)^{3/2}\left(5x^4-x^3+3x^2+x+2\right)}{2x+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5),x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2 - x + 3\right)^{\frac{3}{2}}\left(5x^4 - x^3 + 3x^2 + x + 2\right)}{2x + 5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
, x)
```

3.337 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx$

Optimal. Leaf size=172

$$\frac{5}{96}(2x+5)\left(2x^2-x+3\right)^{5/2}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}-\frac{839}{960}\left(2x^2-x+3\right)^{5/2}-\frac{(909513-226052x)\left(2x^2-x+3\right)^{3/2}}{18432}-\cdots$$

```
[Out] -1/18432*(909513-226052*x)*(2*x^2-x+3)^(3/2)-839/960*(2*x^2-x+3)^(5/2)-3667
/576*(2*x^2-x+3)^(5/2)/(5+2*x)+5/96*(5+2*x)*(2*x^2-x+3)^(5/2)-982669459/131
072*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+959625/128*arctanh(1/24*(17-22*x
)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-1/32768*(85448933-14243732*x)*(2*x^2-x
+3)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1650, 1653, 814, 843, 619, 215, 724, 206}

$$\frac{5}{96}(2x+5)\left(2x^2-x+3\right)^{5/2}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}-\frac{839}{960}\left(2x^2-x+3\right)^{5/2}-\frac{(909513-226052x)\left(2x^2-x+3\right)^{3/2}}{18432}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x]
```

```
[Out] -((85448933 - 14243732*x)*Sqrt[3 - x + 2*x^2])/32768 - ((909513 - 226052*x)
*(3 - x + 2*x^2)^(3/2))/18432 - (839*(3 - x + 2*x^2)^(5/2))/960 - (3667*(3
- x + 2*x^2)^(5/2))/(576*(5 + 2*x)) + (5*(5 + 2*x)*(3 - x + 2*x^2)^(5/2))/9
6 - (982669459*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2]) + (959625*ArcTa
nh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(64*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
```

```
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{576(5+2x)} - \frac{1}{72}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{26675}{16}-4990\right.}{5+2x}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{576(5+2x)} + \frac{5}{96}(5+2x)\left(3-x+2x^2\right)^{5/2} - \frac{\int \frac{(3-x}{}}{}$$

$$= -\frac{839}{960}\left(3-x+2x^2\right)^{5/2} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{576(5+2x)} + \frac{5}{96}(5+2x)\left(3\right.$$

$$= -\frac{(909513-226052x)\left(3-x+2x^2\right)^{3/2}}{18432} - \frac{839}{960}\left(3-x+2x^2\right)^{5/2}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052}{184}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052}{184}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052}{184}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052}{184}$$

Mathematica [A] time = 0.21, size = 108, normalized size = 0.63

$$\frac{14739840000\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{4\sqrt{2x^2-x+3}\left(409600x^6-1798144x^5+8283904x^4-35369408x^3+182033816x^2-1404323114x-6\right.}{2x+5}}{1966080}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2
,x]
```

```
[Out] ((4*Sqrt[3 - x + 2*x^2]*(-6814208295 - 1404323114*x + 182033816*x^2 - 35369
408*x^3 + 8283904*x^4 - 1798144*x^5 + 409600*x^6))/(5 + 2*x) - 14740041885*
Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 14739840000*Sqrt[2]*ArcTanh[(17 - 22*
x)/(12*Sqrt[6 - 2*x + 4*x^2])])/1966080
```

fricas [A] time = 0.88, size = 153, normalized size = 0.89

$$14740041885\sqrt{2}(2x+5)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+14739840000\sqrt{2}(2x+5)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="f
ricas")
```

```
[Out] 1/3932160*(14740041885*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)
*(4*x - 1) - 32*x^2 + 16*x - 25) + 14739840000*sqrt(2)*(2*x + 5)*log((24*sq
rt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 +
```

```
20*x + 25)) + 8*(409600*x^6 - 1798144*x^5 + 8283904*x^4 - 35369408*x^3 + 18
2033816*x^2 - 1404323114*x - 6814208295)*sqrt(2*x^2 - x + 3))/(2*x + 5)
```

giac [B] time = 0.45, size = 707, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="g
iac")
```

```
[Out] 1/1966080*sqrt(2)*(14739840000*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 14740041885*log(abs(sqrt(-11/(2
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 147400
41885*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*
sgn(1/(2*x + 5)) - 2027704320*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)*sgn(
1/(2*x + 5)) + 2*(45496763235*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6
/(2*x + 5))^11*sgn(1/(2*x + 5)) - 126553743360*(sqrt(-11/(2*x + 5) + 36/(2*
x + 5)^2 + 1) + 6/(2*x + 5))^10*sgn(1/(2*x + 5)) + 44062768335*(sqrt(-11/(2
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^9*sgn(1/(2*x + 5)) + 331789824
00*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^8*sgn(1/(2*x +
5)) + 294206421582*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))
^7*sgn(1/(2*x + 5)) - 463672074240*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1
) + 6/(2*x + 5))^6*sgn(1/(2*x + 5)) + 35099942478*(sqrt(-11/(2*x + 5) + 36/
(2*x + 5)^2 + 1) + 6/(2*x + 5))^5*sgn(1/(2*x + 5)) + 171324610560*(sqrt(-11
/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*sgn(1/(2*x + 5)) + 600592
81615*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3*sgn(1/(2*x
 + 5)) - 105051009024*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
5))^2*sgn(1/(2*x + 5)) - 5210329245*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
1) + 6/(2*x + 5))*sgn(1/(2*x + 5)) + 17058392064*sgn(1/(2*x + 5)))/((sqrt(-
11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^6)
```

maple [A] time = 0.01, size = 208, normalized size = 1.21

$$\frac{5\left(2x^2-x+3\right)^{\frac{5}{2}}x}{48}+\frac{982669459\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{131072}+\frac{959625\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{128}-\frac{589\left(2\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x)
```

```
[Out] 5/48*(2*x^2-x+3)^(5/2)*x-589/960*(2*x^2-x+3)^(5/2)+9059/6144*(4*x-1)*(2*x^2
-x+3)^(3/2)+208357/32768*(4*x-1)*(2*x^2-x+3)^(1/2)+982669459/131072*2^(1/2)
*arcsinh(4/23*23^(1/2)*(x-1/4))-3667/1152/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^
(5/2)-106625/2304*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+1637/16*(4*x-1)*(-11*x+2*(
x+5/2)^2-19/2)^(1/2)-319875/128*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+959625/128*2
^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))+36
67/2304*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(3/2)
```

maxima [A] time = 1.01, size = 161, normalized size = 0.94

$$\frac{5}{48}\left(2x^2-x+3\right)^{\frac{5}{2}}x-\frac{589}{960}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{9059}{1536}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{185827}{6144}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{3560933}{8192}\sqrt{2x^2-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="m
axima")
```

```
[Out] 5/48*(2*x^2 - x + 3)^(5/2)*x - 589/960*(2*x^2 - x + 3)^(5/2) + 9059/1536*(2
*x^2 - x + 3)^(3/2)*x - 185827/6144*(2*x^2 - x + 3)^(3/2) + 3560933/8192*sq
rt(2*x^2 - x + 3)*x + 982669459/131072*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/
23*sqrt(23)) - 959625/128*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 1
7/23*sqrt(23)/abs(2*x + 5)) - 85448933/32768*sqrt(2*x^2 - x + 3) - 3667/32*
(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{\left(2\,x+5\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2,x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{2} - x + 3\right)^{\frac{3}{2}} \left(5x^{4} - x^{3} + 3x^{2} + x + 2\right)}{\left(2x + 5\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**2, x)
```

3.338 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$

Optimal. Leaf size=174

$$\frac{438065(2x^2-x+3)^{5/2}}{82944(2x+5)}-\frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2}+\frac{1}{16}(2x^2-x+3)^{5/2}+\frac{(2154633-534617x)(2x^2-x+3)^{3/2}}{82944}+$$

```
[Out] 1/82944*(2154633-534617*x)*(2*x^2-x+3)^(3/2)+1/16*(2*x^2-x+3)^(5/2)-3667/11
52*(2*x^2-x+3)^(5/2)/(5+2*x)^2+438065/82944*(2*x^2-x+3)^(5/2)/(5+2*x)+12934
2063/32768*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-8083915/2048*arctanh(1/24
*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/24576*(33741483-5623292*x)*
(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1650, 1653, 814, 843, 619, 215, 724, 206}

$$\frac{438065(2x^2-x+3)^{5/2}}{82944(2x+5)}-\frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2}+\frac{1}{16}(2x^2-x+3)^{5/2}+\frac{(2154633-534617x)(2x^2-x+3)^{3/2}}{82944}+$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x]
```

```
[Out] ((33741483 - 5623292*x)*Sqrt[3 - x + 2*x^2])/24576 + ((2154633 - 534617*x)*
(3 - x + 2*x^2)^(3/2))/82944 + (3 - x + 2*x^2)^(5/2)/16 - (3667*(3 - x + 2*
x^2)^(5/2))/(1152*(5 + 2*x)^2) + (438065*(3 - x + 2*x^2)^(5/2))/(82944*(5 +
 2*x)) + (129342063*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16384*Sqrt[2]) - (8083915
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1024*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
```

```
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}dx = -\frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} - \frac{1}{144}\int \frac{(3-x+2x^2)^{3/2}\left(\frac{35015}{16} - 2\right.}{(5+2}$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} + \frac{438065(3-x+2x^2)^{5/2}}{82944(5+2x)} + \int \frac{(3-x}{}$$

$$= \frac{1}{16}(3-x+2x^2)^{5/2} - \frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} + \frac{438065(3-}{82944(5}$$

$$= \frac{(2154633-534617x)(3-x+2x^2)^{3/2}}{82944} + \frac{1}{16}(3-x+2x^2)^{5/2}$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633-53461}{82}$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633-53461}{82}$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633-53461}{82}$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633-53461}{82}$$

Mathematica [A] time = 0.22, size = 108, normalized size = 0.62

$$\frac{-129342640\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{4\sqrt{2x^2-x+3}\left(8192x^6-43520x^5+253312x^4-1620944x^3+16667188x^2+181223072x+2989667\right.}{(2x+5)^2}}{32768}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3
,x]
```

```
[Out] ((4*Sqrt[3 - x + 2*x^2]*(298966737 + 181223072*x + 16667188*x^2 - 1620944*x
^3 + 253312*x^4 - 43520*x^5 + 8192*x^6))/(5 + 2*x)^2 + 129342063*Sqrt[2]*Ar
cSinh[(1 - 4*x)/Sqrt[23]] - 129342640*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[
6 - 2*x + 4*x^2])])/32768
```

fricas [A] time = 0.92, size = 169, normalized size = 0.97

$$\frac{129342063\sqrt{2}(4x^2+20x+25)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+129342640\sqrt{2}(4x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="f
ricas")
```

```
[Out] 1/65536*(129342063*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 129342640*sqrt(2)*(4*x^2 + 20*x + 2
5)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1
```

```
153)/(4*x^2 + 20*x + 25)) + 8*(8192*x^6 - 43520*x^5 + 253312*x^4 - 1620944*
x^3 + 16667188*x^2 + 181223072*x + 298966737)*sqrt(2*x^2 - x + 3))/(4*x^2 +
 20*x + 25)
```

giac [A] time = 0.30, size = 268, normalized size = 1.54

$$\frac{1}{8192}\left(4\left(8\left(4\left(16x-165\right)x+4879\right)x-263469\right)x+8460377\right)\sqrt{2x^2-x+3}+\frac{129342063}{32768}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="g
iac")
```

```
[Out] 1/8192*(4*(8*(4*(16*x - 165)*x + 4879)*x - 263469)*x + 8460377)*sqrt(2*x^2
- x + 3) + 129342063/32768*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 -
 x + 3)) + 1) - 8083915/2048*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqr
t(2*x^2 - x + 3))) + 8083915/2048*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2)
 + 2*sqrt(2*x^2 - x + 3))) + 1/512*sqrt(2)*(14243182*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3))^3 + 109906674*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 170
996871*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 110506087)/(2*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) -
11)^2
```

maple [A] time = 0.02, size = 214, normalized size = 1.23

$$-\frac{129342063\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{32768}-\frac{8083915\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{2048}+\frac{\left(2x^2-x+3\right)^{\frac{5}{2}}}{16}+\frac{8083915\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x)
```

```
[Out] 1/16*(2*x^2-x+3)^(5/2)+8083915/6144*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+8083915/
331776*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-3667/4608/(x+5/2)^2*(-11*x+2*(x+5/2)^
2-19/2)^(5/2)-438065/331776*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+438065/1
65888/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(5/2)-343745/6144*(4*x-1)*(-11*x+2*(
x+5/2)^2-19/2)^(1/2)-8083915/2048*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)
/(-11*x+2*(x+5/2)^2-19/2)^(1/2))-149/512*(4*x-1)*(2*x^2-x+3)^(3/2)-12934206
3/32768*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-10281/8192*(4*x-1)*(2*x^2-x+
3)^(1/2)
```

maxima [A] time = 1.00, size = 172, normalized size = 0.99

$$\frac{1}{16}\left(2x^2-x+3\right)^{\frac{5}{2}}-\frac{149}{128}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{46691}{4608}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^2-x+3\right)^{\frac{5}{2}}}{1152\left(4x^2+20x+25\right)}-\frac{1405823}{6144}\sqrt{2x^2-x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="m
axima")
```

```
[Out] 1/16*(2*x^2 - x + 3)^(5/2) - 149/128*(2*x^2 - x + 3)^(3/2)*x + 46691/4608*(
2*x^2 - x + 3)^(3/2) - 3667/1152*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25)
- 1405823/6144*sqrt(2*x^2 - x + 3)*x - 129342063/32768*sqrt(2)*arcsinh(4/23
```

```
*sqrt(23)*x - 1/23*sqrt(23)) + 8083915/2048*sqrt(2)*arcsinh(22/23*sqrt(23)*
x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 11247161/8192*sqrt(2*x^2 -
x + 3) + 438065/4608*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3,x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{2} - x + 3\right)^{\frac{3}{2}} \left(5x^{4} - x^{3} + 3x^{2} + x + 2\right)}{\left(2x + 5\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**3, x)
```

3.339 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$

Optimal. Leaf size=181

$$-\frac{32865365\left(2x^2-x+3\right)^{5/2}}{17915904(2x+5)}+\frac{556255\left(2x^2-x+3\right)^{5/2}}{248832(2x+5)^2}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}-\frac{(138006843-34265045x)\left(2x^2-\right.}{17915904}$$

```
[Out] -1/17915904*(138006843-34265045*x)*(2*x^2-x+3)^(3/2)-3667/1728*(2*x^2-x+3)^
(5/2)/(5+2*x)^3+556255/248832*(2*x^2-x+3)^(5/2)/(5+2*x)^2-32865365/17915904
*(2*x^2-x+3)^(5/2)/(5+2*x)-19176431/16384*arcsinh(1/23*(1-4*x)*23^(1/2))*2^
(1/2)+517762327/442368*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^
(1/2)-1/331776*(135068604-22512089*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{32865365\left(2x^2-x+3\right)^{5/2}}{17915904(2x+5)}+\frac{556255\left(2x^2-x+3\right)^{5/2}}{248832(2x+5)^2}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}-\frac{(138006843-34265045x)\left(2x^2-\right.}{17915904}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x]
```

```
[Out] -((135068604 - 22512089*x)*Sqrt[3 - x + 2*x^2])/331776 - ((138006843 - 3426
5045*x)*(3 - x + 2*x^2)^(3/2))/17915904 - (3667*(3 - x + 2*x^2)^(5/2))/(172
8*(5 + 2*x)^3) + (556255*(3 - x + 2*x^2)^(5/2))/(248832*(5 + 2*x)^2) - (328
65365*(3 - x + 2*x^2)^(5/2))/(17915904*(5 + 2*x)) - (19176431*ArcSinh[(1 -
4*x)/Sqrt[23]])/(8192*Sqrt[2]) + (517762327*ArcTanh[(17 - 22*x)/(12*Sqrt[2]
*Sqrt[3 - x + 2*x^2])])/(221184*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3} - \frac{1}{216}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{43355}{16}-\frac{1160}{2}\right.}{(5+2x)}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3} + \frac{556255\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^2} + \int \frac{(3-x+2x}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3} + \frac{556255\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^2} - \frac{32865365}{17915}$$

$$= -\frac{(138006843-34265045x)\left(3-x+2x^2\right)^{3/2}}{17915904} - \frac{3667\left(3-x+2\right.}{1728(5+2}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(138006843-342}{1}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(138006843-342}{1}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(138006843-342}{1}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(138006843-342}{1}$$

Mathematica [**A**] time = 0.22, size = 108, normalized size = 0.60

$$\frac{517762327\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{12\sqrt{2x^2-x+3}\left(46080x^6-315648x^5+2626848x^4-33595416x^3-594798908x^2-2006873194x-19946\right.}{(2x+5)^3}}{442368}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4
,x]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-1994650739 - 2006873194*x - 594798908*x^2 - 3359
5416*x^3 + 2626848*x^4 - 315648*x^5 + 46080*x^6))/(5 + 2*x)^3 - 517763637*S
qrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 517762327*Sqrt[2]*ArcTanh[(17 - 22*x)/
(12*Sqrt[6 - 2*x + 4*x^2])])/442368
```

fricas [**A**] time = 0.97, size = 183, normalized size = 1.01

$$\frac{517763637\sqrt{2}\left(8x^3+60x^2+150x+125\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+517762327}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="f
ricas")
```

```
[Out] 1/884736*(517763637*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 517762327*sqrt(2)*(8*x
^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17)
```

```
- 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 24*(46080*x^6 - 315648*x
^5 + 2626848*x^4 - 33595416*x^3 - 594798908*x^2 - 2006873194*x - 1994650739
)*sqrt(2*x^2 - x + 3))/(8*x^3 + 60*x^2 + 150*x + 125)
```

giac [B] time = 0.27, size = 314, normalized size = 1.73

$$\frac{1}{4096}\left(4\left(8\left(20x-287\right)x+23341\right)x-1004633\right)\sqrt{2x^2-x+3}-\frac{19176431}{16384}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="g
iac")
```

```
[Out] 1/4096*(4*(8*(20*x - 287)*x + 23341)*x - 1004633)*sqrt(2*x^2 - x + 3) - 191
76431/16384*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) +
 517762327/442368*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x
 + 3))) - 517762327/442368*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sq
rt(2*x^2 - x + 3))) - 1/36864*sqrt(2)*(1092794276*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3))^5 + 18284336132*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 2031
4214356*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 151449344092*(sqrt(2)
*x - sqrt(2*x^2 - x + 3))^2 + 102529692109*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
- x + 3)) - 41882448755)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

maple [A] time = 0.02, size = 221, normalized size = 1.22

$$\frac{19176431\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{16384}+\frac{517762327\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{442368}-\frac{517762327\sqrt{-11x+2\left(x\right.}}{1327104}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x)
```

```
[Out] -517762327/1327104*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-517762327/71663616*(-11*x
+2*(x+5/2)^2-19/2)^(3/2)-3667/13824/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(5/2
)+556255/995328/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(5/2)+32865365/71663616*
(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-32865365/35831808/(x+5/2)*(-11*x+2*(
x+5/2)^2-19/2)^(5/2)+22400309/1327104*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2
)+517762327/442368*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/
2)^2-19/2)^(1/2))+5/256*(4*x-1)*(2*x^2-x+3)^(3/2)+19176431/16384*2^(1/2)*ar
csinh(4/23*23^(1/2)*(x-1/4))+345/4096*(4*x-1)*(2*x^2-x+3)^(1/2)
```

maxima [A] time = 1.04, size = 189, normalized size = 1.04

$$\frac{5}{64}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{1094743}{497664}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^2-x+3\right)^{\frac{5}{2}}}{1728\left(8x^3+60x^2+150x+125\right)}+\frac{556255\left(2x^2-x+3\right)^{\frac{5}{2}}}{248832\left(4x^2+20x+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="m
axima")
```

```
[Out] 5/64*(2*x^2 - x + 3)^(3/2)*x - 1094743/497664*(2*x^2 - x + 3)^(3/2) - 3667/
1728*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 556255/248832*(
```

```
2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25) + 22512089/331776*sqrt(2*x^2 - x +
 3)*x + 19176431/16384*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 5
17762327/442368*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(
23)/abs(2*x + 5)) - 11255717/27648*sqrt(2*x^2 - x + 3) - 32865365/995328*(2
*x^2 - x + 3)^(3/2)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4,x)
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{2} - x + 3\right)^{\frac{3}{2}} \left(5x^{4} - x^{3} + 3x^{2} + x + 2\right)}{\left(2x + 5\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4,x)
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**4, x)
```

3.340 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx$

Optimal. Leaf size=188

$$-\frac{14477995\left(2x^2-x+3\right)^{5/2}}{23887872(2x+5)^2}+\frac{224815\left(2x^2-x+3\right)^{5/2}}{165888(2x+5)^3}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}+\frac{(67865260x+762984903)\left(2x\right.}{95551488(2x+5)}$$

```
[Out] 1/95551488*(762984903+67865260*x)*(2*x^2-x+3)^(3/2)/(5+2*x)-3667/2304*(2*x^
2-x+3)^(5/2)/(5+2*x)^4+224815/165888*(2*x^2-x+3)^(5/2)/(5+2*x)^3-14477995/2
3887872*(2*x^2-x+3)^(5/2)/(5+2*x)^2+432565/2048*arcsinh(1/23*(1-4*x)*23^(1/
2))*2^(1/2)-8969688643/42467328*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^
(1/2))*2^(1/2)+1/31850496*(2339916063-389975609*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1650, 812, 814, 843, 619, 215, 724, 206}

$$-\frac{14477995\left(2x^2-x+3\right)^{5/2}}{23887872(2x+5)^2}+\frac{224815\left(2x^2-x+3\right)^{5/2}}{165888(2x+5)^3}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}+\frac{(67865260x+762984903)\left(2x\right.}{95551488(2x+5)}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x]
```

```
[Out] ((2339916063 - 389975609*x)*Sqrt[3 - x + 2*x^2])/31850496 + ((762984903 + 6
7865260*x)*(3 - x + 2*x^2)^(3/2))/(95551488*(5 + 2*x)) - (3667*(3 - x + 2*x
^2)^(5/2))/(2304*(5 + 2*x)^4) + (224815*(3 - x + 2*x^2)^(5/2))/(165888*(5 +
 2*x)^3) - (14477995*(3 - x + 2*x^2)^(5/2))/(23887872*(5 + 2*x)^2) + (43256
5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*Sqrt[2]) - (8969688643*ArcTanh[(17 - 2
2*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(21233664*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2304(5+2x)^4} - \frac{1}{288}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{51695}{16} - \ldots\right.}{(5+\ldots}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2304(5+2x)^4} + \frac{224815\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^3} + \int \frac{(3-x\ldots}{\ldots}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2304(5+2x)^4} + \frac{224815\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^3} - \frac{14477\ldots}{238\ldots}$$

$$= \frac{(762984903+67865260x)\left(3-x+2x^2\right)^{3/2}}{95551488(5+2x)} - \frac{3667\left(3-x+\ldots\right.}{2304(5+\ldots}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903\ldots}{9\ldots}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903\ldots}{9\ldots}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903\ldots}{9\ldots}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903\ldots}{9\ldots}$$

Mathematica [A] time = 0.23, size = 108, normalized size = 0.57

$$\frac{-8969688643\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{24\sqrt{2x^2-x+3}\left(2949120x^6-29270016x^5+468043776x^4+11761910072x^3+60528581892x^2+\ldots\right.}{(2x+5)^4}}{42467328}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5
,x]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(86386856771 + 121473790266*x + 60528581892*x^2 +
11761910072*x^3 + 468043776*x^4 - 29270016*x^5 + 2949120*x^6))/(5 + 2*x)^4
+ 8969667840*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - 8969688643*Sqrt[2]*ArcTa
nh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/42467328
```

fricas [A] time = 0.96, size = 199, normalized size = 1.06

$$\frac{8969667840\sqrt{2}\left(16x^4+160x^3+600x^2+1000x+625\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="f
ricas")
```

```
[Out] 1/84934656*(8969667840*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*
log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8969688
```

```
643*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*log(-(24*sqrt(2)*sq
rt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 2
5)) + 48*(2949120*x^6 - 29270016*x^5 + 468043776*x^4 + 11761910072*x^3 + 60
528581892*x^2 + 121473790266*x + 86386856771)*sqrt(2*x^2 - x + 3))/(16*x^4
+ 160*x^3 + 600*x^2 + 1000*x + 625)
```

giac [B] time = 0.42, size = 503, normalized size = 2.68

$$-\frac{1}{42467328}\sqrt{2}\left(8969688643\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)\mathrm{sgn}\left(\frac{1}{2x+5}\right)+8969667840\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="g
iac")
```

```
[Out] -1/42467328*sqrt(2)*(8969688643*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2
+ 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 8969667840*log(abs(sqrt(-11/(2
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 896966
7840*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*s
gn(1/(2*x + 5)) + 12*(24*(1296*(29336*sgn(1/(2*x + 5))/(2*x + 5) - 42907*sg
n(1/(2*x + 5)))/(2*x + 5) + 39923563*sgn(1/(2*x + 5)))/(2*x + 5) - 54131203
9*sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 13824*(80624
1*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^5*sgn(1/(2*x + 5
)) - 1152288*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*sgn
(1/(2*x + 5)) - 957352*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
 5))^3*sgn(1/(2*x + 5)) + 1529280*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)
 + 6/(2*x + 5))^2*sgn(1/(2*x + 5)) + 394431*(sqrt(-11/(2*x + 5) + 36/(2*x +
 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x + 5)) - 620352*sgn(1/(2*x + 5)))/((sqr
t(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^3)
```

maple [A] time = 0.02, size = 204, normalized size = 1.09

$$-\frac{432565\sqrt{2}\ \mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2048}-\frac{8969688643\sqrt{2}\ \mathrm{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{42467328}+\frac{8969688643\sqrt{-11x+2\left(x\right.}}{127401984}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x)
```

```
[Out] 8969688643/127401984*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+8969688643/6879707136*(
-11*x+2*(x+5/2)^2-19/2)^(3/2)-3667/36864/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)
^(5/2)+224815/1327104/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(5/2)-14477995/955
51488/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(5/2)-593321753/6879707136*(4*x-1)
*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+593321753/3439853568/(x+5/2)*(-11*x+2*(x+5/
2)^2-19/2)^(5/2)-389975609/127401984*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)
-8969688643/42467328*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+
5/2)^2-19/2)^(1/2))-432565/2048*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 1.04, size = 210, normalized size = 1.12

$$\frac{16966315}{47775744}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^2-x+3\right)^{\frac{5}{2}}}{2304\left(16x^4+160x^3+600x^2+1000x+625\right)}+\frac{224815\left(2x^2-x+3\right)^{\frac{5}{2}}}{165888\left(8x^3+60x^2+150x+12\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="m
axima")
```

```
[Out] 16966315/47775744*(2*x^2 - x + 3)^(3/2) - 3667/2304*(2*x^2 - x + 3)^(5/2)/(
16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 224815/165888*(2*x^2 - x + 3)^
(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 14477995/23887872*(2*x^2 - x + 3)^(5
/2)/(4*x^2 + 20*x + 25) - 389975609/31850496*sqrt(2*x^2 - x + 3)*x - 432565
/2048*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 8969688643/4246732
8*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x +
5)) + 779972021/10616832*sqrt(2*x^2 - x + 3) + 593321753/95551488*(2*x^2 -
x + 3)^(3/2)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2x^2-x+3\right)^{3/2}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5,x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**5,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**5, x)
```

3.341 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx$

Optimal. Leaf size=195

$$-\frac{3730507\left(2x^2-x+3\right)^{5/2}}{11943936(2x+5)^3}+\frac{158527\left(2x^2-x+3\right)^{5/2}}{165888(2x+5)^4}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}+\frac{(44773976x+246012435)\left(2x^2-x\right.}{95551488(2x+5)^2}$$

```
[Out] 1/95551488*(246012435+44773976*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^2-3667/2880*(2*
x^2-x+3)^(5/2)/(5+2*x)^5+158527/165888*(2*x^2-x+3)^(5/2)/(5+2*x)^4-3730507/
11943936*(2*x^2-x+3)^(5/2)/(5+2*x)^3-23775/1024*arcsinh(1/23*(1-4*x)*23^(1/
2))*2^(1/2)+70991525167/3057647616*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+
3)^(1/2))*2^(1/2)-1/127401984*(5658774871+1028823716*x)*(2*x^2-x+3)^(1/2)/(
5+2*x)
```

Rubi [A] time = 0.26, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 812, 843, 619, 215, 724, 206}

$$-\frac{3730507\left(2x^2-x+3\right)^{5/2}}{11943936(2x+5)^3}+\frac{158527\left(2x^2-x+3\right)^{5/2}}{165888(2x+5)^4}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}+\frac{(44773976x+246012435)\left(2x^2-x\right.}{95551488(2x+5)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x]
```

```
[Out] -((5658774871 + 1028823716*x)*Sqrt[3 - x + 2*x^2])/(127401984*(5 + 2*x)) +
((246012435 + 44773976*x)*(3 - x + 2*x^2)^(3/2))/(95551488*(5 + 2*x)^2) - (
3667*(3 - x + 2*x^2)^(5/2))/(2880*(5 + 2*x)^5) + (158527*(3 - x + 2*x^2)^(5
/2))/(165888*(5 + 2*x)^4) - (3730507*(3 - x + 2*x^2)^(5/2))/(11943936*(5 +
2*x)^3) - (23775*ArcSinh[(1 - 4*x)/Sqrt[23]])/(512*Sqrt[2]) + (70991525167*
ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1528823808*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2880(5+2x)^5} - \frac{1}{360}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{60035}{16} - 661\right.}{(5+2x)}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2880(5+2x)^5} + \frac{158527\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^4} + \frac{\int \frac{(3-x+2x^2}{}}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2880(5+2x)^5} + \frac{158527\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^4} - \frac{3730507}{119439}$$

$$= \frac{(246012435+44773976x)\left(3-x+2x^2\right)^{3/2}}{95551488(5+2x)^2} - \frac{3667\left(3-x+2x\right.}{2880(5+2x)}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435+}{95}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435+}{95}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435+}{95}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435+}{95}$$

Mathematica [A] time = 0.24, size = 108, normalized size = 0.55

$$\frac{354957625835\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{24\sqrt{2x^2-x+3}\left(1592524800x^6-30496849920x^5-1023534029552x^4-7117092892448x^3-21590\right.}{(2x+5)^5}}{15288238080}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6
,x]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(-17093312738327 - 30872393829992*x - 215904397970
64*x^2 - 7117092892448*x^3 - 1023534029552*x^4 - 30496849920*x^5 + 15925248
00*x^6))/(5 + 2*x)^5 - 354958848000*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 3
54957625835*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/152882
38080
```

fricas [A] time = 1.03, size = 213, normalized size = 1.09

$$354958848000\sqrt{2}\left(32x^5+400x^4+2000x^3+5000x^2+6250x+3125\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="f
ricas")
```

```
[Out] 1/30576476160*(354958848000*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2
 + 6250*x + 3125)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 1
6*x - 25) + 354957625835*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 +
6250*x + 3125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 +
 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(1592524800*x^6 - 30496849920*x^5
 - 1023534029552*x^4 - 7117092892448*x^3 - 21590439797064*x^2 - 30872393829
992*x - 17093312738327)*sqrt(2*x^2 - x + 3))/(32*x^5 + 400*x^4 + 2000*x^3 +
 5000*x^2 + 6250*x + 3125)
```

giac [B] time = 0.32, size = 406, normalized size = 2.08

$$\frac{1}{256}\sqrt{2x^2-x+3}\,(20x-633)-\frac{23775}{1024}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{70991525167}{3057647616}\sqrt{2}\log\left(\left|\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="g
iac")
```

```
[Out] 1/256*sqrt(2*x^2 - x + 3)*(20*x - 633) - 23775/1024*sqrt(2)*log(-2*sqrt(2)*
(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 70991525167/3057647616*sqrt(2)*log
(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 70991525167/3057647
616*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1
/1274019840*sqrt(2)*(8281387393360*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^9 + 275661428628240*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 156038270334576
0*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 + 4938646760855520*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^6 - 9673562837036232*sqrt(2)*(sqrt(2)*x - sqrt(2*x^
2 - x + 3))^5 - 30647310393849000*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 700
60241036847960*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 97730658088823
880*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 30180638363071845*sqrt(2)*(sqrt(2
)*x - sqrt(2*x^2 - x + 3)) - 7096913381268319)/(2*(sqrt(2)*x - sqrt(2*x^2 -
 x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^5
```

maple [A] time = 0.02, size = 225, normalized size = 1.15

$$\frac{23775\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{1024}+\frac{70991525167\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{3057647616}-\frac{70991525167\sqrt{-11x+2\left(\right.}}{9172942848}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x)
```

```
[Out] -70991525167/9172942848*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-70991525167/49533891
3792*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-3667/92160/(x+5/2)^5*(-11*x+2*(x+5/2)^2
-19/2)^(5/2)+158527/2654208/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^(5/2)-373050
7/95551488/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(5/2)+134077495/6879707136/(x
+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(5/2)+4698578717/495338913792*(4*x-1)*(-11
*x+2*(x+5/2)^2-19/2)^(3/2)-4698578717/247669456896/(x+5/2)*(-11*x+2*(x+5/2)
^2-19/2)^(5/2)+3086715581/9172942848*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)
+70991525167/3057647616*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*
(x+5/2)^2-19/2)^(1/2))+23775/1024*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 1.06, size = 251, normalized size = 1.29

$$-\frac{134077495}{3439853568}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^2-x+3\right)^{\frac{5}{2}}}{2880\left(32x^5+400x^4+2000x^3+5000x^2+6250x+3125\right)}+\frac{158}{165888\left(16x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="m
axima")
```

```
[Out] -134077495/3439853568*(2*x^2 - x + 3)^(3/2) - 3667/2880*(2*x^2 - x + 3)^(5/
2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 158527/165888
*(2*x^2 - x + 3)^(5/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) - 373050
7/11943936*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 134077495
/1719926784*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25) + 3086715581/22932357
12*sqrt(2*x^2 - x + 3)*x + 23775/1024*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/2
3*sqrt(23)) - 70991525167/3057647616*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2
*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 6173186729/764411904*sqrt(2*x^2 -
x + 3) - 4698578717/6879707136*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6,x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{2} - x + 3\right)^{\frac{3}{2}} \left(5x^{4} - x^{3} + 3x^{2} + x + 2\right)}{\left(2x + 5\right)^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**6,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**6, x)
```

3.342 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx$

Optimal. Leaf size=195

$$-\frac{14087245\left(2x^2-x+3\right)^{5/2}}{71663616(2x+5)^4}+\frac{182165\left(2x^2-x+3\right)^{5/2}}{248832(2x+5)^5}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{3456(2x+5)^6}-\frac{(6793718806x+9802984711)\left(\right.}{13759414272(2x+}$$

```
[Out] -1/13759414272*(9802984711+6793718806*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^3-3667/3
456*(2*x^2-x+3)^(5/2)/(5+2*x)^6+182165/248832*(2*x^2-x+3)^(5/2)/(5+2*x)^5-1
4087245/71663616*(2*x^2-x+3)^(5/2)/(5+2*x)^4+369/256*arcsinh(1/23*(1-4*x)*2
3^(1/2))*2^(1/2)-1903976002333/1320903770112*arctanh(1/24*(17-22*x)*2^(1/2)
/(2*x^2-x+3)^(1/2))*2^(1/2)+1/55037657088*(151764102421+27596573612*x)*(2*x
^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A] time = 0.27, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1650, 810, 812, 843, 619, 215, 724, 206}

$$-\frac{14087245\left(2x^2-x+3\right)^{5/2}}{71663616(2x+5)^4}+\frac{182165\left(2x^2-x+3\right)^{5/2}}{248832(2x+5)^5}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{3456(2x+5)^6}-\frac{(6793718806x+9802984711)\left(\right.}{13759414272(2x+}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x]
```

```
[Out] ((151764102421 + 27596573612*x)*Sqrt[3 - x + 2*x^2])/(55037657088*(5 + 2*x)
) - ((9802984711 + 6793718806*x)*(3 - x + 2*x^2)^(3/2))/(13759414272*(5 + 2
*x)^3) - (3667*(3 - x + 2*x^2)^(5/2))/(3456*(5 + 2*x)^6) + (182165*(3 - x +
 2*x^2)^(5/2))/(248832*(5 + 2*x)^5) - (14087245*(3 - x + 2*x^2)^(5/2))/(716
63616*(5 + 2*x)^4) + (369*ArcSinh[(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2]) - (190
3976002333*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(66045188
5056*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{3456(5+2x)^6} - \frac{1}{432}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{68375}{16} - \ldots\right.}{(5+\ldots}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{3456(5+2x)^6} + \frac{182165\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^5} + \int \frac{(3-x\ldots}{\ldots}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{3456(5+2x)^6} + \frac{182165\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^5} - \frac{140872\ldots}{716\ldots}$$

$$= -\frac{(9802984711+6793718806x)\left(3-x+2x^2\right)^{3/2}}{13759414272(5+2x)^3} - \frac{3667\left(3\ldots\right.}{3456\ldots}$$

$$= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(980298\ldots}{\ldots}$$

$$= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(980298\ldots}{\ldots}$$

$$= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(980298\ldots}{\ldots}$$

$$= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(980298\ldots}{\ldots}$$

Mathematica [A] time = 0.25, size = 108, normalized size = 0.55

$$\frac{-1903976002333\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{24\sqrt{2x^2-x+3}\left(275188285440x^6+11854023276320x^5+103803827945872x^4+4225541\ldots\right.}{(2x+5)^6}}{1320903770112\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7
,x]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(458411625354581 + 1011372787716826*x + 9102568424
73992*x^2 + 422554114856528*x^3 + 103803827945872*x^4 + 11854023276320*x^5
+ 275188285440*x^6))/(5 + 2*x)^6 + 1903958949888*Sqrt[2]*ArcSinh[(1 - 4*x)/
Sqrt[23]] - 1903976002333*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*
x^2])])/1320903770112
```

fricas [A] time = 1.06, size = 229, normalized size = 1.17

$$\frac{1903958949888\sqrt{2}\left(64x^6+960x^5+6000x^4+20000x^3+37500x^2+37500x+15625\right)\log\left(4\sqrt{2}\sqrt{2x^2-x\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="f
ricas")
```

```
[Out] 1/2641807540224*(1903958949888*sqrt(2)*(64*x^6 + 960*x^5 + 6000*x^4 + 20000
*x^3 + 37500*x^2 + 37500*x + 15625)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25) + 1903976002333*sqrt(2)*(64*x^6 + 960*x^5 + 6000
*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15625)*log(-(24*sqrt(2)*sqrt(2*x^2
 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)) + 48
*(275188285440*x^6 + 11854023276320*x^5 + 103803827945872*x^4 + 42255411485
6528*x^3 + 910256842473992*x^2 + 1011372787716826*x + 458411625354581)*sqrt
(2*x^2 - x + 3))/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 375
00*x + 15625)
```

giac [B] time = 0.34, size = 452, normalized size = 2.32

$$\frac{369}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)-\frac{1903976002333}{1320903770112}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="g
iac")
```

```
[Out] 369/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 190
3976002333/1320903770112*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*
x^2 - x + 3))) + 1903976002333/1320903770112*sqrt(2)*log(abs(-2*sqrt(2)*x -
 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 5/64*sqrt(2*x^2 - x + 3) + 1/110075
314176*sqrt(2)*(159278433934432*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^1
1 + 6347903280912544*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^10 + 485445268408334
24*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 305716670132783088*(sqrt(2
)*x - sqrt(2*x^2 - x + 3))^8 + 88313821135911024*sqrt(2)*(sqrt(2)*x - sqrt(
2*x^2 - x + 3))^7 - 2423668581998843376*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6
 - 397211131697032056*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 1170889
7232532299576*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 - 12803484860728491138*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 12593033197867577234*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 - 3042533760672408875*sqrt(2)*(sqrt(2)*x - sqrt(2
*x^2 - x + 3)) + 589526263249780195)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2
 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^6
```

maple [A] time = 0.02, size = 246, normalized size = 1.26

$$-\frac{369\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{256}-\frac{1903976002333\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{1320903770112}+\frac{1903976002333\sqrt{-11x+2}}{396271131033}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x)
```

```
[Out] 1903976002333/3962711310336*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+1903976002333/21
3986410758144*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-3667/221184/(x+5/2)^6*(-11*x+2
*(x+5/2)^2-19/2)^(5/2)+182165/7962624/(x+5/2)^5*(-11*x+2*(x+5/2)^2-19/2)^(5
/2)-14087245/1146617856/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^(5/2)+149610673/
41278242816/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(5/2)-3607708597/29720334827
52/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(5/2)-125860542215/213986410758144*(4
*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+125860542215/106993205379072/(x+5/2)*(
-11*x+2*(x+5/2)^2-19/2)^(5/2)-82772668391/3962711310336*(4*x-1)*(-11*x+2*(x
+5/2)^2-19/2)^(1/2)-1903976002333/1320903770112*2^(1/2)*arctanh(1/12*(-11*x
+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))-369/256*2^(1/2)*arcsinh(4/23
*23^(1/2)*(x-1/4))
```

maxima [A] time = 1.07, size = 297, normalized size = 1.52

$$\frac{3607708597}{1486016741376}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^2-x+3\right)^{\frac{5}{2}}}{3456\left(64x^6+960x^5+6000x^4+20000x^3+37500x^2+37500x+15625\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="m
axima")
```

```
[Out] 3607708597/1486016741376*(2*x^2 - x + 3)^(3/2) - 3667/3456*(2*x^2 - x + 3)^
(5/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 1562
5) + 182165/248832*(2*x^2 - x + 3)^(5/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 500
0*x^2 + 6250*x + 3125) - 14087245/71663616*(2*x^2 - x + 3)^(5/2)/(16*x^4 +
160*x^3 + 600*x^2 + 1000*x + 625) + 149610673/5159780352*(2*x^2 - x + 3)^(5
/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 3607708597/743008370688*(2*x^2 - x + 3
)^(5/2)/(4*x^2 + 20*x + 25) - 82772668391/990677827584*sqrt(2*x^2 - x + 3)*
x - 369/256*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 190397600233
3/1320903770112*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(
23)/abs(2*x + 5)) + 165562389227/330225942528*sqrt(2*x^2 - x + 3) + 1258605
42215/2972033482752*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2x^2-x+3\right)^{3/2}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7,x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**7,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**7, x)
```

3.343 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$

Optimal. Leaf size=195

$$-\frac{1930441\left(2x^2-x+3\right)^{5/2}}{13934592(2x+5)^5}+\frac{114335\left(2x^2-x+3\right)^{5/2}}{193536(2x+5)^6}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}-\frac{(411822458x+463558457)\left(2x^2-\right.}{2293235712(2x+5)^4}$$

```
[Out] -1/2293235712*(463558457+411822458*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^4-3667/4032
*(2*x^2-x+3)^(5/2)/(5+2*x)^7+114335/193536*(2*x^2-x+3)^(5/2)/(5+2*x)^6-1930
441/13934592*(2*x^2-x+3)^(5/2)/(5+2*x)^5-5/128*arcsinh(1/23*(1-4*x)*23^(1/2
))*2^(1/2)+412760561351/10567230160896*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^
2-x+3)^(1/2))*2^(1/2)-1/440301256704*(146583836191+101679102454*x)*(2*x^2-x
+3)^(1/2)/(5+2*x)^2
```

Rubi [A] time = 0.26, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1650, 810, 843, 619, 215, 724, 206}

$$-\frac{1930441\left(2x^2-x+3\right)^{5/2}}{13934592(2x+5)^5}+\frac{114335\left(2x^2-x+3\right)^{5/2}}{193536(2x+5)^6}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}-\frac{(411822458x+463558457)\left(2x^2-\right.}{2293235712(2x+5)^4}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x]
```

```
[Out] -((146583836191 + 101679102454*x)*Sqrt[3 - x + 2*x^2])/(440301256704*(5 + 2
*x)^2) - ((463558457 + 411822458*x)*(3 - x + 2*x^2)^(3/2))/(2293235712*(5 +
 2*x)^4) - (3667*(3 - x + 2*x^2)^(5/2))/(4032*(5 + 2*x)^7) + (114335*(3 - x
 + 2*x^2)^(5/2))/(193536*(5 + 2*x)^6) - (1930441*(3 - x + 2*x^2)^(5/2))/(13
934592*(5 + 2*x)^5) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2]) + (41276
0561351*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(52836150804
48*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^8}\,dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{4032(5+2x)^7} - \frac{1}{504}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{76715}{16}-\frac{1485}{2}\right.}{(5+2x}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{4032(5+2x)^7} + \frac{114335\left(3-x+2x^2\right)^{5/2}}{193536(5+2x)^6} + \frac{\int \frac{(3-x+2x^2}{}}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{4032(5+2x)^7} + \frac{114335\left(3-x+2x^2\right)^{5/2}}{193536(5+2x)^6} - \frac{1930441\,(}{139345}$$

$$= -\frac{(463558457+411822458x)\left(3-x+2x^2\right)^{3/2}}{2293235712(5+2x)^4} - \frac{3667\left(3-x+\right.}{4032(5+}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(4635584}{}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(4635584}{}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(4635584}{}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(4635584}{}$$

Mathematica [A] time = 0.26, size = 108, normalized size = 0.55

$$\frac{2889323929457\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) - \frac{24\sqrt{2x^2-x+3}\left(38463671680832x^6+402255822731712x^5+2069947287085104x^4+59663296\right.}{(2x+5)^7}}{73970611126272}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8
,x]
```

```
[Out] ((-24*Sqrt[3 - x + 2*x^2]*(3479517268702637 + 9065154700300572*x + 99760653
67498188*x^2 + 5966329646300704*x^3 + 2069947287085104*x^4 + 40225582273171
2*x^5 + 38463671680832*x^6))/(5 + 2*x)^7 - 2889476997120*Sqrt[2]*ArcSinh[(1
 - 4*x)/Sqrt[23]] + 2889323929457*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 -
2*x + 4*x^2])])/73970611126272
```

fricas [A] time = 0.91, size = 243, normalized size = 1.25

$$\frac{2889476997120\sqrt{2}\left(128x^7+2240x^6+16800x^5+70000x^4+175000x^3+262500x^2+218750x+78125\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="f
ricas")
```

```
[Out] 1/147941222252544*(2889476997120*sqrt(2)*(128*x^7 + 2240*x^6 + 16800*x^5 +
70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125)*log(-4*sqrt(2)*sqrt
(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 2889323929457*sqrt(2)*(12
8*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750
*x + 78125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 10
36*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(38463671680832*x^6 + 40225582273171
2*x^5 + 2069947287085104*x^4 + 5966329646300704*x^3 + 9976065367498188*x^2
+ 9065154700300572*x + 3479517268702637)*sqrt(2*x^2 - x + 3))/(128*x^7 + 22
40*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125
)
```

giac [B] time = 0.33, size = 489, normalized size = 2.51

$$-\frac{5}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{412760561351}{10567230160896}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="g
iac")
```

```
[Out] -5/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 4127
60561351/10567230160896*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x
^2 - x + 3))) - 412760561351/10567230160896*sqrt(2)*log(abs(-2*sqrt(2)*x -
11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/6164217593856*sqrt(2)*(11218973984
12224*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^13 + 48260296303776704*(sqr
t(2)*x - sqrt(2*x^2 - x + 3))^12 + 444673458321712704*sqrt(2)*(sqrt(2)*x -
sqrt(2*x^2 - x + 3))^11 + 3996455936659982656*(sqrt(2)*x - sqrt(2*x^2 - x +
 3))^10 + 6725227967167489360*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 -
 17132661028483948080*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 - 637130120947372
46112*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 + 106515880136064432096*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 + 226947197958946260516*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3))^5 - 856601202771483308188*(sqrt(2)*x - sqrt(2*x^2
 - x + 3))^4 + 617998258357377713732*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^3 - 467121785339763351756*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 9229208
0735560562227*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 15161716093827501
349)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(
2*x^2 - x + 3)) - 11)^7
```

maple [A] time = 0.02, size = 267, normalized size = 1.37

$$\frac{5\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128}+\frac{412760561351\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{10567230160896}-\frac{412760561351\sqrt{-11x+2\left(x-\right.}}{31701690482688}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x)
```

```
[Out] -412760561351/31701690482688*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-412760561351/17
11891286065152*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-3667/516096/(x+5/2)^7*(-11*x+
2*(x+5/2)^2-19/2)^(5/2)+114335/12386304/(x+5/2)^6*(-11*x+2*(x+5/2)^2-19/2)^
(5/2)-1930441/445906944/(x+5/2)^5*(-11*x+2*(x+5/2)^2-19/2)^(5/2)+7861079/91
72942848/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^(5/2)-32967491/330225942528/(x+
5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(5/2)+769352975/23776267862016/(x+5/2)^2*(-
11*x+2*(x+5/2)^2-19/2)^(5/2)+27452157541/1711891286065152*(4*x-1)*(-11*x+2*
(x+5/2)^2-19/2)^(3/2)-27452157541/855945643032576/(x+5/2)*(-11*x+2*(x+5/2)^
```

```
2-19/2)^(5/2)+17957520133/31701690482688*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(
1/2)+412760561351/10567230160896*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/
(-11*x+2*(x+5/2)^2-19/2)^(1/2))+5/128*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4)
)
```

maxima [B] time = 1.07, size = 348, normalized size = 1.78

$$-\frac{769352975}{11888133931008}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^2-x+3\right)^{\frac{5}{2}}}{4032\left(128x^7+2240x^6+16800x^5+70000x^4+175000x^3+262500x^2+21\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="m
axima")
```

```
[Out] -769352975/11888133931008*(2*x^2 - x + 3)^(3/2) - 3667/4032*(2*x^2 - x + 3)
^(5/2)/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^
2 + 218750*x + 78125) + 114335/193536*(2*x^2 - x + 3)^(5/2)/(64*x^6 + 960*x
^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15625) - 1930441/13934592
*(2*x^2 - x + 3)^(5/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3
125) + 7861079/573308928*(2*x^2 - x + 3)^(5/2)/(16*x^4 + 160*x^3 + 600*x^2
+ 1000*x + 625) - 32967491/41278242816*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^
2 + 150*x + 125) + 769352975/5944066965504*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 2
0*x + 25) + 17957520133/7925422620672*sqrt(2*x^2 - x + 3)*x + 5/128*sqrt(2)
*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 412760561351/10567230160896*sqr
t(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) -
 35893173457/2641807540224*sqrt(2*x^2 - x + 3) - 27452157541/23776267862016
*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2x^2-x+3\right)^{3/2}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8,x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**8,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**8, x)
```

3.344 $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=120

$$\frac{1}{16}\sqrt{2x^2-x+3}\,(2x+5)^4-\frac{105}{128}\sqrt{2x^2-x+3}\,(2x+5)^3+\frac{761}{256}\sqrt{2x^2-x+3}\,(2x+5)^2-\frac{(4676x+19227)\sqrt{2x^2-x+}}{2048}$$

```
[Out] -85429/8192*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+761/256*(5+2*x)^2*(2*x^2
-x+3)^(1/2)-105/128*(5+2*x)^3*(2*x^2-x+3)^(1/2)+1/16*(5+2*x)^4*(2*x^2-x+3)^
(1/2)-1/2048*(19227+4676*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1653, 779, 619, 215}

$$\frac{1}{16}\sqrt{2x^2-x+3}\,(2x+5)^4-\frac{105}{128}\sqrt{2x^2-x+3}\,(2x+5)^3+\frac{761}{256}\sqrt{2x^2-x+3}\,(2x+5)^2-\frac{(4676x+19227)\sqrt{2x^2-x+}}{2048}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (761*(5 + 2*x)^2*Sqrt[3 - x + 2*x^2])/256 - (105*(5 + 2*x)^3*Sqrt[3 - x + 2
*x^2])/128 + ((5 + 2*x)^4*Sqrt[3 - x + 2*x^2])/16 - ((19227 + 4676*x)*Sqrt[
3 - x + 2*x^2])/2048 - (85429*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4096*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} + \frac{1}{160}\int \frac{(5+2x)\left(-5055-4390x-5580x\right.}{\sqrt{3-x+2x^2}}$$

$$= -\frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} + \frac{\int \frac{(5+2x)(32}{}}{}$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x$$

Mathematica [A] time = 0.12, size = 60, normalized size = 0.50

$$\frac{4\sqrt{2x^2-x+3}\left(2048x^4+7040x^3+352x^2-6916x+2973\right)-85429\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(2973 - 6916*x + 352*x^2 + 7040*x^3 + 2048*x^4) - 85
429*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/8192
```

fricas [A] time = 0.88, size = 73, normalized size = 0.61

$$\frac{1}{2048}\left(2048\,x^4+7040\,x^3+352\,x^2-6916\,x+2973\right)\sqrt{2\,x^2-x+3}+\frac{85429}{16384}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}\,(4\,x-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2048*(2048*x^4 + 7040*x^3 + 352*x^2 - 6916*x + 2973)*sqrt(2*x^2 - x + 3)
+ 85429/16384*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2
 + 16*x - 25)
```

giac [A] time = 0.19, size = 68, normalized size = 0.57

$$\frac{1}{2048}\,(4\,(8\,(4\,(16\,x+55)x+11)x-1729)x+2973)\sqrt{2\,x^2-x+3}-\frac{85429}{8192}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/2048*(4*(8*(4*(16*x + 55)*x + 11)*x - 1729)*x + 2973)*sqrt(2*x^2 - x + 3)
 - 85429/8192*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 95, normalized size = 0.79

$$\sqrt{2x^2-x+3}\,x^4+\frac{55\sqrt{2x^2-x+3}\,x^3}{16}+\frac{11\sqrt{2x^2-x+3}\,x^2}{64}-\frac{1729\sqrt{2x^2-x+3}\,x}{512}+\frac{85429\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\right.}{23}\right.}{8192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] x^4*(2*x^2-x+3)^(1/2)+55/16*x^3*(2*x^2-x+3)^(1/2)+11/64*x^2*(2*x^2-x+3)^(1/
2)-1729/512*x*(2*x^2-x+3)^(1/2)+2973/2048*(2*x^2-x+3)^(1/2)+85429/8192*2^(1
/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 0.97, size = 96, normalized size = 0.80

$$\sqrt{2\,x^2-x+3}\,x^4+\frac{55}{16}\,\sqrt{2\,x^2-x+3}\,x^3+\frac{11}{64}\,\sqrt{2\,x^2-x+3}\,x^2-\frac{1729}{512}\,\sqrt{2\,x^2-x+3}\,x+\frac{85429}{8192}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="max
ima")
```

```
[Out] sqrt(2*x^2 - x + 3)*x^4 + 55/16*sqrt(2*x^2 - x + 3)*x^3 + 11/64*sqrt(2*x^2
- x + 3)*x^2 - 1729/512*sqrt(2*x^2 - x + 3)*x + 85429/8192*sqrt(2)*arcsinh(
1/23*sqrt(23)*(4*x - 1)) + 2973/2048*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+5)\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/sqrt(2*x**2 - x + 3), x
)
```

3.345 $\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=101

$$\frac{19}{96}\sqrt{2x^2-x+3}\,x^2-\frac{409}{768}\sqrt{2x^2-x+3}\,x-\frac{505\sqrt{2x^2-x+3}}{1024}+\frac{5}{8}\sqrt{2x^2-x+3}\,x^3-\frac{6863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

```
[Out] -6863/4096*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-505/1024*(2*x^2-x+3)^(1/2
)-409/768*x*(2*x^2-x+3)^(1/2)+19/96*x^2*(2*x^2-x+3)^(1/2)+5/8*x^3*(2*x^2-x+
3)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1661, 640, 619, 215}

$$\frac{5}{8}\sqrt{2x^2-x+3}\,x^3+\frac{19}{96}\sqrt{2x^2-x+3}\,x^2-\frac{409}{768}\sqrt{2x^2-x+3}\,x-\frac{505\sqrt{2x^2-x+3}}{1024}-\frac{6863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (-505*Sqrt[3 - x + 2*x^2])/1024 - (409*x*Sqrt[3 - x + 2*x^2])/768 + (19*x^2
*Sqrt[3 - x + 2*x^2])/96 + (5*x^3*Sqrt[3 - x + 2*x^2])/8 - (6863*ArcSinh[(1
 - 4*x)/Sqrt[23]])/(2048*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}dx = \frac{5}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{8}\int \frac{16+8x-21x^2+\frac{19x^3}{2}}{\sqrt{3-x+2x^2}}dx$$

$$= \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{48}\int \frac{96-9x-\frac{409x^2}{4}}{\sqrt{3-x+2x^2}}dx$$

$$= -\frac{409}{768}x\sqrt{3-x+2x^2} + \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{192}\int \frac{\frac{276\ldots}{4}}{\sqrt{3-\ldots}}$$

$$= -\frac{505\sqrt{3-x+2x^2}}{1024} - \frac{409}{768}x\sqrt{3-x+2x^2} + \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x\ldots}$$

$$= -\frac{505\sqrt{3-x+2x^2}}{1024} - \frac{409}{768}x\sqrt{3-x+2x^2} + \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x\ldots}$$

$$= -\frac{505\sqrt{3-x+2x^2}}{1024} - \frac{409}{768}x\sqrt{3-x+2x^2} + \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x\ldots}$$

Mathematica [A] time = 0.07, size = 55, normalized size = 0.54

$$\frac{4\sqrt{2x^2-x+3}\left(1920x^3+608x^2-1636x-1515\right)-20589\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{12288}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-1515 - 1636*x + 608*x^2 + 1920*x^3) - 20589*Sqrt[2
]*ArcSinh[(1 - 4*x)/Sqrt[23]])/12288
```

fricas [A] time = 0.78, size = 68, normalized size = 0.67

$$\frac{1}{3072}\left(1920\,x^3+608\,x^2-1636\,x-1515\right)\sqrt{2\,x^2-x+3}+\frac{6863}{8192}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3072*(1920*x^3 + 608*x^2 - 1636*x - 1515)*sqrt(2*x^2 - x + 3) + 6863/8192
*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.20, size = 63, normalized size = 0.62

$$\frac{1}{3072}\left(4\left(8\left(60\,x+19\right)x-409\right)x-1515\right)\sqrt{2\,x^2-x+3}-\frac{6863}{4096}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3072*(4*(8*(60*x + 19)*x - 409)*x - 1515)*sqrt(2*x^2 - x + 3) - 6863/4096
*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 79, normalized size = 0.78

$$\frac{5\sqrt{2x^2-x+3}\;x^3}{8}+\frac{19\sqrt{2x^2-x+3}\;x^2}{96}-\frac{409\sqrt{2x^2-x+3}\;x}{768}+\frac{6863\sqrt{2}\;\mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}-\frac{505\sqrt{2x^2-x+\ldots}}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/8*(2*x^2-x+3)^(1/2)*x^3+19/96*(2*x^2-x+3)^(1/2)*x^2-409/768*(2*x^2-x+3)^(
1/2)*x-505/1024*(2*x^2-x+3)^(1/2)+6863/4096*2^(1/2)*arcsinh(4/23*23^(1/2)*(
x-1/4))
```

maxima [A] time = 0.96, size = 80, normalized size = 0.79

$$\frac{5}{8}\sqrt{2x^2-x+3}\,x^3+\frac{19}{96}\sqrt{2x^2-x+3}\,x^2-\frac{409}{768}\sqrt{2x^2-x+3}\,x+\frac{6863}{4096}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{505}{1024}\sqrt{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/8*sqrt(2*x^2 - x + 3)*x^3 + 19/96*sqrt(2*x^2 - x + 3)*x^2 - 409/768*sqrt(
2*x^2 - x + 3)*x + 6863/4096*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 505
/1024*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4-x^3+3x^2+x+2}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/sqrt(2*x**2 - x + 3), x)
```

3.346 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=126

$$\frac{5}{48}\sqrt{2x^2-x+3}\,(2x+5)^2-\frac{337}{192}\sqrt{2x^2-x+3}\,(2x+5)+\frac{1669}{128}\sqrt{2x^2-x+3}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{96\sqrt{2}}+\frac{9657}{}$$

```
[Out] 9657/512*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-3667/192*arctanh(1/24*(17-2
2*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1669/128*(2*x^2-x+3)^(1/2)-337/192*
(5+2*x)*(2*x^2-x+3)^(1/2)+5/48*(5+2*x)^2*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1653, 843, 619, 215, 724, 206}

$$\frac{5}{48}\sqrt{2x^2-x+3}\,(2x+5)^2-\frac{337}{192}\sqrt{2x^2-x+3}\,(2x+5)+\frac{1669}{128}\sqrt{2x^2-x+3}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{96\sqrt{2}}+\frac{9657}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (1669*Sqrt[3 - x + 2*x^2])/128 - (337*(5 + 2*x)*Sqrt[3 - x + 2*x^2])/192 +
(5*(5 + 2*x)^2*Sqrt[3 - x + 2*x^2])/48 + (9657*ArcSinh[(1 - 4*x)/Sqrt[23]])
/(256*Sqrt[2]) - (3667*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])
])/(96*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} + \frac{1}{96}\int \frac{-2183-3054x-4092x^2-2696x^3}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} + \frac{\int \frac{24504+128736x+160224}{(5+2x)\sqrt{3-x+2x^2}}}{3072}$$

$$= \frac{1669}{128}\sqrt{3-x+2x^2} - \frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} + \cdots$$

$$= \frac{1669}{128}\sqrt{3-x+2x^2} - \frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} - \frac{9}{2}\cdots$$

$$= \frac{1669}{128}\sqrt{3-x+2x^2} - \frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} - \frac{3}{}\cdots$$

$$= \frac{1669}{128}\sqrt{3-x+2x^2} - \frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} + \frac{9}{}\cdots$$

Mathematica [A] time = 0.10, size = 81, normalized size = 0.64

$$\frac{4\sqrt{2x^2-x+3}\left(160x^2-548x+2637\right)-29336\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+28971\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1536}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(2637 - 548*x + 160*x^2) + 28971*Sqrt[2]*ArcSinh[(1
- 4*x)/Sqrt[23]] - 29336*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x
^2])])/1536
```

fricas [A] time = 0.74, size = 115, normalized size = 0.91

$$\frac{1}{384}\left(160\,x^2-548\,x+2637\right)\sqrt{2\,x^2-x+3}+\frac{9657}{1024}\sqrt{2}\,\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+\frac{366}{38}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/384*(160*x^2 - 548*x + 2637)*sqrt(2*x^2 - x + 3) + 9657/1024*sqrt(2)*log(
4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 3667/384*sq
rt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x
+ 1153)/(4*x^2 + 20*x + 25))
```

giac [A] time = 0.23, size = 119, normalized size = 0.94

$$\frac{1}{384}\left(4\left(40\,x-137\right)x+2637\right)\sqrt{2\,x^2-x+3}+\frac{9657}{512}\,\sqrt{2}\,\log\left(-4\,\sqrt{2}x+\sqrt{2}+4\,\sqrt{2\,x^2-x+3}\right)-\frac{3667}{192}\,\sqrt{2}\,\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/384*(4*(40*x - 137)*x + 2637)*sqrt(2*x^2 - x + 3) + 9657/512*sqrt(2)*log(
-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 3667/192*sqrt(2)*log(abs(
-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 3667/192*sqrt(2)*log(abs
(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
```

maple [A] time = 0.01, size = 92, normalized size = 0.73

$$\frac{5\sqrt{2x^2-x+3}\,x^2}{12}-\frac{137\sqrt{2x^2-x+3}\,x}{96}-\frac{9657\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{512}-\frac{3667\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\cdots}}\right.}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/12*(2*x^2-x+3)^(1/2)*x^2-137/96*(2*x^2-x+3)^(1/2)*x+879/128*(2*x^2-x+3)^(
1/2)-9657/512*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-3667/192*2^(1/2)*arcta
nh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))
```

maxima [A] time = 0.98, size = 99, normalized size = 0.79

$$\frac{5}{12}\,\sqrt{2\,x^2-x+3}\,x^2-\frac{137}{96}\,\sqrt{2\,x^2-x+3}\,x-\frac{9657}{512}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\,\sqrt{23}\,x-\frac{1}{23}\,\sqrt{23}\right)+\frac{3667}{192}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\,\sqrt{23}}{23\,|2\,x\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="max
ima")
```

```
[Out] 5/12*sqrt(2*x^2 - x + 3)*x^2 - 137/96*sqrt(2*x^2 - x + 3)*x - 9657/512*sqrt
(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 3667/192*sqrt(2)*arcsinh(22/
23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 879/128*sqrt(2*
x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{\left(2\,x+5\right)\,\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(1/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)\sqrt{2x^2 - x + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*sqrt(2*x**2 - x + 3)),
 x)
```

3.347 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=126

$$\frac{5}{32}\sqrt{2x^2-x+3}\,(2x+5)-\frac{243}{64}\sqrt{2x^2-x+3}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}+\frac{158527\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6912\sqrt{2}}-\frac{2943\sinh^{-}}{128\sqrt{}}$$

```
[Out] -2943/256*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+158527/13824*arctanh(1/24*
(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-243/64*(2*x^2-x+3)^(1/2)-3667/
576*(2*x^2-x+3)^(1/2)/(5+2*x)+5/32*(5+2*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 1653, 843, 619, 215, 724, 206}

$$\frac{5}{32}\sqrt{2x^2-x+3}\,(2x+5)-\frac{243}{64}\sqrt{2x^2-x+3}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}+\frac{158527\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6912\sqrt{2}}-\frac{2943\sinh^{-}}{128\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (-243*Sqrt[3 - x + 2*x^2])/64 - (3667*Sqrt[3 - x + 2*x^2])/(576*(5 + 2*x))
+ (5*(5 + 2*x)*Sqrt[3 - x + 2*x^2])/32 - (2943*ArcSinh[(1 - 4*x)/Sqrt[23]])
/(128*Sqrt[2]) + (158527*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2
])])/(6912*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} - \frac{1}{72}\int \frac{\frac{12007}{16}-1323x+486x^2-180x^3}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} - \frac{\int \frac{30314-27216x+34992x^2}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{2304}$$

$$= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} - \frac{\int \frac{417472-}{(5+2x)\sqrt{}}}{18}$$

$$= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} + \frac{2943}{128}\int$$

$$= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} + \frac{158527}{576}\,\mathrm{S}$$

$$= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} - \frac{2943\sin}{128}$$

Mathematica [A] time = 0.12, size = 88, normalized size = 0.70

$$\frac{\frac{48\sqrt{2x^2-x+3}\left(180x^2-1287x-6176\right)}{2x+5}+158527\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)-158922\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{13824}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((48*Sqrt[3 - x + 2*x^2]*(-6176 - 1287*x + 180*x^2))/(5 + 2*x) - 158922*Sqr
t[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 158527*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*S
qrt[6 - 2*x + 4*x^2])])/13824
```

fricas [A] time = 0.97, size = 133, normalized size = 1.06

$$\frac{158922\,\sqrt{2}\,(2\,x+5)\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^{2}-x+3}\,(4\,x-1)-32\,x^{2}+16\,x-25\right)+158527\,\sqrt{2}\,(2\,x+5)\log\left(\frac{24\,\sqrt{2}\,\sqrt{}}{}\right.}{27648\,(2\,x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/27648*(158922*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x -
 1) - 32*x^2 + 16*x - 25) + 158527*sqrt(2)*(2*x + 5)*log((24*sqrt(2)*sqrt(2
*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25))
+ 96*(180*x^2 - 1287*x - 6176)*sqrt(2*x^2 - x + 3))/(2*x + 5)
```

giac [B] time = 0.40, size = 339, normalized size = 2.69

$$\frac{1}{13824}\sqrt{2}\left(\frac{158527\log\left(12\sqrt{-\frac{11}{2\,x+5}+\frac{36}{(2\,x+5)^{2}}+1}+\frac{72}{2\,x+5}-11\right)}{\operatorname{sgn}\left(\frac{1}{2\,x+5}\right)}+\frac{158922\log\left(\left|\sqrt{-\frac{11}{2\,x+5}+\frac{36}{(2\,x+5)^{2}}+1}+\frac{6}{2\,x+}\right.\right.}{\operatorname{sgn}\left(\frac{1}{2\,x+5}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/13824*sqrt(2)*(158527*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 7
2/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 158922*log(abs(sqrt(-11/(2*x + 5) + 36
/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))/sgn(1/(2*x + 5)) - 158922*log(abs(sqr
t(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))/sgn(1/(2*x + 5))
- 44004*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)/sgn(1/(2*x + 5)) + 108*(33
93*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3 - 4896*(sqrt(
-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 743*sqrt(-11/(2*x +
5) + 36/(2*x + 5)^2 + 1) - 4458/(2*x + 5) + 2256)/(((sqrt(-11/(2*x + 5) + 3
6/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^2*sgn(1/(2*x + 5))))
```

maple [A] time = 0.01, size = 96, normalized size = 0.76

$$\frac{5\sqrt{2x^{2}-x+3}\;x}{16}+\frac{2943\sqrt{2}\;\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{256}+\frac{158527\sqrt{2}\;\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^{2}-\frac{19}{2}}}\right)}{13824}-\frac{193\sqrt{2x^{2}-x+}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/16*(2*x^2-x+3)^(1/2)*x-193/64*(2*x^2-x+3)^(1/2)+2943/256*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))-3667/1152/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+158
527/13824*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2
)^(1/2))
```

maxima [A] time = 0.98, size = 103, normalized size = 0.82

$$\frac{5}{16}\sqrt{2x^2-x+3}\,x+\frac{2943}{256}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{158527}{13824}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right)-\frac{19}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] 5/16*sqrt(2*x^2 - x + 3)*x + 2943/256*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/2
3*sqrt(23)) - 158527/13824*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) -
17/23*sqrt(23)/abs(2*x + 5)) - 193/64*sqrt(2*x^2 - x + 3) - 3667/576*sqrt(2
*x^2 - x + 3)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(1/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*sqrt(2*x**2 - x + 3
)), x)
```

3.348 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=128

$$\frac{92239\sqrt{2x^2-x+3}}{27648(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}+\frac{5}{16}\sqrt{2x^2-x+3}-\frac{1546507\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{331776\sqrt{2}}+\frac{149\sinh^{-1}\left(\frac{1-}{\sqrt{}}\right.}{32\sqrt{2}}$$

```
[Out] 149/64*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1546507/663552*arctanh(1/24*(
17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+5/16*(2*x^2-x+3)^(1/2)-3667/115
2*(2*x^2-x+3)^(1/2)/(5+2*x)^2+92239/27648*(2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A] time = 0.21, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 1653, 843, 619, 215, 724, 206}

$$\frac{92239\sqrt{2x^2-x+3}}{27648(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}+\frac{5}{16}\sqrt{2x^2-x+3}-\frac{1546507\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{331776\sqrt{2}}+\frac{149\sinh^{-1}\left(\frac{1-}{\sqrt{}}\right.}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (5*Sqrt[3 - x + 2*x^2])/16 - (3667*Sqrt[3 - x + 2*x^2])/(1152*(5 + 2*x)^2)
+ (92239*Sqrt[3 - x + 2*x^2])/(27648*(5 + 2*x)) + (149*ArcSinh[(1 - 4*x)/Sq
rt[23]])/(32*Sqrt[2]) - (1546507*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x
 + 2*x^2])])/(331776*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} - \frac{1}{144}\int \frac{\frac{20347}{16}-\frac{6917x}{4}+972x^2-360x^3}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} + \frac{\int \frac{\frac{647841}{16}-67392x+12960x^2}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{10368}$$

$$= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} + \frac{\int \frac{\frac{777441}{2}-772416x}{(5+2x)\sqrt{3-x+2x^2}}}{82944}$$

$$= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} - \frac{149}{32}\int \frac{1}{\sqrt{3-x+}}$$

$$= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} - \frac{1546507\,\mathrm{Subst}\,(}{}$$

$$= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} + \frac{149\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right.}{32\sqrt{2}}$$

Mathematica [A] time = 0.13, size = 88, normalized size = 0.69

$$\frac{\frac{24\sqrt{2x^2-x+3}\left(34560x^2+357278x+589187\right)}{(2x+5)^2} - 1546507\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + 1544832\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{663552}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(589187 + 357278*x + 34560*x^2))/(5 + 2*x)^2 + 154
4832*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - 1546507*Sqrt[2]*ArcTanh[(17 - 22
*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/663552
```

fricas [A] time = 0.96, size = 149, normalized size = 1.16

$$\frac{1544832\sqrt{2}\left(4x^2+20x+25\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+1546507\sqrt{2}\left(4x^2+20\right.}{1327104\left(4x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/1327104*(1544832*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 1546507*sqrt(2)*(4*x^2 + 20*x + 25)
*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 115
3)/(4*x^2 + 20*x + 25)) + 48*(34560*x^2 + 357278*x + 589187)*sqrt(2*x^2 - x
 + 3))/(4*x^2 + 20*x + 25)
```

giac [B] time = 0.26, size = 248, normalized size = 1.94

$$\frac{149}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)-\frac{1546507}{663552}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{154}{663}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] 149/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 1546
507/663552*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
 + 1546507/663552*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2
- x + 3))) + 5/16*sqrt(2*x^2 - x + 3) + 1/55296*sqrt(2)*(2381290*sqrt(2)*(s
qrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 16628406*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^2 - 25697445*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 16720645)/(2*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
 + 3)) - 11)^2
```

maple [A] time = 0.01, size = 102, normalized size = 0.80

$$-\frac{149\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{64}-\frac{1546507\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{663552}+\frac{5\sqrt{2x^2-x+3}}{16}+\frac{92239\sqrt{-11x}}{55296}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/16*(2*x^2-x+3)^(1/2)-149/64*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+92239/
55296/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-1546507/663552*2^(1/2)*arctanh
(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))-3667/4608/(x+5/2
)^2*(-11*x+2*(x+5/2)^2-19/2)^(1/2)
```

maxima [A] time = 0.98, size = 114, normalized size = 0.89

$$-\frac{149}{64}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{1546507}{663552}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{5}{16}\sqrt{2\,x^2-x+3}-\frac{3}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] -149/64*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 1546507/663552*s
qrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5))
 + 5/16*sqrt(2*x^2 - x + 3) - 3667/1152*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x +
 25) + 92239/27648*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)^3\,\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(1/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\,\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*sqrt(2*x**2 - x + 3
)), x)
```

3.349 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4 \sqrt{3-x+2x^2}} \, dx$

Optimal. Leaf size=135

$$-\frac{3163415\sqrt{2x^2-x+3}}{5971968(2x+5)}+\frac{394907\sqrt{2x^2-x+3}}{248832(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}+\frac{22389491\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{71663616\sqrt{2}}-\frac{5\sin}{}$$

```
[Out] -5/32*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+22389491/143327232*arctanh(1/2
4*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-3667/1728*(2*x^2-x+3)^(1/2)/
(5+2*x)^3+394907/248832*(2*x^2-x+3)^(1/2)/(5+2*x)^2-3163415/5971968*(2*x^2-
x+3)^(1/2)/(5+2*x)
```

Rubi [A] time = 0.20, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1650, 843, 619, 215, 724, 206}

$$-\frac{3163415\sqrt{2x^2-x+3}}{5971968(2x+5)}+\frac{394907\sqrt{2x^2-x+3}}{248832(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}+\frac{22389491\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{71663616\sqrt{2}}-\frac{5\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (-3667*Sqrt[3 - x + 2*x^2])/(1728*(5 + 2*x)^3) + (394907*Sqrt[3 - x + 2*x^2
])/(248832*(5 + 2*x)^2) - (3163415*Sqrt[3 - x + 2*x^2])/(5971968*(5 + 2*x))
 - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16*Sqrt[2]) + (22389491*ArcTanh[(17 - 2
2*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(71663616*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} - \frac{1}{216}\int \frac{\frac{28687}{16}-\frac{4271x}{2}+1458x^2-540x^3}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} + \frac{\int \frac{\frac{1464275}{16}-\frac{413797x}{4}+38880x^2}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx}{31104}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} - \frac{\int \frac{\frac{111812\ldots}{16}\ldots}{(5+2x\ldots}}{2\ldots}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} + \frac{5}{16}\int \ldots$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} + \frac{223894\ldots}{\ldots}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} - \frac{5\sinh^{\ldots}}{16\ldots}$$

Mathematica [A] time = 0.15, size = 88, normalized size = 0.65

$$\frac{-\frac{24\sqrt{2x^2-x+3}\left(12653660x^2+44312764x+44369687\right)}{(2x+5)^3}+22389491\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)-22394880\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{143327232}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((-24*Sqrt[3 - x + 2*x^2]*(44369687 + 44312764*x + 12653660*x^2))/(5 + 2*x)
^3 - 22394880*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 22389491*Sqrt[2]*ArcTan
h[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/143327232
```

fricas [A] time = 0.85, size = 163, normalized size = 1.21

$$\frac{22394880\sqrt{2}\left(8x^3+60x^2+150x+125\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+22389491\sqrt{\ldots}}{2866544\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/286654464*(22394880*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)
*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 22389491*sqrt(2)*(8*
x^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17)
 - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(12653660*x^2 + 4431
2764*x + 44369687)*sqrt(2*x^2 - x + 3))/(8*x^3 + 60*x^2 + 150*x + 125)
```

giac [B] time = 0.26, size = 285, normalized size = 2.11

$$-\frac{5}{32}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{22389491}{143327232}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] -5/32*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 22389
491/143327232*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3
))) - 22389491/143327232*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt
(2*x^2 - x + 3))) - 1/11943936*sqrt(2)*(215012404*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3))^5 + 3010410772*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 27408
02468*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 21459328844*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^2 + 14434519361*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
+ 3)) - 5957650879)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sq
rt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

maple [A] time = 0.01, size = 109, normalized size = 0.81

$$\frac{5\sqrt{2}\ \mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{32}+\frac{22389491\sqrt{2}\ \mathrm{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{143327232}-\frac{3163415\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{11943936\left(x+\frac{5}{2}\right)}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/32*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-3163415/11943936/(x+5/2)*(-11*x
+2*(x+5/2)^2-19/2)^(1/2)+22389491/143327232*2^(1/2)*arctanh(1/12*(-11*x+17/
2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))+394907/995328/(x+5/2)^2*(-11*x+2
*(x+5/2)^2-19/2)^(1/2)-3667/13824/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(1/2)
```

maxima [A] time = 1.00, size = 131, normalized size = 0.97

$$\frac{5}{32}\sqrt{2}\ \mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)-\frac{22389491}{143327232}\sqrt{2}\ \mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|}-\frac{17\sqrt{23}}{23|2x+5|}\right)-\frac{3667\sqrt{2x^2}\ldots}{1728\left(8x^3+60x^2\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] 5/32*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 22389491/143327232*
sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)
```

```
) - 3667/1728*sqrt(2*x^2 - x + 3)/(8*x^3 + 60*x^2 + 150*x + 125) + 394907/2
48832*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x + 25) - 3163415/5971968*sqrt(2*x^2
- x + 3)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^4\,\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(1/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*sqrt(2*x**2 - x + 3
)), x)
```

3.350 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=139

$$\frac{26800085\sqrt{2x^2-x+3}}{1719926784(2x+5)}-\frac{16295969\sqrt{2x^2-x+3}}{71663616(2x+5)^2}+\frac{513097\sqrt{2x^2-x+3}}{497664(2x+5)^3}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}+\frac{2053207\tanh}{20639}$$

```
[Out] 2053207/41278242816*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/
2)-3667/2304*(2*x^2-x+3)^(1/2)/(5+2*x)^4+513097/497664*(2*x^2-x+3)^(1/2)/(5
+2*x)^3-16295969/71663616*(2*x^2-x+3)^(1/2)/(5+2*x)^2+26800085/1719926784*(
2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A] time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1650, 806, 724, 206}

$$\frac{26800085\sqrt{2x^2-x+3}}{1719926784(2x+5)}-\frac{16295969\sqrt{2x^2-x+3}}{71663616(2x+5)^2}+\frac{513097\sqrt{2x^2-x+3}}{497664(2x+5)^3}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}+\frac{2053207\tanh}{20639}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^5*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (-3667*Sqrt[3 - x + 2*x^2])/(2304*(5 + 2*x)^4) + (513097*Sqrt[3 - x + 2*x^2
])/(497664*(5 + 2*x)^3) - (16295969*Sqrt[3 - x + 2*x^2])/(71663616*(5 + 2*x
)^2) + (26800085*Sqrt[3 - x + 2*x^2])/(1719926784*(5 + 2*x)) + (2053207*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(20639121408*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
```

```
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} - \frac{1}{288}\int \frac{\frac{37027}{16}-\frac{10167x}{4}+1944x^2-720x^3}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} + \frac{\int \frac{\frac{2607829}{16}-\frac{295607x}{2}+77760x^2}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx}{62208}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2} - \frac{\int \frac{1941}{1}}{(5+2}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2} + \frac{26800}{1719}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2} + \frac{26800}{1719}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2} + \frac{26800}{1719}$$

Mathematica [A] time = 0.13, size = 81, normalized size = 0.58

$$\frac{2053207\sqrt{2}(2x+5)^4\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+24\sqrt{2x^2-x+3}\left(214400680x^3+43592076x^2-255525906x-29\right.}{41278242816(2x+5)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^5*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] (24*Sqrt[3 - x + 2*x^2]*(-298655447 - 255525906*x + 43592076*x^2 + 21440068
0*x^3) + 2053207*Sqrt[2]*(5 + 2*x)^4*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x +
 4*x^2])])/(41278242816*(5 + 2*x)^4)
```

fricas [A] time = 0.95, size = 125, normalized size = 0.90

$$\frac{2053207\sqrt{2}\left(16x^4+160x^3+600x^2+1000x+625\right)\log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)-1060x^2+1036x-1153}{4x^2+20x+25}\right)+48\left(214400\right.}{82556485632\left(16x^4+160x^3+600x^2+1000x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/82556485632*(2053207*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*
log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)
/(4*x^2 + 20*x + 25)) + 48*(214400680*x^3 + 43592076*x^2 - 255525906*x - 29
8655447)*sqrt(2*x^2 - x + 3))/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)
```

giac [A] time = 0.28, size = 164, normalized size = 1.18

$$\frac{1}{41278242816}\sqrt{2}\left(12\left(\frac{24\left(\frac{144\left(\frac{513097}{\mathrm{sgn}\left(\frac{1}{2x+5}\right)}-\frac{792072}{(2x+5)\mathrm{sgn}\left(\frac{1}{2x+5}\right)}\right)}{2x+5}-\frac{16295969}{\mathrm{sgn}\left(\frac{1}{2x+5}\right)}\right)}{2x+5}+\frac{26800085}{\mathrm{sgn}\left(\frac{1}{2x+5}\right)}\right)\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/41278242816*sqrt(2)*(12*(24*(144*(513097/sgn(1/(2*x + 5)) - 792072/((2*x
+ 5)*sgn(1/(2*x + 5))))/(2*x + 5) - 16295969/sgn(1/(2*x + 5)))/(2*x + 5) +
26800085/sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 20532
07*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 72/(2*x + 5) - 11)/sgn
(1/(2*x + 5)) - 321601020*sgn(1/(2*x + 5)))
```

maple [A] time = 0.01, size = 116, normalized size = 0.83

$$\frac{2053207\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{41278242816}+\frac{26800085\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{3439853568\left(x+\frac{5}{2}\right)}-\frac{16295969\sqrt{-11x+2\left(x+\right.}}{286654464\left(x+\frac{5}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 26800085/3439853568/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+2053207/41278242
816*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2
))-16295969/286654464/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-3667/36864/(
x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+513097/3981312/(x+5/2)^3*(-11*x+2*(
x+5/2)^2-19/2)^(1/2)
```

maxima [A] time = 1.02, size = 149, normalized size = 1.07

$$-\frac{2053207}{41278242816}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right)-\frac{3667\sqrt{2x^2-x+3}}{2304\left(16x^4+160x^3+600x^2+1000x+625\right)}+\frac{}{4976}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] -2053207/41278242816*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
sqrt(23)/abs(2*x + 5)) - 3667/2304*sqrt(2*x^2 - x + 3)/(16*x^4 + 160*x^3 +
600*x^2 + 1000*x + 625) + 513097/497664*sqrt(2*x^2 - x + 3)/(8*x^3 + 60*x^2
 + 150*x + 125) - 16295969/71663616*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x + 25)
 + 26800085/1719926784*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^5\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^5*(2*x^2 - x + 3)^(1/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^5*(2*x^2 - x + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^5 \sqrt{2x^2 - x + 3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**5/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**5*sqrt(2*x**2 - x + 3
)), x)
```

3.351 $\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=124

$$\frac{153}{16}\sqrt{2x^2-x+3}\,x^2+\frac{2645}{128}\sqrt{2x^2-x+3}\,x-\frac{13153}{512}\sqrt{2x^2-x+3}-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}\,x^3+\frac{144217}{}$$

```
[Out] 144217/2048*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-4/23*(346-533*x)/(2*x^2-
x+3)^(1/2)-13153/512*(2*x^2-x+3)^(1/2)+2645/128*x*(2*x^2-x+3)^(1/2)+153/16*
x^2*(2*x^2-x+3)^(1/2)+5/4*x^3*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{5}{4}\sqrt{2x^2-x+3}\,x^3+\frac{153}{16}\sqrt{2x^2-x+3}\,x^2+\frac{2645}{128}\sqrt{2x^2-x+3}\,x-\frac{13153}{512}\sqrt{2x^2-x+3}-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{144217}{}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-4*(346 - 533*x))/(23*Sqrt[3 - x + 2*x^2]) - (13153*Sqrt[3 - x + 2*x^2])/5
12 + (2645*x*Sqrt[3 - x + 2*x^2])/128 + (153*x^2*Sqrt[3 - x + 2*x^2])/16 +
(5*x^3*Sqrt[3 - x + 2*x^2])/4 + (144217*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*
Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeqQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{-759-\frac{575x}{2}+805x^2+\frac{1219x^3}{2}+115x^4}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}+\frac{5}{4}x^3\sqrt{3-x+2x^2}+\frac{1}{92}\int \frac{-6072-2300x+54\ldots}{\sqrt{3-x+2\ldots}}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}+\frac{153}{16}x^2\sqrt{3-x+2x^2}+\frac{5}{4}x^3\sqrt{3-x+2x^2}+\frac{1}{552}\ldots$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}+\frac{2645}{128}x\sqrt{3-x+2x^2}+\frac{153}{16}x^2\sqrt{3-x+2x^2}+\frac{5}{4}\ldots$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}-\frac{13153}{512}\sqrt{3-x+2x^2}+\frac{2645}{128}x\sqrt{3-x+2x^2}+\frac{15\ldots}{1\ldots}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}-\frac{13153}{512}\sqrt{3-x+2x^2}+\frac{2645}{128}x\sqrt{3-x+2x^2}+\frac{15\ldots}{1\ldots}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}-\frac{13153}{512}\sqrt{3-x+2x^2}+\frac{2645}{128}x\sqrt{3-x+2x^2}+\frac{15\ldots}{1\ldots}$$

Mathematica [A] time = 0.47, size = 74, normalized size = 0.60

$$\frac{3316991\sqrt{4x^2-2x+6}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)+4\left(29440x^5+210496x^4+418232x^3-510554x^2+2124123x-1616165\right)}{47104\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2)
,x]
```

```
[Out] (4*(-1616165 + 2124123*x - 510554*x^2 + 418232*x^3 + 210496*x^4 + 29440*x^5
) + 3316991*Sqrt[6 - 2*x + 4*x^2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/(47104*Sqrt[
3 - x + 2*x^2])
```

fricas [A] time = 0.99, size = 102, normalized size = 0.82

$$\frac{3316991\,\sqrt{2}\left(2\,x^2-x+3\right)\log\left(4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+8\left(29440\,x^5+210496\,x^4+41\ldots\right.}{94208\left(2\,x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/94208*(3316991*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*
(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(29440*x^5 + 210496*x^4 + 418232*x^3 -
510554*x^2 + 2124123*x - 1616165)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

giac [A] time = 0.22, size = 72, normalized size = 0.58

$$\frac{144217}{2048}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{(46(4(8(20x+143)x+2273)x-11099)x+2124123)x}{11776\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] 144217/2048*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) +
 1/11776*((46*(4*(8*(20*x + 143)*x + 2273)*x - 11099)*x + 2124123)*x - 1616
165)/sqrt(2*x^2 - x + 3)
```

maple [A] time = 0.02, size = 132, normalized size = 1.06

$$\frac{5x^5}{2\sqrt{2x^2-x+3}}+\frac{143x^4}{8\sqrt{2x^2-x+3}}+\frac{2273x^3}{64\sqrt{2x^2-x+3}}-\frac{11099x^2}{256\sqrt{2x^2-x+3}}+\frac{144217x}{1024\sqrt{2x^2-x+3}}-\frac{144217\sqrt{2}\ \text{arcsi}}{204}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 931255/94208*(4*x-1)/(2*x^2-x+3)^(1/2)-144217/2048*2^(1/2)*arcsinh(4/23*23^
(1/2)*(x-1/4))+5/2*x^5/(2*x^2-x+3)^(1/2)+143/8*x^4/(2*x^2-x+3)^(1/2)+2273/6
4*x^3/(2*x^2-x+3)^(1/2)-11099/256*x^2/(2*x^2-x+3)^(1/2)+144217/1024*x/(2*x^
2-x+3)^(1/2)-521655/4096/(2*x^2-x+3)^(1/2)
```

maxima [A] time = 0.97, size = 114, normalized size = 0.92

$$\frac{5x^5}{2\sqrt{2x^2-x+3}}+\frac{143x^4}{8\sqrt{2x^2-x+3}}+\frac{2273x^3}{64\sqrt{2x^2-x+3}}-\frac{11099x^2}{256\sqrt{2x^2-x+3}}-\frac{144217}{2048}\sqrt{2}\ \text{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 5/2*x^5/sqrt(2*x^2 - x + 3) + 143/8*x^4/sqrt(2*x^2 - x + 3) + 2273/64*x^3/s
qrt(2*x^2 - x + 3) - 11099/256*x^2/sqrt(2*x^2 - x + 3) - 144217/2048*sqrt(2
)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 2124123/11776*x/sqrt(2*x^2 - x + 3) -
1616165/11776/sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+5)^2\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)^2 \left(5x^4 - x^3 + 3x^2 + x + 2\right)}{\left(2x^2 - x + 3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)**2*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((2*x + 5)**2*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3
/2), x)
```

3.352 $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{5}{6}\sqrt{2x^2-x+3}\,x^2+\frac{193}{48}\sqrt{2x^2-x+3}\,x+\frac{33}{64}\sqrt{2x^2-x+3}+\frac{373x-53}{23\sqrt{2x^2-x+3}}+\frac{3111\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

```
[Out] 3111/256*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+1/23*(-53+373*x)/(2*x^2-x+3
)^(1/2)+33/64*(2*x^2-x+3)^(1/2)+193/48*x*(2*x^2-x+3)^(1/2)+5/6*x^2*(2*x^2-x
+3)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{5}{6}\sqrt{2x^2-x+3}\,x^2+\frac{193}{48}\sqrt{2x^2-x+3}\,x+\frac{33}{64}\sqrt{2x^2-x+3}-\frac{53-373x}{23\sqrt{2x^2-x+3}}+\frac{3111\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] -(53 - 373*x)/(23*Sqrt[3 - x + 2*x^2]) + (33*Sqrt[3 - x + 2*x^2])/64 + (193
*x*Sqrt[3 - x + 2*x^2])/48 + (5*x^2*Sqrt[3 - x + 2*x^2])/6 + (3111*ArcSinh[
(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = -\frac{53-373x}{23\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{-\frac{575}{4}+161x^2+\frac{115x^3}{2}}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}}+\frac{5}{6}x^2\sqrt{3-x+2x^2}+\frac{1}{69}\int \frac{-\frac{1725}{2}-345x+\frac{4439x^2}{4}}{\sqrt{3-x+2x^2}}$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}}+\frac{193}{48}x\sqrt{3-x+2x^2}+\frac{5}{6}x^2\sqrt{3-x+2x^2}+\frac{1}{276}\int$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}}+\frac{33}{64}\sqrt{3-x+2x^2}+\frac{193}{48}x\sqrt{3-x+2x^2}+\frac{5}{6}x^2\sqrt{3}$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}}+\frac{33}{64}\sqrt{3-x+2x^2}+\frac{193}{48}x\sqrt{3-x+2x^2}+\frac{5}{6}x^2\sqrt{3}$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}}+\frac{33}{64}\sqrt{3-x+2x^2}+\frac{193}{48}x\sqrt{3-x+2x^2}+\frac{5}{6}x^2\sqrt{3}$$

Mathematica [A] time = 0.19, size = 60, normalized size = 0.58

$$\frac{7360x^4+31832x^3-2162x^2+122607x-3345}{4416\sqrt{2x^2-x+3}}-\frac{3111\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x
]
```

```
[Out] (-3345 + 122607*x - 2162*x^2 + 31832*x^3 + 7360*x^4)/(4416*Sqrt[3 - x + 2*x
^2]) - (3111*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(128*Sqrt[2])
```

fricas [A] time = 0.86, size = 97, normalized size = 0.94

$$\frac{214659\sqrt{2}\left(2x^2-x+3\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)+8\left(7360x^4+31832x^3-2162x\right.}{35328\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/35328*(214659*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(
4*x - 1) - 32*x^2 + 16*x - 25) + 8*(7360*x^4 + 31832*x^3 - 2162*x^2 + 12260
7*x - 3345)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

giac [A] time = 0.22, size = 67, normalized size = 0.65

$$\frac{3111}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{(46(4(40x+173)x-47)x+122607)x-3345}{4416\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="gia
c")
```

```
[Out] 3111/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/
4416*((46*(4*(40*x + 173)*x - 47)*x + 122607)*x - 3345)/sqrt(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 115, normalized size = 1.12

$$\frac{5x^4}{3\sqrt{2x^2-x+3}}+\frac{173x^3}{24\sqrt{2x^2-x+3}}-\frac{47x^2}{96\sqrt{2x^2-x+3}}+\frac{3111x}{128\sqrt{2x^2-x+3}}-\frac{3111\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{256}+\frac{\frac{10185}{2944}\ldots}{\sqrt{2x^2\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 10185/11776*(4*x-1)/(2*x^2-x+3)^(1/2)-3111/256*2^(1/2)*arcsinh(4/23*23^(1/2
)*(x-1/4))+5/3/(2*x^2-x+3)^(1/2)*x^4+173/24/(2*x^2-x+3)^(1/2)*x^3-47/96/(2*
x^2-x+3)^(1/2)*x^2+3111/128/(2*x^2-x+3)^(1/2)*x+55/512/(2*x^2-x+3)^(1/2)
```

maxima [A] time = 0.97, size = 97, normalized size = 0.94

$$\frac{5x^4}{3\sqrt{2x^2-x+3}}+\frac{173x^3}{24\sqrt{2x^2-x+3}}-\frac{47x^2}{96\sqrt{2x^2-x+3}}-\frac{3111}{256}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{40869x}{1472\sqrt{2x^2-x+\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="max
ima")
```

```
[Out] 5/3*x^4/sqrt(2*x^2 - x + 3) + 173/24*x^3/sqrt(2*x^2 - x + 3) - 47/96*x^2/sq
rt(2*x^2 - x + 3) - 3111/256*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 408
69/1472*x/sqrt(2*x^2 - x + 3) - 1115/1472/sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3/2)
, x)
```

3.353 $\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{5}{8}\sqrt{2x^2-x+3}\,x+\frac{27}{32}\sqrt{2x^2-x+3}+\frac{219x+89}{92\sqrt{2x^2-x+3}}+\frac{213\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

```
[Out] 213/128*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+1/92*(89+219*x)/(2*x^2-x+3)^
(1/2)+27/32*(2*x^2-x+3)^(1/2)+5/8*x*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{5}{8}\sqrt{2x^2-x+3}\,x+\frac{27}{32}\sqrt{2x^2-x+3}+\frac{219x+89}{92\sqrt{2x^2-x+3}}+\frac{213\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (89 + 219*x)/(92*Sqrt[3 - x + 2*x^2]) + (27*Sqrt[3 - x + 2*x^2])/32 + (5*x*
Sqrt[3 - x + 2*x^2])/8 + (213*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
```

```
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{89+219x}{92\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{-\frac{345}{16}+\frac{69x}{8}+\frac{115x^2}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{89+219x}{92\sqrt{3-x+2x^2}} + \frac{5}{8}x\sqrt{3-x+2x^2} + \frac{1}{46}\int \frac{-\frac{345}{2}+\frac{621x}{8}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{89+219x}{92\sqrt{3-x+2x^2}} + \frac{27}{32}\sqrt{3-x+2x^2} + \frac{5}{8}x\sqrt{3-x+2x^2} - \frac{213}{64}\int \frac{1}{\sqrt{3-x+2x}}$$

$$= \frac{89+219x}{92\sqrt{3-x+2x^2}} + \frac{27}{32}\sqrt{3-x+2x^2} + \frac{5}{8}x\sqrt{3-x+2x^2} - \frac{213\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}}\right.}{64\sqrt{4}}$$

$$= \frac{89+219x}{92\sqrt{3-x+2x^2}} + \frac{27}{32}\sqrt{3-x+2x^2} + \frac{5}{8}x\sqrt{3-x+2x^2} + \frac{213\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Mathematica [A] time = 0.13, size = 55, normalized size = 0.67

$$\frac{920x^3+782x^2+2511x+2575}{736\sqrt{2x^2-x+3}} - \frac{213\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (2575 + 2511*x + 782*x^2 + 920*x^3)/(736*Sqrt[3 - x + 2*x^2]) - (213*ArcSin
h[(-1 + 4*x)/Sqrt[23]])/(64*Sqrt[2])
```

fricas [A] time = 0.86, size = 92, normalized size = 1.12

$$\frac{4899\sqrt{2}\left(2x^2-x+3\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)+8\left(920x^3+782x^2+2511x+\right.}{5888\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5888*(4899*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
 - 1) - 32*x^2 + 16*x - 25) + 8*(920*x^3 + 782*x^2 + 2511*x + 2575)*sqrt(2*
x^2 - x + 3))/(2*x^2 - x + 3)
```

giac [A] time = 0.22, size = 62, normalized size = 0.76

$$\frac{213}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) + \frac{(46(20x+17)x+2511)x+2575}{736\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] 213/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/7
36*((46*(20*x + 17)*x + 2511)*x + 2575)/sqrt(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 98, normalized size = 1.20

$$\frac{5x^3}{4\sqrt{2x^2-x+3}}+\frac{17x^2}{16\sqrt{2x^2-x+3}}+\frac{213x}{64\sqrt{2x^2-x+3}}-\frac{213\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128}+\frac{901}{256\sqrt{2x^2-x+3}}+\frac{\frac{123x}{1472}-}{\sqrt{2x^2-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 5/4/(2*x^2-x+3)^(1/2)*x^3+17/16/(2*x^2-x+3)^(1/2)*x^2+213/64/(2*x^2-x+3)^(1
/2)*x+901/256/(2*x^2-x+3)^(1/2)+123/5888*(4*x-1)/(2*x^2-x+3)^(1/2)-213/128*
2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 0.95, size = 80, normalized size = 0.98

$$\frac{5\,x^3}{4\,\sqrt{2\,x^2-x+3}}+\frac{17\,x^2}{16\,\sqrt{2\,x^2-x+3}}-\frac{213}{128}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{2511\,x}{736\,\sqrt{2\,x^2-x+3}}+\frac{2575}{736\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/4*x^3/sqrt(2*x^2 - x + 3) + 17/16*x^2/sqrt(2*x^2 - x + 3) - 213/128*sqrt(
2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 2511/736*x/sqrt(2*x^2 - x + 3) + 2575
/736/sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x^2-x+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{\left(2x^2 - x + 3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3/2), x)
```

3.354 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{917x+1191}{3312\sqrt{2x^2-x+3}}+\frac{5}{8}\sqrt{2x^2-x+3}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1728\sqrt{2}}+\frac{39\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}$$

```
[Out] 39/32*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-3667/3456*arctanh(1/24*(17-22*
x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/3312*(1191+917*x)/(2*x^2-x+3)^(1/2)
+5/8*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1646, 1653, 843, 619, 215, 724, 206}

$$\frac{917x+1191}{3312\sqrt{2x^2-x+3}}+\frac{5}{8}\sqrt{2x^2-x+3}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1728\sqrt{2}}+\frac{39\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (1191 + 917*x)/(3312*Sqrt[3 - x + 2*x^2]) + (5*Sqrt[3 - x + 2*x^2])/8 + (39
*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16*Sqrt[2]) - (3667*ArcTanh[(17 - 22*x)/(12*
Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1728*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{-\frac{6739}{576}+\frac{69x}{8}+\frac{115x^2}{4}}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} + \frac{1}{92}\int \frac{\frac{3611}{72}-\frac{897x}{2}}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} - \frac{39}{16}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx + \frac{3667}{288}\int \frac{}{(5+}$$

$$= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} - \frac{3667}{144}\,\mathrm{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-}{\sqrt{3-x}}\right.$$

$$= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} + \frac{39\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}} - \frac{3667\tanh^{-1}\left(\frac{1}{12\sqrt{2}}\right.}{1728\sqrt{2}}$$

Mathematica [A] time = 0.35, size = 86, normalized size = 0.85

$$\frac{\frac{12\left(4140x^2-1153x+7401\right)}{23\sqrt{x^2-\frac{x}{2}+\frac{3}{2}}} - 3667\log\left(12\sqrt{4x^2-2x+6}-22x+17\right) + 3667\log(2x+5) - 4212\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{1728\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(3/2)),x
]
```

```
[Out] ((12*(7401 - 1153*x + 4140*x^2))/(23*Sqrt[3/2 - x/2 + x^2]) - 4212*ArcSinh[
(-1 + 4*x)/Sqrt[23]] + 3667*Log[5 + 2*x] - 3667*Log[17 - 22*x + 12*Sqrt[6 -
 2*x + 4*x^2]])/(1728*Sqrt[2])
```

fricas [A] time = 1.01, size = 149, normalized size = 1.48

$$\frac{96876\sqrt{2}\left(2x^2-x+3\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)+84341\sqrt{2}\left(2x^2-x+3\right)\log\left(\right.}{158976\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/158976*(96876*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(
4*x - 1) - 32*x^2 + 16*x - 25) + 84341*sqrt(2)*(2*x^2 - x + 3)*log(-(24*sqr
t(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 2
0*x + 25)) + 48*(4140*x^2 - 1153*x + 7401)*sqrt(2*x^2 - x + 3))/(2*x^2 - x
+ 3)
```

giac [A] time = 0.40, size = 118, normalized size = 1.17

$$\frac{39}{32}\sqrt{2}\log\left(-4\sqrt{2}x+\sqrt{2}+4\sqrt{2x^2-x+3}\right)-\frac{3667}{3456}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{3667}{3456}\sqrt{2}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="gia
c")
```

```
[Out] 39/32*sqrt(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 3667/34
56*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 3667/
3456*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) +
1/3312*((4140*x - 1153)*x + 7401)/sqrt(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 148, normalized size = 1.47

$$\frac{5x^2}{4\sqrt{2x^2-x+3}}+\frac{39x}{16\sqrt{2x^2-x+3}}-\frac{39\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{32}-\frac{3667\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{3456}-\frac{}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 5/4/(2*x^2-x+3)^(1/2)*x^2+39/16/(2*x^2-x+3)^(1/2)*x-309/64/(2*x^2-x+3)^(1/2
)-5507/1472*(4*x-1)/(2*x^2-x+3)^(1/2)-39/32*2^(1/2)*arcsinh(4/23*23^(1/2)*(
x-1/4))+3667/576/(-11*x+2*(x+5/2)^2-19/2)^(1/2)+40337/13248*(4*x-1)/(-11*x+
2*(x+5/2)^2-19/2)^(1/2)-3667/3456*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)
/(-11*x+2*(x+5/2)^2-19/2)^(1/2))
```

maxima [A] time = 0.98, size = 99, normalized size = 0.98

$$\frac{5x^2}{4\sqrt{2x^2-x+3}}-\frac{39}{32}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)+\frac{3667}{3456}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|}-\frac{17\sqrt{23}}{23|2x+5|}\right)-\frac{}{3312}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="max
ima")
```

```
[Out] 5/4*x^2/sqrt(2*x^2 - x + 3) - 39/32*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*
sqrt(23)) + 3667/3456*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23
*sqrt(23)/abs(2*x + 5)) - 1153/3312*x/sqrt(2*x^2 - x + 3) + 2467/1104/sqrt(
2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)\left(2x^2 - x + 3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(3/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)\left(2x^2 - x + 3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*(2*x**2 - x + 3)**(3/2
)), x)
```

3.355 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{10368(2x+5)}+\frac{25951\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{41472\sqrt{2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

```
[Out] -5/16*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+25951/82944*arctanh(1/24*(17-2
2*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/119232*(9897+2203*x)/(2*x^2-x+3)^
(1/2)-3667/10368*(2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A] time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1646, 1650, 843, 619, 215, 724, 206}

$$\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{10368(2x+5)}+\frac{25951\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{41472\sqrt{2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (9897 + 2203*x)/(119232*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(
10368*(5 + 2*x)) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(8*Sqrt[2]) + (25951*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(41472*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{9897+2203x}{119232\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{-\frac{33649}{20736}+\frac{131215x}{10368}+\frac{115x^2}{4}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$$

$$=\frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}-\frac{1}{828}\int \frac{\frac{100073}{192}-1035x}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$=\frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}+\frac{5}{8}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx-\frac{25951}{}$$

$$=\frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}+\frac{25951\,\text{Subst}\left(\int \frac{1}{288-x^2}\,dx,x,\frac{17}{\sqrt{3-}}\right.}{3456}$$

$$=\frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}+\frac{25951\tanh^{-1}}{414}$$

Mathematica [A] time = 0.32, size = 104, normalized size = 0.96

$$\frac{\frac{8(2203x+9897)}{23\sqrt{x^2-\frac{x}{2}+\frac{3}{2}}}-\frac{14668\sqrt{4x^2-2x+6}}{2x+5}+25951\log\left(12\sqrt{4x^2-2x+6}-22x+17\right)-25951\log(2x+5)+25920\sinh^{-1}\left(\right.}{41472\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(3/2))
,x]
```

```
[Out] ((8*(9897 + 2203*x))/(23*Sqrt[3/2 - x/2 + x^2]) - (14668*Sqrt[6 - 2*x + 4*x
^2])/(5 + 2*x) + 25920*ArcSinh[(-1 + 4*x)/Sqrt[23]] - 25951*Log[5 + 2*x] +
25951*Log[17 - 22*x + 12*Sqrt[6 - 2*x + 4*x^2]])/(41472*Sqrt[2])
```

fricas [A] time = 0.96, size = 157, normalized size = 1.45

$$\frac{596160 \sqrt{2} \left(4 x^{3} + 8 x^{2} + x + 15\right) \log \left(-4 \sqrt{2} \sqrt{2 x^{2} - x + 3} (4 x - 1) - 32 x^{2} + 16 x - 25\right) + 596873 \sqrt{2} \left(4 x^{3} + \cdots\right.}{3815424 \left(4 x^{3} \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/3815424*(596160*sqrt(2)*(4*x^3 + 8*x^2 + x + 15)*log(-4*sqrt(2)*sqrt(2*x^
2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 596873*sqrt(2)*(4*x^3 + 8*x^2
+ x + 15)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036
*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(53290*x^2 - 48653*x + 51351)*sqrt(2*x
^2 - x + 3))/(4*x^3 + 8*x^2 + x + 15)
```

giac [B] time = 0.42, size = 225, normalized size = 2.08

$$\frac{1}{1907712} \sqrt{2} \left(\frac{12 \left(\frac{\frac{315103}{\mathrm{sgn}\left(\frac{1}{2x+5}\right)} - \frac{1012092}{(2x+5)\mathrm{sgn}\left(\frac{1}{2x+5}\right)}}{2x+5} - \frac{26645}{\mathrm{sgn}\left(\frac{1}{2x+5}\right)} \right)}{\sqrt{-\frac{11}{2x+5} + \frac{36}{(2x+5)^2} + 1}} + \frac{596873 \log \left(12 \sqrt{-\frac{11}{2x+5} + \frac{36}{(2x+5)^2} + 1} + \frac{72}{2x+5} - 11\right)}{\mathrm{sgn}\left(\frac{1}{2x+5}\right)} \cdots \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] 1/1907712*sqrt(2)*(12*((315103/sgn(1/(2*x + 5)) - 1012092/((2*x + 5)*sgn(1/
(2*x + 5))))/(2*x + 5) - 26645/sgn(1/(2*x + 5)))/sqrt(-11/(2*x + 5) + 36/(2
*x + 5)^2 + 1) + 596873*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 7
2/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 596160*log(abs(sqrt(-11/(2*x + 5) + 36
/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))/sgn(1/(2*x + 5)) - 596160*log(abs(sqr
t(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))/sgn(1/(2*x + 5)))
```

maple [A] time = 0.01, size = 152, normalized size = 1.41

$$-\frac{5x}{8\sqrt{2x^2 - x + 3}} + \frac{5\sqrt{2}\ \mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x - \frac{1}{4}\right)}{23}\right)}{16} + \frac{25951\sqrt{2}\ \mathrm{arctanh}\left(\frac{\left(-11x + \frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x + 2\left(x + \frac{5}{2}\right)^2 - \frac{19}{2}}}\right)}{82944} + \frac{99}{32\sqrt{2x^2 - x + 3}} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x)
```

```
[Out] -5/8/(2*x^2-x+3)^(1/2)*x+99/32/(2*x^2-x+3)^(1/2)+1529/736*(4*x-1)/(2*x^2-x+
3)^(1/2)+5/16*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-3667/1152/(x+5/2)/(-11
*x+2*(x+5/2)^2-19/2)^(1/2)-25951/13824/(-11*x+2*(x+5/2)^2-19/2)^(1/2)-63749
3/317952*(4*x-1)/(-11*x+2*(x+5/2)^2-19/2)^(1/2)+25951/82944*2^(1/2)*arctanh
(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))
```

maxima [A] time = 1.01, size = 116, normalized size = 1.07

$$\frac{5}{16} \sqrt{2}\ \mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}\,x - \frac{1}{23}\sqrt{23}\right) - \frac{25951}{82944}\sqrt{2}\ \mathrm{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2x+5|} - \frac{17\sqrt{23}}{23\,|2x+5|}\right) - \frac{26645\,x}{79488\sqrt{2x^2 - x + 3}} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 5/16*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 25951/82944*sqrt(2)
*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 266
45/79488*x/sqrt(2*x^2 - x + 3) + 30313/26496/sqrt(2*x^2 - x + 3) - 3667/576
/(2*sqrt(2*x^2 - x + 3)*x + 5*sqrt(2*x^2 - x + 3))
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^2\,{\left(2\,x^2-x+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(3/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(3/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^{4} - x^{3} + 3x^{2} + x + 2}{\left(2x + 5\right)^{2} \left(2x^{2} - x + 3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*(2*x**2 - x + 3)**(
3/2)), x)
```

3.356 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=112

$$\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}+\frac{115369\sqrt{2x^2-x+3}}{1492992(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{20736(2x+5)^2}-\frac{52631\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{5971968\sqrt{2}}$$

```
[Out] -52631/11943936*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1
/4292352*(65991-8779*x)/(2*x^2-x+3)^(1/2)-3667/20736*(2*x^2-x+3)^(1/2)/(5+2
*x)^2+115369/1492992*(2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A] time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 1650, 806, 724, 206}

$$\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}+\frac{115369\sqrt{2x^2-x+3}}{1492992(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{20736(2x+5)^2}-\frac{52631\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{5971968\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (65991 - 8779*x)/(4292352*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])
/(20736*(5 + 2*x)^2) + (115369*Sqrt[3 - x + 2*x^2])/(1492992*(5 + 2*x)) - (
52631*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(5971968*Sqrt[
2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
```

```
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{\frac{5168261}{746496}+\frac{3637795x}{186624}+\frac{5620625x^2}{186624}}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} - \frac{\int \frac{\frac{842237}{1296}-\frac{4102487x}{2592}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx}{1656}$$

$$= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} + \frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)} + \frac{52631}{\cdots}$$

$$= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} + \frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)} - \frac{52631}{\cdots}$$

$$= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} + \frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)} - \frac{52631}{\cdots}$$

Mathematica [A] time = 0.30, size = 84, normalized size = 0.75

$$\frac{-52631\log\left(12\sqrt{4x^2-2x+6}-22x+17\right)+\frac{12\left(3444340x^3+3263288x^2+5842933x+11594283\right)}{23(2x+5)^2\sqrt{x^2-\frac{x}{2}+\frac{3}{2}}}+52631\log(2x+5)}{5971968\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(3/2))
,x]
```

```
[Out] ((12*(11594283 + 5842933*x + 3263288*x^2 + 3444340*x^3))/(23*(5 + 2*x)^2*Sq
rt[3/2 - x/2 + x^2]) + 52631*Log[5 + 2*x] - 52631*Log[17 - 22*x + 12*Sqrt[6
 - 2*x + 4*x^2]])/(5971968*Sqrt[2])
```

fricas [A] time = 0.92, size = 126, normalized size = 1.12

$$\frac{1210513\sqrt{2}\left(8x^4+36x^3+42x^2+35x+75\right)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}\,(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right)+48\left(3444340x^3\cdots\right.}{549421056\left(8x^4+36x^3+42x^2+35x+75\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/549421056*(1210513*sqrt(2)*(8*x^4 + 36*x^3 + 42*x^2 + 35*x + 75)*log(-(24
*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2
 + 20*x + 25)) + 48*(3444340*x^3 + 3263288*x^2 + 5842933*x + 11594283)*sqrt
(2*x^2 - x + 3))/(8*x^4 + 36*x^3 + 42*x^2 + 35*x + 75)
```

giac [B] time = 0.25, size = 220, normalized size = 1.96

$$-\frac{52631}{11943936}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{52631}{11943936}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] -52631/11943936*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x +
 3))) + 52631/11943936*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2
*x^2 - x + 3))) - 1/4292352*(8779*x - 65991)/sqrt(2*x^2 - x + 3) + 1/298598
4*sqrt(2)*(3594214*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 19874490*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 30140067*sqrt(2)*(sqrt(2)*x - sqrt(2*x
^2 - x + 3)) + 19989859)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2
```

maple [A] time = 0.01, size = 144, normalized size = 1.29

$$-\frac{52631\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{11943936}-\frac{5}{16\sqrt{2x^2-x+3}}-\frac{149\,(4x-1)}{368\sqrt{2x^2-x+3}}+\frac{196043}{165888\left(x+\frac{5}{2}\right)\sqrt{-11x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x)
```

```
[Out] -5/16/(2*x^2-x+3)^(1/2)-149/368*(4*x-1)/(2*x^2-x+3)^(1/2)+196043/165888/(x+
5/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2)+52631/1990656/(-11*x+2*(x+5/2)^2-19/2)^
(1/2)+19399069/45785088*(4*x-1)/(-11*x+2*(x+5/2)^2-19/2)^(1/2)-52631/119439
36*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2)
)-3667/4608/(x+5/2)^2/(-11*x+2*(x+5/2)^2-19/2)^(1/2)
```

maxima [A] time = 0.98, size = 149, normalized size = 1.33

$$\frac{52631}{11943936}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|}-\frac{17\sqrt{23}}{23|2x+5|}\right)+\frac{861085x}{11446272\sqrt{2x^2-x+3}}-\frac{1163201}{3815424\sqrt{2x^2-x+3}}-\frac{}{1152\left(4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 52631/11943936*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(2
3)/abs(2*x + 5)) + 861085/11446272*x/sqrt(2*x^2 - x + 3) - 1163201/3815424/
sqrt(2*x^2 - x + 3) - 3667/1152/(4*sqrt(2*x^2 - x + 3)*x^2 + 20*sqrt(2*x^2
```

```
- x + 3)*x + 25*sqrt(2*x^2 - x + 3)) + 196043/82944/(2*sqrt(2*x^2 - x + 3)*
x + 5*sqrt(2*x^2 - x + 3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^3\,{\left(2\,x^2-x+3\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(3/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*(2*x**2 - x + 3)**(
3/2)), x)
```

3.357 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}+\frac{430799\sqrt{2x^2-x+3}}{107495424(2x+5)}+\frac{152885\sqrt{2x^2-x+3}}{4478976(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{31104(2x+5)^3}-\frac{3505819\tanh^{-1}}{128994}$$

```
[Out] -3505819/2579890176*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/
2)+1/154524672*(369609-175877*x)/(2*x^2-x+3)^(1/2)-3667/31104*(2*x^2-x+3)^(
1/2)/(5+2*x)^3+152885/4478976*(2*x^2-x+3)^(1/2)/(5+2*x)^2+430799/107495424*
(2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A] time = 0.20, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 1650, 806, 724, 206}

$$\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}+\frac{430799\sqrt{2x^2-x+3}}{107495424(2x+5)}+\frac{152885\sqrt{2x^2-x+3}}{4478976(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{31104(2x+5)^3}-\frac{3505819\tanh^{-1}}{128994}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (369609 - 175877*x)/(154524672*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*
x^2])/(31104*(5 + 2*x)^3) + (152885*Sqrt[3 - x + 2*x^2])/(4478976*(5 + 2*x)
^2) + (430799*Sqrt[3 - x + 2*x^2])/(107495424*(5 + 2*x)) - (3505819*ArcTanh
[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1289945088*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
```

```
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{\frac{348877271}{26873856}+\frac{119871055x}{4478976}+\frac{73960295x^2}{2239488}+\frac{1302559x}{3359232}}{(5+2x)^4\sqrt{3-x+2x^2}}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}-\frac{\int \frac{\frac{79609325}{124416}-\frac{71248733x}{31104}-\frac{1302559x^2}{31104}}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx}{2484}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}+\frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2}+\int$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}+\frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2}+\frac{430}{10}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}+\frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2}+\frac{430}{10}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}+\frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2}+\frac{430}{10}$$

Mathematica [A] time = 0.16, size = 95, normalized size = 0.69

$$\frac{24\left(56754760x^4+572739684x^3+441046842x^2+1257975811x+1873786587\right)-80633837(2x+5)^3\sqrt{4x^2-2x}}{59337474048(2x+5)^3\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(3/2))
,x]
```

```
[Out] (24*(1873786587 + 1257975811*x + 441046842*x^2 + 572739684*x^3 + 56754760*x
^4) - 80633837*(5 + 2*x)^3*Sqrt[6 - 2*x + 4*x^2]*ArcTanh[(17 - 22*x)/(12*Sq
rt[6 - 2*x + 4*x^2])])/(59337474048*(5 + 2*x)^3*Sqrt[3 - x + 2*x^2])
```

fricas [A] time = 0.88, size = 141, normalized size = 1.03

$$\frac{80633837\sqrt{2}\left(16x^5+112x^4+264x^3+280x^2+325x+375\right)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right)}{118674948096\left(16x^5+112x^4+264\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/118674948096*(80633837*sqrt(2)*(16*x^5 + 112*x^4 + 264*x^3 + 280*x^2 + 32
5*x + 375)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 10
36*x + 1153)/(4*x^2 + 20*x + 25)) + 48*(56754760*x^4 + 572739684*x^3 + 4410
46842*x^2 + 1257975811*x + 1873786587)*sqrt(2*x^2 - x + 3))/(16*x^5 + 112*x
^4 + 264*x^3 + 280*x^2 + 325*x + 375)
```

giac [B] time = 0.29, size = 271, normalized size = 1.98

$$-\frac{3505819}{2579890176}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{3505819}{2579890176}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] -3505819/2579890176*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) + 3505819/2579890176*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) +
2*sqrt(2*x^2 - x + 3))) - 1/154524672*(175877*x - 369609)/sqrt(2*x^2 - x +
3) - 1/214990848*sqrt(2)*(10398764*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^5 - 303070900*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 - 529738052*sqrt(2)*(sq
rt(2)*x - sqrt(2*x^2 - x + 3))^3 + 3644644652*(sqrt(2)*x - sqrt(2*x^2 - x +
 3))^2 - 2612608649*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1052284471)
/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^
2 - x + 3)) - 11)^3
```

maple [A] time = 0.01, size = 151, normalized size = 1.10

$$-\frac{3505819\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{2579890176}+\frac{\frac{5x}{46}-\frac{5}{184}}{\sqrt{2x^2-x+3}}-\frac{3127169}{35831808\left(x+\frac{5}{2}\right)\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 5/184*(4*x-1)/(2*x^2-x+3)^(1/2)-3127169/35831808/(x+5/2)/(-11*x+2*(x+5/2)^2
-19/2)^(1/2)+3505819/429981696/(-11*x+2*(x+5/2)^2-19/2)^(1/2)-261644215/988
9579008*(4*x-1)/(-11*x+2*(x+5/2)^2-19/2)^(1/2)-3505819/2579890176*2^(1/2)*a
rctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))+314233/995
328/(x+5/2)^2/(-11*x+2*(x+5/2)^2-19/2)^(1/2)-3667/13824/(x+5/2)^3/(-11*x+2*
(x+5/2)^2-19/2)^(1/2)
```

maxima [A] time = 1.01, size = 217, normalized size = 1.58

$$\frac{3505819}{2579890176}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|}-\frac{17\sqrt{23}}{23|2x+5|}\right)+\frac{7094345x}{2472394752\sqrt{2x^2-x+3}}+\frac{6128291}{824131584\sqrt{2x^2-x+3}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 3505819/2579890176*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sq
rt(23)/abs(2*x + 5)) + 7094345/2472394752*x/sqrt(2*x^2 - x + 3) + 6128291/8
24131584/sqrt(2*x^2 - x + 3) - 3667/1728/(8*sqrt(2*x^2 - x + 3)*x^3 + 60*sq
rt(2*x^2 - x + 3)*x^2 + 150*sqrt(2*x^2 - x + 3)*x + 125*sqrt(2*x^2 - x + 3)
) + 314233/248832/(4*sqrt(2*x^2 - x + 3)*x^2 + 20*sqrt(2*x^2 - x + 3)*x + 2
5*sqrt(2*x^2 - x + 3)) - 3127169/17915904/(2*sqrt(2*x^2 - x + 3)*x + 5*sqrt
(2*x^2 - x + 3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^4\,{\left(2\,x^2-x+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(3/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^{4} - x^{3} + 3x^{2} + x + 2}{\left(2x + 5\right)^{4} \left(2x^{2} - x + 3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*(2*x**2 - x + 3)**(
3/2)), x)
```

3.358 $\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=105

$$\frac{4(18982-20383x)}{1587\sqrt{2x^2-x+3}}+\frac{5}{4}x\sqrt{2x^2-x+3}+\frac{247}{16}\sqrt{2x^2-x+3}-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}-\frac{1471\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

```
[Out] -4/69*(346-533*x)/(2*x^2-x+3)^(3/2)-1471/64*arcsinh(1/23*(1-4*x)*23^(1/2))*
2^(1/2)+4/1587*(18982-20383*x)/(2*x^2-x+3)^(1/2)+247/16*(2*x^2-x+3)^(1/2)+5
/4*x*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{4(18982-20383x)}{1587\sqrt{2x^2-x+3}}+\frac{5}{4}x\sqrt{2x^2-x+3}+\frac{247}{16}\sqrt{2x^2-x+3}-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}-\frac{1471\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-4*(346 - 533*x))/(69*(3 - x + 2*x^2)^(3/2)) + (4*(18982 - 20383*x))/(1587
*Sqrt[3 - x + 2*x^2]) + (247*Sqrt[3 - x + 2*x^2])/16 + (5*x*Sqrt[3 - x + 2*
x^2])/4 - (1471*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-145-\frac{1725x}{2}+2415x^2+\frac{3657x^3}{2}+345x}{\left(3-x+2x^2\right)^{3/2}}$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{4\int \frac{\frac{33327}{2}+\frac{46023x}{4}+\frac{7935x^2}{4}}{\sqrt{3-x+2x^2}}\,dx}{1587}$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{5}{4}x\sqrt{3-x+2x^2}+\int$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{247}{16}\sqrt{3-x+2x^2}+\frac{5}{4}x$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{247}{16}\sqrt{3-x+2x^2}+\frac{5}{4}x$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{247}{16}\sqrt{3-x+2x^2}+\frac{5}{4}x$$

Mathematica [A] time = 0.69, size = 65, normalized size = 0.62

$$\frac{126960x^5+1440996x^4-3764360x^3+8639625x^2-6410082x+6663133}{25392\left(2x^2-x+3\right)^{3/2}}-\frac{1471\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2)
,x]
```

```
[Out] (6663133 - 6410082*x + 8639625*x^2 - 3764360*x^3 + 1440996*x^4 + 126960*x^5
)/(25392*(3 - x + 2*x^2)^(3/2)) - (1471*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sq
rt[2])
```

fricas [A] time = 0.86, size = 122, normalized size = 1.16

$$\frac{2334477\sqrt{2}\left(4x^4-4x^3+13x^2-6x+9\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+8\left(126960x\right.}{203136\left(4x^4-4x^3+13x^2-6x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/203136*(2334477*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)
*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(126960*x^5 + 1440
996*x^4 - 3764360*x^3 + 8639625*x^2 - 6410082*x + 6663133)*sqrt(2*x^2 - x +
 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.22, size = 71, normalized size = 0.68

$$-\frac{1471}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{((4(1587(20x+227)x-941090)x+8639625)x-6410}{25392\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] -1471/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/
25392*(((4*(1587*(20*x + 227)*x - 941090)*x + 8639625)*x - 6410082)*x + 666
3133)/(2*x^2 - x + 3)^(3/2)
```

maple [B] time = 0.02, size = 180, normalized size = 1.71

$$\frac{5x^5}{\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{227x^4}{4\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{1471x^3}{48\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{19073x^2}{64\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{32257x}{512\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{1471x}{32\sqrt{2x^2-x+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -162931/50784*(4*x-1)/(2*x^2-x+3)^(1/2)-753223/141312*(4*x-1)/(2*x^2-x+3)^(
3/2)+5*x^5/(2*x^2-x+3)^(3/2)+227/4*x^4/(2*x^2-x+3)^(3/2)-1471/48*x^3/(2*x^2
-x+3)^(3/2)+19073/64*x^2/(2*x^2-x+3)^(3/2)-32257/512*x/(2*x^2-x+3)^(3/2)+14
71/64*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-1471/32/(2*x^2-x+3)^(1/2)*x+57
7397/2048/(2*x^2-x+3)^(3/2)-1471/128/(2*x^2-x+3)^(1/2)
```

maxima [B] time = 0.98, size = 219, normalized size = 2.09

$$\frac{5x^5}{\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{227x^4}{4\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{1471}{50784}x\left(\frac{284x}{\sqrt{2x^2-x+3}}-\frac{3174x^2}{\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2x^2-x+3}}+\frac{805x}{\left(2x^2-x+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] 5*x^5/(2*x^2 - x + 3)^(3/2) + 227/4*x^4/(2*x^2 - x + 3)^(3/2) + 1471/50784*
x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/sqrt(2*x
^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/2)) + 1
471/64*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 104441/25392*sqrt(2*x^2 -
 x + 3) - 383581/12696*x/sqrt(2*x^2 - x + 3) + 321*x^2/(2*x^2 - x + 3)^(3/2
) - 15965/4232/sqrt(2*x^2 - x + 3) - 4147/46*x/(2*x^2 - x + 3)^(3/2) + 4288
3/138/(2*x^2 - x + 3)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+5)^2\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)^2 \left(5x^4 - x^3 + 3x^2 + x + 2\right)}{\left(2x^2 - x + 3\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)**2*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((2*x + 5)**2*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5
/2), x)
```

3.359 $\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=86

$$\frac{6055-28981x}{3174\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}+\frac{373x-53}{69\left(2x^2-x+3\right)^{3/2}}-\frac{71\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

```
[Out] 1/69*(-53+373*x)/(2*x^2-x+3)^(3/2)-71/16*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(
1/2)+1/3174*(6055-28981*x)/(2*x^2-x+3)^(1/2)+5/4*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1660, 640, 619, 215}

$$\frac{6055-28981x}{3174\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}-\frac{71\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] -(53 - 373*x)/(69*(3 - x + 2*x^2)^(3/2)) + (6055 - 28981*x)/(3174*Sqrt[3 -
x + 2*x^2]) + (5*Sqrt[3 - x + 2*x^2])/4 - (71*ArcSinh[(1 - 4*x)/Sqrt[23]])/
(8*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{233}{4}+483x^2+\frac{345x^3}{2}}{\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{4\int \frac{\frac{52371}{16}+\frac{7935x}{8}}{\sqrt{3-x+2x^2}}\,dx}{1587}$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{5}{4}\sqrt{3-x+2x^2}+\frac{71}{8}\int$$

$$71\,\mathrm{Sub}$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{5}{4}\sqrt{3-x+2x^2}+$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{5}{4}\sqrt{3-x+2x^2}-\frac{71\,\mathrm{sinh}}{8}$$

Mathematica [A] time = 0.25, size = 60, normalized size = 0.70

$$\frac{31740x^4-147664x^3+185337x^2-199290x+102869}{6348\left(2x^2-x+3\right)^{3/2}}+\frac{71\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x
]
```

```
[Out] (102869 - 199290*x + 185337*x^2 - 147664*x^3 + 31740*x^4)/(6348*(3 - x + 2*
x^2)^(3/2)) + (71*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(8*Sqrt[2])
```

fricas [A] time = 0.72, size = 117, normalized size = 1.36

$$\frac{112677\sqrt{2}\left(4x^4-4x^3+13x^2-6x+9\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)+8\left(31740x^4-\right.}{50784\left(4x^4-4x^3+13x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/50784*(112677*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(31740*x^4 - 147664*
x^3 + 185337*x^2 - 199290*x + 102869)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 +
 13*x^2 - 6*x + 9)
```

giac [A] time = 0.21, size = 66, normalized size = 0.77

$$-\frac{71}{16}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{((4(7935x-36916)x+185337)x-199290)x+102869}{6348\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="gia
c")
```

```
[Out] -71/16*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/63
48*(((4*(7935*x - 36916)*x + 185337)*x - 199290)*x + 102869)/(2*x^2 - x + 3
)^(3/2)
```

maple [B] time = 0.01, size = 163, normalized size = 1.90

$$\frac{5x^4}{\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{71x^3}{12\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{401x^2}{16\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{945x}{128\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{71x}{8\sqrt{2x^2-x+3}}+\frac{71\sqrt{2}\,\operatorname{arcsinh}\left(\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] 643/12696*(4*x-1)/(2*x^2-x+3)^(1/2)-2327/35328*(4*x-1)/(2*x^2-x+3)^(3/2)+5/
(2*x^2-x+3)^(3/2)*x^4-71/12/(2*x^2-x+3)^(3/2)*x^3+401/16/(2*x^2-x+3)^(3/2)*
x^2-945/128/(2*x^2-x+3)^(3/2)*x+71/16*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4)
)-71/8/(2*x^2-x+3)^(1/2)*x+11749/512/(2*x^2-x+3)^(3/2)-71/32/(2*x^2-x+3)^(1
/2)
```

maxima [B] time = 0.97, size = 202, normalized size = 2.35

$$\frac{5\,x^4}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}+\frac{71}{12696}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\frac{3174\,x^2}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^2-x+3}}+\frac{805\,x}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{3243}{\left(2\,x^2-x+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="max
ima")
```

```
[Out] 5*x^4/(2*x^2 - x + 3)^(3/2) + 71/12696*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*
x^2/(2*x^2 - x + 3)^(3/2) - 71/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^
(3/2) - 3243/(2*x^2 - x + 3)^(3/2)) + 71/16*sqrt(2)*arcsinh(1/23*sqrt(23)*(
4*x - 1)) - 5041/6348*sqrt(2*x^2 - x + 3) - 10007/3174*x/sqrt(2*x^2 - x + 3
) + 59/2*x^2/(2*x^2 - x + 3)^(3/2) - 2959/2116/sqrt(2*x^2 - x + 3) - 807/92
*x/(2*x^2 - x + 3)^(3/2) + 7603/276/(2*x^2 - x + 3)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+5)\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{\left(2\,x^2-x+3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5/2)
, x)
```

3.360 $\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=68

$$\frac{219x+89}{276\left(2x^2-x+3\right)^{3/2}}-\frac{2604x+1465}{2116\sqrt{2x^2-x+3}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

```
[Out] 1/276*(89+219*x)/(2*x^2-x+3)^(3/2)-5/8*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/
2)+1/2116*(-1465-2604*x)/(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1660, 12, 619, 215}

$$\frac{219x+89}{276\left(2x^2-x+3\right)^{3/2}}-\frac{2604x+1465}{2116\sqrt{2x^2-x+3}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (89 + 219*x)/(276*(3 - x + 2*x^2)^(3/2)) - (1465 + 2604*x)/(2116*Sqrt[3 - x
 + 2*x^2]) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{159}{16}+\frac{207x}{8}+\frac{345x^2}{4}}{\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}+\frac{4\int \frac{7935}{16\sqrt{3-x+2x^2}}\,dx}{1587}$$

$$= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}+\frac{5}{4}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}+\frac{5\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{4\sqrt{46}}$$

$$= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

Mathematica [A] time = 0.23, size = 55, normalized size = 0.81

$$\frac{5\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{7812x^3+489x^2+7002x+5569}{3174\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] -1/3174*(5569 + 7002*x + 489*x^2 + 7812*x^3)/(3 - x + 2*x^2)^(3/2) + (5*Arc
Sinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

fricas [B] time = 0.89, size = 112, normalized size = 1.65

$$\frac{7935\,\sqrt{2}\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)-8\left(7812\,x^3+489\right.}{25392\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/25392*(7935*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*sqr
t(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) - 8*(7812*x^3 + 489*x^2 +
7002*x + 5569)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.30, size = 62, normalized size = 0.91

$$-\frac{5}{8}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)-\frac{3\,((2604\,x+163)x+2334)x+5569}{3174\left(2\,x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 1/3174
*(3*((2604*x + 163)*x + 2334)*x + 5569)/(2*x^2 - x + 3)^(3/2)
```

maple [B] time = 0.01, size = 146, normalized size = 2.15

$$-\frac{5x^3}{6\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{x^2}{8\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{47x}{64\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{5x}{4\sqrt{2x^2-x+3}}+\frac{5\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8}-\frac{}{768\left(2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -5/6/(2*x^2-x+3)^(3/2)*x^3-1/8/(2*x^2-x+3)^(3/2)*x^2-47/64/(2*x^2-x+3)^(3/2
)*x-271/768/(2*x^2-x+3)^(3/2)+2423/17664*(4*x-1)/(2*x^2-x+3)^(3/2)+173/1587
*(4*x-1)/(2*x^2-x+3)^(1/2)-5/4/(2*x^2-x+3)^(1/2)*x-5/16/(2*x^2-x+3)^(1/2)+5
/8*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [B] time = 0.98, size = 185, normalized size = 2.72

$$\frac{5}{6348}x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\frac{3174\,x^2}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^2-x+3}}+\frac{805\,x}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{3243}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\right)+\frac{5}{8}\sqrt{2}\,\text{arsinl}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/6348*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/s
qrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/
2)) + 5/8*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 355/3174*sqrt(2*x^2 -
x + 3) - 58/1587*x/sqrt(2*x^2 - x + 3) + 1/2*x^2/(2*x^2 - x + 3)^(3/2) - 18
97/6348/sqrt(2*x^2 - x + 3) - 95/276*x/(2*x^2 - x + 3)^(3/2) + 41/276/(2*x^
2 - x + 3)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{\left(2\,x^2-x+3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5/2), x)
```

3.361 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}-\frac{146729x+335337}{1371168\sqrt{2x^2-x+3}}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{31104\sqrt{2}}$$

```
[Out] 1/9936*(1191+917*x)/(2*x^2-x+3)^(3/2)-3667/62208*arctanh(1/24*(17-22*x)*2^(
1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/1371168*(-335337-146729*x)/(2*x^2-x+3)^(1
/2)
```

Rubi [A] time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1646, 12, 724, 206}

$$\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}-\frac{146729x+335337}{1371168\sqrt{2x^2-x+3}}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{31104\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(5/2)),x]
```

```
[Out] (1191 + 917*x)/(9936*(3 - x + 2*x^2)^(3/2)) - (335337 + 146729*x)/(1371168*
Sqrt[3 - x + 2*x^2]) - (3667*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2
*x^2])])/(31104*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{1877}{576}+\frac{695x}{18}+\frac{345x^2}{4}}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}+\frac{4\int \frac{1939843}{6912(5+2x)\sqrt{3-x+2x^2}}\,dx}{1587}$$

$$= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}+\frac{3667\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{5184}$$

$$= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}-\frac{3667\,\text{Subst}\left(\int \frac{1}{288-x^2}\,dx,x$$

$$2592$$

$$= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x}}\right.$$

$$31104\sqrt{2}$$

Mathematica [A] time = 0.47, size = 80, normalized size = 0.94

$$\frac{-3667\log\left(12\sqrt{4x^2-2x+6}-22x+17\right)-\frac{12\sqrt{2}\left(293458x^3+523945x^2-21696x+841653\right)}{529\left(2x^2-x+3\right)^{3/2}}+3667\log(2x+5)}{31104\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(5/2)),x
]
```

```
[Out] ((-12*Sqrt[2]*(841653 - 21696*x + 523945*x^2 + 293458*x^3))/(529*(3 - x + 2
*x^2)^(3/2)) + 3667*Log[5 + 2*x] - 3667*Log[17 - 22*x + 12*Sqrt[6 - 2*x + 4
*x^2]])/(31104*Sqrt[2])
```

fricas [A] time = 0.84, size = 126, normalized size = 1.48

$$\frac{1939843\sqrt{2}\left(4x^4-4x^3+13x^2-6x+9\right)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right)-48\left(293458x^3+5\right.}{65816064\left(4x^4-4x^3+13x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/65816064*(1939843*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-(24*sqr
t(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 2
0*x + 25)) - 48*(293458*x^3 + 523945*x^2 - 21696*x + 841653)*sqrt(2*x^2 - x
 + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.23, size = 92, normalized size = 1.08

$$-\frac{3667}{62208}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{3667}{62208}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="gia
c")
```

```
[Out] -3667/62208*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))
) + 3667/62208*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x
 + 3))) - 1/1371168*(((293458*x + 523945)*x - 21696)*x + 841653)/(2*x^2 - x
 + 3)^(3/2)
```

maple [B] time = 0.01, size = 190, normalized size = 2.24

$$-\frac{5x^2}{4\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{59x}{32\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{3667\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{62208}-\frac{1597}{384\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{3817\,(4}{2944\left(2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -5/4/(2*x^2-x+3)^(3/2)*x^2+59/32/(2*x^2-x+3)^(3/2)*x-1597/384/(2*x^2-x+3)^(
3/2)-3817/2944*(4*x-1)/(2*x^2-x+3)^(3/2)-3817/4232*(4*x-1)/(2*x^2-x+3)^(1/2
)+3667/1728/(-11*x+2*(x+5/2)^2-19/2)^(3/2)+40337/39744*(4*x-1)/(-11*x+2*(x+
5/2)^2-19/2)^(3/2)+4800103/5484672*(4*x-1)/(-11*x+2*(x+5/2)^2-19/2)^(1/2)+3
667/10368/(-11*x+2*(x+5/2)^2-19/2)^(1/2)-3667/62208*2^(1/2)*arctanh(1/12*(-
11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))
```

maxima [A] time = 0.97, size = 110, normalized size = 1.29

$$\frac{3667}{62208}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{146729\,x}{1371168\sqrt{2\,x^2-x+3}}-\frac{5\,x^2}{4\left(2\,x^2-x+3\right)^{\frac{3}{2}}}+\frac{173881}{457056\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="max
ima")
```

```
[Out] 3667/62208*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/a
bs(2*x + 5)) - 146729/1371168*x/sqrt(2*x^2 - x + 3) - 5/4*x^2/(2*x^2 - x +
3)^(3/2) + 173881/457056/sqrt(2*x^2 - x + 3) + 7127/9936*x/(2*x^2 - x + 3)^
(3/2) - 5813/3312/(2*x^2 - x + 3)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)\,\left(2\,x^2-x+3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(5/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*(2*x**2 - x + 3)**(5/2
)), x)
```

3.362 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{1255878-62021x}{24681024\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{186624(2x+5)}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}-\frac{2821\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{2239488\sqrt{2}}$$

```
[Out] 1/357696*(9897+2203*x)/(2*x^2-x+3)^(3/2)-2821/4478976*arctanh(1/24*(17-22*x
)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/24681024*(-1255878+62021*x)/(2*x^2-x
+3)^(1/2)-3667/186624*(2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A] time = 0.15, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1646, 806, 724, 206}

$$-\frac{1255878-62021x}{24681024\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{186624(2x+5)}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}-\frac{2821\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{2239488\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(5/2)),x]
```

```
[Out] (9897 + 2203*x)/(357696*(3 - x + 2*x^2)^(3/2)) - (1255878 - 62021*x)/(24681
024*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(186624*(5 + 2*x)) -
(2821*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(2239488*Sqrt[
2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
```

```
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{119353}{20736}+\frac{481765x}{10368}+\frac{113983x^2}{1296}}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}-\frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}}+\frac{4\int \frac{\frac{10109719}{124416}-\frac{4961491x}{62208}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx}{1587}$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}-\frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)}+$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}-\frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)}-$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}-\frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)}-$$

Mathematica [A] time = 0.41, size = 92, normalized size = 0.84

$$\frac{-2821\log\left(12\sqrt{4x^2-2x+6}-22x+17\right)-\frac{12\sqrt{2}\left(6767036x^4+10350004x^3+63941915x^2-18840090x+79153407\right)}{529(2x+5)\left(2x^2-x+3\right)^{3/2}}+2821\log(2}{2239488\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))
,x]
```

```
[Out] ((-12*Sqrt[2]*(79153407 - 18840090*x + 63941915*x^2 + 10350004*x^3 + 676703
6*x^4))/(529*(5 + 2*x)*(3 - x + 2*x^2)^(3/2)) + 2821*Log[5 + 2*x] - 2821*Lo
g[17 - 22*x + 12*Sqrt[6 - 2*x + 4*x^2]])/(2239488*Sqrt[2])
```

fricas [A] time = 0.85, size = 141, normalized size = 1.28

$$\frac{1492309\sqrt{2}\left(8x^5+12x^4+6x^3+53x^2-12x+45\right)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right)-48\left(676\right.}{4738756608\left(8x^5+12x^4+6x^3+53x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/4738756608*(1492309*sqrt(2)*(8*x^5 + 12*x^4 + 6*x^3 + 53*x^2 - 12*x + 45)
*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 115
3)/(4*x^2 + 20*x + 25)) - 48*(6767036*x^4 + 10350004*x^3 + 63941915*x^2 - 1
```

```
8840090*x + 79153407)*sqrt(2*x^2 - x + 3))/(8*x^5 + 12*x^4 + 6*x^3 + 53*x^2
 - 12*x + 45)
```

giac [B] time = 0.40, size = 206, normalized size = 1.87

$$-\frac{1}{2369378304}\sqrt{2}\left(\frac{1492309\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}+\frac{12\left(\frac{\frac{48\left(\frac{23642785}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}-\frac{52375761}{(2x+5)\operatorname{sgn}\left(\frac{1}{2x+5}\right)}\right)}{2x+5}-\frac{2400807\ldots}{\operatorname{sgn}\left(\frac{1}{2x+\ldots}\right)}}{2x+5}}{2x+5}\right.}{\left(\frac{11}{2x+5}-\frac{36}{(2x+5)^2}-1\right)\sqrt{-\frac{11}{2x+\ldots}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] -1/2369378304*sqrt(2)*(1492309*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
 1) + 72/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 12*(((48*(23642785/sgn(1/(2*x +
 5)) - 52375761/((2*x + 5)*sgn(1/(2*x + 5))))/(2*x + 5) - 240080735/sgn(1/(
2*x + 5)))/(2*x + 5) + 28660178/sgn(1/(2*x + 5)))/(2*x + 5) - 1691759/sgn(1
/(2*x + 5)))/((11/(2*x + 5) - 36/(2*x + 5)^2 - 1)*sqrt(-11/(2*x + 5) + 36/(
2*x + 5)^2 + 1)) - 20301108*sgn(1/(2*x + 5)))
```

maple [B] time = 0.01, size = 194, normalized size = 1.76

$$-\frac{5x}{16\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{2821\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{4478976}+\frac{203}{192\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{\frac{3173x}{1104}-\frac{3173}{4416}}{\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{\frac{3173x}{1587}-\frac{317\ldots}{634\ldots}}{\sqrt{2x^2-x+\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -5/16/(2*x^2-x+3)^(3/2)*x+203/192/(2*x^2-x+3)^(3/2)+3173/4416*(4*x-1)/(2*x^
2-x+3)^(3/2)+3173/6348*(4*x-1)/(2*x^2-x+3)^(1/2)-3667/1152/(x+5/2)/(-11*x+2
*(x+5/2)^2-19/2)^(3/2)+2821/124416/(-11*x+2*(x+5/2)^2-19/2)^(3/2)-2081161/2
861568*(4*x-1)/(-11*x+2*(x+5/2)^2-19/2)^(3/2)-199077743/394896384*(4*x-1)/(
-11*x+2*(x+5/2)^2-19/2)^(1/2)+2821/746496/(-11*x+2*(x+5/2)^2-19/2)^(1/2)-28
21/4478976*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/
2)^(1/2))
```

maxima [A] time = 1.01, size = 127, normalized size = 1.15

$$\frac{2821}{4478976}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right)-\frac{1691759\,x}{98724096\sqrt{2x^2-x+3}}+\frac{265339}{32908032\sqrt{2x^2-x+3}}-\frac{24\ldots}{715392\,(2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] 2821/4478976*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)
/abs(2*x + 5)) - 1691759/98724096*x/sqrt(2*x^2 - x + 3) + 265339/32908032/s
qrt(2*x^2 - x + 3) - 248617/715392*x/(2*x^2 - x + 3)^(3/2) - 3667/576/(2*(2
*x^2 - x + 3)^(3/2)*x + 5*(2*x^2 - x + 3)^(3/2)) + 259621/238464/(2*x^2 - x
 + 3)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^2\,{\left(2\,x^2-x+3\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(5/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^{4} - x^{3} + 3x^{2} + x + 2}{\left(2x + 5\right)^{2} \left(2x^{2} - x + 3\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*(2*x**2 - x + 3)**(
5/2)), x)
```

3.363 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{4679797-2148263x}{592344576\sqrt{2x^2-x+3}}-\frac{45979\sqrt{2x^2-x+3}}{26873856(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{373248(2x+5)^2}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}+\frac{774079\tanh}{3224}$$

```
[Out] 1/12877056*(65991-8779*x)/(2*x^2-x+3)^(3/2)+774079/644972544*arctanh(1/24*(
17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/592344576*(-4679797+2148263*x
)/(2*x^2-x+3)^(1/2)-3667/373248*(2*x^2-x+3)^(1/2)/(5+2*x)^2-45979/26873856*
(2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A] time = 0.22, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 1650, 806, 724, 206}

$$-\frac{4679797-2148263x}{592344576\sqrt{2x^2-x+3}}-\frac{45979\sqrt{2x^2-x+3}}{26873856(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{373248(2x+5)^2}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}+\frac{774079\tanh}{3224}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(5/2)),x]
```

```
[Out] (65991 - 8779*x)/(12877056*(3 - x + 2*x^2)^(3/2)) - (4679797 - 2148263*x)/(
592344576*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(373248*(5 + 2*
x)^2) - (45979*Sqrt[3 - x + 2*x^2])/(26873856*(5 + 2*x)) + (774079*ArcTanh[
(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(322486272*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
```

```
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{11115283}{746496}+\frac{3198845x}{62208}+\frac{605005x^2}{6912}-\frac{8779x^3}{23328}}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,d$$

$$= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}}-\frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}}+\frac{4\int \frac{-\frac{171639869}{2985984}-\frac{14239}{746}}{(5+2x)^3\sqrt{}}}{158}$$

$$= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}}-\frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2}$$

$$= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}}-\frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2}$$

$$= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}}-\frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2}$$

$$= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}}-\frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2}$$

Mathematica [A] time = 0.31, size = 97, normalized size = 0.72

$$\frac{774079\log\left(12\sqrt{4x^2-2x+6}-22x+17\right)+\frac{12\sqrt{2}\left(217883368x^5+107028732x^4-1503926130x^3-5919924791x^2+2280511668x-89\right)}{529(2x+5)^2\left(2x^2-x+3\right)^{3/2}}}{322486272\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(5/2))
,x]
```

```
[Out] ((12*Sqrt[2]*(-8953831359 + 2280511668*x - 5919924791*x^2 - 1503926130*x^3
+ 107028732*x^4 + 217883368*x^5))/(529*(5 + 2*x)^2*(3 - x + 2*x^2)^(3/2)) -
```

```
 774079*Log[5 + 2*x] + 774079*Log[17 - 22*x + 12*Sqrt[6 - 2*x + 4*x^2]])/(3
22486272*Sqrt[2])
```

fricas [A] time = 0.89, size = 155, normalized size = 1.15

$$\frac{409487791\sqrt{2}\left(16x^6+64x^5+72x^4+136x^3+241x^2+30x+225\right)\log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}\,(22x-17)-1060x^2+1036x-115}{4x^2+20x+25}\right.}{682380951552\left(16x^6+64x^5+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/682380951552*(409487791*sqrt(2)*(16*x^6 + 64*x^5 + 72*x^4 + 136*x^3 + 241
*x^2 + 30*x + 225)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x
^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(217883368*x^5 + 107028732*x^
4 - 1503926130*x^3 - 5919924791*x^2 + 2280511668*x - 8953831359)*sqrt(2*x^2
 - x + 3))/(16*x^6 + 64*x^5 + 72*x^4 + 136*x^3 + 241*x^2 + 30*x + 225)
```

giac [B] time = 0.27, size = 228, normalized size = 1.69

$$\frac{774079}{644972544}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)-\frac{774079}{644972544}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] 774079/644972544*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x
+ 3))) - 774079/644972544*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqr
t(2*x^2 - x + 3))) + 1/53747712*sqrt(2)*(44558*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^3 - 10136238*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 16812201*s
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 10182217)/(2*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2 + 1
/592344576*(((4296526*x - 11507857)*x + 10720752)*x - 11003805)/(2*x^2 - x
+ 3)^(3/2)
```

maple [A] time = 0.01, size = 200, normalized size = 1.48

$$\frac{774079\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{644972544}-\frac{5}{48\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{149\,(4x-1)}{1104\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{149\,(4x-1)}{1587\sqrt{2x^2-x+3}}+\frac{\cdots}{16588\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -5/48/(2*x^2-x+3)^(3/2)-149/1104*(4*x-1)/(2*x^2-x+3)^(3/2)-149/1587*(4*x-1)
/(2*x^2-x+3)^(1/2)+115369/165888/(x+5/2)/(-11*x+2*(x+5/2)^2-19/2)^(3/2)-774
079/17915904/(-11*x+2*(x+5/2)^2-19/2)^(3/2)+57937675/412065792*(4*x-1)/(-11
*x+2*(x+5/2)^2-19/2)^(3/2)+5366174813/56865079296*(4*x-1)/(-11*x+2*(x+5/2)^
2-19/2)^(1/2)-774079/107495424/(-11*x+2*(x+5/2)^2-19/2)^(1/2)+774079/644972
544*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2
))-3667/4608/(x+5/2)^2/(-11*x+2*(x+5/2)^2-19/2)^(3/2)
```

maxima [A] time = 1.00, size = 178, normalized size = 1.32

$$-\frac{774079}{644972544}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{27235421\,x}{14216269824\sqrt{2\,x^2-x+3}}-\frac{36393601}{4738756608\sqrt{2\,x^2-x+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] -774079/644972544*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqr
t(23)/abs(2*x + 5)) + 27235421/14216269824*x/sqrt(2*x^2 - x + 3) - 36393601
/4738756608/sqrt(2*x^2 - x + 3) + 2323723/103016448*x/(2*x^2 - x + 3)^(3/2)
 - 3667/1152/(4*(2*x^2 - x + 3)^(3/2)*x^2 + 20*(2*x^2 - x + 3)^(3/2)*x + 25
*(2*x^2 - x + 3)^(3/2)) + 115369/82944/(2*(2*x^2 - x + 3)^(3/2)*x + 5*(2*x^
2 - x + 3)^(3/2)) - 5254255/34338816/(2*x^2 - x + 3)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)^3\,\left(2\,x^2-x+3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(5/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*(2*x**2 - x + 3)**(
5/2)), x)
```

3.364 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=160

$$-\frac{27754539-31190998x}{31986607104\sqrt{2x^2-x+3}}+\frac{475357\sqrt{2x^2-x+3}}{1934917632(2x+5)}-\frac{89137\sqrt{2x^2-x+3}}{80621568(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{559872(2x+5)^3}+\frac{369609-1\ldots}{463574016\left(2x^2\ldots\right.}$$

```
[Out] 1/463574016*(369609-175877*x)/(2*x^2-x+3)^(3/2)+4778789/15479341056*arctanh
(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/31986607104*(-27754539
+31190998*x)/(2*x^2-x+3)^(1/2)-3667/559872*(2*x^2-x+3)^(1/2)/(5+2*x)^3-8913
7/80621568*(2*x^2-x+3)^(1/2)/(5+2*x)^2+475357/1934917632*(2*x^2-x+3)^(1/2)/
(5+2*x)
```

Rubi [A] time = 0.28, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 1650, 806, 724, 206}

$$-\frac{27754539-31190998x}{31986607104\sqrt{2x^2-x+3}}+\frac{475357\sqrt{2x^2-x+3}}{1934917632(2x+5)}-\frac{89137\sqrt{2x^2-x+3}}{80621568(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{559872(2x+5)^3}+\frac{369609-1\ldots}{463574016\left(2x^2\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(5/2)),x]
```

```
[Out] (369609 - 175877*x)/(463574016*(3 - x + 2*x^2)^(3/2)) - (27754539 - 3119099
8*x)/(31986607104*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(559872
*(5 + 2*x)^3) - (89137*Sqrt[3 - x + 2*x^2])/(80621568*(5 + 2*x)^2) + (47535
7*Sqrt[3 - x + 2*x^2])/(1934917632*(5 + 2*x)) + (4778789*ArcTanh[(17 - 22*x
)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(7739670528*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
```

```
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{606939313}{26873856}+\frac{727085495x}{13436928}+\frac{186705485x^2}{2239488}-1}{(5+2x)^4\left(3-x+2x^2\right.}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}}-\frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}}+\frac{4\int \frac{-\frac{4811736919}{40310784}}{}}{}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}}-\frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+}}{559872(5+2}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}}-\frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+}}{559872(5+2}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}}-\frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+}}{559872(5+2}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}}-\frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+}}{559872(5+2}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}}-\frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+}}{559872(5+2}$$

Mathematica [A] time = 0.24, size = 89, normalized size = 0.56

$$\frac{2527979381\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+\frac{24\left(6664404208x^6+34872810880x^5+46210466520x^4+27484986184x^3-6702882569x^2+7362\right.}{(2x+5)^3\left(2x^2-x+3\right)^{3/2}}}{8188571418624}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(5/2))
,x]
```

```
[Out] ((24*(-95241881529 + 73621973154*x - 6702882569*x^2 + 27484986184*x^3 + 462
10466520*x^4 + 34872810880*x^5 + 6664404208*x^6))/((5 + 2*x)^3*(3 - x + 2*x
^2)^(3/2)) + 2527979381*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^
2])])/8188571418624
```

fricas [A] time = 0.92, size = 170, normalized size = 1.06

$$\frac{2527979381 \sqrt{2} \left(32 x^{7} + 208 x^{6} + 464 x^{5} + 632 x^{4} + 1162 x^{3} + 1265 x^{2} + 600 x + 1125\right) \log\left(\frac{24 \sqrt{2} \sqrt{2 x^{2}-x+3} (22 x-17 \cdots}{4 x^{2}+20 \cdots}\right.}{16377142837248 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/16377142837248*(2527979381*sqrt(2)*(32*x^7 + 208*x^6 + 464*x^5 + 632*x^4
+ 1162*x^3 + 1265*x^2 + 600*x + 1125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(
22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(666440420
8*x^6 + 34872810880*x^5 + 46210466520*x^4 + 27484986184*x^3 - 6702882569*x^
2 + 73621973154*x - 95241881529)*sqrt(2*x^2 - x + 3))/(32*x^7 + 208*x^6 + 4
64*x^5 + 632*x^4 + 1162*x^3 + 1265*x^2 + 600*x + 1125)
```

giac [B] time = 0.28, size = 279, normalized size = 1.74

$$\frac{4778789}{15479341056} \sqrt{2} \log\left(\left|-2 \sqrt{2} x + \sqrt{2} + 2 \sqrt{2 x^{2} - x + 3}\right|\right) - \frac{4778789}{15479341056} \sqrt{2} \log\left(\left|-2 \sqrt{2} x - 11 \sqrt{2} + 2 \sqrt{2 x^{2} - \cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] 4778789/15479341056*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) - 4778789/15479341056*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) +
 2*sqrt(2*x^2 - x + 3))) + 1/7996651776*(((15595499*x - 21675019)*x + 27298
005)*x - 14440149)/(2*x^2 - x + 3)^(3/2) + 1/3869835264*sqrt(2)*(38030012*s
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 734231900*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^4 + 122834956*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 2
154595396*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 1659431083*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3)) - 760577429)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

maple [A] time = 0.01, size = 207, normalized size = 1.29

$$\frac{4778789 \sqrt{2}\, \operatorname{arctanh}\left(\frac{\left(-11 x + \frac{17}{2}\right) \sqrt{2}}{12 \sqrt{-11 x + 2 \left(x + \frac{5}{2}\right)^{2} - \frac{19}{2}}}\right)}{15479341056} - \frac{4778789}{429981696 \left(-11 x + 2 \left(x + \frac{5}{2}\right)^{2} - \frac{19}{2}\right)^{\frac{3}{2}}} - \frac{4778789}{2579890176 \sqrt{-11 x + 2 \left(x \cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -4778789/429981696/(-11*x+2*(x+5/2)^2-19/2)^(3/2)-4778789/2579890176/(-11*x
+2*(x+5/2)^2-19/2)^(1/2)-3667/13824/(x+5/2)^3/(-11*x+2*(x+5/2)^2-19/2)^(3/2
```

```
)+25951/110592/(x+5/2)^2/(-11*x+2*(x+5/2)^2-19/2)^(3/2)-34861/3981312/(x+5/
2)/(-11*x+2*(x+5/2)^2-19/2)^(3/2)-72646615/9889579008*(4*x-1)/(-11*x+2*(x+5
/2)^2-19/2)^(3/2)+4778789/15479341056*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(
1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))+10/1587*(4*x-1)/(2*x^2-x+3)^(1/2)+5/55
2*(4*x-1)/(2*x^2-x+3)^(3/2)-8183108657/1364761903104*(4*x-1)/(-11*x+2*(x+5/
2)^2-19/2)^(1/2)
```

maxima [A] time = 1.01, size = 246, normalized size = 1.54

$$-\frac{4778789}{15479341056}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{416525263\,x}{341190475776\,\sqrt{2\,x^2-x+3}}-\frac{245375387}{113730158592\,\sqrt{2\,x^2-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] -4778789/15479341056*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
sqrt(23)/abs(2*x + 5)) + 416525263/341190475776*x/sqrt(2*x^2 - x + 3) - 245
375387/113730158592/sqrt(2*x^2 - x + 3) + 16932905/2472394752*x/(2*x^2 - x
+ 3)^(3/2) - 3667/1728/(8*(2*x^2 - x + 3)^(3/2)*x^3 + 60*(2*x^2 - x + 3)^(3
/2)*x^2 + 150*(2*x^2 - x + 3)^(3/2)*x + 125*(2*x^2 - x + 3)^(3/2)) + 25951/
27648/(4*(2*x^2 - x + 3)^(3/2)*x^2 + 20*(2*x^2 - x + 3)^(3/2)*x + 25*(2*x^2
 - x + 3)^(3/2)) - 34861/1990656/(2*(2*x^2 - x + 3)^(3/2)*x + 5*(2*x^2 - x
+ 3)^(3/2)) - 10570421/824131584/(2*x^2 - x + 3)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^4\,{\left(2\,x^2-x+3\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(5/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^{4} - x^{3} + 3x^{2} + x + 2}{\left(2x + 5\right)^{4} \left(2x^{2} - x + 3\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*(2*x**2 - x + 3)**(
5/2)), x)
```

3.365 $\int \frac{f+gx+hx^2+ix^3+jx^4}{\left(a+bx+cx^2\right)^{5/2}} dx$

Optimal. Leaf size=354

$$\frac{2\left(-x\left(c^2\left(2a^2j+3abi+b^2h\right)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j+ach+c^2f\right)-ab^3j+ab^2ci\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] 2/3*(a*b^2*c*i+2*a*c^2*(-a*i+c*g)-a*b^3*j-b*c*(-3*a^2*j+a*c*h+c^2*f)-(2*c^4
*f-c^3*(2*a*h+b*g)+b^4*j-b^2*c*(4*a*j+b*i)+c^2*(2*a^2*j+3*a*b*i+b^2*h))*x)/
c^3/(-4*a*c+b^2)/(c*x^2+b*x+a)^(3/2)+j*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2
+b*x+a)^(1/2))/c^(5/2)-2/3*(b^4*c*i+24*a^2*c^3*i+2*b^2*c^2*(-3*a*i+2*c*g)-b
^5*j-b^3*c*(-10*a*j+c*h)-4*b*c^2*(8*a^2*j+a*c*h+2*c^2*f)-c*(16*c^4*f-c^3*(-
8*a*h+8*b*g)-4*b^4*j+b^2*c*(28*a*j+b*i)+2*c^2*(-16*a^2*j-6*a*b*i+b^2*h))*x)
/c^3/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1660, 12, 621, 206}

$$-\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)+b^2c(28aj+bi)-c^3(8bg-8ah)-4b^4j+16c^4f\right)-4bc^2\left(8a^2j+ach+2c^2f\right)\right.}{3c^3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(a*b^2*c*i + 2*a*c^2*(c*g - a*i) - a*b^3*j - b*c*(c^2*f + a*c*h - 3*a^2*
j) - (2*c^4*f - c^3*(b*g + 2*a*h) + b^4*j - b^2*c*(b*i + 4*a*j) + c^2*(b^2*
h + 3*a*b*i + 2*a^2*j))*x))/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) -
 (2*(b^4*c*i + 24*a^2*c^3*i + 2*b^2*c^2*(2*c*g - 3*a*i) - b^5*j - b^3*c*(c*
h - 10*a*j) - 4*b*c^2*(2*c^2*f + a*c*h + 8*a^2*j) - c*(16*c^4*f - c^3*(8*b*
g - 8*a*h) - 4*b^4*j + b^2*c*(b*i + 28*a*j) + 2*c^2*(b^2*h - 6*a*b*i - 16*a
^2*j))*x))/(3*c^3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) + (j*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{f+gx+hx^2+365x^3+jx^4}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a\left(365b^2c+2c^3g-bc^2h-b^3j\right)}{c^3}\right)-\left(365b^3c-bc^2(1095a-\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+c\right.}$$

$$= -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a\left(365b^2c+2c^3g-bc^2h-b^3j\right)}{c^3}\right)-\left(365b^3c-bc^2(1095a-\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+c\right.}$$

$$= -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a\left(365b^2c+2c^3g-bc^2h-b^3j\right)}{c^3}\right)-\left(365b^3c-bc^2(1095a-\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+c\right.}$$

$$= -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a\left(365b^2c+2c^3g-bc^2h-b^3j\right)}{c^3}\right)-\left(365b^3c-bc^2(1095a-\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+c\right.}$$

$$= -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a\left(365b^2c+2c^3g-bc^2h-b^3j\right)}{c^3}\right)-\left(365b^3c-bc^2(1095a-\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+c\right.}$$

Mathematica [A] time = 1.14, size = 316, normalized size = 0.89

$$-\frac{2\left(bc\left(-3a^2j+ac(h+3ix)+c^2(f-gx)\right)+2c^2\left(a^2(i+jx)-ac(g+hx)+c^2fx\right)+b^3(aj-cix)+b^2c(chx-a(i+4jx))+b^4jx\right)}{\left(b^2-4ac\right)(a+x(b+cx))^{3/2}}+\frac{2\left(4bc^2\left(8a^2j+ac(h-3ix)+2c^2(f-gx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] ((-2*(b^4*j*x + b^3*(a*j - c*i*x) + b*c*(-3*a^2*j + c^2*(f - g*x) + a*c*(h
+ 3*i*x)) + 2*c^2*(c^2*f*x - a*c*(g + h*x) + a^2*(i + j*x)) + b^2*c*(c*h*x
- a*(i + 4*j*x))))/((b^2 - 4*a*c)*(a + x*(b + c*x))^(3/2)) + (2*(b^5*j - b^
4*c*(i + 4*j*x) + 2*b^2*c^2*(-2*c*g + 3*a*i + c*h*x + 14*a*j*x) + 4*b*c^2*(
8*a^2*j + 2*c^2*(f - g*x) + a*c*(h - 3*i*x)) + b^3*c*(-10*a*j + c*(h + i*x)
) + 8*c^3*(2*c^2*f*x + a*c*h*x - a^2*(3*i + 4*j*x))))/((b^2 - 4*a*c)^2*Sqrt
[a + x*(b + c*x)]) + 3*Sqrt[c]*j*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b +
c*x)]])/(3*c^3)
```

fricas [B] time = 88.99, size = 1373, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/6*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 + 2*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*j*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*j*x^2 + 2*(a*b^5 -
 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*j)*
sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b
)*sqrt(c) - 4*a*c) + 4*(8*a^2*b*c^3*h - 16*a^3*c^3*i + (16*c^6*f - 8*b*c^5*
g + 2*(b^2*c^4 + 4*a*c^5)*h + (b^3*c^3 - 12*a*b*c^4)*i - 4*(b^4*c^2 - 7*a*b
^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f - 4*b^2*c^4*g + (b^3*c^3 + 4*a*b*
c^4)*h - 2*(a*b^2*c^3 + 4*a^2*c^4)*i - (b^5*c - 6*a*b^3*c^2)*j)*x^2 - (b^3*
c^3 - 12*a*b*c^4)*f - 2*(a*b^2*c^3 + 4*a^2*c^4)*g - (3*a^2*b^3*c - 20*a^3*b
*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i + 2*(b^2*c^4 + 4*a*c^5)*f - (b^3
*c^3 + 4*a*b*c^4)*g - 2*(a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)*x)*sqrt(c*
x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 +
(b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 1
6*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 + 2*(b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*j
*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2)*j)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt
(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(8*a^2*b*c^3*h - 16*a^3*c^3*i + (16*c^6*f
 - 8*b*c^5*g + 2*(b^2*c^4 + 4*a*c^5)*h + (b^3*c^3 - 12*a*b*c^4)*i - 4*(b^4*
c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f - 4*b^2*c^4*g + (b^3*c
^3 + 4*a*b*c^4)*h - 2*(a*b^2*c^3 + 4*a^2*c^4)*i - (b^5*c - 6*a*b^3*c^2)*j)*
x^2 - (b^3*c^3 - 12*a*b*c^4)*f - 2*(a*b^2*c^3 + 4*a^2*c^4)*g - (3*a^2*b^3*c
 - 20*a^3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i + 2*(b^2*c^4 + 4*a*c^
5)*f - (b^3*c^3 + 4*a*b*c^4)*g - 2*(a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)
*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4
*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*
c^6)*x^3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*
b^3*c^4 + 16*a^3*b*c^5)*x)]
```

giac [A] time = 0.31, size = 465, normalized size = 1.31

$$\frac{2\left(\left(\left(\frac{\left(16c^5f-8bc^4g+2b^2c^3h+8ac^4h+b^3c^2i-12abc^3i-4b^4cj+28ab^2c^2j-32a^2c^3j\right)x}{b^4c^2-8ab^2c^3+16a^2c^4}+\frac{3\left(8bc^4f-4b^2c^3g+b^3c^2h+4abc^3h-2ab^2c^2i-8a^2c^3i-b^5j+6ab^3c\ldots\right.}{b^4c^2-8ab^2c^3+16a^2c^4}\right.\right.\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac"
)
```

```
[Out] 2/3*((((16*c^5*f - 8*b*c^4*g + 2*b^2*c^3*h + 8*a*c^4*h + b^3*c^2*i - 12*a*b
*c^3*i - 4*b^4*c*j + 28*a*b^2*c^2*j - 32*a^2*c^3*j)*x/(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4) + 3*(8*b*c^4*f - 4*b^2*c^3*g + b^3*c^2*h + 4*a*b*c^3*h - 2*
a*b^2*c^2*i - 8*a^2*c^3*i - b^5*j + 6*a*b^3*c*j)/(b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4))*x + 3*(2*b^2*c^3*f + 8*a*c^4*f - b^3*c^2*g - 4*a*b*c^3*g + 4*a*
b^2*c^2*h - 8*a^2*b*c^2*i - 2*a*b^4*j + 14*a^2*b^2*c*j - 8*a^3*c^2*j)/(b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x - (b^3*c^2*f - 12*a*b*c^3*f + 2*a*b^2*c^
2*g + 8*a^2*c^3*g - 8*a^2*b*c^2*h + 16*a^3*c^2*i + 3*a^2*b^3*j - 20*a^3*b*c
*j)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(3/2) - j*log(a
bs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2)
```

maple [B] time = 0.02, size = 1406, normalized size = 3.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] j/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-1/24*j/c^3*b^4/(4*a*c-b^2)/(c*x
^2+b*x+a)^(3/2)*x-1/3*j/c^2*b^4/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/4*j/c
^3*b^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+2*j/c^2*b^3*a/(4*a*c-b^2)^2/(c*x^2
+b*x+a)^(1/2)+2/3*i/c*b^3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/3*h*a/c/(4*
a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b+16/3*h*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)
*x-1/2*i/c^2*b^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+1/12*i/c^2*b^3/(4*a*c-b^
2)/(c*x^2+b*x+a)^(3/2)*x-16/3*g*c*b/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-8*i
*b*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-4*i/c*b^2*a/(4*a*c-b^2)^2/(c*x^2+b
*x+a)^(1/2)+1/6*h/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+2/3*f/(4*a*c-b^2)
/(c*x^2+b*x+a)^(3/2)*b-i*x^2/c/(c*x^2+b*x+a)^(3/2)+1/24*i/c^3*b^2/(c*x^2+b*
x+a)^(3/2)-2/3*i*a/c^2/(c*x^2+b*x+a)^(3/2)-1/2*h*x/c/(c*x^2+b*x+a)^(3/2)+1/
12*h/c^2*b/(c*x^2+b*x+a)^(3/2)-8/3*g*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)-
1/3*j*x^3/c/(c*x^2+b*x+a)^(3/2)-1/48*j/c^4*b^3/(c*x^2+b*x+a)^(3/2)-j/c^2*x/
(c*x^2+b*x+a)^(1/2)+1/2*j/c^3*b/(c*x^2+b*x+a)^(1/2)+4*j/c*b^2*a/(4*a*c-b^2)
^2/(c*x^2+b*x+a)^(1/2)*x-i/c*b*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+1/2*j/c^
2*b^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-2/3*g*b/(4*a*c-b^2)/(c*x^2+b*x+a)
^(3/2)*x-1/3*g/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+4/3*f/(4*a*c-b^2)/(c*x
^2+b*x+a)^(3/2)*x*c+32/3*f*c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+16/3*f*c
/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b+1/12*h/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+
a)^(3/2)+4/3*h*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+2/3*h/c*b^3/(4*a*c-b
^2)^2/(c*x^2+b*x+a)^(1/2)+2/3*h*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+8/3*h*a
/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b+1/2*j/c^2*b*x^2/(c*x^2+b*x+a)^(3/2)+1/
8*j/c^3*b^2*x/(c*x^2+b*x+a)^(3/2)-1/48*j/c^4*b^5/(4*a*c-b^2)/(c*x^2+b*x+a)^
(3/2)-1/6*j/c^3*b^5/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)-1/4*i/c^2*b*x/(c*x^2+
b*x+a)^(3/2)+1/24*i/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+1/3*i/c^2*b^4/(
4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+1/2*j/c^3*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1
/2)-1/3*g/c/(c*x^2+b*x+a)^(3/2)+j/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))+1/3*j/c^3*b*a/(c*x^2+b*x+a)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{j\,x^4+i\,x^3+h\,x^2+g\,x+f}{{\left(c\,x^2+b\,x+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**4+i*x**3+h*x**2+g*x+f)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

3.366 $\int \frac{f+gx+hx^2+ix^3+jx^4}{\left(a+bx-cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=353

$$\frac{2\left(x\left(c^2\left(2a^2j+3abi+b^2h\right)+b^2c(4aj+bi)+c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j-ach+c^2f\right)+ab^3j+ab^2ci\right.}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}}$$

```
[Out] 2/3*(a*b^2*c*i+2*a*c^2*(a*i+c*g)+a*b^3*j-b*c*(-3*a^2*j-a*c*h+c^2*f)+(2*c^4*
f+c^3*(2*a*h+b*g)+b^4*j+b^2*c*(4*a*j+b*i)+c^2*(2*a^2*j+3*a*b*i+b^2*h))*x)/c
^3/(4*a*c+b^2)/(-c*x^2+b*x+a)^(3/2)-j*arctan(1/2*(-2*c*x+b)/c^(1/2)/(-c*x^2
+b*x+a)^(1/2))/c^(5/2)-2/3*(b^4*c*i+24*a^2*c^3*i+2*b^2*c^2*(3*a*i+2*c*g)+b^
5*j+b^3*c*(10*a*j+c*h)+4*b*c^2*(8*a^2*j-a*c*h+2*c^2*f)-c*(16*c^4*f+8*c^3*(-
a*h+b*g)-4*b^4*j-b^2*c*(28*a*j+b*i)+2*c^2*(-16*a^2*j-6*a*b*i+b^2*h))*x)/c^3
/(4*a*c+b^2)^2/(-c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1660, 12, 621, 204}

$$-\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)-b^2c(28aj+bi)+8c^3(bg-ah)-4b^4j+16c^4f\right)+4bc^2\left(8a^2j-ach+2c^2f\right)+\right.}{3c^3\left(4ac+b^2\right)^2\sqrt{a+bx-cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x]
```

```
[Out] (2*(a*b^2*c*i + 2*a*c^2*(c*g + a*i) + a*b^3*j - b*c*(c^2*f - a*c*h - 3*a^2*
j) + (2*c^4*f + c^3*(b*g + 2*a*h) + b^4*j + b^2*c*(b*i + 4*a*j) + c^2*(b^2*
h + 3*a*b*i + 2*a^2*j))*x))/(3*c^3*(b^2 + 4*a*c)*(a + b*x - c*x^2)^(3/2)) -
 (2*(b^4*c*i + 24*a^2*c^3*i + 2*b^2*c^2*(2*c*g + 3*a*i) + b^5*j + b^3*c*(c*
h + 10*a*j) + 4*b*c^2*(2*c^2*f - a*c*h + 8*a^2*j) - c*(16*c^4*f + 8*c^3*(b*
g - a*h) - 4*b^4*j - b^2*c*(b*i + 28*a*j) + 2*c^2*(b^2*h - 6*a*b*i - 16*a^2
*j))*x))/(3*c^3*(b^2 + 4*a*c)^2*Sqrt[a + b*x - c*x^2]) - (j*ArcTan[(b - 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x - c*x^2])])/c^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{f+gx+hx^2+366x^3+jx^4}{\left(a+bx-cx^2\right)^{5/2}}\,dx = -\frac{2\left(c^3\left(bf-\frac{3a^2(244c+bj)}{c^2}-\frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right)-\left(366b^3c+bc^2(1098a+\cdots\right.\right.}{3c^3\left(b^2+4ac\right)\left(a+bx-\cdots\right.}$$

$$= -\frac{2\left(c^3\left(bf-\frac{3a^2(244c+bj)}{c^2}-\frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right)-\left(366b^3c+bc^2(1098a+\cdots\right.\right.}{3c^3\left(b^2+4ac\right)\left(a+bx-\cdots\right.}$$

$$= -\frac{2\left(c^3\left(bf-\frac{3a^2(244c+bj)}{c^2}-\frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right)-\left(366b^3c+bc^2(1098a+\cdots\right.\right.}{3c^3\left(b^2+4ac\right)\left(a+bx-\cdots\right.}$$

$$= -\frac{2\left(c^3\left(bf-\frac{3a^2(244c+bj)}{c^2}-\frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right)-\left(366b^3c+bc^2(1098a+\cdots\right.\right.}{3c^3\left(b^2+4ac\right)\left(a+bx-\cdots\right.}$$

$$= -\frac{2\left(c^3\left(bf-\frac{3a^2(244c+bj)}{c^2}-\frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right)-\left(366b^3c+bc^2(1098a+\cdots\right.\right.}{3c^3\left(b^2+4ac\right)\left(a+bx-\cdots\right.}$$

Mathematica [C] time = 1.11, size = 319, normalized size = 0.90

$$-\frac{2\left(b^3\left(3a^2j+18acjx^2+c^2\left(f+3gx-\left(x^2(3h+ix)\right)\right)\right)+2b^2c\left(21a^2jx+ac\left(g+x\left(-6h+3ix-14jx^2\right)\right)+c^2x(\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x]
```

```
[Out] (-2*(3*b^5*j*x^2 + b^4*(6*a*j*x - 4*c*j*x^3) + b^3*(3*a^2*j + 18*a*c*j*x^2
+ c^2*(f + 3*g*x - x^2*(3*h + i*x))) + 8*c^2*(2*c^3*f*x^3 + a^3*(2*i + 3*j*
x) - a*c^2*x*(3*f + h*x^2) - a^2*c*(g + x^2*(3*i + 4*j*x))) + 4*b*c*(5*a^3*
j + 2*c^3*x^2*(-3*f + g*x) - 2*a^2*c*(h - 3*i*x) + 3*a*c^2*(f - x*(g - h*x
+ i*x^2))) + 2*b^2*c*(21*a^2*j*x + c^2*x*(3*f + x*(-6*g + h*x)) + a*c*(g +
x*(-6*h + 3*i*x - 14*j*x^2)))))/(3*c^2*(b^2 + 4*a*c)^2*(a + x*(b - c*x))^(3
/2)) + (I*j*Log[(I*(b - 2*c*x))/Sqrt[c] + 2*Sqrt[a + x*(b - c*x)]])/c^(5/2)
```

fricas [B] time = 95.78, size = 1385, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="fric
as")
```

```
[Out] [-1/6*(3*((b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 - 2*(b^5*c + 8*a*b^3*c
^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 + 6*a*b^4*c - 32*a^3*c^3)*j*x^2 + 2*(a*b^5
+ 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 + 8*a^3*b^2*c + 16*a^4*c^2)*j)
*sqrt(-c)*log(8*c^2*x^2 - 8*b*c*x + b^2 - 4*sqrt(-c*x^2 + b*x + a)*(2*c*x -
 b)*sqrt(-c) - 4*a*c) - 4*(8*a^2*b*c^3*h - 16*a^3*c^3*i - (16*c^6*f + 8*b*c
^5*g + 2*(b^2*c^4 - 4*a*c^5)*h - (b^3*c^3 + 12*a*b*c^4)*i - 4*(b^4*c^2 + 7*
a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f + 4*b^2*c^4*g + (b^3*c^3 - 4*a
*b*c^4)*h - 2*(a*b^2*c^3 - 4*a^2*c^4)*i - (b^5*c + 6*a*b^3*c^2)*j)*x^2 - (b
^3*c^3 + 12*a*b*c^4)*f - 2*(a*b^2*c^3 - 4*a^2*c^4)*g - (3*a^2*b^3*c + 20*a^
3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i - 2*(b^2*c^4 - 4*a*c^5)*f - (
b^3*c^3 - 4*a*b*c^4)*g - 2*(a*b^4*c + 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)*x)*sqrt
(-c*x^2 + b*x + a))/(a^2*b^4*c^3 + 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 +
8*a*b^2*c^6 + 16*a^2*c^7)*x^4 - 2*(b^5*c^4 + 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^
3 + (b^6*c^3 + 6*a*b^4*c^4 - 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 + 8*a^2*b^3*c^4
 + 16*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 -
2*(b^5*c + 8*a*b^3*c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 + 6*a*b^4*c - 32*a^3*c^
3)*j*x^2 + 2*(a*b^5 + 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 + 8*a^3*b^
2*c + 16*a^4*c^2)*j)*sqrt(c)*arctan(1/2*sqrt(-c*x^2 + b*x + a)*(2*c*x - b)*
sqrt(c)/(c^2*x^2 - b*c*x - a*c)) - 2*(8*a^2*b*c^3*h - 16*a^3*c^3*i - (16*c^
6*f + 8*b*c^5*g + 2*(b^2*c^4 - 4*a*c^5)*h - (b^3*c^3 + 12*a*b*c^4)*i - 4*(b
^4*c^2 + 7*a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f + 4*b^2*c^4*g + (b^
3*c^3 - 4*a*b*c^4)*h - 2*(a*b^2*c^3 - 4*a^2*c^4)*i - (b^5*c + 6*a*b^3*c^2)*
j)*x^2 - (b^3*c^3 + 12*a*b*c^4)*f - 2*(a*b^2*c^3 - 4*a^2*c^4)*g - (3*a^2*b^
3*c + 20*a^3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i - 2*(b^2*c^4 - 4*a
*c^5)*f - (b^3*c^3 - 4*a*b*c^4)*g - 2*(a*b^4*c + 7*a^2*b^2*c^2 + 4*a^3*c^3)
*j)*x)*sqrt(-c*x^2 + b*x + a))/(a^2*b^4*c^3 + 8*a^3*b^2*c^4 + 16*a^4*c^5 +
(b^4*c^5 + 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 - 2*(b^5*c^4 + 8*a*b^3*c^5 + 16*a^
2*b*c^6)*x^3 + (b^6*c^3 + 6*a*b^4*c^4 - 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 + 8*
a^2*b^3*c^4 + 16*a^3*b*c^5)*x)]
```

giac [**A**] time = 0.45, size = 488, normalized size = 1.38

$$-\frac{2\sqrt{-cx^2+bx+a}\left(\left(\left(\frac{\left(16c^5f+8bc^4g+2b^2c^3h-8ac^4h-b^3c^2i-12abc^3i-4b^4cj-28ab^2c^2j-32a^2c^3j\right)x}{b^4c^2+8ab^2c^3+16a^2c^4}-\frac{3\left(8bc^4f+4b^2c^3g+b^3c^2h-4abc^3h-2ab^2\cdots\right.}{b^4c^2+8ab^2c^3+16\cdots}\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="giac
")
```

```
[Out] -2/3*sqrt(-c*x^2 + b*x + a)*((((16*c^5*f + 8*b*c^4*g + 2*b^2*c^3*h - 8*a*c^
4*h - b^3*c^2*i - 12*a*b*c^3*i - 4*b^4*c*j - 28*a*b^2*c^2*j - 32*a^2*c^3*j)
*x/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4) - 3*(8*b*c^4*f + 4*b^2*c^3*g + b^3*
c^2*h - 4*a*b*c^3*h - 2*a*b^2*c^2*i + 8*a^2*c^3*i - b^5*j - 6*a*b^3*c*j)/(b
^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))*x + 3*(2*b^2*c^3*f - 8*a*c^4*f + b^3*c^
2*g - 4*a*b*c^3*g - 4*a*b^2*c^2*h + 8*a^2*b*c^2*i + 2*a*b^4*j + 14*a^2*b^2*
c*j + 8*a^3*c^2*j)/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))*x + (b^3*c^2*f + 1
2*a*b*c^3*f + 2*a*b^2*c^2*g - 8*a^2*c^3*g - 8*a^2*b*c^2*h + 16*a^3*c^2*i +
3*a^2*b^3*j + 20*a^3*b*c*j)/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 -
b*x - a)^2 - j*log(abs(2*(sqrt(-c)*x - sqrt(-c*x^2 + b*x + a))*sqrt(-c) + b
))/(sqrt(-c)*c^2)
```

maple [**B**] time = 0.02, size = 1453, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x)
```

[Out] 1/3*g/c/(-c*x^2+b*x+a)^(3/2)+j/c^(5/2)*arctan(c^(1/2)*(x-1/2/c*b)/(-c*x^2+b*x+a)^(1/2))+16/3*g*c*b/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x+1/12*i/c^2*b^3/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x-2/3*i/c*b^3/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x-1/2*i/c^2*b^2*a/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-8*i*b*a/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x+4*i/c*b^2*a/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)-1/6*h/c*b^2/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x-1/3*h*a/c/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*b-16/3*h*a*c/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x+2*j/c^2*b^3*a/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)+j/c^2*b^2/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(1/2)*x+1/24*j/c^3*b^4/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x-1/3*j/c^2*b^4/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x-1/4*j/c^3*b^3*a/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)+1/2*h*x/c/(-c*x^2+b*x+a)^(3/2)+1/12*h/c^2*b/(-c*x^2+b*x+a)^(3/2)-8/3*g*b^2/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)+2/3*f/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*b+i*x^2/c/(-c*x^2+b*x+a)^(3/2)-1/24*i/c^3*b^2/(-c*x^2+b*x+a)^(3/2)-2/3*i*a/c^2/(-c*x^2+b*x+a)^(3/2)+1/3*j*x^3/c/(-c*x^2+b*x+a)^(3/2)-1/48*j/c^4*b^3/(-c*x^2+b*x+a)^(3/2)-j/c^2*x/(-c*x^2+b*x+a)^(1/2)-1/2*j/c^3*b/(-c*x^2+b*x+a)^(1/2)+i/c*b*a/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x+1/2*j/c^2*b^2*a/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x-4*j/c*b^2*a/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x-1/3*j/c^3*b*a/(-c*x^2+b*x+a)^(3/2)-1/2*j/c^3*b^3/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(1/2)-1/4*i/c^2*b*x/(-c*x^2+b*x+a)^(3/2)-1/24*i/c^3*b^4/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)+1/3*i/c^2*b^4/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)-4/3*f/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x*c+32/3*f*c^2/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x-16/3*f*c/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*b-2/3*g*b/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x+1/3*g/c*b^2/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)+1/12*h/c^2*b^3/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)+4/3*h*b^2/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x-2/3*h/c*b^3/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)+2/3*h*a/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x+8/3*h*a/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*b+1/2*j/c^2*b*x^2/(-c*x^2+b*x+a)^(3/2)-1/8*j/c^3*b^2*x/(-c*x^2+b*x+a)^(3/2)-1/48*j/c^4*b^5/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)+1/6*j/c^3*b^5/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}i\left(\frac{32\,abx}{\sqrt{-cx^2+bx+a}\left(b^2+4\,ac\right)^2}-\frac{16\,ab^2}{\sqrt{-cx^2+bx+a}\left(b^2+4\,ac\right)^2c}+\frac{b^3x}{\left(-cx^2+bx+a\right)^{\frac{3}{2}}\left(b^2+4\,ac\right)c^2}+\frac{}{\sqrt{-cx^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="maxima")

[Out] -1/3*i*(32*a*b*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) - 16*a*b^2/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2*c) + b^3*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c^2) + 2*(b^2 - 4*a*c)*b*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2*c) + 6*a*b*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c) - 3*x^2/((-c*x^2 + b*x + a)^(3/2)*c) - (b^2 - 4*a*c)*b^2/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2*c^2) - a*b^2/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c^2) + 2*a/((-c*x^2 + b*x + a)^(3/2)*c^2)) + 1/3*g*(16*b*c*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) - 8*b^2/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) + 2*b*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)) - b^2/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c) + 1/((-c*x^2 + b*x + a)^(3/2)*c)) + 2/3*f*(16*c^2*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) - 8*b*c/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) + 2*c*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)) - b/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c))) + 2/3*h*(2*(b^2 - 4*a*c)*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) + 2*a*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)) + b^2*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c) - (b^2 - 4*a*c)*b/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2*c) + a*b/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c)) + j*integrate(x^4/((c^2*x^4 - 2*b*c*x^3 + 2*a*b*x + (b^2 - 2*a*c)*x^2 + a^2)*sqrt(-c*x^2 + b*x + a)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{j\,x^4+i\,x^3+h\,x^2+g\,x+f}{\left(-c\,x^2+b\,x+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x)
[Out] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**4+i*x**3+h*x**2+g*x+f)/(-c*x**2+b*x+a)**(5/2),x)
[Out] Timed out
```

3.367 $\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal. Leaf size=588

$$\frac{45\left(500d^2+5de+17e^2\right)(d+ex)^{m+9}}{e^{11}(m+9)}-\frac{2\left(30000d^3+450d^2e+3060de^2+49e^3\right)(d+ex)^{m+8}}{e^{11}(m+8)}+\frac{\left(5d^2-2de+3e^2\right)}{}$$

```
[Out] (5*d^2-2*d*e+3*e^2)^3*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/e
^11/(1+m)-(5*d^2-2*d*e+3*e^2)^2*(200*d^5+169*d^4*e+108*d^3*e^2-20*d^2*e^3+8
6*d*e^4-15*e^5)*(e*x+d)^(2+m)/e^11/(2+m)+3*(5*d^2-2*d*e+3*e^2)*(1500*d^6+66
0*d^5*e+792*d^4*e^2+58*d^3*e^3+547*d^2*e^4-156*d*e^5+53*e^6)*(e*x+d)^(3+m)/
e^11/(3+m)-2*(30000*d^7+1050*d^6*e+21420*d^5*e^2+1715*d^4*e^3+9990*d^3*e^4-
2550*d^2*e^5+2218*d*e^6-287*e^7)*(e*x+d)^(4+m)/e^11/(4+m)+(105000*d^6+3150*
d^5*e+53550*d^4*e^2+3430*d^3*e^3+14985*d^2*e^4-2550*d*e^5+1109*e^6)*(e*x+d)
^(5+m)/e^11/(5+m)-6*(21000*d^5+525*d^4*e+7140*d^3*e^2+343*d^2*e^3+999*d*e^4
-85*e^5)*(e*x+d)^(6+m)/e^11/(6+m)+(105000*d^4+2100*d^3*e+21420*d^2*e^2+686*
d*e^3+999*e^4)*(e*x+d)^(7+m)/e^11/(7+m)-2*(30000*d^3+450*d^2*e+3060*d*e^2+4
9*e^3)*(e*x+d)^(8+m)/e^11/(8+m)+45*(500*d^2+5*d*e+17*e^2)*(e*x+d)^(9+m)/e^1
1/(9+m)-25*(200*d+e)*(e*x+d)^(10+m)/e^11/(10+m)+500*(e*x+d)^(11+m)/e^11/(11
+m)
```

Rubi [A] time = 0.36, antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1628}

$$\frac{\left(5d^2-2de+3e^2\right)^3\left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)(d+ex)^{m+1}}{e^{11}(m+1)}-\frac{\left(5d^2-2de+3e^2\right)^2\left(108d^3e^2-20d^2e^3+169\right.}{e^{11}(m}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^3*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^(1 + m))/(e^11*(1 + m)) - ((5*d^2 - 2*d*e + 3*e^2)^2*(200*d^5 + 169
*d^4*e + 108*d^3*e^2 - 20*d^2*e^3 + 86*d*e^4 - 15*e^5)*(d + e*x)^(2 + m))/(
e^11*(2 + m)) + (3*(5*d^2 - 2*d*e + 3*e^2)*(1500*d^6 + 660*d^5*e + 792*d^4*
e^2 + 58*d^3*e^3 + 547*d^2*e^4 - 156*d*e^5 + 53*e^6)*(d + e*x)^(3 + m))/(e^
11*(3 + m)) - (2*(30000*d^7 + 1050*d^6*e + 21420*d^5*e^2 + 1715*d^4*e^3 + 9
990*d^3*e^4 - 2550*d^2*e^5 + 2218*d*e^6 - 287*e^7)*(d + e*x)^(4 + m))/(e^11
*(4 + m)) + ((105000*d^6 + 3150*d^5*e + 53550*d^4*e^2 + 3430*d^3*e^3 + 1498
5*d^2*e^4 - 2550*d*e^5 + 1109*e^6)*(d + e*x)^(5 + m))/(e^11*(5 + m)) - (6*(
21000*d^5 + 525*d^4*e + 7140*d^3*e^2 + 343*d^2*e^3 + 999*d*e^4 - 85*e^5)*(d
 + e*x)^(6 + m))/(e^11*(6 + m)) + ((105000*d^4 + 2100*d^3*e + 21420*d^2*e^2
 + 686*d*e^3 + 999*e^4)*(d + e*x)^(7 + m))/(e^11*(7 + m)) - (2*(30000*d^3 +
 450*d^2*e + 3060*d*e^2 + 49*e^3)*(d + e*x)^(8 + m))/(e^11*(8 + m)) + (45*(
500*d^2 + 5*d*e + 17*e^2)*(d + e*x)^(9 + m))/(e^11*(9 + m)) - (25*(200*d +
e)*(d + e*x)^(10 + m))/(e^11*(10 + m)) + (500*(d + e*x)^(11 + m))/(e^11*(11
 + m))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) dx = \int \left(\frac{\left(5d^2-2de+3e^2\right)^3 \left(4d^4+5d^3e+3d^2e^2-de^3+\right.}{e^{10}} \right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)^3 \left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{e^{11}(1+m)}$$

Mathematica [A] time = 0.38, size = 537, normalized size = 0.91

$$(d+ex)^{m+1} \left(\frac{45\left(500d^2+5de+17e^2\right)(d+ex)^8}{m+9} - \frac{2\left(30000d^3+450d^2e+3060de^2+49e^3\right)(d+ex)^7}{m+8} + \frac{\left(105000d^4+2100d^3e+21420d^2e^2+686de^3+999e^4\right)(d}{m+7} \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] ((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)^3*(4*d^4 + 5*d^3*e + 3*d^2*e^2
 - d*e^3 + 2*e^4))/(1 + m) - ((5*d^2 - 2*d*e + 3*e^2)^2*(200*d^5 + 169*d^4*
e + 108*d^3*e^2 - 20*d^2*e^3 + 86*d*e^4 - 15*e^5)*(d + e*x))/(2 + m) + (3*(
5*d^2 - 2*d*e + 3*e^2)*(1500*d^6 + 660*d^5*e + 792*d^4*e^2 + 58*d^3*e^3 + 5
47*d^2*e^4 - 156*d*e^5 + 53*e^6)*(d + e*x)^2)/(3 + m) - (2*(30000*d^7 + 105
0*d^6*e + 21420*d^5*e^2 + 1715*d^4*e^3 + 9990*d^3*e^4 - 2550*d^2*e^5 + 2218
*d*e^6 - 287*e^7)*(d + e*x)^3)/(4 + m) + ((105000*d^6 + 3150*d^5*e + 53550*
d^4*e^2 + 3430*d^3*e^3 + 14985*d^2*e^4 - 2550*d*e^5 + 1109*e^6)*(d + e*x)^4
)/(5 + m) - (6*(21000*d^5 + 525*d^4*e + 7140*d^3*e^2 + 343*d^2*e^3 + 999*d*
e^4 - 85*e^5)*(d + e*x)^5)/(6 + m) + ((105000*d^4 + 2100*d^3*e + 21420*d^2*
e^2 + 686*d*e^3 + 999*e^4)*(d + e*x)^6)/(7 + m) - (2*(30000*d^3 + 450*d^2*e
 + 3060*d*e^2 + 49*e^3)*(d + e*x)^7)/(8 + m) + (45*(500*d^2 + 5*d*e + 17*e^
2)*(d + e*x)^8)/(9 + m) - (25*(200*d + e)*(d + e*x)^9)/(10 + m) + (500*(d +
 e*x)^10)/(11 + m)))/e^11
```

fricas [B] time = 1.20, size = 4795, normalized size = 8.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] (54*d*e^10*m^10 + 500*(e^11*m^10 + 55*e^11*m^9 + 1320*e^11*m^8 + 18150*e^11
*m^7 + 157773*e^11*m^6 + 902055*e^11*m^5 + 3416930*e^11*m^4 + 8409500*e^11*
m^3 + 12753576*e^11*m^2 + 10628640*e^11*m + 3628800*e^11)*x^11 + 1814400000
*d^11 + 99792000*d^10*e + 3392928000*d^9*e^2 + 488980800*d^8*e^3 + 56966976
00*d^7*e^4 - 3392928000*d^6*e^5 + 8853546240*d^5*e^6 - 5728060800*d^4*e^7 +
 6346771200*d^3*e^8 - 2694384000*d^2*e^9 + 2155507200*d*e^10 - 25*(3991680*
e^11 - (20*d*e^10 - e^11)*m^10 - 4*(225*d*e^10 - 14*e^11)*m^9 - 15*(1160*d*
e^10 - 91*e^11)*m^8 - 60*(3150*d*e^10 - 317*e^11)*m^7 - 21*(60260*d*e^10 -
7963*e^11)*m^6 - 84*(64125*d*e^10 - 11492*e^11)*m^5 - 5*(2894720*d*e^10 - 7
37251*e^11)*m^4 - 20*(1172700*d*e^10 - 456659*e^11)*m^3 - 36*(570320*d*e^10
 - 386841*e^11)*m^2 - 144*(50400*d*e^10 - 80939*e^11)*m)*x^10 - 135*(d^2*e^
9 - 26*d*e^10)*m^9 + 5*(678585600*e^11 - (5*d*e^10 - 153*e^11)*m^10 - (1000
*d^2*e^9 + 235*d*e^10 - 8721*e^11)*m^9 - 6*(6000*d^2*e^9 + 785*d*e^10 - 360
06*e^11)*m^8 - 6*(91000*d^2*e^9 + 8785*d*e^10 - 509031*e^11)*m^7 - 105*(432
00*d^2*e^9 + 3445*d*e^10 - 259029*e^11)*m^6 - 21*(1069000*d^2*e^9 + 74815*d
*e^10 - 7560189*e^11)*m^5 - 2*(33642000*d^2*e^9 + 2145620*d*e^10 - 30603656
```

```
7*e^11)*m^4 - 4*(29531000*d^2*e^9 + 1761185*d*e^10 - 382172121*e^11)*m^3 -
72*(1522000*d^2*e^9 + 86510*d*e^10 - 32587351*e^11)*m^2 - 1440*(28000*d^2*e
^9 + 1540*d*e^10 - 1370727*e^11)*m)*x^9 + 9*(106*d^3*e^8 - 945*d^2*e^9 + 11
160*d*e^10)*m^8 - (488980800*e^11 - (765*d*e^10 - 98*e^11)*m^10 - (225*d^2*
e^9 + 37485*d*e^10 - 5684*e^11)*m^9 - 3*(15000*d^3*e^8 + 2925*d^2*e^9 + 260
100*d*e^10 - 47726*e^11)*m^8 - 42*(30000*d^3*e^8 + 3375*d^2*e^9 + 214965*d*
e^10 - 48958*e^11)*m^7 - 63*(230000*d^3*e^8 + 19650*d^2*e^9 + 1012095*d*e^1
0 - 294882*e^11)*m^6 - 63*(1400000*d^3*e^8 + 101175*d^2*e^9 + 4503555*d*e^1
0 - 1743812*e^11)*m^5 - (304605000*d^3*e^8 + 19707975*d^2*e^9 + 790573950*d
*e^10 - 428393182*e^11)*m^4 - 4*(147735000*d^3*e^8 + 8860500*d^2*e^9 + 3297
12705*d*e^10 - 270109021*e^11)*m^3 - 36*(16335000*d^3*e^8 + 929925*d^2*e^9
+ 32795550*d*e^10 - 46438966*e^11)*m^2 - 5040*(45000*d^3*e^8 + 2475*d^2*e^9
 + 84150*d*e^10 - 280861*e^11)*m)*x^8 - 6*(574*d^4*e^7 - 9540*d^3*e^8 + 390
15*d^2*e^9 - 277290*d*e^10)*m^7 + (5696697600*e^11 - (98*d*e^10 - 999*e^11)
*m^10 - 3*(2040*d^2*e^9 + 1666*d*e^10 - 19647*e^11)*m^9 - 24*(75*d^3*e^8 +
10710*d^2*e^9 + 4508*d*e^10 - 62937*e^11)*m^8 - 6*(60000*d^4*e^7 + 9600*d^3
*e^8 + 740520*d^2*e^9 + 216482*d*e^10 - 3677319*e^11)*m^7 - 3*(2520000*d^4*
e^7 + 243600*d^3*e^8 + 13708800*d^2*e^9 + 3161774*d*e^10 - 67539393*e^11)*m
^6 - 21*(3000000*d^4*e^7 + 228000*d^3*e^8 + 10581480*d^2*e^9 + 2069662*d*e^
10 - 57933009*e^11)*m^5 - 2*(132300000*d^4*e^7 + 8738100*d^3*e^8 + 35715708
0*d^2*e^9 + 62076434*d*e^10 - 2405021571*e^11)*m^4 - 36*(16240000*d^4*e^7 +
 981400*d^3*e^8 + 36788680*d^2*e^9 + 5871278*d*e^10 - 341095341*e^11)*m^3 -
 72*(8820000*d^4*e^7 + 503100*d^3*e^8 + 17778600*d^2*e^9 + 2670010*d*e^10 -
 266622111*e^11)*m^2 - 12960*(20000*d^4*e^7 + 1100*d^3*e^8 + 37400*d^2*e^9
+ 5390*d*e^10 - 1264623*e^11)*m)*x^7 + 6*(4436*d^5*e^6 - 32144*d^4*e^7 + 24
7086*d^3*e^8 - 615195*d^2*e^9 + 2939517*d*e^10)*m^6 + (3392928000*e^11 + 3*
(333*d*e^10 + 170*e^11)*m^10 + (686*d^2*e^9 + 52947*d*e^10 + 30600*e^11)*m^
9 + 6*(7140*d^3*e^8 + 5145*d^2*e^9 + 198801*d*e^10 + 133025*e^11)*m^8 + 6*(
2100*d^4*e^7 + 257040*d^3*e^8 + 95354*d^2*e^9 + 2484513*d*e^10 + 1978800*e^
11)*m^7 + 3*(840000*d^5*e^6 + 109200*d^4*e^7 + 7282800*d^3*e^8 + 1886500*d^
2*e^9 + 37725237*d*e^10 + 37016310*e^11)*m^6 + 3*(12600000*d^5*e^6 + 105000
0*d^4*e^7 + 52264800*d^3*e^8 + 10813418*d^2*e^9 + 179179641*d*e^10 + 226287
000*e^11)*m^5 + 42*(5100000*d^5*e^6 + 348000*d^4*e^7 + 14635980*d^3*e^8 + 2
609495*d^2*e^9 + 37733562*d*e^10 + 64999925*e^11)*m^4 + 4*(141750000*d^5*e^
6 + 8659350*d^4*e^7 + 327983040*d^3*e^8 + 52869334*d^2*e^9 + 692643663*d*e^
10 + 1769460300*e^11)*m^3 + 120*(5754000*d^5*e^6 + 329070*d^4*e^7 + 1165962
0*d^3*e^8 + 1755817*d^2*e^9 + 21444534*d*e^10 + 93454763*e^11)*m^2 + 7200*(
42000*d^5*e^6 + 2310*d^4*e^7 + 78540*d^3*e^8 + 11319*d^2*e^9 + 131868*d*e^1
0 + 1344547*e^11)*m)*x^6 - 3*(20400*d^6*e^5 - 452472*d^5*e^6 + 1526840*d^4*
e^7 - 7212240*d^3*e^8 + 12236805*d^2*e^9 - 41597010*d*e^10)*m^5 + (88535462
40*e^11 + (510*d*e^10 + 1109*e^11)*m^10 - (5994*d^2*e^9 - 28050*d*e^10 - 67
649*e^11)*m^9 - 12*(343*d^3*e^8 + 23976*d^2*e^9 - 54825*d*e^10 - 149715*e^1
1)*m^8 - 6*(42840*d^4*e^7 + 27440*d^3*e^8 + 953046*d^2*e^9 - 1430550*d*e^10
 - 4541355*e^11)*m^7 - 3*(25200*d^5*e^6 + 2656080*d^4*e^7 + 869848*d^3*e^8
+ 20283696*d^2*e^9 - 22710810*d*e^10 - 86713819*e^11)*m^6 - 3*(5040000*d^6*
e^5 + 529200*d^5*e^6 + 30416400*d^4*e^7 + 6969760*d^3*e^8 + 124932942*d^2*e
^9 - 112732950*d*e^10 - 541448179*e^11)*m^5 - 2*(75600000*d^6*e^5 + 5481000
*d^5*e^6 + 242260200*d^4*e^7 + 45047562*d^3*e^8 + 675619704*d^2*e^9 - 51950
1300*d*e^10 - 3335910815*e^11)*m^4 - 4*(132300000*d^6*e^5 + 8221500*d^5*e^6
 + 316416240*d^4*e^7 + 51779280*d^3*e^8 + 688165146*d^2*e^9 - 470707050*d*e
^10 - 4412539105*e^11)*m^3 - 72*(10500000*d^6*e^5 + 602700*d^5*e^6 + 214342
80*d^4*e^7 + 3239978*d^3*e^8 + 39724236*d^2*e^9 - 25005980*d*e^10 - 3955614
47*e^11)*m^2 - 288*(1260000*d^6*e^5 + 69300*d^5*e^6 + 2356200*d^4*e^7 + 339
570*d^3*e^8 + 3956040*d^2*e^9 - 2356200*d*e^10 - 86687203*e^11)*m)*x^5 + 3*
(239760*d^7*e^4 - 918000*d^6*e^5 + 9537400*d^5*e^6 - 19929280*d^4*e^7 + 648
36702*d^3*e^8 - 79518915*d^2*e^9 + 198514620*d*e^10)*m^4 + (5728060800*e^11
 + (1109*d*e^10 + 574*e^11)*m^10 - (2550*d^2*e^9 - 63213*d*e^10 - 35588*e^1
1)*m^9 + 6*(4995*d^3*e^8 - 21675*d^2*e^9 + 257288*d*e^10 + 160433*e^11)*m^8
 + 6*(3430*d^4*e^7 + 219780*d^3*e^8 - 461550*d^2*e^9 + 3512203*d*e^10 + 248
```

```
3698*e^11)*m^7 + 15*(85680*d^5*e^6 + 49392*d^4*e^7 + 1554444*d^3*e^8 - 2122
620*d^2*e^9 + 11723239*d*e^10 + 9703470*e^11)*m^6 + 3*(126000*d^6*e^5 + 115
66800*d^5*e^6 + 3361400*d^4*e^7 + 70329600*d^3*e^8 - 71101650*d^2*e^9 + 306
983399*d*e^10 + 310583364*e^11)*m^5 + 2*(37800000*d^7*e^4 + 3213000*d^6*e^5
 + 158722200*d^5*e^6 + 32104800*d^4*e^7 + 515019465*d^3*e^8 - 418887225*d^2
*e^9 + 1494010421*d*e^10 + 1964946361*e^11)*m^4 + 4*(113400000*d^7*e^4 + 72
76500*d^6*e^5 + 288206100*d^5*e^6 + 48409305*d^4*e^7 + 659010330*d^3*e^8 -
460978800*d^2*e^9 + 1424518263*d*e^10 + 2670494533*e^11)*m^3 + 72*(11550000
*d^7*e^4 + 666750*d^6*e^5 + 23847600*d^5*e^6 + 3625510*d^4*e^7 + 44710245*d
^3*e^8 - 28312225*d^2*e^9 + 79001833*d*e^10 + 245697543*e^11)*m^2 + 720*(63
0000*d^7*e^4 + 34650*d^6*e^5 + 1178100*d^5*e^6 + 169785*d^4*e^7 + 1978020*d
^3*e^8 - 1178100*d^2*e^9 + 3074148*d*e^10 + 22036147*e^11)*m)*x^4 + 12*(411
60*d^8*e^3 + 2277720*d^7*e^4 - 4105500*d^6*e^5 + 26582730*d^5*e^6 - 3858686
3*d^4*e^7 + 91855890*d^3*e^8 - 84312180*d^2*e^9 + 157352130*d*e^10)*m^3 + (
6346771200*e^11 + (574*d*e^10 + 477*e^11)*m^10 - (4436*d^2*e^9 - 33866*d*e^
10 - 30051*e^11)*m^9 + 24*(425*d^3*e^8 - 9981*d^2*e^9 + 35875*d*e^10 + 3450
3*e^11)*m^8 - 6*(19980*d^4*e^7 - 81600*d^3*e^8 + 909380*d^2*e^9 - 2053198*d
*e^10 - 2183229*e^11)*m^7 - 3*(27440*d^5*e^6 + 1638360*d^4*e^7 - 3202800*d^
3*e^8 + 22641344*d^2*e^9 - 36198162*d*e^10 - 43730883*e^11)*m^6 - 3*(171360
0*d^6*e^5 + 905520*d^5*e^6 + 26173800*d^4*e^7 - 32844000*d^3*e^8 + 16654074
8*d^2*e^9 - 201988878*d*e^10 - 288179073*e^11)*m^5 - 2*(756000*d^7*e^4 + 61
689600*d^6*e^5 + 16093560*d^5*e^6 + 304195500*d^4*e^7 - 278811900*d^3*e^8 +
 1092467028*d^2*e^9 - 1055996410*d*e^10 - 1884673269*e^11)*m^4 - 4*(7560000
0*d^8*e^3 + 5292000*d^7*e^4 + 224910000*d^6*e^5 + 40069260*d^5*e^6 + 573745
680*d^4*e^7 - 419556600*d^3*e^8 + 1349320300*d^2*e^9 - 1086499918*d*e^10 -
2657980899*e^11)*m^3 - 24*(37800000*d^8*e^3 + 2205000*d^7*e^4 + 79682400*d^
6*e^5 + 12238240*d^5*e^6 + 152467380*d^4*e^7 - 97540900*d^3*e^8 + 275018692
*d^2*e^9 - 193842670*d*e^10 - 763013811*e^11)*m^2 - 4320*(140000*d^8*e^3 +
7700*d^7*e^4 + 261800*d^6*e^5 + 37730*d^5*e^6 + 439560*d^4*e^7 - 261800*d^3
*e^8 + 683144*d^2*e^9 - 441980*d*e^10 - 3946963*e^11)*m)*x^3 + 12*(2570400*
d^9*e^2 + 1234800*d^8*e^3 + 32307660*d^7*e^4 - 36490500*d^6*e^5 + 165294232
*d^5*e^6 - 177258088*d^4*e^7 + 320238402*d^3*e^8 - 224755965*d^2*e^9 + 3163
09212*d*e^10)*m^2 + 3*(898128000*e^11 + 3*(53*d*e^10 + 15*e^11)*m^10 - (574
*d^2*e^9 - 9699*d*e^10 - 2880*e^11)*m^9 + (4436*d^3*e^8 - 32718*d^2*e^9 + 2
56626*d*e^10 + 80865*e^11)*m^8 - 2*(5100*d^4*e^7 - 115336*d^3*e^8 + 397782*
d^2*e^9 - 1926603*d*e^10 - 654210*e^11)*m^7 + (119880*d^5*e^6 - 469200*d^4*
e^7 + 4994936*d^3*e^8 - 10728060*d^2*e^9 + 36024471*d*e^10 + 13467195*e^11)
*m^6 + (82320*d^6*e^5 + 4675320*d^5*e^6 - 8670000*d^4*e^7 + 57934160*d^3*e^
8 - 87138366*d^2*e^9 + 216130131*d*e^10 + 91755720*e^11)*m^5 + (5140800*d^7
*e^4 + 2551920*d^6*e^5 + 69170760*d^5*e^6 - 81192000*d^4*e^7 + 383753924*d^
3*e^8 - 431689902*d^2*e^9 + 824188584*d*e^10 + 416767635*e^11)*m^4 + 4*(378
000*d^8*e^3 + 28274400*d^7*e^4 + 6770820*d^6*e^5 + 117512370*d^5*e^6 - 9880
9950*d^4*e^7 + 354356552*d^3*e^8 - 312153254*d^2*e^9 + 473899341*d*e^10 + 3
09068145*e^11)*m^3 + 12*(25200000*d^9*e^2 + 1512000*d^8*e^3 + 56120400*d^7*
e^4 + 8842540*d^6*e^5 + 112906980*d^5*e^6 - 73978900*d^4*e^7 + 213535732*d^
3*e^8 - 154064470*d^2*e^9 + 192742980*d*e^10 + 188672355*e^11)*m^2 + 2160*(
140000*d^9*e^2 + 7700*d^8*e^3 + 261800*d^7*e^4 + 37730*d^6*e^5 + 439560*d^5
*e^6 - 261800*d^4*e^7 + 683144*d^3*e^8 - 441980*d^2*e^9 + 489720*d*e^10 + 1
047765*e^11)*m)*x^2 + 144*(63000*d^10*e + 4498200*d^9*e^2 + 1025570*d^8*e^3
 + 16893090*d^7*e^4 - 13427450*d^6*e^5 + 45284906*d^5*e^6 - 37254035*d^4*e^
7 + 52296690*d^3*e^8 - 28438425*d^2*e^9 + 30235140*d*e^10)*m + 3*(718502400
*e^11 + 9*(5*d*e^10 + 2*e^11)*m^10 - 3*(106*d^2*e^9 - 945*d*e^10 - 390*e^11
)*m^9 + 2*(574*d^3*e^8 - 9540*d^2*e^9 + 39015*d*e^10 + 16740*e^11)*m^8 - 2*
(4436*d^4*e^7 - 32144*d^3*e^8 + 247086*d^2*e^9 - 615195*d*e^10 - 277290*e^1
1)*m^7 + (20400*d^5*e^6 - 452472*d^4*e^7 + 1526840*d^3*e^8 - 7212240*d^2*e^
9 + 12236805*d*e^10 + 5879034*e^11)*m^6 - (239760*d^6*e^5 - 918000*d^5*e^6
+ 9537400*d^4*e^7 - 19929280*d^3*e^8 + 64836702*d^2*e^9 - 79518915*d*e^10 -
 41597010*e^11)*m^5 - 4*(41160*d^7*e^4 + 2277720*d^6*e^5 - 4105500*d^5*e^6
+ 26582730*d^4*e^7 - 38586863*d^3*e^8 + 91855890*d^2*e^9 - 84312180*d*e^10
```

```
- 49628655*e^11)*m^4 - 4*(2570400*d^8*e^3 + 1234800*d^7*e^4 + 32307660*d^6*
e^5 - 36490500*d^5*e^6 + 165294232*d^4*e^7 - 177258088*d^3*e^8 + 320238402*
d^2*e^9 - 224755965*d*e^10 - 157352130*e^11)*m^3 - 48*(63000*d^9*e^2 + 4498
200*d^8*e^3 + 1025570*d^7*e^4 + 16893090*d^6*e^5 - 13427450*d^5*e^6 + 45284
906*d^4*e^7 - 37254035*d^3*e^8 + 52296690*d^2*e^9 - 28438425*d*e^10 - 26359
101*e^11)*m^2 - 8640*(70000*d^10*e + 3850*d^9*e^2 + 130900*d^8*e^3 + 18865*
d^7*e^4 + 219780*d^6*e^5 - 130900*d^5*e^6 + 341572*d^4*e^7 - 220990*d^3*e^8
 + 244860*d^2*e^9 - 103950*d*e^10 - 167973*e^11)*m)*x)*(e*x + d)^m/(e^11*m^
11 + 66*e^11*m^10 + 1925*e^11*m^9 + 32670*e^11*m^8 + 357423*e^11*m^7 + 2637
558*e^11*m^6 + 13339535*e^11*m^5 + 45995730*e^11*m^4 + 105258076*e^11*m^3 +
 150917976*e^11*m^2 + 120543840*e^11*m + 39916800*e^11)
```

giac [B] time = 0.62, size = 10960, normalized size = 18.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] (500*(x*e + d)^m*m^10*x^11*e^11 + 500*(x*e + d)^m*d*m^10*x^10*e^10 - 25*(x*
e + d)^m*m^10*x^10*e^11 + 27500*(x*e + d)^m*m^9*x^11*e^11 - 25*(x*e + d)^m*
d*m^10*x^9*e^10 + 22500*(x*e + d)^m*d*m^9*x^10*e^10 - 5000*(x*e + d)^m*d^2*
m^9*x^9*e^9 + 765*(x*e + d)^m*m^10*x^9*e^11 - 1400*(x*e + d)^m*m^9*x^10*e^1
1 + 660000*(x*e + d)^m*m^8*x^11*e^11 + 765*(x*e + d)^m*d*m^10*x^8*e^10 - 11
75*(x*e + d)^m*d*m^9*x^9*e^10 + 435000*(x*e + d)^m*d*m^8*x^10*e^10 + 225*(x
*e + d)^m*d^2*m^9*x^8*e^9 - 180000*(x*e + d)^m*d^2*m^8*x^9*e^9 + 45000*(x*e
 + d)^m*d^3*m^8*x^8*e^8 - 98*(x*e + d)^m*m^10*x^8*e^11 + 43605*(x*e + d)^m*
m^9*x^9*e^11 - 34125*(x*e + d)^m*m^8*x^10*e^11 + 9075000*(x*e + d)^m*m^7*x^
11*e^11 - 98*(x*e + d)^m*d*m^10*x^7*e^10 + 37485*(x*e + d)^m*d*m^9*x^8*e^10
 - 23550*(x*e + d)^m*d*m^8*x^9*e^10 + 4725000*(x*e + d)^m*d*m^7*x^10*e^10 -
 6120*(x*e + d)^m*d^2*m^9*x^7*e^9 + 8775*(x*e + d)^m*d^2*m^8*x^8*e^9 - 2730
000*(x*e + d)^m*d^2*m^7*x^9*e^9 - 1800*(x*e + d)^m*d^3*m^8*x^7*e^8 + 126000
0*(x*e + d)^m*d^3*m^7*x^8*e^8 - 360000*(x*e + d)^m*d^4*m^7*x^7*e^7 + 999*(x
*e + d)^m*m^10*x^7*e^11 - 5684*(x*e + d)^m*m^9*x^8*e^11 + 1080180*(x*e + d)
^m*m^8*x^9*e^11 - 475500*(x*e + d)^m*m^7*x^10*e^11 + 78886500*(x*e + d)^m*m
^6*x^11*e^11 + 999*(x*e + d)^m*d*m^10*x^6*e^10 - 4998*(x*e + d)^m*d*m^9*x^7
*e^10 + 780300*(x*e + d)^m*d*m^8*x^8*e^10 - 263550*(x*e + d)^m*d*m^7*x^9*e^
10 + 31636500*(x*e + d)^m*d*m^6*x^10*e^10 + 686*(x*e + d)^m*d^2*m^9*x^6*e^9
 - 257040*(x*e + d)^m*d^2*m^8*x^7*e^9 + 141750*(x*e + d)^m*d^2*m^7*x^8*e^9
- 22680000*(x*e + d)^m*d^2*m^6*x^9*e^9 + 42840*(x*e + d)^m*d^3*m^8*x^6*e^8
- 57600*(x*e + d)^m*d^3*m^7*x^7*e^8 + 14490000*(x*e + d)^m*d^3*m^6*x^8*e^8
+ 12600*(x*e + d)^m*d^4*m^7*x^6*e^7 - 7560000*(x*e + d)^m*d^4*m^6*x^7*e^7 +
 2520000*(x*e + d)^m*d^5*m^6*x^6*e^6 + 510*(x*e + d)^m*m^10*x^6*e^11 + 5894
1*(x*e + d)^m*m^9*x^7*e^11 - 143178*(x*e + d)^m*m^8*x^8*e^11 + 15270930*(x*
e + d)^m*m^7*x^9*e^11 - 4180575*(x*e + d)^m*m^6*x^10*e^11 + 451027500*(x*e
+ d)^m*m^5*x^11*e^11 + 510*(x*e + d)^m*d*m^10*x^5*e^10 + 52947*(x*e + d)^m*
d*m^9*x^6*e^10 - 108192*(x*e + d)^m*d*m^8*x^7*e^10 + 9028530*(x*e + d)^m*d*
m^7*x^8*e^10 - 1808625*(x*e + d)^m*d*m^6*x^9*e^10 + 134662500*(x*e + d)^m*d
*m^5*x^10*e^10 - 5994*(x*e + d)^m*d^2*m^9*x^5*e^9 + 30870*(x*e + d)^m*d^2*m
^8*x^6*e^9 - 4443120*(x*e + d)^m*d^2*m^7*x^7*e^9 + 1237950*(x*e + d)^m*d^2*
m^6*x^8*e^9 - 112245000*(x*e + d)^m*d^2*m^5*x^9*e^9 - 4116*(x*e + d)^m*d^3*
m^8*x^5*e^8 + 1542240*(x*e + d)^m*d^3*m^7*x^6*e^8 - 730800*(x*e + d)^m*d^3*
m^6*x^7*e^8 + 88200000*(x*e + d)^m*d^3*m^5*x^8*e^8 - 257040*(x*e + d)^m*d^4
*m^7*x^5*e^7 + 327600*(x*e + d)^m*d^4*m^6*x^6*e^7 - 63000000*(x*e + d)^m*d^
4*m^5*x^7*e^7 - 75600*(x*e + d)^m*d^5*m^6*x^5*e^6 + 37800000*(x*e + d)^m*d^
5*m^5*x^6*e^6 - 15120000*(x*e + d)^m*d^6*m^5*x^5*e^5 + 1109*(x*e + d)^m*m^1
0*x^5*e^11 + 30600*(x*e + d)^m*m^9*x^6*e^11 + 1510488*(x*e + d)^m*m^8*x^7*e
^11 - 2056236*(x*e + d)^m*m^7*x^8*e^11 + 135990225*(x*e + d)^m*m^6*x^9*e^11
 - 24133200*(x*e + d)^m*m^5*x^10*e^11 + 1708465000*(x*e + d)^m*m^4*x^11*e^1
```

```
1 + 1109*(x*e + d)^m*d*m^10*x^4*e^10 + 28050*(x*e + d)^m*d*m^9*x^5*e^10 + 1
192806*(x*e + d)^m*d*m^8*x^6*e^10 - 1298892*(x*e + d)^m*d*m^7*x^7*e^10 + 63
761985*(x*e + d)^m*d*m^6*x^8*e^10 - 7855575*(x*e + d)^m*d*m^5*x^9*e^10 + 36
1840000*(x*e + d)^m*d*m^4*x^10*e^10 - 2550*(x*e + d)^m*d^2*m^9*x^4*e^9 - 28
7712*(x*e + d)^m*d^2*m^8*x^5*e^9 + 572124*(x*e + d)^m*d^2*m^7*x^6*e^9 - 411
26400*(x*e + d)^m*d^2*m^6*x^7*e^9 + 6374025*(x*e + d)^m*d^2*m^5*x^8*e^9 - 3
36420000*(x*e + d)^m*d^2*m^4*x^9*e^9 + 29970*(x*e + d)^m*d^3*m^8*x^4*e^8 -
164640*(x*e + d)^m*d^3*m^7*x^5*e^8 + 21848400*(x*e + d)^m*d^3*m^6*x^6*e^8 -
 4788000*(x*e + d)^m*d^3*m^5*x^7*e^8 + 304605000*(x*e + d)^m*d^3*m^4*x^8*e^
8 + 20580*(x*e + d)^m*d^4*m^7*x^4*e^7 - 7968240*(x*e + d)^m*d^4*m^6*x^5*e^7
 + 3150000*(x*e + d)^m*d^4*m^5*x^6*e^7 - 264600000*(x*e + d)^m*d^4*m^4*x^7*
e^7 + 1285200*(x*e + d)^m*d^5*m^6*x^4*e^6 - 1587600*(x*e + d)^m*d^5*m^5*x^5
*e^6 + 214200000*(x*e + d)^m*d^5*m^4*x^6*e^6 + 378000*(x*e + d)^m*d^6*m^5*x
^4*e^5 - 151200000*(x*e + d)^m*d^6*m^4*x^5*e^5 + 75600000*(x*e + d)^m*d^7*m
^4*x^4*e^4 + 574*(x*e + d)^m*m^10*x^4*e^11 + 67649*(x*e + d)^m*m^9*x^5*e^11
 + 798150*(x*e + d)^m*m^8*x^6*e^11 + 22063914*(x*e + d)^m*m^7*x^7*e^11 - 18
577566*(x*e + d)^m*m^6*x^8*e^11 + 793819845*(x*e + d)^m*m^5*x^9*e^11 - 9215
6375*(x*e + d)^m*m^4*x^10*e^11 + 4204750000*(x*e + d)^m*m^3*x^11*e^11 + 574
*(x*e + d)^m*d*m^10*x^3*e^10 + 63213*(x*e + d)^m*d*m^9*x^4*e^10 + 657900*(x
*e + d)^m*d*m^8*x^5*e^10 + 14907078*(x*e + d)^m*d*m^7*x^6*e^10 - 9485322*(x
*e + d)^m*d*m^6*x^7*e^10 + 283723965*(x*e + d)^m*d*m^5*x^8*e^10 - 21456200*
(x*e + d)^m*d*m^4*x^9*e^10 + 586350000*(x*e + d)^m*d*m^3*x^10*e^10 - 4436*(
x*e + d)^m*d^2*m^9*x^3*e^9 - 130050*(x*e + d)^m*d^2*m^8*x^4*e^9 - 5718276*(
x*e + d)^m*d^2*m^7*x^5*e^9 + 5659500*(x*e + d)^m*d^2*m^6*x^6*e^9 - 22221108
0*(x*e + d)^m*d^2*m^5*x^7*e^9 + 19707975*(x*e + d)^m*d^2*m^4*x^8*e^9 - 5906
20000*(x*e + d)^m*d^2*m^3*x^9*e^9 + 10200*(x*e + d)^m*d^3*m^8*x^3*e^8 + 131
8680*(x*e + d)^m*d^3*m^7*x^4*e^8 - 2609544*(x*e + d)^m*d^3*m^6*x^5*e^8 + 15
6794400*(x*e + d)^m*d^3*m^5*x^6*e^8 - 17476200*(x*e + d)^m*d^3*m^4*x^7*e^8
+ 590940000*(x*e + d)^m*d^3*m^3*x^8*e^8 - 119880*(x*e + d)^m*d^4*m^7*x^3*e^
7 + 740880*(x*e + d)^m*d^4*m^6*x^4*e^7 - 91249200*(x*e + d)^m*d^4*m^5*x^5*e
^7 + 14616000*(x*e + d)^m*d^4*m^4*x^6*e^7 - 584640000*(x*e + d)^m*d^4*m^3*x
^7*e^7 - 82320*(x*e + d)^m*d^5*m^6*x^3*e^6 + 34700400*(x*e + d)^m*d^5*m^5*x
^4*e^6 - 10962000*(x*e + d)^m*d^5*m^4*x^5*e^6 + 567000000*(x*e + d)^m*d^5*m
^3*x^6*e^6 - 5140800*(x*e + d)^m*d^6*m^5*x^3*e^5 + 6426000*(x*e + d)^m*d^6*
m^4*x^4*e^5 - 529200000*(x*e + d)^m*d^6*m^3*x^5*e^5 - 1512000*(x*e + d)^m*d
^7*m^4*x^3*e^4 + 453600000*(x*e + d)^m*d^7*m^3*x^4*e^4 - 302400000*(x*e + d
)^m*d^8*m^3*x^3*e^3 + 477*(x*e + d)^m*m^10*x^3*e^11 + 35588*(x*e + d)^m*m^9
*x^4*e^11 + 1796580*(x*e + d)^m*m^8*x^5*e^11 + 11872800*(x*e + d)^m*m^7*x^6
*e^11 + 202618179*(x*e + d)^m*m^6*x^7*e^11 - 109860156*(x*e + d)^m*m^5*x^8*
e^11 + 3060365670*(x*e + d)^m*m^4*x^9*e^11 - 228329500*(x*e + d)^m*m^3*x^10
*e^11 + 6376788000*(x*e + d)^m*m^2*x^11*e^11 + 477*(x*e + d)^m*d*m^10*x^2*e
^10 + 33866*(x*e + d)^m*d*m^9*x^3*e^10 + 1543728*(x*e + d)^m*d*m^8*x^4*e^10
 + 8583300*(x*e + d)^m*d*m^7*x^5*e^10 + 113175711*(x*e + d)^m*d*m^6*x^6*e^1
0 - 43462902*(x*e + d)^m*d*m^5*x^7*e^10 + 790573950*(x*e + d)^m*d*m^4*x^8*e
^10 - 35223700*(x*e + d)^m*d*m^3*x^9*e^10 + 513288000*(x*e + d)^m*d*m^2*x^1
0*e^10 - 1722*(x*e + d)^m*d^2*m^9*x^2*e^9 - 239544*(x*e + d)^m*d^2*m^8*x^3*
e^9 - 2769300*(x*e + d)^m*d^2*m^7*x^4*e^9 - 60851088*(x*e + d)^m*d^2*m^6*x^
5*e^9 + 32440254*(x*e + d)^m*d^2*m^5*x^6*e^9 - 714314160*(x*e + d)^m*d^2*m^
4*x^7*e^9 + 35442000*(x*e + d)^m*d^2*m^3*x^8*e^9 - 547920000*(x*e + d)^m*d^
2*m^2*x^9*e^9 + 13308*(x*e + d)^m*d^3*m^8*x^2*e^8 + 489600*(x*e + d)^m*d^3*
m^7*x^3*e^8 + 23316660*(x*e + d)^m*d^3*m^6*x^4*e^8 - 20909280*(x*e + d)^m*d
^3*m^5*x^5*e^8 + 614711160*(x*e + d)^m*d^3*m^4*x^6*e^8 - 35330400*(x*e + d)
^m*d^3*m^3*x^7*e^8 + 588060000*(x*e + d)^m*d^3*m^2*x^8*e^8 - 30600*(x*e + d
)^m*d^4*m^7*x^2*e^7 - 4915080*(x*e + d)^m*d^4*m^6*x^3*e^7 + 10084200*(x*e +
 d)^m*d^4*m^5*x^4*e^7 - 484520400*(x*e + d)^m*d^4*m^4*x^5*e^7 + 34637400*(x
*e + d)^m*d^4*m^3*x^6*e^7 - 635040000*(x*e + d)^m*d^4*m^2*x^7*e^7 + 359640*
(x*e + d)^m*d^5*m^6*x^2*e^6 - 2716560*(x*e + d)^m*d^5*m^5*x^3*e^6 + 3174444
00*(x*e + d)^m*d^5*m^4*x^4*e^6 - 32886000*(x*e + d)^m*d^5*m^3*x^5*e^6 + 690
480000*(x*e + d)^m*d^5*m^2*x^6*e^6 + 246960*(x*e + d)^m*d^6*m^5*x^2*e^5 - 1
```

```
23379200*(x*e + d)^m*d^6*m^4*x^3*e^5 + 29106000*(x*e + d)^m*d^6*m^3*x^4*e^5
 - 756000000*(x*e + d)^m*d^6*m^2*x^5*e^5 + 15422400*(x*e + d)^m*d^7*m^4*x^2
*e^4 - 21168000*(x*e + d)^m*d^7*m^3*x^3*e^4 + 831600000*(x*e + d)^m*d^7*m^2
*x^4*e^4 + 4536000*(x*e + d)^m*d^8*m^3*x^2*e^3 - 907200000*(x*e + d)^m*d^8*
m^2*x^3*e^3 + 907200000*(x*e + d)^m*d^9*m^2*x^2*e^2 + 135*(x*e + d)^m*m^10*
x^2*e^11 + 30051*(x*e + d)^m*m^9*x^3*e^11 + 962598*(x*e + d)^m*m^8*x^4*e^11
 + 27248130*(x*e + d)^m*m^7*x^5*e^11 + 111048930*(x*e + d)^m*m^6*x^6*e^11 +
 1216593189*(x*e + d)^m*m^5*x^7*e^11 - 428393182*(x*e + d)^m*m^4*x^8*e^11 +
 7643442420*(x*e + d)^m*m^3*x^9*e^11 - 348156900*(x*e + d)^m*m^2*x^10*e^11
+ 5314320000*(x*e + d)^m*m*x^11*e^11 + 135*(x*e + d)^m*d*m^10*x*e^10 + 2909
7*(x*e + d)^m*d*m^9*x^2*e^10 + 861000*(x*e + d)^m*d*m^8*x^3*e^10 + 21073218
*(x*e + d)^m*d*m^7*x^4*e^10 + 68132430*(x*e + d)^m*d*m^6*x^5*e^10 + 5375389
23*(x*e + d)^m*d*m^5*x^6*e^10 - 124152868*(x*e + d)^m*d*m^4*x^7*e^10 + 1318
850820*(x*e + d)^m*d*m^3*x^8*e^10 - 31143600*(x*e + d)^m*d*m^2*x^9*e^10 + 1
81440000*(x*e + d)^m*d*m*x^10*e^10 - 954*(x*e + d)^m*d^2*m^9*x*e^9 - 98154*
(x*e + d)^m*d^2*m^8*x^2*e^9 - 5456280*(x*e + d)^m*d^2*m^7*x^3*e^9 - 3183930
0*(x*e + d)^m*d^2*m^6*x^4*e^9 - 374798826*(x*e + d)^m*d^2*m^5*x^5*e^9 + 109
598790*(x*e + d)^m*d^2*m^4*x^6*e^9 - 1324392480*(x*e + d)^m*d^2*m^3*x^7*e^9
 + 33477300*(x*e + d)^m*d^2*m^2*x^8*e^9 - 201600000*(x*e + d)^m*d^2*m*x^9*e
^9 + 3444*(x*e + d)^m*d^3*m^8*x*e^8 + 692016*(x*e + d)^m*d^3*m^7*x^2*e^8 +
9608400*(x*e + d)^m*d^3*m^6*x^3*e^8 + 210988800*(x*e + d)^m*d^3*m^5*x^4*e^8
 - 90095124*(x*e + d)^m*d^3*m^4*x^5*e^8 + 1311932160*(x*e + d)^m*d^3*m^3*x^
6*e^8 - 36223200*(x*e + d)^m*d^3*m^2*x^7*e^8 + 226800000*(x*e + d)^m*d^3*m*
x^8*e^8 - 26616*(x*e + d)^m*d^4*m^7*x*e^7 - 1407600*(x*e + d)^m*d^4*m^6*x^2
*e^7 - 78521400*(x*e + d)^m*d^4*m^5*x^3*e^7 + 64209600*(x*e + d)^m*d^4*m^4*
x^4*e^7 - 1265664960*(x*e + d)^m*d^4*m^3*x^5*e^7 + 39488400*(x*e + d)^m*d^4
*m^2*x^6*e^7 - 259200000*(x*e + d)^m*d^4*m*x^7*e^7 + 61200*(x*e + d)^m*d^5*
m^6*x*e^6 + 14025960*(x*e + d)^m*d^5*m^5*x^2*e^6 - 32187120*(x*e + d)^m*d^5
*m^4*x^3*e^6 + 1152824400*(x*e + d)^m*d^5*m^3*x^4*e^6 - 43394400*(x*e + d)^
m*d^5*m^2*x^5*e^6 + 302400000*(x*e + d)^m*d^5*m*x^6*e^6 - 719280*(x*e + d)^
m*d^6*m^5*x*e^5 + 7655760*(x*e + d)^m*d^6*m^4*x^2*e^5 - 899640000*(x*e + d)
^m*d^6*m^3*x^3*e^5 + 48006000*(x*e + d)^m*d^6*m^2*x^4*e^5 - 362880000*(x*e
+ d)^m*d^6*m*x^5*e^5 - 493920*(x*e + d)^m*d^7*m^4*x*e^4 + 339292800*(x*e +
d)^m*d^7*m^3*x^2*e^4 - 52920000*(x*e + d)^m*d^7*m^2*x^3*e^4 + 453600000*(x*
e + d)^m*d^7*m*x^4*e^4 - 30844800*(x*e + d)^m*d^8*m^3*x*e^3 + 54432000*(x*e
 + d)^m*d^8*m^2*x^2*e^3 - 604800000*(x*e + d)^m*d^8*m*x^3*e^3 - 9072000*(x*
e + d)^m*d^9*m^2*x*e^2 + 907200000*(x*e + d)^m*d^9*m*x^2*e^2 - 1814400000*(
x*e + d)^m*d^10*m*x*e + 54*(x*e + d)^m*m^10*x*e^11 + 8640*(x*e + d)^m*m^9*x
^2*e^11 + 828072*(x*e + d)^m*m^8*x^3*e^11 + 14902188*(x*e + d)^m*m^7*x^4*e^
11 + 260141457*(x*e + d)^m*m^6*x^5*e^11 + 678861000*(x*e + d)^m*m^5*x^6*e^1
1 + 4810043142*(x*e + d)^m*m^4*x^7*e^11 - 1080436084*(x*e + d)^m*m^3*x^8*e^
11 + 11731446360*(x*e + d)^m*m^2*x^9*e^11 - 291380400*(x*e + d)^m*m*x^10*e^
11 + 1814400000*(x*e + d)^m*x^11*e^11 + 54*(x*e + d)^m*d*m^10*e^10 + 8505*(
x*e + d)^m*d*m^9*x*e^10 + 769878*(x*e + d)^m*d*m^8*x^2*e^10 + 12319188*(x*e
 + d)^m*d*m^7*x^3*e^10 + 175848585*(x*e + d)^m*d*m^6*x^4*e^10 + 338198850*(
x*e + d)^m*d*m^5*x^5*e^10 + 1584809604*(x*e + d)^m*d*m^4*x^6*e^10 - 2113660
08*(x*e + d)^m*d*m^3*x^7*e^10 + 1180639800*(x*e + d)^m*d*m^2*x^8*e^10 - 110
88000*(x*e + d)^m*d*m*x^9*e^10 - 135*(x*e + d)^m*d^2*m^9*e^9 - 57240*(x*e +
 d)^m*d^2*m^8*x*e^9 - 2386692*(x*e + d)^m*d^2*m^7*x^2*e^9 - 67924032*(x*e +
 d)^m*d^2*m^6*x^3*e^9 - 213304950*(x*e + d)^m*d^2*m^5*x^4*e^9 - 1351239408*
(x*e + d)^m*d^2*m^4*x^5*e^9 + 211477336*(x*e + d)^m*d^2*m^3*x^6*e^9 - 12800
59200*(x*e + d)^m*d^2*m^2*x^7*e^9 + 12474000*(x*e + d)^m*d^2*m*x^8*e^9 + 95
4*(x*e + d)^m*d^3*m^8*e^8 + 192864*(x*e + d)^m*d^3*m^7*x*e^8 + 14984808*(x*
e + d)^m*d^3*m^6*x^2*e^8 + 98532000*(x*e + d)^m*d^3*m^5*x^3*e^8 + 103003893
0*(x*e + d)^m*d^3*m^4*x^4*e^8 - 207117120*(x*e + d)^m*d^3*m^3*x^5*e^8 + 139
9154400*(x*e + d)^m*d^3*m^2*x^6*e^8 - 14256000*(x*e + d)^m*d^3*m*x^7*e^8 -
3444*(x*e + d)^m*d^4*m^7*e^7 - 1357416*(x*e + d)^m*d^4*m^6*x*e^7 - 26010000
*(x*e + d)^m*d^4*m^5*x^2*e^7 - 608391000*(x*e + d)^m*d^4*m^4*x^3*e^7 + 1936
37220*(x*e + d)^m*d^4*m^3*x^4*e^7 - 1543268160*(x*e + d)^m*d^4*m^2*x^5*e^7
```

+ 16632000*(x*e + d)^m*d^4*m*x^6*e^7 + 26616*(x*e + d)^m*d^5*m^6*e^6 + 2754
000*(x*e + d)^m*d^5*m^5*x*e^6 + 207512280*(x*e + d)^m*d^5*m^4*x^2*e^6 - 160
277040*(x*e + d)^m*d^5*m^3*x^3*e^6 + 1717027200*(x*e + d)^m*d^5*m^2*x^4*e^6
 - 19958400*(x*e + d)^m*d^5*m*x^5*e^6 - 61200*(x*e + d)^m*d^6*m^5*e^5 - 273
32640*(x*e + d)^m*d^6*m^4*x*e^5 + 81249840*(x*e + d)^m*d^6*m^3*x^2*e^5 - 19
12377600*(x*e + d)^m*d^6*m^2*x^3*e^5 + 24948000*(x*e + d)^m*d^6*m*x^4*e^5 +
 719280*(x*e + d)^m*d^7*m^4*e^4 - 14817600*(x*e + d)^m*d^7*m^3*x*e^4 + 2020
334400*(x*e + d)^m*d^7*m^2*x^2*e^4 - 33264000*(x*e + d)^m*d^7*m*x^3*e^4 + 4
93920*(x*e + d)^m*d^8*m^3*e^3 - 647740800*(x*e + d)^m*d^8*m^2*x*e^3 + 49896
000*(x*e + d)^m*d^8*m*x^2*e^3 + 30844800*(x*e + d)^m*d^9*m^2*e^2 - 99792000
*(x*e + d)^m*d^9*m*x*e^2 + 9072000*(x*e + d)^m*d^10*m*e + 1814400000*(x*e +
 d)^m*d^11 + 3510*(x*e + d)^m*m^9*x*e^11 + 242595*(x*e + d)^m*m^8*x^2*e^11
+ 13099374*(x*e + d)^m*m^7*x^3*e^11 + 145552050*(x*e + d)^m*m^6*x^4*e^11 +
1624344537*(x*e + d)^m*m^5*x^5*e^11 + 2729996850*(x*e + d)^m*m^4*x^6*e^11 +
 12279432276*(x*e + d)^m*m^3*x^7*e^11 - 1671802776*(x*e + d)^m*m^2*x^8*e^11
 + 9869234400*(x*e + d)^m*m*x^9*e^11 - 99792000*(x*e + d)^m*x^10*e^11 + 351
0*(x*e + d)^m*d*m^9*e^10 + 234090*(x*e + d)^m*d*m^8*x*e^10 + 11559618*(x*e
+ d)^m*d*m^7*x^2*e^10 + 108594486*(x*e + d)^m*d*m^6*x^3*e^10 + 920950197*(x
*e + d)^m*d*m^5*x^4*e^10 + 1039002600*(x*e + d)^m*d*m^4*x^5*e^10 + 27705746
52*(x*e + d)^m*d*m^3*x^6*e^10 - 192240720*(x*e + d)^m*d*m^2*x^7*e^10 + 4241
16000*(x*e + d)^m*d*m*x^8*e^10 - 8505*(x*e + d)^m*d^2*m^8*e^9 - 1482516*(x*
e + d)^m*d^2*m^7*x*e^9 - 32184180*(x*e + d)^m*d^2*m^6*x^2*e^9 - 499622244*(
x*e + d)^m*d^2*m^5*x^3*e^9 - 837774450*(x*e + d)^m*d^2*m^4*x^4*e^9 - 275266
0584*(x*e + d)^m*d^2*m^3*x^5*e^9 + 210698040*(x*e + d)^m*d^2*m^2*x^6*e^9 -
484704000*(x*e + d)^m*d^2*m*x^7*e^9 + 57240*(x*e + d)^m*d^3*m^7*e^8 + 45805
20*(x*e + d)^m*d^3*m^6*x*e^8 + 173802480*(x*e + d)^m*d^3*m^5*x^2*e^8 + 5576
23800*(x*e + d)^m*d^3*m^4*x^3*e^8 + 2636041320*(x*e + d)^m*d^3*m^3*x^4*e^8
- 233278416*(x*e + d)^m*d^3*m^2*x^5*e^8 + 565488000*(x*e + d)^m*d^3*m*x^6*e
^8 - 192864*(x*e + d)^m*d^4*m^6*e^7 - 28612200*(x*e + d)^m*d^4*m^5*x*e^7 -
243576000*(x*e + d)^m*d^4*m^4*x^2*e^7 - 2294982720*(x*e + d)^m*d^4*m^3*x^3*
e^7 + 261036720*(x*e + d)^m*d^4*m^2*x^4*e^7 - 678585600*(x*e + d)^m*d^4*m*x
^5*e^7 + 1357416*(x*e + d)^m*d^5*m^5*e^6 + 49266000*(x*e + d)^m*d^5*m^4*x*e
^6 + 1410148440*(x*e + d)^m*d^5*m^3*x^2*e^6 - 293717760*(x*e + d)^m*d^5*m^2
*x^3*e^6 + 848232000*(x*e + d)^m*d^5*m*x^4*e^6 - 2754000*(x*e + d)^m*d^6*m^
4*e^5 - 387691920*(x*e + d)^m*d^6*m^3*x*e^5 + 318331440*(x*e + d)^m*d^6*m^2
*x^2*e^5 - 1130976000*(x*e + d)^m*d^6*m*x^3*e^5 + 27332640*(x*e + d)^m*d^7*
m^3*e^4 - 147682080*(x*e + d)^m*d^7*m^2*x*e^4 + 1696464000*(x*e + d)^m*d^7*
m*x^2*e^4 + 14817600*(x*e + d)^m*d^8*m^2*e^3 - 3392928000*(x*e + d)^m*d^8*m
*x*e^3 + 647740800*(x*e + d)^m*d^9*m*e^2 + 99792000*(x*e + d)^m*d^10*e + 10
0440*(x*e + d)^m*m^8*x*e^11 + 3925260*(x*e + d)^m*m^7*x^2*e^11 + 131192649*
(x*e + d)^m*m^6*x^3*e^11 + 931750092*(x*e + d)^m*m^5*x^4*e^11 + 6671821630*
(x*e + d)^m*m^4*x^5*e^11 + 7077841200*(x*e + d)^m*m^3*x^6*e^11 + 1919679199
2*(x*e + d)^m*m^2*x^7*e^11 - 1415539440*(x*e + d)^m*m*x^8*e^11 + 3392928000
*(x*e + d)^m*x^9*e^11 + 100440*(x*e + d)^m*d*m^8*e^10 + 3691170*(x*e + d)^m
*d*m^7*x*e^10 + 108073413*(x*e + d)^m*d*m^6*x^2*e^10 + 605966634*(x*e + d)^
m*d*m^5*x^3*e^10 + 2988020842*(x*e + d)^m*d*m^4*x^4*e^10 + 1882828200*(x*e
+ d)^m*d*m^3*x^5*e^10 + 2573344080*(x*e + d)^m*d*m^2*x^6*e^10 - 69854400*(x
*e + d)^m*d*m*x^7*e^10 - 234090*(x*e + d)^m*d^2*m^7*e^9 - 21636720*(x*e + d
)^m*d^2*m^6*x*e^9 - 261415098*(x*e + d)^m*d^2*m^5*x^2*e^9 - 2184934056*(x*e
 + d)^m*d^2*m^4*x^3*e^9 - 1843915200*(x*e + d)^m*d^2*m^3*x^4*e^9 - 28601449
92*(x*e + d)^m*d^2*m^2*x^5*e^9 + 81496800*(x*e + d)^m*d^2*m*x^6*e^9 + 14825
16*(x*e + d)^m*d^3*m^6*e^8 + 59787840*(x*e + d)^m*d^3*m^5*x*e^8 + 115126177
2*(x*e + d)^m*d^3*m^4*x^2*e^8 + 1678226400*(x*e + d)^m*d^3*m^3*x^3*e^8 + 32
19137640*(x*e + d)^m*d^3*m^2*x^4*e^8 - 97796160*(x*e + d)^m*d^3*m*x^5*e^8 -
 4580520*(x*e + d)^m*d^4*m^5*e^7 - 318992760*(x*e + d)^m*d^4*m^4*x*e^7 - 11
85719400*(x*e + d)^m*d^4*m^3*x^2*e^7 - 3659217120*(x*e + d)^m*d^4*m^2*x^3*e
^7 + 122245200*(x*e + d)^m*d^4*m*x^4*e^7 + 28612200*(x*e + d)^m*d^5*m^4*e^6
 + 437886000*(x*e + d)^m*d^5*m^3*x*e^6 + 4064651280*(x*e + d)^m*d^5*m^2*x^2
e^6 - 162993600(x*e + d)^m*d^5*m*x^3*e^6 - 49266000*(x*e + d)^m*d^6*m^3*e

^5 - 2432604960*(x*e + d)^m*d^6*m^2*x*e^5 + 244490400*(x*e + d)^m*d^6*m*x^2
e^5 + 387691920(x*e + d)^m*d^7*m^2*e^4 - 488980800*(x*e + d)^m*d^7*m*x*e^
4 + 147682080*(x*e + d)^m*d^8*m*e^3 + 3392928000*(x*e + d)^m*d^9*e^2 + 1663
740*(x*e + d)^m*m^7*x*e^11 + 40401585*(x*e + d)^m*m^6*x^2*e^11 + 864537219*
(x*e + d)^m*m^5*x^3*e^11 + 3929892722*(x*e + d)^m*m^4*x^4*e^11 + 1765015642
0*(x*e + d)^m*m^3*x^5*e^11 + 11214571560*(x*e + d)^m*m^2*x^6*e^11 + 1638951
4080*(x*e + d)^m*m*x^7*e^11 - 488980800*(x*e + d)^m*x^8*e^11 + 1663740*(x*e
 + d)^m*d*m^7*e^10 + 36710415*(x*e + d)^m*d*m^6*x*e^10 + 648390393*(x*e + d
)^m*d*m^5*x^2*e^10 + 2111992820*(x*e + d)^m*d*m^4*x^3*e^10 + 5698073052*(x*
e + d)^m*d*m^3*x^4*e^10 + 1800430560*(x*e + d)^m*d*m^2*x^5*e^10 + 949449600
*(x*e + d)^m*d*m*x^6*e^10 - 3691170*(x*e + d)^m*d^2*m^6*e^9 - 194510106*(x*
e + d)^m*d^2*m^5*x*e^9 - 1295069706*(x*e + d)^m*d^2*m^4*x^2*e^9 - 539728120
0*(x*e + d)^m*d^2*m^3*x^3*e^9 - 2038480200*(x*e + d)^m*d^2*m^2*x^4*e^9 - 11
39339520*(x*e + d)^m*d^2*m*x^5*e^9 + 21636720*(x*e + d)^m*d^3*m^5*e^8 + 463
042356*(x*e + d)^m*d^3*m^4*x*e^8 + 4252278624*(x*e + d)^m*d^3*m^3*x^2*e^8 +
 2340981600*(x*e + d)^m*d^3*m^2*x^3*e^8 + 1424174400*(x*e + d)^m*d^3*m*x^4*
e^8 - 59787840*(x*e + d)^m*d^4*m^4*e^7 - 1983530784*(x*e + d)^m*d^4*m^3*x*e
^7 - 2663240400*(x*e + d)^m*d^4*m^2*x^2*e^7 - 1898899200*(x*e + d)^m*d^4*m*
x^3*e^7 + 318992760*(x*e + d)^m*d^5*m^3*e^6 + 1933552800*(x*e + d)^m*d^5*m^
2*x*e^6 + 2848348800*(x*e + d)^m*d^5*m*x^2*e^6 - 437886000*(x*e + d)^m*d^6*
m^2*e^5 - 5696697600*(x*e + d)^m*d^6*m*x*e^5 + 2432604960*(x*e + d)^m*d^7*m
e^4 + 488980800(x*e + d)^m*d^8*e^3 + 17637102*(x*e + d)^m*m^6*x*e^11 + 27
5267160*(x*e + d)^m*m^5*x^2*e^11 + 3769346538*(x*e + d)^m*m^4*x^3*e^11 + 10
681978132*(x*e + d)^m*m^3*x^4*e^11 + 28480424184*(x*e + d)^m*m^2*x^5*e^11 +
 9680738400*(x*e + d)^m*m*x^6*e^11 + 5696697600*(x*e + d)^m*x^7*e^11 + 1763
7102*(x*e + d)^m*d*m^6*e^10 + 238556745*(x*e + d)^m*d*m^5*x*e^10 + 24725657
52*(x*e + d)^m*d*m^4*x^2*e^10 + 4345999672*(x*e + d)^m*d*m^3*x^3*e^10 + 568
8131976*(x*e + d)^m*d*m^2*x^4*e^10 + 678585600*(x*e + d)^m*d*m*x^5*e^10 - 3
6710415*(x*e + d)^m*d^2*m^5*e^9 - 1102270680*(x*e + d)^m*d^2*m^4*x*e^9 - 37
45839048*(x*e + d)^m*d^2*m^3*x^2*e^9 - 6600448608*(x*e + d)^m*d^2*m^2*x^3*e
^9 - 848232000*(x*e + d)^m*d^2*m*x^4*e^9 + 194510106*(x*e + d)^m*d^3*m^4*e^
8 + 2127097056*(x*e + d)^m*d^3*m^3*x*e^8 + 7687286352*(x*e + d)^m*d^3*m^2*x
^2*e^8 + 1130976000*(x*e + d)^m*d^3*m*x^3*e^8 - 463042356*(x*e + d)^m*d^4*m
^3*e^7 - 6521026464*(x*e + d)^m*d^4*m^2*x*e^7 - 1696464000*(x*e + d)^m*d^4*
m*x^2*e^7 + 1983530784*(x*e + d)^m*d^5*m^2*e^6 + 3392928000*(x*e + d)^m*d^5
*m*x*e^6 - 1933552800*(x*e + d)^m*d^6*m*e^5 + 5696697600*(x*e + d)^m*d^7*e^
4 + 124791030*(x*e + d)^m*m^5*x*e^11 + 1250302905*(x*e + d)^m*m^4*x^2*e^11
+ 10631923596*(x*e + d)^m*m^3*x^3*e^11 + 17690223096*(x*e + d)^m*m^2*x^4*e^
11 + 24965914464*(x*e + d)^m*m*x^5*e^11 + 3392928000*(x*e + d)^m*x^6*e^11 +
 124791030*(x*e + d)^m*d*m^5*e^10 + 1011746160*(x*e + d)^m*d*m^4*x*e^10 + 5
686792092*(x*e + d)^m*d*m^3*x^2*e^10 + 4652224080*(x*e + d)^m*d*m^2*x^3*e^1
0 + 2213386560*(x*e + d)^m*d*m*x^4*e^10 - 238556745*(x*e + d)^m*d^2*m^4*e^9
 - 3842860824*(x*e + d)^m*d^2*m^3*x*e^9 - 5546320920*(x*e + d)^m*d^2*m^2*x^
2*e^9 - 2951182080*(x*e + d)^m*d^2*m*x^3*e^9 + 1102270680*(x*e + d)^m*d^3*m
^3*e^8 + 5364581040*(x*e + d)^m*d^3*m^2*x*e^8 + 4426773120*(x*e + d)^m*d^3*
m*x^2*e^8 - 2127097056*(x*e + d)^m*d^4*m^2*e^7 - 8853546240*(x*e + d)^m*d^4
*m*x*e^7 + 6521026464*(x*e + d)^m*d^5*m*e^6 - 3392928000*(x*e + d)^m*d^6*e^
5 + 595543860*(x*e + d)^m*m^4*x*e^11 + 3708817740*(x*e + d)^m*m^3*x^2*e^11
+ 18312331464*(x*e + d)^m*m^2*x^3*e^11 + 15866025840*(x*e + d)^m*m*x^4*e^11
 + 8853546240*(x*e + d)^m*x^5*e^11 + 595543860*(x*e + d)^m*d*m^4*e^10 + 269
7071580*(x*e + d)^m*d*m^3*x*e^10 + 6938747280*(x*e + d)^m*d*m^2*x^2*e^10 +
1909353600*(x*e + d)^m*d*m*x^3*e^10 - 1011746160*(x*e + d)^m*d^2*m^3*e^9 -
7530723360*(x*e + d)^m*d^2*m^2*x*e^9 - 2864030400*(x*e + d)^m*d^2*m*x^2*e^9
 + 3842860824*(x*e + d)^m*d^3*m^2*e^8 + 5728060800*(x*e + d)^m*d^3*m*x*e^8
- 5364581040*(x*e + d)^m*d^4*m*e^7 + 8853546240*(x*e + d)^m*d^5*e^6 + 18882
25560*(x*e + d)^m*m^3*x*e^11 + 6792204780*(x*e + d)^m*m^2*x^2*e^11 + 170508
80160*(x*e + d)^m*m*x^3*e^11 + 5728060800*(x*e + d)^m*x^4*e^11 + 1888225560
*(x*e + d)^m*d*m^3*e^10 + 4095133200*(x*e + d)^m*d*m^2*x*e^10 + 3173385600*
(x*e + d)^m*d*m*x^2*e^10 - 2697071580*(x*e + d)^m*d^2*m^2*e^9 - 6346771200*

```
(x*e + d)^m*d^2*m*x*e^9 + 7530723360*(x*e + d)^m*d^3*m*e^8 - 5728060800*(x*
e + d)^m*d^4*e^7 + 3795710544*(x*e + d)^m*m^2*x*e^11 + 6789517200*(x*e + d)
^m*m*x^2*e^11 + 6346771200*(x*e + d)^m*x^3*e^11 + 3795710544*(x*e + d)^m*d*
m^2*e^10 + 2694384000*(x*e + d)^m*d*m*x*e^10 - 4095133200*(x*e + d)^m*d^2*m
*e^9 + 6346771200*(x*e + d)^m*d^3*e^8 + 4353860160*(x*e + d)^m*m*x*e^11 + 2
694384000*(x*e + d)^m*x^2*e^11 + 4353860160*(x*e + d)^m*d*m*e^10 - 26943840
00*(x*e + d)^m*d^2*e^9 + 2155507200*(x*e + d)^m*x*e^11 + 2155507200*(x*e +
d)^m*d*e^10)/(m^11*e^11 + 66*m^10*e^11 + 1925*m^9*e^11 + 32670*m^8*e^11 + 3
57423*m^7*e^11 + 2637558*m^6*e^11 + 13339535*m^5*e^11 + 45995730*m^4*e^11 +
 105258076*m^3*e^11 + 150917976*m^2*e^11 + 120543840*m*e^11 + 39916800*e^11
)
```

maple [**B**] time = 0.08, size = 5924, normalized size = 10.07

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] result too large to display
```

maxima [**B**] time = 0.74, size = 2292, normalized size = 3.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 135*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) + 5
4*(e*x + d)^(m + 1)/(e*(m + 1)) + 477*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*
d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e^3)
 + 574*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 -
3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*m^3 +
 35*m^2 + 50*m + 24)*e^4) + 1109*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x
^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3
*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m/((m^5
+ 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + 510*((m^5 + 15*m^4 + 85*m
^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*
m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2
 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(
e*x + d)^m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^
6) + 999*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^7*
x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*x^6 - 6*(m^
5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 + 11*m^2
 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2 + m)*d
^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 + 28*m^6 + 322*m^5
+ 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7) - 98*((m^7 + 28*m^
6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^8*x^8 + (
m^7 + 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d*e^7*x^7 -
 7*(m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^2*e^6*x^6 + 42*(m^
5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^3*e^5*x^5 - 210*(m^4 + 6*m^3 + 11*m^
2 + 6*m)*d^4*e^4*x^4 + 840*(m^3 + 3*m^2 + 2*m)*d^5*e^3*x^3 - 2520*(m^2 + m)
*d^6*e^2*x^2 + 5040*d^7*e*m*x - 5040*d^8)*(e*x + d)^m/((m^8 + 36*m^7 + 546*
m^6 + 4536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^2 + 109584*m + 40320)*e^8
) + 765*((m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67284*m^3 + 11812
4*m^2 + 109584*m + 40320)*e^9*x^9 + (m^8 + 28*m^7 + 322*m^6 + 1960*m^5 + 67
69*m^4 + 13132*m^3 + 13068*m^2 + 5040*m)*d*e^8*x^8 - 8*(m^7 + 21*m^6 + 175*
m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d^2*e^7*x^7 + 56*(m^6 + 15*m^5
 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^3*e^6*x^6 - 336*(m^5 + 10*m^4 + 35
```

```
*m^3 + 50*m^2 + 24*m)*d^4*e^5*x^5 + 1680*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^5*e
^4*x^4 - 6720*(m^3 + 3*m^2 + 2*m)*d^6*e^3*x^3 + 20160*(m^2 + m)*d^7*e^2*x^2
 - 40320*d^8*e*m*x + 40320*d^9)*(e*x + d)^m/((m^9 + 45*m^8 + 870*m^7 + 9450
*m^6 + 63273*m^5 + 269325*m^4 + 723680*m^3 + 1172700*m^2 + 1026576*m + 3628
80)*e^9) - 25*((m^9 + 45*m^8 + 870*m^7 + 9450*m^6 + 63273*m^5 + 269325*m^4
+ 723680*m^3 + 1172700*m^2 + 1026576*m + 362880)*e^10*x^10 + (m^9 + 36*m^8
+ 546*m^7 + 4536*m^6 + 22449*m^5 + 67284*m^4 + 118124*m^3 + 109584*m^2 + 40
320*m)*d*e^9*x^9 - 9*(m^8 + 28*m^7 + 322*m^6 + 1960*m^5 + 6769*m^4 + 13132*
m^3 + 13068*m^2 + 5040*m)*d^2*e^8*x^8 + 72*(m^7 + 21*m^6 + 175*m^5 + 735*m^
4 + 1624*m^3 + 1764*m^2 + 720*m)*d^3*e^7*x^7 - 504*(m^6 + 15*m^5 + 85*m^4 +
 225*m^3 + 274*m^2 + 120*m)*d^4*e^6*x^6 + 3024*(m^5 + 10*m^4 + 35*m^3 + 50*
m^2 + 24*m)*d^5*e^5*x^5 - 15120*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^6*e^4*x^4 +
60480*(m^3 + 3*m^2 + 2*m)*d^7*e^3*x^3 - 181440*(m^2 + m)*d^8*e^2*x^2 + 3628
80*d^9*e*m*x - 362880*d^10)*(e*x + d)^m/((m^10 + 55*m^9 + 1320*m^8 + 18150*
m^7 + 157773*m^6 + 902055*m^5 + 3416930*m^4 + 8409500*m^3 + 12753576*m^2 +
10628640*m + 3628800)*e^10) + 500*((m^10 + 55*m^9 + 1320*m^8 + 18150*m^7 +
157773*m^6 + 902055*m^5 + 3416930*m^4 + 8409500*m^3 + 12753576*m^2 + 106286
40*m + 3628800)*e^11*x^11 + (m^10 + 45*m^9 + 870*m^8 + 9450*m^7 + 63273*m^6
 + 269325*m^5 + 723680*m^4 + 1172700*m^3 + 1026576*m^2 + 362880*m)*d*e^10*x
^10 - 10*(m^9 + 36*m^8 + 546*m^7 + 4536*m^6 + 22449*m^5 + 67284*m^4 + 11812
4*m^3 + 109584*m^2 + 40320*m)*d^2*e^9*x^9 + 90*(m^8 + 28*m^7 + 322*m^6 + 19
60*m^5 + 6769*m^4 + 13132*m^3 + 13068*m^2 + 5040*m)*d^3*e^8*x^8 - 720*(m^7
+ 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d^4*e^7*x^7 + 5
040*(m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^5*e^6*x^6 - 30240
*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^6*e^5*x^5 + 151200*(m^4 + 6*m^3
+ 11*m^2 + 6*m)*d^7*e^4*x^4 - 604800*(m^3 + 3*m^2 + 2*m)*d^8*e^3*x^3 + 1814
400*(m^2 + m)*d^9*e^2*x^2 - 3628800*d^10*e*m*x + 3628800*d^11)*(e*x + d)^m/
((m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 + 2637558*m^6 + 133395
35*m^5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2 + 120543840*m + 39916
800)*e^11)
```

mupad [B] time = 8.39, size = 4341, normalized size = 7.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(2*x + 5*x^2 + 3)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] (500*x^11*(d + e*x)^m*(10628640*m + 12753576*m^2 + 8409500*m^3 + 3416930*m^
4 + 902055*m^5 + 157773*m^6 + 18150*m^7 + 1320*m^8 + 55*m^9 + m^10 + 362880
0))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*
m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39
916800) + ((d + e*x)^m*(2155507200*d*e^10 + 99792000*d^10*e + 1814400000*d^
11 - 2694384000*d^2*e^9 + 6346771200*d^3*e^8 - 5728060800*d^4*e^7 + 8853546
240*d^5*e^6 - 3392928000*d^6*e^5 + 5696697600*d^7*e^4 + 488980800*d^8*e^3 +
 3392928000*d^9*e^2 - 4095133200*d^2*e^9*m + 7530723360*d^3*e^8*m - 5364581
040*d^4*e^7*m + 6521026464*d^5*e^6*m - 1933552800*d^6*e^5*m + 2432604960*d^
7*e^4*m + 147682080*d^8*e^3*m + 647740800*d^9*e^2*m + 3795710544*d*e^10*m^2
 + 1888225560*d*e^10*m^3 + 595543860*d*e^10*m^4 + 124791030*d*e^10*m^5 + 17
637102*d*e^10*m^6 + 1663740*d*e^10*m^7 + 100440*d*e^10*m^8 + 3510*d*e^10*m^
9 + 54*d*e^10*m^10 - 2697071580*d^2*e^9*m^2 + 3842860824*d^3*e^8*m^2 - 2127
097056*d^4*e^7*m^2 + 1983530784*d^5*e^6*m^2 - 437886000*d^6*e^5*m^2 + 38769
1920*d^7*e^4*m^2 + 14817600*d^8*e^3*m^2 + 30844800*d^9*e^2*m^2 - 1011746160
*d^2*e^9*m^3 + 1102270680*d^3*e^8*m^3 - 463042356*d^4*e^7*m^3 + 318992760*d
^5*e^6*m^3 - 49266000*d^6*e^5*m^3 + 27332640*d^7*e^4*m^3 + 493920*d^8*e^3*m
^3 - 238556745*d^2*e^9*m^4 + 194510106*d^3*e^8*m^4 - 59787840*d^4*e^7*m^4 +
 28612200*d^5*e^6*m^4 - 2754000*d^6*e^5*m^4 + 719280*d^7*e^4*m^4 - 36710415
*d^2*e^9*m^5 + 21636720*d^3*e^8*m^5 - 4580520*d^4*e^7*m^5 + 1357416*d^5*e^6
*m^5 - 61200*d^6*e^5*m^5 - 3691170*d^2*e^9*m^6 + 1482516*d^3*e^8*m^6 - 1928
64*d^4*e^7*m^6 + 26616*d^5*e^6*m^6 - 234090*d^2*e^9*m^7 + 57240*d^3*e^8*m^7
```

```
 - 3444*d^4*e^7*m^7 - 8505*d^2*e^9*m^8 + 954*d^3*e^8*m^8 - 135*d^2*e^9*m^9
+ 4353860160*d*e^10*m + 9072000*d^10*e*m))/(e^11*(120543840*m + 150917976*m
^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7
 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (x*(d + e*x)^m*(435
3860160*e^11*m + 2155507200*e^11 + 3795710544*e^11*m^2 + 1888225560*e^11*m^
3 + 595543860*e^11*m^4 + 124791030*e^11*m^5 + 17637102*e^11*m^6 + 1663740*e
^11*m^7 + 100440*e^11*m^8 + 3510*e^11*m^9 + 54*e^11*m^10 - 6346771200*d^2*e
^9*m + 5728060800*d^3*e^8*m - 8853546240*d^4*e^7*m + 3392928000*d^5*e^6*m -
 5696697600*d^6*e^5*m - 488980800*d^7*e^4*m - 3392928000*d^8*e^3*m - 997920
00*d^9*e^2*m + 4095133200*d*e^10*m^2 + 2697071580*d*e^10*m^3 + 1011746160*d
*e^10*m^4 + 238556745*d*e^10*m^5 + 36710415*d*e^10*m^6 + 3691170*d*e^10*m^7
 + 234090*d*e^10*m^8 + 8505*d*e^10*m^9 + 135*d*e^10*m^10 - 7530723360*d^2*e
^9*m^2 + 5364581040*d^3*e^8*m^2 - 6521026464*d^4*e^7*m^2 + 1933552800*d^5*e
^6*m^2 - 2432604960*d^6*e^5*m^2 - 147682080*d^7*e^4*m^2 - 647740800*d^8*e^3
*m^2 - 9072000*d^9*e^2*m^2 - 3842860824*d^2*e^9*m^3 + 2127097056*d^3*e^8*m^
3 - 1983530784*d^4*e^7*m^3 + 437886000*d^5*e^6*m^3 - 387691920*d^6*e^5*m^3
- 14817600*d^7*e^4*m^3 - 30844800*d^8*e^3*m^3 - 1102270680*d^2*e^9*m^4 + 46
3042356*d^3*e^8*m^4 - 318992760*d^4*e^7*m^4 + 49266000*d^5*e^6*m^4 - 273326
40*d^6*e^5*m^4 - 493920*d^7*e^4*m^4 - 194510106*d^2*e^9*m^5 + 59787840*d^3*
e^8*m^5 - 28612200*d^4*e^7*m^5 + 2754000*d^5*e^6*m^5 - 719280*d^6*e^5*m^5 -
 21636720*d^2*e^9*m^6 + 4580520*d^3*e^8*m^6 - 1357416*d^4*e^7*m^6 + 61200*d
^5*e^6*m^6 - 1482516*d^2*e^9*m^7 + 192864*d^3*e^8*m^7 - 26616*d^4*e^7*m^7 -
 57240*d^2*e^9*m^8 + 3444*d^3*e^8*m^8 - 954*d^2*e^9*m^9 + 2694384000*d*e^10
*m - 1814400000*d^10*e*m))/(e^11*(120543840*m + 150917976*m^2 + 105258076*m
^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1
925*m^9 + 66*m^10 + m^11 + 39916800)) + (x^8*(d + e*x)^m*(13068*m + 13132*m
^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)*(45000*d^3*m - 29
302*e^3*m - 97020*e^3 - 2940*e^3*m^2 - 98*e^3*m^3 + 16065*d*e^2*m^2 + 225*d
^2*e*m^2 + 765*d*e^2*m^3 + 84150*d*e^2*m + 2475*d^2*e*m))/(e^3*(120543840*m
 + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^
6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (3*x^
2*(m + 1)*(d + e*x)^m*(302400000*d^9*m + 1365044400*e^9*m + 898128000*e^9 +
 899023860*e^9*m^2 + 337248720*e^9*m^3 + 79518915*e^9*m^4 + 12236805*e^9*m^
5 + 1230390*e^9*m^6 + 78030*e^9*m^7 + 2835*e^9*m^8 + 45*e^9*m^9 - 954676800
*d^2*e^7*m + 1475591040*d^3*e^6*m - 565488000*d^4*e^5*m + 949449600*d^5*e^4
*m + 81496800*d^6*e^3*m + 565488000*d^7*e^2*m + 1255120560*d*e^8*m^2 + 1512
000*d^8*e*m^2 + 640476804*d*e^8*m^3 + 183711780*d*e^8*m^4 + 32418351*d*e^8*
m^5 + 3606120*d*e^8*m^6 + 247086*d*e^8*m^7 + 9540*d*e^8*m^8 + 159*d*e^8*m^9
 - 894096840*d^2*e^7*m^2 + 1086837744*d^3*e^6*m^2 - 322258800*d^4*e^5*m^2 +
 405434160*d^5*e^4*m^2 + 24613680*d^6*e^3*m^2 + 107956800*d^7*e^2*m^2 - 354
516176*d^2*e^7*m^3 + 330588464*d^3*e^6*m^3 - 72981000*d^4*e^5*m^3 + 6461532
0*d^5*e^4*m^3 + 2469600*d^6*e^3*m^3 + 5140800*d^7*e^2*m^3 - 77173726*d^2*e^
7*m^4 + 53165460*d^3*e^6*m^4 - 8211000*d^4*e^5*m^4 + 4555440*d^5*e^4*m^4 +
82320*d^6*e^3*m^4 - 9964640*d^2*e^7*m^5 + 4768700*d^3*e^6*m^5 - 459000*d^4*
e^5*m^5 + 119880*d^5*e^4*m^5 - 763420*d^2*e^7*m^6 + 226236*d^3*e^6*m^6 - 10
200*d^4*e^5*m^6 - 32144*d^2*e^7*m^7 + 4436*d^3*e^6*m^7 - 574*d^2*e^7*m^8 +
1057795200*d*e^8*m + 16632000*d^8*e*m))/(e^9*(120543840*m + 150917976*m^2 +
 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 3
2670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (x^6*(d + e*x)^m*(274*m
 + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)*(2520000*d^5*m + 16112940*e^5*m +
 28274400*e^5 + 3649050*e^5*m^2 + 410550*e^5*m^3 + 22950*e^5*m^4 + 510*e^5*
m^5 + 679140*d^2*e^3*m + 4712400*d^3*e^2*m + 3378618*d*e^4*m^2 + 12600*d^4*
e*m^2 + 538461*d*e^4*m^3 + 37962*d*e^4*m^4 + 999*d*e^4*m^5 + 205114*d^2*e^3
*m^2 + 899640*d^3*e^2*m^2 + 20580*d^2*e^3*m^3 + 42840*d^3*e^2*m^3 + 686*d^2
*e^3*m^4 + 7912080*d*e^4*m + 138600*d^4*e*m))/(e^5*(120543840*m + 150917976
*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m
^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (x^3*(d + e*x)^m*
(3*m + m^2 + 2)*(3765361680*e^8*m - 302400000*d^8*m + 3173385600*e^8 + 1921
430412*e^8*m^2 + 551135340*e^8*m^3 + 97255053*e^8*m^4 + 10818360*e^8*m^5 +
```

```
741258*e^8*m^6 + 28620*e^8*m^7 + 477*e^8*m^8 - 1475591040*d^2*e^6*m + 56548
8000*d^3*e^5*m - 949449600*d^4*e^4*m - 81496800*d^5*e^3*m - 565488000*d^6*e
^2*m + 894096840*d*e^7*m^2 - 1512000*d^7*e*m^2 + 354516176*d*e^7*m^3 + 7717
3726*d*e^7*m^4 + 9964640*d*e^7*m^5 + 763420*d*e^7*m^6 + 32144*d*e^7*m^7 + 5
74*d*e^7*m^8 - 1086837744*d^2*e^6*m^2 + 322258800*d^3*e^5*m^2 - 405434160*d
^4*e^4*m^2 - 24613680*d^5*e^3*m^2 - 107956800*d^6*e^2*m^2 - 330588464*d^2*e
^6*m^3 + 72981000*d^3*e^5*m^3 - 64615320*d^4*e^4*m^3 - 2469600*d^5*e^3*m^3
- 5140800*d^6*e^2*m^3 - 53165460*d^2*e^6*m^4 + 8211000*d^3*e^5*m^4 - 455544
0*d^4*e^4*m^4 - 82320*d^5*e^3*m^4 - 4768700*d^2*e^6*m^5 + 459000*d^3*e^5*m^
5 - 119880*d^4*e^4*m^5 - 226236*d^2*e^6*m^6 + 10200*d^3*e^5*m^6 - 4436*d^2*
e^6*m^7 + 954676800*d*e^7*m - 16632000*d^7*e*m))/(e^8*(120543840*m + 150917
976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 35742
3*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (x^4*(d + e*x)
^m*(11*m + 6*m^2 + m^3 + 6)*(75600000*d^7*m + 894096840*e^7*m + 954676800*e
^7 + 354516176*e^7*m^2 + 77173726*e^7*m^3 + 9964640*e^7*m^4 + 763420*e^7*m^
5 + 32144*e^7*m^6 + 574*e^7*m^7 - 141372000*d^2*e^5*m + 237362400*d^3*e^4*m
 + 20374200*d^4*e^3*m + 141372000*d^5*e^2*m + 271709436*d*e^6*m^2 + 378000*
d^6*e*m^2 + 82647116*d*e^6*m^3 + 13291365*d*e^6*m^4 + 1192175*d*e^6*m^5 + 5
6559*d*e^6*m^6 + 1109*d*e^6*m^7 - 80564700*d^2*e^5*m^2 + 101358540*d^3*e^4*
m^2 + 6153420*d^4*e^3*m^2 + 26989200*d^5*e^2*m^2 - 18245250*d^2*e^5*m^3 + 1
6153830*d^3*e^4*m^3 + 617400*d^4*e^3*m^3 + 1285200*d^5*e^2*m^3 - 2052750*d^
2*e^5*m^4 + 1138860*d^3*e^4*m^4 + 20580*d^4*e^3*m^4 - 114750*d^2*e^5*m^5 +
29970*d^3*e^4*m^5 - 2550*d^2*e^5*m^6 + 368897760*d*e^6*m + 4158000*d^6*e*m)
)/(e^7*(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 133395
35*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 +
 39916800)) - (25*x^10*(d + e*x)^m*(11*e - 20*d*m + e*m)*(1026576*m + 11727
00*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8
+ m^9 + 362880))/(e*(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730
*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*
m^10 + m^11 + 39916800)) - (x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 +
 24)*(15120000*d^6*m - 271709436*e^6*m - 368897760*e^6 - 82647116*e^6*m^2 -
 13291365*e^6*m^3 - 1192175*e^6*m^4 - 56559*e^6*m^5 - 1109*e^6*m^6 + 474724
80*d^2*e^4*m + 4074840*d^3*e^3*m + 28274400*d^4*e^2*m - 16112940*d*e^5*m^2
+ 75600*d^5*e*m^2 - 3649050*d*e^5*m^3 - 410550*d*e^5*m^4 - 22950*d*e^5*m^5
- 510*d*e^5*m^6 + 20271708*d^2*e^4*m^2 + 1230684*d^3*e^3*m^2 + 5397840*d^4*
e^2*m^2 + 3230766*d^2*e^4*m^3 + 123480*d^3*e^3*m^3 + 257040*d^4*e^2*m^3 + 2
27772*d^2*e^4*m^4 + 4116*d^3*e^3*m^4 + 5994*d^2*e^4*m^5 - 28274400*d*e^5*m
+ 831600*d^5*e*m))/(e^6*(120543840*m + 150917976*m^2 + 105258076*m^3 + 4599
5730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 +
 66*m^10 + m^11 + 39916800)) - (x^7*(d + e*x)^m*(1764*m + 1624*m^2 + 735*m^
3 + 175*m^4 + 21*m^5 + m^6 + 720)*(360000*d^4*m - 3378618*e^4*m - 7912080*e
^4 - 538461*e^4*m^2 - 37962*e^4*m^3 - 999*e^4*m^4 + 673200*d^2*e^2*m + 2930
2*d*e^3*m^2 + 1800*d^3*e*m^2 + 2940*d*e^3*m^3 + 98*d*e^3*m^4 + 128520*d^2*e
^2*m^2 + 6120*d^2*e^2*m^3 + 97020*d*e^3*m + 19800*d^3*e*m))/(e^4*(120543840
*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*
m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) - (5*
x^9*(d + e*x)^m*(1000*d^2*m - 3213*e^2*m - 16830*e^2 - 153*e^2*m^2 + 55*d*e
*m + 5*d*e*m^2)*(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 +
 546*m^6 + 36*m^7 + m^8 + 40320))/(e^2*(120543840*m + 150917976*m^2 + 10525
8076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m
^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(5*x**2+2*x+3)**3*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] Timed out
```

3.368 $\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx$

Optimal. Leaf size=432

$$\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{m+7}}{e^9(m+7)}-\frac{\left(5600d^3+945d^2e+666de^2+37e^3\right)(d+ex)^{m+6}}{e^9(m+6)}+\frac{\left(5d^2-2de+3e^2\right)^2\left(4d\right.}{}$$

```
[Out] (5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/e
^9/(1+m)-(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*d^3*e^2-4*d^2*e^3+64*d*e
^4-11*e^5)*(e*x+d)^(2+m)/e^9/(2+m)+(2800*d^6+945*d^5*e+1665*d^4*e^2+370*d^3
*e^3+888*d^2*e^4-195*d*e^5+107*e^6)*(e*x+d)^(3+m)/e^9/(3+m)-(5600*d^5+1575*
d^4*e+2220*d^3*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*(e*x+d)^(4+m)/e^9/(4+m)+(7
000*d^4+1575*d^3*e+1665*d^2*e^2+185*d*e^3+148*e^4)*(e*x+d)^(5+m)/e^9/(5+m)-
(5600*d^3+945*d^2*e+666*d*e^2+37*e^3)*(e*x+d)^(6+m)/e^9/(6+m)+(2800*d^2+315
*d*e+111*e^2)*(e*x+d)^(7+m)/e^9/(7+m)-5*(160*d+9*e)*(e*x+d)^(8+m)/e^9/(8+m)
+100*(e*x+d)^(9+m)/e^9/(9+m)
```

Rubi [A] time = 0.24, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1628}

$$\frac{\left(5d^2-2de+3e^2\right)^2\left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)(d+ex)^{m+1}}{e^9(m+1)}-\frac{\left(5d^2-2de+3e^2\right)\left(88d^3e^2-4d^2e^3+127d^4e+1\right.}{e^9(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^(1 + m))/(e^9*(1 + m)) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^
4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x)^(2 + m))/(e^9*(
2 + m)) + ((2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4
 - 195*d*e^5 + 107*e^6)*(d + e*x)^(3 + m))/(e^9*(3 + m)) - ((5600*d^5 + 157
5*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*(d + e*x)^(4 + m
))/(e^9*(4 + m)) + ((7000*d^4 + 1575*d^3*e + 1665*d^2*e^2 + 185*d*e^3 + 148
*e^4)*(d + e*x)^(5 + m))/(e^9*(5 + m)) - ((5600*d^3 + 945*d^2*e + 666*d*e^2
 + 37*e^3)*(d + e*x)^(6 + m))/(e^9*(6 + m)) + ((2800*d^2 + 315*d*e + 111*e^
2)*(d + e*x)^(7 + m))/(e^9*(7 + m)) - (5*(160*d + 9*e)*(d + e*x)^(8 + m))/(
e^9*(8 + m)) + (100*(d + e*x)^(9 + m))/(e^9*(9 + m))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx = \int \left(\frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+\right.}{e^8}\right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{e^9(1+m)}$$

Mathematica [A] time = 0.24, size = 391, normalized size = 0.91

$$(d+ex)^{m+1}\left(\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^6}{m+7}-\frac{\left(5600d^3+945d^2e+666de^2+37e^3\right)(d+ex)^5}{m+6}+\frac{\left(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4\right)(d+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] ((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2
 - d*e^3 + 2*e^4))/(1 + m) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e
+ 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x))/(2 + m) + ((2800*d
^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107
*e^6)*(d + e*x)^2)/(3 + m) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d
^2*e^3 + 592*d*e^4 - 65*e^5)*(d + e*x)^3)/(4 + m) + ((7000*d^4 + 1575*d^3*e
 + 1665*d^2*e^2 + 185*d*e^3 + 148*e^4)*(d + e*x)^4)/(5 + m) - ((5600*d^3 +
945*d^2*e + 666*d*e^2 + 37*e^3)*(d + e*x)^5)/(6 + m) + ((2800*d^2 + 315*d*e
 + 111*e^2)*(d + e*x)^6)/(7 + m) - (5*(160*d + 9*e)*(d + e*x)^7)/(8 + m) +
(100*(d + e*x)^8)/(9 + m)))/e^9
```

fricas [B] time = 0.97, size = 2796, normalized size = 6.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] (18*d*e^8*m^8 + 100*(e^9*m^8 + 36*e^9*m^7 + 546*e^9*m^6 + 4536*e^9*m^5 + 22
449*e^9*m^4 + 67284*e^9*m^3 + 118124*e^9*m^2 + 109584*e^9*m + 40320*e^9)*x^
9 + 4032000*d^9 + 2041200*d^8*e + 5754240*d^7*e^2 + 2237760*d^6*e^3 + 10741
248*d^5*e^4 - 5896800*d^4*e^5 + 12942720*d^3*e^6 - 5987520*d^2*e^7 + 653184
0*d*e^8 - 5*(408240*e^9 - (20*d*e^8 - 9*e^9)*m^8 - (560*d*e^8 - 333*e^9)*m^
7 - 14*(460*d*e^8 - 369*e^9)*m^6 - 14*(2800*d*e^8 - 3123*e^9)*m^5 - 7*(1934
0*d*e^8 - 31383*e^9)*m^4 - 7*(37520*d*e^8 - 95211*e^9)*m^3 - 216*(1210*d*e^
8 - 5469*e^9)*m^2 - 36*(2800*d*e^8 - 30663*e^9)*m)*x^8 - 33*(d^2*e^7 - 24*d
*e^8)*m^7 + (5754240*e^9 - 3*(15*d*e^8 - 37*e^9)*m^8 - 2*(400*d^2*e^7 + 675
*d*e^8 - 2109*e^9)*m^7 - 12*(1400*d^2*e^7 + 1365*d*e^8 - 5587*e^9)*m^6 - 14
*(10000*d^2*e^7 + 7425*d*e^8 - 41403*e^9)*m^5 - 21*(28000*d^2*e^7 + 17655*d
*e^8 - 141229*e^9)*m^4 - 28*(46400*d^2*e^7 + 26325*d*e^8 - 326229*e^9)*m^3
- 36*(39200*d^2*e^7 + 20745*d*e^8 - 455211*e^9)*m^2 - 144*(4000*d^2*e^7 + 2
025*d*e^8 - 107337*e^9)*m)*x^7 + 2*(107*d^3*e^6 - 693*d^2*e^7 + 7434*d*e^8)
*m^6 - (2237760*e^9 - 37*(3*d*e^8 - e^9)*m^8 - 3*(105*d^2*e^7 + 1184*d*e^8
- 481*e^9)*m^7 - 4*(1400*d^3*e^6 + 1890*d^2*e^7 + 11433*d*e^8 - 5883*e^9)*m
^6 - 6*(14000*d^3*e^6 + 11550*d^2*e^7 + 50875*d*e^8 - 34743*e^9)*m^5 - (476
000*d^3*e^6 + 311850*d^2*e^7 + 1134309*d*e^8 - 1090353*e^9)*m^4 - 3*(420000
*d^3*e^6 + 241395*d^2*e^7 + 776186*d*e^8 - 1140969*e^9)*m^3 - 2*(767200*d^3
*e^6 + 407295*d^2*e^7 + 1208124*d*e^8 - 3119359*e^9)*m^2 - 24*(28000*d^3*e^
6 + 14175*d^2*e^7 + 39960*d*e^8 - 248233*e^9)*m)*x^6 - 6*(65*d^4*e^5 - 1391
*d^3*e^6 + 4081*d^2*e^7 - 25872*d*e^8)*m^5 + (10741248*e^9 - 37*(d*e^8 - 4*
e^9)*m^8 - 74*(9*d^2*e^7 + 17*d*e^8 - 80*e^9)*m^7 - 2*(945*d^3*e^6 + 8991*d
^2*e^7 + 8621*d*e^8 - 49580*e^9)*m^6 - 2*(16800*d^4*e^5 + 17955*d^3*e^6 + 9
2241*d^2*e^7 + 61124*d*e^8 - 451400*e^9)*m^5 - (336000*d^4*e^5 + 236250*d^3
*e^6 + 909090*d^2*e^7 + 479113*d*e^8 - 4850404*e^9)*m^4 - 2*(588000*d^4*e^5
 + 344925*d^3*e^6 + 1130202*d^2*e^7 + 513671*d*e^8 - 7804040*e^9)*m^3 - 12*
(140000*d^4*e^5 + 74655*d^3*e^6 + 222444*d^2*e^7 + 91834*d*e^8 - 2422020*e^
9)*m^2 - 144*(5600*d^4*e^5 + 2835*d^3*e^6 + 7992*d^2*e^7 + 3108*d*e^8 - 196
100*e^9)*m)*x^5 + 2*(1776*d^5*e^4 - 6825*d^4*e^5 + 66875*d^3*e^6 - 117810*d
```

```
^2*e^7 + 491841*d*e^8)*m^4 + (5896800*e^9 + (148*d*e^8 + 65*e^9)*m^8 + (185
*d^2*e^7 + 5328*d*e^8 + 2665*e^9)*m^7 + 2*(1665*d^3*e^6 + 2775*d^2*e^7 + 38
924*d*e^8 + 22945*e^9)*m^6 + 2*(4725*d^4*e^5 + 38295*d^3*e^6 + 32005*d^2*e^
7 + 295704*d*e^8 + 215345*e^9)*m^5 + (168000*d^5*e^4 + 141750*d^4*e^5 + 616
050*d^3*e^6 + 355200*d^2*e^7 + 2484772*d*e^8 + 2389985*e^9)*m^4 + (1008000*
d^5*e^4 + 614250*d^4*e^5 + 2081250*d^3*e^6 + 974765*d^2*e^7 + 5668992*d*e^8
 + 7946185*e^9)*m^3 + 6*(308000*d^5*e^4 + 165375*d^4*e^5 + 496170*d^3*e^6 +
 206275*d^2*e^7 + 1064712*d*e^8 + 2542410*e^9)*m^2 + 36*(28000*d^5*e^4 + 14
175*d^4*e^5 + 39960*d^3*e^6 + 15540*d^2*e^7 + 74592*d*e^8 + 422435*e^9)*m)*
x^4 + 3*(1480*d^6*e^3 + 35520*d^5*e^4 - 63050*d^4*e^5 + 375570*d^3*e^6 - 44
4059*d^2*e^7 + 1288056*d*e^8)*m^3 + (12942720*e^9 + (65*d*e^8 + 107*e^9)*m^
8 - 2*(296*d^2*e^7 - 1235*d*e^8 - 2247*e^9)*m^7 - 4*(185*d^3*e^6 + 4884*d^2
*e^7 - 9620*d*e^8 - 19902*e^9)*m^6 - 2*(6660*d^4*e^5 + 9990*d^3*e^6 + 12639
2*d^2*e^7 - 157625*d*e^8 - 386163*e^9)*m^5 - (37800*d^5*e^4 + 266400*d^4*e^
5 + 196100*d^3*e^6 + 1607280*d^2*e^7 - 1444235*d*e^8 - 4453233*e^9)*m^4 - 4
*(168000*d^6*e^3 + 113400*d^5*e^4 + 416250*d^4*e^5 + 208125*d^3*e^6 + 12793
12*d^2*e^7 - 903370*d*e^8 - 3864519*e^9)*m^3 - 4*(504000*d^6*e^3 + 274050*d
^5*e^4 + 832500*d^4*e^5 + 350390*d^3*e^6 + 1831056*d^2*e^7 - 1103505*d*e^8
- 7764883*e^9)*m^2 - 48*(28000*d^6*e^3 + 14175*d^5*e^4 + 39960*d^4*e^5 + 15
540*d^3*e^6 + 74592*d^2*e^7 - 40950*d*e^8 - 672923*e^9)*m)*x^3 + 2*(39960*d
^7*e^2 + 53280*d^6*e^3 + 594960*d^5*e^4 - 648375*d^4*e^5 + 2629418*d^3*e^6
- 2209977*d^2*e^7 + 4581036*d*e^8)*m^2 + (5987520*e^9 + (107*d*e^8 + 33*e^9
)*m^8 - (195*d^2*e^7 - 4280*d*e^8 - 1419*e^9)*m^7 + 4*(444*d^3*e^6 - 1755*d
^2*e^7 + 17762*d*e^8 + 6468*e^9)*m^6 + 2*(1110*d^4*e^5 + 27528*d^3*e^6 - 50
700*d^2*e^7 + 315115*d*e^8 + 130053*e^9)*m^5 + (39960*d^5*e^4 + 55500*d^4*e
^5 + 648240*d^3*e^6 - 742950*d^2*e^7 + 3192773*d*e^8 + 1567797*e^9)*m^4 + (
113400*d^6*e^3 + 719280*d^5*e^4 + 477300*d^4*e^5 + 3525360*d^3*e^6 - 284680
5*d^2*e^7 + 9072530*d*e^8 + 5752131*e^9)*m^3 + 6*(336000*d^7*e^2 + 189000*d
^6*e^3 + 592740*d^5*e^4 + 257150*d^4*e^5 + 1383504*d^3*e^6 - 857805*d^2*e^7
 + 2152412*d*e^8 + 2062863*e^9)*m^2 + 72*(28000*d^7*e^2 + 14175*d^6*e^3 + 3
9960*d^5*e^4 + 15540*d^4*e^5 + 74592*d^3*e^6 - 40950*d^2*e^7 + 89880*d*e^8
+ 193677*e^9)*m)*x^2 + 12*(18900*d^8*e + 113220*d^7*e^2 + 70670*d^6*e^3 + 4
88400*d^5*e^4 - 366405*d^4*e^5 + 1073852*d^3*e^6 - 663102*d^2*e^7 + 995544*
d*e^8)*m + (6531840*e^9 + 3*(11*d*e^8 + 6*e^9)*m^8 - 2*(107*d^2*e^7 - 693*d
*e^8 - 396*e^9)*m^7 + 6*(65*d^3*e^6 - 1391*d^2*e^7 + 4081*d*e^8 + 2478*e^9)
*m^6 - 2*(1776*d^4*e^5 - 6825*d^3*e^6 + 66875*d^2*e^7 - 117810*d*e^8 - 7761
6*e^9)*m^5 - 3*(1480*d^5*e^4 + 35520*d^4*e^5 - 63050*d^3*e^6 + 375570*d^2*e
^7 - 444059*d*e^8 - 327894*e^9)*m^4 - 2*(39960*d^6*e^3 + 53280*d^5*e^4 + 59
4960*d^4*e^5 - 648375*d^3*e^6 + 2629418*d^2*e^7 - 2209977*d*e^8 - 1932084*e
^9)*m^3 - 12*(18900*d^7*e^2 + 113220*d^6*e^3 + 70670*d^5*e^4 + 488400*d^4*e
^5 - 366405*d^3*e^6 + 1073852*d^2*e^7 - 663102*d*e^8 - 763506*e^9)*m^2 - 14
4*(28000*d^8*e + 14175*d^7*e^2 + 39960*d^6*e^3 + 15540*d^5*e^4 + 74592*d^4*
e^5 - 40950*d^3*e^6 + 89880*d^2*e^7 - 41580*d*e^8 - 82962*e^9)*m)*x)*(e*x +
 d)^m/(e^9*m^9 + 45*e^9*m^8 + 870*e^9*m^7 + 9450*e^9*m^6 + 63273*e^9*m^5 +
269325*e^9*m^4 + 723680*e^9*m^3 + 1172700*e^9*m^2 + 1026576*e^9*m + 362880*
e^9)
```

giac [B] time = 0.39, size = 6223, normalized size = 14.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] (100*(x*e + d)^m*m^8*x^9*e^9 + 100*(x*e + d)^m*d*m^8*x^8*e^8 - 45*(x*e + d)
^m*m^8*x^8*e^9 + 3600*(x*e + d)^m*m^7*x^9*e^9 - 45*(x*e + d)^m*d*m^8*x^7*e^
8 + 2800*(x*e + d)^m*d*m^7*x^8*e^8 - 800*(x*e + d)^m*d^2*m^7*x^7*e^7 + 111*
(x*e + d)^m*m^8*x^7*e^9 - 1665*(x*e + d)^m*m^7*x^8*e^9 + 54600*(x*e + d)^m*
m^6*x^9*e^9 + 111*(x*e + d)^m*d*m^8*x^6*e^8 - 1350*(x*e + d)^m*d*m^7*x^7*e^
```

```
8 + 32200*(x*e + d)^m*d*m^6*x^8*e^8 + 315*(x*e + d)^m*d^2*m^7*x^6*e^7 - 168
00*(x*e + d)^m*d^2*m^6*x^7*e^7 + 5600*(x*e + d)^m*d^3*m^6*x^6*e^6 - 37*(x*e
 + d)^m*m^8*x^6*e^9 + 4218*(x*e + d)^m*m^7*x^7*e^9 - 25830*(x*e + d)^m*m^6*
x^8*e^9 + 453600*(x*e + d)^m*m^5*x^9*e^9 - 37*(x*e + d)^m*d*m^8*x^5*e^8 + 3
552*(x*e + d)^m*d*m^7*x^6*e^8 - 16380*(x*e + d)^m*d*m^6*x^7*e^8 + 196000*(x
*e + d)^m*d*m^5*x^8*e^8 - 666*(x*e + d)^m*d^2*m^7*x^5*e^7 + 7560*(x*e + d)^
m*d^2*m^6*x^6*e^7 - 140000*(x*e + d)^m*d^2*m^5*x^7*e^7 - 1890*(x*e + d)^m*d
^3*m^6*x^5*e^6 + 84000*(x*e + d)^m*d^3*m^5*x^6*e^6 - 33600*(x*e + d)^m*d^4*
m^5*x^5*e^5 + 148*(x*e + d)^m*m^8*x^5*e^9 - 1443*(x*e + d)^m*m^7*x^6*e^9 +
67044*(x*e + d)^m*m^6*x^7*e^9 - 218610*(x*e + d)^m*m^5*x^8*e^9 + 2244900*(x
*e + d)^m*m^4*x^9*e^9 + 148*(x*e + d)^m*d*m^8*x^4*e^8 - 1258*(x*e + d)^m*d*
m^7*x^5*e^8 + 45732*(x*e + d)^m*d*m^6*x^6*e^8 - 103950*(x*e + d)^m*d*m^5*x^
7*e^8 + 676900*(x*e + d)^m*d*m^4*x^8*e^8 + 185*(x*e + d)^m*d^2*m^7*x^4*e^7
- 17982*(x*e + d)^m*d^2*m^6*x^5*e^7 + 69300*(x*e + d)^m*d^2*m^5*x^6*e^7 - 5
88000*(x*e + d)^m*d^2*m^4*x^7*e^7 + 3330*(x*e + d)^m*d^3*m^6*x^4*e^6 - 3591
0*(x*e + d)^m*d^3*m^5*x^5*e^6 + 476000*(x*e + d)^m*d^3*m^4*x^6*e^6 + 9450*(
x*e + d)^m*d^4*m^5*x^4*e^5 - 336000*(x*e + d)^m*d^4*m^4*x^5*e^5 + 168000*(x
*e + d)^m*d^5*m^4*x^4*e^4 + 65*(x*e + d)^m*m^8*x^4*e^9 + 5920*(x*e + d)^m*m
^7*x^5*e^9 - 23532*(x*e + d)^m*m^6*x^6*e^9 + 579642*(x*e + d)^m*m^5*x^7*e^9
 - 1098405*(x*e + d)^m*m^4*x^8*e^9 + 6728400*(x*e + d)^m*m^3*x^9*e^9 + 65*(
x*e + d)^m*d*m^8*x^3*e^8 + 5328*(x*e + d)^m*d*m^7*x^4*e^8 - 17242*(x*e + d)
^m*d*m^6*x^5*e^8 + 305250*(x*e + d)^m*d*m^5*x^6*e^8 - 370755*(x*e + d)^m*d*
m^4*x^7*e^8 + 1313200*(x*e + d)^m*d*m^3*x^8*e^8 - 592*(x*e + d)^m*d^2*m^7*x
^3*e^7 + 5550*(x*e + d)^m*d^2*m^6*x^4*e^7 - 184482*(x*e + d)^m*d^2*m^5*x^5*
e^7 + 311850*(x*e + d)^m*d^2*m^4*x^6*e^7 - 1299200*(x*e + d)^m*d^2*m^3*x^7*
e^7 - 740*(x*e + d)^m*d^3*m^6*x^3*e^6 + 76590*(x*e + d)^m*d^3*m^5*x^4*e^6 -
 236250*(x*e + d)^m*d^3*m^4*x^5*e^6 + 1260000*(x*e + d)^m*d^3*m^3*x^6*e^6 -
 13320*(x*e + d)^m*d^4*m^5*x^3*e^5 + 141750*(x*e + d)^m*d^4*m^4*x^4*e^5 - 1
176000*(x*e + d)^m*d^4*m^3*x^5*e^5 - 37800*(x*e + d)^m*d^5*m^4*x^3*e^4 + 10
08000*(x*e + d)^m*d^5*m^3*x^4*e^4 - 672000*(x*e + d)^m*d^6*m^3*x^3*e^3 + 10
7*(x*e + d)^m*m^8*x^3*e^9 + 2665*(x*e + d)^m*m^7*x^4*e^9 + 99160*(x*e + d)^
m*m^6*x^5*e^9 - 208458*(x*e + d)^m*m^5*x^6*e^9 + 2965809*(x*e + d)^m*m^4*x^
7*e^9 - 3332385*(x*e + d)^m*m^3*x^8*e^9 + 11812400*(x*e + d)^m*m^2*x^9*e^9
+ 107*(x*e + d)^m*d*m^8*x^2*e^8 + 2470*(x*e + d)^m*d*m^7*x^3*e^8 + 77848*(x
*e + d)^m*d*m^6*x^4*e^8 - 122248*(x*e + d)^m*d*m^5*x^5*e^8 + 1134309*(x*e +
 d)^m*d*m^4*x^6*e^8 - 737100*(x*e + d)^m*d*m^3*x^7*e^8 + 1306800*(x*e + d)^
m*d*m^2*x^8*e^8 - 195*(x*e + d)^m*d^2*m^7*x^2*e^7 - 19536*(x*e + d)^m*d^2*m
^6*x^3*e^7 + 64010*(x*e + d)^m*d^2*m^5*x^4*e^7 - 909090*(x*e + d)^m*d^2*m^4
*x^5*e^7 + 724185*(x*e + d)^m*d^2*m^3*x^6*e^7 - 1411200*(x*e + d)^m*d^2*m^2
*x^7*e^7 + 1776*(x*e + d)^m*d^3*m^6*x^2*e^6 - 19980*(x*e + d)^m*d^3*m^5*x^3
*e^6 + 616050*(x*e + d)^m*d^3*m^4*x^4*e^6 - 689850*(x*e + d)^m*d^3*m^3*x^5*
e^6 + 1534400*(x*e + d)^m*d^3*m^2*x^6*e^6 + 2220*(x*e + d)^m*d^4*m^5*x^2*e^
5 - 266400*(x*e + d)^m*d^4*m^4*x^3*e^5 + 614250*(x*e + d)^m*d^4*m^3*x^4*e^5
 - 1680000*(x*e + d)^m*d^4*m^2*x^5*e^5 + 39960*(x*e + d)^m*d^5*m^4*x^2*e^4
- 453600*(x*e + d)^m*d^5*m^3*x^3*e^4 + 1848000*(x*e + d)^m*d^5*m^2*x^4*e^4
+ 113400*(x*e + d)^m*d^6*m^3*x^2*e^3 - 2016000*(x*e + d)^m*d^6*m^2*x^3*e^3
+ 2016000*(x*e + d)^m*d^7*m^2*x^2*e^2 + 33*(x*e + d)^m*m^8*x^2*e^9 + 4494*(
x*e + d)^m*m^7*x^3*e^9 + 45890*(x*e + d)^m*m^6*x^4*e^9 + 902800*(x*e + d)^m
*m^5*x^5*e^9 - 1090353*(x*e + d)^m*m^4*x^6*e^9 + 9134412*(x*e + d)^m*m^3*x^
7*e^9 - 5906520*(x*e + d)^m*m^2*x^8*e^9 + 10958400*(x*e + d)^m*m*x^9*e^9 +
33*(x*e + d)^m*d*m^8*x*e^8 + 4280*(x*e + d)^m*d*m^7*x^2*e^8 + 38480*(x*e +
d)^m*d*m^6*x^3*e^8 + 591408*(x*e + d)^m*d*m^5*x^4*e^8 - 479113*(x*e + d)^m*
d*m^4*x^5*e^8 + 2328558*(x*e + d)^m*d*m^3*x^6*e^8 - 746820*(x*e + d)^m*d*m^
2*x^7*e^8 + 504000*(x*e + d)^m*d*m*x^8*e^8 - 214*(x*e + d)^m*d^2*m^7*x*e^7
- 7020*(x*e + d)^m*d^2*m^6*x^2*e^7 - 252784*(x*e + d)^m*d^2*m^5*x^3*e^7 + 3
55200*(x*e + d)^m*d^2*m^4*x^4*e^7 - 2260404*(x*e + d)^m*d^2*m^3*x^5*e^7 + 8
14590*(x*e + d)^m*d^2*m^2*x^6*e^7 - 576000*(x*e + d)^m*d^2*m*x^7*e^7 + 390*
(x*e + d)^m*d^3*m^6*x*e^6 + 55056*(x*e + d)^m*d^3*m^5*x^2*e^6 - 196100*(x*e
 + d)^m*d^3*m^4*x^3*e^6 + 2081250*(x*e + d)^m*d^3*m^3*x^4*e^6 - 895860*(x*e
```

+ d)^m*d^3*m^2*x^5*e^6 + 672000*(x*e + d)^m*d^3*m*x^6*e^6 - 3552*(x*e + d)
^m*d^4*m^5*x*e^5 + 55500*(x*e + d)^m*d^4*m^4*x^2*e^5 - 1665000*(x*e + d)^m*
d^4*m^3*x^3*e^5 + 992250*(x*e + d)^m*d^4*m^2*x^4*e^5 - 806400*(x*e + d)^m*d
^4*m*x^5*e^5 - 4440*(x*e + d)^m*d^5*m^4*x*e^4 + 719280*(x*e + d)^m*d^5*m^3*
x^2*e^4 - 1096200*(x*e + d)^m*d^5*m^2*x^3*e^4 + 1008000*(x*e + d)^m*d^5*m*x
^4*e^4 - 79920*(x*e + d)^m*d^6*m^3*x*e^3 + 1134000*(x*e + d)^m*d^6*m^2*x^2*
e^3 - 1344000*(x*e + d)^m*d^6*m*x^3*e^3 - 226800*(x*e + d)^m*d^7*m^2*x*e^2
+ 2016000*(x*e + d)^m*d^7*m*x^2*e^2 - 4032000*(x*e + d)^m*d^8*m*x*e + 18*(x
*e + d)^m*m^8*x*e^9 + 1419*(x*e + d)^m*m^7*x^2*e^9 + 79608*(x*e + d)^m*m^6*
x^3*e^9 + 430690*(x*e + d)^m*m^5*x^4*e^9 + 4850404*(x*e + d)^m*m^4*x^5*e^9
- 3422907*(x*e + d)^m*m^3*x^6*e^9 + 16387596*(x*e + d)^m*m^2*x^7*e^9 - 5519
340*(x*e + d)^m*m*x^8*e^9 + 4032000*(x*e + d)^m*x^9*e^9 + 18*(x*e + d)^m*d*
m^8*e^8 + 1386*(x*e + d)^m*d*m^7*x*e^8 + 71048*(x*e + d)^m*d*m^6*x^2*e^8 +
315250*(x*e + d)^m*d*m^5*x^3*e^8 + 2484772*(x*e + d)^m*d*m^4*x^4*e^8 - 1027
342*(x*e + d)^m*d*m^3*x^5*e^8 + 2416248*(x*e + d)^m*d*m^2*x^6*e^8 - 291600*
(x*e + d)^m*d*m*x^7*e^8 - 33*(x*e + d)^m*d^2*m^7*e^7 - 8346*(x*e + d)^m*d^2
*m^6*x*e^7 - 101400*(x*e + d)^m*d^2*m^5*x^2*e^7 - 1607280*(x*e + d)^m*d^2*m
^4*x^3*e^7 + 974765*(x*e + d)^m*d^2*m^3*x^4*e^7 - 2669328*(x*e + d)^m*d^2*m
^2*x^5*e^7 + 340200*(x*e + d)^m*d^2*m*x^6*e^7 + 214*(x*e + d)^m*d^3*m^6*e^6
+ 13650*(x*e + d)^m*d^3*m^5*x*e^6 + 648240*(x*e + d)^m*d^3*m^4*x^2*e^6 - 8
32500*(x*e + d)^m*d^3*m^3*x^3*e^6 + 2977020*(x*e + d)^m*d^3*m^2*x^4*e^6 - 4
08240*(x*e + d)^m*d^3*m*x^5*e^6 - 390*(x*e + d)^m*d^4*m^5*e^5 - 106560*(x*e
+ d)^m*d^4*m^4*x*e^5 + 477300*(x*e + d)^m*d^4*m^3*x^2*e^5 - 3330000*(x*e +
d)^m*d^4*m^2*x^3*e^5 + 510300*(x*e + d)^m*d^4*m*x^4*e^5 + 3552*(x*e + d)^m
*d^5*m^4*e^4 - 106560*(x*e + d)^m*d^5*m^3*x*e^4 + 3556440*(x*e + d)^m*d^5*m
^2*x^2*e^4 - 680400*(x*e + d)^m*d^5*m*x^3*e^4 + 4440*(x*e + d)^m*d^6*m^3*e^
3 - 1358640*(x*e + d)^m*d^6*m^2*x*e^3 + 1020600*(x*e + d)^m*d^6*m*x^2*e^3 +
79920*(x*e + d)^m*d^7*m^2*e^2 - 2041200*(x*e + d)^m*d^7*m*x*e^2 + 226800*(
x*e + d)^m*d^8*m*e + 4032000*(x*e + d)^m*d^9 + 792*(x*e + d)^m*m^7*x*e^9 +
25872*(x*e + d)^m*m^6*x^2*e^9 + 772326*(x*e + d)^m*m^5*x^3*e^9 + 2389985*(x
*e + d)^m*m^4*x^4*e^9 + 15608080*(x*e + d)^m*m^3*x^5*e^9 - 6238718*(x*e + d
)^m*m^2*x^6*e^9 + 15456528*(x*e + d)^m*m*x^7*e^9 - 2041200*(x*e + d)^m*x^8*
e^9 + 792*(x*e + d)^m*d*m^7*e^8 + 24486*(x*e + d)^m*d*m^6*x*e^8 + 630230*(x
*e + d)^m*d*m^5*x^2*e^8 + 1444235*(x*e + d)^m*d*m^4*x^3*e^8 + 5668992*(x*e
+ d)^m*d*m^3*x^4*e^8 - 1102008*(x*e + d)^m*d*m^2*x^5*e^8 + 959040*(x*e + d)
^m*d*m*x^6*e^8 - 1386*(x*e + d)^m*d^2*m^6*e^7 - 133750*(x*e + d)^m*d^2*m^5*
x*e^7 - 742950*(x*e + d)^m*d^2*m^4*x^2*e^7 - 5117248*(x*e + d)^m*d^2*m^3*x^
3*e^7 + 1237650*(x*e + d)^m*d^2*m^2*x^4*e^7 - 1150848*(x*e + d)^m*d^2*m*x^5
e^7 + 8346(x*e + d)^m*d^3*m^5*e^6 + 189150*(x*e + d)^m*d^3*m^4*x*e^6 + 35
25360*(x*e + d)^m*d^3*m^3*x^2*e^6 - 1401560*(x*e + d)^m*d^3*m^2*x^3*e^6 + 1
438560*(x*e + d)^m*d^3*m*x^4*e^6 - 13650*(x*e + d)^m*d^4*m^4*e^5 - 1189920*
(x*e + d)^m*d^4*m^3*x*e^5 + 1542900*(x*e + d)^m*d^4*m^2*x^2*e^5 - 1918080*(
x*e + d)^m*d^4*m*x^3*e^5 + 106560*(x*e + d)^m*d^5*m^3*e^4 - 848040*(x*e + d
)^m*d^5*m^2*x*e^4 + 2877120*(x*e + d)^m*d^5*m*x^2*e^4 + 106560*(x*e + d)^m*
d^6*m^2*e^3 - 5754240*(x*e + d)^m*d^6*m*x*e^3 + 1358640*(x*e + d)^m*d^7*m*e
^2 + 2041200*(x*e + d)^m*d^8*e + 14868*(x*e + d)^m*m^6*x*e^9 + 260106*(x*e
+ d)^m*m^5*x^2*e^9 + 4453233*(x*e + d)^m*m^4*x^3*e^9 + 7946185*(x*e + d)^m*
m^3*x^4*e^9 + 29064240*(x*e + d)^m*m^2*x^5*e^9 - 5957592*(x*e + d)^m*m*x^6*
e^9 + 5754240*(x*e + d)^m*x^7*e^9 + 14868*(x*e + d)^m*d*m^6*e^8 + 235620*(x
*e + d)^m*d*m^5*x*e^8 + 3192773*(x*e + d)^m*d*m^4*x^2*e^8 + 3613480*(x*e +
d)^m*d*m^3*x^3*e^8 + 6388272*(x*e + d)^m*d*m^2*x^4*e^8 - 447552*(x*e + d)^m
*d*m*x^5*e^8 - 24486*(x*e + d)^m*d^2*m^5*e^7 - 1126710*(x*e + d)^m*d^2*m^4*
x*e^7 - 2846805*(x*e + d)^m*d^2*m^3*x^2*e^7 - 7324224*(x*e + d)^m*d^2*m^2*x
^3*e^7 + 559440*(x*e + d)^m*d^2*m*x^4*e^7 + 133750*(x*e + d)^m*d^3*m^4*e^6
+ 1296750*(x*e + d)^m*d^3*m^3*x*e^6 + 8301024*(x*e + d)^m*d^3*m^2*x^2*e^6 -
745920*(x*e + d)^m*d^3*m*x^3*e^6 - 189150*(x*e + d)^m*d^4*m^3*e^5 - 586080
0*(x*e + d)^m*d^4*m^2*x*e^5 + 1118880*(x*e + d)^m*d^4*m*x^2*e^5 + 1189920*(
x*e + d)^m*d^5*m^2*e^4 - 2237760*(x*e + d)^m*d^5*m*x*e^4 + 848040*(x*e + d)
^m*d^6*m*e^3 + 5754240*(x*e + d)^m*d^7*e^2 + 155232*(x*e + d)^m*m^5*x*e^9 +

```
 1567797*(x*e + d)^m*m^4*x^2*e^9 + 15458076*(x*e + d)^m*m^3*x^3*e^9 + 15254
460*(x*e + d)^m*m^2*x^4*e^9 + 28238400*(x*e + d)^m*m*x^5*e^9 - 2237760*(x*e
 + d)^m*x^6*e^9 + 155232*(x*e + d)^m*d*m^5*e^8 + 1332177*(x*e + d)^m*d*m^4*
x*e^8 + 9072530*(x*e + d)^m*d*m^3*x^2*e^8 + 4414020*(x*e + d)^m*d*m^2*x^3*e
^8 + 2685312*(x*e + d)^m*d*m*x^4*e^8 - 235620*(x*e + d)^m*d^2*m^4*e^7 - 525
8836*(x*e + d)^m*d^2*m^3*x*e^7 - 5146830*(x*e + d)^m*d^2*m^2*x^2*e^7 - 3580
416*(x*e + d)^m*d^2*m*x^3*e^7 + 1126710*(x*e + d)^m*d^3*m^3*e^6 + 4396860*(
x*e + d)^m*d^3*m^2*x*e^6 + 5370624*(x*e + d)^m*d^3*m*x^2*e^6 - 1296750*(x*e
 + d)^m*d^4*m^2*e^5 - 10741248*(x*e + d)^m*d^4*m*x*e^5 + 5860800*(x*e + d)^
m*d^5*m*e^4 + 2237760*(x*e + d)^m*d^6*e^3 + 983682*(x*e + d)^m*m^4*x*e^9 +
5752131*(x*e + d)^m*m^3*x^2*e^9 + 31059532*(x*e + d)^m*m^2*x^3*e^9 + 152076
60*(x*e + d)^m*m*x^4*e^9 + 10741248*(x*e + d)^m*x^5*e^9 + 983682*(x*e + d)^
m*d*m^4*e^8 + 4419954*(x*e + d)^m*d*m^3*x*e^8 + 12914472*(x*e + d)^m*d*m^2*
x^2*e^8 + 1965600*(x*e + d)^m*d*m*x^3*e^8 - 1332177*(x*e + d)^m*d^2*m^3*e^7
 - 12886224*(x*e + d)^m*d^2*m^2*x*e^7 - 2948400*(x*e + d)^m*d^2*m*x^2*e^7 +
 5258836*(x*e + d)^m*d^3*m^2*e^6 + 5896800*(x*e + d)^m*d^3*m*x*e^6 - 439686
0*(x*e + d)^m*d^4*m*e^5 + 10741248*(x*e + d)^m*d^5*e^4 + 3864168*(x*e + d)^
m*m^3*x*e^9 + 12377178*(x*e + d)^m*m^2*x^2*e^9 + 32300304*(x*e + d)^m*m*x^3
*e^9 + 5896800*(x*e + d)^m*x^4*e^9 + 3864168*(x*e + d)^m*d*m^3*e^8 + 795722
4*(x*e + d)^m*d*m^2*x*e^8 + 6471360*(x*e + d)^m*d*m*x^2*e^8 - 4419954*(x*e
+ d)^m*d^2*m^2*e^7 - 12942720*(x*e + d)^m*d^2*m*x*e^7 + 12886224*(x*e + d)^
m*d^3*m*e^6 - 5896800*(x*e + d)^m*d^4*e^5 + 9162072*(x*e + d)^m*m^2*x*e^9 +
 13944744*(x*e + d)^m*m*x^2*e^9 + 12942720*(x*e + d)^m*x^3*e^9 + 9162072*(x
*e + d)^m*d*m^2*e^8 + 5987520*(x*e + d)^m*d*m*x*e^8 - 7957224*(x*e + d)^m*d
^2*m*e^7 + 12942720*(x*e + d)^m*d^3*e^6 + 11946528*(x*e + d)^m*m*x*e^9 + 59
87520*(x*e + d)^m*x^2*e^9 + 11946528*(x*e + d)^m*d*m*e^8 - 5987520*(x*e + d
)^m*d^2*e^7 + 6531840*(x*e + d)^m*x*e^9 + 6531840*(x*e + d)^m*d*e^8)/(m^9*e
^9 + 45*m^8*e^9 + 870*m^7*e^9 + 9450*m^6*e^9 + 63273*m^5*e^9 + 269325*m^4*e
^9 + 723680*m^3*e^9 + 1172700*m^2*e^9 + 1026576*m*e^9 + 362880*e^9)
```

maple [B] time = 0.04, size = 3222, normalized size = 7.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] (e*x+d)^(1+m)*(100*e^8*m^8*x^8-45*e^8*m^8*x^7+3600*e^8*m^7*x^8-800*d*e^7*m^
7*x^7+111*e^8*m^8*x^6-1665*e^8*m^7*x^7+54600*e^8*m^6*x^8+315*d*e^7*m^7*x^6-
22400*d*e^7*m^6*x^7-37*e^8*m^8*x^5+4218*e^8*m^7*x^6-25830*e^8*m^6*x^7+45360
0*e^8*m^5*x^8+5600*d^2*e^6*m^6*x^6-666*d*e^7*m^7*x^5+9450*d*e^7*m^6*x^6-257
600*d*e^7*m^5*x^7+148*e^8*m^8*x^4-1443*e^8*m^7*x^5+67044*e^8*m^6*x^6-218610
*e^8*m^5*x^7+2244900*e^8*m^4*x^8-1890*d^2*e^6*m^6*x^5+117600*d^2*e^6*m^5*x^
6+185*d*e^7*m^7*x^4-21312*d*e^7*m^6*x^5+114660*d*e^7*m^5*x^6-1568000*d*e^7*
m^4*x^7+65*e^8*m^8*x^3+5920*e^8*m^7*x^4-23532*e^8*m^6*x^5+579642*e^8*m^5*x^
6-1098405*e^8*m^4*x^7+6728400*e^8*m^3*x^8-33600*d^3*e^5*m^5*x^5+3330*d^2*e^
6*m^6*x^4-45360*d^2*e^6*m^5*x^5+980000*d^2*e^6*m^4*x^6-592*d*e^7*m^7*x^3+62
90*d*e^7*m^6*x^4-274392*d*e^7*m^5*x^5+727650*d*e^7*m^4*x^6-5415200*d*e^7*m^
3*x^7+107*e^8*m^8*x^2+2665*e^8*m^7*x^3+99160*e^8*m^6*x^4-208458*e^8*m^5*x^5
+2965809*e^8*m^4*x^6-3332385*e^8*m^3*x^7+11812400*e^8*m^2*x^8+9450*d^3*e^5*
m^5*x^4-504000*d^3*e^5*m^4*x^5-740*d^2*e^6*m^6*x^3+89910*d^2*e^6*m^5*x^4-41
5800*d^2*e^6*m^4*x^5+4116000*d^2*e^6*m^3*x^6-195*d*e^7*m^7*x^2-21312*d*e^7*
m^6*x^3+86210*d*e^7*m^5*x^4-1831500*d*e^7*m^4*x^5+2595285*d*e^7*m^3*x^6-105
05600*d*e^7*m^2*x^7+33*e^8*m^8*x+4494*e^8*m^7*x^2+45890*e^8*m^6*x^3+902800*
e^8*m^5*x^4-1090353*e^8*m^4*x^5+9134412*e^8*m^3*x^6-5906520*e^8*m^2*x^7+109
58400*e^8*m*x^8+168000*d^4*e^4*m^4*x^4-13320*d^3*e^5*m^5*x^3+179550*d^3*e^5
*m^4*x^4-2856000*d^3*e^5*m^3*x^5+1776*d^2*e^6*m^6*x^2-22200*d^2*e^6*m^5*x^3
+922410*d^2*e^6*m^4*x^4-1871100*d^2*e^6*m^3*x^5+9094400*d^2*e^6*m^2*x^6-214
*d*e^7*m^7*x-7410*d*e^7*m^6*x^2-311392*d*e^7*m^5*x^3+611240*d*e^7*m^4*x^4-6
805854*d*e^7*m^3*x^5+5159700*d*e^7*m^2*x^6-10454400*d*e^7*m*x^7+18*e^8*m^8+
```

```
1419*e^8*m^7*x+79608*e^8*m^6*x^2+430690*e^8*m^5*x^3+4850404*e^8*m^4*x^4-342
2907*e^8*m^3*x^5+16387596*e^8*m^2*x^6-5519340*e^8*m*x^7+4032000*e^8*x^8-378
00*d^4*e^4*m^4*x^3+1680000*d^4*e^4*m^3*x^4+2220*d^3*e^5*m^5*x^2-306360*d^3*
e^5*m^4*x^3+1181250*d^3*e^5*m^3*x^4-7560000*d^3*e^5*m^2*x^5+390*d^2*e^6*m^6
*x+58608*d^2*e^6*m^5*x^2-256040*d^2*e^6*m^4*x^3+4545450*d^2*e^6*m^3*x^4-434
5110*d^2*e^6*m^2*x^5+9878400*d^2*e^6*m*x^6-33*d*e^7*m^7-8560*d*e^7*m^6*x-11
5440*d*e^7*m^5*x^2-2365632*d*e^7*m^4*x^3+2395565*d*e^7*m^3*x^4-13971348*d*e
^7*m^2*x^5+5227740*d*e^7*m*x^6-4032000*d*e^7*x^7+792*e^8*m^7+25872*e^8*m^6*
x+772326*e^8*m^5*x^2+2389985*e^8*m^4*x^3+15608080*e^8*m^3*x^4-6238718*e^8*m
^2*x^5+15456528*e^8*m*x^6-2041200*e^8*x^7-672000*d^5*e^3*m^3*x^3+39960*d^4*
e^4*m^4*x^2-567000*d^4*e^4*m^3*x^3+5880000*d^4*e^4*m^2*x^4-3552*d^3*e^5*m^5
*x+59940*d^3*e^5*m^4*x^2-2464200*d^3*e^5*m^3*x^3+3449250*d^3*e^5*m^2*x^4-92
06400*d^3*e^5*m*x^5+214*d^2*e^6*m^6+14040*d^2*e^6*m^5*x+758352*d^2*e^6*m^4*
x^2-1420800*d^2*e^6*m^3*x^3+11302020*d^2*e^6*m^2*x^4-4887540*d^2*e^6*m*x^5+
4032000*d^2*e^6*x^6-1386*d*e^7*m^6-142096*d*e^7*m^5*x-945750*d*e^7*m^4*x^2-
9939088*d*e^7*m^3*x^3+5136710*d*e^7*m^2*x^4-14497488*d*e^7*m*x^5+2041200*d*
e^7*x^6+14868*e^8*m^6+260106*e^8*m^5*x+4453233*e^8*m^4*x^2+7946185*e^8*m^3*
x^3+29064240*e^8*m^2*x^4-5957592*e^8*m*x^5+5754240*e^8*x^6+113400*d^5*e^3*m
^3*x^2-4032000*d^5*e^3*m^2*x^3-4440*d^4*e^4*m^4*x+799200*d^4*e^4*m^3*x^2-24
57000*d^4*e^4*m^2*x^3+8400000*d^4*e^4*m*x^4-390*d^3*e^5*m^5-110112*d^3*e^5*
m^4*x+588300*d^3*e^5*m^3*x^2-8325000*d^3*e^5*m^2*x^3+4479300*d^3*e^5*m*x^4-
4032000*d^3*e^5*x^5+8346*d^2*e^6*m^5+202800*d^2*e^6*m^4*x+4821840*d^2*e^6*m
^3*x^2-3899060*d^2*e^6*m^2*x^3+13346640*d^2*e^6*m*x^4-2041200*d^2*e^6*x^5-2
4486*d*e^7*m^5-1260460*d*e^7*m^4*x-4332705*d*e^7*m^3*x^2-22675968*d*e^7*m^2
*x^3+5510040*d*e^7*m*x^4-5754240*d*e^7*x^5+155232*e^8*m^5+1567797*e^8*m^4*x
+15458076*e^8*m^3*x^2+15254460*e^8*m^2*x^3+28238400*e^8*m*x^4-2237760*e^8*x
^5+2016000*d^6*e^2*m^2*x^2-79920*d^5*e^3*m^3*x+1360800*d^5*e^3*m^2*x^2-7392
000*d^5*e^3*m*x^3+3552*d^4*e^4*m^4-111000*d^4*e^4*m^3*x+4995000*d^4*e^4*m^2
*x^2-3969000*d^4*e^4*m*x^3+4032000*d^4*e^4*x^4-13650*d^3*e^5*m^4-1296480*d^
3*e^5*m^3*x+2497500*d^3*e^5*m^2*x^2-11908080*d^3*e^5*m*x^3+2041200*d^3*e^5*
x^4+133750*d^2*e^6*m^4+1485900*d^2*e^6*m^3*x+15351744*d^2*e^6*m^2*x^2-49506
00*d^2*e^6*m*x^3+5754240*d^2*e^6*x^4-235620*d*e^7*m^4-6385546*d*e^7*m^3*x-1
0840440*d*e^7*m^2*x^2-25553088*d*e^7*m*x^3+2237760*d*e^7*x^4+983682*e^8*m^4
+5752131*e^8*m^3*x+31059532*e^8*m^2*x^2+15207660*e^8*m*x^3+10741248*e^8*x^4
-226800*d^6*e^2*m^2*x+6048000*d^6*e^2*m*x^2+4440*d^5*e^3*m^3-1438560*d^5*e^
3*m^2*x+3288600*d^5*e^3*m*x^2-4032000*d^5*e^3*x^3+106560*d^4*e^4*m^3-954600
*d^4*e^4*m^2*x+9990000*d^4*e^4*m*x^2-2041200*d^4*e^4*x^3-189150*d^3*e^5*m^3
-7050720*d^3*e^5*m^2*x+4204680*d^3*e^5*m*x^2-5754240*d^3*e^5*x^3+1126710*d^
2*e^6*m^3+5693610*d^2*e^6*m^2*x+21972672*d^2*e^6*m*x^2-2237760*d^2*e^6*x^3-
1332177*d*e^7*m^3-18145060*d*e^7*m^2*x-13242060*d*e^7*m*x^2-10741248*d*e^7*
x^3+3864168*e^8*m^3+12377178*e^8*m^2*x+32300304*e^8*m*x^2+5896800*e^8*x^3-4
032000*d^7*e*m*x+79920*d^6*e^2*m^2-2268000*d^6*e^2*m*x+4032000*d^6*e^2*x^2+
106560*d^5*e^3*m^2-7112880*d^5*e^3*m*x+2041200*d^5*e^3*x^2+1189920*d^4*e^4*
m^2-3085800*d^4*e^4*m*x+5754240*d^4*e^4*x^2-1296750*d^3*e^5*m^2-16602048*d^
3*e^5*m*x+2237760*d^3*e^5*x^2+5258836*d^2*e^6*m^2+10293660*d^2*e^6*m*x+1074
1248*d^2*e^6*x^2-4419954*d*e^7*m^2-25828944*d*e^7*m*x-5896800*d*e^7*x^2+916
2072*e^8*m^2+13944744*e^8*m*x+12942720*e^8*x^2+226800*d^7*e*m-4032000*d^7*e
*x+1358640*d^6*e^2*m-2041200*d^6*e^2*x+848040*d^5*e^3*m-5754240*d^5*e^3*x+5
860800*d^4*e^4*m-2237760*d^4*e^4*x-4396860*d^3*e^5*m-10741248*d^3*e^5*x+128
86224*d^2*e^6*m+5896800*d^2*e^6*x-7957224*d*e^7*m-12942720*d*e^7*x+11946528
*e^8*m+5987520*e^8*x+4032000*d^8+2041200*d^7*e+5754240*d^6*e^2+2237760*d^5*
e^3+10741248*d^4*e^4-5896800*d^3*e^5+12942720*d^2*e^6-5987520*d*e^7+6531840
*e^8)/e^9/(m^9+45*m^8+870*m^7+9450*m^6+63273*m^5+269325*m^4+723680*m^3+1172
700*m^2+1026576*m+362880)
```

maxima [B] time = 0.64, size = 1414, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 33*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) + 18
*(e*x + d)^(m + 1)/(e*(m + 1)) + 107*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d
*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e^3)
+ 65*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*
(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*m^3 + 3
5*m^2 + 50*m + 24)*e^4) + 148*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5
+ (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^
3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m/((m^5 + 1
5*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) - 37*((m^5 + 15*m^4 + 85*m^3 +
 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d
*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2
*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x
+ d)^m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6) +
 111*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^7*x^7
+ (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*x^6 - 6*(m^5 +
10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 + 11*m^2 + 6
*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2 + m)*d^5*e
^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 + 28*m^6 + 322*m^5 + 19
60*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7) - 45*((m^7 + 28*m^6 +
322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^8*x^8 + (m^7
+ 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d*e^7*x^7 - 7*(
m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^2*e^6*x^6 + 42*(m^5 +
10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^3*e^5*x^5 - 210*(m^4 + 6*m^3 + 11*m^2 +
6*m)*d^4*e^4*x^4 + 840*(m^3 + 3*m^2 + 2*m)*d^5*e^3*x^3 - 2520*(m^2 + m)*d^6
*e^2*x^2 + 5040*d^7*e*m*x - 5040*d^8)*(e*x + d)^m/((m^8 + 36*m^7 + 546*m^6
+ 4536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^2 + 109584*m + 40320)*e^8) +
100*((m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^
2 + 109584*m + 40320)*e^9*x^9 + (m^8 + 28*m^7 + 322*m^6 + 1960*m^5 + 6769*m
^4 + 13132*m^3 + 13068*m^2 + 5040*m)*d*e^8*x^8 - 8*(m^7 + 21*m^6 + 175*m^5
+ 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d^2*e^7*x^7 + 56*(m^6 + 15*m^5 + 8
5*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^3*e^6*x^6 - 336*(m^5 + 10*m^4 + 35*m^3
 + 50*m^2 + 24*m)*d^4*e^5*x^5 + 1680*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^5*e^4*x
^4 - 6720*(m^3 + 3*m^2 + 2*m)*d^6*e^3*x^3 + 20160*(m^2 + m)*d^7*e^2*x^2 - 4
0320*d^8*e*m*x + 40320*d^9)*(e*x + d)^m/((m^9 + 45*m^8 + 870*m^7 + 9450*m^6
 + 63273*m^5 + 269325*m^4 + 723680*m^3 + 1172700*m^2 + 1026576*m + 362880)*
e^9)
```

mupad [B] time = 6.05, size = 2625, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] ((d + e*x)^m*(6531840*d*e^8 + 2041200*d^8*e + 4032000*d^9 - 5987520*d^2*e^7
 + 12942720*d^3*e^6 - 5896800*d^4*e^5 + 10741248*d^5*e^4 + 2237760*d^6*e^3
+ 5754240*d^7*e^2 - 7957224*d^2*e^7*m + 12886224*d^3*e^6*m - 4396860*d^4*e^
5*m + 5860800*d^5*e^4*m + 848040*d^6*e^3*m + 1358640*d^7*e^2*m + 9162072*d*
e^8*m^2 + 3864168*d*e^8*m^3 + 983682*d*e^8*m^4 + 155232*d*e^8*m^5 + 14868*d
*e^8*m^6 + 792*d*e^8*m^7 + 18*d*e^8*m^8 - 4419954*d^2*e^7*m^2 + 5258836*d^3
*e^6*m^2 - 1296750*d^4*e^5*m^2 + 1189920*d^5*e^4*m^2 + 106560*d^6*e^3*m^2 +
 79920*d^7*e^2*m^2 - 1332177*d^2*e^7*m^3 + 1126710*d^3*e^6*m^3 - 189150*d^4
*e^5*m^3 + 106560*d^5*e^4*m^3 + 4440*d^6*e^3*m^3 - 235620*d^2*e^7*m^4 + 133
750*d^3*e^6*m^4 - 13650*d^4*e^5*m^4 + 3552*d^5*e^4*m^4 - 24486*d^2*e^7*m^5
+ 8346*d^3*e^6*m^5 - 390*d^4*e^5*m^5 - 1386*d^2*e^7*m^6 + 214*d^3*e^6*m^6 -
 33*d^2*e^7*m^7 + 11946528*d*e^8*m + 226800*d^8*e*m))/(e^9*(1026576*m + 117
2700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^
```

```
8 + m^9 + 362880)) + (100*x^9*(d + e*x)^m*(109584*m + 118124*m^2 + 67284*m^
3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320))/(1026576*m + 11
72700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m
^8 + m^9 + 362880) + (x*(d + e*x)^m*(11946528*e^9*m + 6531840*e^9 + 9162072
*e^9*m^2 + 3864168*e^9*m^3 + 983682*e^9*m^4 + 155232*e^9*m^5 + 14868*e^9*m^
6 + 792*e^9*m^7 + 18*e^9*m^8 - 12942720*d^2*e^7*m + 5896800*d^3*e^6*m - 107
41248*d^4*e^5*m - 2237760*d^5*e^4*m - 5754240*d^6*e^3*m - 2041200*d^7*e^2*m
 + 7957224*d*e^8*m^2 + 4419954*d*e^8*m^3 + 1332177*d*e^8*m^4 + 235620*d*e^8
*m^5 + 24486*d*e^8*m^6 + 1386*d*e^8*m^7 + 33*d*e^8*m^8 - 12886224*d^2*e^7*m
^2 + 4396860*d^3*e^6*m^2 - 5860800*d^4*e^5*m^2 - 848040*d^5*e^4*m^2 - 13586
40*d^6*e^3*m^2 - 226800*d^7*e^2*m^2 - 5258836*d^2*e^7*m^3 + 1296750*d^3*e^6
*m^3 - 1189920*d^4*e^5*m^3 - 106560*d^5*e^4*m^3 - 79920*d^6*e^3*m^3 - 11267
10*d^2*e^7*m^4 + 189150*d^3*e^6*m^4 - 106560*d^4*e^5*m^4 - 4440*d^5*e^4*m^4
 - 133750*d^2*e^7*m^5 + 13650*d^3*e^6*m^5 - 3552*d^4*e^5*m^5 - 8346*d^2*e^7
*m^6 + 390*d^3*e^6*m^6 - 214*d^2*e^7*m^7 + 5987520*d*e^8*m - 4032000*d^8*e*
m))/(e^9*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9
450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (x^5*(d + e*x)^m*(50*m + 35*m
^2 + 10*m^3 + m^4 + 24)*(33600*d^4*m - 244200*e^4*m - 447552*e^4 - 49580*e^
4*m^2 - 4440*e^4*m^3 - 148*e^4*m^4 + 47952*d^2*e^2*m + 7067*d*e^3*m^2 + 189
0*d^3*e*m^2 + 888*d*e^3*m^3 + 37*d*e^3*m^4 + 11322*d^2*e^2*m^2 + 666*d^2*e^
2*m^3 + 18648*d*e^3*m + 17010*d^3*e*m))/(e^4*(1026576*m + 1172700*m^2 + 723
680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 3628
80)) - (x^7*(d + e*x)^m*(800*d^2*m - 1887*e^2*m - 7992*e^2 - 111*e^2*m^2 +
405*d*e*m + 45*d*e*m^2)*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m
^6 + 720))/(e^2*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*
m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) + (x^6*(d + e*x)^m*(274*
m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)*(5600*d^3*m - 7067*e^3*m - 18648
*e^3 - 888*e^3*m^2 - 37*e^3*m^3 + 1887*d*e^2*m^2 + 315*d^2*e*m^2 + 111*d*e^
2*m^3 + 7992*d*e^2*m + 2835*d^2*e*m))/(e^3*(1026576*m + 1172700*m^2 + 72368
0*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880
)) + (x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6)*(168000*d^5*m + 732810*e^5*m
 + 982800*e^5 + 216125*e^5*m^2 + 31525*e^5*m^3 + 2275*e^5*m^4 + 65*e^5*m^5
+ 93240*d^2*e^3*m + 239760*d^3*e^2*m + 244200*d*e^4*m^2 + 9450*d^4*e*m^2 +
49580*d*e^4*m^3 + 4440*d*e^4*m^4 + 148*d*e^4*m^5 + 35335*d^2*e^3*m^2 + 5661
0*d^3*e^2*m^2 + 4440*d^2*e^3*m^3 + 3330*d^3*e^2*m^3 + 185*d^2*e^3*m^4 + 447
552*d*e^4*m + 85050*d^4*e*m))/(e^5*(1026576*m + 1172700*m^2 + 723680*m^3 +
269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (5*
x^8*(d + e*x)^m*(81*e - 20*d*m + 9*e*m)*(13068*m + 13132*m^2 + 6769*m^3 + 1
960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040))/(e*(1026576*m + 1172700*m^2 + 723
680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 3628
80)) + (x^2*(m + 1)*(d + e*x)^m*(2016000*d^7*m + 7957224*e^7*m + 5987520*e^
7 + 4419954*e^7*m^2 + 1332177*e^7*m^3 + 235620*e^7*m^4 + 24486*e^7*m^5 + 13
86*e^7*m^6 + 33*e^7*m^7 - 2948400*d^2*e^5*m + 5370624*d^3*e^4*m + 1118880*d
^4*e^3*m + 2877120*d^5*e^2*m + 6443112*d*e^6*m^2 + 113400*d^6*e*m^2 + 26294
18*d*e^6*m^3 + 563355*d*e^6*m^4 + 66875*d*e^6*m^5 + 4173*d*e^6*m^6 + 107*d*
e^6*m^7 - 2198430*d^2*e^5*m^2 + 2930400*d^3*e^4*m^2 + 424020*d^4*e^3*m^2 +
679320*d^5*e^2*m^2 - 648375*d^2*e^5*m^3 + 594960*d^3*e^4*m^3 + 53280*d^4*e^
3*m^3 + 39960*d^5*e^2*m^3 - 94575*d^2*e^5*m^4 + 53280*d^3*e^4*m^4 + 2220*d^
4*e^3*m^4 - 6825*d^2*e^5*m^5 + 1776*d^3*e^4*m^5 - 195*d^2*e^5*m^6 + 6471360
*d*e^6*m + 1020600*d^6*e*m))/(e^7*(1026576*m + 1172700*m^2 + 723680*m^3 + 2
69325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (x^3
*(d + e*x)^m*(3*m + m^2 + 2)*(672000*d^6*m - 6443112*e^6*m - 6471360*e^6 -
2629418*e^6*m^2 - 563355*e^6*m^3 - 66875*e^6*m^4 - 4173*e^6*m^5 - 107*e^6*m
^6 + 1790208*d^2*e^4*m + 372960*d^3*e^3*m + 959040*d^4*e^2*m - 732810*d*e^5
*m^2 + 37800*d^5*e*m^2 - 216125*d*e^5*m^3 - 31525*d*e^5*m^4 - 2275*d*e^5*m^
5 - 65*d*e^5*m^6 + 976800*d^2*e^4*m^2 + 141340*d^3*e^3*m^2 + 226440*d^4*e^2
*m^2 + 198320*d^2*e^4*m^3 + 17760*d^3*e^3*m^3 + 13320*d^4*e^2*m^3 + 17760*d
^2*e^4*m^4 + 740*d^3*e^3*m^4 + 592*d^2*e^4*m^5 - 982800*d*e^5*m + 340200*d^
5*e*m))/(e^6*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5
```

```
 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
[Out] Timed out
```

3.369 $\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx$

Optimal. Leaf size=292

$$\frac{\left(300d^2+85de+17e^2\right)(d+ex)^{m+5}}{e^7(m+5)}-\frac{2\left(200d^3+85d^2e+34de^2+2e^3\right)(d+ex)^{m+4}}{e^7(m+4)}+\frac{\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3\right.}{e^7(}$$

```
[Out] (5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/e^7
/(1+m)-(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*(e*x+d)^(2+m
)/e^7/(2+m)+(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e^4)*(e*x+d)^(3+m)/e
^7/(3+m)-2*(200*d^3+85*d^2*e+34*d*e^2+2*e^3)*(e*x+d)^(4+m)/e^7/(4+m)+(300*d
^2+85*d*e+17*e^2)*(e*x+d)^(5+m)/e^7/(5+m)-(120*d+17*e)*(e*x+d)^(6+m)/e^7/(6
+m)+20*(e*x+d)^(7+m)/e^7/(7+m)
```

Rubi [A] time = 0.19, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1628}

$$\frac{\left(5d^2-2de+3e^2\right)\left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)(d+ex)^{m+1}}{e^7(m+1)}-\frac{\left(68d^3e^2+12d^2e^3+85d^4e+120d^5+42de^4-7e\right.}{e^7(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d +
 e*x)^(1 + m))/(e^7*(1 + m)) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e
^3 + 42*d*e^4 - 7*e^5)*(d + e*x)^(2 + m))/(e^7*(2 + m)) + ((300*d^4 + 170*d
^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^(3 + m))/(e^7*(3 + m)) -
(2*(200*d^3 + 85*d^2*e + 34*d*e^2 + 2*e^3)*(d + e*x)^(4 + m))/(e^7*(4 + m))
 + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^(5 + m))/(e^7*(5 + m)) - ((120*d
+ 17*e)*(d + e*x)^(6 + m))/(e^7*(6 + m)) + (20*(d + e*x)^(7 + m))/(e^7*(7 +
 m))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^m \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right) dx = \int \left(\frac{\left(20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5\right.}{e^6}\right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(}{e^7(1+m)}$$

Mathematica [A] time = 0.17, size = 261, normalized size = 0.89

$$\frac{(d+ex)^{m+1}\left(\frac{\left(300d^2+85de+17e^2\right)(d+ex)^4}{m+5}-\frac{2\left(200d^3+85d^2e+34de^2+2e^3\right)(d+ex)^3}{m+4}+\frac{\left(300d^4+170d^3e+102d^2e^2+12de^3+21e^4\right)(d+ex)^2}{m+3}+\frac{\left(5d^2-2d\right.}{}\right.}{e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 -
 d*e^3 + 2*e^4))/(1 + m) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 +
 42*d*e^4 - 7*e^5)*(d + e*x))/(2 + m) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2
 + 12*d*e^3 + 21*e^4)*(d + e*x)^2)/(3 + m) - (2*(200*d^3 + 85*d^2*e + 34*d*
e^2 + 2*e^3)*(d + e*x)^3)/(4 + m) + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^
4)/(5 + m) - ((120*d + 17*e)*(d + e*x)^5)/(6 + m) + (20*(d + e*x)^6)/(7 + m
)))/e^7
```

fricas [B] time = 0.61, size = 1448, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] (6*d*e^6*m^6 + 20*(e^7*m^6 + 21*e^7*m^5 + 175*e^7*m^4 + 735*e^7*m^3 + 1624*
e^7*m^2 + 1764*e^7*m + 720*e^7)*x^7 + 14400*d^7 + 14280*d^6*e + 17136*d^5*e
^2 + 5040*d^4*e^3 + 35280*d^3*e^4 - 17640*d^2*e^5 + 30240*d*e^6 - (14280*e^
7 - (20*d*e^6 - 17*e^7)*m^6 - 2*(150*d*e^6 - 187*e^7)*m^5 - 170*(10*d*e^6 -
 19*e^7)*m^4 - 20*(225*d*e^6 - 697*e^7)*m^3 - (5480*d*e^6 - 31433*e^7)*m^2
- 2*(1200*d*e^6 - 17323*e^7)*m)*x^6 - (7*d^2*e^5 - 162*d*e^6)*m^5 + (17136*
e^7 - 17*(d*e^6 - e^7)*m^6 - (120*d^2*e^5 + 289*d*e^6 - 391*e^7)*m^5 - 3*(4
00*d^2*e^5 + 595*d*e^6 - 1173*e^7)*m^4 - 5*(840*d^2*e^5 + 1003*d*e^6 - 3145
*e^7)*m^3 - 2*(3000*d^2*e^5 + 3179*d*e^6 - 18224*e^7)*m^2 - 12*(240*d^2*e^5
 + 238*d*e^6 - 3417*e^7)*m)*x^5 + (42*d^3*e^4 - 175*d^2*e^5 + 1770*d*e^6)*m
^4 - (5040*e^7 - (17*d*e^6 - 4*e^7)*m^6 - (85*d^2*e^5 + 323*d*e^6 - 96*e^7)
*m^5 - (600*d^3*e^4 + 1105*d^2*e^5 + 2227*d*e^6 - 904*e^7)*m^4 - (3600*d^3*
e^4 + 4505*d^2*e^5 + 6817*d*e^6 - 4224*e^7)*m^3 - 5*(1320*d^3*e^4 + 1411*d^
2*e^5 + 1836*d*e^6 - 2036*e^7)*m^2 - 6*(600*d^3*e^4 + 595*d^2*e^5 + 714*d*e
^6 - 1968*e^7)*m)*x^4 + (24*d^4*e^3 + 924*d^3*e^4 - 1715*d^2*e^5 + 9990*d*e
^6)*m^3 + (35280*e^7 - (4*d*e^6 - 21*e^7)*m^6 - (68*d^2*e^5 + 84*d*e^6 - 52
5*e^7)*m^5 - (340*d^3*e^4 + 1088*d^2*e^5 + 652*d*e^6 - 5187*e^7)*m^4 - (240
0*d^4*e^3 + 3400*d^3*e^4 + 5644*d^2*e^5 + 2268*d*e^6 - 25599*e^7)*m^3 - 4*(
1800*d^4*e^3 + 1955*d^3*e^4 + 2584*d^2*e^5 + 844*d*e^6 - 16338*e^7)*m^2 - 4
*(1200*d^4*e^3 + 1190*d^3*e^4 + 1428*d^2*e^5 + 420*d*e^6 - 19929*e^7)*m)*x^
3 + (408*d^5*e^2 + 432*d^4*e^3 + 7518*d^3*e^4 - 8225*d^2*e^5 + 30624*d*e^6)
*m^2 + (17640*e^7 + 7*(3*d*e^6 + e^7)*m^6 + (12*d^2*e^5 + 483*d*e^6 + 182*e
^7)*m^5 + 3*(68*d^3*e^4 + 76*d^2*e^5 + 1407*d*e^6 + 630*e^7)*m^4 + (1020*d^
4*e^3 + 2856*d^3*e^4 + 1500*d^2*e^5 + 17157*d*e^6 + 9940*e^7)*m^3 + (7200*d
^5*e^2 + 8160*d^4*e^3 + 11220*d^3*e^4 + 3804*d^2*e^5 + 31038*d*e^6 + 27503*
e^7)*m^2 + 6*(1200*d^5*e^2 + 1190*d^4*e^3 + 1428*d^3*e^4 + 420*d^2*e^5 + 29
40*d*e^6 + 6153*e^7)*m)*x^2 + 6*(340*d^6*e + 884*d^5*e^2 + 428*d^4*e^3 + 44
66*d^3*e^4 - 3213*d^2*e^5 + 8028*d*e^6)*m + (30240*e^7 + (7*d*e^6 + 6*e^7)*
m^6 - (42*d^2*e^5 - 175*d*e^6 - 162*e^7)*m^5 - (24*d^3*e^4 + 924*d^2*e^5 -
1715*d*e^6 - 1770*e^7)*m^4 - (408*d^4*e^3 + 432*d^3*e^4 + 7518*d^2*e^5 - 82
25*d*e^6 - 9990*e^7)*m^3 - 6*(340*d^5*e^2 + 884*d^4*e^3 + 428*d^3*e^4 + 446
6*d^2*e^5 - 3213*d*e^6 - 5104*e^7)*m^2 - 24*(600*d^6*e + 595*d^5*e^2 + 714*
d^4*e^3 + 210*d^3*e^4 + 1470*d^2*e^5 - 735*d*e^6 - 2007*e^7)*m)*x)*(e*x + d
)^m/(e^7*m^7 + 28*e^7*m^6 + 322*e^7*m^5 + 1960*e^7*m^4 + 6769*e^7*m^3 + 131
32*e^7*m^2 + 13068*e^7*m + 5040*e^7)
```

giac [B] time = 0.27, size = 3098, normalized size = 10.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] (20*(x*e + d)^m*m^6*x^7*e^7 + 20*(x*e + d)^m*d*m^6*x^6*e^6 - 17*(x*e + d)^m
*m^6*x^6*e^7 + 420*(x*e + d)^m*m^5*x^7*e^7 - 17*(x*e + d)^m*d*m^6*x^5*e^6 +
 300*(x*e + d)^m*d*m^5*x^6*e^6 - 120*(x*e + d)^m*d^2*m^5*x^5*e^5 + 17*(x*e
+ d)^m*m^6*x^5*e^7 - 374*(x*e + d)^m*m^5*x^6*e^7 + 3500*(x*e + d)^m*m^4*x^7
*e^7 + 17*(x*e + d)^m*d*m^6*x^4*e^6 - 289*(x*e + d)^m*d*m^5*x^5*e^6 + 1700*
(x*e + d)^m*d*m^4*x^6*e^6 + 85*(x*e + d)^m*d^2*m^5*x^4*e^5 - 1200*(x*e + d)
^m*d^2*m^4*x^5*e^5 + 600*(x*e + d)^m*d^3*m^4*x^4*e^4 - 4*(x*e + d)^m*m^6*x^
4*e^7 + 391*(x*e + d)^m*m^5*x^5*e^7 - 3230*(x*e + d)^m*m^4*x^6*e^7 + 14700*
(x*e + d)^m*m^3*x^7*e^7 - 4*(x*e + d)^m*d*m^6*x^3*e^6 + 323*(x*e + d)^m*d*m
^5*x^4*e^6 - 1785*(x*e + d)^m*d*m^4*x^5*e^6 + 4500*(x*e + d)^m*d*m^3*x^6*e^
6 - 68*(x*e + d)^m*d^2*m^5*x^3*e^5 + 1105*(x*e + d)^m*d^2*m^4*x^4*e^5 - 420
0*(x*e + d)^m*d^2*m^3*x^5*e^5 - 340*(x*e + d)^m*d^3*m^4*x^3*e^4 + 3600*(x*e
 + d)^m*d^3*m^3*x^4*e^4 - 2400*(x*e + d)^m*d^4*m^3*x^3*e^3 + 21*(x*e + d)^m
*m^6*x^3*e^7 - 96*(x*e + d)^m*m^5*x^4*e^7 + 3519*(x*e + d)^m*m^4*x^5*e^7 -
13940*(x*e + d)^m*m^3*x^6*e^7 + 32480*(x*e + d)^m*m^2*x^7*e^7 + 21*(x*e + d
)^m*d*m^6*x^2*e^6 - 84*(x*e + d)^m*d*m^5*x^3*e^6 + 2227*(x*e + d)^m*d*m^4*x
^4*e^6 - 5015*(x*e + d)^m*d*m^3*x^5*e^6 + 5480*(x*e + d)^m*d*m^2*x^6*e^6 +
12*(x*e + d)^m*d^2*m^5*x^2*e^5 - 1088*(x*e + d)^m*d^2*m^4*x^3*e^5 + 4505*(x
*e + d)^m*d^2*m^3*x^4*e^5 - 6000*(x*e + d)^m*d^2*m^2*x^5*e^5 + 204*(x*e + d
)^m*d^3*m^4*x^2*e^4 - 3400*(x*e + d)^m*d^3*m^3*x^3*e^4 + 6600*(x*e + d)^m*d
^3*m^2*x^4*e^4 + 1020*(x*e + d)^m*d^4*m^3*x^2*e^3 - 7200*(x*e + d)^m*d^4*m^
2*x^3*e^3 + 7200*(x*e + d)^m*d^5*m^2*x^2*e^2 + 7*(x*e + d)^m*m^6*x^2*e^7 +
525*(x*e + d)^m*m^5*x^3*e^7 - 904*(x*e + d)^m*m^4*x^4*e^7 + 15725*(x*e + d)
^m*m^3*x^5*e^7 - 31433*(x*e + d)^m*m^2*x^6*e^7 + 35280*(x*e + d)^m*m*x^7*e^
7 + 7*(x*e + d)^m*d*m^6*x*e^6 + 483*(x*e + d)^m*d*m^5*x^2*e^6 - 652*(x*e +
d)^m*d*m^4*x^3*e^6 + 6817*(x*e + d)^m*d*m^3*x^4*e^6 - 6358*(x*e + d)^m*d*m^
2*x^5*e^6 + 2400*(x*e + d)^m*d*m*x^6*e^6 - 42*(x*e + d)^m*d^2*m^5*x*e^5 + 2
28*(x*e + d)^m*d^2*m^4*x^2*e^5 - 5644*(x*e + d)^m*d^2*m^3*x^3*e^5 + 7055*(x
*e + d)^m*d^2*m^2*x^4*e^5 - 2880*(x*e + d)^m*d^2*m*x^5*e^5 - 24*(x*e + d)^m
*d^3*m^4*x*e^4 + 2856*(x*e + d)^m*d^3*m^3*x^2*e^4 - 7820*(x*e + d)^m*d^3*m^
2*x^3*e^4 + 3600*(x*e + d)^m*d^3*m*x^4*e^4 - 408*(x*e + d)^m*d^4*m^3*x*e^3
+ 8160*(x*e + d)^m*d^4*m^2*x^2*e^3 - 4800*(x*e + d)^m*d^4*m*x^3*e^3 - 2040*
(x*e + d)^m*d^5*m^2*x*e^2 + 7200*(x*e + d)^m*d^5*m*x^2*e^2 - 14400*(x*e + d
)^m*d^6*m*x*e + 6*(x*e + d)^m*m^6*x*e^7 + 182*(x*e + d)^m*m^5*x^2*e^7 + 518
7*(x*e + d)^m*m^4*x^3*e^7 - 4224*(x*e + d)^m*m^3*x^4*e^7 + 36448*(x*e + d)^
m*m^2*x^5*e^7 - 34646*(x*e + d)^m*m*x^6*e^7 + 14400*(x*e + d)^m*x^7*e^7 + 6
*(x*e + d)^m*d*m^6*e^6 + 175*(x*e + d)^m*d*m^5*x*e^6 + 4221*(x*e + d)^m*d*m
^4*x^2*e^6 - 2268*(x*e + d)^m*d*m^3*x^3*e^6 + 9180*(x*e + d)^m*d*m^2*x^4*e^
6 - 2856*(x*e + d)^m*d*m*x^5*e^6 - 7*(x*e + d)^m*d^2*m^5*e^5 - 924*(x*e + d
)^m*d^2*m^4*x*e^5 + 1500*(x*e + d)^m*d^2*m^3*x^2*e^5 - 10336*(x*e + d)^m*d^
2*m^2*x^3*e^5 + 3570*(x*e + d)^m*d^2*m*x^4*e^5 + 42*(x*e + d)^m*d^3*m^4*e^4
 - 432*(x*e + d)^m*d^3*m^3*x*e^4 + 11220*(x*e + d)^m*d^3*m^2*x^2*e^4 - 4760
*(x*e + d)^m*d^3*m*x^3*e^4 + 24*(x*e + d)^m*d^4*m^3*e^3 - 5304*(x*e + d)^m*
d^4*m^2*x*e^3 + 7140*(x*e + d)^m*d^4*m*x^2*e^3 + 408*(x*e + d)^m*d^5*m^2*e^
2 - 14280*(x*e + d)^m*d^5*m*x*e^2 + 2040*(x*e + d)^m*d^6*m*e + 14400*(x*e +
 d)^m*d^7 + 162*(x*e + d)^m*m^5*x*e^7 + 1890*(x*e + d)^m*m^4*x^2*e^7 + 2559
9*(x*e + d)^m*m^3*x^3*e^7 - 10180*(x*e + d)^m*m^2*x^4*e^7 + 41004*(x*e + d)
^m*m*x^5*e^7 - 14280*(x*e + d)^m*x^6*e^7 + 162*(x*e + d)^m*d*m^5*e^6 + 1715
*(x*e + d)^m*d*m^4*x*e^6 + 17157*(x*e + d)^m*d*m^3*x^2*e^6 - 3376*(x*e + d)
^m*d*m^2*x^3*e^6 + 4284*(x*e + d)^m*d*m*x^4*e^6 - 175*(x*e + d)^m*d^2*m^4*e
^5 - 7518*(x*e + d)^m*d^2*m^3*x*e^5 + 3804*(x*e + d)^m*d^2*m^2*x^2*e^5 - 57
12*(x*e + d)^m*d^2*m*x^3*e^5 + 924*(x*e + d)^m*d^3*m^3*e^4 - 2568*(x*e + d)
^m*d^3*m^2*x*e^4 + 8568*(x*e + d)^m*d^3*m*x^2*e^4 + 432*(x*e + d)^m*d^4*m^2
*e^3 - 17136*(x*e + d)^m*d^4*m*x*e^3 + 5304*(x*e + d)^m*d^5*m*e^2 + 14280*(
x*e + d)^m*d^6*e + 1770*(x*e + d)^m*m^4*x*e^7 + 9940*(x*e + d)^m*m^3*x^2*e^
7 + 65352*(x*e + d)^m*m^2*x^3*e^7 - 11808*(x*e + d)^m*m*x^4*e^7 + 17136*(x*
e + d)^m*x^5*e^7 + 1770*(x*e + d)^m*d*m^4*e^6 + 8225*(x*e + d)^m*d*m^3*x*e^
6 + 31038*(x*e + d)^m*d*m^2*x^2*e^6 - 1680*(x*e + d)^m*d*m*x^3*e^6 - 1715*(
x*e + d)^m*d^2*m^3*e^5 - 26796*(x*e + d)^m*d^2*m^2*x*e^5 + 2520*(x*e + d)^m
```

```
*d^2*m*x^2*e^5 + 7518*(x*e + d)^m*d^3*m^2*e^4 - 5040*(x*e + d)^m*d^3*m*x*e^
4 + 2568*(x*e + d)^m*d^4*m*e^3 + 17136*(x*e + d)^m*d^5*e^2 + 9990*(x*e + d)
^m*m^3*x*e^7 + 27503*(x*e + d)^m*m^2*x^2*e^7 + 79716*(x*e + d)^m*m*x^3*e^7
- 5040*(x*e + d)^m*x^4*e^7 + 9990*(x*e + d)^m*d*m^3*e^6 + 19278*(x*e + d)^m
*d*m^2*x*e^6 + 17640*(x*e + d)^m*d*m*x^2*e^6 - 8225*(x*e + d)^m*d^2*m^2*e^5
 - 35280*(x*e + d)^m*d^2*m*x*e^5 + 26796*(x*e + d)^m*d^3*m*e^4 + 5040*(x*e
+ d)^m*d^4*e^3 + 30624*(x*e + d)^m*m^2*x*e^7 + 36918*(x*e + d)^m*m*x^2*e^7
+ 35280*(x*e + d)^m*x^3*e^7 + 30624*(x*e + d)^m*d*m^2*e^6 + 17640*(x*e + d)
^m*d*m*x*e^6 - 19278*(x*e + d)^m*d^2*m*e^5 + 35280*(x*e + d)^m*d^3*e^4 + 48
168*(x*e + d)^m*m*x*e^7 + 17640*(x*e + d)^m*x^2*e^7 + 48168*(x*e + d)^m*d*m
*e^6 - 17640*(x*e + d)^m*d^2*e^5 + 30240*(x*e + d)^m*x*e^7 + 30240*(x*e + d
)^m*d*e^6)/(m^7*e^7 + 28*m^6*e^7 + 322*m^5*e^7 + 1960*m^4*e^7 + 6769*m^3*e^
7 + 13132*m^2*e^7 + 13068*m*e^7 + 5040*e^7)
```

maple [B] time = 0.02, size = 1504, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] (e*x+d)^(1+m)*(20*e^6*m^6*x^6-17*e^6*m^6*x^5+420*e^6*m^5*x^6-120*d*e^5*m^5*
x^5+17*e^6*m^6*x^4-374*e^6*m^5*x^5+3500*e^6*m^4*x^6+85*d*e^5*m^5*x^4-1800*d
*e^5*m^4*x^5-4*e^6*m^6*x^3+391*e^6*m^5*x^4-3230*e^6*m^4*x^5+14700*e^6*m^3*x
^6+600*d^2*e^4*m^4*x^4-68*d*e^5*m^5*x^3+1445*d*e^5*m^4*x^4-10200*d*e^5*m^3*
x^5+21*e^6*m^6*x^2-96*e^6*m^5*x^3+3519*e^6*m^4*x^4-13940*e^6*m^3*x^5+32480*
e^6*m^2*x^6-340*d^2*e^4*m^4*x^3+6000*d^2*e^4*m^3*x^4+12*d*e^5*m^5*x^2-1292*
d*e^5*m^4*x^3+8925*d*e^5*m^3*x^4-27000*d*e^5*m^2*x^5+7*e^6*m^6*x+525*e^6*m^
5*x^2-904*e^6*m^4*x^3+15725*e^6*m^3*x^4-31433*e^6*m^2*x^5+35280*e^6*m*x^6-2
400*d^3*e^3*m^3*x^3+204*d^2*e^4*m^4*x^2-4420*d^2*e^4*m^3*x^3+21000*d^2*e^4*
m^2*x^4-42*d*e^5*m^5*x+252*d*e^5*m^4*x^2-8908*d*e^5*m^3*x^3+25075*d*e^5*m^2
*x^4-32880*d*e^5*m*x^5+6*e^6*m^6+182*e^6*m^5*x+5187*e^6*m^4*x^2-4224*e^6*m^
3*x^3+36448*e^6*m^2*x^4-34646*e^6*m*x^5+14400*e^6*x^6+1020*d^3*e^3*m^3*x^2-
14400*d^3*e^3*m^2*x^3-24*d^2*e^4*m^4*x+3264*d^2*e^4*m^3*x^2-18020*d^2*e^4*m
^2*x^3+30000*d^2*e^4*m*x^4-7*d*e^5*m^5-966*d*e^5*m^4*x+1956*d*e^5*m^3*x^2-2
7268*d*e^5*m^2*x^3+31790*d*e^5*m*x^4-14400*d*e^5*x^5+162*e^6*m^5+1890*e^6*m
^4*x+25599*e^6*m^3*x^2-10180*e^6*m^2*x^3+41004*e^6*m*x^4-14280*e^6*x^5+7200
*d^4*e^2*m^2*x^2-408*d^3*e^3*m^3*x+10200*d^3*e^3*m^2*x^2-26400*d^3*e^3*m*x^
3+42*d^2*e^4*m^4-456*d^2*e^4*m^3*x+16932*d^2*e^4*m^2*x^2-28220*d^2*e^4*m*x^
3+14400*d^2*e^4*x^4-175*d*e^5*m^4-8442*d*e^5*m^3*x+6804*d*e^5*m^2*x^2-36720
*d*e^5*m*x^3+14280*d*e^5*x^4+1770*e^6*m^4+9940*e^6*m^3*x+65352*e^6*m^2*x^2-
11808*e^6*m*x^3+17136*e^6*x^4-2040*d^4*e^2*m^2*x+21600*d^4*e^2*m*x^2+24*d^3
*e^3*m^3-5712*d^3*e^3*m^2*x+23460*d^3*e^3*m*x^2-14400*d^3*e^3*x^3+924*d^2*e
^4*m^3-3000*d^2*e^4*m^2*x+31008*d^2*e^4*m*x^2-14280*d^2*e^4*x^3-1715*d*e^5*
m^3-34314*d*e^5*m^2*x+10128*d*e^5*m*x^2-17136*d*e^5*x^3+9990*e^6*m^3+27503*
e^6*m^2*x+79716*e^6*m*x^2-5040*e^6*x^3-14400*d^5*e*m*x+408*d^4*e^2*m^2-1632
0*d^4*e^2*m*x+14400*d^4*e^2*x^2+432*d^3*e^3*m^2-22440*d^3*e^3*m*x+14280*d^3
*e^3*x^2+7518*d^2*e^4*m^2-7608*d^2*e^4*m*x+17136*d^2*e^4*x^2-8225*d*e^5*m^2
-62076*d*e^5*m*x+5040*d*e^5*x^2+30624*e^6*m^2+36918*e^6*m*x+35280*e^6*x^2+2
040*d^5*e*m-14400*d^5*e*x+5304*d^4*e^2*m-14280*d^4*e^2*x+2568*d^3*e^3*m-171
36*d^3*e^3*x+26796*d^2*e^4*m-5040*d^2*e^4*x-19278*d*e^5*m-35280*d*e^5*x+481
68*e^6*m+17640*e^6*x+14400*d^6+14280*d^5*e+17136*d^4*e^2+5040*d^3*e^3+35280
*d^2*e^4-17640*d*e^5+30240*e^6)/e^7/(m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+1
3132*m^2+13068*m+5040)
```

maxima [B] time = 0.55, size = 788, normalized size = 2.70

$$\frac{7\left(e^2(m+1)x^2+demx-d^2\right)(ex+d)^m}{\left(m^2+3m+2\right)e^2}+\frac{6(ex+d)^{m+1}}{e(m+1)}+\frac{21\left(\left(m^2+3m+2\right)e^3x^3+\left(m^2+m\right)de^2x^2-2d^2emx+\right.}{\left(m^3+6m^2+11m+6\right)e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 7*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) + 6*(
e*x + d)^(m + 1)/(e*(m + 1)) + 21*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^
2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e^3) - 4
*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2
 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*m^3 + 35*m^
2 + 50*m + 24)*e^4) + 17*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^
4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 1
2*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m/((m^5 + 15*m^4
 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) - 17*((m^5 + 15*m^4 + 85*m^3 + 225*
m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d*e^5*
x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2*m)*d
^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x + d)^
m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6) + 20*(
(m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^7*x^7 + (m^6
 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*x^6 - 6*(m^5 + 10*m^4
 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^
3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2 + m)*d^5*e^2*x^2
 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 + 28*m^6 + 322*m^5 + 1960*m^4
 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7)
```

mupad [B] time = 5.09, size = 1425, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] ((d + e*x)^m*(30240*d*e^6 + 14280*d^6*e + 14400*d^7 - 17640*d^2*e^5 + 35280
*d^3*e^4 + 5040*d^4*e^3 + 17136*d^5*e^2 - 19278*d^2*e^5*m + 26796*d^3*e^4*m
 + 2568*d^4*e^3*m + 5304*d^5*e^2*m + 30624*d*e^6*m^2 + 9990*d*e^6*m^3 + 177
0*d*e^6*m^4 + 162*d*e^6*m^5 + 6*d*e^6*m^6 - 8225*d^2*e^5*m^2 + 7518*d^3*e^4
*m^2 + 432*d^4*e^3*m^2 + 408*d^5*e^2*m^2 - 1715*d^2*e^5*m^3 + 924*d^3*e^4*m
^3 + 24*d^4*e^3*m^3 - 175*d^2*e^5*m^4 + 42*d^3*e^4*m^4 - 7*d^2*e^5*m^5 + 48
168*d*e^6*m + 2040*d^6*e*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^
4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (20*x^7*(d + e*x)^m*(1764*m + 1624*m^
2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))/(13068*m + 13132*m^2 + 6769*m^
3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) - (x*(d + e*x)^m*(35280*d^2*e
^5*m - 30240*e^7 - 30624*e^7*m^2 - 9990*e^7*m^3 - 1770*e^7*m^4 - 162*e^7*m^
5 - 6*e^7*m^6 - 48168*e^7*m + 5040*d^3*e^4*m + 17136*d^4*e^3*m + 14280*d^5*
e^2*m - 19278*d*e^6*m^2 - 8225*d*e^6*m^3 - 1715*d*e^6*m^4 - 175*d*e^6*m^5 -
 7*d*e^6*m^6 + 26796*d^2*e^5*m^2 + 2568*d^3*e^4*m^2 + 5304*d^4*e^3*m^2 + 20
40*d^5*e^2*m^2 + 7518*d^2*e^5*m^3 + 432*d^3*e^4*m^3 + 408*d^4*e^3*m^3 + 924
*d^2*e^5*m^4 + 24*d^3*e^4*m^4 + 42*d^2*e^5*m^5 - 17640*d*e^6*m + 14400*d^6*
e*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 +
m^7 + 5040)) - (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(2400*d^4*m - 13398*e^4*m -
 17640*e^4 - 3759*e^4*m^2 - 462*e^4*m^3 - 21*e^4*m^4 + 2856*d^2*e^2*m + 428
*d*e^3*m^2 + 340*d^3*e*m^2 + 72*d*e^3*m^3 + 4*d*e^3*m^4 + 884*d^2*e^2*m^2 +
 68*d^2*e^2*m^3 + 840*d*e^3*m + 2380*d^3*e*m))/(e^4*(13068*m + 13132*m^2 +
6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) - (x^5*(d + e*x)^m*(5
0*m + 35*m^2 + 10*m^3 + m^4 + 24)*(120*d^2*m - 221*e^2*m - 714*e^2 - 17*e^2
*m^2 + 119*d*e*m + 17*d*e*m^2))/(e^2*(13068*m + 13132*m^2 + 6769*m^3 + 1960
*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^4*(d + e*x)^m*(11*m + 6*m^2 + m
^3 + 6)*(600*d^3*m - 428*e^3*m - 840*e^3 - 72*e^3*m^2 - 4*e^3*m^3 + 221*d*e
^2*m^2 + 85*d^2*e*m^2 + 17*d*e^2*m^3 + 714*d*e^2*m + 595*d^2*e*m))/(e^3*(13
068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) +
```

```
 (x^2*(m + 1)*(d + e*x)^m*(7200*d^5*m + 19278*e^5*m + 17640*e^5 + 8225*e^5*
m^2 + 1715*e^5*m^3 + 175*e^5*m^4 + 7*e^5*m^5 + 2520*d^2*e^3*m + 8568*d^3*e^
2*m + 13398*d*e^4*m^2 + 1020*d^4*e*m^2 + 3759*d*e^4*m^3 + 462*d*e^4*m^4 + 2
1*d*e^4*m^5 + 1284*d^2*e^3*m^2 + 2652*d^3*e^2*m^2 + 216*d^2*e^3*m^3 + 204*d
^3*e^2*m^3 + 12*d^2*e^3*m^4 + 17640*d*e^4*m + 7140*d^4*e*m))/(e^5*(13068*m
+ 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) - (x^6*
(d + e*x)^m*(119*e - 20*d*m + 17*e*m)*(274*m + 225*m^2 + 85*m^3 + 15*m^4 +
m^5 + 120))/(e*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^
6 + m^7 + 5040))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] Timed out
```

3.370 $\int \frac{(d+ex)^m\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx$

Optimal. Leaf size=255

$$\frac{\left(100d^2+165de+81e^2\right)(d+ex)^{m+1}}{125e^3(m+1)}-\frac{(40d+33e)(d+ex)^{m+2}}{25e^3(m+2)}+\frac{4(d+ex)^{m+3}}{5e^3(m+3)}-\frac{\left(-423\sqrt{14}+6412i\right)(d+ex)^{m+1}\,{}_2F}{3500(m+1)\left(5id-\right.}$$

```
[Out] 1/125*(100*d^2+165*d*e+81*e^2)*(e*x+d)^(1+m)/e^3/(1+m)-1/25*(40*d+33*e)*(e*
x+d)^(2+m)/e^3/(2+m)+4/5*(e*x+d)^(3+m)/e^3/(3+m)-1/3500*(e*x+d)^(1+m)*hyper
geom([1, 1+m],[2+m],5*(e*x+d)/(5*d-e*(1+I*14^(1/2))))*(6412*I+423*14^(1/2))
/(1+m)/(5*I*d-e*(I-14^(1/2)))-1/3500*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m]
,5*(e*x+d)/(5*d-e+I*14^(1/2)*e))*(6412*I-423*14^(1/2))/(1+m)/(5*I*d-e*(I+14
^(1/2)))
```

Rubi [A] time = 0.48, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1628, 68}

$$\frac{\left(100d^2+165de+81e^2\right)(d+ex)^{m+1}}{125e^3(m+1)}-\frac{(40d+33e)(d+ex)^{m+2}}{25e^3(m+2)}+\frac{4(d+ex)^{m+3}}{5e^3(m+3)}-\frac{\left(-423\sqrt{14}+6412i\right)(d+ex)^{m+1}\,{}_2F}{3500(m+1)\left(5id-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] ((100*d^2 + 165*d*e + 81*e^2)*(d + e*x)^(1 + m))/(125*e^3*(1 + m)) - ((40*d
 + 33*e)*(d + e*x)^(2 + m))/(25*e^3*(2 + m)) + (4*(d + e*x)^(3 + m))/(5*e^3
*(3 + m)) - ((6412*I - 423*Sqrt[14])*(d + e*x)^(1 + m)*Hypergeometric2F1[1,
 1 + m, 2 + m, (5*(d + e*x))/(5*d - e + I*Sqrt[14]*e)])/(3500*((5*I)*d - (I
 + Sqrt[14])*e)*(1 + m)) - ((6412*I + 423*Sqrt[14])*(d + e*x)^(1 + m)*Hyper
geometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - (1 + I*Sqrt[14])*e)])/(3
500*((5*I)*d - (I - Sqrt[14])*e)*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^m \left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2} dx = \int \left(\frac{\left(100d^2+165de+81e^2\right)(d+ex)^m}{125e^2} + \frac{\left(\frac{458}{125}+\frac{423i}{125\sqrt{14}}\right)(d+ex)^m}{2-2i\sqrt{14}+10x} \right.$$

$$= \frac{\left(100d^2+165de+81e^2\right)(d+ex)^{1+m}}{125e^3(1+m)} - \frac{(40d+33e)(d+ex)^{2+m}}{25e^3(2+m)} +$$

$$= \frac{\left(100d^2+165de+81e^2\right)(d+ex)^{1+m}}{125e^3(1+m)} - \frac{(40d+33e)(d+ex)^{2+m}}{25e^3(2+m)} +$$

Mathematica [A] time = 0.73, size = 221, normalized size = 0.87

$$\frac{(d+ex)^{m+1}\left(\frac{28\left(100d^2+165de+81e^2\right)}{e^3(m+1)} + \frac{2800(d+ex)^2}{e^3(m+3)} - \frac{140(40d+33e)(d+ex)}{e^3(m+2)} - \frac{\left(423\sqrt{14}+6412i\right){}_2F_1\left(1,m+1;m+2;\frac{5(d+ex)}{5d+\left(-1-i\sqrt{14}\right)e}\right)}{(m+1)\left(5id+\left(\sqrt{14}-i\right)e\right)} - \frac{(423}{}\right.}{3500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x
]
```

```
[Out] ((d + e*x)^(1 + m)*((28*(100*d^2 + 165*d*e + 81*e^2))/(e^3*(1 + m)) - (140*
(40*d + 33*e)*(d + e*x))/(e^3*(2 + m)) + (2800*(d + e*x)^2)/(e^3*(3 + m)) -
 ((6412*I + 423*Sqrt[14])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/
(5*d + (-1 - I*Sqrt[14])*e)])/(((5*I)*d + (-I + Sqrt[14])*e)*(1 + m)) - ((-
6412*I + 423*Sqrt[14])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*
d + I*(I + Sqrt[14])*e)])/(((-5*I)*d + (I + Sqrt[14])*e)*(1 + m))))/3500
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(4x^4-5x^3+3x^2+x+2\right)(ex+d)^m}{5x^2+2x+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] integral((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4x^4-5x^3+3x^2+x+2\right)(ex+d)^m}{5x^2+2x+3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(4x^4-5x^3+3x^2+x+2\right)(ex+d)^m}{5x^2+2x+3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^{4} - 5\,x^{3} + 3\,x^{2} + x + 2\right)(ex + d)^{m}}{5\,x^{2} + 2\,x + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d + e\,x)^{m}\,\left(4\,x^{4} - 5\,x^{3} + 3\,x^{2} + x + 2\right)}{5\,x^{2} + 2\,x + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

```
[Out] int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

3.371 $\int \frac{(d+ex)^m\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$

Optimal. Leaf size=377

$$\frac{\left(i\sqrt{14}\left(6565d^2-2de(1313-3206m)+e^2(3939-98m)\right)+80360d^2-5922dem-32144de+19138e^2m+482\right.}{19600(m+1)\left(5d+i\left(\sqrt{14}+i\right)e\right)\left(5d^2-2de+3e^2\right)}$$

```
[Out] 4/25*(e*x+d)^(1+m)/e/(1+m)-1/700*(1367*d-293*e+(423*d-1367*e)*x)*(e*x+d)^(1
+m)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)+1/19600*(e*x+d)^(1+m)*hypergeom([1, 1
+m],[2+m],5*(e*x+d)/(5*d-e*(1+I*14^(1/2))))*(80360*d^2-32144*d*e+48216*e^2-
5922*d*e*m+19138*e^2*m-I*(6565*d^2-2*d*e*(1313-3206*m)+e^2*(3939-98*m))*14^
(1/2))/(5*d^2-2*d*e+3*e^2)/(1+m)/(5*d-e*(1+I*14^(1/2)))+1/19600*(e*x+d)^(1+
m)*hypergeom([1, 1+m],[2+m],5*(e*x+d)/(5*d-e+I*14^(1/2)*e))*(80360*d^2-3214
4*d*e+48216*e^2-5922*d*e*m+19138*e^2*m+I*(6565*d^2-2*d*e*(1313-3206*m)+e^2*
(3939-98*m))*14^(1/2))/(5*d^2-2*d*e+3*e^2)/(1+m)/(5*d+I*e*(I+14^(1/2)))
```

Rubi [A] time = 0.90, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {1648, 1628, 68}

$$\frac{\left(i\sqrt{14}\left(6565d^2-2de(1313-3206m)+e^2(3939-98m)\right)+80360d^2-5922dem-32144de+19138e^2m+482\right.}{19600(m+1)\left(5d+i\left(\sqrt{14}+i\right)e\right)\left(5d^2-2de+3e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] (4*(d + e*x)^(1 + m))/(25*e*(1 + m)) - ((1367*d - 293*e + (423*d - 1367*e)*
x)*(d + e*x)^(1 + m))/(700*(5*d^2 - 2*d*e + 3*e^2)*(3 + 2*x + 5*x^2)) + ((8
0360*d^2 - 32144*d*e + 48216*e^2 + I*Sqrt[14]*(6565*d^2 - 2*d*e*(1313 - 320
6*m) + e^2*(3939 - 98*m)) - 5922*d*e*m + 19138*e^2*m)*(d + e*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - e + I*Sqrt[14]*e)])/(1
9600*(5*d + I*(I + Sqrt[14])*e)*(5*d^2 - 2*d*e + 3*e^2)*(1 + m)) + ((80360*
d^2 - 32144*d*e + 48216*e^2 - I*Sqrt[14]*(6565*d^2 - 2*d*e*(1313 - 3206*m)
+ e^2*(3939 - 98*m)) - 5922*d*e*m + 19138*e^2*m)*(d + e*x)^(1 + m)*Hypergeo
metric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - (1 + I*Sqrt[14])*e)])/(1960
0*(5*d - (1 + I*Sqrt[14])*e)*(5*d^2 - 2*d*e + 3*e^2)*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1648

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
```

```
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^m \left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx = -\frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{\int \frac{(d+ex)^m\left(\frac{2}{25}(1845d^{\cdots}\right.}{\cdots}}{\cdots}$$

$$= -\frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{\int\left(\frac{224}{25}\left(5d^2-2d\cdots\right.\right.}{\cdots}$$

$$= \frac{4(d+ex)^{1+m}}{25e(1+m)} - \frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} - \frac{(8\cdots}{\cdots}$$

$$= \frac{4(d+ex)^{1+m}}{25e(1+m)} - \frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{(8\cdots}{\cdots}$$

Mathematica [A] time = 1.80, size = 441, normalized size = 1.17

$$(d+ex)^{m+1}\left(-\frac{\sqrt{14}\left(\frac{\left(2115d^2+de\left(-846+\left(-6412+423i\sqrt{14}\right)m\right)+e^2\left(1269+\left(98-1367i\sqrt{14}\right)m\right)\right){}_2F_1\left(1,m+1;m+2;\frac{5(d+ex)}{5d+\left(-1-i\sqrt{14}\right)e}\right)}{5id+\left(\sqrt{14}-i\right)e} - \frac{\left(2115d^2-de\left(846+\left(6412+423i\sqrt{14}\right)m\right)\cdots\right.}{\cdots}\right.}{(m+1)\left(5d^2-2de+3e^2\right)}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2
,x]
```

```
[Out] ((d + e*x)^(1 + m)*(3136/(e + e*m) - (28*(d*(1367 + 423*x) - e*(293 + 1367*
x)))/((5*d^2 - 2*d*e + 3*e^2)*(3 + 2*x + 5*x^2)) + (56*(287*I + 31*Sqrt[14]
)*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + (-1 - I*Sqrt[14])
*e)])/(((5*I)*d + (-I + Sqrt[14])*e)*(1 + m)) + (56*(-287*I + 31*Sqrt[14])*
Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + I*(I + Sqrt[14])*e)
])/(((-5*I)*d + (I + Sqrt[14])*e)*(1 + m)) - (Sqrt[14]*(((2115*d^2 + d*e*(-
846 + (-6412 + (423*I)*Sqrt[14])*m) + e^2*(1269 + (98 - (1367*I)*Sqrt[14])*
m))*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + (-1 - I*Sqrt[14
])*e)])/((5*I)*d + (-I + Sqrt[14])*e) - ((2115*d^2 - d*e*(846 + (6412 + (42
3*I)*Sqrt[14])*m) + e^2*(1269 + (98 + (1367*I)*Sqrt[14])*m))*Hypergeometric
```

```
2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + I*(I + Sqrt[14])*e)])/((5*I)*d -
(I + Sqrt[14])*e)))/((5*d^2 - 2*d*e + 3*e^2)*(1 + m))))/19600
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(4\,x^4-5\,x^3+3\,x^2+x+2\right)(ex+d)^m}{25\,x^4+20\,x^3+34\,x^2+12\,x+9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] integral((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(25*x^4 + 20*x^3 + 34*
x^2 + 12*x + 9), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^4-5\,x^3+3\,x^2+x+2\right)(ex+d)^m}{\left(5\,x^2+2\,x+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3)^2,
x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(4x^4-5x^3+3x^2+x+2\right)(ex+d)^m}{\left(5x^2+2x+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^4-5\,x^3+3\,x^2+x+2\right)(ex+d)^m}{\left(5\,x^2+2\,x+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3)^2,
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^m\,\left(4\,x^4-5\,x^3+3\,x^2+x+2\right)}{\left(5\,x^2+2\,x+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
[Out] int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
[Out] Timed out
```

3.372 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(a+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=528

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2c^3\left(6a^2h-3abg+b^2f\right)-30a^2bc^2i+10ab^3ci-c^4(6be-4af)+b^5(-i)+12c^5d\right)}{c^3\left(b^2-4ac\right)^{5/2}}-\frac{x\left(c^3\left(2\right.\right.}{}$$

```
[Out] 1/2*(-a*b^3*c*h-b*c^2*(-3*a^2*h+a*c*f+c^2*d)+a*b^4*i+a*b^2*c*(-4*a*i+c*g)+2
*a*c^2*(a^2*i-a*c*g+c^2*e)-(2*c^5*d-c^4*(2*a*f+b*e)+c^3*(2*a^2*h+3*a*b*g+b^
2*f)-b^5*i+b^3*c*(5*a*i+b*h)-b*c^2*(5*a^2*i+4*a*b*h+b^2*g))*x)/c^4/(-4*a*c+
b^2)/(c*x^2+b*x+a)^2+1/2*(b^5*c*h+b^3*c^2*(-8*a*h+c*f)+2*b*c^3*(11*a^2*h+a*
c*f+3*c^2*d)-b^6*i-b^4*c*(-11*a*i+c*g)-16*a^2*c^3*(-2*a*i+c*g)-b^2*c^2*(39*
a^2*i-5*a*c*g+3*c^2*e)+2*c*(6*c^5*d-c^4*(-2*a*f+3*b*e)+c^3*(-10*a^2*h-3*a*b
*g+b^2*f)+2*b^5*i-b^3*c*(15*a*i+b*h)+a*b*c^2*(25*a*i+8*b*h))*x)/c^4/(-4*a*c
+b^2)^2/(c*x^2+b*x+a)-(12*c^5*d-c^4*(-4*a*f+6*b*e)+2*c^3*(6*a^2*h-3*a*b*g+b
^2*f)-b^5*i+10*a*b^3*c*i-30*a^2*b*c^2*i)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/
2))/c^3/(-4*a*c+b^2)^(5/2)+1/2*i*ln(c*x^2+b*x+a)/c^3
```

Rubi [A] time = 1.31, antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1660, 634, 618, 206, 628}

$$\frac{2cx\left(c^3\left(-10a^2h-3abg+b^2f\right)-b^3c(15ai+bh)-c^4(3be-2af)+abc^2(25ai+8bh)+2b^5i+6c^5d\right)-b^2c^2\left(39a\right.}{2c^4\left(b^2-4a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -(a*b^3*c*h + b*c^2*(c^2*d + a*c*f - 3*a^2*h) - a*b^4*i - a*b^2*c*(c*g - 4*
a*i) - 2*a*c^2*(c^2*e - a*c*g + a^2*i) + (2*c^5*d - c^4*(b*e + 2*a*f) + c^3
*(b^2*f + 3*a*b*g + 2*a^2*h) - b^5*i + b^3*c*(b*h + 5*a*i) - b*c^2*(b^2*g +
 4*a*b*h + 5*a^2*i))*x)/(2*c^4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (b^5*c*
h + b^3*c^2*(c*f - 8*a*h) + 2*b*c^3*(3*c^2*d + a*c*f + 11*a^2*h) - b^6*i -
b^4*c*(c*g - 11*a*i) - 16*a^2*c^3*(c*g - 2*a*i) - b^2*c^2*(3*c^2*e - 5*a*c*
g + 39*a^2*i) + 2*c*(6*c^5*d - c^4*(3*b*e - 2*a*f) + c^3*(b^2*f - 3*a*b*g -
 10*a^2*h) + 2*b^5*i - b^3*c*(b*h + 15*a*i) + a*b*c^2*(8*b*h + 25*a*i))*x)/
(2*c^4*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - ((12*c^5*d - c^4*(6*b*e - 4*a*f
) + 2*c^3*(b^2*f - 3*a*b*g + 6*a^2*h) - b^5*i + 10*a*b^3*c*i - 30*a^2*b*c^2
*i)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + (i*
Log[a + b*x + c*x^2])/(2*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+372x^5}{\left(a+bx+cx^2\right)^3}\,dx = \frac{744a^3c^2-bc^4d-a^2c\left(1488b^2+2c^2g-3bch\right)+a\left(372b^4+2c^4e-bc^3\right.}{}$$

$$= \frac{744a^3c^2-bc^4d-a^2c\left(1488b^2+2c^2g-3bch\right)+a\left(372b^4+2c^4e-bc^3\right.}{}$$

$$= \frac{744a^3c^2-bc^4d-a^2c\left(1488b^2+2c^2g-3bch\right)+a\left(372b^4+2c^4e-bc^3\right.}{}$$

$$= \frac{744a^3c^2-bc^4d-a^2c\left(1488b^2+2c^2g-3bch\right)+a\left(372b^4+2c^4e-bc^3\right.}{}$$

$$= \frac{744a^3c^2-bc^4d-a^2c\left(1488b^2+2c^2g-3bch\right)+a\left(372b^4+2c^4e-bc^3\right.}{}$$

Mathematica [A] time = 1.08, size = 488, normalized size = 0.92

$$\frac{\frac{2c\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(2c^3\left(6a^2h-3abg+b^2f\right)-30a^2bc^2i+10ab^3ci+c^4(4af-6be)+b^5(-i)+12c^5d\right)}{\left(4ac-b^2\right)^{5/2}}+\frac{b^2c\left(-4a^2i+ac(g+4hx)-c^2fx\right)+bc^2\left(a^2(3h+5ix)-ac(f+3g\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x]
```

```
[Out] ((b^5*i*x + b^4*(a*i - c*h*x) + 2*c^2*(a^3*i - c^3*d*x + a*c^2*(e + f*x) -
a^2*c*(g + h*x)) + b^2*c*(-4*a^2*i - c^2*f*x + a*c*(g + 4*h*x)) + b^3*c*(c*
```

```
g*x - a*(h + 5*i*x)) + b*c^2*(c^2*(-d + e*x) - a*c*(f + 3*g*x) + a^2*(3*h +
 5*i*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (-(b^6*i) + b^5*c*(h + 4*i*
x) + b^3*c^2*(c*f - 8*a*h - 30*a*i*x) - b^4*c*(-11*a*i + c*(g + 2*h*x)) + 4
*c^3*(8*a^3*i + 3*c^3*d*x + a*c^2*f*x - a^2*c*(4*g + 5*h*x)) + b^2*c^2*(-39
*a^2*i + c^2*(-3*e + 2*f*x) + a*c*(5*g + 16*h*x)) + 2*b*c^3*(3*c^2*(d - e*x
) + a*c*(f - 3*g*x) + a^2*(11*h + 25*i*x)))/((b^2 - 4*a*c)^2*(a + x*(b + c*
x))) + (2*c*(12*c^5*d + c^4*(-6*b*e + 4*a*f) + 2*c^3*(b^2*f - 3*a*b*g + 6*a
^2*h) - b^5*i + 10*a*b^3*c*i - 30*a^2*b*c^2*i)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*i*Log[a + x*(b + c*x)])/(2*c^4)
```

fricas [B] time = 0.99, size = 3480, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="fri
cas")
```

```
[Out] [1/2*(2*(6*(b^2*c^6 - 4*a*c^7)*d - 3*(b^3*c^5 - 4*a*b*c^6)*e + (b^4*c^4 - 2
*a*b^2*c^5 - 8*a^2*c^6)*f - 3*(a*b^3*c^4 - 4*a^2*b*c^5)*g - (b^6*c^2 - 12*a
*b^4*c^3 + 42*a^2*b^2*c^4 - 40*a^3*c^5)*h + (2*b^7*c - 23*a*b^5*c^2 + 85*a^
2*b^3*c^3 - 100*a^3*b*c^4)*i)*x^3 + (18*(b^3*c^5 - 4*a*b*c^6)*d - 9*(b^4*c^
4 - 4*a*b^2*c^5)*e + 3*(b^5*c^3 - 2*a*b^3*c^4 - 8*a^2*b*c^5)*f - (b^6*c^2 -
 3*a*b^4*c^3 + 12*a^2*b^2*c^4 - 64*a^3*c^5)*g - (b^7*c - 12*a*b^5*c^2 + 30*
a^2*b^3*c^3 + 8*a^3*b*c^4)*h + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^
3*b^2*c^3 - 128*a^4*c^4)*i)*x^2 - (12*a^2*c^5*d - 6*a^2*b*c^4*e - 6*a^3*b*c
^3*g + 12*a^4*c^3*h + (12*c^7*d - 6*b*c^6*e - 6*a*b*c^5*g + 12*a^2*c^5*h +
2*(b^2*c^5 + 2*a*c^6)*f - (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*i)*x^4 +
2*(12*b*c^6*d - 6*b^2*c^5*e - 6*a*b^2*c^4*g + 12*a^2*b*c^4*h + 2*(b^3*c^4 +
 2*a*b*c^5)*f - (b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*i)*x^3 + (12*(b^2*c
^5 + 2*a*c^6)*d - 6*(b^3*c^4 + 2*a*b*c^5)*e + 2*(b^4*c^3 + 4*a*b^2*c^4 + 4*
a^2*c^5)*f - 6*(a*b^3*c^3 + 2*a^2*b*c^4)*g + 12*(a^2*b^2*c^3 + 2*a^3*c^4)*h
 - (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*i)*x^2 + 2*(a^2*b^2*c^
3 + 2*a^3*c^4)*f - (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2)*i + 2*(12*a*b*c^
5*d - 6*a*b^2*c^4*e - 6*a^2*b^2*c^3*g + 12*a^3*b*c^3*h + 2*(a*b^3*c^3 + 2*a
^2*b*c^4)*f - (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*i)*x)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/
(c*x^2 + b*x + a)) - (b^5*c^3 - 14*a*b^3*c^4 + 40*a^2*b*c^5)*d - (a*b^4*c^3
 + 4*a^2*b^2*c^4 - 32*a^3*c^5)*e + 6*(a^2*b^3*c^3 - 4*a^3*b*c^4)*f - (a^2*b
^4*c^2 + 4*a^3*b^2*c^3 - 32*a^4*c^4)*g - (a^2*b^5*c - 14*a^3*b^3*c^2 + 40*a
^4*b*c^3)*h + 3*(a^2*b^6 - 11*a^3*b^4*c + 36*a^4*b^2*c^2 - 32*a^5*c^3)*i +
2*(2*(b^4*c^4 + a*b^2*c^5 - 20*a^2*c^6)*d - (b^5*c^3 + a*b^3*c^4 - 20*a^2*b
*c^5)*e + (5*a*b^4*c^3 - 22*a^2*b^2*c^4 + 8*a^3*c^5)*f - (a*b^5*c^2 + a^2*b
^3*c^3 - 20*a^3*b*c^4)*g - (a*b^6*c - 14*a^2*b^4*c^2 + 46*a^3*b^2*c^3 - 24*
a^4*c^4)*h + (3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*i)*
x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*i*x^4 + 2*(b^7*
c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*i*x^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*i*x^2 + 2*(a*b^7 - 12*a^2
*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*i*x + (a^2*b^6 - 12*a^3*b^4*c + 48*
a^4*b^2*c^2 - 64*a^5*c^3)*i)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^
4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*
c^7 - 64*a^3*c^8)*x^4 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3
*b*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 1
28*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b
*c^6)*x), 1/2*(2*(6*(b^2*c^6 - 4*a*c^7)*d - 3*(b^3*c^5 - 4*a*b*c^6)*e + (b^
4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*f - 3*(a*b^3*c^4 - 4*a^2*b*c^5)*g - (b^6*c
^2 - 12*a*b^4*c^3 + 42*a^2*b^2*c^4 - 40*a^3*c^5)*h + (2*b^7*c - 23*a*b^5*c^
2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*i)*x^3 + (18*(b^3*c^5 - 4*a*b*c^6)*d -
9*(b^4*c^4 - 4*a*b^2*c^5)*e + 3*(b^5*c^3 - 2*a*b^3*c^4 - 8*a^2*b*c^5)*f - (
b^6*c^2 - 3*a*b^4*c^3 + 12*a^2*b^2*c^4 - 64*a^3*c^5)*g - (b^7*c - 12*a*b^5*
```

```
c^2 + 30*a^2*b^3*c^3 + 8*a^3*b*c^4)*h + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^
2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*i)*x^2 - 2*(12*a^2*c^5*d - 6*a^2*b*c^4*e
- 6*a^3*b*c^3*g + 12*a^4*c^3*h + (12*c^7*d - 6*b*c^6*e - 6*a*b*c^5*g + 12*a
^2*c^5*h + 2*(b^2*c^5 + 2*a*c^6)*f - (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4
)*i)*x^4 + 2*(12*b*c^6*d - 6*b^2*c^5*e - 6*a*b^2*c^4*g + 12*a^2*b*c^4*h + 2
*(b^3*c^4 + 2*a*b*c^5)*f - (b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*i)*x^3 +
 (12*(b^2*c^5 + 2*a*c^6)*d - 6*(b^3*c^4 + 2*a*b*c^5)*e + 2*(b^4*c^3 + 4*a*b
^2*c^4 + 4*a^2*c^5)*f - 6*(a*b^3*c^3 + 2*a^2*b*c^4)*g + 12*(a^2*b^2*c^3 + 2
*a^3*c^4)*h - (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*i)*x^2 + 2*
(a^2*b^2*c^3 + 2*a^3*c^4)*f - (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2)*i + 2
*(12*a*b*c^5*d - 6*a*b^2*c^4*e - 6*a^2*b^2*c^3*g + 12*a^3*b*c^3*h + 2*(a*b^
3*c^3 + 2*a^2*b*c^4)*f - (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*i)*x)*sqrt
(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^5
*c^3 - 14*a*b^3*c^4 + 40*a^2*b*c^5)*d - (a*b^4*c^3 + 4*a^2*b^2*c^4 - 32*a^3
*c^5)*e + 6*(a^2*b^3*c^3 - 4*a^3*b*c^4)*f - (a^2*b^4*c^2 + 4*a^3*b^2*c^3 -
32*a^4*c^4)*g - (a^2*b^5*c - 14*a^3*b^3*c^2 + 40*a^4*b*c^3)*h + 3*(a^2*b^6
- 11*a^3*b^4*c + 36*a^4*b^2*c^2 - 32*a^5*c^3)*i + 2*(2*(b^4*c^4 + a*b^2*c^5
 - 20*a^2*c^6)*d - (b^5*c^3 + a*b^3*c^4 - 20*a^2*b*c^5)*e + (5*a*b^4*c^3 -
22*a^2*b^2*c^4 + 8*a^3*c^5)*f - (a*b^5*c^2 + a^2*b^3*c^3 - 20*a^3*b*c^4)*g
- (a*b^6*c - 14*a^2*b^4*c^2 + 46*a^3*b^2*c^3 - 24*a^4*c^4)*h + (3*a*b^7 - 3
4*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*i)*x + ((b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*i*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*i*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^
3*b^2*c^3 - 128*a^4*c^4)*i*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 -
 64*a^4*b*c^3)*i*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
*i)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 -
64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 + 2
*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3 -
10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*b^
7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x)]
```

giac [**A**] time = 0.22, size = 657, normalized size = 1.24

$$\frac{\left(12\,c^5di + 2\,b^2c^3fi + 4\,ac^4fi - 6\,abc^3gi + 12\,a^2c^3hi - 6\,bc^4ie + b^5 - 10\,ab^3c + 30\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4c^3i - 8\,ab^2c^4i + 16\,a^2c^5i\right)\sqrt{-b^2+4\,ac}} + \frac{i\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="gia
c")
```

```
[Out] (12*c^5*d*i + 2*b^2*c^3*f*i + 4*a*c^4*f*i - 6*a*b*c^3*g*i + 12*a^2*c^3*h*i
- 6*b*c^4*i*e + b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b
^2 + 4*a*c))/((b^4*c^3*i - 8*a*b^2*c^4*i + 16*a^2*c^5*i)*sqrt(-b^2 + 4*a*c)
) + 1/2*i*log(c*x^2 + b*x + a)/c^3 - 1/2*(b^3*c^3*d - 10*a*b*c^4*d - 6*a^2*
b*c^3*f + a^2*b^2*c^2*g + 8*a^3*c^3*g + a^2*b^3*c*h - 10*a^3*b*c^2*h - 3*a^
2*b^4*i + 21*a^3*b^2*c*i - 24*a^4*c^2*i + a*b^2*c^3*e + 8*a^2*c^4*e - 2*(6*
c^6*d + b^2*c^4*f + 2*a*c^5*f - 3*a*b*c^4*g - b^4*c^2*h + 8*a*b^2*c^3*h - 1
0*a^2*c^4*h + 2*b^5*c*i - 15*a*b^3*c^2*i + 25*a^2*b*c^3*i - 3*b*c^5*e)*x^3
- (18*b*c^5*d + 3*b^3*c^3*f + 6*a*b*c^4*f - b^4*c^2*g - a*b^2*c^3*g - 16*a^
2*c^4*g - b^5*c*h + 8*a*b^3*c^2*h + 2*a^2*b*c^3*h + 3*b^6*i - 19*a*b^4*c*i
+ 11*a^2*b^2*c^2*i + 32*a^3*c^3*i - 9*b^2*c^4*e)*x^2 - 2*(2*b^2*c^4*d + 10*
a*c^5*d + 5*a*b^2*c^3*f - 2*a^2*c^4*f - a*b^3*c^2*g - 5*a^2*b*c^3*g - a*b^4
*c*h + 10*a^2*b^2*c^2*h - 6*a^3*c^3*h + 3*a*b^5*i - 22*a^2*b^3*c*i + 31*a^3
*b*c^2*i - b^3*c^3*e - 5*a*b*c^4*e)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2
*c^3)
```

maple [B] time = 0.02, size = 1244, normalized size = 2.36

$$-\frac{30a^2bi\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}\;c}+\frac{12a^2h\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}+\frac{10a\,b^3i\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}\;c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x)
```

```
[Out] ((25*a^2*b*c^2*i-10*a^2*c^3*h-15*a*b^3*c*i+8*a*b^2*c^2*h-3*a*b*c^3*g+2*a*c^
4*f+2*b^5*i-b^4*c*h+b^2*c^3*f-3*b*c^4*e+6*c^5*d)/c^2/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^3+1/2*(32*a^3*c^3*i+11*a^2*b^2*c^2*i+2*a^2*b*c^3*h-16*a^2*c^4*g-19*a
*b^4*c*i+8*a*b^3*c^2*h-a*b^2*c^3*g+6*a*b*c^4*f+3*b^6*i-b^5*c*h-b^4*c^2*g+3*
b^3*c^3*f-9*b^2*c^4*e+18*b*c^5*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x^2+(31*a^
3*b*c^2*i-6*a^3*c^3*h-22*a^2*b^3*c*i+10*a^2*b^2*c^2*h-5*a^2*b*c^3*g-2*a^2*c
^4*f+3*a*b^5*i-a*b^4*c*h-a*b^3*c^2*g+5*a*b^2*c^3*f-5*a*b*c^4*e+10*a*c^5*d-b
^3*c^3*e+2*b^2*c^4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x+1/2/c^3*(24*a^4*c^2*
i-21*a^3*b^2*c*i+10*a^3*b*c^2*h-8*a^3*c^3*g+3*a^2*b^4*i-a^2*b^3*c*h-a^2*b^2
*c^2*g+6*a^2*b*c^3*f-8*a^2*c^4*e-a*b^2*c^3*e+10*a*b*c^4*d-b^3*c^3*d)/(16*a^
2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+8/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x
^2+b*x+a)*a^2*i-4/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*a*b^2*i+1/
2/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^4*i-30/c/(16*a^2*c^2-8*a
*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*i+1
2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*a^2*h+10/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*i-6/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)
^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*g+4*c/(16*a^2*c^2-8*a*b^2*c+
b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*f+2/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*
f-6*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))*b*e+12*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x+b)/(4*a*c-b^2)^(1/2))*d-1/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)
^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5*i
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 6.17, size = 1027, normalized size = 1.95

$$\frac{\mathrm{atan}\left(\frac{x\left(32\,a^2\,c^5\,{\left(4\,a\,c-b^2\right)}^{5/2}+2\,b^4\,c^3\,{\left(4\,a\,c-b^2\right)}^{5/2}-16\,a\,b^2\,c^4\,{\left(4\,a\,c-b^2\right)}^{5/2}\right)}{c^2\,{\left(4\,a\,c-b^2\right)}^5}+\frac{\left(32\,a^2\,c^5\,{\left(4\,a\,c-b^2\right)}^{5/2}+2\,b^4\,c^3\,{\left(4\,a\,c-b^2\right)}^{5/2}-16\,a\,b^2\,c^4\,{\left(4\,a\,c-b^2\right)}^{5/}\right.}{2\,c^5\,{\left(4\,a\,c-b^2\right)}^5\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right.}\right.}{c^3\,\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x)
```

```
[Out] (atan((x*(32*a^2*c^5*(4*a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) -
16*a*b^2*c^4*(4*a*c - b^2)^(5/2)))/(c^2*(4*a*c - b^2)^5) + ((32*a^2*c^5*(4*
a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) - 16*a*b^2*c^4*(4*a*c - b^
2)^(5/2))*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4))/(2*c^5*(4*a*c - b^2)^5*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(12*c^5*d - b^5*i + 2*b^2*c^3*f + 12*a^2*c^3
*h + 4*a*c^4*f - 6*b*c^4*e - 6*a*b*c^3*g + 10*a*b^3*c*i - 30*a^2*b*c^2*i))/
(c^3*(4*a*c - b^2)^(5/2)) - (log(a + b*x + c*x^2)*(b^10*i - 1024*a^5*c^5*i
+ 160*a^2*b^6*c^2*i - 640*a^3*b^4*c^3*i + 1280*a^4*b^2*c^4*i - 20*a*b^8*c*i
))/(2*(1024*a^5*c^8 - b^10*c^3 + 20*a*b^8*c^4 - 160*a^2*b^6*c^5 + 640*a^3*b
^4*c^6 - 1280*a^4*b^2*c^7)) - ((8*a^2*c^4*e + b^3*c^3*d + 8*a^3*c^3*g - 3*a
^2*b^4*i - 24*a^4*c^2*i + a^2*b^2*c^2*g - 10*a*b*c^4*d + a*b^2*c^3*e - 6*a^
2*b*c^3*f + a^2*b^3*c*h - 10*a^3*b*c^2*h + 21*a^3*b^2*c*i)/(2*c^3*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) - (x^2*(3*b^6*i - 9*b^2*c^4*e - 16*a^2*c^4*g + 3*b^3
*c^3*f - b^4*c^2*g + 32*a^3*c^3*i + 18*b*c^5*d - b^5*c*h + 11*a^2*b^2*c^2*i
 + 6*a*b*c^4*f - 19*a*b^4*c*i - a*b^2*c^3*g + 8*a*b^3*c^2*h + 2*a^2*b*c^3*h
))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(2*a^2*c^4*f - 2*b^2*c^4*d +
 b^3*c^3*e + 6*a^3*c^3*h - 10*a*c^5*d - 3*a*b^5*i - 10*a^2*b^2*c^2*h + 5*a*
b*c^4*e + a*b^4*c*h - 5*a*b^2*c^3*f + a*b^3*c^2*g + 5*a^2*b*c^3*g + 22*a^2*
b^3*c*i - 31*a^3*b*c^2*i))/(c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^3*(6*c
^5*d + 2*b^5*i + b^2*c^3*f - 10*a^2*c^3*h + 2*a*c^4*f - 3*b*c^4*e - b^4*c*h
 - 3*a*b*c^3*g - 15*a*b^3*c*i + 8*a*b^2*c^2*h + 25*a^2*b*c^2*i))/(c^2*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x +
2*b*c*x^3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

3.373 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx$

Optimal. Leaf size=765

$$\frac{x^3\left(c^2\left(a^2m+2abl+b^2k\right)-b^2c(3am+bl)-c^3(ak+bj)+b^4m+c^4h\right)}{3c^5}+\frac{x^2\left(c^3\left(a^2l+2abk+b^2j\right)-bc^2\left(3a^2m+\right.\right.}{}$$

```
[Out] (c^6*f-c^5*(a*h+b*g)+c^4*(a^2*k+2*a*b*j+b^2*h)+b^6*m-b^4*c*(5*a*m+b*l)+b^2*
c^2*(6*a^2*m+4*a*b*l+b^2*k)-c^3*(a^3*m+3*a^2*b*l+3*a*b^2*k+b^3*j))*x/c^7+1/
2*(c^5*g-c^4*(a*j+b*h)+c^3*(a^2*l+2*a*b*k+b^2*j)-b^5*m+b^3*c*(4*a*m+b*l)-b*
c^2*(3*a^2*m+3*a*b*l+b^2*k))*x^2/c^6+1/3*(c^4*h-c^3*(a*k+b*j)+b^4*m-b^2*c*(
3*a*m+b*l)+c^2*(a^2*m+2*a*b*l+b^2*k))*x^3/c^5+1/4*(c^3*j-c^2*(a*l+b*k)-b^3*
m+b*c*(2*a*m+b*l))*x^4/c^4+1/5*(c^2*k+b^2*m-c*(a*m+b*l))*x^5/c^3+1/6*(-b*m+
c*l)*x^6/c^2+1/7*m*x^7/c+1/2*(c^7*e-c^6*(a*g+b*f)+c^5*(a^2*j+2*a*b*h+b^2*g)
-c^4*(a^3*l+3*a^2*b*k+3*a*b^2*j+b^3*h)-b^7*m+b^5*c*(6*a*m+b*l)-b^3*c^2*(10*
a^2*m+5*a*b*l+b^2*k)+b*c^3*(4*a^3*m+6*a^2*b*l+4*a*b^2*k+b^3*j))*ln(c*x^2+b*
x+a)/c^8-(2*c^8*d-c^7*(2*a*f+b*e)+c^6*(2*a^2*h+3*a*b*g+b^2*f)-c^5*(2*a^3*k+
5*a^2*b*j+4*a*b^2*h+b^3*g)+b^8*m-b^6*c*(8*a*m+b*l)+b^4*c^2*(20*a^2*m+7*a*b*
l+b^2*k)-b^2*c^3*(16*a^3*m+14*a^2*b*l+6*a*b^2*k+b^3*j)+c^4*(2*a^4*m+7*a^3*b
*l+9*a^2*b^2*k+5*a*b^3*j+b^4*h))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^8/
(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 5.83, antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1657, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^6\left(2a^2h+3abg+b^2f\right)-c^5\left(5a^2bj+2a^3k+4ab^2h+b^3g\right)+c^4\left(9a^2b^2k+7a^3bl+2a^4m+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x + c*x^2),x]
```

```
[Out] ((c^6*f - c^5*(b*g + a*h) + c^4*(b^2*h + 2*a*b*j + a^2*k) + b^6*m - b^4*c*(
b*l + 5*a*m) + b^2*c^2*(b^2*k + 4*a*b*l + 6*a^2*m) - c^3*(b^3*j + 3*a*b^2*k
 + 3*a^2*b*l + a^3*m))*x)/c^7 + ((c^5*g - c^4*(b*h + a*j) + c^3*(b^2*j + 2*
a*b*k + a^2*l) - b^5*m + b^3*c*(b*l + 4*a*m) - b*c^2*(b^2*k + 3*a*b*l + 3*a
^2*m))*x^2)/(2*c^6) + ((c^4*h - c^3*(b*j + a*k) + b^4*m - b^2*c*(b*l + 3*a*
m) + c^2*(b^2*k + 2*a*b*l + a^2*m))*x^3)/(3*c^5) + ((c^3*j - c^2*(b*k + a*l
) - b^3*m + b*c*(b*l + 2*a*m))*x^4)/(4*c^4) + ((c^2*k + b^2*m - c*(b*l + a*
m))*x^5)/(5*c^3) + ((c*l - b*m)*x^6)/(6*c^2) + (m*x^7)/(7*c) - ((2*c^8*d -
c^7*(b*e + 2*a*f) + c^6*(b^2*f + 3*a*b*g + 2*a^2*h) - c^5*(b^3*g + 4*a*b^2*
h + 5*a^2*b*j + 2*a^3*k) + b^8*m - b^6*c*(b*l + 8*a*m) + b^4*c^2*(b^2*k + 7
*a*b*l + 20*a^2*m) - b^2*c^3*(b^3*j + 6*a*b^2*k + 14*a^2*b*l + 16*a^3*m) +
c^4*(b^4*h + 5*a*b^3*j + 9*a^2*b^2*k + 7*a^3*b*l + 2*a^4*m))*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(c^8*Sqrt[b^2 - 4*a*c]) + ((c^7*e - c^6*(b*f + a*
g) + c^5*(b^2*g + 2*a*b*h + a^2*j) - c^4*(b^3*h + 3*a*b^2*j + 3*a^2*b*k + a
^3*l) - b^7*m + b^5*c*(b*l + 6*a*m) - b^3*c^2*(b^2*k + 5*a*b*l + 10*a^2*m)
+ b*c^3*(b^3*j + 4*a*b^2*k + 6*a^2*b*l + 4*a^3*m))*Log[a + b*x + c*x^2])/(2
*c^8)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx = \int \left(\frac{c^6f-c^5(bg+ah)+c^4\left(b^2h+2abj+a^2k\right)+b^6m-\right.$$

$$= \frac{\left(c^6f-c^5(bg+ah)+c^4\left(b^2h+2abj+a^2k\right)+b^6m-b^4\right.}{}$$

$$= \frac{\left(c^6f-c^5(bg+ah)+c^4\left(b^2h+2abj+a^2k\right)+b^6m-b^4\right.}{}$$

$$= \frac{\left(c^6f-c^5(bg+ah)+c^4\left(b^2h+2abj+a^2k\right)+b^6m-b^4\right.}{}$$

$$= \frac{\left(c^6f-c^5(bg+ah)+c^4\left(b^2h+2abj+a^2k\right)+b^6m-b^4\right.}{}$$

Mathematica [A] time = 0.73, size = 754, normalized size = 0.99

$$\frac{140c^3x^3\left(c^2\left(a^2m+2abl+b^2k\right)-b^2c(3am+bl)-c^3(ak+bj)+b^4m+c^4h\right)+210c^2x^2\left(c^3\left(a^2l+2abk+b^2j\right)-bc^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)
/(a + b*x + c*x^2),x]
```

```
[Out] (420*c*(c^6*f - c^5*(b*g + a*h) + c^4*(b^2*h + 2*a*b*j + a^2*k) + b^6*m - b
^4*c*(b*l + 5*a*m) + b^2*c^2*(b^2*k + 4*a*b*l + 6*a^2*m) - c^3*(b^3*j + 3*a
*b^2*k + 3*a^2*b*l + a^3*m))*x + 210*c^2*(c^5*g - c^4*(b*h + a*j) + c^3*(b^
2*j + 2*a*b*k + a^2*l) - b^5*m + b^3*c*(b*l + 4*a*m) - b*c^2*(b^2*k + 3*a*b
```

```
*l + 3*a^2*m))*x^2 + 140*c^3*(c^4*h - c^3*(b*j + a*k) + b^4*m - b^2*c*(b*l
+ 3*a*m) + c^2*(b^2*k + 2*a*b*l + a^2*m))*x^3 + 105*c^4*(c^3*j - c^2*(b*k +
 a*l) - b^3*m + b*c*(b*l + 2*a*m))*x^4 + 84*c^5*(c^2*k + b^2*m - c*(b*l + a
*m))*x^5 + 70*c^6*(c*l - b*m)*x^6 + 60*c^7*m*x^7 + (420*(2*c^8*d - c^7*(b*e
 + 2*a*f) + c^6*(b^2*f + 3*a*b*g + 2*a^2*h) - c^5*(b^3*g + 4*a*b^2*h + 5*a^
2*b*j + 2*a^3*k) + b^8*m - b^6*c*(b*l + 8*a*m) + b^4*c^2*(b^2*k + 7*a*b*l +
 20*a^2*m) - b^2*c^3*(b^3*j + 6*a*b^2*k + 14*a^2*b*l + 16*a^3*m) + c^4*(b^4
*h + 5*a*b^3*j + 9*a^2*b^2*k + 7*a^3*b*l + 2*a^4*m))*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 210*(c^7*e - c^6*(b*f + a*g) + c^5*(
b^2*g + 2*a*b*h + a^2*j) - c^4*(b^3*h + 3*a*b^2*j + 3*a^2*b*k + a^3*l) - b^
7*m + b^5*c*(b*l + 6*a*m) - b^3*c^2*(b^2*k + 5*a*b*l + 10*a^2*m) + b*c^3*(b
^3*j + 4*a*b^2*k + 6*a^2*b*l + 4*a^3*m))*Log[a + x*(b + c*x)])/(420*c^8)
```

fricas [A] time = 1.84, size = 2643, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x
, algorithm="fricas")
```

```
[Out] [1/420*(60*(b^2*c^7 - 4*a*c^8)*m*x^7 + 70*((b^2*c^7 - 4*a*c^8)*l - (b^3*c^6
 - 4*a*b*c^7)*m)*x^6 + 84*((b^2*c^7 - 4*a*c^8)*k - (b^3*c^6 - 4*a*b*c^7)*l
+ (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*m)*x^5 + 105*((b^2*c^7 - 4*a*c^8)*j -
 (b^3*c^6 - 4*a*b*c^7)*k + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*l - (b^5*c^4
 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*m)*x^4 + 140*((b^2*c^7 - 4*a*c^8)*h - (b^3*c^
6 - 4*a*b*c^7)*j + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*k - (b^5*c^4 - 6*a*b
^3*c^5 + 8*a^2*b*c^6)*l + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c
^6)*m)*x^3 + 210*((b^2*c^7 - 4*a*c^8)*g - (b^3*c^6 - 4*a*b*c^7)*h + (b^4*c^
5 - 5*a*b^2*c^6 + 4*a^2*c^7)*j - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*k +
(b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*l - (b^7*c^2 - 8*a*b^5
*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*m)*x^2 + 210*(2*c^8*d - b*c^7*e + (b^
2*c^6 - 2*a*c^7)*f - (b^3*c^5 - 3*a*b*c^6)*g + (b^4*c^4 - 4*a*b^2*c^5 + 2*a
^2*c^6)*h - (b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5)*j + (b^6*c^2 - 6*a*b^4*c^
3 + 9*a^2*b^2*c^4 - 2*a^3*c^5)*k - (b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 -
7*a^3*b*c^4)*l + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*a^4
*c^4)*m)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 420*((b^2*c^7 - 4*a*c^8)*f - (
b^3*c^6 - 4*a*b*c^7)*g + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*h - (b^5*c^4 -
 6*a*b^3*c^5 + 8*a^2*b*c^6)*j + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4
*a^3*c^6)*k - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*l + (
b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*b^2*c^4 + 4*a^4*c^5)*m)*x + 2
10*((b^2*c^7 - 4*a*c^8)*e - (b^3*c^6 - 4*a*b*c^7)*f + (b^4*c^5 - 5*a*b^2*c^
6 + 4*a^2*c^7)*g - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*h + (b^6*c^3 - 7*a
*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*j - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*
b^3*c^4 - 12*a^3*b*c^5)*k + (b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*
b^2*c^4 + 4*a^4*c^5)*l - (b^9 - 10*a*b^7*c + 34*a^2*b^5*c^2 - 44*a^3*b^3*c^
3 + 16*a^4*b*c^4)*m)*log(c*x^2 + b*x + a))/(b^2*c^8 - 4*a*c^9), 1/420*(60*(
b^2*c^7 - 4*a*c^8)*m*x^7 + 70*((b^2*c^7 - 4*a*c^8)*l - (b^3*c^6 - 4*a*b*c^7
)*m)*x^6 + 84*((b^2*c^7 - 4*a*c^8)*k - (b^3*c^6 - 4*a*b*c^7)*l + (b^4*c^5 -
 5*a*b^2*c^6 + 4*a^2*c^7)*m)*x^5 + 105*((b^2*c^7 - 4*a*c^8)*j - (b^3*c^6 -
4*a*b*c^7)*k + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*l - (b^5*c^4 - 6*a*b^3*c
^5 + 8*a^2*b*c^6)*m)*x^4 + 140*((b^2*c^7 - 4*a*c^8)*h - (b^3*c^6 - 4*a*b*c^
7)*j + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*k - (b^5*c^4 - 6*a*b^3*c^5 + 8*a
^2*b*c^6)*l + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*m)*x^3 +
 210*((b^2*c^7 - 4*a*c^8)*g - (b^3*c^6 - 4*a*b*c^7)*h + (b^4*c^5 - 5*a*b^2*
c^6 + 4*a^2*c^7)*j - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*k + (b^6*c^3 - 7
*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*l - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^
2*b^3*c^4 - 12*a^3*b*c^5)*m)*x^2 - 420*(2*c^8*d - b*c^7*e + (b^2*c^6 - 2*a*
c^7)*f - (b^3*c^5 - 3*a*b*c^6)*g + (b^4*c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*h -
```

```
(b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5)*j + (b^6*c^2 - 6*a*b^4*c^3 + 9*a^2*b^
2*c^4 - 2*a^3*c^5)*k - (b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 7*a^3*b*c^4)
*l + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*a^4*c^4)*m)*sqr
t(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 420
*((b^2*c^7 - 4*a*c^8)*f - (b^3*c^6 - 4*a*b*c^7)*g + (b^4*c^5 - 5*a*b^2*c^6
+ 4*a^2*c^7)*h - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*j + (b^6*c^3 - 7*a*b
^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*k - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*b^
3*c^4 - 12*a^3*b*c^5)*l + (b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*b^
2*c^4 + 4*a^4*c^5)*m)*x + 210*((b^2*c^7 - 4*a*c^8)*e - (b^3*c^6 - 4*a*b*c^7
)*f + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*g - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^
2*b*c^6)*h + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*j - (b^7*
c^2 - 8*a*b^5*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*k + (b^8*c - 9*a*b^6*c^2
 + 26*a^2*b^4*c^3 - 25*a^3*b^2*c^4 + 4*a^4*c^5)*l - (b^9 - 10*a*b^7*c + 34*
a^2*b^5*c^2 - 44*a^3*b^3*c^3 + 16*a^4*b*c^4)*m)*log(c*x^2 + b*x + a))/(b^2*
c^8 - 4*a*c^9)]
```

giac [A] time = 0.18, size = 982, normalized size = 1.28

$$\frac{60\,c^6mx^7+70\,c^6lx^6-70\,bc^5mx^6+84\,c^6kx^5-84\,bc^5lx^5+84\,b^2c^4mx^5-84\,ac^5mx^5+105\,c^6jx^4-105\,bc^5kx^4+10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x
, algorithm="giac")
```

```
[Out] 1/420*(60*c^6*m*x^7 + 70*c^6*l*x^6 - 70*b*c^5*m*x^6 + 84*c^6*k*x^5 - 84*b*c
^5*l*x^5 + 84*b^2*c^4*m*x^5 - 84*a*c^5*m*x^5 + 105*c^6*j*x^4 - 105*b*c^5*k*
x^4 + 105*b^2*c^4*l*x^4 - 105*a*c^5*l*x^4 - 105*b^3*c^3*m*x^4 + 210*a*b*c^4
*m*x^4 + 140*c^6*h*x^3 - 140*b*c^5*j*x^3 + 140*b^2*c^4*k*x^3 - 140*a*c^5*k*
x^3 - 140*b^3*c^3*l*x^3 + 280*a*b*c^4*l*x^3 + 140*b^4*c^2*m*x^3 - 420*a*b^2
*c^3*m*x^3 + 140*a^2*c^4*m*x^3 + 210*c^6*g*x^2 - 210*b*c^5*h*x^2 + 210*b^2*
c^4*j*x^2 - 210*a*c^5*j*x^2 - 210*b^3*c^3*k*x^2 + 420*a*b*c^4*k*x^2 + 210*b
^4*c^2*l*x^2 - 630*a*b^2*c^3*l*x^2 + 210*a^2*c^4*l*x^2 - 210*b^5*c*m*x^2 +
840*a*b^3*c^2*m*x^2 - 630*a^2*b*c^3*m*x^2 + 420*c^6*f*x - 420*b*c^5*g*x + 4
20*b^2*c^4*h*x - 420*a*c^5*h*x - 420*b^3*c^3*j*x + 840*a*b*c^4*j*x + 420*b^
4*c^2*k*x - 1260*a*b^2*c^3*k*x + 420*a^2*c^4*k*x - 420*b^5*c*l*x + 1680*a*b
^3*c^2*l*x - 1260*a^2*b*c^3*l*x + 420*b^6*m*x - 2100*a*b^4*c*m*x + 2520*a^2
*b^2*c^2*m*x - 420*a^3*c^3*m*x)/c^7 - 1/2*(b*c^6*f - b^2*c^5*g + a*c^6*g +
b^3*c^4*h - 2*a*b*c^5*h - b^4*c^3*j + 3*a*b^2*c^4*j - a^2*c^5*j + b^5*c^2*k
 - 4*a*b^3*c^3*k + 3*a^2*b*c^4*k - b^6*c*l + 5*a*b^4*c^2*l - 6*a^2*b^2*c^3*
l + a^3*c^4*l + b^7*m - 6*a*b^5*c*m + 10*a^2*b^3*c^2*m - 4*a^3*b*c^3*m - c^
7*e)*log(c*x^2 + b*x + a)/c^8 + (2*c^8*d + b^2*c^6*f - 2*a*c^7*f - b^3*c^5*
g + 3*a*b*c^6*g + b^4*c^4*h - 4*a*b^2*c^5*h + 2*a^2*c^6*h - b^5*c^3*j + 5*a
*b^3*c^4*j - 5*a^2*b*c^5*j + b^6*c^2*k - 6*a*b^4*c^3*k + 9*a^2*b^2*c^4*k -
2*a^3*c^5*k - b^7*c*l + 7*a*b^5*c^2*l - 14*a^2*b^3*c^3*l + 7*a^3*b*c^4*l +
b^8*m - 8*a*b^6*c*m + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 2*a^4*c^4*m - b
*c^7*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^8)
```

maple [B] time = 0.01, size = 1960, normalized size = 2.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x)
```

```
[Out] 1/7*m*x^7/c-1/2/c^2*ln(c*x^2+b*x+a)*b*f-1/2/c^4*ln(c*x^2+b*x+a)*a^3*l+1/2/c
^3*ln(c*x^2+b*x+a)*a^2*j-1/2/c^2*ln(c*x^2+b*x+a)*a*g-1/2/c^8*ln(c*x^2+b*x+a
)*b^7*m+1/2/c^7*ln(c*x^2+b*x+a)*b^6*l-1/6/c^2*x^6*b*m+1/c^7*b^6*m*x-1/4/c^2
```

```
*x^4*a*l-1/4/c^4*x^4*b^3*m+1/5/c^3*x^5*b^2*m-1/5/c^2*x^5*b*l-1/5/c^2*x^5*a*
m+1/c^5*b^4*k*x-1/c^4*b^3*j*x+1/c^3*b^2*h*x-1/c^2*b*g*x+1/2/c^5*x^2*b^4*l-1
/2/c^4*x^2*b^3*k-1/2/c^2*x^2*a*j-1/2/c^6*x^2*b^5*m-1/3/c^2*x^3*a*k+1/3/c^5*
x^3*b^4*m-1/3/c^4*x^3*b^3*l+1/3/c^3*x^3*b^2*k-1/3/c^2*x^3*b*j+1/2/c^3*x^2*a
^2*l-1/4/c^2*x^4*b*k+1/3/c^3*x^3*a^2*m+1/4/c^3*x^4*b^2*l+1/2/c^3*x^2*b^2*j-
1/2/c^2*x^2*b*h-1/c^4*a^3*m*x+1/c^3*a^2*k*x-1/c^2*a*h*x-1/c^6*b^5*l*x-1/2/c
^6*ln(c*x^2+b*x+a)*b^5*k+1/2/c^5*ln(c*x^2+b*x+a)*b^4*j-1/2/c^4*ln(c*x^2+b*x
+a)*b^3*h+1/2/c^3*ln(c*x^2+b*x+a)*b^2*g+20/c^6/(4*a*c-b^2)^(1/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^4*m-14/c^5/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*a^2*b^3*l+9/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*a^2*b^2*k-16/c^5/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*a^3*b^2*m+7/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*a^3*b*l-5/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
*a^2*b*j+7/c^6/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^5*
l-6/c^5/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^4*k+2/(4*
a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d+1/2/c*ln(c*x^2+b*x+a)*
e+1/c*f*x+1/2/c*x^2*g+1/3/c*x^3*h+1/6/c*x^6*l+1/5/c*x^5*k+1/4/c*x^4*j+3/c^5
*ln(c*x^2+b*x+a)*a^2*b^2*l-3/2/c^4*ln(c*x^2+b*x+a)*a^2*b*k+3/c^7*ln(c*x^2+b
*x+a)*a*b^5*m-5/2/c^6*ln(c*x^2+b*x+a)*a*b^4*l+2/c^5*ln(c*x^2+b*x+a)*a*b^3*k
-1/c^7/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^7*l+2/c^5*ln
(c*x^2+b*x+a)*a^3*b*m-5/c^6*ln(c*x^2+b*x+a)*a^2*b^3*m-3/2/c^4*ln(c*x^2+b*x+
a)*a*b^2*j+1/c^3*ln(c*x^2+b*x+a)*a*b*h+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*b^2*f-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*b*e+1/c^8/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*b^8*m-3/c^4*a*b^2*k*x+2/c^3*a*b*j*x+6/c^5*a^2*b^2*m*x-3/c^4*a^2*b*l*x+2/c
^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^4*m-2/c/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*f+1/c^6/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^6*k-1/c^5/(4*a*c-b^2)^(1/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*b^5*j+1/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*b^4*h-1/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*b^3*g+1/c^3*x^2*a*b*k-5/c^6*a*b^4*m*x+4/c^5*a*b^3*l*x-3/2/c^4*x^2*a
^2*b*m+2/c^5*x^2*a*b^3*m-3/2/c^4*x^2*a*b^2*l-1/c^4*x^3*a*b^2*m+2/3/c^3*x^3*
a*b*l+1/2/c^3*x^4*a*b*m+5/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2
)^(1/2))*a*b^3*j-4/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*a*b^2*h+3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*g
-8/c^7/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^6*m-2/c^3/
(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3*k+2/c^2/(4*a*c-b^
2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*h
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 7.26, size = 2779, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x + c*x^2),x)
```

```
[Out] x^6*(l/(6*c) - (b*m)/(6*c^2)) + x*(f/c + (b*((a*(j/c - (a*(l/c - (b*m)/c^2)
)/c + (b*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/c - g/c + (b*(h/c
 - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l/c - (b*m)/c^2))/c - k/c +
(a*m)/c^2))/c))/c + (a*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/c))
/c - (a*(h/c - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l/c - (b*m)/c^2)
)/c - k/c + (a*m)/c^2))/c))/c + (a*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c
^2))/c))/c) + x^4*(j/(4*c) - (a*(l/c - (b*m)/c^2))/(4*c) + (b*((b*(l/c - (b
*m)/c^2))/c - k/c + (a*m)/c^2))/(4*c)) - x^2*((a*(j/c - (a*(l/c - (b*m)/c^2
))/c + (b*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/(2*c) - g/(2*c)
+ (b*(h/c - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l/c - (b*m)/c^2))/c
 - k/c + (a*m)/c^2))/c))/c + (a*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2)
)/c))/(2*c)) + x^3*(h/(3*c) - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l
/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/(3*c) + (a*((b*(l/c - (b*m)/c^2)
)/c - k/c + (a*m)/c^2))/(3*c)) - x^5*((b*(l/c - (b*m)/c^2))/(5*c) - k/(5*c)
 + (a*m)/(5*c^2)) + (log((2*c^9*x*(-(2*c^8*d + b^8*m + b^2*c^6*f + 2*a^2*c^
6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b^6*c^2*k + 2*a^4*c
^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^2*c^4*k - 14*a^2*b^3*c^3*l +
 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^6*g - 8*a*b^6*c*m - 4*a*b^2*
c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4*c^3*k + 7*a*b^5*c^2*l + 7*a
^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) - b^8*m - 2*c^8*d - b^2*c^6*f - 2
*a^2*c^6*h + b^3*c^5*g - b^4*c^4*h + 2*a^3*c^5*k + b^5*c^3*j - b^6*c^2*k -
2*a^4*c^4*m + 2*a*c^7*f + b*c^7*e + b^7*c*l + b*c^8*(-(2*c^8*d + b^8*m + b^
2*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b
^6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^2*c^4*k -
14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^6*g - 8*a*
b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4*c^3*k + 7
*a*b^5*c^2*l + 7*a^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) - 9*a^2*b^2*c^4
*k + 14*a^2*b^3*c^3*l - 20*a^2*b^4*c^2*m + 16*a^3*b^2*c^3*m - 3*a*b*c^6*g +
 8*a*b^6*c*m + 4*a*b^2*c^5*h - 5*a*b^3*c^4*j + 5*a^2*b*c^5*j + 6*a*b^4*c^3*
k - 7*a*b^5*c^2*l - 7*a^3*b*c^4*l)*(2*c^8*d + b^8*m + 2*c^9*x*(-(2*c^8*d +
b^8*m + b^2*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5
*c^3*j + b^6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^
2*c^4*k - 14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^
6*g - 8*a*b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4
*c^3*k + 7*a*b^5*c^2*l + 7*a^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) + b^2
*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b^
6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + b*c^8*(-(2*c^8*d +
b^8*m + b^2*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5
*c^3*j + b^6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^
2*c^4*k - 14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^
6*g - 8*a*b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4
*c^3*k + 7*a*b^5*c^2*l + 7*a^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) + 9*a
^2*b^2*c^4*k - 14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a
*b*c^6*g - 8*a*b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*
a*b^4*c^3*k + 7*a*b^5*c^2*l + 7*a^3*b*c^4*l))*(b^9*m - b^2*c^7*e - 4*a^2*c^
7*g + b^3*c^6*f - b^4*c^5*g + b^5*c^4*h + 4*a^3*c^6*j - b^6*c^3*j - 4*a^4*c
^5*l + b^7*c^2*k + 4*a*c^8*e - b^8*c*l - 13*a^2*b^2*c^5*j + 19*a^2*b^3*c^4*
k - 26*a^2*b^4*c^3*l + 25*a^3*b^2*c^4*l + 34*a^2*b^5*c^2*m - 44*a^3*b^3*c^3
*m - 4*a*b*c^7*f - 10*a*b^7*c*m + 5*a*b^2*c^6*g - 6*a*b^3*c^5*h + 8*a^2*b*c
^6*h + 7*a*b^4*c^4*j - 8*a*b^5*c^3*k - 12*a^3*b*c^5*k + 9*a*b^6*c^2*l + 16*
a^4*b*c^4*m))/(2*(4*a*c^9 - b^2*c^8)) + (m*x^7)/(7*c) + (atan(b/(4*a*c - b^
2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(2*c^8*d + b^8*m + b^2*c^6*f + 2*a^
2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b^6*c^2*k + 2*a
^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^2*c^4*k - 14*a^2*b^3*c^3
*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^6*g - 8*a*b^6*c*m - 4*a*
b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4*c^3*k + 7*a*b^5*c^2*l +
 7*a^3*b*c^4*l))/(c^8*(4*a*c - b^2)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**2+
b*x+a),x)
```

```
[Out] Timed out
```

3.374 $\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx$

Optimal. Leaf size=208

$$\frac{98060877\left(5x^2+2x+3\right)^{3/2}x^2}{4375000}+\frac{1045360143\left(5x^2+2x+3\right)^{3/2}x}{43750000}-\frac{1968340667\left(5x^2+2x+3\right)^{3/2}}{131250000}-\frac{77159983(5x+}{312}$$

```
[Out] -1968340667/131250000*(5*x^2+2*x+3)^(3/2)+1045360143/43750000*x*(5*x^2+2*x+
3)^(3/2)+98060877/4375000*x^2*(5*x^2+2*x+3)^(3/2)-90960857/1575000*x^3*(5*x
^2+2*x+3)^(3/2)-888751/105000*x^4*(5*x^2+2*x+3)^(3/2)+190939/3000*x^5*(5*x^
2+2*x+3)^(3/2)-50519/2250*x^6*(5*x^2+2*x+3)^(3/2)-343/50*x^7*(5*x^2+2*x+3)^
(3/2)-540119881/78125000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-77159983/31
250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1661, 640, 612, 619, 215}

$$-\frac{343}{50}\left(5x^2+2x+3\right)^{3/2}x^7-\frac{50519\left(5x^2+2x+3\right)^{3/2}x^6}{2250}+\frac{190939\left(5x^2+2x+3\right)^{3/2}x^5}{3000}-\frac{888751\left(5x^2+2x+3\right)^{3/2}x}{105000}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-77159983*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/31250000 - (1968340667*(3 + 2*x
 + 5*x^2)^(3/2))/131250000 + (1045360143*x*(3 + 2*x + 5*x^2)^(3/2))/4375000
0 + (98060877*x^2*(3 + 2*x + 5*x^2)^(3/2))/4375000 - (90960857*x^3*(3 + 2*x
 + 5*x^2)^(3/2))/1575000 - (888751*x^4*(3 + 2*x + 5*x^2)^(3/2))/105000 + (1
90939*x^5*(3 + 2*x + 5*x^2)^(3/2))/3000 - (50519*x^6*(3 + 2*x + 5*x^2)^(3/2
))/2250 - (343*x^7*(3 + 2*x + 5*x^2)^(3/2))/50 - (540119881*ArcSinh[(1 + 5*
x)/Sqrt[14]])/(15625000*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx = -\frac{343}{50}x^7\left(3+2x+5x^2\right)^{3/2}+\frac{1}{50}\int \sqrt{3+2x+5x^2}\,\left(100\right.$$

$$= -\frac{50519x^6\left(3+2x+5x^2\right)^{3/2}}{2250}-\frac{343}{50}x^7\left(3+2x+5x^2\right)^{3/2}$$

$$= \frac{190939x^5\left(3+2x+5x^2\right)^{3/2}}{3000}-\frac{50519x^6\left(3+2x+5x^2\right)^{3/2}}{2250}$$

$$= -\frac{888751x^4\left(3+2x+5x^2\right)^{3/2}}{105000}+\frac{190939x^5\left(3+2x+5x^2\right.}{3000}$$

$$= -\frac{90960857x^3\left(3+2x+5x^2\right)^{3/2}}{1575000}-\frac{888751x^4\left(3+2x+5\right.}{105000}$$

$$= \frac{98060877x^2\left(3+2x+5x^2\right)^{3/2}}{4375000}-\frac{90960857x^3\left(3+2x+\right.}{1575000}$$

$$= \frac{1045360143x\left(3+2x+5x^2\right)^{3/2}}{43750000}+\frac{98060877x^2\left(3+2x\right.}{4375000}$$

$$= -\frac{1968340667\left(3+2x+5x^2\right)^{3/2}}{131250000}+\frac{1045360143x\left(3+2x\right.}{43750000}$$

$$= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000}-\frac{1968340667\left(3+\right.}{1312500}$$

$$= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000}-\frac{1968340667\left(3+\right.}{1312500}$$

$$= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000}-\frac{1968340667\left(3+\right.}{1312500}$$

Mathematica [A] time = 0.31, size = 85, normalized size = 0.41

$$-5\sqrt{5x^2+2x+3}\,\left(67528125000x^9+248031875000x^8-497593468750x^7-34674656250x^6+22592236250\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-5*Sqrt[3 + 2*x + 5*x^2]*(93436408944 - 57768004650*x - 78839046795*x^2 +
17642392275*x^3 + 56757413000*x^4 + 225922362500*x^5 - 34674656250*x^6 - 49
7593468750*x^7 + 248031875000*x^8 + 67528125000*x^9) - 68055105006*Sqrt[5]*
ArcSinh[(1 + 5*x)/Sqrt[14]])/9843750000
```

fricas [A] time = 0.86, size = 97, normalized size = 0.47

$$-\frac{1}{1968750000}\left(67528125000\,x^9+248031875000\,x^8-497593468750\,x^7-34674656250\,x^6+225922362500\,x^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/1968750000*(67528125000*x^9 + 248031875000*x^8 - 497593468750*x^7 - 3467
4656250*x^6 + 225922362500*x^5 + 56757413000*x^4 + 17642392275*x^3 - 788390
46795*x^2 - 57768004650*x + 93436408944)*sqrt(5*x^2 + 2*x + 3) + 540119881/
156250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10
*x - 8)
```

giac [A] time = 0.21, size = 92, normalized size = 0.44

$$-\frac{1}{1968750000}\left(5\left(\left(5\left(10\left(25\left(5\left(49\left(140\left(315\,x+1157\right)x-324959\right)x-1109589\right)x+36147578\right)x+227029652\right)x+\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/1968750000*(5*((5*(10*(25*(5*(49*(140*(315*x + 1157)*x - 324959)*x - 110
9589)*x + 36147578)*x + 227029652)*x + 705695691)*x - 15767809359)*x - 1155
3600930)*x + 93436408944)*sqrt(5*x^2 + 2*x + 3) + 540119881/78125000*sqrt(5
)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

maple [A] time = 0.03, size = 166, normalized size = 0.80

$$-\frac{343\left(5x^2+2x+3\right)^{\frac{3}{2}}x^7}{50}-\frac{50519\left(5x^2+2x+3\right)^{\frac{3}{2}}x^6}{2250}+\frac{190939\left(5x^2+2x+3\right)^{\frac{3}{2}}x^5}{3000}-\frac{888751\left(5x^2+2x+3\right)^{\frac{3}{2}}x^4}{105000}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -1968340667/131250000*(5*x^2+2*x+3)^(3/2)-343/50*x^7*(5*x^2+2*x+3)^(3/2)-50
519/2250*x^6*(5*x^2+2*x+3)^(3/2)+190939/3000*x^5*(5*x^2+2*x+3)^(3/2)-888751
/105000*x^4*(5*x^2+2*x+3)^(3/2)-90960857/1575000*x^3*(5*x^2+2*x+3)^(3/2)+98
060877/4375000*x^2*(5*x^2+2*x+3)^(3/2)+1045360143/43750000*x*(5*x^2+2*x+3)^
(3/2)-540119881/78125000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-77159983/62
500000*(10*x+2)*(5*x^2+2*x+3)^(1/2)
```

maxima [A] time = 0.99, size = 177, normalized size = 0.85

$$-\frac{343}{50}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}x^7-\frac{50519}{2250}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}x^6+\frac{190939}{3000}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}x^5-\frac{888751}{105000}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}x^4-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -343/50*(5*x^2 + 2*x + 3)^(3/2)*x^7 - 50519/2250*(5*x^2 + 2*x + 3)^(3/2)*x^
6 + 190939/3000*(5*x^2 + 2*x + 3)^(3/2)*x^5 - 888751/105000*(5*x^2 + 2*x +
3)^(3/2)*x^4 - 90960857/1575000*(5*x^2 + 2*x + 3)^(3/2)*x^3 + 98060877/4375
000*(5*x^2 + 2*x + 3)^(3/2)*x^2 + 1045360143/43750000*(5*x^2 + 2*x + 3)^(3/
```

```
2)*x - 1968340667/131250000*(5*x^2 + 2*x + 3)^(3/2) - 77159983/6250000*sqrt
(5*x^2 + 2*x + 3)*x - 540119881/78125000*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x
 + 1)) - 77159983/31250000*sqrt(5*x^2 + 2*x + 3)
```

mupad [B] time = 6.31, size = 221, normalized size = 1.06

$$\frac{98060877\,x^2\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{4375000}-\frac{90960857\,x^3\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{1575000}-\frac{888751\,x^4\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{105000}+\frac{190939\,x^5\,(5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3,x)
```

```
[Out] (98060877*x^2*(2*x + 5*x^2 + 3)^(3/2))/4375000 - (90960857*x^3*(2*x + 5*x^2
 + 3)^(3/2))/1575000 - (888751*x^4*(2*x + 5*x^2 + 3)^(3/2))/105000 + (19093
9*x^5*(2*x + 5*x^2 + 3)^(3/2))/3000 - (50519*x^6*(2*x + 5*x^2 + 3)^(3/2))/2
250 - (343*x^7*(2*x + 5*x^2 + 3)^(3/2))/50 - (3048580429*5^(1/2)*log((2*x +
 5*x^2 + 3)^(1/2) + (5^(1/2)*(5*x + 1))/5))/156250000 - (3048580429*(x/2 +
1/10)*(2*x + 5*x^2 + 3)^(1/2))/43750000 - (1968340667*(2*x + 5*x^2 + 3)^(1/
2)*(20*x + 200*x^2 + 108))/5250000000 + (1045360143*x*(2*x + 5*x^2 + 3)^(3/
2))/43750000 + (1968340667*5^(1/2)*log(2*(2*x + 5*x^2 + 3)^(1/2) + (5^(1/2)
*(10*x + 2))/5))/156250000
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-29x\sqrt{5x^2+2x+3}\right)dx-\int\left(-115x^2\sqrt{5x^2+2x+3}\right)dx-\int 61x^3\sqrt{5x^2+2x+3}\,dx-\int 871x^4\sqrt{5x^2+2.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-29*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-115*x**2*sqrt(5*x**2
 + 2*x + 3), x) - Integral(61*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(87
1*x**4*sqrt(5*x**2 + 2*x + 3), x) - Integral(-127*x**5*sqrt(5*x**2 + 2*x +
3), x) - Integral(-2065*x**6*sqrt(5*x**2 + 2*x + 3), x) - Integral(1127*x**
7*sqrt(5*x**2 + 2*x + 3), x) - Integral(343*x**8*sqrt(5*x**2 + 2*x + 3), x)
 - Integral(-2*sqrt(5*x**2 + 2*x + 3), x)
```

3.375 $\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx$

Optimal. Leaf size=166

$$-\frac{77509\left(5x^2+2x+3\right)^{3/2}x^2}{25000}+\frac{1781669\left(5x^2+2x+3\right)^{3/2}x}{250000}+\frac{198439\left(5x^2+2x+3\right)^{3/2}}{750000}-\frac{2521723(5x+1)\sqrt{5x^2+2}}{1250000}$$

```
[Out] 198439/750000*(5*x^2+2*x+3)^(3/2)+1781669/250000*x*(5*x^2+2*x+3)^(3/2)-7750
9/25000*x^2*(5*x^2+2*x+3)^(3/2)-25277/3000*x^3*(5*x^2+2*x+3)^(3/2)+989/200*
x^4*(5*x^2+2*x+3)^(3/2)+49/40*x^5*(5*x^2+2*x+3)^(3/2)-17652061/3125000*arcs
inh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-2521723/1250000*(1+5*x)*(5*x^2+2*x+3)^(1
/2)
```

Rubi [A] time = 0.20, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5+\frac{989}{200}\left(5x^2+2x+3\right)^{3/2}x^4-\frac{25277\left(5x^2+2x+3\right)^{3/2}x^3}{3000}-\frac{77509\left(5x^2+2x+3\right)^{3/2}x^2}{25000}+\frac{1781}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-2521723*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/1250000 + (198439*(3 + 2*x + 5*x
^2)^(3/2))/750000 + (1781669*x*(3 + 2*x + 5*x^2)^(3/2))/250000 - (77509*x^2
*(3 + 2*x + 5*x^2)^(3/2))/25000 - (25277*x^3*(3 + 2*x + 5*x^2)^(3/2))/3000
+ (989*x^4*(3 + 2*x + 5*x^2)^(3/2))/200 + (49*x^5*(3 + 2*x + 5*x^2)^(3/2))/
40 - (17652061*ArcSinh[(1 + 5*x)/Sqrt[14]])/(625000*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
```

```
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx = \frac{49}{40}x^5\left(3+2x+5x^2\right)^{3/2}+\frac{1}{40}\int\sqrt{3+2x+5x^2}\,\left(80+84\right.$$

$$=\frac{989}{200}x^4\left(3+2x+5x^2\right)^{3/2}+\frac{49}{40}x^5\left(3+2x+5x^2\right)^{3/2}+\frac{\int\sqrt{}}{}$$

$$=-\frac{25277x^3\left(3+2x+5x^2\right)^{3/2}}{3000}+\frac{989}{200}x^4\left(3+2x+5x^2\right)^{3/2}$$

$$=-\frac{77509x^2\left(3+2x+5x^2\right)^{3/2}}{25000}-\frac{25277x^3\left(3+2x+5x^2\right)^{3/}}{3000}$$

$$=\frac{1781669x\left(3+2x+5x^2\right)^{3/2}}{250000}-\frac{77509x^2\left(3+2x+5x^2\right)^{3/}}{25000}$$

$$=\frac{198439\left(3+2x+5x^2\right)^{3/2}}{750000}+\frac{1781669x\left(3+2x+5x^2\right)^{3/2}}{250000}$$

$$=-\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000}+\frac{198439\left(3+2x+5\right.}{750000}$$

$$=-\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000}+\frac{198439\left(3+2x+5\right.}{750000}$$

$$=-\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000}+\frac{198439\left(3+2x+5\right.}{750000}$$

Mathematica [A] time = 0.19, size = 75, normalized size = 0.45

$$\frac{5\sqrt{5x^2+2x+3}\,\left(22968750x^7+101906250x^6-107112500x^5-65693000x^4+15583725x^3+23531995x^2+4\right.}{18750000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (5*Sqrt[3 + 2*x + 5*x^2]*(-4588584 + 44333650*x + 23531995*x^2 + 15583725*x
^3 - 65693000*x^4 - 107112500*x^5 + 101906250*x^6 + 22968750*x^7) - 1059123
66*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/18750000
```

fricas [A] time = 0.89, size = 87, normalized size = 0.52

$$\frac{1}{3750000}\left(22968750\,x^7+101906250\,x^6-107112500\,x^5-65693000\,x^4+15583725\,x^3+23531995\,x^2+4433\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/3750000*(22968750*x^7 + 101906250*x^6 - 107112500*x^5 - 65693000*x^4 + 15
583725*x^3 + 23531995*x^2 + 44333650*x - 4588584)*sqrt(5*x^2 + 2*x + 3) + 1
```

```
7652061/6250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^
2 - 10*x - 8)
```

giac [A] time = 0.20, size = 82, normalized size = 0.49

$$\frac{1}{3750000}\left(5\left(\left(5\left(10\left(25\left(15\left(245x+1087\right)x-17138\right)x-262772\right)x+623349\right)x+4706399\right)x+8866730\right)x-45885\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/3750000*(5*((5*(10*(25*(15*(245*x + 1087)*x - 17138)*x - 262772)*x + 6233
49)*x + 4706399)*x + 8866730)*x - 4588584)*sqrt(5*x^2 + 2*x + 3) + 17652061
/3125000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

maple [A] time = 0.01, size = 132, normalized size = 0.80

$$\frac{49\left(5x^2+2x+3\right)^{\frac{3}{2}}x^5}{40}+\frac{989\left(5x^2+2x+3\right)^{\frac{3}{2}}x^4}{200}-\frac{25277\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3}{3000}-\frac{77509\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2}{25000}+\frac{1781669}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] 198439/750000*(5*x^2+2*x+3)^(3/2)+49/40*(5*x^2+2*x+3)^(3/2)*x^5+989/200*(5*
x^2+2*x+3)^(3/2)*x^4-25277/3000*(5*x^2+2*x+3)^(3/2)*x^3-77509/25000*(5*x^2+
2*x+3)^(3/2)*x^2+1781669/250000*(5*x^2+2*x+3)^(3/2)*x-17652061/3125000*5^(1
/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-2521723/2500000*(10*x+2)*(5*x^2+2*x+3)^(
1/2)
```

maxima [A] time = 0.97, size = 143, normalized size = 0.86

$$\frac{49}{40}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^5+\frac{989}{200}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^4-\frac{25277}{3000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3-\frac{77509}{25000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2+\frac{178166}{25000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 49/40*(5*x^2 + 2*x + 3)^(3/2)*x^5 + 989/200*(5*x^2 + 2*x + 3)^(3/2)*x^4 - 2
5277/3000*(5*x^2 + 2*x + 3)^(3/2)*x^3 - 77509/25000*(5*x^2 + 2*x + 3)^(3/2)
*x^2 + 1781669/250000*(5*x^2 + 2*x + 3)^(3/2)*x + 198439/750000*(5*x^2 + 2*
x + 3)^(3/2) - 2521723/250000*sqrt(5*x^2 + 2*x + 3)*x - 17652061/3125000*sq
rt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 2521723/1250000*sqrt(5*x^2 + 2*x +
 3)
```

mupad [B] time = 6.01, size = 187, normalized size = 1.13

$$\frac{989x^4\left(5x^2+2x+3\right)^{3/2}}{200}-\frac{25277x^3\left(5x^2+2x+3\right)^{3/2}}{3000}-\frac{77509x^2\left(5x^2+2x+3\right)^{3/2}}{25000}+\frac{49x^5\left(5x^2+2x+3\right)^{3/2}}{40}-\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2,x)
```

```
[Out] (989*x^4*(2*x + 5*x^2 + 3)^(3/2))/200 - (25277*x^3*(2*x + 5*x^2 + 3)^(3/2))
/3000 - (77509*x^2*(2*x + 5*x^2 + 3)^(3/2))/25000 + (49*x^5*(2*x + 5*x^2 +
3)^(3/2))/40 - (33915049*5^(1/2)*log((2*x + 5*x^2 + 3)^(1/2) + (5^(1/2)*(5*
```

```
x + 1))/5))/6250000 - (4845007*(x/2 + 1/10)*(2*x + 5*x^2 + 3)^(1/2))/250000
 + (198439*(2*x + 5*x^2 + 3)^(1/2)*(20*x + 200*x^2 + 108))/30000000 + (1781
669*x*(2*x + 5*x^2 + 3)^(3/2))/250000 - (1389073*5^(1/2)*log(2*(2*x + 5*x^2
 + 3)^(1/2) + (5^(1/2)*(10*x + 2))/5))/6250000
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+5x+2\right)\sqrt{5x^2+2x+3}\left(7x^2-4x-1\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*sqrt(5*x**2 + 2*x + 3)*(7*x**2 - 4*x - 1)**2, x)
```

3.376 $\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx$

Optimal. Leaf size=124

$$-\frac{289}{250}\left(5x^2+2x+3\right)^{3/2}x^2+\frac{2149\left(5x^2+2x+3\right)^{3/2}x}{2500}+\frac{7819\left(5x^2+2x+3\right)^{3/2}}{7500}-\frac{4633(5x+1)\sqrt{5x^2+2x+3}}{12500}-\frac{7}{30}\left(\right.$$

```
[Out] 7819/7500*(5*x^2+2*x+3)^(3/2)+2149/2500*x*(5*x^2+2*x+3)^(3/2)-289/250*x^2*(
5*x^2+2*x+3)^(3/2)-7/30*x^3*(5*x^2+2*x+3)^(3/2)-32431/31250*arcsinh(1/14*(1
+5*x)*14^(1/2))*5^(1/2)-4633/12500*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1661, 640, 612, 619, 215}

$$-\frac{7}{30}\left(5x^2+2x+3\right)^{3/2}x^3-\frac{289}{250}\left(5x^2+2x+3\right)^{3/2}x^2+\frac{2149\left(5x^2+2x+3\right)^{3/2}x}{2500}+\frac{7819\left(5x^2+2x+3\right)^{3/2}}{7500}-\frac{4633(5x\ }{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-4633*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/12500 + (7819*(3 + 2*x + 5*x^2)^(3/
2))/7500 + (2149*x*(3 + 2*x + 5*x^2)^(3/2))/2500 - (289*x^2*(3 + 2*x + 5*x^
2)^(3/2))/250 - (7*x^3*(3 + 2*x + 5*x^2)^(3/2))/30 - (32431*ArcSinh[(1 + 5*
x)/Sqrt[14]])/(6250*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
```

```
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx = -\frac{7}{30}x^3\left(3+2x+5x^2\right)^{3/2}+\frac{1}{30}\int \sqrt{3+2x+5x^2}\,(60+3$$

$$= -\frac{289}{250}x^2\left(3+2x+5x^2\right)^{3/2}-\frac{7}{30}x^3\left(3+2x+5x^2\right)^{3/2}+\frac{1}{75}$$

$$= \frac{2149x\left(3+2x+5x^2\right)^{3/2}}{2500}-\frac{289}{250}x^2\left(3+2x+5x^2\right)^{3/2}-\frac{7}{30}$$

$$= \frac{7819\left(3+2x+5x^2\right)^{3/2}}{7500}+\frac{2149x\left(3+2x+5x^2\right)^{3/2}}{2500}-\frac{289}{250}$$

$$= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500}+\frac{7819\left(3+2x+5x^2\right)^{3/2}}{7500}$$

$$= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500}+\frac{7819\left(3+2x+5x^2\right)^{3/2}}{7500}$$

$$= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500}+\frac{7819\left(3+2x+5x^2\right)^{3/2}}{7500}$$

Mathematica [A] time = 0.10, size = 65, normalized size = 0.52

$$\frac{5\sqrt{5x^2+2x+3}\left(-43750x^5-234250x^4+48225x^3+129895x^2+105400x+103386\right)-194586\sqrt{5}\sinh^{-1}\left(\frac{5}{}\right.}{187500}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (5*Sqrt[3 + 2*x + 5*x^2]*(103386 + 105400*x + 129895*x^2 + 48225*x^3 - 2342
50*x^4 - 43750*x^5) - 194586*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/187500
```

fricas [A] time = 0.87, size = 77, normalized size = 0.62

$$-\frac{1}{37500}\left(43750\,x^5+234250\,x^4-48225\,x^3-129895\,x^2-105400\,x-103386\right)\sqrt{5\,x^2+2\,x+3}+\frac{32431}{62500}\sqrt{5}\,\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fric
as")
```

```
[Out] -1/37500*(43750*x^5 + 234250*x^4 - 48225*x^3 - 129895*x^2 - 105400*x - 1033
86)*sqrt(5*x^2 + 2*x + 3) + 32431/62500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*
x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

giac [A] time = 0.19, size = 72, normalized size = 0.58

$$-\frac{1}{37500}\left(5\left(\left(5\left(10\left(175\,x+937\right)x-1929\right)x-25979\right)x-21080\right)x-103386\right)\sqrt{5\,x^2+2\,x+3}+\frac{32431}{31250}\sqrt{5}\,\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/37500*(5*((5*(10*(175*x + 937)*x - 1929)*x - 25979)*x - 21080)*x - 10338
6)*sqrt(5*x^2 + 2*x + 3) + 32431/31250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sq
rt(5*x^2 + 2*x + 3)) - 1)
```

maple [A] time = 0.01, size = 98, normalized size = 0.79

$$-\frac{7\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3}{30}-\frac{289\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2}{250}+\frac{2149\left(5x^2+2x+3\right)^{\frac{3}{2}}x}{2500}-\frac{32431\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{31250}+\frac{7819}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -7/30*(5*x^2+2*x+3)^(3/2)*x^3-289/250*(5*x^2+2*x+3)^(3/2)*x^2+2149/2500*(5*
x^2+2*x+3)^(3/2)*x+7819/7500*(5*x^2+2*x+3)^(3/2)-4633/25000*(10*x+2)*(5*x^2
+2*x+3)^(1/2)-32431/31250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

maxima [A] time = 0.96, size = 109, normalized size = 0.88

$$-\frac{7}{30}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3-\frac{289}{250}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2+\frac{2149}{2500}\left(5x^2+2x+3\right)^{\frac{3}{2}}x+\frac{7819}{7500}\left(5x^2+2x+3\right)^{\frac{3}{2}}-\frac{4633}{2500}\sqrt{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -7/30*(5*x^2 + 2*x + 3)^(3/2)*x^3 - 289/250*(5*x^2 + 2*x + 3)^(3/2)*x^2 + 2
149/2500*(5*x^2 + 2*x + 3)^(3/2)*x + 7819/7500*(5*x^2 + 2*x + 3)^(3/2) - 46
33/2500*sqrt(5*x^2 + 2*x + 3)*x - 32431/31250*sqrt(5)*arcsinh(1/14*sqrt(14)
*(5*x + 1)) - 4633/12500*sqrt(5*x^2 + 2*x + 3)
```

mupad [B] time = 5.38, size = 153, normalized size = 1.23

$$\frac{7819\sqrt{5x^2+2x+3}\ \left(200x^2+20x+108\right)}{300000}-\frac{7x^3\left(5x^2+2x+3\right)^{3/2}}{30}-\frac{10129\sqrt{5}\ \ln\left(\sqrt{5x^2+2x+3}+\frac{\sqrt{5}\,(5x+1)}{5}\right.}{62500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1),x)
```

```
[Out] (7819*(2*x + 5*x^2 + 3)^(1/2)*(20*x + 200*x^2 + 108))/300000 - (7*x^3*(2*x
+ 5*x^2 + 3)^(3/2))/30 - (10129*5^(1/2)*log((2*x + 5*x^2 + 3)^(1/2) + (5^(1
/2)*(5*x + 1))/5))/62500 - (1447*(x/2 + 1/10)*(2*x + 5*x^2 + 3)^(1/2))/2500
 - (289*x^2*(2*x + 5*x^2 + 3)^(3/2))/250 + (2149*x*(2*x + 5*x^2 + 3)^(3/2))
/2500 - (54733*5^(1/2)*log(2*(2*x + 5*x^2 + 3)^(1/2) + (5^(1/2)*(10*x + 2))
/5))/62500
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-13x\sqrt{5x^2+2x+3}\right)dx-\int\left(-7x^2\sqrt{5x^2+2x+3}\right)dx-\int 31x^3\sqrt{5x^2+2x+3}\,dx-\int 7x^4\sqrt{5x^2+2x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-13*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-7*x**2*sqrt(5*x**2 +
 2*x + 3), x) - Integral(31*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(7*x*
*4*sqrt(5*x**2 + 2*x + 3), x) - Integral(-2*sqrt(5*x**2 + 2*x + 3), x)
```

3.377 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx$

Optimal. Leaf size=187

$$-\frac{1}{490}\sqrt{5x^2+2x+3}\,(35x+397)-\frac{3}{343}\sqrt{\frac{1}{11}\left(497041-146555\sqrt{11}\right)}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right.$$

```
[Out] -8233/8575*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-1/490*(397+35*x)*(5*x^2+2
*x+3)^(1/2)-3/3773*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1
/2)/(250-34*11^(1/2))^(1/2))*(5467451-1612105*11^(1/2))^(1/2)+3/3773*arctan
h((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/
2))*(5467451+1612105*11^(1/2))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1066, 1076, 619, 215, 1032, 724, 206}

$$-\frac{1}{490}\sqrt{5x^2+2x+3}\,(35x+397)-\frac{3}{343}\sqrt{\frac{1}{11}\left(497041-146555\sqrt{11}\right)}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2),x]
```

```
[Out] -((397 + 35*x)*Sqrt[3 + 2*x + 5*x^2])/490 - (8233*ArcSinh[(1 + 5*x)/Sqrt[14
]])/(1715*Sqrt[5]) - (3*Sqrt[(497041 - 146555*Sqrt[11])/11]*ArcTanh[(23 - S
qrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x +
5*x^2])])/343 + (3*Sqrt[(497041 + 146555*Sqrt[11])/11]*ArcTanh[(23 + Sqrt[1
1] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2
])])/343
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx = -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2}-\frac{1}{490}\int \frac{-3442-13408x-16466x^2}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,d\ldots$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2}+\frac{\int \frac{40560+159720x}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx}{3430}-\frac{8233\int \frac{\ldots}{\sqrt{3+\ldots}}}{171\ldots}$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2}-\frac{8233\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx,x,2+10x\right)}{3430\sqrt{70}}+\ldots$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2}-\frac{8233\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1715\sqrt{5}}-\frac{\left(24\left(14641-50\ldots\right.\right.}{\ldots}$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2}-\frac{8233\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1715\sqrt{5}}-\frac{3\sqrt{5467451-16\ldots}}{\ldots}$$

Mathematica [A] time = 0.89, size = 189, normalized size = 1.01

$$\frac{-385\sqrt{5x^2+2x+3}\,(35x+397)-75\sqrt{250-34\sqrt{11}}\,\left(61\sqrt{11}-143\right)\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{250-34\sqrt{11}}\,\sqrt{5x^2+2x+3}}\right)+75\sqrt{25}}{188650}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2),x]
```

```
[Out] (-385*(397 + 35*x)*Sqrt[3 + 2*x + 5*x^2] - 181126*Sqrt[5]*ArcSinh[(1 + 5*x)
/Sqrt[14]] - 75*Sqrt[250 - 34*Sqrt[11]]*(-143 + 61*Sqrt[11])*ArcTanh[(23 -
Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x
^2])] + 75*Sqrt[250 + 34*Sqrt[11]]*(143 + 61*Sqrt[11])*ArcTanh[(23 + Sqrt[1
1] + (17 + 5*Sqrt[11])*x)/(Sqrt[250 + 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])])
/188650
```

fricas [B] time = 0.91, size = 304, normalized size = 1.63

$$\frac{3}{7546}\sqrt{11}\sqrt{146555\sqrt{11}+497041}\log\left(\frac{6\left(\sqrt{5x^2+2x+3}\sqrt{146555\sqrt{11}+497041}\left(87\sqrt{11}-265\right)+6517\sqrt{}\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="fric
as")
```

```
[Out] 3/7546*sqrt(11)*sqrt(146555*sqrt(11) + 497041)*log(6*(sqrt(5*x^2 + 2*x + 3)
*sqrt(146555*sqrt(11) + 497041)*(87*sqrt(11) - 265) + 6517*sqrt(11)*(x + 3)
 + 19551*x - 32585)/x) - 3/7546*sqrt(11)*sqrt(146555*sqrt(11) + 497041)*log
(-6*(sqrt(5*x^2 + 2*x + 3)*sqrt(146555*sqrt(11) + 497041)*(87*sqrt(11) - 26
5) - 6517*sqrt(11)*(x + 3) - 19551*x + 32585)/x) - 1/15092*sqrt(11)*sqrt(-5
275980*sqrt(11) + 17893476)*log(-(sqrt(5*x^2 + 2*x + 3)*(87*sqrt(11) + 265)
*sqrt(-5275980*sqrt(11) + 17893476) + 39102*sqrt(11)*(x + 3) - 117306*x + 1
95510)/x) + 1/15092*sqrt(11)*sqrt(-5275980*sqrt(11) + 17893476)*log((sqrt(5
*x^2 + 2*x + 3)*(87*sqrt(11) + 265)*sqrt(-5275980*sqrt(11) + 17893476) - 39
102*sqrt(11)*(x + 3) + 117306*x - 195510)/x) - 1/490*sqrt(5*x^2 + 2*x + 3)*
(35*x + 397) + 8233/17150*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x +
1) - 25*x^2 - 10*x - 8)
```

giac [A] time = 0.27, size = 144, normalized size = 0.77

$$-\frac{1}{490}\sqrt{5x^2+2x+3}\,(35x+397)+\frac{8233}{8575}\sqrt{5}\log\left(-5\sqrt{5}x-\sqrt{5}+5\sqrt{5x^2+2x+3}\right)+2.61475869687464\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="giac
")
```

```
[Out] -1/490*sqrt(5*x^2 + 2*x + 3)*(35*x + 397) + 8233/8575*sqrt(5)*log(-5*sqrt(5
)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*x + 3)) + 2.61475869687464*log(-sqrt(5)*x
+ sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.276245077121866*log(-sqrt(5
)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 2.61475869687464*log(-sqr
t(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.276245077121866*log(
-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

maple [B] time = 0.09, size = 403, normalized size = 2.16

$$-\frac{\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{25}-\frac{(10x+2)\sqrt{5x^{2}+2x+3}}{140}-\frac{3\left(-61+13\sqrt{11}\right)\sqrt{11}\left(\frac{\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\sqrt{5}\,\operatorname{arcsinh}\left(\frac{\sqrt{5}\left(x+\cdots\right.}{\sqrt{\frac{250}{49}-\frac{34\sqrt{11}}{49}-\cdots}}\right.}{70}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x)
```

```
[Out] -1/140*(10*x+2)*(5*x^2+2*x+3)^(1/2)-1/25*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1
/5))-3/154*(-61+13*11^(1/2))*11^(1/2)*(1/49*(245*(x-2/7+1/7*11^(1/2))^2+49*
(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/70*(34/7
-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-1
0/7*11^(1/2))^2)^(1/2)*(x+1/5))-(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(
1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11
^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*
11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))-3/154*11^(1/2)*(61+
13*11^(1/2))*(1/49*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2
/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/70*(34/7+10/7*11^(1/2))*5^(1/2)*a
rcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x
+1/5))-(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49
+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2)
)^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(
1/2))+250+34*11^(1/2))^(1/2)))
```

maxima [B] time = 1.17, size = 500, normalized size = 2.67

$$\frac{1}{188650}\sqrt{11}\left(975\sqrt{11}\sqrt{2}\sqrt{17\sqrt{11}+125}\,\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7\left|14x-2\sqrt{11}-4\right|}+\frac{17\sqrt{7}\sqrt{2}x}{7\left|14x-2\sqrt{11}-4\right|}+\frac{\sqrt{11}\sqrt{7}\sqrt{2}}{7\left|14x-2\sqrt{11}\right.}\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="maxi
ma")
```

```
[Out] 1/188650*sqrt(11)*(975*sqrt(11)*sqrt(2)*sqrt(17*sqrt(11) + 125)*arcsinh(5/7
*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(
2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2
*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) - 1225*sq
rt(11)*sqrt(5*x^2 + 2*x + 3)*x - 16466*sqrt(11)*sqrt(5)*arcsinh(5/14*sqrt(7
)*sqrt(2)*x + 1/14*sqrt(7)*sqrt(2)) - 6825*sqrt(11)*sqrt(-34/49*sqrt(11) +
250/49)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) -
 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*s
qrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(
11) - 4)) + 4575*sqrt(2)*sqrt(17*sqrt(11) + 125)*arcsinh(5/7*sqrt(11)*sqrt(
7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x -
 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)
+ 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) + 32025*sqrt(-34/49*sqrt
(11) + 250/49)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11)
 - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sq
rt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x +
2*sqrt(11) - 4)) - 13895*sqrt(11)*sqrt(5*x^2 + 2*x + 3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5\,x+2\right)\,\sqrt{5\,x^2+2\,x+3}}{-7\,x^2+4\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1),x)
```

```
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx-\int \frac{5x\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx-\int \frac{x^2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1),x)
```

```
[Out] -Integral(2*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(5*x*sq
rt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(x**2*sqrt(5*x**2 + 2
*x + 3)/(7*x**2 - 4*x - 1), x)
```

3.378 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx$

Optimal. Leaf size=199

$$\frac{3\sqrt{5x^2+2x+3}\,(61x+3)}{154\left(-7x^2+4x+1\right)}-\frac{\sqrt{\frac{325022311+39132731\sqrt{11}}{1397}}\,\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\,\sqrt{5x^2+2x+3}}\right)}{2156}+\frac{\sqrt{\frac{325022311-39132731\sqrt{11}}{1397}}\ \mathrm{ta}}{}$$

```
[Out] 1/49*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)+3/154*(3+61*x)*(5*x^2+2*x+3)^(1
/2)/(-7*x^2+4*x+1)+1/3011932*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2
+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(454056168467-54668425207*11^(1/2))^
(1/2)-1/3011932*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)
/(250-34*11^(1/2))^(1/2))*(454056168467+54668425207*11^(1/2))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1054, 1076, 619, 215, 1032, 724, 206}

$$\frac{3\sqrt{5x^2+2x+3}\,(61x+3)}{154\left(-7x^2+4x+1\right)}-\frac{\sqrt{\frac{325022311+39132731\sqrt{11}}{1397}}\,\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\,\sqrt{5x^2+2x+3}}\right)}{2156}+\frac{\sqrt{\frac{325022311-39132731\sqrt{11}}{1397}}\ \mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] (3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(154*(1 + 4*x - 7*x^2)) + (Sqrt[5]*Arc
Sinh[(1 + 5*x)/Sqrt[14]])/49 - (Sqrt[(325022311 + 39132731*Sqrt[11])/1397]*
ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*
Sqrt[3 + 2*x + 5*x^2])])/2156 + (Sqrt[(325022311 - 39132731*Sqrt[11])/1397]
*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]
*Sqrt[3 + 2*x + 5*x^2])])/2156
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1054

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((A*b*c - 2
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}}{\left(1+4x-7x^2\right)^2}\,dx = \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\left(1+4x-7x^2\right)} - \frac{1}{308}\int \frac{-948-188x+220x^2}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\left(1+4x-7x^2\right)} + \frac{\int \frac{6416+436x}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx}{2156} + \frac{5}{49}\int \frac{1}{\sqrt{3+2x}}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\left(1+4x-7x^2\right)} + \frac{1}{98}\sqrt{\frac{5}{14}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right.$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\left(1+4x-7x^2\right)} + \frac{1}{49}\sqrt{5}\,\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right) - \frac{\left(2\left(1199-11446\right.\right.}{}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\left(1+4x-7x^2\right)} + \frac{1}{49}\sqrt{5}\,\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right) - \frac{\sqrt{\frac{325022311+39132}{1397}}}{}$$

Mathematica [A] time = 1.28, size = 354, normalized size = 1.78

$$\frac{56364\sqrt{5x^2+2x+3}\,x}{-7x^2+4x+1} + \frac{2772\sqrt{5x^2+2x+3}}{-7x^2+4x+1} + 22892\sqrt{\frac{22}{125+17\sqrt{11}}}\,\log\left(\sqrt{2750+374\sqrt{11}}\,\sqrt{5x^2+2x+3} + \left(55+17\sqrt{11}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] ((2772*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2) + (56364*x*Sqrt[3 + 2*x + 5
*x^2])/(1 + 4*x - 7*x^2) + 968*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]] + 2*Sqrt
[2/(125 - 17*Sqrt[11])]*(-1199 + 11446*Sqrt[11])*ArcTanh[(Sqrt[250 - 34*Sqr
t[11]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[11] - 17*x + 5*Sqrt[11]*x)] - 239
8*Sqrt[2/(125 + 17*Sqrt[11])]*Log[2 + Sqrt[11] - 7*x] - 22892*Sqrt[22/(125
+ 17*Sqrt[11])]*Log[2 + Sqrt[11] - 7*x] + 2398*Sqrt[2/(125 + 17*Sqrt[11])]*
Log[11 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqr
t[3 + 2*x + 5*x^2]] + 22892*Sqrt[22/(125 + 17*Sqrt[11])]*Log[11 + 23*Sqrt[1
1] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2]
])/47432
```

fricas [B] time = 0.99, size = 378, normalized size = 1.90

$$-\frac{\sqrt{1397}\left(7x^2-4x-1\right)\sqrt{39132731\sqrt{11}+325022311}\,\log\left(-\frac{\sqrt{1397}\sqrt{5x^2+2x+3}\sqrt{39132731\sqrt{11}+325022311}\left(16943\sqrt{11}+23\ldots\right.}{x}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="fr
icas")
```

```
[Out] -1/6023864*(sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(39132731*sqrt(11) + 325022311
)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(39132731*sqrt(11) + 325022311
)*(16943*sqrt(11) + 235367) + 26119953475*sqrt(11)*(x + 3) - 78359860425*x
+ 130599767375)/x) - sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(39132731*sqrt(11) +
325022311)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(39132731*sqrt(11) + 3
25022311)*(16943*sqrt(11) + 235367) - 26119953475*sqrt(11)*(x + 3) + 783598
60425*x - 130599767375)/x) + sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(-39132731*sq
rt(11) + 325022311)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(16943*sqrt(11) -
 235367)*sqrt(-39132731*sqrt(11) + 325022311) + 26119953475*sqrt(11)*(x + 3
) + 78359860425*x - 130599767375)/x) - sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(-3
9132731*sqrt(11) + 325022311)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(16943
*sqrt(11) - 235367)*sqrt(-39132731*sqrt(11) + 325022311) - 26119953475*sqrt
(11)*(x + 3) - 78359860425*x + 130599767375)/x) - 61468*sqrt(5)*(7*x^2 - 4*
x - 1)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) +
117348*sqrt(5*x^2 + 2*x + 3)*(61*x + 3))/(7*x^2 - 4*x - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{184473632,[8]%%%}+%%%{%%{[421654016,0]:[1,0,-5]%%},[7]%%%}+%%%{-2484
746880,[6]%%%}+%%%{%%{[-5059848192,0]:[1,0,-5]%%},[5]%%%}+%%%{18003120576,[
4]%%%}+%%%{%%{[13432692224,0]:[1,0,-5]%%},[3]%%%}+%%%{-38927701120,[2]%%%}+
%%%{%%{[-9999223808,0]:[1,0,-5]%%},[1]%%%}+%%%{25935486752,[0]%%%} / %%%{24
5,[8]%%%}+%%%{%%{poly1[560,0]:[1,0,-5]%%},[7]%%%}+%%%{-3300,[6]%%%}+%%%{%%{
poly1[-6720,0]:[1,0,-5]%%},[5]%%%}+%%%{23910,[4]%%%}+%%%{%%{poly1[17840,0]:
[1,0,-5]%%},[3]%%%}+%%%{-51700,[2]%%%}+%%%{%%{poly1[-13280,0]:[1,0,-5]%%},[
1]%%%}+%%%{34445,[0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.03, size = 1084, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x)
```

```
[Out] (183/44+39/44*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))
*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49
+34/49*11^(1/2))^(3/2)+1/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1
/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))
+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(
250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49
+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2
)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(
x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*1
1^(1/2))^(1/2)))+10/49/(250/49+34/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7
*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2
))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arc
sinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1
/5))))+161/484*11^(1/2)*(1/49*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^
(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/70*(34/7-10/7*11^(1/2)
)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^
2)^(1/2)*(x+1/5))-(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(4
9/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-
34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2
/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))+(183/44-39/44*11^(1/2))*(-1/49/(2
50/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-
10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/98*(34/7
-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49
*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/
7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-
10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2)
)^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7
*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10
/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))+10/49/(250/49-34
/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))
*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11
^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/
2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))))-161/484*11^(1/2)*(1/49*(24
5*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+3
4*11^(1/2))^(1/2)+1/70*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49
+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-(250/49+34/49*1
1^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+
10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/
7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))
^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{5\,x^{2}+2\,x+3}\,\left(x^{2}+5\,x+2\right)}{\left(7\,x^{2}-4\,x-1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(5*x^2 + 2*x + 3)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5\,x+2\right)\,\sqrt{5\,x^2+2\,x+3}}{{\left(-7\,x^2+4\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^2,x)
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2} + 5x + 2\right)\sqrt{5x^{2} + 2x + 3}}{\left(7x^{2} - 4x - 1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1)**2,x)
[Out] Integral((x**2 + 5*x + 2)*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1)**2, x)
```

3.379 $\int \frac{\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}}{\left(1+4x-7x^2\right)^3}\,dx$

Optimal. Leaf size=213

$$-\frac{\sqrt{5x^2+2x+3}\,(272941-813113x)}{1721104\left(-7x^2+4x+1\right)}+\frac{3(61x+3)\sqrt{5x^2+2x+3}}{308\left(-7x^2+4x+1\right)^2}-\frac{\sqrt{\frac{6492253020949-11879169071\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(1}{\sqrt{2(1}}\right.}{491744}$$

```
[Out] 3/308*(3+61*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2-1/1721104*(272941-81311
3*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)-1/686966368*arctanh((23+x*(17-5*11^
(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(906967747026
5753-16595199192187*11^(1/2))^(1/2)+1/686966368*arctanh((23+11^(1/2)+x*(17+
5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(9069677470265753
+16595199192187*11^(1/2))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1054, 1060, 1032, 724, 206}

$$-\frac{\sqrt{5x^2+2x+3}\,(272941-813113x)}{1721104\left(-7x^2+4x+1\right)}+\frac{3(61x+3)\sqrt{5x^2+2x+3}}{308\left(-7x^2+4x+1\right)^2}-\frac{\sqrt{\frac{6492253020949-11879169071\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(1}{\sqrt{2(1}}\right.}{491744}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] (3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(308*(1 + 4*x - 7*x^2)^2) - ((272941 -
 813113*x)*Sqrt[3 + 2*x + 5*x^2])/(1721104*(1 + 4*x - 7*x^2)) - (Sqrt[(6492
253020949 - 11879169071*Sqrt[11])/1397]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sq
rt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])]])/491744 + (
Sqrt[(6492253020949 + 11879169071*Sqrt[11])/1397]*ArcTanh[(23 + Sqrt[11] +
(17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])]])/
491744
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1054

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((A*b*c - 2
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx = \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308(1+4x-7x^2)^2} - \frac{1}{616}\int \frac{-3012-1564x-3220x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104(1+4x-7x^2)} + \ldots$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104(1+4x-7x^2)} + \ldots$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104(1+4x-7x^2)} + \ldots$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104(1+4x-7x^2)} - \ldots$$

Mathematica [A] time = 1.40, size = 334, normalized size = 1.57

$$-\sqrt{\frac{22}{125-17\sqrt{11}}}\left(126542\sqrt{11}-1740003\right)\log\left(49x^2+14\left(\sqrt{11}-2\right)x-4\sqrt{11}+15\right)+2\sqrt{\frac{22}{125+17\sqrt{11}}}\left(1740003\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] ((-44*Sqrt[3 + 2*x + 5*x^2]*(31807 - 106279*x - 737577*x^2 + 813113*x^3))/(
1 + 4*x - 7*x^2)^2 - 2*Sqrt[22/(125 - 17*Sqrt[11])]*(-1740003 + 126542*Sqrt
[11])*ArcTanh[(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[1
1] + (-17 + 5*Sqrt[11])*x)] - 2*Sqrt[22/(125 + 17*Sqrt[11])]*(1740003 + 126
542*Sqrt[11])*Log[2 + Sqrt[11] - 7*x] + Sqrt[22/(125 - 17*Sqrt[11])]*(-1740
003 + 126542*Sqrt[11])*Log[(-2 + Sqrt[11] + 7*x)^2] - Sqrt[22/(125 - 17*Sqr
t[11])]*(-1740003 + 126542*Sqrt[11])*Log[15 - 4*Sqrt[11] + 14*(-2 + Sqrt[11
])*x + 49*x^2] + 2*Sqrt[22/(125 + 17*Sqrt[11])]*(1740003 + 126542*Sqrt[11])
*Log[11 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sq
rt[3 + 2*x + 5*x^2]])/10818368
```

fricas [B] time = 0.96, size = 390, normalized size = 1.83

$$-\frac{\sqrt{1397}\left(49x^4-56x^3+2x^2+8x+1\right)\sqrt{11879169071\sqrt{11}+6492253020949}\log\left(\frac{\sqrt{1397}\sqrt{5x^2+2x+3}\sqrt{11879169\ldots}}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="fr
icas")
```

```
[Out] -1/1373932736*(sqrt(1397)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(11879169
071*sqrt(11) + 6492253020949)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(11
879169071*sqrt(11) + 6492253020949)*(4822219*sqrt(11) - 37335441) + 5690716
98870455*sqrt(11)*(x + 3) + 1707215096611365*x - 2845358494352275)/x) - sqr
```

```
t(1397)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(11879169071*sqrt(11) + 649
2253020949)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(11879169071*sqrt(11
) + 6492253020949)*(4822219*sqrt(11) - 37335441) - 569071698870455*sqrt(11)
*(x + 3) - 1707215096611365*x + 2845358494352275)/x) + sqrt(1397)*(49*x^4 -
 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-11879169071*sqrt(11) + 6492253020949)*log(
-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(4822219*sqrt(11) + 37335441)*sqrt(-1187
9169071*sqrt(11) + 6492253020949) + 569071698870455*sqrt(11)*(x + 3) - 1707
215096611365*x + 2845358494352275)/x) - sqrt(1397)*(49*x^4 - 56*x^3 + 2*x^2
 + 8*x + 1)*sqrt(-11879169071*sqrt(11) + 6492253020949)*log((sqrt(1397)*sqr
t(5*x^2 + 2*x + 3)*(4822219*sqrt(11) + 37335441)*sqrt(-11879169071*sqrt(11)
 + 6492253020949) - 569071698870455*sqrt(11)*(x + 3) + 1707215096611365*x -
 2845358494352275)/x) + 5588*(813113*x^3 - 737577*x^2 - 106279*x + 31807)*s
qrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

giac [B] time = 0.26, size = 378, normalized size = 1.77

$$\frac{6200558\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^7-835775\,\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^6-190947036\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+\cdots}\right.}{430276\left(7\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,\cdots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="gi
ac")
```

```
[Out] 1/430276*(6200558*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 - 835775*sqrt(5)*(s
qrt(5)*x - sqrt(5*x^2 + 2*x + 3))^6 - 190947036*(sqrt(5)*x - sqrt(5*x^2 + 2
*x + 3))^5 - 92732607*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 + 81632
1374*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 419437335*sqrt(5)*(sqrt(5)*x -
 sqrt(5*x^2 + 2*x + 3))^2 - 765111048*sqrt(5)*x - 376983161*sqrt(5) + 76511
1048*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sq
rt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 +
2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83)^2 + 0.13
9051039089329*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) -
0.138209741946100*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000
) - 0.139051039089329*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.0225803811
3000) + 0.138209741946100*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.094112
35400000)
```

maple [B] time = 0.03, size = 2342, normalized size = 11.00

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x)
```

```
[Out] -21/968*(61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49+34/49*11^(1/2))/(x-2/7-1/
7*11^(1/2))^2*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
(1/2))+250/49+34/49*11^(1/2))^(3/2)-1/1372*(34/7+10/7*11^(1/2))/(250/49+34/
49*11^(1/2))*(-1/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7
*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2
))^(3/2)+1/2*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-
1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2
))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11
^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2)
)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11
^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1
/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
)+10/(250/49+34/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7
+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(50
```

```
00/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/
49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))))+5/686/(250/
49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))
*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1
/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/
2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*
(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*1
1^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1
/7*11^(1/2))+250+34*11^(1/2))^(1/2))))+3535/21296*11^(1/2)*(1/49*(245*(x-2/
7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1
/2))^(1/2)+1/70*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*
11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-(250/49-34/49*11^(1/2)
)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11
^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1
/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))
)-21/968*(-61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49-34/49*11^(1/2))/(x-2/7+
1/7*11^(1/2))^2*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*1
1^(1/2))+250/49-34/49*11^(1/2))^(3/2)-1/1372*(34/7-10/7*11^(1/2))/(250/49-3
4/49*11^(1/2))*(-1/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1
/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1
/2))^(3/2)+1/2*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/
7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1
/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*
11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/
2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*
11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^
(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2
)))+10/(250/49-34/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34
/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(
5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(25
0/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))))+5/686/(25
0/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2
))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^
(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(
1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/
2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34
*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7
+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))-(-3535/1936+273/1936*11^(1/2))*(-1
/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+
(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/98
*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2)
)^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/1
0*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*
(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11
^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2
/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(3
4/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))+10/49/(250
/49-34/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^
(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680
/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*
11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))))-(-3535/1936-273/1936
*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/
7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/
2))^(3/2)+1/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/
7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1
/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*
11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/
2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*
11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^
```

```
(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2
)))+10/49/(250/49+34/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+
(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/20
0*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/
(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))))-3535/2
1296*11^(1/2)*(1/49*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-
2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/70*(34/7+10/7*11^(1/2))*5^(1/2)*
arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(
x+1/5))-(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/4
9+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2
))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
(1/2))+250+34*11^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{5 x^2 + 2 x + 3}\left(x^2 + 5 x + 2\right)}{\left(7 x^2 - 4 x - 1\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="ma
xima")
```

```
[Out] -integrate(sqrt(5*x^2 + 2*x + 3)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2 + 5 x + 2\right) \sqrt{5 x^2 + 2 x + 3}}{\left(-7 x^2 + 4 x + 1\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^3,x)
```

```
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\sqrt{5x^2 + 2x + 3}}{343x^6 - 588x^5 + 189x^4 + 104x^3 - 27x^2 - 12x - 1}\, dx - \int \frac{5x\sqrt{5x^2 + 2x + 3}}{343x^6 - 588x^5 + 189x^4 + 104x^3 - 27x^2 - 12x - 1}\, d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1)**3,x)
```

```
[Out] -Integral(2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x*
*3 - 27*x**2 - 12*x - 1), x) - Integral(5*x*sqrt(5*x**2 + 2*x + 3)/(343*x**
6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral(x**
2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x*
*2 - 12*x - 1), x)
```

3.380 $\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} dx$

Optimal. Leaf size=231

$$\frac{2173004363 \left(5x^2+2x+3\right)^{5/2} x^2}{173250000}+\frac{837379699 \left(5x^2+2x+3\right)^{5/2} x}{72187500}-\frac{6133820867 \left(5x^2+2x+3\right)^{5/2}}{1203125000}-\frac{22840599}{}$$

```
[Out] -22840599/62500000*(1+5*x)*(5*x^2+2*x+3)^(3/2)-6133820867/1203125000*(5*x^2
+2*x+3)^(5/2)+837379699/72187500*x*(5*x^2+2*x+3)^(5/2)+2173004363/173250000
*x^2*(5*x^2+2*x+3)^(5/2)-190236913/4950000*x^3*(5*x^2+2*x+3)^(5/2)-796559/1
23750*x^4*(5*x^2+2*x+3)^(5/2)+1031177/20625*x^5*(5*x^2+2*x+3)^(5/2)-61103/3
300*x^6*(5*x^2+2*x+3)^(5/2)-343/60*x^7*(5*x^2+2*x+3)^(5/2)-3357568053/78125
0000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-479652579/312500000*(1+5*x)*(5*
x^2+2*x+3)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1661, 640, 612, 619, 215}

$$-\frac{343}{60}\left(5x^2+2x+3\right)^{5/2} x^7-\frac{61103 \left(5x^2+2x+3\right)^{5/2} x^6}{3300}+\frac{1031177 \left(5x^2+2x+3\right)^{5/2} x^5}{20625}-\frac{796559 \left(5x^2+2x+3\right)}{123750}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-479652579*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/312500000 - (22840599*(1 + 5*x
)*(3 + 2*x + 5*x^2)^(3/2))/62500000 - (6133820867*(3 + 2*x + 5*x^2)^(5/2))/
1203125000 + (837379699*x*(3 + 2*x + 5*x^2)^(5/2))/72187500 + (2173004363*x
^2*(3 + 2*x + 5*x^2)^(5/2))/173250000 - (190236913*x^3*(3 + 2*x + 5*x^2)^(5
/2))/4950000 - (796559*x^4*(3 + 2*x + 5*x^2)^(5/2))/123750 + (1031177*x^5*(
3 + 2*x + 5*x^2)^(5/2))/20625 - (61103*x^6*(3 + 2*x + 5*x^2)^(5/2))/3300 -
(343*x^7*(3 + 2*x + 5*x^2)^(5/2))/60 - (3357568053*ArcSinh[(1 + 5*x)/Sqrt[1
4]])/(156250000*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx = -\frac{343}{60}x^7\left(3+2x+5x^2\right)^{5/2}+\frac{1}{60}\int \left(3+2x+5x^2\right)^{3/2}\left(120\right.$$

$$= -\frac{61103x^6\left(3+2x+5x^2\right)^{5/2}}{3300}-\frac{343}{60}x^7\left(3+2x+5x^2\right)^{5/2}+$$

$$= \frac{1031177x^5\left(3+2x+5x^2\right)^{5/2}}{20625}-\frac{61103x^6\left(3+2x+5x^2\right)^{5/}}{3300}$$

$$= -\frac{796559x^4\left(3+2x+5x^2\right)^{5/2}}{123750}+\frac{1031177x^5\left(3+2x+5x\right.}{20625}$$

$$= -\frac{190236913x^3\left(3+2x+5x^2\right)^{5/2}}{4950000}-\frac{796559x^4\left(3+2x+\right.}{123750}$$

$$= \frac{2173004363x^2\left(3+2x+5x^2\right)^{5/2}}{173250000}-\frac{190236913x^3\left(3+2x\right.}{4950000}$$

$$= \frac{837379699x\left(3+2x+5x^2\right)^{5/2}}{72187500}+\frac{2173004363x^2\left(3+2x\right.}{173250000}$$

$$= -\frac{6133820867\left(3+2x+5x^2\right)^{5/2}}{1203125000}+\frac{837379699x\left(3+2x\right.}{72187500}$$

$$= -\frac{22840599(1+5x)\left(3+2x+5x^2\right)^{3/2}}{62500000}-\frac{6133820867\left(3\right.}{120312}$$

$$= -\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000}-\frac{22840599(1+5x}{625}$$

$$= -\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000}-\frac{22840599(1+5x}{625}$$

$$= -\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000}-\frac{22840599(1+5x}{625}$$

Mathematica [A] time = 0.42, size = 95, normalized size = 0.41

$$\frac{-4653589321458\sqrt{5}\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)-5\sqrt{5x^2+2x+3}\left(30950390625000x^{11}+125007421875000x^{10}-1483937\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-5*Sqrt[3 + 2*x + 5*x^2]*(10506617068392 - 6352777129950*x - 1586584440868
5*x^2 - 19041688239675*x^3 - 2573089891000*x^4 + 85130334087500*x^5 + 52106
830406250*x^6 - 72918247281250*x^7 - 30505457500000*x^8 - 148393743750000*x
^9 + 125007421875000*x^10 + 30950390625000*x^11) - 4653589321458*Sqrt[5]*Ar
cSinh[(1 + 5*x)/Sqrt[14]])/1082812500000
```

fricas [A] time = 0.88, size = 107, normalized size = 0.46

$$-\frac{1}{216562500000}\left(30950390625000\,x^{11}+125007421875000\,x^{10}-148393743750000\,x^{9}-30505457500000\,x^{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/216562500000*(30950390625000*x^11 + 125007421875000*x^10 - 1483937437500
00*x^9 - 30505457500000*x^8 - 72918247281250*x^7 + 52106830406250*x^6 + 851
30334087500*x^5 - 2573089891000*x^4 - 19041688239675*x^3 - 15865844408685*x
^2 - 6352777129950*x + 10506617068392)*sqrt(5*x^2 + 2*x + 3) + 3357568053/1
562500000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10
*x - 8)
```

giac [A] time = 0.23, size = 102, normalized size = 0.44

$$-\frac{1}{216562500000}\left(5\left(\left(5\left(10\left(25\left(5\left(7\left(20\left(105\left(875\left(77\,x+311\right)x-323034\right)x-6972676\right)x-333340559\right)x+166\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/216562500000*(5*((5*(10*(25*(5*(7*(20*(105*(875*(77*x + 311)*x - 323034)
*x - 6972676)*x - 333340559)*x + 1667418573)*x + 13620853454)*x - 102923595
64)*x - 761667529587)*x - 3173168881737)*x - 1270555425990)*x + 10506617068
392)*sqrt(5*x^2 + 2*x + 3) + 3357568053/781250000*sqrt(5)*log(-sqrt(5)*(sqr
t(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

maple [A] time = 0.04, size = 185, normalized size = 0.80

$$-\frac{343\left(5x^{2}+2x+3\right)^{\frac{5}{2}}x^{7}}{60}-\frac{61103\left(5x^{2}+2x+3\right)^{\frac{5}{2}}x^{6}}{3300}+\frac{1031177\left(5x^{2}+2x+3\right)^{\frac{5}{2}}x^{5}}{20625}-\frac{796559\left(5x^{2}+2x+3\right)^{\frac{5}{2}}x^{4}}{123750}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] -6133820867/1203125000*(5*x^2+2*x+3)^(5/2)-343/60*x^7*(5*x^2+2*x+3)^(5/2)-6
1103/3300*x^6*(5*x^2+2*x+3)^(5/2)+1031177/20625*x^5*(5*x^2+2*x+3)^(5/2)-796
559/123750*x^4*(5*x^2+2*x+3)^(5/2)-190236913/4950000*x^3*(5*x^2+2*x+3)^(5/2
)+2173004363/173250000*x^2*(5*x^2+2*x+3)^(5/2)-3357568053/781250000*5^(1/2)
*arcsinh(5/14*14^(1/2)*(x+1/5))-479652579/625000000*(10*x+2)*(5*x^2+2*x+3)^
(1/2)+837379699/72187500*x*(5*x^2+2*x+3)^(5/2)-22840599/125000000*(10*x+2)*
(5*x^2+2*x+3)^(3/2)
```

maxima [A] time = 1.01, size = 206, normalized size = 0.89

$$-\frac{343}{60}\left(5\,x^{2}+2\,x+3\right)^{\frac{5}{2}}x^{7}-\frac{61103}{3300}\left(5\,x^{2}+2\,x+3\right)^{\frac{5}{2}}x^{6}+\frac{1031177}{20625}\left(5\,x^{2}+2\,x+3\right)^{\frac{5}{2}}x^{5}-\frac{796559}{123750}\left(5\,x^{2}+2\,x+3\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] -343/60*(5*x^2 + 2*x + 3)^(5/2)*x^7 - 61103/3300*(5*x^2 + 2*x + 3)^(5/2)*x^
6 + 1031177/20625*(5*x^2 + 2*x + 3)^(5/2)*x^5 - 796559/123750*(5*x^2 + 2*x
+ 3)^(5/2)*x^4 - 190236913/4950000*(5*x^2 + 2*x + 3)^(5/2)*x^3 + 2173004363
/173250000*(5*x^2 + 2*x + 3)^(5/2)*x^2 + 837379699/72187500*(5*x^2 + 2*x +
3)^(5/2)*x - 6133820867/1203125000*(5*x^2 + 2*x + 3)^(5/2) - 22840599/12500
000*(5*x^2 + 2*x + 3)^(3/2)*x - 22840599/62500000*(5*x^2 + 2*x + 3)^(3/2) -
 479652579/62500000*sqrt(5*x^2 + 2*x + 3)*x - 3357568053/781250000*sqrt(5)*
arcsinh(1/14*sqrt(14)*(5*x + 1)) - 479652579/312500000*sqrt(5*x^2 + 2*x + 3
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(x^2+5\,x+2\right)\,\left(5\,x^2+2\,x+3\right)^{3/2}\,\left(-7\,x^2+4\,x+1\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3,x)
```

```
[Out] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-91x\sqrt{5x^2+2x+3}\right)dx-\int \left(-413x^2\sqrt{5x^2+2x+3}\right)dx-\int \left(-192x^3\sqrt{5x^2+2x+3}\right)dx-\int 2160x^4\sqrt{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-91*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-413*x**2*sqrt(5*x**2
 + 2*x + 3), x) - Integral(-192*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(
2160*x**4*sqrt(5*x**2 + 2*x + 3), x) - Integral(1666*x**5*sqrt(5*x**2 + 2*x
 + 3), x) - Integral(-2094*x**6*sqrt(5*x**2 + 2*x + 3), x) - Integral(-1384
*x**7*sqrt(5*x**2 + 2*x + 3), x) - Integral(-7042*x**8*sqrt(5*x**2 + 2*x +
3), x) - Integral(6321*x**9*sqrt(5*x**2 + 2*x + 3), x) - Integral(1715*x**1
0*sqrt(5*x**2 + 2*x + 3), x) - Integral(-6*sqrt(5*x**2 + 2*x + 3), x)
```

3.381 $\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx$

Optimal. Leaf size=189

$$-\frac{219271 \left(5x^2+2x+3\right)^{5/2} x^2}{105000}+\frac{86721 \left(5x^2+2x+3\right)^{5/2} x}{21875}+\frac{505667 \left(5x^2+2x+3\right)^{5/2}}{2187500}-\frac{690561(5x+1) \left(5x^2+2\right.}{1250000}$$

```
[Out] -690561/1250000*(1+5*x)*(5*x^2+2*x+3)^(3/2)+505667/2187500*(5*x^2+2*x+3)^(5
/2)+86721/21875*x*(5*x^2+2*x+3)^(5/2)-219271/105000*x^2*(5*x^2+2*x+3)^(5/2)
-18379/3000*x^3*(5*x^2+2*x+3)^(5/2)+581/150*x^4*(5*x^2+2*x+3)^(5/2)+49/50*x
^5*(5*x^2+2*x+3)^(5/2)-101512467/15625000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^
(1/2)-14501781/6250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1661, 640, 612, 619, 215}

$$\frac{49}{50} \left(5x^2+2x+3\right)^{5/2} x^5+\frac{581}{150} \left(5x^2+2x+3\right)^{5/2} x^4-\frac{18379 \left(5x^2+2x+3\right)^{5/2} x^3}{3000}-\frac{219271 \left(5x^2+2x+3\right)^{5/2} x^2}{105000}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-14501781*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/6250000 - (690561*(1 + 5*x)*(3
+ 2*x + 5*x^2)^(3/2))/1250000 + (505667*(3 + 2*x + 5*x^2)^(5/2))/2187500 +
(86721*x*(3 + 2*x + 5*x^2)^(5/2))/21875 - (219271*x^2*(3 + 2*x + 5*x^2)^(5/
2))/105000 - (18379*x^3*(3 + 2*x + 5*x^2)^(5/2))/3000 + (581*x^4*(3 + 2*x +
 5*x^2)^(5/2))/150 + (49*x^5*(3 + 2*x + 5*x^2)^(5/2))/50 - (101512467*ArcSi
nh[(1 + 5*x)/Sqrt[14]])/(3125000*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} dx = \frac{49}{50}x^5 \left(3+2x+5x^2\right)^{5/2} + \frac{1}{50}\int \left(3+2x+5x^2\right)^{3/2} \left(100+\right.$$

$$= \frac{581}{150}x^4 \left(3+2x+5x^2\right)^{5/2} + \frac{49}{50}x^5 \left(3+2x+5x^2\right)^{5/2} + \frac{\int (3}{}$$

$$= -\frac{18379x^3 \left(3+2x+5x^2\right)^{5/2}}{3000} + \frac{581}{150}x^4 \left(3+2x+5x^2\right)^{5/2} +$$

$$= -\frac{219271x^2 \left(3+2x+5x^2\right)^{5/2}}{105000} - \frac{18379x^3 \left(3+2x+5x^2\right)^{5}}{3000}$$

$$= \frac{86721x \left(3+2x+5x^2\right)^{5/2}}{21875} - \frac{219271x^2 \left(3+2x+5x^2\right)^{5/2}}{105000}$$

$$= \frac{505667 \left(3+2x+5x^2\right)^{5/2}}{2187500} + \frac{86721x \left(3+2x+5x^2\right)^{5/2}}{21875} -$$

$$= -\frac{690561(1+5x) \left(3+2x+5x^2\right)^{3/2}}{1250000} + \frac{505667 \left(3+2x+\right.}{2187500}$$

$$= -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1+5x) \left(3\right.}{12500}$$

$$= -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1+5x) \left(3\right.}{12500}$$

$$= -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1+5x) \left(3\right.}{12500}$$

Mathematica [A] time = 0.25, size = 85, normalized size = 0.45

$$\frac{5\sqrt{5x^2+2x+3} \left(3215625000x^9 + 15281875000x^8 - 5561281250x^7 - 4105593750x^6 - 12554262500x^5 - 32275\right.}{6562500}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (5*Sqrt[3 + 2*x + 5*x^2]*(-249003936 + 2291675850*x + 3721040355*x^2 + 5959
365525*x^3 - 3227597000*x^4 - 12554262500*x^5 - 4105593750*x^6 - 5561281250
*x^7 + 15281875000*x^8 + 3215625000*x^9) - 4263523614*Sqrt[5]*ArcSinh[(1 +
5*x)/Sqrt[14]])/656250000
```

fricas [A] time = 0.70, size = 97, normalized size = 0.51

$$\frac{1}{131250000} \left(3215625000\,x^9 + 15281875000\,x^8 - 5561281250\,x^7 - 4105593750\,x^6 - 12554262500\,x^5 - 32275970\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/131250000*(3215625000*x^9 + 15281875000*x^8 - 5561281250*x^7 - 4105593750
*x^6 - 12554262500*x^5 - 3227597000*x^4 + 5959365525*x^3 + 3721040355*x^2 +
 2291675850*x - 249003936)*sqrt(5*x^2 + 2*x + 3) + 101512467/31250000*sqrt(
5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

giac [A] time = 0.22, size = 92, normalized size = 0.49

$$\frac{1}{131250000}\left(5\left(\left(5\left(10\left(25\left(5\left(7\left(140\left(105\,x+499\right)x-25423\right)x-131379\right)x-2008682\right)x-12910388\right)x+23837\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/131250000*(5*((5*(10*(25*(5*(7*(140*(105*x + 499)*x - 25423)*x - 131379)*
x - 2008682)*x - 12910388)*x + 238374621)*x + 744208071)*x + 458335170)*x -
 249003936)*sqrt(5*x^2 + 2*x + 3) + 101512467/15625000*sqrt(5)*log(-sqrt(5)
*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

maple [A] time = 0.01, size = 151, normalized size = 0.80

$$\frac{49\left(5x^{2}+2x+3\right)^{\frac{5}{2}}x^{5}}{50}+\frac{581\left(5x^{2}+2x+3\right)^{\frac{5}{2}}x^{4}}{150}-\frac{18379\left(5x^{2}+2x+3\right)^{\frac{5}{2}}x^{3}}{3000}-\frac{219271\left(5x^{2}+2x+3\right)^{\frac{5}{2}}x^{2}}{105000}+\frac{8672}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] 505667/2187500*(5*x^2+2*x+3)^(5/2)+49/50*(5*x^2+2*x+3)^(5/2)*x^5+581/150*(5
*x^2+2*x+3)^(5/2)*x^4-18379/3000*(5*x^2+2*x+3)^(5/2)*x^3-219271/105000*(5*x
^2+2*x+3)^(5/2)*x^2-101512467/15625000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5
))-14501781/12500000*(10*x+2)*(5*x^2+2*x+3)^(1/2)+86721/21875*(5*x^2+2*x+3)
^(5/2)*x-690561/2500000*(10*x+2)*(5*x^2+2*x+3)^(3/2)
```

maxima [A] time = 0.98, size = 172, normalized size = 0.91

$$\frac{49}{50}\left(5\,x^{2}+2\,x+3\right)^{\frac{5}{2}}x^{5}+\frac{581}{150}\left(5\,x^{2}+2\,x+3\right)^{\frac{5}{2}}x^{4}-\frac{18379}{3000}\left(5\,x^{2}+2\,x+3\right)^{\frac{5}{2}}x^{3}-\frac{219271}{105000}\left(5\,x^{2}+2\,x+3\right)^{\frac{5}{2}}x^{2}+\frac{867}{218}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] 49/50*(5*x^2 + 2*x + 3)^(5/2)*x^5 + 581/150*(5*x^2 + 2*x + 3)^(5/2)*x^4 - 1
8379/3000*(5*x^2 + 2*x + 3)^(5/2)*x^3 - 219271/105000*(5*x^2 + 2*x + 3)^(5/
2)*x^2 + 86721/21875*(5*x^2 + 2*x + 3)^(5/2)*x + 505667/2187500*(5*x^2 + 2*
x + 3)^(5/2) - 690561/250000*(5*x^2 + 2*x + 3)^(3/2)*x - 690561/1250000*(5*
x^2 + 2*x + 3)^(3/2) - 14501781/1250000*sqrt(5*x^2 + 2*x + 3)*x - 101512467
/15625000*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 14501781/6250000*sqrt(
5*x^2 + 2*x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2+5\,x+2\right)\,{\left(5\,x^2+2\,x+3\right)}^{3/2}\,{\left(-7\,x^2+4\,x+1\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2,x)
```

```
[Out] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{2}+5x+2\right)\left(5x^{2}+2x+3\right)^{\frac{3}{2}}\left(7x^{2}-4x-1\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*(5*x**2 + 2*x + 3)**(3/2)*(7*x**2 - 4*x - 1)**2,
x)
```

3.382 $\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx$

Optimal. Leaf size=147

$$-\frac{1163\left(5x^2+2x+3\right)^{5/2}x^2}{1400}+\frac{2809\left(5x^2+2x+3\right)^{5/2}x}{5250}+\frac{149509\left(5x^2+2x+3\right)^{5/2}}{262500}-\frac{18397(5x+1)\left(5x^2+2x+3\right)^{3/2}}{150000}$$

```
[Out] -18397/150000*(1+5*x)*(5*x^2+2*x+3)^(3/2)+149509/262500*(5*x^2+2*x+3)^(5/2)
+2809/5250*x*(5*x^2+2*x+3)^(5/2)-1163/1400*x^2*(5*x^2+2*x+3)^(5/2)-7/40*x^3
*(5*x^2+2*x+3)^(5/2)-901453/625000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-1
28779/250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1661, 640, 612, 619, 215}

$$-\frac{7}{40}\left(5x^2+2x+3\right)^{5/2}x^3-\frac{1163\left(5x^2+2x+3\right)^{5/2}x^2}{1400}+\frac{2809\left(5x^2+2x+3\right)^{5/2}x}{5250}+\frac{149509\left(5x^2+2x+3\right)^{5/2}}{262500}-\frac{18}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-128779*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/250000 - (18397*(1 + 5*x)*(3 + 2*
x + 5*x^2)^(3/2))/150000 + (149509*(3 + 2*x + 5*x^2)^(5/2))/262500 + (2809*
x*(3 + 2*x + 5*x^2)^(5/2))/5250 - (1163*x^2*(3 + 2*x + 5*x^2)^(5/2))/1400 -
 (7*x^3*(3 + 2*x + 5*x^2)^(5/2))/40 - (901453*ArcSinh[(1 + 5*x)/Sqrt[14]])/
(125000*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
```

```
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx = -\frac{7}{40}x^3\left(3+2x+5x^2\right)^{5/2}+\frac{1}{40}\int \left(3+2x+5x^2\right)^{3/2}\left(80+\right.$$

$$= -\frac{1163x^2\left(3+2x+5x^2\right)^{5/2}}{1400}-\frac{7}{40}x^3\left(3+2x+5x^2\right)^{5/2}+\int$$

$$= \frac{2809x\left(3+2x+5x^2\right)^{5/2}}{5250}-\frac{1163x^2\left(3+2x+5x^2\right)^{5/2}}{1400}-\frac{7}{40}$$

$$= \frac{149509\left(3+2x+5x^2\right)^{5/2}}{262500}+\frac{2809x\left(3+2x+5x^2\right)^{5/2}}{5250}-\frac{11}{}$$

$$= -\frac{18397(1+5x)\left(3+2x+5x^2\right)^{3/2}}{150000}+\frac{149509\left(3+2x+5x\right.}{262500}$$

$$= -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x)\left(3+2x\right.}{150000}$$

$$= -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x)\left(3+2x\right.}{150000}$$

$$= -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x)\left(3+2x\right.}{150000}$$

Mathematica [A] time = 0.15, size = 75, normalized size = 0.51

$$\frac{-5\sqrt{5x^2+2x+3}\left(22968750x^7+127406250x^6+48237500x^5+28373000x^4-78608475x^3-86464445x^2-36\right.}{26250000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-5*Sqrt[3 + 2*x + 5*x^2]*(-22275576 - 36695150*x - 86464445*x^2 - 78608475
*x^3 + 28373000*x^4 + 48237500*x^5 + 127406250*x^6 + 22968750*x^7) - 378610
26*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/26250000
```

fricas [A] time = 0.94, size = 87, normalized size = 0.59

$$-\frac{1}{5250000}\left(22968750\,x^7+127406250\,x^6+48237500\,x^5+28373000\,x^4-78608475\,x^3-86464445\,x^2-366951\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/5250000*(22968750*x^7 + 127406250*x^6 + 48237500*x^5 + 28373000*x^4 - 78
608475*x^3 - 86464445*x^2 - 36695150*x - 22275576)*sqrt(5*x^2 + 2*x + 3) +
901453/1250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2
 - 10*x - 8)
```

giac [A] time = 0.20, size = 82, normalized size = 0.56

$$-\frac{1}{5250000}\left(5\left(\left(5\left(10\left(25\left(15\left(245\,x+1359\right)x+7718\right)x+113492\right)x-3144339\right)x-17292889\right)x-7339030\right)x-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="giac
")
```

```
[Out] -1/5250000*(5*((5*(10*(25*(15*(245*x + 1359)*x + 7718)*x + 113492)*x - 3144
339)*x - 17292889)*x - 7339030)*x - 22275576)*sqrt(5*x^2 + 2*x + 3) + 90145
3/625000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

maple [A] time = 0.01, size = 117, normalized size = 0.80

$$-\frac{7\left(5x^{2}+2x+3\right)^{\frac{5}{2}}x^{3}}{40}-\frac{1163\left(5x^{2}+2x+3\right)^{\frac{5}{2}}x^{2}}{1400}+\frac{2809\left(5x^{2}+2x+3\right)^{\frac{5}{2}}x}{5250}-\frac{901453\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{625000}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] 149509/262500*(5*x^2+2*x+3)^(5/2)-7/40*(5*x^2+2*x+3)^(5/2)*x^3-1163/1400*(5
*x^2+2*x+3)^(5/2)*x^2-901453/625000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-
128779/500000*(10*x+2)*(5*x^2+2*x+3)^(1/2)+2809/5250*(5*x^2+2*x+3)^(5/2)*x-
18397/300000*(10*x+2)*(5*x^2+2*x+3)^(3/2)
```

maxima [A] time = 0.96, size = 138, normalized size = 0.94

$$-\frac{7}{40}\left(5\,x^{2}+2\,x+3\right)^{\frac{5}{2}}x^{3}-\frac{1163}{1400}\left(5\,x^{2}+2\,x+3\right)^{\frac{5}{2}}x^{2}+\frac{2809}{5250}\left(5\,x^{2}+2\,x+3\right)^{\frac{5}{2}}x+\frac{149509}{262500}\left(5\,x^{2}+2\,x+3\right)^{\frac{5}{2}}-\frac{1839}{3000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -7/40*(5*x^2 + 2*x + 3)^(5/2)*x^3 - 1163/1400*(5*x^2 + 2*x + 3)^(5/2)*x^2 +
 2809/5250*(5*x^2 + 2*x + 3)^(5/2)*x + 149509/262500*(5*x^2 + 2*x + 3)^(5/2
) - 18397/30000*(5*x^2 + 2*x + 3)^(3/2)*x - 18397/150000*(5*x^2 + 2*x + 3)^
(3/2) - 128779/50000*sqrt(5*x^2 + 2*x + 3)*x - 901453/625000*sqrt(5)*arcsin
h(1/14*sqrt(14)*(5*x + 1)) - 128779/250000*sqrt(5*x^2 + 2*x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^{2}+5\,x+2\right)\,{\left(5\,x^{2}+2\,x+3\right)}^{3/2}\,\left(-7\,x^{2}+4\,x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1),x)
```

```
[Out] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-43x\sqrt{5x^{2}+2x+3}\right)dx-\int\left(-57x^{2}\sqrt{5x^{2}+2x+3}\right)dx-\int 14x^{3}\sqrt{5x^{2}+2x+3}\,dx-\int 48x^{4}\sqrt{5x^{2}+2x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-43*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-57*x**2*sqrt(5*x**2
+ 2*x + 3), x) - Integral(14*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(48*
x**4*sqrt(5*x**2 + 2*x + 3), x) - Integral(169*x**5*sqrt(5*x**2 + 2*x + 3),
 x) - Integral(35*x**6*sqrt(5*x**2 + 2*x + 3), x) - Integral(-6*sqrt(5*x**2
 + 2*x + 3), x)
```

3.383 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx$

Optimal. Leaf size=210

$$-\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}-\frac{3(196105x+571621)\sqrt{5x^2+2x+3}}{240100}-\frac{6\sqrt{\frac{2}{11}}\left(8098902607-2434122235\right.}{1}$$

```
[Out] -1/980*(267+35*x)*(5*x^2+2*x+3)^(3/2)-34425687/4201750*arcsinh(1/14*(1+5*x)
*14^(1/2))*5^(1/2)-3/240100*(571621+196105*x)*(5*x^2+2*x+3)^(1/2)-6/184877*
arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2
))^(1/2))*(178175857354-53550689170*11^(1/2))^(1/2)+6/184877*arctanh((23+11
^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(178
175857354+53550689170*11^(1/2))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1066, 1076, 619, 215, 1032, 724, 206}

$$-\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}-\frac{3(196105x+571621)\sqrt{5x^2+2x+3}}{240100}-\frac{6\sqrt{\frac{2}{11}}\left(8098902607-2434122235\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2),x]
```

```
[Out] (-3*(571621 + 196105*x)*Sqrt[3 + 2*x + 5*x^2])/240100 - ((267 + 35*x)*(3 +
2*x + 5*x^2)^(3/2))/980 - (34425687*ArcSinh[(1 + 5*x)/Sqrt[14]])/(840350*Sq
rt[5]) - (6*Sqrt[(2*(8098902607 - 2434122235*Sqrt[11]))/11]*ArcTanh[(23 - S
qrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x +
5*x^2])])/16807 + (6*Sqrt[(2*(8098902607 + 2434122235*Sqrt[11]))/11]*ArcTan
h[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3
 + 2*x + 5*x^2])])/16807
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx = -\frac{1}{980}(267+35x)(3+2x+5x^2)^{3/2} - \frac{\int \frac{(-20358-79272x-100854x^2)\sqrt{3+2x+}}{1+4x-7x^2}}{2940}$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)(3+2x+5$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)(3+2x+5$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)(3+2x+5$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)(3+2x+5$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)(3+2x+5$$

Mathematica [A] time = 0.94, size = 202, normalized size = 0.96

$$-5\left(600\sqrt{1572625-425459\sqrt{11}}\,(61\sqrt{11}-143)\tanh^{-1}\left(\frac{-5\sqrt{11}\,x+17x-\sqrt{11}+23}{\sqrt{250-34\sqrt{11}}\,\sqrt{5x^2+2x+3}}\right)-600\,(143+61\sqrt{11})\,\sqrt{1572}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2),x]
```

```
[Out] (-757365114*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]] - 5*(77*Sqrt[3 + 2*x + 5*x^
2]*(1911108 + 744870*x + 344225*x^2 + 42875*x^3) + 600*Sqrt[1572625 - 42545
9*Sqrt[11]]*(-143 + 61*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + 17*x - 5*Sqrt[11]
*x)/(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])] - 600*(143 + 61*Sqrt[1
1])*Sqrt[1572625 + 425459*Sqrt[11]]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[1
1])*x)/(Sqrt[250 + 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])]))/92438500
```

fricas [B] time = 0.98, size = 326, normalized size = 1.55

$$\frac{3}{184877}\sqrt{11}\,\sqrt{2}\,\sqrt{2434122235\,\sqrt{11}+8098902607}\,\log\left(\frac{12\left(\sqrt{2}\,\sqrt{5\,x^2+2\,x+3}\,\sqrt{2434122235\,\sqrt{11}+809890}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="fric
as")
```

```
[Out] 3/184877*sqrt(11)*sqrt(2)*sqrt(2434122235*sqrt(11) + 8098902607)*log(12*(sq
rt(2)*sqrt(5*x^2 + 2*x + 3)*sqrt(2434122235*sqrt(11) + 8098902607)*(7690*sq
rt(11) - 24697) + 40555291*sqrt(11)*(x + 3) + 121665873*x - 202776455)/x) -
 3/184877*sqrt(11)*sqrt(2)*sqrt(2434122235*sqrt(11) + 8098902607)*log(-12*(
sqrt(2)*sqrt(5*x^2 + 2*x + 3)*sqrt(2434122235*sqrt(11) + 8098902607)*(7690*
sqrt(11) - 24697) - 40555291*sqrt(11)*(x + 3) - 121665873*x + 202776455)/x)
```

```
 - 1/739508*sqrt(11)*sqrt(-701027203680*sqrt(11) + 2332483950816)*log(-(sqr
t(5*x^2 + 2*x + 3)*(7690*sqrt(11) + 24697)*sqrt(-701027203680*sqrt(11) + 23
32483950816) + 486663492*sqrt(11)*(x + 3) - 1459990476*x + 2433317460)/x) +
 1/739508*sqrt(11)*sqrt(-701027203680*sqrt(11) + 2332483950816)*log((sqrt(5
*x^2 + 2*x + 3)*(7690*sqrt(11) + 24697)*sqrt(-701027203680*sqrt(11) + 23324
83950816) - 486663492*sqrt(11)*(x + 3) + 1459990476*x - 2433317460)/x) - 1/
240100*(42875*x^3 + 344225*x^2 + 744870*x + 1911108)*sqrt(5*x^2 + 2*x + 3)
+ 34425687/8403500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25
*x^2 - 10*x - 8)
```

giac [A] time = 0.28, size = 154, normalized size = 0.73

$$-\frac{1}{240100}\left(35\left(35\left(35\,x+281\right)x+21282\right)x+1911108\right)\sqrt{5\,x^2+2\,x+3}+\frac{34425687}{4201750}\,\sqrt{5}\,\log\left(-5\,\sqrt{5}\,x-\sqrt{5}\,+5\,\sqrt{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="giac
")
```

```
[Out] -1/240100*(35*(35*(35*x + 281)*x + 21282)*x + 1911108)*sqrt(5*x^2 + 2*x + 3
) + 34425687/4201750*sqrt(5)*log(-5*sqrt(5)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*
x + 3)) + 19.3580321168561*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924
736459000) - 0.773682164624264*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.2
5295163054000) - 19.3580321168561*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) -
1.02258038113000) + 0.773682164625454*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3
) - 2.09411235400000)
```

maple [B] time = 0.02, size = 730, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x)
```

```
[Out] -1/280*(10*x+2)*(5*x^2+2*x+3)^(3/2)-3/200*(10*x+2)*(5*x^2+2*x+3)^(1/2)-21/2
50*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-3/154*(-61+13*11^(1/2))*11^(1/2)*
(1/21*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+2
50/49-34/49*11^(1/2))^(3/2)+1/14*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-
2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*
11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1
/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/
2)*(x+1/5)))+1/7*(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+4
9*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34
/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7
-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2
))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/
7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-1
0/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))-3/154*11^(1/2)
*(61+13*11^(1/2))*(1/21*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2
/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+1/14*(34/7+10/7*11^(1/2))*(1/
20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1
/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7
*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/
7*11^(1/2))^2)^(1/2)*(x+1/5)))+1/7*(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7
-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/
2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*1
1^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2
))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*1
1^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(
```

```
1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)
)))
```

maxima [B] time = 1.31, size = 535, normalized size = 2.55

$$\frac{1}{92438500}\sqrt{11}\left(19500\sqrt{11}\sqrt{2}\left(17\sqrt{11}+125\right)^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7\left|14x-2\sqrt{11}-4\right|}+\frac{17\sqrt{7}\sqrt{2}x}{7\left|14x-2\sqrt{11}-4\right|}+\frac{\sqrt{1}}{7\left|14x\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="maxi
ma")
```

```
[Out] 1/92438500*sqrt(11)*(19500*sqrt(11)*sqrt(2)*(17*sqrt(11) + 125)^(3/2)*arcsi
nh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)
*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14
*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) - 3
00125*sqrt(11)*(5*x^2 + 2*x + 3)^(3/2)*x - 3344250*sqrt(11)*(-34/49*sqrt(11
) + 250/49)^(3/2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(
11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)
*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x
 + 2*sqrt(11) - 4)) + 91500*sqrt(2)*(17*sqrt(11) + 125)^(3/2)*arcsinh(5/7*s
qrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)
*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*s
qrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) + 15692250*
(-34/49*sqrt(11) + 250/49)^(3/2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs
(14*x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4)
 + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*s
qrt(2)/abs(14*x + 2*sqrt(11) - 4)) - 2289525*sqrt(11)*(5*x^2 + 2*x + 3)^(3/
2) - 20591025*sqrt(11)*sqrt(5*x^2 + 2*x + 3)*x - 68851374*sqrt(11)*sqrt(5)*
arcsinh(5/14*sqrt(7)*sqrt(2)*x + 1/14*sqrt(7)*sqrt(2)) - 60020205*sqrt(11)*
sqrt(5*x^2 + 2*x + 3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2+5x+2\right)\left(5x^2+2x+3\right)^{3/2}}{-7x^2+4x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1),x)
```

```
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{6\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx-\int \frac{19x\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx-\int \frac{23x^2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx-\int \frac{27x^3\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx-\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1),x)
```

```
[Out] -Integral(6*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(19*x*s
qrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(23*x**2*sqrt(5*x**2
 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(27*x**3*sqrt(5*x**2 + 2*x + 3
)/(7*x**2 - 4*x - 1), x) - Integral(5*x**4*sqrt(5*x**2 + 2*x + 3)/(7*x**2 -
 4*x - 1), x)
```

3.384 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx$

Optimal. Leaf size=222

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}+\frac{(3395x+5826)\sqrt{5x^2+2x+3}}{3773}-\frac{\sqrt{\frac{1}{22}\left(52175400311-13155376531\sqrt{11}\right)}\tanh^{-1}\left(\right.}{26411}$$

```
[Out] 3/154*(3+61*x)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)+16691/12005*arcsinh(1/14*
(1+5*x)*14^(1/2))*5^(1/2)+1/3773*(5826+3395*x)*(5*x^2+2*x+3)^(1/2)-1/581042
*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/
2))^(1/2))*(1147858806842-289418283682*11^(1/2))^(1/2)-1/581042*arctanh((23
+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(
1147858806842+289418283682*11^(1/2))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1054, 1066, 1076, 619, 215, 1032, 724, 206}

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}+\frac{(3395x+5826)\sqrt{5x^2+2x+3}}{3773}-\frac{\sqrt{\frac{1}{22}\left(52175400311-13155376531\sqrt{11}\right)}\tanh^{-1}\left(\right.}{26411}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] ((5826 + 3395*x)*Sqrt[3 + 2*x + 5*x^2])/3773 + (3*(3 + 61*x)*(3 + 2*x + 5*x
^2)^(3/2))/(154*(1 + 4*x - 7*x^2)) + (16691*ArcSinh[(1 + 5*x)/Sqrt[14]])/(2
401*Sqrt[5]) - (Sqrt[(52175400311 - 13155376531*Sqrt[11])/22]*ArcTanh[(23 -
 Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x
+ 5*x^2])])/26411 - (Sqrt[(52175400311 + 13155376531*Sqrt[11])/22]*ArcTanh[
(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 +
 2*x + 5*x^2])])/26411
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1054

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((A*b*c - 2
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} - \frac{1}{308}\int \frac{\sqrt{3+2x+5x^2}\,(-912+724x+3\ldots}{1+4x-7x^2}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} + \frac{\int \frac{70020\ldots}{(1\ldots}}{\ldots}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} - \frac{\int \frac{244\ldots}{(1+4\ldots}}{\ldots}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} + \frac{16691\ldots}{\ldots}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} + \frac{16691\ldots}{2\ldots}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} + \frac{16691\ldots}{2\ldots}$$

Mathematica [A] time = 1.83, size = 354, normalized size = 1.59

$$\frac{5\sqrt{\frac{22}{125-17\sqrt{11}}}\left(743879\sqrt{11}-1701489\right)\log\left(49x^2+14\left(\sqrt{11}-2\right)x-4\sqrt{11}+15\right)-10\sqrt{\frac{22}{125+17\sqrt{11}}}\left(1701489+7\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] ((770*Sqrt[3 + 2*x + 5*x^2]*(-12975 - 81181*x + 34265*x^2 + 2695*x^3))/(-1
- 4*x + 7*x^2) + 8078444*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]] + 10*Sqrt[22/(
125 - 17*Sqrt[11])]*(-1701489 + 743879*Sqrt[11])*ArcTanh[(Sqrt[250 - 34*Sqr
t[11]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[11] + (-17 + 5*Sqrt[11])*x)] + 10
*Sqrt[22/(125 + 17*Sqrt[11])]*(1701489 + 743879*Sqrt[11])*Log[2 + Sqrt[11]
- 7*x] - 5*Sqrt[22/(125 - 17*Sqrt[11])]*(-1701489 + 743879*Sqrt[11])*Log[(-
2 + Sqrt[11] + 7*x)^2] + 5*Sqrt[22/(125 - 17*Sqrt[11])]*(-1701489 + 743879*
Sqrt[11])*Log[15 - 4*Sqrt[11] + 14*(-2 + Sqrt[11])*x + 49*x^2] - 10*Sqrt[22
/(125 + 17*Sqrt[11])]*(1701489 + 743879*Sqrt[11])*Log[11 + 23*Sqrt[11] + (5
5 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2]])/5810
420
```

fricas [B] time = 0.90, size = 378, normalized size = 1.70

$$\frac{5\sqrt{11}\left(7x^2-4x-1\right)\sqrt{26310753062\sqrt{11}+104350800622}\,\log\left(\frac{\sqrt{5x^2+2x+3}\sqrt{26310753062\sqrt{11}+104350800622}\left(16206\sqrt{11}\ldots\right.}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="fr
icas")
```

```
[Out] 1/5810420*(5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(26310753062*sqrt(11) + 1043508
00622)*log((sqrt(5*x^2 + 2*x + 3)*sqrt(26310753062*sqrt(11) + 104350800622)
*(16206*sqrt(11) - 68441) + 1795191685*sqrt(11)*(x + 3) + 5385575055*x - 89
75958425)/x) - 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(26310753062*sqrt(11) + 104
350800622)*log(-(sqrt(5*x^2 + 2*x + 3)*sqrt(26310753062*sqrt(11) + 10435080
0622)*(16206*sqrt(11) - 68441) - 1795191685*sqrt(11)*(x + 3) - 5385575055*x
 + 8975958425)/x) - 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(-26310753062*sqrt(11)
 + 104350800622)*log(-(sqrt(5*x^2 + 2*x + 3)*(16206*sqrt(11) + 68441)*sqrt(
-26310753062*sqrt(11) + 104350800622) + 1795191685*sqrt(11)*(x + 3) - 53855
75055*x + 8975958425)/x) + 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(-26310753062*s
qrt(11) + 104350800622)*log((sqrt(5*x^2 + 2*x + 3)*(16206*sqrt(11) + 68441)
*sqrt(-26310753062*sqrt(11) + 104350800622) - 1795191685*sqrt(11)*(x + 3) +
 5385575055*x - 8975958425)/x) + 4039222*sqrt(5)*(7*x^2 - 4*x - 1)*log(-sqr
t(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) + 770*(2695*x^3 +
 34265*x^2 - 81181*x - 12975)*sqrt(5*x^2 + 2*x + 3))/(7*x^2 - 4*x - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{63274455776,[8]%%%}+%%%{%%{[144627327488,0]:[1,0,-5]%%},[7]%%%}+%%%{
-852268179840,[6]%%%}+%%%{%%{[-1735527929856,0]:[1,0,-5]%%},[5]%%%}+%%%{617
5070357568,[4]%%%}+%%%{%%{[4607413432832,0]:[1,0,-5]%%},[3]%%%}+%%%{-133522
01484160,[2]%%%}+%%%{%%{[-3429733766144,0]:[1,0,-5]%%},[1]%%%}+%%%{88958719
55936,[0]%%%} / %%%{245,[8]%%%}+%%%{%%{poly1[560,0]:[1,0,-5]%%},[7]%%%}+%%%
{-3300,[6]%%%}+%%%{%%{poly1[-6720,0]:[1,0,-5]%%},[5]%%%}+%%%{23910,[4]%%%}+
%%%{%%{poly1[17840,0]:[1,0,-5]%%},[3]%%%}+%%%{-51700,[2]%%%}+%%%{%%{poly1[-
13280,0]:[1,0,-5]%%},[1]%%%}+%%%{34445,[0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.02, size = 1828, normalized size = 8.23

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x)
```

```
[Out] 161/484*11^(1/2)*(1/21*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/
7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/14*(34/7-10/7*11^(1/2))*(1/2
0*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/
2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*
11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7
*11^(1/2))^2)^(1/2)*(x+1/5)))+1/7*(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+
1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2
))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11
^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2)
)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11
^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1
/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))
))+(183/44-39/44*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/
2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250
/49-34/49*11^(1/2))^(5/2)+3/98*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))
*(1/3*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+2
50/49-34/49*11^(1/2))^(3/2)+1/2*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2
```

```
/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*1
1^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/
2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2
)*(x+1/5)))+(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34
/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10
/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7
*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1
/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^
(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*1
1^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))+20/49/(250/49-34/49
*11^(1/2))*(1/40*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x
-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+3/80*(5000/49-680/49*11^(1/
2)-(34/7-10/7*11^(1/2))^2)*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-1
0/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000
/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49
-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))+(183/44+39/4
4*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1
/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1
/2))^(5/2)+3/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/3*(5*(x-2/7
-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^
(1/2))^(3/2)+1/2*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2)
)^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+
1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1
/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(25
0/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2
))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^
(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(
1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/
2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34
*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7
-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))))+20/49/(250/49+34/49*11^(1/2))*(1/4
0*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/
2))+250/49+34/49*11^(1/2))^(3/2)+3/80*(5000/49+680/49*11^(1/2)-(34/7+10/7*1
1^(1/2))^2)*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(
x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(
1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)
-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))-161/484*11^(1/2)*(1/21*(5*(
x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/4
9*11^(1/2))^(3/2)+1/14*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11
^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^
(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsin
h(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)
))+1/7*(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10
/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11
^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(
1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*a
rctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)
))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/
2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}\left(x^2+5\,x+2\right)}{\left(7\,x^2-4\,x-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="ma
xima")
```

```
[Out] integrate((5*x^2 + 2*x + 3)^(3/2)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x^2+5x+2)\,(5x^2+2x+3)^{3/2}}{(-7x^2+4x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^2,x)
```

```
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2+5x+2)\,(5x^2+2x+3)^{\frac{3}{2}}}{(7x^2-4x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1)**2,x)
```

```
[Out] Integral((x**2 + 5*x + 2)*(5*x**2 + 2*x + 3)**(3/2)/(7*x**2 - 4*x - 1)**2,
x)
```

3.385 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx$

Optimal. Leaf size=234

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}-\frac{(9495-37088x)\sqrt{5x^2+2x+3}}{23716\left(-7x^2+4x+1\right)}-\frac{\sqrt{\frac{62294197250171-2085440742055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{(17-\ldots}{\sqrt{2(125-\ldots}}\right.}{332024}$$

```
[Out] 3/308*(3+61*x)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2-5/343*arcsinh(1/14*(1+5
*x)*14^(1/2))*5^(1/2)-1/23716*(9495-37088*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*
x+1)-1/927675056*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2
)/(250-34*11^(1/2))^(1/2))*(17404998711697774-5826721433301670*11^(1/2))^(
1/2)+1/927675056*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2
)/(250+34*11^(1/2))^(1/2))*(17404998711697774+5826721433301670*11^(1/2))^(
1/2)
```

Rubi [A] time = 0.32, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1054, 1076, 619, 215, 1032, 724, 206}

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}-\frac{(9495-37088x)\sqrt{5x^2+2x+3}}{23716\left(-7x^2+4x+1\right)}-\frac{\sqrt{\frac{62294197250171-2085440742055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{(17-\ldots}{\sqrt{2(125-\ldots}}\right.}{332024}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] -((9495 - 37088*x)*Sqrt[3 + 2*x + 5*x^2])/(23716*(1 + 4*x - 7*x^2)) + (3*(3
 + 61*x)*(3 + 2*x + 5*x^2)^(3/2))/(308*(1 + 4*x - 7*x^2)^2) - (5*Sqrt[5]*Ar
cSinh[(1 + 5*x)/Sqrt[14]])/343 - (Sqrt[(62294197250171 - 2085440742055*Sqrt
[11])/2794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17
*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/332024 + (Sqrt[(62294197250171 + 20854
40742055*Sqrt[11])/2794]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqr
t[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/332024
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1054

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((A*b*c - 2
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}dx = \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{308(1+4x-7x^2)^2} - \frac{1}{616}\int \frac{\sqrt{3+2x+5x^2}(-2976-652x+}{(1+4x-7x^2)^2}$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716(1+4x-7x^2)} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{308(1+4x-7x^2)^2} + \frac{\int 10}{}$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716(1+4x-7x^2)} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{308(1+4x-7x^2)^2} - \frac{\int \frac{}{(1}}{}$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716(1+4x-7x^2)} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{308(1+4x-7x^2)^2} - \frac{1}{686}$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716(1+4x-7x^2)} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{308(1+4x-7x^2)^2} - \frac{5}{343}$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716(1+4x-7x^2)} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{308(1+4x-7x^2)^2} - \frac{5}{343}$$

Mathematica [A] time = 2.24, size = 376, normalized size = 1.61

$$\frac{\frac{88\sqrt{5x^2+2x+3}(138372-189161x)}{7x^2-4x-1} + \frac{11616(5028x+655)\sqrt{5x^2+2x+3}}{(-7x^2+4x+1)^2} - \sqrt{\frac{22}{125-17\sqrt{11}}}\left(674221\sqrt{11}-7706073\right)\log\left(49x^2+14\left(\sqrt{\right.\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] ((11616*(655 + 5028*x)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2 + (88*(13
8372 - 189161*x)*Sqrt[3 + 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) - 212960*Sqrt[5]
*ArcSinh[(1 + 5*x)/Sqrt[14]] - 2*Sqrt[22/(125 - 17*Sqrt[11])]*(-7706073 + 6
74221*Sqrt[11])*ArcTanh[(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])/(-2
3 + Sqrt[11] + (-17 + 5*Sqrt[11])*x)] - 2*Sqrt[22/(125 + 17*Sqrt[11])]*(770
6073 + 674221*Sqrt[11])*Log[2 + Sqrt[11] - 7*x] + Sqrt[22/(125 - 17*Sqrt[11
])]*(-7706073 + 674221*Sqrt[11])*Log[(-2 + Sqrt[11] + 7*x)^2] - Sqrt[22/(12
5 - 17*Sqrt[11])]*(-7706073 + 674221*Sqrt[11])*Log[15 - 4*Sqrt[11] + 14*(-2
 + Sqrt[11])*x + 49*x^2] + 2*Sqrt[22/(125 + 17*Sqrt[11])]*(7706073 + 674221
*Sqrt[11])*Log[11 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sq
rt[11]]*Sqrt[3 + 2*x + 5*x^2]])/14609056
```

fricas [B] time = 0.99, size = 447, normalized size = 1.91

$$-\frac{\sqrt{2794}\left(49x^4-56x^3+2x^2+8x+1\right)\sqrt{2085440742055\sqrt{11}+62294197250171}\log\left(\frac{\sqrt{2794}\sqrt{5x^2+2x+3}\sqrt{2085440}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="fr
icas")
```

```
[Out] -1/1855350112*(sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(20854407
42055*sqrt(11) + 62294197250171)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt
(2085440742055*sqrt(11) + 62294197250171)*(11840590*sqrt(11) - 83479737) +
5426671202560069*sqrt(11)*(x + 3) + 16280013607680207*x - 27133356012800345
)/x) - sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(2085440742055*sq
rt(11) + 62294197250171)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(208544
0742055*sqrt(11) + 62294197250171)*(11840590*sqrt(11) - 83479737) - 5426671
202560069*sqrt(11)*(x + 3) - 16280013607680207*x + 27133356012800345)/x) +
sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-2085440742055*sqrt(11)
 + 62294197250171)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(11840590*sqrt(11
) + 83479737)*sqrt(-2085440742055*sqrt(11) + 62294197250171) + 542667120256
0069*sqrt(11)*(x + 3) - 16280013607680207*x + 27133356012800345)/x) - sqrt(
2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-2085440742055*sqrt(11) + 62
294197250171)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(11840590*sqrt(11) + 83
479737)*sqrt(-2085440742055*sqrt(11) + 62294197250171) - 5426671202560069*s
qrt(11)*(x + 3) + 16280013607680207*x - 27133356012800345)/x) - 13522960*sq
rt(5)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)
*(5*x + 1) - 25*x^2 - 10*x - 8) + 78232*(189161*x^3 - 246464*x^2 - 42767*x
+ 7416)*sqrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{-1771684761728,[12]%%%}+%%%{%%{[-6074347754496,0]:[1,0,-5]%%},[11]%%
%}+%%%{18439984254720,[10]%%%}+%%%{%%{[120412580657152,0]:[1,0,-5]%%},[9]%%
%}+%%%{-108578966111616,[8]%%%}+%%%{%%{[-915119084156928,0]:[1,0,-5]%%},[7]
%%%}+%%%{1093279290575360,[6]%%%}+%%%{%%{[2784778529734656,0]:[1,0,-5]%%},[
5]%%%}+%%%{-4014694487954304,[4]%%%}+%%%{%%{[-3629195511796736,0]:[1,0,-5]%
%},[3]%%%}+%%%{5826260235237120,[2]%%%}+%%%{%%{[1708007415539712,0]:[1,0,-5
]%%},[1]%%%}+%%%{-2953429489370752,[0]%%%} / %%%{%%{[1715,0]:[1,0,-5]%%},[1
2]%%%}+%%%{29400,[11]%%%}+%%%{%%{[-17850,0]:[1,0,-5]%%},[10]%%%}+%%%{-58280
0,[9]%%%}+%%%{%%{[105105,0]:[1,0,-5]%%},[8]%%%}+%%%{4429200,[7]%%%}+%%%{%%{
[-1058300,0]:[1,0,-5]%%},[6]%%%}+%%%{-13478400,[5]%%%}+%%%{%%{[3886245,0]:[
1,0,-5]%%},[4]%%%}+%%%{17565400,[3]%%%}+%%%{%%{[-5639850,0]:[1,0,-5]%%},[2]
%%%}+%%%{-8266800,[1]%%%}+%%%{%%{[2858935,0]:[1,0,-5]%%},[0]%%%} Error: Bad
 Argument Value
```

maple [B] time = 0.02, size = 3828, normalized size = 16.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x)
```

```
[Out] 3535/21296*11^(1/2)*(1/21*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x
-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/14*(34/7-10/7*11^(1/2))*(
1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^
(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10
/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-1
0/7*11^(1/2))^2)^(1/2)*(x+1/5)))+1/7*(250/49-34/49*11^(1/2))*(1/7*(245*(x-2
/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(
1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49
```

11^(1/2)-1/20(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))-21/968*(-61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))^2*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(5/2)+1/1372*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(-1/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(5/2)+3/2*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/3*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/2*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))+20/(250/49-34/49*11^(1/2))*(1/40*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+3/80*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))+15/686/(250/49-34/49*11^(1/2))*(1/3*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/2*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))))-(-3535/1936-273/1936*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(5/2)+3/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/3*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+1/2*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))))+20/49/(250/49+34/49*11^(1/2))*(1/40*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+3/80*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*

11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(
1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))-(-3535/1936+273/1936*11
^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*1
1^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))
^(5/2)+3/98*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/3*(5*(x-2/7+1/7
*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2
))^(3/2)+1/2*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+
(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/20
0*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/
(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49
-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(
x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2
)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)
(x+1/5))-7(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(5
00/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^
(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7
*11^(1/2))+250-34*11^(1/2))^(1/2))))+20/49/(250/49-34/49*11^(1/2))*(1/40*(1
0*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+
250/49-34/49*11^(1/2))^(3/2)+3/80*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1
/2))^2)*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/
7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)
-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/2
0*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))-3535/21296*11^(1/2)*(1/21*(5*(x
-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49
11^(1/2))^(3/2)+1/14(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^
(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(
1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh
(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))
)+1/7*(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/
7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^
(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1
/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*ar
ctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))
)/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2
))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))))-21/968*(61+13*11^(1/2))*1
1^(1/2)*(-1/686/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))^2*(5*(x-2/7-1/
7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/
2))^(5/2)+1/1372*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(-1/(250/49+3
4/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11
^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(5/2)+3/2*(34/7+10/7*11
^(1/2))/(250/49+34/49*11^(1/2))*(1/3*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*1
1^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+1/2*(34/7+10/7*1
1^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2
/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2
)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/
20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49+34/49*11^(1/2))*(1/7*(24
5*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+3
4*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49
+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49
*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/
7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-
1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2
))^(1/2))))+20/(250/49+34/49*11^(1/2))*(1/40*(10*x+2)*(5*(x-2/7-1/7*11^(1/2
))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)
+3/80*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*(1/20*(10*x+2)*(5*(x
-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49
11^(1/2))^(1/2)+1/200(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(
1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1
/2)*(x+1/5)))))+15/686/(250/49+34/49*11^(1/2))*(1/3*(5*(x-2/7-1/7*11^(1/2))

```
^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+1
/2*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7
*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49
+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34
/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49+34/49*11^
(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*
11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(
5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-
7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/4
9*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/
2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))
+250+34*11^(1/2))^(1/2)))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}\left(x^2+5\,x+2\right)}{\left(7\,x^2-4\,x-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="ma
xima")
```

```
[Out] -integrate((5*x^2 + 2*x + 3)^(3/2)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2+5\,x+2\right)\,\left(5\,x^2+2\,x+3\right)^{3/2}}{\left(-7\,x^2+4\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^3,x)
```

```
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{6\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx-\int \frac{19x\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1)**3,x)
```

```
[Out] -Integral(6*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x*
*3 - 27*x**2 - 12*x - 1), x) - Integral(19*x*sqrt(5*x**2 + 2*x + 3)/(343*x*
*6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral(23
*x**2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 2
7*x**2 - 12*x - 1), x) - Integral(27*x**3*sqrt(5*x**2 + 2*x + 3)/(343*x**6
- 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral(5*x**
4*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x*
*2 - 12*x - 1), x)
```

3.386 $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

Optimal. Leaf size=185

$$\frac{40722851\sqrt{5x^2+2x+3}\,x^2}{750000}+\frac{5793077\sqrt{5x^2+2x+3}\,x}{75000}-\frac{16515809\sqrt{5x^2+2x+3}}{156250}-\frac{343}{40}\sqrt{5x^2+2x+3}\,x^7-\frac{114}{40}$$

```
[Out] -77513689/3125000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-16515809/156250*(5
*x^2+2*x+3)^(1/2)+5793077/75000*x*(5*x^2+2*x+3)^(1/2)+40722851/750000*x^2*(
5*x^2+2*x+3)^(1/2)-5160533/50000*x^3*(5*x^2+2*x+3)^(1/2)-47807/3750*x^4*(5*
x^2+2*x+3)^(1/2)+26159/300*x^5*(5*x^2+2*x+3)^(1/2)-1141/40*x^6*(5*x^2+2*x+3
)^(1/2)-343/40*x^7*(5*x^2+2*x+3)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1661, 640, 619, 215}

$$-\frac{343}{40}\sqrt{5x^2+2x+3}\,x^7-\frac{1141}{40}\sqrt{5x^2+2x+3}\,x^6+\frac{26159}{300}\sqrt{5x^2+2x+3}\,x^5-\frac{47807\sqrt{5x^2+2x+3}\,x^4}{3750}-\frac{5160533\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-16515809*Sqrt[3 + 2*x + 5*x^2])/156250 + (5793077*x*Sqrt[3 + 2*x + 5*x^2]
)/75000 + (40722851*x^2*Sqrt[3 + 2*x + 5*x^2])/750000 - (5160533*x^3*Sqrt[3
 + 2*x + 5*x^2])/50000 - (47807*x^4*Sqrt[3 + 2*x + 5*x^2])/3750 + (26159*x^
5*Sqrt[3 + 2*x + 5*x^2])/300 - (1141*x^6*Sqrt[3 + 2*x + 5*x^2])/40 - (343*x
^7*Sqrt[3 + 2*x + 5*x^2])/40 - (77513689*ArcSinh[(1 + 5*x)/Sqrt[14]])/(6250
00*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{343}{40}x^7\sqrt{3+2x+5x^2} + \frac{1}{40}\int \frac{80+1160x+4600x^2-2440x^3-34840x^{4}\cdots}{\sqrt{3+2x+5\cdots}}$$

$$= -\frac{1141}{40}x^6\sqrt{3+2x+5x^2} - \frac{343}{40}x^7\sqrt{3+2x+5x^2} + \frac{\int \frac{2800+40600x+161000x^2-\cdots}{\cdots}}{\cdots}$$

$$= \frac{26159}{300}x^5\sqrt{3+2x+5x^2} - \frac{1141}{40}x^6\sqrt{3+2x+5x^2} - \frac{343}{40}x^7\sqrt{3+2x+5x^2}\cdots$$

$$= -\frac{47807x^4\sqrt{3+2x+5x^2}}{3750} + \frac{26159}{300}x^5\sqrt{3+2x+5x^2} - \frac{1141}{40}x^6\sqrt{3+2x+5\cdots}$$

$$= -\frac{5160533x^3\sqrt{3+2x+5x^2}}{50000} - \frac{47807x^4\sqrt{3+2x+5x^2}}{3750} + \frac{26159}{300}x^5\sqrt{3+2\cdots}$$

$$= \frac{40722851x^2\sqrt{3+2x+5x^2}}{750000} - \frac{5160533x^3\sqrt{3+2x+5x^2}}{50000} - \frac{47807x^4\sqrt{3+\cdots}}{3750}$$

$$= \frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{40722851x^2\sqrt{3+2x+5x^2}}{750000} - \frac{5160533x^3\sqrt{3\cdots}}{5000\cdots}$$

$$= -\frac{16515809\sqrt{3+2x+5x^2}}{156250} + \frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{40722851x^2\sqrt{3\cdots}}{7500\cdots}$$

$$= -\frac{16515809\sqrt{3+2x+5x^2}}{156250} + \frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{40722851x^2\sqrt{3\cdots}}{7500\cdots}$$

$$= -\frac{16515809\sqrt{3+2x+5x^2}}{156250} + \frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{40722851x^2\sqrt{3\cdots}}{7500\cdots}$$

Mathematica [A] time = 0.25, size = 75, normalized size = 0.41

$$\frac{-5\sqrt{5x^2+2x+3}\left(32156250x^7+106968750x^6-326987500x^5+47807000x^4+387039975x^3-203614255x^2-\cdots\right.}{18750000}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-5*Sqrt[3 + 2*x + 5*x^2]*(396379416 - 289653850*x - 203614255*x^2 + 387039
975*x^3 + 47807000*x^4 - 326987500*x^5 + 106968750*x^6 + 32156250*x^7) - 46
5082134*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/18750000
```

fricas [A] time = 0.81, size = 87, normalized size = 0.47

$$-\frac{1}{3750000}\left(32156250\,x^7+106968750\,x^6-326987500\,x^5+47807000\,x^4+387039975\,x^3-203614255\,x^2-289\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/3750000*(32156250*x^7 + 106968750*x^6 - 326987500*x^5 + 47807000*x^4 + 3
87039975*x^3 - 203614255*x^2 - 289653850*x + 396379416)*sqrt(5*x^2 + 2*x +
```

```
3) + 77513689/6250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) -
 25*x^2 - 10*x - 8)
```

giac [A] time = 0.25, size = 82, normalized size = 0.44

$$-\frac{1}{3750000}\left(5\left(\left(5\left(10\left(175\left(15\left(49\,x+163\right)x-7474\right)x+191228\right)x+15481599\right)x-40722851\right)x-57930770\right)x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/3750000*(5*((5*(10*(175*(15*(49*x + 163)*x - 7474)*x + 191228)*x + 15481
599)*x - 40722851)*x - 57930770)*x + 396379416)*sqrt(5*x^2 + 2*x + 3) + 775
13689/3125000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

maple [A] time = 0.03, size = 147, normalized size = 0.79

$$-\frac{343\sqrt{5x^2+2x+3}\,x^7}{40}-\frac{1141\sqrt{5x^2+2x+3}\,x^6}{40}+\frac{26159\sqrt{5x^2+2x+3}\,x^5}{300}-\frac{47807\sqrt{5x^2+2x+3}\,x^4}{3750}-\frac{5160533}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -16515809/156250*(5*x^2+2*x+3)^(1/2)-77513689/3125000*5^(1/2)*arcsinh(5/14*
14^(1/2)*(x+1/5))-343/40*x^7*(5*x^2+2*x+3)^(1/2)-1141/40*x^6*(5*x^2+2*x+3)^
(1/2)+26159/300*x^5*(5*x^2+2*x+3)^(1/2)-47807/3750*x^4*(5*x^2+2*x+3)^(1/2)-
5160533/50000*x^3*(5*x^2+2*x+3)^(1/2)+40722851/750000*x^2*(5*x^2+2*x+3)^(1/
2)+5793077/75000*x*(5*x^2+2*x+3)^(1/2)
```

maxima [A] time = 0.99, size = 148, normalized size = 0.80

$$-\frac{343}{40}\sqrt{5\,x^2+2\,x+3}\,x^7-\frac{1141}{40}\sqrt{5\,x^2+2\,x+3}\,x^6+\frac{26159}{300}\sqrt{5\,x^2+2\,x+3}\,x^5-\frac{47807}{3750}\sqrt{5\,x^2+2\,x+3}\,x^4-\frac{5160}{500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -343/40*sqrt(5*x^2 + 2*x + 3)*x^7 - 1141/40*sqrt(5*x^2 + 2*x + 3)*x^6 + 261
59/300*sqrt(5*x^2 + 2*x + 3)*x^5 - 47807/3750*sqrt(5*x^2 + 2*x + 3)*x^4 - 5
160533/50000*sqrt(5*x^2 + 2*x + 3)*x^3 + 40722851/750000*sqrt(5*x^2 + 2*x +
 3)*x^2 + 5793077/75000*sqrt(5*x^2 + 2*x + 3)*x - 77513689/3125000*sqrt(5)*
arcsinh(1/14*sqrt(14)*(5*x + 1)) - 16515809/156250*sqrt(5*x^2 + 2*x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5\,x+2\right)\,{\left(-7\,x^2+4\,x+1\right)}^3}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(1/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{29x}{\sqrt{5x^2+2x+3}}\right)dx-\int\left(-\frac{115x^2}{\sqrt{5x^2+2x+3}}\right)dx-\int\frac{61x^3}{\sqrt{5x^2+2x+3}}\,dx-\int\frac{871x^4}{\sqrt{5x^2+2x+3}}\,dx-\int\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-29*x/sqrt(5*x**2 + 2*x + 3), x) - Integral(-115*x**2/sqrt(5*x**2
 + 2*x + 3), x) - Integral(61*x**3/sqrt(5*x**2 + 2*x + 3), x) - Integral(87
1*x**4/sqrt(5*x**2 + 2*x + 3), x) - Integral(-127*x**5/sqrt(5*x**2 + 2*x +
3), x) - Integral(-2065*x**6/sqrt(5*x**2 + 2*x + 3), x) - Integral(1127*x**
7/sqrt(5*x**2 + 2*x + 3), x) - Integral(343*x**8/sqrt(5*x**2 + 2*x + 3), x)
 - Integral(-2/sqrt(5*x**2 + 2*x + 3), x)
```

3.387 $\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

Optimal. Leaf size=143

$$-\frac{207427\sqrt{5x^2+2x+3}\,x^2}{37500}+\frac{36073\sqrt{5x^2+2x+3}\,x}{1875}-\frac{22053\sqrt{5x^2+2x+3}}{31250}+\frac{49}{30}\sqrt{5x^2+2x+3}\,x^5+\frac{5131}{750}\sqrt{5x^2$$

```
[Out] -1719097/156250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-22053/31250*(5*x^2+2
*x+3)^(1/2)+36073/1875*x*(5*x^2+2*x+3)^(1/2)-207427/37500*x^2*(5*x^2+2*x+3)
^(1/2)-33259/2500*x^3*(5*x^2+2*x+3)^(1/2)+5131/750*x^4*(5*x^2+2*x+3)^(1/2)+
49/30*x^5*(5*x^2+2*x+3)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1661, 640, 619, 215}

$$\frac{49}{30}\sqrt{5x^2+2x+3}\,x^5+\frac{5131}{750}\sqrt{5x^2+2x+3}\,x^4-\frac{33259\sqrt{5x^2+2x+3}\,x^3}{2500}-\frac{207427\sqrt{5x^2+2x+3}\,x^2}{37500}+\frac{36073\sqrt{5x^2}}{18}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-22053*Sqrt[3 + 2*x + 5*x^2])/31250 + (36073*x*Sqrt[3 + 2*x + 5*x^2])/1875
 - (207427*x^2*Sqrt[3 + 2*x + 5*x^2])/37500 - (33259*x^3*Sqrt[3 + 2*x + 5*x
^2])/2500 + (5131*x^4*Sqrt[3 + 2*x + 5*x^2])/750 + (49*x^5*Sqrt[3 + 2*x + 5
*x^2])/30 - (1719097*ArcSinh[(1 + 5*x)/Sqrt[14]])/(31250*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)}{\sqrt{3+2x+5x^2}}\,dx = \frac{49}{30}x^5\sqrt{3+2x+5x^2} + \frac{1}{30}\int \frac{60+630x+1350x^2-2820x^3-6135x^4+51}{\sqrt{3+2x+5x^2}}$$

$$= \frac{5131}{750}x^4\sqrt{3+2x+5x^2} + \frac{49}{30}x^5\sqrt{3+2x+5x^2} + \frac{1}{750}\int \frac{1500+15750x+}{}$$

$$= -\frac{33259x^3\sqrt{3+2x+5x^2}}{2500} + \frac{5131}{750}x^4\sqrt{3+2x+5x^2} + \frac{49}{30}x^5\sqrt{3+2x+5x^2}$$

$$= -\frac{207427x^2\sqrt{3+2x+5x^2}}{37500} - \frac{33259x^3\sqrt{3+2x+5x^2}}{2500} + \frac{5131}{750}x^4\sqrt{3+2x+}$$

$$= \frac{36073x\sqrt{3+2x+5x^2}}{1875} - \frac{207427x^2\sqrt{3+2x+5x^2}}{37500} - \frac{33259x^3\sqrt{3+2x+}}{2500}$$

$$= -\frac{22053\sqrt{3+2x+5x^2}}{31250} + \frac{36073x\sqrt{3+2x+5x^2}}{1875} - \frac{207427x^2\sqrt{3+2x+5}}{37500}$$

$$= -\frac{22053\sqrt{3+2x+5x^2}}{31250} + \frac{36073x\sqrt{3+2x+5x^2}}{1875} - \frac{207427x^2\sqrt{3+2x+5}}{37500}$$

$$= -\frac{22053\sqrt{3+2x+5x^2}}{31250} + \frac{36073x\sqrt{3+2x+5x^2}}{1875} - \frac{207427x^2\sqrt{3+2x+5}}{37500}$$

Mathematica [A] time = 0.14, size = 65, normalized size = 0.45

$$\frac{5\sqrt{5x^2+2x+3}\left(306250x^5+1282750x^4-2494425x^3-1037135x^2+3607300x-132318\right)-10314582\sqrt{5}\,\sinh}{937500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (5*Sqrt[3 + 2*x + 5*x^2]*(-132318 + 3607300*x - 1037135*x^2 - 2494425*x^3 +
 1282750*x^4 + 306250*x^5) - 10314582*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/
937500
```

fricas [A] time = 0.87, size = 77, normalized size = 0.54

$$\frac{1}{187500}\left(306250\,x^5+1282750\,x^4-2494425\,x^3-1037135\,x^2+3607300\,x-132318\right)\sqrt{5\,x^2+2\,x+3}+\frac{1719097}{312500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/187500*(306250*x^5 + 1282750*x^4 - 2494425*x^3 - 1037135*x^2 + 3607300*x
- 132318)*sqrt(5*x^2 + 2*x + 3) + 1719097/312500*sqrt(5)*log(sqrt(5)*sqrt(5
*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

giac [A] time = 0.39, size = 72, normalized size = 0.50

$$\frac{1}{187500}\left(5\left(\left(5\left(70\left(175\,x+733\right)x-99777\right)x-207427\right)x+721460\right)x-132318\right)\sqrt{5\,x^2+2\,x+3}+\frac{1719097}{156250}\sqrt{5}\,\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/187500*(5*((5*(70*(175*x + 733)*x - 99777)*x - 207427)*x + 721460)*x - 13
2318)*sqrt(5*x^2 + 2*x + 3) + 1719097/156250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

maple [A] time = 0.01, size = 113, normalized size = 0.79

$$\frac{49\sqrt{5x^2+2x+3}\,x^5}{30}+\frac{5131\sqrt{5x^2+2x+3}\,x^4}{750}-\frac{33259\sqrt{5x^2+2x+3}\,x^3}{2500}-\frac{207427\sqrt{5x^2+2x+3}\,x^2}{37500}+\frac{36073\sqrt{5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -22053/31250*(5*x^2+2*x+3)^(1/2)-1719097/156250*5^(1/2)*arcsinh(5/14*14^(1/
2)*(x+1/5))+49/30*(5*x^2+2*x+3)^(1/2)*x^5+5131/750*(5*x^2+2*x+3)^(1/2)*x^4-
33259/2500*(5*x^2+2*x+3)^(1/2)*x^3-207427/37500*(5*x^2+2*x+3)^(1/2)*x^2+360
73/1875*(5*x^2+2*x+3)^(1/2)*x
```

maxima [A] time = 0.97, size = 114, normalized size = 0.80

$$\frac{49}{30}\sqrt{5\,x^2+2\,x+3}\,x^5+\frac{5131}{750}\sqrt{5\,x^2+2\,x+3}\,x^4-\frac{33259}{2500}\sqrt{5\,x^2+2\,x+3}\,x^3-\frac{207427}{37500}\sqrt{5\,x^2+2\,x+3}\,x^2+\frac{36073}{1875}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 49/30*sqrt(5*x^2 + 2*x + 3)*x^5 + 5131/750*sqrt(5*x^2 + 2*x + 3)*x^4 - 3325
9/2500*sqrt(5*x^2 + 2*x + 3)*x^3 - 207427/37500*sqrt(5*x^2 + 2*x + 3)*x^2 +
 36073/1875*sqrt(5*x^2 + 2*x + 3)*x - 1719097/156250*sqrt(5)*arcsinh(1/14*s
qrt(14)*(5*x + 1)) - 22053/31250*sqrt(5*x^2 + 2*x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5\,x+2\right)\,{\left(-7\,x^2+4\,x+1\right)}^2}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(1/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+5x+2\right)\left(7x^2-4x-1\right)^2}{\sqrt{5x^2+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*(7*x**2 - 4*x - 1)**2/sqrt(5*x**2 + 2*x + 3), x)
```

3.388 $\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

Optimal. Leaf size=101

$$-\frac{571}{300}\sqrt{5x^2+2x+3}\,x^2+\frac{59}{30}\sqrt{5x^2+2x+3}\,x+\frac{463}{125}\sqrt{5x^2+2x+3}-\frac{7}{20}\sqrt{5x^2+2x+3}\,x^3-\frac{1901\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{250\sqrt{5}}$$

```
[Out] -1901/1250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)+463/125*(5*x^2+2*x+3)^(1/
2)+59/30*x*(5*x^2+2*x+3)^(1/2)-571/300*x^2*(5*x^2+2*x+3)^(1/2)-7/20*x^3*(5*
x^2+2*x+3)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1661, 640, 619, 215}

$$-\frac{7}{20}\sqrt{5x^2+2x+3}\,x^3-\frac{571}{300}\sqrt{5x^2+2x+3}\,x^2+\frac{59}{30}\sqrt{5x^2+2x+3}\,x+\frac{463}{125}\sqrt{5x^2+2x+3}-\frac{1901\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{250\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (463*Sqrt[3 + 2*x + 5*x^2])/125 + (59*x*Sqrt[3 + 2*x + 5*x^2])/30 - (571*x^
2*Sqrt[3 + 2*x + 5*x^2])/300 - (7*x^3*Sqrt[3 + 2*x + 5*x^2])/20 - (1901*Arc
Sinh[(1 + 5*x)/Sqrt[14]])/(250*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}dx = -\frac{7}{20}x^3\sqrt{3+2x+5x^2} + \frac{1}{20}\int \frac{40+260x+203x^2-571x^3}{\sqrt{3+2x+5x^2}}dx$$
$$= -\frac{571}{300}x^2\sqrt{3+2x+5x^2} - \frac{7}{20}x^3\sqrt{3+2x+5x^2} + \frac{1}{300}\int \frac{600+7326x+}{\sqrt{3+2x+}}$$
$$= \frac{59}{30}x\sqrt{3+2x+5x^2} - \frac{571}{300}x^2\sqrt{3+2x+5x^2} - \frac{7}{20}x^3\sqrt{3+2x+5x^2} + \int$$
$$= \frac{463}{125}\sqrt{3+2x+5x^2} + \frac{59}{30}x\sqrt{3+2x+5x^2} - \frac{571}{300}x^2\sqrt{3+2x+5x^2} - \frac{7}{20}$$
$$= \frac{463}{125}\sqrt{3+2x+5x^2} + \frac{59}{30}x\sqrt{3+2x+5x^2} - \frac{571}{300}x^2\sqrt{3+2x+5x^2} - \frac{7}{20}$$
$$= \frac{463}{125}\sqrt{3+2x+5x^2} + \frac{59}{30}x\sqrt{3+2x+5x^2} - \frac{571}{300}x^2\sqrt{3+2x+5x^2} - \frac{7}{20}$$

Mathematica [A] time = 0.08, size = 55, normalized size = 0.54

$$\frac{-5\sqrt{5x^2+2x+3}\left(525x^3+2855x^2-2950x-5556\right)-11406\sqrt{5}\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{7500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-5*Sqrt[3 + 2*x + 5*x^2]*(-5556 - 2950*x + 2855*x^2 + 525*x^3) - 11406*Sqr
t[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/7500
```

fricas [A] time = 0.73, size = 67, normalized size = 0.66

$$-\frac{1}{1500}\left(525\,x^3+2855\,x^2-2950\,x-5556\right)\sqrt{5\,x^2+2\,x+3}+\frac{1901}{2500}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fric
as")
```

```
[Out] -1/1500*(525*x^3 + 2855*x^2 - 2950*x - 5556)*sqrt(5*x^2 + 2*x + 3) + 1901/2
500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8
)
```

giac [A] time = 0.22, size = 62, normalized size = 0.61

$$-\frac{1}{1500}\left(5\left((105\,x+571)x-590\right)x-5556\right)\sqrt{5\,x^2+2\,x+3}+\frac{1901}{1250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/1500*(5*((105*x + 571)*x - 590)*x - 5556)*sqrt(5*x^2 + 2*x + 3) + 1901/1
250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

maple [A] time = 0.01, size = 79, normalized size = 0.78

$$-\frac{7\sqrt{5x^2+2x+3}\,x^3}{20}-\frac{571\sqrt{5x^2+2x+3}\,x^2}{300}+\frac{59\sqrt{5x^2+2x+3}\,x}{30}-\frac{1901\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{1250}+\frac{463\sqrt{5x^2+2}}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -7/20*(5*x^2+2*x+3)^(1/2)*x^3-571/300*(5*x^2+2*x+3)^(1/2)*x^2+59/30*(5*x^2+
2*x+3)^(1/2)*x+463/125*(5*x^2+2*x+3)^(1/2)-1901/1250*5^(1/2)*arcsinh(5/14*1
4^(1/2)*(x+1/5))
```

maxima [A] time = 0.96, size = 80, normalized size = 0.79

$$-\frac{7}{20}\sqrt{5x^2+2x+3}\,x^3-\frac{571}{300}\sqrt{5x^2+2x+3}\,x^2+\frac{59}{30}\sqrt{5x^2+2x+3}\,x-\frac{1901}{1250}\sqrt{5}\,\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}\,(5x+1)\right)+\frac{463}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -7/20*sqrt(5*x^2 + 2*x + 3)*x^3 - 571/300*sqrt(5*x^2 + 2*x + 3)*x^2 + 59/30
*sqrt(5*x^2 + 2*x + 3)*x - 1901/1250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1
)) + 463/125*sqrt(5*x^2 + 2*x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5x+2\right)\left(-7x^2+4x+1\right)}{\sqrt{5x^2+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(1/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{13x}{\sqrt{5x^2+2x+3}}\right)dx-\int\left(-\frac{7x^2}{\sqrt{5x^2+2x+3}}\right)dx-\int\frac{31x^3}{\sqrt{5x^2+2x+3}}\,dx-\int\frac{7x^4}{\sqrt{5x^2+2x+3}}\,dx-\int\left(-\frac{}{\sqrt{5x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-13*x/sqrt(5*x**2 + 2*x + 3), x) - Integral(-7*x**2/sqrt(5*x**2 +
 2*x + 3), x) - Integral(31*x**3/sqrt(5*x**2 + 2*x + 3), x) - Integral(7*x*
*4/sqrt(5*x**2 + 2*x + 3), x) - Integral(-2/sqrt(5*x**2 + 2*x + 3), x)
```

3.389 $\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx$

Optimal. Leaf size=164

$$-\frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)+\frac{3}{14}\sqrt{\frac{4091+1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{(17}{\sqrt{2(1}}\right.$$

```
[Out] -1/35*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-3/39116*arctanh((23+x*(17-5*11
^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(11430254-29
47670*11^(1/2))^(1/2)+3/39116*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^
2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(11430254+2947670*11^(1/2))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1076, 619, 215, 1032, 724, 206}

$$-\frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)+\frac{3}{14}\sqrt{\frac{4091+1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{(17}{\sqrt{2(1}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] -ArcSinh[(1 + 5*x)/Sqrt[14]]/(7*Sqrt[5]) - (3*Sqrt[(4091 - 1055*Sqrt[11]])/2
794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[1
1])]*Sqrt[3 + 2*x + 5*x^2])])/14 + (3*Sqrt[(4091 + 1055*Sqrt[11])/2794]*Arc
Tanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqr
t[3 + 2*x + 5*x^2])])/14
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
```

```
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx = -\left(\frac{1}{7}\int \frac{1}{\sqrt{3+2x+5x^2}}\,dx\right)-\frac{1}{7}\int \frac{-15-39x}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right)}{14\sqrt{70}}+\frac{1}{77}\left(3\left(143-61\sqrt{11}\right)\right)\int \frac{}{\left(4-2\sqrt{11}\right.}$$

$$= -\frac{\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{7\sqrt{5}}-\frac{1}{77}\left(6\left(143-61\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{}{2352+112\left(4-2\sqrt{11}\right.}\right.$$

$$= -\frac{\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{7\sqrt{5}}-\frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\right.}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3}}\right.$$

Mathematica [A] time = 0.47, size = 157, normalized size = 0.96

$$-\frac{3\left(\sqrt{4091-1055\sqrt{11}}\tanh^{-1}\left(\frac{-5\sqrt{11}x+17x-\sqrt{11}+23}{\sqrt{250-34\sqrt{11}}\sqrt{5x^2+2x+3}}\right)-\sqrt{4091+1055\sqrt{11}}\tanh^{-1}\left(\frac{5\sqrt{11}x+17x+\sqrt{11}+23}{\sqrt{250+34\sqrt{11}}\sqrt{5x^2+2x+3}}\right)\right)}{14\sqrt{2794}}-\frac{\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] -1/7*ArcSinh[(1 + 5*x)/Sqrt[14]]/Sqrt[5] - (3*(Sqrt[4091 - 1055*Sqrt[11]]*A
rcTanh[(23 - Sqrt[11] + 17*x - 5*Sqrt[11]*x)/(Sqrt[250 - 34*Sqrt[11]]*Sqrt[
3 + 2*x + 5*x^2])] - Sqrt[4091 + 1055*Sqrt[11]]*ArcTanh[(23 + Sqrt[11] + 17
*x + 5*Sqrt[11]*x)/(Sqrt[250 + 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])]))/(14*S
qrt[2794])
```

fricas [B] time = 0.86, size = 297, normalized size = 1.81

$$-\frac{3}{78232}\sqrt{2794}\sqrt{1055\sqrt{11}+4091}\log\left(\frac{3\left(\sqrt{2794}\sqrt{5x^2+2x+3}\sqrt{1055\sqrt{11}+4091}\left(172\sqrt{11}-715\right)+18580\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="fric
as")
```

```
[Out] -3/78232*sqrt(2794)*sqrt(1055*sqrt(11) + 4091)*log(3*(sqrt(2794)*sqrt(5*x^2
 + 2*x + 3)*sqrt(1055*sqrt(11) + 4091)*(172*sqrt(11) - 715) + 185801*sqrt(1
1)*(x + 3) + 557403*x - 929005)/x) + 3/78232*sqrt(2794)*sqrt(1055*sqrt(11)
+ 4091)*log(-3*(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(1055*sqrt(11) + 4091)
*(172*sqrt(11) - 715) - 185801*sqrt(11)*(x + 3) - 557403*x + 929005)/x) - 1
/78232*sqrt(2794)*sqrt(-9495*sqrt(11) + 36819)*log(-(sqrt(2794)*sqrt(5*x^2
+ 2*x + 3)*(172*sqrt(11) + 715)*sqrt(-9495*sqrt(11) + 36819) + 557403*sqrt(
11)*(x + 3) - 1672209*x + 2787015)/x) + 1/78232*sqrt(2794)*sqrt(-9495*sqrt(
11) + 36819)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(172*sqrt(11) + 715)*sqr
t(-9495*sqrt(11) + 36819) - 557403*sqrt(11)*(x + 3) + 1672209*x - 2787015)/
x) + 1/70*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10
*x - 8)
```

giac [A] time = 0.27, size = 125, normalized size = 0.76

$$\frac{1}{35}\sqrt{5}\log\left(-5\sqrt{5}x-\sqrt{5}+5\sqrt{5x^2+2x+3}\right)+0.353184817631429\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}+4.4192\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac
")
```

```
[Out] 1/35*sqrt(5)*log(-5*sqrt(5)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*x + 3)) + 0.3531
84817631429*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.
0986339689905714*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000)
 - 0.353184817631429*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113
000) + 0.0986339689905714*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.094112
35400000)
```

maple [A] time = 0.02, size = 204, normalized size = 1.24

$$-\frac{\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{35}+\frac{3\left(-61+13\sqrt{11}\right)\sqrt{11}\,\operatorname{arctanh}\left(\frac{250-34\sqrt{11}+\frac{49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250-34\sqrt{11}}\,\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right.}}\right.}{154\sqrt{250-34\sqrt{11}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -1/35*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+3/154*(-61+13*11^(1/2))*11^(1/
2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*1
1^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(
1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)
)+3/154*11^(1/2)*(61+13*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500
/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1
/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*1
1^(1/2))+250+34*11^(1/2))^(1/2))
```

maxima [B] time = 1.14, size = 465, normalized size = 2.84

$$-\frac{1}{10780}\sqrt{11}\left(28\sqrt{11}\sqrt{5}\,\operatorname{arsinh}\left(\frac{5}{14}\sqrt{7}\sqrt{2}x+\frac{1}{14}\sqrt{7}\sqrt{2}\right)-\frac{1365\sqrt{11}\sqrt{2}\,\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7\left|14x-2\sqrt{11}-4\right|}+\frac{17\sqrt{7}\cdots}{7\left|14x-2\cdots\right.}\right.}{\sqrt{17\sqrt{11}+\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/10780*sqrt(11)*(28*sqrt(11)*sqrt(5)*arcsinh(5/14*sqrt(7)*sqrt(2)*x + 1/1
4*sqrt(7)*sqrt(2)) - 1365*sqrt(11)*sqrt(2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqr
t(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqr
t(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4) + 23/7
*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/sqrt(17*sqrt(11) + 125) + 390*
sqrt(11)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4)
- 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*
sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt
(11) - 4))/sqrt(-34/49*sqrt(11) + 250/49) - 6405*sqrt(2)*arcsinh(5/7*sqrt(1
1)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/ab
s(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(1
1) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/sqrt(17*sqrt(11)
 + 125) - 1830*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11)
 - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sq
rt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x +
2*sqrt(11) - 4))/sqrt(-34/49*sqrt(11) + 250/49))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+5\,x+2}{\sqrt{5\,x^2+2\,x+3}\,\left(-7\,x^2+4\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)),x)
```

```
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{5x}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{5x^2+2x+3}}\,dx-\int \frac{x^2}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(5*x/(7*x**2*sqrt(5*x**2 + 2*x + 3) - 4*x*sqrt(5*x**2 + 2*x + 3) -
 sqrt(5*x**2 + 2*x + 3)), x) - Integral(x**2/(7*x**2*sqrt(5*x**2 + 2*x + 3)
 - 4*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(7
*x**2*sqrt(5*x**2 + 2*x + 3) - 4*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2
*x + 3)), x)
```

3.390 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx$

Optimal. Leaf size=178

$$-\frac{3\sqrt{5x^2+2x+3}\,(40-371x)}{5588\left(-7x^2+4x+1\right)}-\frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{11176}+\frac{\sqrt{\frac{3027900955-14035}{2794}}\ldots}{}$$

```
[Out] -3/5588*(40-371*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)+1/31225744*arctanh((2
3+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*
(8459955268270-39215692714*11^(1/2))^(1/2)-1/31225744*arctanh((23+x*(17-5*1
1^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(8459955268
270+39215692714*11^(1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1060, 1032, 724, 206}

$$-\frac{3\sqrt{5x^2+2x+3}\,(40-371x)}{5588\left(-7x^2+4x+1\right)}-\frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{11176}+\frac{\sqrt{\frac{3027900955-14035}{2794}}\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] (-3*(40 - 371*x)*Sqrt[3 + 2*x + 5*x^2])/(5588*(1 + 4*x - 7*x^2)) - (Sqrt[(3
027900955 + 14035681*Sqrt[11])/2794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[
11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/11176 + (Sqrt
[(3027900955 - 14035681*Sqrt[11])/2794]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sq
rt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/11176
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
```

```
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}}\,dx = -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\left(1+4x-7x^2\right)} - \frac{\int \frac{-52136-29544x}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx}{44704}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\left(1+4x-7x^2\right)} - \frac{\left(-40623+53005\sqrt{11}\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{\cdots}}}{61468}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\left(1+4x-7x^2\right)} - \frac{\left(40623-53005\sqrt{11}\right)\text{Subst}\left(\int \frac{\cdots}{2352+112\cdots}\right.}{\cdots}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\left(1+4x-7x^2\right)} - \frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{23-\cdots}{\sqrt{2(125\cdots}}\right.}{11176}$$

Mathematica [A] time = 1.00, size = 313, normalized size = 1.76

$$\frac{\frac{48972\sqrt{5x^2+2x+3}\,x}{-7x^2+4x+1}+\frac{5280\sqrt{5x^2+2x+3}}{7x^2-4x-1}+53005\sqrt{\frac{22}{125+17\sqrt{11}}}\log\left(\sqrt{2750+374\sqrt{11}}\sqrt{5x^2+2x+3}+\left(55+17\sqrt{11}\right)x\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] ((48972*x*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2) + (5280*Sqrt[3 + 2*x + 5
*x^2])/(-1 - 4*x + 7*x^2) + Sqrt[2/(125 - 17*Sqrt[11])]*(-40623 + 53005*Sqr
t[11])*ArcTanh[(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[
11] + (-17 + 5*Sqrt[11])*x)] - Sqrt[2/(125 + 17*Sqrt[11])]*(40623 + 53005*S
qrt[11])*Log[2 + Sqrt[11] - 7*x] + 40623*Sqrt[2/(125 + 17*Sqrt[11])]*Log[11
 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 +
2*x + 5*x^2]] + 53005*Sqrt[22/(125 + 17*Sqrt[11])]*Log[11 + 23*Sqrt[11] + (
55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2]])/245
872
```

fricas [B] time = 0.87, size = 330, normalized size = 1.85

$$-\frac{\sqrt{2794}\left(7x^2-4x-1\right)\sqrt{14035681\sqrt{11}+3027900955}\,\log\left(-\frac{\sqrt{2794}\sqrt{5x^2+2x+3}\sqrt{14035681\sqrt{11}+3027900955}\left(71796\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/62451488*(sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(14035681*sqrt(11) + 30279009
55)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(14035681*sqrt(11) + 3027900
955)*(71796*sqrt(11) + 567523) + 265381033753*sqrt(11)*(x + 3) - 7961431012
59*x + 1326905168765)/x) - sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(14035681*sqrt(
11) + 3027900955)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(14035681*sqrt(
11) + 3027900955)*(71796*sqrt(11) + 567523) - 265381033753*sqrt(11)*(x + 3)
 + 796143101259*x - 1326905168765)/x) + sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(-
14035681*sqrt(11) + 3027900955)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(7179
6*sqrt(11) - 567523)*sqrt(-14035681*sqrt(11) + 3027900955) + 265381033753*s
qrt(11)*(x + 3) + 796143101259*x - 1326905168765)/x) - sqrt(2794)*(7*x^2 -
4*x - 1)*sqrt(-14035681*sqrt(11) + 3027900955)*log(-(sqrt(2794)*sqrt(5*x^2
+ 2*x + 3)*(71796*sqrt(11) - 567523)*sqrt(-14035681*sqrt(11) + 3027900955)
- 265381033753*sqrt(11)*(x + 3) - 796143101259*x + 1326905168765)/x) + 3352
8*sqrt(5*x^2 + 2*x + 3)*(371*x - 40))/(7*x^2 - 4*x - 1)
```

giac [B] time = 0.28, size = 276, normalized size = 1.55

$$\frac{3\left(1231\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^3+1735\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^2-3913\sqrt{5}x-3989\sqrt{5}+\cdots\right.}{2794\left(7\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^4-8\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^3-70\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^2+16\sqrt{5}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 3/2794*(1231*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 1735*sqrt(5)*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3))^2 - 3913*sqrt(5)*x - 3989*sqrt(5) + 3913*sqrt(5*
x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sqrt(5)*(sqrt(
5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2
+ 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83) + 0.0924287071106453
*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.09386080346
04765*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.092428
7071106453*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.0
938608034604765*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

maple [B] time = 0.02, size = 510, normalized size = 2.87

$$-\frac{161\sqrt{11}\,\operatorname{arctanh}\left(\frac{250-34\sqrt{11}+\frac{49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250-34\sqrt{11}}\,\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)+250-34\sqrt{11}}}\right)}{484\sqrt{250-34\sqrt{11}}}+\frac{161\sqrt{11}\,\operatorname{arctanh}\left(\frac{\cdots}{\sqrt{250+34\cdots}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -161/484*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/
2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*
(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*
11^(1/2))^(1/2))+(183/44-39/44*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-
2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7
*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/14*(34/7-10/7*11^(1/2))/(250/49-3
4/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+
(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-
2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^
(1/2))^(1/2)))+161/484*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/4
9+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2
))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
(1/2))+250+34*11^(1/2))^(1/2))+(183/44+39/44*11^(1/2))*(-1/49/(250/49+34/49
*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/
2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/14*(34/7+10/7*11^(1
/2))/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+6
8/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^
(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/
2))+250+34*11^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 5\,x + 2}{\left(7\,x^2 - 4\,x - 1\right)^2\sqrt{5\,x^2 + 2\,x + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^2*sqrt(5*x^2 + 2*x + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 + 5\,x + 2}{\sqrt{5\,x^2 + 2\,x + 3}\,\left(-7\,x^2 + 4\,x + 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2),x)
```

```
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 5x + 2}{\sqrt{5x^2 + 2x + 3}\,\left(7x^2 - 4x - 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**2/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)/(sqrt(5*x**2 + 2*x + 3)*(7*x**2 - 4*x - 1)**2), x
)
```

3.391 $\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3 \sqrt{3+2x+5x^2}} \, dx$

Optimal. Leaf size=227

$$-\frac{7\sqrt{5x^2+2x+3}\,(409769-1189370x)}{62451488\left(-7x^2+4x+1\right)}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}-\frac{7\left(39370231-2538725\sqrt{11}\right)\tanh^{-1}\Big(}{124902976\sqrt{22\left(125-\right.}}$$

```
[Out] -3/11176*(40-371*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2-7/62451488*(409769
-1189370*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)-7/124902976*arctanh((23+x*(1
7-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(39370
231-2538725*11^(1/2))/(2750-374*11^(1/2))^(1/2)+7/124902976*arctanh((23+11^
(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(3937
0231+2538725*11^(1/2))/(2750+374*11^(1/2))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1060, 1032, 724, 206}

$$-\frac{7\sqrt{5x^2+2x+3}\,(409769-1189370x)}{62451488\left(-7x^2+4x+1\right)}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}-\frac{7\left(39370231-2538725\sqrt{11}\right)\tanh^{-1}\Big(}{124902976\sqrt{22\left(125-\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] (-3*(40 - 371*x)*Sqrt[3 + 2*x + 5*x^2])/(11176*(1 + 4*x - 7*x^2)^2) - (7*(4
09769 - 1189370*x)*Sqrt[3 + 2*x + 5*x^2])/(62451488*(1 + 4*x - 7*x^2)) - (7
*(39370231 - 2538725*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x
)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(124902976*Sqrt[22*
(125 - 17*Sqrt[11])]) + (7*(39370231 + 2538725*Sqrt[11])*ArcTanh[(23 + Sqrt
[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x
^2])])/(124902976*Sqrt[22*(125 + 17*Sqrt[11])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3\sqrt{3+2x+5x^2}}\,dx = -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{\int \frac{-130024-81000x-89040x^2}{\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}}\,dx}{89408}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)} +$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)} +$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)} -$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)} -$$

Mathematica [A] time = 1.17, size = 371, normalized size = 1.63

$$\frac{\frac{732651920\sqrt{5x^2+2x+3}\,x}{-7x^2+4x+1} + \frac{547311072\sqrt{5x^2+2x+3}\,x}{\left(-7x^2+4x+1\right)^2} - \frac{59009280\sqrt{5x^2+2x+3}}{\left(-7x^2+4x+1\right)^2} + \frac{252417704\sqrt{5x^2+2x+3}}{7x^2-4x-1} + 551183234\sqrt{\frac{22}{125+17\sqrt{11}}}\,\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] ((-59009280*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2 + (547311072*x*Sqrt[
3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2 + (732651920*x*Sqrt[3 + 2*x + 5*x^2])
/(1 + 4*x - 7*x^2) + (252417704*Sqrt[3 + 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) +
 14*Sqrt[2/(125 - 17*Sqrt[11])]*(-27925975 + 39370231*Sqrt[11])*ArcTanh[(Sq
rt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[11] + (-17 + 5*Sqr
t[11])*x)] - 14*Sqrt[2/(125 + 17*Sqrt[11])]*(27925975 + 39370231*Sqrt[11])*
Log[2 + Sqrt[11] - 7*x] + 390963650*Sqrt[2/(125 + 17*Sqrt[11])]*Log[11 + 23
*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*x +
 5*x^2]] + 551183234*Sqrt[22/(125 + 17*Sqrt[11])]*Log[11 + 23*Sqrt[11] + (5
5 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2]])/5495
730944
```

fricas [B] time = 1.10, size = 390, normalized size = 1.72

$$-\frac{\sqrt{2794}\left(49\,x^4-56\,x^3+2\,x^2+8\,x+1\right)\sqrt{1283973697005131\,\sqrt{11}+82616280769148425}\,\log\left(-\frac{\sqrt{2794}\,\sqrt{5\,x^2+2}\cdots}{\cdots}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/697957829888*(sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(128397
3697005131*sqrt(11) + 82616280769148425)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x
+ 3)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*(358684877*sqrt(11
) + 2940638404) + 7232150972206110797*sqrt(11)*(x + 3) - 216964529166183323
91*x + 36160754861030553985)/x) - sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x
 + 1)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*log((sqrt(2794)*s
qrt(5*x^2 + 2*x + 3)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*(3
58684877*sqrt(11) + 2940638404) - 7232150972206110797*sqrt(11)*(x + 3) + 21
696452916618332391*x - 36160754861030553985)/x) + sqrt(2794)*(49*x^4 - 56*x
^3 + 2*x^2 + 8*x + 1)*sqrt(-1283973697005131*sqrt(11) + 82616280769148425)*
log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(358684877*sqrt(11) - 2940638404)*sqr
t(-1283973697005131*sqrt(11) + 82616280769148425) + 7232150972206110797*sqr
t(11)*(x + 3) + 21696452916618332391*x - 36160754861030553985)/x) - sqrt(27
94)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-1283973697005131*sqrt(11) + 8
2616280769148425)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(358684877*sqrt(11
) - 2940638404)*sqrt(-1283973697005131*sqrt(11) + 82616280769148425) - 7232
150972206110797*sqrt(11)*(x + 3) - 21696452916618332391*x + 361607548610305
53985)/x) + 11176*(58279130*x^3 - 53381041*x^2 - 3071502*x + 3538943)*sqrt(
5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

giac [B] time = 0.32, size = 378, normalized size = 1.67

$$\frac{124397525\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^7+26796567\,\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^6-3595807617\left(\sqrt{5}\,x-\sqrt{5\,x\cdots}\right.\cdots}{31225744\left(7\left(\sqrt{5}\,x\cdots\right.\right.\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/31225744*(124397525*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 + 26796567*sqrt
(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^6 - 3595807617*(sqrt(5)*x - sqrt(5*
x^2 + 2*x + 3))^5 - 1719888775*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^
4 + 17096132999*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 8328401413*sqrt(5)*
(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 - 16383202915*sqrt(5)*x - 7800623485*
sqrt(5) + 16383202915*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2
```

```
*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3
)) + 83)^2 + 0.0423989586659649*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.
41924736459000) - 0.0446437606656958*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3)
 + 1.25295163054000) - 0.0423989586659649*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x
 + 3) - 1.02258038113000) + 0.0446437606656958*log(-sqrt(5)*x + sqrt(5*x^2
+ 2*x + 3) - 2.09411235400000)
```

maple [B] time = 0.02, size = 1194, normalized size = 5.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -3535/21296*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^
(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(2
45*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-
34*11^(1/2))^(1/2))-21/968*(-61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49-34/49
*11^(1/2))/(x-2/7+1/7*11^(1/2))^2*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(
1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-3/1372*(34/7-10/7*1
1^(1/2))/(250/49-34/49*11^(1/2))*(-1/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^
(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+
250/49-34/49*11^(1/2))^(1/2)+7/2*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2
))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*1
1^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(
1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)
))+5/98/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/4
9-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2
))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^
(1/2))+250-34*11^(1/2))^(1/2)))-(-3535/1936+273/1936*11^(1/2))*(-1/49/(250/
49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/
7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/14*(34/7-10
/7*11^(1/2))/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(
500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11
^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/
7*11^(1/2))+250-34*11^(1/2))^(1/2)))+3535/21296*11^(1/2)/(250+34*11^(1/2))^
(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*1
1^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7
*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))-(-3535/1936-273/193
6*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1
/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1
/2))^(1/2)+1/14*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))/(250+34*11^(1/
2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1
/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+
10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-21/968*(61+13*
11^(1/2))*11^(1/2)*(-1/686/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))^2*(
5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+3
4/49*11^(1/2))^(1/2)-3/1372*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(-
1/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7*11^(1/2))^2+(3
4/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+7/2*(3
4/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(
49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250
+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-
2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))+5/98/(250/49+34/49*11^(1/2))/(25
0+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2
))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^
2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 + 5x + 2}{\left(7x^2 - 4x - 1\right)^3 \sqrt{5x^2 + 2x + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^3*sqrt(5*x^2 + 2*x + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 + 5x + 2}{\sqrt{5x^2 + 2x + 3}\left(-7x^2 + 4x + 1\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3),x)
```

```
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{5x}{343x^6\sqrt{5x^2 + 2x + 3} - 588x^5\sqrt{5x^2 + 2x + 3} + 189x^4\sqrt{5x^2 + 2x + 3} + 104x^3\sqrt{5x^2 + 2x + 3} - 27x^2\sqrt{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**3/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(5*x/(343*x**6*sqrt(5*x**2 + 2*x + 3) - 588*x**5*sqrt(5*x**2 + 2*x
 + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3) + 104*x**3*sqrt(5*x**2 + 2*x + 3) -
 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2
 + 2*x + 3)), x) - Integral(x**2/(343*x**6*sqrt(5*x**2 + 2*x + 3) - 588*x**
5*sqrt(5*x**2 + 2*x + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3) + 104*x**3*sqrt(
5*x**2 + 2*x + 3) - 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*sqrt(5*x**2 + 2*x
 + 3) - sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(343*x**6*sqrt(5*x**2 + 2*
x + 3) - 588*x**5*sqrt(5*x**2 + 2*x + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3)
+ 104*x**3*sqrt(5*x**2 + 2*x + 3) - 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*s
qrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x)
```

3.392 $\int \frac{\left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)}{\left(3+2x+5x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{2583293\sqrt{5x^2+2x+3}\,x^2}{187500}-\frac{3192602\sqrt{5x^2+2x+3}\,x}{46875}+\frac{15715799\sqrt{5x^2+2x+3}}{156250}+\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}-\frac{34}{15}$$

```
[Out] 50047657/781250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)+16/546875*(6122807-5
338217*x)/(5*x^2+2*x+3)^(1/2)+15715799/156250*(5*x^2+2*x+3)^(1/2)-3192602/4
6875*x*(5*x^2+2*x+3)^(1/2)-2583293/187500*x^2*(5*x^2+2*x+3)^(1/2)+393659/12
500*x^3*(5*x^2+2*x+3)^(1/2)-25921/3750*x^4*(5*x^2+2*x+3)^(1/2)-343/150*x^5*
(5*x^2+2*x+3)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1660, 1661, 640, 619, 215}

$$-\frac{343}{150}\sqrt{5x^2+2x+3}\,x^5-\frac{25921\sqrt{5x^2+2x+3}\,x^4}{3750}+\frac{393659\sqrt{5x^2+2x+3}\,x^3}{12500}-\frac{2583293\sqrt{5x^2+2x+3}\,x^2}{187500}-\frac{319260}{}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (16*(6122807 - 5338217*x))/(546875*Sqrt[3 + 2*x + 5*x^2]) + (15715799*Sqrt[
3 + 2*x + 5*x^2])/156250 - (3192602*x*Sqrt[3 + 2*x + 5*x^2])/46875 - (25832
93*x^2*Sqrt[3 + 2*x + 5*x^2])/187500 + (393659*x^3*Sqrt[3 + 2*x + 5*x^2])/1
2500 - (25921*x^4*Sqrt[3 + 2*x + 5*x^2])/3750 - (343*x^5*Sqrt[3 + 2*x + 5*x
^2])/150 + (50047657*ArcSinh[(1 + 5*x)/Sqrt[14]])/(156250*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
```

```
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)}{\left(3+2x+5x^2\right)^{3/2}}\,dx = \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}}+\frac{1}{28}\int \frac{\frac{473724104}{78125}+\frac{94462228x}{15625}-\frac{40822404x^2}{3125}-1\ldots}{\sqrt{3+2\ldots}}$$

$$=\frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}}-\frac{343}{150}x^5\sqrt{3+2x+5x^2}+\frac{1}{840}\int \frac{\frac{2842344624}{15625}\ldots}{\ldots}$$

$$=\frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}}-\frac{25921x^4\sqrt{3+2x+5x^2}}{3750}-\frac{343}{150}x^5\sqrt{3+2x\ldots}$$

$$=\frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}}+\frac{393659x^3\sqrt{3+2x+5x^2}}{12500}-\frac{25921x^4\sqrt{3+\ldots}}{3750}$$

$$=\frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}}-\frac{2583293x^2\sqrt{3+2x+5x^2}}{187500}+\frac{393659x^3\sqrt{3\ldots}}{125\ldots}$$

$$=\frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}}-\frac{3192602x\sqrt{3+2x+5x^2}}{46875}-\frac{2583293x^2\sqrt{3\ldots}}{1875\ldots}$$

$$=\frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}}+\frac{15715799\sqrt{3+2x+5x^2}}{156250}-\frac{3192602x\sqrt{3+\ldots}}{4687\ldots}$$

$$=\frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}}+\frac{15715799\sqrt{3+2x+5x^2}}{156250}-\frac{3192602x\sqrt{3+\ldots}}{4687\ldots}$$

$$=\frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}}+\frac{15715799\sqrt{3+2x+5x^2}}{156250}-\frac{3192602x\sqrt{3+\ldots}}{4687\ldots}$$

Mathematica [A] time = 0.43, size = 75, normalized size = 0.45

$$\frac{2102001594\sqrt{5}\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)-\frac{5\left(75031250x^7+256821250x^6-897612625x^5+174819575x^4+1795638985x^3-2135143465x^2+10457033\ldots\right)}{\sqrt{5x^2+2x+3}}}{32812500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] ((-5*(-3155769618 + 1045703388*x - 2135143465*x^2 + 1795638985*x^3 + 174819
575*x^4 - 897612625*x^5 + 256821250*x^6 + 75031250*x^7))/Sqrt[3 + 2*x + 5*x
^2] + 2102001594*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/32812500
```

fricas [A] time = 0.83, size = 112, normalized size = 0.67

$$\frac{1051000797 \sqrt{5} \left(5 x^2 + 2 x + 3\right) \log\left(-\sqrt{5} \sqrt{5 x^2 + 2 x + 3} (5 x + 1) - 25 x^2 - 10 x - 8\right) - 5 \left(75031250 x^7 + 25682\right.}{328125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/32812500*(1051000797*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 +
2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) - 5*(75031250*x^7 + 256821250*x^6 -
 897612625*x^5 + 174819575*x^4 + 1795638985*x^3 - 2135143465*x^2 + 10457033
88*x - 3155769618)*sqrt(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

giac [A] time = 0.26, size = 81, normalized size = 0.49

$$-\frac{50047657}{781250} \sqrt{5} \log\left(-\sqrt{5} \left(\sqrt{5} x - \sqrt{5 x^2 + 2 x + 3}\right) - 1\right) - \frac{(35 ((5 (35 (70 (175 x + 599) x - 146549) x + 998969) x}{6562500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] -50047657/781250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) -
 1) - 1/6562500*((35*((5*(35*(70*(175*x + 599)*x - 146549)*x + 998969)*x +
51303971)*x - 61004099)*x + 1045703388)*x - 3155769618)/sqrt(5*x^2 + 2*x +
3)
```

maple [A] time = 0.03, size = 166, normalized size = 1.00

$$-\frac{343 x^7}{30 \sqrt{5 x^2 + 2 x + 3}} - \frac{29351 x^6}{750 \sqrt{5 x^2 + 2 x + 3}} + \frac{1025843 x^5}{7500 \sqrt{5 x^2 + 2 x + 3}} - \frac{998969 x^4}{37500 \sqrt{5 x^2 + 2 x + 3}} - \frac{51303971 x^3}{187500 \sqrt{5 x^2 + 2 x + 3}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] 50047657/781250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+176049701/10937500*(
10*x+2)/(5*x^2+2*x+3)^(1/2)-998969/37500*x^4/(5*x^2+2*x+3)^(1/2)-51303971/1
87500*x^3/(5*x^2+2*x+3)^(1/2)+61004099/187500*x^2/(5*x^2+2*x+3)^(1/2)-50047
657/156250*x/(5*x^2+2*x+3)^(1/2)-343/30*x^7/(5*x^2+2*x+3)^(1/2)-29351/750*x
^6/(5*x^2+2*x+3)^(1/2)+1025843/7500*x^5/(5*x^2+2*x+3)^(1/2)+175268451/39062
5/(5*x^2+2*x+3)^(1/2)
```

maxima [A] time = 0.99, size = 148, normalized size = 0.89

$$-\frac{343 x^7}{30 \sqrt{5 x^2 + 2 x + 3}} - \frac{29351 x^6}{750 \sqrt{5 x^2 + 2 x + 3}} + \frac{1025843 x^5}{7500 \sqrt{5 x^2 + 2 x + 3}} - \frac{998969 x^4}{37500 \sqrt{5 x^2 + 2 x + 3}} - \frac{51303971 x^3}{187500 \sqrt{5 x^2 + 2 x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] -343/30*x^7/sqrt(5*x^2 + 2*x + 3) - 29351/750*x^6/sqrt(5*x^2 + 2*x + 3) + 1
025843/7500*x^5/sqrt(5*x^2 + 2*x + 3) - 998969/37500*x^4/sqrt(5*x^2 + 2*x +
 3) - 51303971/187500*x^3/sqrt(5*x^2 + 2*x + 3) + 61004099/187500*x^2/sqrt(
5*x^2 + 2*x + 3) + 50047657/781250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1))
```

```
- 87141949/546875*x/sqrt(5*x^2 + 2*x + 3) + 525961603/1093750/sqrt(5*x^2 +
2*x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5x+2\right)\left(-7x^2+4x+1\right)^3}{\left(5x^2+2x+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(3/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{29x}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx-\int\left(-\frac{115x^2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-29*x/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
 + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-115*x**2/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(61*x**3/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
+ 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(871*x**4/(5*x**2*sqrt(5*x**2 + 2
*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Inte
gral(-127*x**5/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
+ 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-2065*x**6/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(1127*x**7/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3
) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(343*x**8/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(-2/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*s
qrt(5*x**2 + 2*x + 3)), x)
```

3.393 $\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

Optimal. Leaf size=124

$$\frac{203}{100}\sqrt{5x^2+2x+3}\,x^2-\frac{8749\sqrt{5x^2+2x+3}\,x}{1250}-\frac{5086\sqrt{5x^2+2x+3}}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}+\frac{49}{100}\sqrt{5x^2+2x+3}\,x^3$$

```
[Out] 89583/6250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-8/21875*(12983+136602*x)/
(5*x^2+2*x+3)^(1/2)-5086/3125*(5*x^2+2*x+3)^(1/2)-8749/1250*x*(5*x^2+2*x+3)
^(1/2)+203/100*x^2*(5*x^2+2*x+3)^(1/2)+49/100*x^3*(5*x^2+2*x+3)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{49}{100}\sqrt{5x^2+2x+3}\,x^3+\frac{203}{100}\sqrt{5x^2+2x+3}\,x^2-\frac{8749\sqrt{5x^2+2x+3}\,x}{1250}-\frac{5086\sqrt{5x^2+2x+3}}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-8*(12983 + 136602*x))/(21875*Sqrt[3 + 2*x + 5*x^2]) - (5086*Sqrt[3 + 2*x
+ 5*x^2])/3125 - (8749*x*Sqrt[3 + 2*x + 5*x^2])/1250 + (203*x^2*Sqrt[3 + 2*
x + 5*x^2])/100 + (49*x^3*Sqrt[3 + 2*x + 5*x^2])/100 + (89583*ArcSinh[(1 +
5*x)/Sqrt[14]])/(1250*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)}{\left(3+2x+5x^2\right)^{3/2}}\,dx = -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}}+\frac{1}{28}\int \frac{\frac{4291112}{3125}-\frac{296716x}{625}-\frac{194012x^2}{125}+\frac{23716x^3}{25}+$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}}+\frac{49}{100}x^3\sqrt{3+2x+5x^2}+\frac{1}{560}\int \frac{\frac{17164448}{625}-\frac{11}{}}{}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}}+\frac{203}{100}x^2\sqrt{3+2x+5x^2}+\frac{49}{100}x^3\sqrt{3+2x+5x^2}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}}-\frac{8749x\sqrt{3+2x+5x^2}}{1250}+\frac{203}{100}x^2\sqrt{3+2x+5x^2}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}}-\frac{5086\sqrt{3+2x+5x^2}}{3125}-\frac{8749x\sqrt{3+2x+5x^2}}{1250}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}}-\frac{5086\sqrt{3+2x+5x^2}}{3125}-\frac{8749x\sqrt{3+2x+5x^2}}{1250}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}}-\frac{5086\sqrt{3+2x+5x^2}}{3125}-\frac{8749x\sqrt{3+2x+5x^2}}{1250}$$

Mathematica [A] time = 0.27, size = 65, normalized size = 0.52

$$\frac{\frac{5\left(42875x^5+194775x^4-515655x^3-280805x^2-1298674x-168536\right)}{\sqrt{5x^2+2x+3}}+1254162\sqrt{5}\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{87500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] ((5*(-168536 - 1298674*x - 280805*x^2 - 515655*x^3 + 194775*x^4 + 42875*x^5
))/Sqrt[3 + 2*x + 5*x^2] + 1254162*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/875
00
```

fricas [A] time = 0.69, size = 102, normalized size = 0.82

$$\frac{627081\sqrt{5}\left(5x^2+2x+3\right)\log\left(-\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)+5\left(42875x^5+194775x^4-\right.}{87500\left(5x^2+2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/87500*(627081*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3
)*(5*x + 1) - 25*x^2 - 10*x - 8) + 5*(42875*x^5 + 194775*x^4 - 515655*x^3 -
 280805*x^2 - 1298674*x - 168536)*sqrt(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

giac [A] time = 0.25, size = 71, normalized size = 0.57

$$-\frac{89583}{6250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)+\frac{(35((35(35x+159)x-14733)x-8023)x-1298674)x-}{17500\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] -89583/6250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1) +
 1/17500*((35*((35*(35*x + 159)*x - 14733)*x - 8023)*x - 1298674)*x - 16853
6)/sqrt(5*x^2 + 2*x + 3)
```

maple [A] time = 0.01, size = 132, normalized size = 1.06

$$\frac{49x^5}{20\sqrt{5x^2+2x+3}}+\frac{1113x^4}{100\sqrt{5x^2+2x+3}}-\frac{14733x^3}{500\sqrt{5x^2+2x+3}}-\frac{8023x^2}{500\sqrt{5x^2+2x+3}}-\frac{89583x}{1250\sqrt{5x^2+2x+3}}+\frac{89583\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] 89583/6250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-5564/21875*(10*x+2)/(5*x^
2+2*x+3)^(1/2)+1113/100/(5*x^2+2*x+3)^(1/2)*x^4-14733/500/(5*x^2+2*x+3)^(1/
2)*x^3-8023/500/(5*x^2+2*x+3)^(1/2)*x^2-89583/1250/(5*x^2+2*x+3)^(1/2)*x+49
/20/(5*x^2+2*x+3)^(1/2)*x^5-28506/3125/(5*x^2+2*x+3)^(1/2)
```

maxima [A] time = 0.97, size = 114, normalized size = 0.92

$$\frac{49x^5}{20\sqrt{5x^2+2x+3}}+\frac{1113x^4}{100\sqrt{5x^2+2x+3}}-\frac{14733x^3}{500\sqrt{5x^2+2x+3}}-\frac{8023x^2}{500\sqrt{5x^2+2x+3}}+\frac{89583}{6250}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] 49/20*x^5/sqrt(5*x^2 + 2*x + 3) + 1113/100*x^4/sqrt(5*x^2 + 2*x + 3) - 1473
3/500*x^3/sqrt(5*x^2 + 2*x + 3) - 8023/500*x^2/sqrt(5*x^2 + 2*x + 3) + 8958
3/6250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 649337/8750*x/sqrt(5*x^2
+ 2*x + 3) - 42134/4375/sqrt(5*x^2 + 2*x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5x+2\right)\left(-7x^2+4x+1\right)^2}{\left(5x^2+2x+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(3/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+5x+2\right)\left(7x^2-4x-1\right)^2}{\left(5x^2+2x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*(7*x**2 - 4*x - 1)**2/(5*x**2 + 2*x + 3)**(3/2),
x)
```

3.394 $\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{7}{50}\sqrt{5x^2+2x+3}\,x-\frac{261}{250}\sqrt{5x^2+2x+3}-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}+\frac{149\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{5}}$$

```
[Out] 149/125*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-2/875*(2321+2449*x)/(5*x^2+2
*x+3)^(1/2)-261/250*(5*x^2+2*x+3)^(1/2)-7/50*x*(5*x^2+2*x+3)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1660, 1661, 640, 619, 215}

$$-\frac{7}{50}\sqrt{5x^2+2x+3}\,x-\frac{261}{250}\sqrt{5x^2+2x+3}-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}+\frac{149\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-2*(2321 + 2449*x))/(875*Sqrt[3 + 2*x + 5*x^2]) - (261*Sqrt[3 + 2*x + 5*x^
2])/250 - (7*x*Sqrt[3 + 2*x + 5*x^2])/50 + (149*ArcSinh[(1 + 5*x)/Sqrt[14]]
)/(25*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
```

```
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(1+4x-7x^2\right)\left(2+5x+x^2\right)}{\left(3+2x+5x^2\right)^{3/2}}\,dx = -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}} + \frac{1}{28}\int \frac{\frac{15736}{125}-\frac{3948x}{25}-\frac{196x^2}{5}}{\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}} - \frac{7}{50}x\sqrt{3+2x+5x^2} + \frac{1}{280}\int \frac{\frac{34412}{25}-\frac{7308x}{5}}{\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}} - \frac{261}{250}\sqrt{3+2x+5x^2} - \frac{7}{50}x\sqrt{3+2x+5x^2} + \frac{149}{25}$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}} - \frac{261}{250}\sqrt{3+2x+5x^2} - \frac{7}{50}x\sqrt{3+2x+5x^2} + \frac{149}{}$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}} - \frac{261}{250}\sqrt{3+2x+5x^2} - \frac{7}{50}x\sqrt{3+2x+5x^2} + \frac{149}{}$$

Mathematica [A] time = 0.15, size = 55, normalized size = 0.67

$$\frac{149\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{5}} - \frac{245x^3+1925x^2+2837x+2953}{350\sqrt{5x^2+2x+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] -1/350*(2953 + 2837*x + 1925*x^2 + 245*x^3)/Sqrt[3 + 2*x + 5*x^2] + (149*Ar
cSinh[(1 + 5*x)/Sqrt[14]])/(25*Sqrt[5])
```

fricas [A] time = 0.99, size = 92, normalized size = 1.12

$$\frac{1043\sqrt{5}\left(5x^2+2x+3\right)\log\left(-\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)-5\left(245x^3+1925x^2+2837x\right.}{1750\left(5x^2+2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/1750*(1043*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(
5*x + 1) - 25*x^2 - 10*x - 8) - 5*(245*x^3 + 1925*x^2 + 2837*x + 2953)*sqrt
(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

giac [A] time = 0.22, size = 62, normalized size = 0.76

$$-\frac{149}{125}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right) - \frac{(35(7x+55)x+2837)x+2953}{350\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="giac
")
```

```
[Out] -149/125*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1) - 1/
350*((35*(7*x + 55)*x + 2837)*x + 2953)/sqrt(5*x^2 + 2*x + 3)
```

maple [A] time = 0.01, size = 98, normalized size = 1.20

$$-\frac{7x^3}{10\sqrt{5x^2+2x+3}}-\frac{11x^2}{2\sqrt{5x^2+2x+3}}-\frac{149x}{25\sqrt{5x^2+2x+3}}+\frac{149\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{125}-\frac{1001}{125\sqrt{5x^2+2x+3}}-\frac{}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] -7/10/(5*x^2+2*x+3)^(1/2)*x^3-11/2/(5*x^2+2*x+3)^(1/2)*x^2-149/25/(5*x^2+2*
x+3)^(1/2)*x-1001/125/(5*x^2+2*x+3)^(1/2)-751/3500*(10*x+2)/(5*x^2+2*x+3)^(
1/2)+149/125*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

maxima [A] time = 0.96, size = 80, normalized size = 0.98

$$-\frac{7\,x^3}{10\,\sqrt{5\,x^2+2\,x+3}}-\frac{11\,x^2}{2\,\sqrt{5\,x^2+2\,x+3}}+\frac{149}{125}\,\sqrt{5}\,\operatorname{arsinh}\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)-\frac{2837\,x}{350\,\sqrt{5\,x^2+2\,x+3}}-\frac{2953}{350\,\sqrt{5\,x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -7/10*x^3/sqrt(5*x^2 + 2*x + 3) - 11/2*x^2/sqrt(5*x^2 + 2*x + 3) + 149/125*
sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 2837/350*x/sqrt(5*x^2 + 2*x + 3)
 - 2953/350/sqrt(5*x^2 + 2*x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5\,x+2\right)\,\left(-7\,x^2+4\,x+1\right)}{{\left(5\,x^2+2\,x+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(3/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{13x}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx-\int\left(-\frac{7x^2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-13*x/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
 + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-7*x**2/(5*x**2*sqrt(5*x**2 + 2
*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Inte
gral(31*x**3/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) +
3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(7*x**4/(5*x**2*sqrt(5*x**2 + 2*x +
 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral
(-2/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*
x**2 + 2*x + 3)), x)
```

3.395 $\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{131-605x}{3556\sqrt{5x^2+2x+3}}-\frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{1016}+\frac{3\sqrt{\frac{281693+25015\sqrt{11}}{1397}}\tanh^{-1}\left(\ldots\right.}{1016}$$

```
[Out] 1/3556*(-131+605*x)/(5*x^2+2*x+3)^(1/2)-3/1419352*arctanh((23+x*(17-5*11^(1
/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(393525121-3494
5955*11^(1/2))^(1/2)+3/1419352*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x
^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(393525121+34945955*11^(1/2))^(1/2
)
```

Rubi [A] time = 0.22, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1060, 1032, 724, 206}

$$-\frac{131-605x}{3556\sqrt{5x^2+2x+3}}-\frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{1016}+\frac{3\sqrt{\frac{281693+25015\sqrt{11}}{1397}}\tanh^{-1}\left(\ldots\right.}{1016}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] -(131 - 605*x)/(3556*Sqrt[3 + 2*x + 5*x^2]) - (3*Sqrt[(281693 - 25015*Sqrt[
11])/1397]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*
Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/1016 + (3*Sqrt[(281693 + 25015*Sqrt[11]
)/1397]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqr
t[11])]*Sqrt[3 + 2*x + 5*x^2])])/1016
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
```

```
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)\left(3+2x+5x^2\right)^{3/2}}\,dx = -\frac{131-605x}{3556\sqrt{3+2x+5x^2}}+\frac{\int \frac{13776+14112x}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx}{28448}$$

$$= -\frac{131-605x}{3556\sqrt{3+2x+5x^2}}+\frac{\left(21\left(66-53\sqrt{11}\right)\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{2794}$$

$$= -\frac{131-605x}{3556\sqrt{3+2x+5x^2}}-\frac{\left(21\left(66-53\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{1}{2352+112\left(4-2\sqrt{11}\right)+}\right.}{}$$

$$= -\frac{131-605x}{3556\sqrt{3+2x+5x^2}}-\frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{1}\right.}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2}}\right.}{1016}$$

Mathematica [A] time = 1.13, size = 174, normalized size = 1.05

$$\frac{\frac{2794(605x-131)}{\sqrt{5x^2+2x+3}}-21\sqrt{127\left(125+17\sqrt{11}\right)}\left(53\sqrt{11}-66\right)\tanh^{-1}\left(\frac{-5\sqrt{11}x+17x-\sqrt{11}+23}{\sqrt{250-34\sqrt{11}}\sqrt{5x^2+2x+3}}\right)+21\sqrt{127\left(125-17\sqrt{11}\right)}}{9935464}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] ((2794*(-131 + 605*x))/Sqrt[3 + 2*x + 5*x^2] - 21*Sqrt[127*(125 + 17*Sqrt[1
1])]*(-66 + 53*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + 17*x - 5*Sqrt[11]*x)/(Sqr
t[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])] + 21*Sqrt[127*(125 - 17*Sqrt[1
1])]*(66 + 53*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt
[250 + 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])])/9935464
```

fricas [B] time = 0.78, size = 333, normalized size = 2.01

$$-\frac{21\sqrt{1397}\left(5x^2+2x+3\right)\sqrt{25015\sqrt{11}+281693}\log\left(\frac{3\left(\sqrt{1397}\sqrt{5x^2+2x+3}\sqrt{25015\sqrt{11}+281693}\left(1335\sqrt{11}-8173\right)+2359\right.}{x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/19870928*(21*sqrt(1397)*(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*
log(3*(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*(1335
*sqrt(11) - 8173) + 23596727*sqrt(11)*(x + 3) + 70790181*x - 117983635)/x)
- 21*sqrt(1397)*(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*log(-3*(sqr
t(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*(1335*sqrt(11)
- 8173) - 23596727*sqrt(11)*(x + 3) - 70790181*x + 117983635)/x) + 7*sqrt(1
397)*(5*x^2 + 2*x + 3)*sqrt(-225135*sqrt(11) + 2535237)*log(-(sqrt(1397)*sq
rt(5*x^2 + 2*x + 3)*(1335*sqrt(11) + 8173)*sqrt(-225135*sqrt(11) + 2535237)
 + 70790181*sqrt(11)*(x + 3) - 212370543*x + 353950905)/x) - 7*sqrt(1397)*(
5*x^2 + 2*x + 3)*sqrt(-225135*sqrt(11) + 2535237)*log((sqrt(1397)*sqrt(5*x^
2 + 2*x + 3)*(1335*sqrt(11) + 8173)*sqrt(-225135*sqrt(11) + 2535237) - 7079
0181*sqrt(11)*(x + 3) + 212370543*x - 353950905)/x) - 5588*sqrt(5*x^2 + 2*x
 + 3)*(605*x - 131))/(5*x^2 + 2*x + 3)
```

giac [A] time = 0.25, size = 112, normalized size = 0.67

$$\frac{605x-131}{3556\sqrt{5x^2+2x+3}}+0.0477059376663667\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}+4.41924736459000\right)-0.035217$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="giac
")
```

```
[Out] 1/3556*(605*x - 131)/sqrt(5*x^2 + 2*x + 3) + 0.0477059376663667*log(-sqrt(5
)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.0352174957838020*log(-s
qrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.0477059376663667*l
og(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.0352174957838
020*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

maple [B] time = 0.02, size = 489, normalized size = 2.95

$$-\frac{10x+2}{196\sqrt{5x^2+2x+3}}-\frac{3\left(-61+13\sqrt{11}\right)\sqrt{11}\left(-\frac{\operatorname{arctanh}\left(\frac{250-34\sqrt{11}+\frac{49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250-34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)+250-34\sqrt{11}}}\right)}{\left(\frac{250}{49}-\frac{34\sqrt{11}}{49}\right)\sqrt{250-34\sqrt{11}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] -1/196*(10*x+2)/(5*x^2+2*x+3)^(1/2)-3/154*(-61+13*11^(1/2))*11^(1/2)*(1/7/(
250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/
7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/7*(34/7-10/7*11^(1/2))/(250/
```

```
49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2
)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/4
9-34/49*11^(1/2))^(1/2)-1/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*a
rctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)
))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/
2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))-3/154*(61+13*11^(1/2))*11
^(1/2)*(1/7/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11
^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-1/7*(34/7+10/7*11
^(1/2))/(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/
7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11
^(1/2))+250/49+34/49*11^(1/2))^(1/2)-1/(250/49+34/49*11^(1/2))/(250+34*11^(
1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7
-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/
7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))
```

maxima [B] time = 1.16, size = 777, normalized size = 4.68

$$-\frac{1}{4312}\sqrt{11}\left(\frac{20\sqrt{11}\,x}{\sqrt{5x^2+2x+3}}-\frac{7890\sqrt{11}\,x}{17\sqrt{11}\sqrt{5x^2+2x+3}+125\sqrt{5x^2+2x+3}}+\frac{7890\sqrt{11}\,x}{17\sqrt{11}\sqrt{5x^2+2x+3}-125\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -1/4312*sqrt(11)*(20*sqrt(11)*x/sqrt(5*x^2 + 2*x + 3) - 7890*sqrt(11)*x/(17
*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x + 3)) + 7890*sqrt(11
)*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x + 3)) - 13377
*sqrt(11)*sqrt(2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(
11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)
*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x
 - 2*sqrt(11) - 4))/(17*sqrt(11) + 125)^(3/2) + 4*sqrt(11)/sqrt(5*x^2 + 2*x
 + 3) - 26280*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x +
 3)) - 26280*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x +
3)) + 156*sqrt(11)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt
(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11
)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*
x + 2*sqrt(11) - 4))/(-34/49*sqrt(11) + 250/49)^(3/2) - 62769*sqrt(2)*arcsi
nh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)
*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14
*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/(17
*sqrt(11) + 125)^(3/2) + 2244*sqrt(11)/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) +
 125*sqrt(5*x^2 + 2*x + 3)) - 2244*sqrt(11)/(17*sqrt(11)*sqrt(5*x^2 + 2*x +
 3) - 125*sqrt(5*x^2 + 2*x + 3)) - 732*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)
*x/abs(14*x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11
) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqr
t(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4))/(-34/49*sqrt(11) + 250/49)^(3/2) +
 12678/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x + 3)) + 12
678/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x + 3)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+5x+2}{\left(5x^2+2x+3\right)^{3/2}\,\left(-7x^2+4x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)),x)
```

```
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{5x}{35x^4\sqrt{5x^2+2x+3} - 6x^3\sqrt{5x^2+2x+3} + 8x^2\sqrt{5x^2+2x+3} - 14x\sqrt{5x^2+2x+3} - 3\sqrt{5x^2+2x+3}}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(5*x/(35*x**4*sqrt(5*x**2 + 2*x + 3) - 6*x**3*sqrt(5*x**2 + 2*x +
3) + 8*x**2*sqrt(5*x**2 + 2*x + 3) - 14*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5
*x**2 + 2*x + 3)), x) - Integral(x**2/(35*x**4*sqrt(5*x**2 + 2*x + 3) - 6*x
**3*sqrt(5*x**2 + 2*x + 3) + 8*x**2*sqrt(5*x**2 + 2*x + 3) - 14*x*sqrt(5*x*
*2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(35*x**4*sqrt(5*
x**2 + 2*x + 3) - 6*x**3*sqrt(5*x**2 + 2*x + 3) + 8*x**2*sqrt(5*x**2 + 2*x
+ 3) - 14*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x)
```

3.396 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^2 (3+2x+5x^2)^{3/2}}\,dx$

Optimal. Leaf size=215

$$-\frac{3(40-371x)}{5588(-7x^2+4x+1)\sqrt{5x^2+2x+3}}-\frac{22755x+76567}{19870928\sqrt{5x^2+2x+3}}-\frac{7(541543-5144\sqrt{11})\tanh^{-1}\left(\frac{(17-5\sqrt{11})x}{\sqrt{2(125-17\sqrt{11}}}\right.}{2838704\sqrt{22(125-17\sqrt{11})}}$$

```
[Out] 1/19870928*(-76567-22755*x)/(5*x^2+2*x+3)^(1/2)-3/5588*(40-371*x)/(-7*x^2+4
*x+1)/(5*x^2+2*x+3)^(1/2)-7/2838704*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))
/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(541543-5144*11^(1/2))/(2750-
374*11^(1/2))^(1/2)+7/2838704*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^
2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(541543+5144*11^(1/2))/(2750+374*11
^(1/2))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1060, 1032, 724, 206}

$$-\frac{3(40-371x)}{5588(-7x^2+4x+1)\sqrt{5x^2+2x+3}}-\frac{22755x+76567}{19870928\sqrt{5x^2+2x+3}}-\frac{7(541543-5144\sqrt{11})\tanh^{-1}\left(\frac{(17-5\sqrt{11})x}{\sqrt{2(125-17\sqrt{11}}}\right.}{2838704\sqrt{22(125-17\sqrt{11})}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] -(76567 + 22755*x)/(19870928*Sqrt[3 + 2*x + 5*x^2]) - (3*(40 - 371*x))/(558
8*(1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]) - (7*(541543 - 5144*Sqrt[11])*Ar
cTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sq
rt[3 + 2*x + 5*x^2])])/(2838704*Sqrt[22*(125 - 17*Sqrt[11])]) + (7*(541543
+ 5144*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125
 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(2838704*Sqrt[22*(125 + 17*Sqrt[1
1])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f))))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx = -\frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} - \frac{\int \frac{-50216-37752x-89040x^2}{\left(1+4x-7x^2\right)\left(3+2x+5x^2\right)^{3/2}}\,dx}{44704}$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} -$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} +$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} -$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} -$$

Mathematica [A] time = 1.14, size = 351, normalized size = 1.63

$$\frac{\frac{5084772\sqrt{5x^2+2x+3}\,x}{-7x^2+4x+1} + \frac{24422640x}{7\sqrt{5x^2+2x+3}} + \frac{12968296}{7\sqrt{5x^2+2x+3}} + \frac{1672044\sqrt{5x^2+2x+3}}{7x^2-4x-1} + 7581602\sqrt{\frac{22}{125+17\sqrt{11}}}\log\left(\sqrt{2750+374\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] (12968296/(7*Sqrt[3 + 2*x + 5*x^2]) + (24422640*x)/(7*Sqrt[3 + 2*x + 5*x^2]
) + (5084772*x*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2) + (1672044*Sqrt[3 +
 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) + 14*Sqrt[2/(125 - 17*Sqrt[11])]*(-56584
+ 541543*Sqrt[11])*ArcTanh[(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])/
(-23 + Sqrt[11] + (-17 + 5*Sqrt[11])*x)] - 14*Sqrt[2/(125 + 17*Sqrt[11])]*(
56584 + 541543*Sqrt[11])*Log[2 + Sqrt[11] - 7*x] + 792176*Sqrt[2/(125 + 17*
Sqrt[11])]*Log[11 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sq
rt[11]]*Sqrt[3 + 2*x + 5*x^2]] + 7581602*Sqrt[22/(125 + 17*Sqrt[11])]*Log[1
1 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 +
 2*x + 5*x^2]])/124902976
```

fricas [B] time = 0.88, size = 392, normalized size = 1.82

$$-\frac{7\sqrt{1397}\left(35x^4-6x^3+8x^2-14x-3\right)\sqrt{4294093814065\sqrt{11}+35653135368317}\log\left(-\frac{\sqrt{1397}\sqrt{5x^2+2x+3}\sqrt{4294\cdots}}{\cdots}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/111038745664*(7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(4294
093814065*sqrt(11) + 35653135368317)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)
*sqrt(4294093814065*sqrt(11) + 35653135368317)*(5609479*sqrt(11) + 77949905
) + 2865029444171587*sqrt(11)*(x + 3) - 8595088332514761*x + 14325147220857
935)/x) - 7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(42940938140
65*sqrt(11) + 35653135368317)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(42
94093814065*sqrt(11) + 35653135368317)*(5609479*sqrt(11) + 77949905) - 2865
029444171587*sqrt(11)*(x + 3) + 8595088332514761*x - 14325147220857935)/x)
+ 7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(-4294093814065*sqrt
(11) + 35653135368317)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(5609479*sqrt(
11) - 77949905)*sqrt(-4294093814065*sqrt(11) + 35653135368317) + 2865029444
171587*sqrt(11)*(x + 3) + 8595088332514761*x - 14325147220857935)/x) - 7*sq
rt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(-4294093814065*sqrt(11) +
 35653135368317)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(5609479*sqrt(11) -
 77949905)*sqrt(-4294093814065*sqrt(11) + 35653135368317) - 286502944417158
7*sqrt(11)*(x + 3) - 8595088332514761*x + 14325147220857935)/x) + 5588*(159
285*x^3 + 444949*x^2 + 3628805*x - 503287)*sqrt(5*x^2 + 2*x + 3))/(35*x^4 -
 6*x^3 + 8*x^2 - 14*x - 3)
```

giac [A] time = 0.27, size = 295, normalized size = 1.37

$$\frac{25230x+13397}{903224\sqrt{5x^2+2x+3}}+\frac{3\left(42623\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^3+77302\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^2-27\cdots\right)}{709676\left(7\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^4-8\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^3-70\left(\sqrt{5}x-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/903224*(25230*x + 13397)/sqrt(5*x^2 + 2*x + 3) + 3/709676*(42623*(sqrt(5)
*x - sqrt(5*x^2 + 2*x + 3))^3 + 77302*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x
 + 3))^2 - 275511*sqrt(5)*x - 219860*sqrt(5) + 275511*sqrt(5*x^2 + 2*x + 3)
)/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*
x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(
sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83) + 0.0218058276254033*log(-sqrt(5)*
x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.0332874364433911*log(-sqr
t(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.0218058276254033*log
```

```
(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.033287436443391
1*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

maple [B] time = 0.02, size = 1214, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] 161/484*11^(1/2)*(1/7/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34
/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/7*(34
/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)
-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-
2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/(250/49-34/49*11^(1/2))/(2
50-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/
2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))
^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))+(1
83/44-39/44*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))/(
5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-3
4/49*11^(1/2))^(1/2)-3/98*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/(
250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/
7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-(34/7-10/7*11^(1/2))/(250/49-3
4/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5
*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34
/49*11^(1/2))^(1/2)-7/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arcta
nh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(
250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*
(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))-20/49/(250/49-34/49*11^(1/2))
*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11
^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^
(1/2))+(183/44+39/44*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11
^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))
+250/49+34/49*11^(1/2))^(1/2)-3/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1
/2))*(1/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/
2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-(34/7+10/7*11^(1/2))/
(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/
2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+
250/49+34/49*11^(1/2))^(1/2)-7/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1
/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*1
1^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-20/49/(250/49+34/49*
11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2
/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*1
1^(1/2))^(1/2))-161/484*11^(1/2)*(1/7/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7
*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2
))^(1/2)-1/7*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49
+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10
/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-1/(250/49+34
/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(
34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2
/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(
1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+5\,x+2}{\left(7\,x^2-4\,x-1\right)^2\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^2*(5*x^2 + 2*x + 3)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2+5\,x+2}{{\left(5\,x^2+2\,x+3\right)}^{3/2}\,{\left(-7\,x^2+4\,x+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2),x)
```

```
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} + 5x + 2}{\left(5x^{2} + 2x + 3\right)^{\frac{3}{2}} \left(7x^{2} - 4x - 1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**2/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)/((5*x**2 + 2*x + 3)**(3/2)*(7*x**2 - 4*x - 1)**2)
, x)
```

3.397 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^3(3+2x+5x^2)^{3/2}}\,dx$

Optimal. Leaf size=250

$$-\frac{2701733-9148874x}{62451488\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}-\frac{5(1118731375x+461370781)}{222077491328\sqrt{5x^2+2x+3}}-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+}}$$

```
[Out] -5/222077491328*(461370781+1118731375*x)/(5*x^2+2*x+3)^(1/2)-3/11176*(40-37
1*x)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2)+1/62451488*(-2701733+9148874*x)/(
-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2)-7/31725355904*arctanh((23+x*(17-5*11^(1/2
))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(2792860024-84865
895*11^(1/2))/(2750-374*11^(1/2))^(1/2)+7/31725355904*arctanh((23+11^(1/2)+
x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(2792860024
+84865895*11^(1/2))/(2750+374*11^(1/2))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1060, 1032, 724, 206}

$$-\frac{2701733-9148874x}{62451488\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}-\frac{5(1118731375x+461370781)}{222077491328\sqrt{5x^2+2x+3}}-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] (-5*(461370781 + 1118731375*x))/(222077491328*Sqrt[3 + 2*x + 5*x^2]) - (3*(
40 - 371*x))/(11176*(1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]) - (2701733 -
 9148874*x)/(62451488*(1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]) - (7*(279286
0024 - 84865895*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sq
rt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(31725355904*Sqrt[22*(12
5 - 17*Sqrt[11])]) + (7*(2792860024 + 84865895*Sqrt[11])*ArcTanh[(23 + Sqrt
[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x
^2])])/(31725355904*Sqrt[22*(125 + 17*Sqrt[11])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
```

```
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3\left(3+2x+5x^2\right)^{3/2}}\,dx = -\frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}} - \frac{\int \frac{-128104-89208x-178080x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx}{89408}$$

$$= -\frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}} - \frac{2701733-914887}{62451488\left(1+4x-7x^2\right)\sqrt{3}}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x}}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x}}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x}}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x}}$$

Mathematica [A] time = 1.59, size = 381, normalized size = 1.52

$$\frac{\frac{44\sqrt{5x^2+2x+3}\,(507770113-1167248019x)}{7x^2-4x-1}+\frac{737616(38521x-12667)\sqrt{5x^2+2x+3}}{\left(-7x^2+4x+1\right)^2}+\frac{21296(501205x+1702037)}{7\sqrt{5x^2+2x+3}}-7\sqrt{\frac{22}{125-17\sqrt{11}}}\left(84865\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] ((21296*(1702037 + 501205*x))/(7*Sqrt[3 + 2*x + 5*x^2]) + (737616*(-12667 +
 38521*x)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2 + (44*(507770113 - 116
7248019*x)*Sqrt[3 + 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) - 14*Sqrt[22/(125 - 17
*Sqrt[11])]*(-2792860024 + 84865895*Sqrt[11])*ArcTanh[(Sqrt[250 - 34*Sqrt[1
1]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[11] + (-17 + 5*Sqrt[11])*x)] - 14*Sq
rt[22/(125 + 17*Sqrt[11])]*(2792860024 + 84865895*Sqrt[11])*Log[2 + Sqrt[11
] - 7*x] + 7*Sqrt[22/(125 - 17*Sqrt[11])]*(-2792860024 + 84865895*Sqrt[11])
*Log[(-2 + Sqrt[11] + 7*x)^2] - 7*Sqrt[22/(125 - 17*Sqrt[11])]*(-2792860024
 + 84865895*Sqrt[11])*Log[15 - 4*Sqrt[11] + 14*(-2 + Sqrt[11])*x + 49*x^2]
+ 14*Sqrt[22/(125 + 17*Sqrt[11])]*(2792860024 + 84865895*Sqrt[11])*Log[11 +
 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*
x + 5*x^2]])/1395915659776
```

fricas [B] time = 1.33, size = 452, normalized size = 1.81

$$-\frac{7\sqrt{1397}\left(245x^6-182x^5+45x^4-124x^3+27x^2+26x+3\right)\sqrt{74693314710639641467\sqrt{11}+8962664983}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/1240969021540864*(7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 2
7*x^2 + 26*x + 3)*sqrt(74693314710639641467*sqrt(11) + 89626649837723365785
5)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(74693314710639641467*sqrt(11
) + 896266498377233657855)*(37271563201*sqrt(11) + 407780707037) + 75502120
686844055144479*sqrt(11)*(x + 3) - 226506362060532165433437*x + 37751060343
4220275722395)/x) - 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27
*x^2 + 26*x + 3)*sqrt(74693314710639641467*sqrt(11) + 896266498377233657855
)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(74693314710639641467*sqrt(11)
+ 896266498377233657855)*(37271563201*sqrt(11) + 407780707037) - 7550212068
6844055144479*sqrt(11)*(x + 3) + 226506362060532165433437*x - 3775106034342
20275722395)/x) + 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x
^2 + 26*x + 3)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855)
*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(37271563201*sqrt(11) - 407780707037
)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855) + 7550212068
6844055144479*sqrt(11)*(x + 3) + 226506362060532165433437*x - 3775106034342
20275722395)/x) - 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x
^2 + 26*x + 3)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855)
*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(37271563201*sqrt(11) - 40778070703
7)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855) - 755021206
86844055144479*sqrt(11)*(x + 3) - 226506362060532165433437*x + 377510603434
220275722395)/x) + 5588*(274089186875*x^5 - 200208943655*x^4 + 109737266678
*x^3 - 148022158802*x^2 + 7828199499*x + 14298727813)*sqrt(5*x^2 + 2*x + 3)
)/(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x^2 + 26*x + 3)
```

giac [B] time = 0.32, size = 397, normalized size = 1.59

$$\frac{501205\,x+1702037}{458837792\,\sqrt{5\,x^2+2\,x+3}}+\frac{6871871279\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^7+4012856750\,\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/458837792*(501205*x + 1702037)/sqrt(5*x^2 + 2*x + 3) + 1/7931338976*(6871
871279*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 + 4012856750*sqrt(5)*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3))^6 - 223088535693*(sqrt(5)*x - sqrt(5*x^2 + 2*x +
 3))^5 - 100577598176*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 + 12550
97956673*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 566810398070*sqrt(5)*(sqrt
(5)*x - sqrt(5*x^2 + 2*x + 3))^2 - 1246245909011*sqrt(5)*x - 561299654796*s
qrt(5) + 1246245909011*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 +
2*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)
*x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x +
3)) + 83)^2 + 0.0107382277384513*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4
.41924736459000) - 0.0142619066316905*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3
) + 1.25295163054000) - 0.0107382277384513*log(-sqrt(5)*x + sqrt(5*x^2 + 2*
x + 3) - 1.02258038113000) + 0.0142619066316905*log(-sqrt(5)*x + sqrt(5*x^2
 + 2*x + 3) - 2.09411235400000)
```

maple [B] time = 0.02, size = 2600, normalized size = 10.40

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] 3535/21296*11^(1/2)*(1/7/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+
(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/7*
(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1
/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*
(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/(250/49-34/49*11^(1/2))
/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^
(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/
2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))
-21/968*(-61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49-34/49*11^(1/2))/(x-2/7+1
/7*11^(1/2))^2/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11
^(1/2))+250/49-34/49*11^(1/2))^(1/2)-5/1372*(34/7-10/7*11^(1/2))/(250/49-34
/49*11^(1/2))*(-1/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))/(5*(x-2/7+1/
7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/
2))^(1/2)-3/2*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/(250/49-34/49
*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/
2))+250/49-34/49*11^(1/2))^(1/2)-(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2
))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*
11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2)
)^(1/2)-7/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500
/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1
/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*1
1^(1/2))+250-34*11^(1/2))^(1/2)))-20/(250/49-34/49*11^(1/2))*(10*x+2)/(5000
/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7
-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2))-15/686/(
250/49-34/49*11^(1/2))*(1/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2
+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-(34
```

/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)
-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-
2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-7/(250/49-34/49*11^(1/2))/(2
50-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/
2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))
^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))-(
-3535/1936+273/1936*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^
(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+
250/49-34/49*11^(1/2))^(1/2)-3/98*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/
2))*(1/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2
))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-(34/7-10/7*11^(1/2))/(
250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2
))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+2
50/49-34/49*11^(1/2))^(1/2)-7/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/
2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(
1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11
^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))-20/49/(250/49-34/49*1
1^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/
7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11
^(1/2))^(1/2))-21/968*(61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49+34/49*11^(1
/2))/(x-2/7-1/7*11^(1/2))^2/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*
(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-5/1372*(34/7+10/7*11^(1/2
))/(250/49+34/49*11^(1/2))*(-1/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))
/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49
+34/49*11^(1/2))^(1/2)-3/2*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/
(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2
/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-(34/7+10/7*11^(1/2))/(250/49+
34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(
5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+3
4/49*11^(1/2))^(1/2)-7/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arct
anh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/
(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))
*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-20/(250/49+34/49*11^(1/2))*(
10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(
1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1
/2))-15/686/(250/49+34/49*11^(1/2))*(1/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/
7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/
2))^(1/2)-(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49+68
0/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*
11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-7/(250/49+34/49
*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/
7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-
1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2
))^(1/2))))-(-3535/1936-273/1936*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(
x-2/7-1/7*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1
/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-3/98*(34/7+10/7*11^(1/2))/(250/49
+34/49*11^(1/2))*(1/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7
+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-(34/7+10/
7*11^(1/2))/(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7
+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/
7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-7/(250/49+34/49*11^(1/2))/(250+34*
11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x
-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*
(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-20/49/(2
50/49+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2)
)^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+25
0/49+34/49*11^(1/2))^(1/2))-3535/21296*11^(1/2)*(1/7/(250/49+34/49*11^(1/2)
)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/4
9+34/49*11^(1/2))^(1/2)-1/7*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1

```
0*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1
/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/
2)-1/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+6
8/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^
(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/
2))+250+34*11^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2+5x+2}{\left(7x^2-4x-1\right)^3\left(5x^2+2x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] -integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^3*(5*x^2 + 2*x + 3)^(3/2)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2+5x+2}{\left(5x^2+2x+3\right)^{3/2}\left(-7x^2+4x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3),x)
```

```
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{5x}{1715x^8\sqrt{5x^2+2x+3}-2254x^7\sqrt{5x^2+2x+3}+798x^6\sqrt{5x^2+2x+3}-866x^5\sqrt{5x^2+2x+3}+640x^4\sqrt{5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**3/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(5*x/(1715*x**8*sqrt(5*x**2 + 2*x + 3) - 2254*x**7*sqrt(5*x**2 + 2
*x + 3) + 798*x**6*sqrt(5*x**2 + 2*x + 3) - 866*x**5*sqrt(5*x**2 + 2*x + 3)
 + 640*x**4*sqrt(5*x**2 + 2*x + 3) + 198*x**3*sqrt(5*x**2 + 2*x + 3) - 110*
x**2*sqrt(5*x**2 + 2*x + 3) - 38*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 +
 2*x + 3)), x) - Integral(x**2/(1715*x**8*sqrt(5*x**2 + 2*x + 3) - 2254*x**
7*sqrt(5*x**2 + 2*x + 3) + 798*x**6*sqrt(5*x**2 + 2*x + 3) - 866*x**5*sqrt(
5*x**2 + 2*x + 3) + 640*x**4*sqrt(5*x**2 + 2*x + 3) + 198*x**3*sqrt(5*x**2
+ 2*x + 3) - 110*x**2*sqrt(5*x**2 + 2*x + 3) - 38*x*sqrt(5*x**2 + 2*x + 3)
- 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(1715*x**8*sqrt(5*x**2 + 2*x +
 3) - 2254*x**7*sqrt(5*x**2 + 2*x + 3) + 798*x**6*sqrt(5*x**2 + 2*x + 3) -
866*x**5*sqrt(5*x**2 + 2*x + 3) + 640*x**4*sqrt(5*x**2 + 2*x + 3) + 198*x**
3*sqrt(5*x**2 + 2*x + 3) - 110*x**2*sqrt(5*x**2 + 2*x + 3) - 38*x*sqrt(5*x*
*2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x)
```

3.398 $\int \left(a+cx^2\right)^p \left(A+Cx^2\right) \left(d+fx^2\right)^q \, dx$

Optimal. Leaf size=166

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-}$$

```
[Out] A*x*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(1/2,-p,-q,3/2,-c*x^2/a,-f*x^2/d)/((c*x
^2/a+1)^p)/((1+f*x^2/d)^q)+1/3*C*x^3*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(3/2,-
p,-q,5/2,-c*x^2/a,-f*x^2/d)/((c*x^2/a+1)^p)/((1+f*x^2/d)^q)
```

Rubi [A] time = 0.15, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {531, 430, 429, 511, 510}

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^p*(A + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (C*x^3*(a + c*x^2)^p*
(d + f*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*(1
 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
```

```
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \left(a+cx^2\right)^p \left(A+Cx^2\right)\left(d+fx^2\right)^q\,dx = A\int \left(a+cx^2\right)^p \left(d+fx^2\right)^q\,dx + C\int x^2\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx$$

$$= \left(A\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int\left(1+\frac{cx^2}{a}\right)^p\left(d+fx^2\right)^q\,dx + \left(C\left(a+cx^2\right)^{p}\right.$$

$$= \left(A\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d+fx^2\right)^q\left(1+\frac{fx^2}{d}\right)^{-q}\right)\int\left(1+\frac{cx^2}{a}\right)^p\left(1+\right.$$

$$= Ax\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d+fx^2\right)^q\left(1+\frac{fx^2}{d}\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx}{a}\right.$$

Mathematica [A] time = 0.40, size = 242, normalized size = 1.46

$$\frac{1}{3}x\left(a+cx^2\right)^p\left(d+fx^2\right)^q\left(\frac{9aAdF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{2x^2\left(cdpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+afqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\right)+3adF_1\left(\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*x^2)^p*(A + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (x*(a + c*x^2)^p*(d + f*x^2)^q*((9*a*A*d*AppellF1[1/2, -p, -q, 3/2, -((c*x^
2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -((f*
x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q, 5/2, -((c*x^2)/a), -((f*x^
2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((c*x^2)/a), -((f*x^2)/d)]))
+ (C*x^2*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/((1 + (c*x
^2)/a)^p*(1 + (f*x^2)/d)^q)))/3
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Cx^2+A\right)\left(cx^2+a\right)^p\left(fx^2+d\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Cx^2+A\right)\left(cx^2+a\right)^p\left(fx^2+d\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \left(C\,x^2+A\right)\left(c\,x^2+a\right)^p\left(f\,x^2+d\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x)
[Out] int((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Cx^2 + A\right)\left(cx^2 + a\right)^p \left(fx^2 + d\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="maxima")
[Out] integrate((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\,x^2 + A\right)\,\left(c\,x^2 + a\right)^p\,\left(f\,x^2 + d\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + c*x^2)^p*(d + f*x^2)^q,x)
[Out] int((A + C*x^2)*(a + c*x^2)^p*(d + f*x^2)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**p*(C*x**2+A)*(f*x**2+d)**q,x)
[Out] Timed out
```

3.399 $\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx$

Optimal. Leaf size=167

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{B\left(a+cx^2\right)^{p+1}\left(d+fx^2\right)^q\left(\frac{c(d+\cdots}{cd\cdots}\right.}{\cdots}$$

```
[Out] A*x*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(1/2,-p,-q,3/2,-c*x^2/a,-f*x^2/d)/((c*x
^2/a+1)^p)/((1+f*x^2/d)^q)+1/2*B*(c*x^2+a)^(1+p)*(f*x^2+d)^q*hypergeom([-q,
 1+p],[2+p],-f*(c*x^2+a)/(-a*f+c*d))/c/(1+p)/((c*(f*x^2+d)/(-a*f+c*d))^q)
```

Rubi [A] time = 0.15, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1010, 430, 429, 444, 70, 69}

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{B\left(a+cx^2\right)^{p+1}\left(d+fx^2\right)^q\left(\frac{c(d+\cdots}{cd\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)*(a + c*x^2)^p*(d + f*x^2)^q,x]
```

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (B*(a + c*x^2)^(1 + p
)*(d + f*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((f*(a + c*x^2))/(c*d
- a*f))])/(2*c*(1 + p)*((c*(d + f*x^2))/(c*d - a*f))^q)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx = A\int \left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx + B\int x\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx$$

$$= \frac{1}{2}B\,\text{Subst}\left(\int (a+cx)^p(d+fx)^q\,dx, x, x^2\right) + \left(A\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right.$$

$$= \frac{1}{2}\left(B\left(d+fx^2\right)^q\left(\frac{c\left(d+fx^2\right)}{cd-af}\right)^{-q}\right)\text{Subst}\left(\int (a+cx)^p\left(\frac{cd}{cd-af}+\frac{cfx}{cd-}\right.\right.$$

$$= Ax\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d+fx^2\right)^q\left(1+\frac{fx^2}{d}\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{c}{}\right.$$

Mathematica [A] time = 0.32, size = 236, normalized size = 1.41

$$\frac{1}{2}x\left(a+cx^2\right)^p\left(d+fx^2\right)^q\left(\frac{6aAdF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{2x^2\left(cdpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+afqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\right)+3adF}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*x)*(a + c*x^2)^p*(d + f*x^2)^q,x]
```

```
[Out] (x*(a + c*x^2)^p*(d + f*x^2)^q*((B*x*AppellF1[1, -p, -q, 2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (6*a*A*d*AppellF1[1/2
, -p, -q, 3/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q, 3/
2, -((c*x^2)/a), -((f*x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q, 5/2,
 -((c*x^2)/a), -((f*x^2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((c*x^2
)/a), -((f*x^2)/d)]))))/2
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left((Bx+A)\left(cx^2+a\right)^p\left(fx^2+d\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx+A)\left(cx^2+a\right)^p\left(fx^2+d\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (Bx+A)\left(c\,x^2+a\right)^p \left(f\,x^2+d\right)^q \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x)
```

```
[Out] int((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx+A)\left(cx^2+a\right)^p \left(fx^2+d\right)^q \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,x^2+a\right)^p \left(f\,x^2+d\right)^q \,(A+B\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x),x)
```

```
[Out] int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+a)**p*(f*x**2+d)**q,x)
```

```
[Out] Timed out
```

3.400 $\int \left(a+cx^2\right)^p \left(A+Bx+Cx^2\right) \left(d+fx^2\right)^q \, dx$

Optimal. Leaf size=252

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-}$$

```
[Out] A*x*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(1/2,-p,-q,3/2,-c*x^2/a,-f*x^2/d)/((c*x
^2/a+1)^p)/((1+f*x^2/d)^q)+1/3*C*x^3*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(3/2,-
p,-q,5/2,-c*x^2/a,-f*x^2/d)/((c*x^2/a+1)^p)/((1+f*x^2/d)^q)+1/2*B*(c*x^2+a)
^(1+p)*(f*x^2+d)^q*hypergeom([-q, 1+p],[2+p],-f*(c*x^2+a)/(-a*f+c*d))/c/(1+
p)/((c*(f*x^2+d)/(-a*f+c*d))^q)
```

Rubi [A] time = 0.48, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {6742, 430, 429, 444, 70, 69, 511, 510}

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^p*(A + B*x + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (C*x^3*(a + c*x^2)^p*
(d + f*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*(1
 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (B*(a + c*x^2)^(1 + p)*(d + f*x^2)^q*H
ypergeometric2F1[1 + p, -q, 2 + p, -((f*(a + c*x^2))/(c*d - a*f))])/(2*c*(1
 + p)*((c*(d + f*x^2))/(c*d - a*f))^q)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
```

```
Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \left(a+cx^2\right)^p \left(A+Bx+Cx^2\right) \left(d+fx^2\right)^q \, dx = \int \left(A\left(a+cx^2\right)^p \left(d+fx^2\right)^q + Bx\left(a+cx^2\right)^p \left(d+fx^2\right)^q + Cx^2 \left(a+\right.\right.$$

$$= A\int \left(a+cx^2\right)^p \left(d+fx^2\right)^q \, dx + B\int x\left(a+cx^2\right)^p \left(d+fx^2\right)^q \, dx +$$

$$= \frac{1}{2}B\,\text{Subst}\left(\int (a+cx)^p(d+fx)^q \, dx, x, x^2\right) + \left(A\left(a+cx^2\right)^p \left(1+\frac{cx}{a}\right.\right.$$

$$= \frac{1}{2}\left(B\left(d+fx^2\right)^q \left(\frac{c\left(d+fx^2\right)}{cd-af}\right)^{-q}\right)\text{Subst}\left(\int (a+cx)^p \left(\frac{cd}{cd-af}+\right.\right.$$

$$= Ax\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(d+fx^2\right)^q \left(1+\frac{fx^2}{d}\right)^{-q} F_1\left(\frac{1}{2};-p,-q;\frac{3}{2}\right.$$

Mathematica [A] time = 0.52, size = 302, normalized size = 1.20

$$\frac{1}{6}x\left(a+cx^2\right)^p \left(d+fx^2\right)^q \left(\frac{18aAdF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{2x^2\left(cdpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+afqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\right)+3adF_1\left(\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*x^2)^p*(A + B*x + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (x*(a + c*x^2)^p*(d + f*x^2)^q*((3*B*x*AppellF1[1, -p, -q, 2, -((c*x^2)/a),
 -((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (18*a*A*d*AppellF1[
1/2, -p, -q, 3/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q,
 3/2, -((c*x^2)/a), -((f*x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q, 5
/2, -((c*x^2)/a), -((f*x^2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((c*
x^2)/a), -((f*x^2)/d)])) + (2*C*x^2*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a)
, -((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q)))/6
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Cx^2 + Bx + A\right)\left(cx^2 + a\right)^p\left(fx^2 + d\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Cx^2 + Bx + A\right)\left(cx^2 + a\right)^p\left(fx^2 + d\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(C\,x^2 + Bx + A\right)\left(c\,x^2 + a\right)^p\left(f\,x^2 + d\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x)
```

```
[Out] int((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Cx^2 + Bx + A\right)\left(cx^2 + a\right)^p\left(fx^2 + d\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,x^2+a\right)}^p\,{\left(f\,x^2+d\right)}^q\,\left(C\,x^2+B\,x+A\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x + C*x^2),x)
```

```
[Out] int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x + C*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**p*(C*x**2+B*x+A)*(f*x**2+d)**q,x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```